FUZZY GRADE OF THE COMPLETE HYPERGROUPS

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Abstract. This paper continues the study of the connection between hypergroups and fuzzy sets, investigating the length of the sequence of join spaces associated with a hypergroup. The classes of complete hypergroups and of 1-hypergroups are considered and analyzed in this context. Finally, we give a method to construct a finite hypergroup with the strong fuzzy grade equal to a given natural number.

1. Introduction

Hyperstructure theory and fuzzy set theory are two traditional fields of theoretical and applied mathematics. The study of the connections between them has been an independent direction of research from the '70 of the past century, when Rosenfeld [31] introduced the notion of fuzzy group. Later on, this concept was generalized to obtain hypergroups and join spaces induced by fuzzy sets [1, 5, 6, 9], fuzzy hyperrings [18, 27] or fuzzy hypermodules [26], or fuzzy hypervector spaces[2], fuzzy ideals of semirings [24, 32] or hemirings [34], or general fuzzy hypersystems [36], ecc.

The starting point of this contribution is the connection between hypergroupoids and fuzzy sets established by Corsini [5, 6]: he has associated a join space with a fuzzy set [5] and then a fuzzy set with a hypergroupoid [6], constructing a sequence of join spaces and fuzzy sets, that ends if two consecutive join spaces are isomorphic. Its length is called the fuzzy grade. This sequence has been studied in the general case [15, 33], for the direct product of two hypergroupoids [16], for some particular classes of hypergroups: the hypergroups with partial identities of order less than 8 [7], the non complete 1-hypergroups [8], the complete hypergroups of order less than 7 [14]. Besides, Corsini et al. studied the same sequence associated with a hypergraph [11, 12], or with multivalued functions [13].

The complete hypergroups, studied for example in [19, 20, 29], represent a special class of hypergroups (introduced in 1934 by Marty [28]) closed to groups: they can be constructed starting from a group, as we will see in the next section. In this paper, we will investigate the sequence of join spaces associated with some particular complete hypergroups.

An outline of this paper follows. In section 2 we present some basic notions concerning hypergroups and a short description of the complete hypergroups. In section 3 we briefly recall the construction of the sequence of join spaces and fuzzy

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sets associated with a hypergroupoid. Section 4 includes the computation of the length of the above sequence for some finite hypergroupoids, necessary to determine, in the next section, the fuzzy grades of particular complete hypergroups. Section 6 is dedicated to a method to construct a finite hypergroup with the strong fuzzy grade equals to a determined number. Finally, section 7 concludes the paper, giving also some future lines of our research.

2. **Complete Hypergroups and 1-hypergroups**

We briefly recall some notions and results concerning complete hypergroups and 1-hypergroups; for a comprehensive overview of this theme, the reader is refereed to [4].

Let H be a nonempty set and $\mathcal{P}^*(H)$ the set of all nonempty subsets of H. A set H equipped with a *hyperoperation* $\circ : H^2 \longrightarrow \mathcal{P}^*(H)$ is called a *hypergroupoid*. The image of the pair $(a,b) \in H^2$ is denoted by $a \circ b$ and it is called the *hyperproduct* of a and b. If A and B are nonempty subsets of H, then $A \circ B = \bigcup\limits_{\substack{a\in A\\ b\in B}} a \circ b$.

A *semihypergroup* is a hypergroupoid $(H,\circ)$ such that, for any $(a,b,c) \in H^3$, $(a\circ b)\circ c = a\circ(b\circ c)$ (the associativity). A *quasihypergroup* is a hypergroupoid $(H,\circ)$ which satisfies the *reproduction axiom*: for any $a \in H, a\circ H = H\circ a = H$. A *hypergroup* is a semihypergroup which is also a quasihypergroup.

Notation. For each pair $(a,b) \in H^2$, we denote:
$a/b = \{x \mid x \in H, a \in x\circ b\}$ and $b\backslash a = \{y \mid y \in H, a \in b\circ y\}$.

A commutative hypergroupoid $(H,\circ)$ is called a *join space* if, for any $(a,b,c,d) \in H^4$, the following implication holds:

$$a/b \cap c/d \neq \emptyset \Longrightarrow a\circ d \cap b\circ c \neq \emptyset.$$

A hypergroup H is *regular* if it has at least one identity and each element has at least one inverse. A regular hypergroup $(H,\circ)$ is called *reversible* if, for any $(x,y,a) \in H^3$, it satisfies the following conditions:

1) if $y \in a\circ x$, then there exists an inverse a' of a such that $x \in a'\circ y$;
2) if $y \in x\circ a$, then there exists an inverse a'' of a such that $x \in y\circ a''$.

Several fundamental relations may be defined on a hypergroupoid (see [4, 19, 20, 25]); here we recall one of them strongly related to the notion of completeness.

The relation β on a hypergroupoid $(H,\circ)$ is defined as follows:

$$a\beta b \Longleftrightarrow \exists n \in \mathbb{N}^*, \exists (x_1,x_2,\ldots,x_n) \in H^n : a \in \prod_{i=1}^{n} x_i \ni b.$$

Notice that β is a reflexive and a symmetric relation on H, but generally, not a transitive one. Let us denote by β^* the transitive closure of β.

It is well known that, if H is a hypergroup, then $\beta^* = \beta$ and H/β is a group [4].

One of the most important notions in hypergroup theory is the heart of a hypergroup H. Studying its properties one determines completely the structure of the hypergroup H.

The *heart* of a hypergroup H is the set $\omega_H = \{x \in H \mid \varphi_H(x) = 1\}$, where $\varphi_H : H \longrightarrow H/\beta$ is the canonical projection and 1 is the identity of the group H/β.

Definition 2.1. A hypergroup H is called 1-*hypergroup* if the cardinality of its heart equals 1.

Definition 2.2. A hypergroup $(H, \circ)$ is *complete* if, for any $x, y \in H$, we have $x \circ y = \beta^*(x \circ y)$.

In the following we give a characterization of the complete hypergroups and present some of their properties (see [4]).

Theorem 2.3. *A hypergroup $(H, \circ)$ is complete if H can be written as the union of its subsets $H = \bigcup_{g \in G} A_g$, where*

1) $(G, \cdot)$ *is a group;*
2) *for any $(g_1, g_2) \in G^2$, $g_1 \neq g_2$, we have $A_{g_1} \cap A_{g_2} = \emptyset$;*
3) *if $(a, b) \in A_{g_1} \times A_{g_2}$, then $a \circ b = A_{g_1 g_2}$.*

Theorem 2.4. *If H is a complete hypergroup, then*

(i) *ω_H is the set of all identities of H;*
(ii) *H is a regular reversible hypergroup.*

Theorem 2.5. *All complete commutative hypergroups are join spaces.*

Theorem 2.6. [4]

(i) *Let H be an 1-hypergroup. If $|H| \leq 4$, then H is a group or it is a complete hypergroup.*
(ii) *For every $n \geq 5$, there exist 1-hypergroups of cardinality n which are not complete.*

Proposition 2.7. *Let H be a complete hypergroup and $H = \bigcup_{g \in G} A_g$, with $(G, \cdot)$ a group with the identity element e. Then $A_e = \omega_H$.*

Proof. We know that, for a complete hypergroup H, its heart is the set of all identities of H.

First we prove that $A_e \subset \omega_H$. For any $x \in A_e$ and for any $a \in A_g, g \in G$, we have $a \circ x = A_{ge} = A_g \ni a$ and $x \circ a = A_{eg} = A_g \ni a$, therefore x is an identity for H and thereby $x \in \omega_H$.

Conversely, for any $x \in \omega_H$ and for any $a \in H$, we have $a \in a \circ x \cap x \circ a$. Assuming that $x \in A_{g_1}$ and $a \in A_{g_2}$, we obtain $a \in A_{g_1 \cdot g_2} \cap A_{g_2 \cdot g_1}$ and by the definition of a complete hypergroup, $g_1 \cdot g_2 = g_2 \cdot g_1$, for any $g_2 \in G$; thus $g_1 \in Z(G)$, the center of the group G. More over, $A_{g_2} \cap A_{g_1 \cdot g_2} \neq \emptyset$ and then $g_1 \cdot g_2 = g_2$, for any $g_2 \in G$, which means $g_1 = e$ and $x \in A_e$. $\square$

We conclude this section with a result about the number of non-isomorphic finite complete hypergroups.

Definition 2.8. [14] Let n be a positive integer, $n \geq 3$. A *m-decomposition* of n, $2 \leq m \leq n-1$, is an ordered system of natural numbers $(k_1, k_2, \ldots, k_m)$ such that $k_i \geq 1$, for any i, $1 \leq i \leq m$, $k_1 + k_2 + \ldots + k_m = n$, $k_2 \leq k_3 \leq \ldots \leq k_m$.

We denote $d_m(n)$ the number of m-decompositions of n.

Theorem 2.9. [14] *There are* $\sum_{m=2}^{n-1} s_m d_m(n)$ *non-isomorphic complete hypergroups of order* n*, where* s_m *denotes the number of the non-isomorphic groups of order* m.

3. The Sequences of Join Spaces and Fuzzy Sets Associated with a Hypergroupoid

Several connections between hypergroupoids and fuzzy sets or their generalizations (Atanassov's intuitionistic fuzzy sets, soft sets, rough sets) have been defined and investigated, giving the possibility to study fuzzy sets from an algebraic point of view and to obtain new hyperstructures related to fuzzy sets. We recall here two of them, which lead to a sequence of join spaces and fuzzy sets associated with a hypergroupoid. In this paper H denotes a finite hypergroupoid.

Let X be a nonempty set. A *fuzzy set* A in X is characterized by a *membership function* $\mu_A : X \longrightarrow [0,1]$, where for any $x \in X$, the value $\mu_A(x)$ represents the *grade of membership* of x in A. More generally, any function $\mu : X \longrightarrow [0,1]$ is called a *fuzzy subset* of X.

For any hypergroup $(H, \circ)$, Corsini defined in [6] a fuzzy subset $\widetilde{\mu}$ of H in the following way: for any $u \in H$, one considers:

$$\widetilde{\mu}(u) = \frac{\displaystyle\sum_{(x,y)\in Q(u)} \frac{1}{|x \circ y|}}{q(u)}, \tag{1}$$

where $Q(u) = \{(a,b) \in H^2 \mid u \in a \circ b\}$, $\quad q(u) = |Q(u)|$. If $Q(u) = \emptyset$, set $\widetilde{\mu}(u) = 0$. In other words, $\widetilde{\mu}(u)$ is the average value of the reciprocals of the sizes of $x \circ y$, for all $x \circ y$ containing u.

On the other hand, with any hypergroupoid H endowed with a fuzzy set α, we can associate a join space $(H, \circ_\alpha)$ as follows (see [5]): for any $(x,y) \in H^2$,

$$x \circ_\alpha y = \{z \in H \mid \alpha(x) \wedge \alpha(y) \leq \alpha(z) \leq \alpha(x) \vee \alpha(y)\}. \tag{2}$$

Let $({}^1H, \circ_1)$ be the join space obtained as in (3) for the fuzzy subset $\widetilde{\mu}$. By using the same procedure as in (3), from ${}^1H$ we can obtain a membership function $\widetilde{\mu}_1$ and the associated join space ${}^2H$ and so on. A sequence of fuzzy sets and join spaces $(({}^rH, \circ_r), \widetilde{\mu}_r)_{r\geq 1}$ is determined in this way. We denote $\widetilde{\mu}_0 = \widetilde{\mu}$, ${}^0H = H$. If two consecutive hypergroups of the obtained sequence are isomorphic, then the sequence stops.

Let $(({}^iH, \circ_i), \widetilde{\mu}_i(u))_{i\geq 1}$ be the sequence of fuzzy sets and join spaces associated with H. Then, for any i, there are r, namely $r = r_i$, and a partition $\pi = \{{}^iC_j\}_{j=1}^r$ of ${}^iH$ such that, for any $j \geq 1$ and $x \in {}^iC_j$, ${}^iC_j = \widetilde{\mu}_{i-1}^{-1}(\widetilde{\mu}_{i-1}(x))$ (this means $x, y \in$

${}^iC_j \Longleftrightarrow \widetilde{\mu}_{i-1}(x) = \widetilde{\mu}_{i-1}(y))$. On the set of classes $\{{}^iC_j\}_{j=1}^r$ we define the following ordering relation : $j < k$ if, for $x \in {}^iC_j$ and $y \in {}^iC_k$, we have $\widetilde{\mu}_{i-1}(x) < \widetilde{\mu}_{i-1}(y)$.

We will use some notations, for all j, s:

$$k_j = |{}^iC_j|, \quad {}_sC = \bigcup_{1\le j\le s} {}^iC_j, \quad {}^sC = \bigcup_{s\le j\le r} {}^iC_j, \quad {}_sk = |{}_sC|, \quad {}^sk = |{}^sC|. \tag{3}$$

With any ordered chain $({}^iC_1, {}^iC_2, \ldots, {}^iC_r)$ we may associate an ordered r-tuple $(k_1, k_2, \ldots, k_r)$, where $k_l = |{}^iC_l|$, for all l, $1 \le l \le r$.

A general formula to compute the values of $\widetilde{\mu}$ is given in the following theorem.

Theorem 3.1. [6] *For any $z \in {}^iC_s$, $s = 1, 2, \ldots, r$,*

$$\widetilde{\mu}_i(z) = \frac{2\displaystyle\sum_{\substack{i\le s\le j\\ i\ne j}}^{i,j}\left(\frac{k_ik_j}{\displaystyle\sum_{i\le t\le j} k_t}\right) + k_s}{2_sk^sk - k_s^2}.$$

It is important to know when two consecutive join spaces in the associated sequence of a hypergroupoid are isomorphic or not. In this direction we have the following result.

Theorem 3.2. [9] *Let ${}^iH$ and ${}^{i+1}H$ be the join spaces associated with H determined by the membership functions $\widetilde{\mu}_i$ and $\widetilde{\mu}_{i+1}$, where ${}^iH = \bigcup_{l=1}^{r_1} C_l$, ${}^{i+1}H = \bigcup_{l=1}^{r_2} C'_l$ and $(k_1, k_2, \ldots, k_{r_1})$ is the r_1-tuple associated with ${}^iH$, $(k'_1, k'_2, \ldots, k'_{r_2})$ is the r_2-tuple associated with ${}^{i+1}H$.*
The join spaces ${}^iH$ and ${}^{i+1}H$ are isomorphic if and only if $r_1 = r_2$ and $(k_1, k_2, \ldots, k_{r_1}) = (k'_1, k'_2, \ldots, k'_{r_1})$ or $(k_1, k_2, \ldots, k_{r_1}) = (k'_{r_1}, k'_{r_1-1}, \ldots, k'_1)$.

The length of the sequence of join spaces associated with H is called the fuzzy grade of the hypergroupoid H, more exactly we give the following definition.

Definition 3.3. [7] A hypergroupoid H has the *fuzzy grade* m, $m \in \mathbb{N}^*$, and we write $f.g.(H) = m$ if, for any i, $0 \le i < m$, the join spaces ${}^iH$ and ${}^{i+1}H$ associated with H are not isomorphic (where ${}^0H = H$) and for any s, $s > m$, ${}^sH$ is isomorphic with ${}^mH$. We say that the hypergroupoid H has the *strong fuzzy grade* m and we write $s.f.g.(H) = m$ if $f.g.(H) = m$ and for all $s, s > m$, ${}^sH = {}^mH$.

4. Computation of the Fuzzy Grade of a Hypergroupoid

Let $(H, \circ)$ be a hypergroupoid and $(({}^rH, \circ_r), \widetilde{\mu}_r)_{r\ge 1}$ be the sequence of fuzzy sets and of join spaces associated with H. Let $R_{\widetilde{\mu}_i}$ be the equivalence relation on the join space $({}^iH, \circ_i)$ defined by

$$xR_{\widetilde{\mu}_i}y \Longleftrightarrow \widetilde{\mu}_i(x) = \widetilde{\mu}_i(y).$$

Then, for all $i \ge 0$, $R_{\widetilde{\mu}_i}$ is a regular equivalence and a congruence on ${}^iH$ (see [6]).

It would be useful to know which is the fuzzy grade of a hypergroupoid H on the base of the type of the n-tuple associated with H. We give some results for several types of n-tuples. More precisely, we determine the fuzzy grade of a hypergroupoid H with the property that $|H/R_{\widetilde{\mu}}| \in \{2, 3\}$ (recall that $\widetilde{\mu}_0 = \widetilde{\mu}$).

Proposition 4.1. *Let $(H, \circ)$ be a hypergroupoid of cardinality n.*

1) *If $|H/R_{\widetilde{\mu}}| = 2$, that is with H we associate the pair (k_1, k_2), and*
   a) *if $k_1 = k_2$, then we have $s.f.g.(H) = 2$.*
   b) *if $k_1 \neq k_2$, then we have $f.g.(H) = 1$.*
2) *If $|H/R_{\widetilde{\mu}}| = 3$, that is with H we associate the triple (k_1, k_2, k_3), and*
   a) *if $k_1 = k_2 = k_3$, then we have $f.g.(H) = 2$.*
   b) *for $k_1 = k_3 \neq k_2$ we have $f.g.(H) = 2$ if $k_2 \neq 2k_3$ and $s.f.g.(H) = 3$ if $k_2 = 2k_3$.*
   c) *if $k_1 < k_2 = k_3$ we have $f.g.(H) = 1$.*
   d) *for $k_1 = k_2 < k_3$ we have $f.g.(H) = 1$ if $P = 2k_3^3 - 8k_1^3 - k_1^2k_3 + 5k_1k_3^2 > 0$ and $f.g.(H) = 3$ otherwise.*
   e) *for $k_1 \neq k_2 \neq k_3$, there is no precise order between $\widetilde{\mu}_1(x), \widetilde{\mu}_1(y), \widetilde{\mu}_1(z)$.*

Proof. 1) If the hypergroupoid H has two classes of equivalence, i.e. $H = C_1 \bigcup C_2$, where $|C_1| = k_1$ and $|C_2| = k_2$, with $k_1 + k_2 = n = |H|$, then one distinguishes the following two cases:

a) $k_1 = k_2$; by Theorem 3.1, one finds that $\widetilde{\mu}_1(x) = \widetilde{\mu}_1(y)$, for any $x \in C_1$ and any $y \in C_2$. It follows that the join space ${}^2H$ is a total hypergroup and therefore $s.f.g.(H) = 2$.

b) $k_1 < k_2$; by Theorem 3.1, $\widetilde{\mu}_1(x) = \dfrac{k_1 + 3k_2}{n(k_1 + 2k_2)}$, for any $x \in C_1$ and similarly, $\widetilde{\mu}_1(y) = \dfrac{k_2 + 3k_1}{n(k_2 + 2k_1)}$, for any $y \in C_2$. It is easy to see that $\widetilde{\mu}_1(x) > \widetilde{\mu}_1(y)$ which means that with ${}^1H$ one associates the pair (k_2, k_1). Thereby, by Theorem 3.2, it follows that ${}^2H$ is isomorphic with ${}^1H$, so $f.g.(H) = 1$.

2) We suppose now that the hypergroupoid H has three classes of equivalence: $H = C_1 \bigcup C_2 \bigcup C_3$, where $|C_1| = k_1$, $|C_2| = k_2$ and $|C_3| = k_3$, with $k_1 + k_2 + k_3 = n$. There exist the following possibilities:

a) $k_1 = k_2 = k_3 = k$; by Theorem 3.1 it follows, for any $x \in C_1$, $y \in C_2$, $z \in C_3$, that $\widetilde{\mu}_1(x) = \dfrac{8}{15k}$, $\widetilde{\mu}_1(y) = \dfrac{11}{21k}$ and $\widetilde{\mu}_1(z) = \dfrac{8}{15k}$. Therefore, $\widetilde{\mu}_1(x) = \widetilde{\mu}_1(z) > \widetilde{\mu}_1(y)$ and thus with the join space ${}^1H$ one associates the pair $(k, 2k)$. By the case 1)b), we conclude that $f.g.(H) = 2$.

b) $k_1 = k_3 \neq k_2$; using again Theorem 3.1, it results that $\widetilde{\mu}_1(x) = \widetilde{\mu}_1(z) \neq$ $\neq \widetilde{\mu}_1(y)$ as in the previous case. If $k_2 \neq 2k_3$, then $f.g.(H) = 2$; otherwise, if $k_2 = 2k_3$, it follows that the join space ${}^2H$ is a total hypergroup and thus $s.f.g.(H) = 3$.

c) $k_1 < k_2 = k_3$; by Theorem 3.1 one obtains, for any $x \in C_1$, $y \in C_2$, $z \in C_3$,

$$\widetilde{\mu}_1(x) = \frac{1 + 2k_2\left(\dfrac{1}{k_1+k_2} + \dfrac{1}{k_1+2k_2}\right)}{k_1+4k_2},$$
$$\widetilde{\mu}_1(y) = \frac{2 + 2k_1\left(\dfrac{1}{k_1+k_2} + \dfrac{1}{k_1+2k_2}\right)}{4k_1+3k_2},$$
$$\widetilde{\mu}_1(z) = \frac{2 + \dfrac{2k_1}{k_1+2k_2}}{2k_1+3k_2}.$$

We verify whether $\widetilde{\mu}_1(x) > \widetilde{\mu}_1(y)$; this is equivalent with

$$\frac{1 + 2k_2\left(\dfrac{1}{k_1+k_2} + \dfrac{1}{k_1+2k_2}\right)}{k_1+4k_2} > \frac{2 + 2k_1\left(\dfrac{1}{k_1+k_2} + \dfrac{1}{k_1+2k_2}\right)}{4k_1+3k_2} \Longleftrightarrow$$

$$8k_2^3 - 2k_1^3 - 5k_1^2k_2 + k_1k_2^2 > 0 \Longleftrightarrow$$

$5k_2(k_2^2 - k_1^2) + 3k_2^3 - 2k_1^3 + k_1k_2^2 > 0$ which is obviously true for $k_1 < k_2$.

Now we check if $\widetilde{\mu}_1(x) > \widetilde{\mu}_1(z)$; that is

$$\frac{1 + 2k_2\left(\dfrac{1}{k_1+k_2} + \dfrac{1}{k_1+2k_2}\right)}{k_1+4k_2} > \frac{2 + \dfrac{2k_1}{k_1+2k_2}}{2k_1+3k_2} \Longleftrightarrow$$

$$8k_2^3 - 2k_1^3 - 7k_1^2k_2 + k_1k_2^2 > 0 \Longleftrightarrow$$

$(k_2 - k_1)(2k_1^2 + 9k_1k_2 + 8k_2^2) > 0$ which is clearly true for $k_1 < k_2$.

Finally, we see if $\widetilde{\mu}_1(y) = \widetilde{\mu}_1(z)$, that is

$$\frac{2 + 2k_1\left(\dfrac{1}{k_1+k_2} + \dfrac{1}{k_1+2k_2}\right)}{4k_1+3k_2} = \frac{2 + \dfrac{2k_1}{k_1+2k_2}}{2k_1+3k_2} \Longleftrightarrow$$

$$2k_1^2 + k_1k_2 - 2k_2^2 = 0.$$

Considering it as a second degree equation in $k_1 \in \mathbb{N}^*$, the solutions are $\dfrac{-k_2 \pm \sqrt{17}k_2}{2}$, which are not natural numbers, so $\widetilde{\mu}_1(y) \neq \widetilde{\mu}_1(z)$.
We conclude that $\widetilde{\mu}_1(x) > \widetilde{\mu}_1(y)$ and $\widetilde{\mu}_1(x) > \widetilde{\mu}_1(z)$, with $\widetilde{\mu}_1(y) \neq \widetilde{\mu}_1(z)$; therefore the triple associated with the join space ${}^1H$ is (k_2, k_2, k_1) and, according to Theorem 3.2, the join spaces ${}^2H$ and ${}^1H$ are isomorphic and thus $f.g.(H) = 1$.

d) $k_1 = k_2 < k_3$; again by the same theorem one obtains, for any $x \in C_1$, $y \in C_2$, $z \in C_3$,

$$\widetilde{\mu}_1(x) = \frac{2\left(1+\dfrac{k_3}{2k_1+k_3}\right)}{3k_1+2k_3};$$
$$\widetilde{\mu}_1(y) = \frac{2\left(1+\dfrac{k_3}{k_1+k_3}+\dfrac{k_3}{2k_1+k_3}\right)}{3k_1+4k_3};$$
$$\widetilde{\mu}_1(z) = \frac{1+2k_1\left(\dfrac{1}{k_1+k_3}+\dfrac{1}{2k_1+k_3}\right)}{4k_1+k_3}.$$

After a simple computation one finds $\widetilde{\mu}_1(x) \neq \widetilde{\mu}_1(y)$ and $\widetilde{\mu}_1(x) > \widetilde{\mu}_1(z)$.

Now we see when $\widetilde{\mu}_1(y) > \widetilde{\mu}_1(z)$; this is equivalent to

$$P = P(k_1, k_3) = 2k_3^3 - 8k_1^3 - k_1^2 k_3 + 5k_1 k_3^2 > 0.$$

Since $k_3 > k_1$, set $k_3 = k_1 + l$, with $l \in \mathbb{N}^*$. Then $P(k_1, k_3) = -2k_1^3 + 2l^3 + 15k_1^2 l + 11k_1 l^2$. It is obvious that, for $l \geq k_1$, that is equivalent to $k_3 \geq 2k_1$, one finds $P(k_1, k_3) > 0$. For $l < k_1$ the result is variable: there are several values of (k_1, k_3) such that $P(k_1, k_3) > 0$ and several values such that $P(k_1, k_3) < 0$.

If $P > 0$, then $\widetilde{\mu}_1(z) < \widetilde{\mu}_1(y)$ and $\widetilde{\mu}_1(z) < \widetilde{\mu}_1(x)$, with $\widetilde{\mu}_1(x) \neq \widetilde{\mu}_1(y)$. As in the previous case, it results that $f.g.(H) = 1$.

If $P < 0$, then $\widetilde{\mu}_1(y) < \widetilde{\mu}_1(z) < \widetilde{\mu}_1(x)$ and the triple associated with the join space ${}^1H$ is (k_1, k_3, k_1). By the second case of 2), with the join space ${}^2H$ one associates the pair $(2k_1, k_3)$, with $2k_1 \neq k_3$ (if $2k_1 = k_3$, then $P > 0$) and thus $f.g.(H) = 3$ (for example, if $k_1 = k_2 = 10$ and $k_3 = 11$, then $P < 0$ and $f.g.(H) = 3$).

e) $k_1 \neq k_2 \neq k_3$; using Theorem 3.1, for any $x \in C_1$, $y \in C_2$, $z \in C_3$, we obtain

$$\widetilde{\mu}_1(x) = \frac{1+\dfrac{2k_2}{k_1+k_2}+\dfrac{2k_3}{k_1+k_2+k_3}}{k_1+2k_2+2k_3}$$
$$\widetilde{\mu}_1(y) = \frac{k_2+2\dfrac{k_1k_2}{k_1+k_2}+2\dfrac{k_1k_3}{k_1+k_2+k_3}+2\dfrac{k_2k_3}{k_2+k_3}}{2k_1k_2+2k_1k_3+k_2^2+2k_2k_3}$$
$$\widetilde{\mu}_1(z) = \frac{1+\dfrac{2k_1}{k_1+k_2+k_3}+\dfrac{2k_2}{k_2+k_3}}{2k_1+2k_2+k_3}.$$

It is simple to verify that $\widetilde{\mu}_1(x) > \widetilde{\mu}_1(y)$ and $\widetilde{\mu}_1(x) > \widetilde{\mu}_1(z)$. We can not say that for any triple (k_1, k_2, k_3) there is the same relation between $\widetilde{\mu}_1(y)$ and $\widetilde{\mu}_1(z)$. For example, if $k_1 < k_2 < k_3$,

taking $(k_1, k_2, k_3) = (10, 11, 12)$ we have $\widetilde{\mu}_1(y) > \widetilde{\mu}_1(z)$;

taking $(k_1, k_2, k_3) = (20, 21, 22)$ we have $\widetilde{\mu}_1(y) < \widetilde{\mu}_1(z)$.

Similarly, if $k_1 < k_3 < k_2$, then
for $(k_1, k_2, k_3) = (1, 3, 2)$ we have $\widetilde{\mu}_1(y) < \widetilde{\mu}_1(z)$;
for $(k_1, k_2, k_3) = (10, 20, 19)$ we have $\widetilde{\mu}_1(y) > \widetilde{\mu}_1(z)$.
So, there is no order between $\widetilde{\mu}_1(x), \widetilde{\mu}_1(y), \widetilde{\mu}_1(z)$.

□

We conclude this section with a general result regarding the membership function $\widetilde{\mu}$ for a hypergroupoid with the associated r-tuple of a symmetric form. We distinguish two cases, depending on the parity of r.

Proposition 4.2. *Let r be an odd number and $(k_1, k_2, \ldots, k_{s-1}, k_s, k_{s-1}, \ldots, k_2, k_1)$, with $s = (r+1)/2$, be the r-tuple associated with a hypergroupoid H, that is $H = \bigcup_{i=1}^r C_i$, with $|C_i| = k_i$, for any $i = 1, 2, \ldots, r$ as in Proposition 4.1. Then, for any $x \in C_i$ and $y \in C_{r-i+1}$, we have $\widetilde{\mu}(x) = \widetilde{\mu}(y)$, whenever $i = 1, 2, \ldots, s-1$.*

Proof. It follows directly from Theorem 3.1. □

Proposition 4.3. *Let r be an even number and $(k_1, k_2, \ldots, k_{s-1}, k_s, k_s, k_{s-1}, \ldots, k_2, k_1)$, with $s = r/2$, be the r-tuple associated with a hypergroupoid H, that is $H = \bigcup_{i=1}^r C_i$, with $|C_i| = k_i$, for any $i = 1, 2, \ldots, r$ as in Proposition 4.1. Then, for any $x \in C_i$ and $y \in C_{r-i+1}$, we have $\widetilde{\mu}(x) = \widetilde{\mu}(y)$, whenever $i = 1, 2, \ldots, s$.*

Proof. It follows directly from Theorem 3.1. □

5. **Fuzzy Grade of a Complete Hypergroup**

In this section we determine the fuzzy grade of particular finite complete hypergroups.

Let $(H, \circ)$ be a complete hypergroup, that is H can be partitioned as the union $H = \bigcup\limits_{g \in G} A_g$, as in Theorem 2.3.

If G is a commutative group, then H is a commutative complete hypergroup, that is a join space.

If $|H| = |G| = n$, then $A_{g_i} = \{a_i\}$, for all $i = 1, 2, ..., n$, $a_i \in H$, and thus, for any $(a, b) \in H^2$, $|a \circ b| = 1$. Therefore H is a group. Moreover, any group is a complete hypergroup. In the following we will consider only finite complete hypergroups which are not groups, so $m = |G| < |H| = n$.

Take $G = \{g_1, g_2, ..., g_m\}$. By the definition, it results that the structure of any complete hypergroup H of cardinality n is determined by an m-tuple, $2 \leq m \leq n-1$, denoted by $[k_1, k_2, ..., k_m]$, where, for any $i \in \{1, 2, ..., m\}$, $k_i = |A_{g_i}|$ and $k_2 \leq k_3 \leq \ldots \leq k_m$.

If H is a complete 1-hypergroup, then $|A_{g_1}| = |\omega_H| = 1$, where g_1 is the identity of the group G, and then H is of the type $[1, k_2, k_3, ..., k_m]$.

Theorem 5.1. [4], [14] *Let H be a complete hypergroup. Then, for any $u \in H$, $\widetilde{\mu}(u) = \dfrac{1}{|A_{g_u}|}$.*

Remark 5.2. If G_1 and G_2 are non isomorphic groups of the same cardinality m and H_1, H_2 are the complete hypergroups obtained with G_1 and G_2, respectively, then $f.g.(H_1) = f.g.(H_2)$. Consequently, the fuzzy grade of a complete hypergroup H does not depend on the considered group G, but only on the m-decomposition of n.

Now, we will calculate the fuzzy grade of a complete hypergroup H of cardinality n, constructed from the group G with $|G| \in \{2,3\}$.

Case 1: $|G| = 2$; $H = A_{g_1} \cup A_{g_2}$, with $|A_{g_i}| = k_i$, $i = 1,2$.
The structure of H is represented by $[k_1, k_2], k_1 \neq k_2$, if n is an odd number and thus $\widetilde{\mu}(x) = 1/k_1$, $\widetilde{\mu}(y) = 1/k_2$, for any $x \in A_{g_1}$ and $y \in A_{g_2}$. By Proposition 4.1 it follows that $s.f.g.(H) = 1$. On the other hand, if n is an even number, H is represented by $[k_1, k_2], k_1 \neq k_2$, and $[n/2, n/2]$. In both situations, by Proposition 4.1 it follows that $s.f.g.(H) = 1$; in the last one, the associated join space ${}^1H$ is a total hypergroup.

Case 2: $|G| = 3$; $H = A_{g_1} \cup A_{g_2} \cup A_{g_3}$, with $|A_{g_i}| = k_i$, $i = 1,2,3$.
The structure of H is represented by $[k_1, k_2, k_3]$, with $n = k_1 + k_2 + k_3$, $k_2 \leq k_3$. We have to study four possibilities:

(i) $k_1 = k_2 = k_3 = k \geq 2$, then $\widetilde{\mu}(x) = \dfrac{1}{k}$, for any $x \in H$, so the associated join space ${}^1H$ is a total hypergroup and thus $s.f.g.(H) = 1$.

(ii) Two of the numbers k_i are equal to the half of the third one; with the hypergroup $(H, \widetilde{\mu})$ we associate the pair (k,k). Then the join space ${}^2H$ is a total hypergroup and thus $s.f.g.(H) = 2$.

(iii) Two of the numbers k_i are equal and different from the half of the third one; then the pair associated with the hypergroup $(H, \widetilde{\mu})$ has the form (k,l), $k \neq l$; therefore, by Proposition 4.1 it follows that $f.g.(H) = 1$;

(iv) $k_1 \neq k_2 \neq k_3$; in this case with the hypergroup $(H, \widetilde{\mu})$ it is associated the triple (k_i, k_j, k_l), $i \neq j \neq l \in \{1,2,3\}$, with $k_i \neq k_j \neq k_l$. According to Proposition 4.1, we have no general rule for determining the fuzzy grade of a such hypergroup.

Cristea [14] has determined the fuzzy grade of all complete hypergroups of order less than 7. We recall here this result.

Theorem 5.3. [14] *Let H be a complete hypergroup of order $n \leq 6$.*

(i) *For $n = 3$, $s.f.g.(H) = 1$.*

(ii) *For $n = 4$, there are three hypergroups with $s.f.g.(H) = 1$ and two hypergroups with $s.f.g.(H) = 2$.*

(iii) *For $n = 5$, $s.f.g.(H) = 1$.*

(iv) *For $n = 6$, there are seventeen hypergroups of $s.f.g.(H) = 1$ and four hypergroups of $s.f.g.(H) = 2$.*

Next we will determine the fuzzy grade of some finite complete 1-hypergroups. Then we will generalize their structures, obtaining other complete hypergroups that are no longer 1-hypergroups.

Proposition 5.4. *If H is a complete 1-hypergroup of the type $[\underbrace{1,1,...,1}_{k\ \ times}, k]$, where $n = |H| = 2k$, then $s.f.g.(H)$=2.*

More general, if the structure of the complete hypergroup H, that is no longer an 1-hypergroup, is represented by $[\underbrace{p,p,...,p}_{k\ \ times},kp]$*, where* $n=|H|=2kp$*, then s.f.g.*(H)*=2.*

Proof. We suppose that the structure of H is determined by the $(k+1)$-tuple $[\underbrace{p,p,...,p}_{k\ \ times},kp]$. By Theorem 5.1, it results that the pair associated with the hypergroup $(H,\widetilde{\mu})$ is (kp,kp), therefore, by Proposition 4.1, we conclude that the associated join space ${}^2H$ is a total hypergroup and $s.f.g.(H)=2$. □

Proposition 5.5. *Let H be a complete 1-hypergroup of the following type:* $[\underbrace{1,1,...,1}_{l\ \ times},\underbrace{k,k,...,k}_{p\ \ times},l]$*, with $1<k<l$, $p\geq 1$, $n=|H|=kp+2l$.*

(i) *If $kp=2l$, then $s.f.g.(H)=3$.*
(ii) *If $kp\neq 2l$, then $f.g.(H)=2$.*

Proof. We use again Theorem 5.1.

(i) If $kp=2l$, then with the hypergroupoid $(H,\widetilde{\mu})$ it is associated the triple $(l,2l,l)$ and thus, by Proposition 4.1, it follows that $s.f.g.(H)=3$, with the associated join space ${}^3H$ a total hypergroup.
(ii) If $kp\neq 2l$, then the triple associated with the hypergroup $(H,\widetilde{\mu})$ is (l,kp,l). According to Proposition 4.1, it follows that $f.g.(H)=2$. □

Remark 5.6. Generalizing Proposition 5.5 for the complete hypergroups of the type $[\underbrace{p,p,\dots,p}_{s\ \ times},\underbrace{k,k,\dots,k}_{t\ \ times},ps]$, with $2\leq p<k<ps$, $n=|H|=2ps+kt$, that are no longer 1-hypergroups, we obtain, for $n=4ps$, $s.f.g.(H)=3$, and for $kt\neq 2ps$, $f.g.(H)=2$.

Proposition 5.7. *Let H be a complete 1-hypergroup of the following type:* $[\underbrace{1,1,\dots,1}_{l\ \ times},\underbrace{k,k,\dots,k}_{p\ \ times},\underbrace{p,p,\dots,p}_{k\ \ times},l]$*, with $1<k<p<l$, $n=|H|=2(l+pk)$.*

(i) *If $pk=l$, then $s.f.g.(H)=3$.*
(ii) *If $pk\neq l$, then $f.g.(H)=2$.*

Proof. From Theorem 5.1, with the hypergroupoid $(H,\widetilde{\mu})$ it is associated the 4-tuple (l,pk,pk,l) and then, from Proposition 4.3, with the join space ${}^1H$ it is associated the pair $(2l,2pk)$.

If $l=pk$ then with the join space ${}^2H$ it is associated $(4l)$; thus ${}^3H$ is a total hypergroup and $s.f.g.(H)=3$.

If $pk\neq l$, by Proposition 4.1, we have $f.g.(H)=2$. □

Remark 5.8. Generalizing Proposition 5.7 for the complete hypergroups of the type $[\underbrace{k,k,\dots,k}_{l\ \ times},\underbrace{p,p,\dots,p}_{s\ \ times},\underbrace{s,s,\dots,s}_{p\ \ times},\underbrace{l,l,\dots,l}_{k\ \ times}]$, (which are not 1-hypergroups), with $2 \leq k < p < s < l$, $n = |H| = 2(ps + kl)$, we obtain, for $kl = ps$, that $s.f.g.(H) = 3$ and, for $kl \neq ps$, that $f.g.(H) = 2$.

6. A Method to Construct a Hypergroup H with $s.f.g.(H) = s$

In [15], the second author has investigated the fuzzy grade of a finite hypergroupoid verifying a certain property, obtaining a fundamental result, that we summarize as it follows.

Theorem 6.1. *Let H be the finite hypergroupoid $H = \{x_1, x_2, \ldots, x_n\}$, with $n = 2^s$, $s \in \mathbb{N}^*$, which verifies the relation $\widetilde{\mu}(x_1) < \widetilde{\mu}(x_2) < \ldots < \widetilde{\mu}(x_n)$, i.e. the n-tuple associated with H is $(1, 1, \ldots, 1)$. Then the join space ${}^{s+1}H$ is a total hypergroup and by consequence $s.f.g.(H) = s + 1$.*

In [15], it has proven that, $\widetilde{\mu}_1(x_i) = \widetilde{\mu}_1(x_{n+1-i})$, whenever $i = 1, 2, \ldots, n/2$ and, for any $i < j \leq n/2$, $\widetilde{\mu}_1(x_i) > \widetilde{\mu}_1(x_j)$; therefore with the join space $({}^1H, \widetilde{\mu}_1)$ is associated the r_1-tuple $(2, 2, \ldots, 2)$, with $r_1 = 2^{s-1}$. Then with the join space $({}^2H, \widetilde{\mu}_2)$ is associated the r_2-tuple $(4, 4, ..., 4)$, with $r_2 = 2^{s-2}$ and so on; with the join space $({}^{s-1}H, \widetilde{\mu}_{s-1})$ one associates the pair $(2^{s-1}, 2^{s-1})$; thereby the join space ${}^{s+1}H$ is a total hypergroup. In conclusion, $s.f.g.(H) = s + 1$.

The same procedure can also be used if with the hypergroup $(H, \widetilde{\mu})$ one associates the r-tuple $(k, k, ..., k)$, where $r = 2^s$ and $n = |H| = 2^s k$. Similarly, we obtain $s.f.g.(H) = s + 1$. A such hypergroup always exists; in particular, for $k = 2$, that is $n = 2^{s+1}$, we can consider the hypergroup $H = \{a_1, a_2, ..., a_n\}$ with the hyperproduct defined by

$a_i \circ a_i = a_i$, for any $i \in \{1, 2, ..., n\}$,
$a_i \circ a_j = a_j \circ a_i = \{a_i, a_{i+1}, ..., a_j\}$, for any i, j, $1 \leq i < j \leq n$.

Now we construct a complete 1-hypergroup such that one associates with H the m-tuple $(k, k, ..., k)$, with $m = 2^{s-1}$, so $n = |H| = 2^{s-1}k$. For instance, let H be of the type

$$[\underbrace{1, 1, \ldots, 1}_{k\ \ times}, \underbrace{2, 2, \ldots, 2}_{\frac{k}{2}\ \ times}, \underbrace{3, 3, \ldots, 3}_{\frac{k}{3}\ \ times}, \ldots, \underbrace{m-1, m-1, \ldots, m-1}_{\frac{k}{m-1}\ \ times}, k],$$

where $k = LCM(1, 2, \ldots, m-1)$, i.e. the smallest natural number divisible by any i, $1 \leq i \leq m-1$. It follows that with H one associates the 2^{s-1}-tuple $(k, k, \ldots, k)$ and by the previous method we have $s.f.g.(H) = s$.

7. Conclusions and Future Work

With any hypergroupoid H one may associate a numerical function that computes the fuzzy grade of H. In the case of a complete hypergroup constructed from a group G, its value does not depend on the group G, but only on the m-decomposition of the cardinality of H. In this paper, we have concentrated our study on the sequences of join spaces associated with finite complete hypergroups with particular decomposition.

Since Cristea and Davvaz [17] have defined the notion of Atanassov's intuitionistic fuzzy grade of a hypergroup, it is interesting to study it in the context of the complete hypergroups, doing a comparison between the two grades. This theme will be discussed in future works [3], [21], [22], [23].

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