A NEW SIMILARITY MEASURE BETWEEN TYPE-2 FUZZY NUMBERS AND FUZZY RISK ANALYSIS

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Abstract. In this paper, we present a revised similarity measure based on Chen-and-Chen's similarity measure for fuzzy risk analysis. The revised similarity measure uses the corrected formulae to calculate the centre of gravity points, therefore it is more effective than the Chen-and-Chen's method. The revised similarity measure can overcome the drawbacks of the existing methods. We have also proposed a new similarity measure between type-2 fuzzy numbers and a method for handling fuzzy risk analysis problem.

1. Introduction

The concept of a type-2 fuzzy set was introduced by Zadeh [19] as an extension of the concept of an ordinary fuzzy set. A type-2 fuzzy set is characterized by a fuzzy membership function, i.e., the membership value for each element of this set is a fuzzy set in $[0,1]$, unlike an ordinary fuzzy set where the membership value is a crisp number in $[0,1]$.

Type-2 fuzzy sets allow us to handle linguistic uncertainties (as typified by the adage "words can mean different things to different people [10]) as well as numerical uncertainties. A fuzzy relation of higher type has been regarded as one way to increase the fuzziness of a relation and, according to Hisdal [4] "increased fuzziness in a description means increased ability to handle inexact information in a logically correct manner". According to John [5] "Type-2 fuzzy sets allow for linguistic grades of membership, thus assisting in knowledge representation, and they also offer improvement on inferring with type-2 fuzzy sets". Type-2 fuzzy sets have already been used in a number of applications [7], including decision making, solving fuzzy relation equations, and pre-processing of data. The task of measuring the similarity between fuzzy numbers plays an important role in fuzzy decision making. Chen-and-Chen [1, 2] presented a similarity measure between fuzzy numbers and between interval valued fuzzy numbers based on geometric distance and centre of gravity. The above method has the drawbacks of getting incorrect results in some situations. We have presented a revised similarity measure based on the corrected centre of gravity formulae [12, 14, 18] for generalized fuzzy numbers, interval-valued fuzzy numbers,type-2 fuzzy numbers and apply these on fuzzy risk analysis. In section 2 definitions of the interval-valued fuzzy numbers and their

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arithmetic operations are given. In section 3 we have presented the revised similarity measure between trapezoidal fuzzy numbers. In section 4 we have presented the revised similarity measure between interval-valued fuzzy numbers. In sections 5 and 6 we have proposed a new similarity measure between type-2 fuzzy numbers and a risk analysis algorithm. In the last section conclusion is discussed.

2. Generalized Trapezoidal Fuzzy Number

The membership function $\mu_A(x)$ of the generalized fuzzy number $(a_1, a_2, a_3, a_4; w_A)$ can be expressed as

$$\mu_A(x) = \begin{cases} \mu_{AL}(x) & a_1 \le x \le a_2 \\ w_A & a_2 \le x \le a_3 \\ \mu_{AR}(x) & a_3 \le x \le a_4 \\ 0 & \text{otherwise} \end{cases}$$

where $\mu_{AL}(x) : [a_1,\ a_2] \to [0, w_A]$ and $\mu_{AR}(x) : [a_3,\ a_4] \to [0, w_A]$ is continuous and strictly increasing, and $\mu_{AR}(x) : [a_3,\ a_4] \to [0, w_A]$ is continuous and strictly decreasing. Since $\mu_{AL}^{-1} : [a_1, a_2] \to [0, w_A]$ is continuous and strictly increasing, the inverse function of μ_{AR} also exists. The inverse function of μ_{AL} and μ_{AR} can be denoted by μ_{AL}^{-1} and μ_{AR}^{-1} respectively. Since μ_{AL}^{-1} and μ_{AR}^{-1} are continuous on $[0, w_A]$, $\int_0^{w_A} \mu_{AL}^{-1}(y)dy$ and $\int_0^{w_A} \mu_{AR}^{-1}(y)dy$ exist.

2.1. **Arithmetic on Trapezoidal Fuzzy Numbers** [9, 11, 13, 15]**.** Let $A = (a_1,\ a_2,\ a_3,\ a_4;\ w_A)$ and $B = (b_1,\ b_2,\ b_3,\ b_4; w_B)$ be two generalized trapezoidal fuzzy numbers. Define

$$\begin{aligned} A &= (ka_1,\ ka_2,\ ka_3,\ ka_4; w_A); \quad \text{if } k > 0,\ k \text{ in } R \\ kA &= (ka_4, ka_3, ka_2, ka_1; w_B) \quad \text{if } k < 0, k \text{ in } R \\ A + B &= (a_1 + b_1,\ a_2 + b_2,\ a_3 + b_3,\ a_4 + b_4; \min\{w_A,\ w_B\}) \\ A - B &= (a_1 - b_4,\ a_2 - b_3,\ a_3 - b_2,\ a_4 - b_1; \min\{w_A,\ w_B\}) \\ A \times B &= (a_1b_1,\ a_2b_2,\ a_3b_3,\ a_4b_4;\ \min\{w_A,\ w_B\}) \\ A/B &= (a_1/b_4,\ a_2/b_3,\ a_3/b_2,\ a_4/b_1; \min\{w_A,\ w_B\}). \end{aligned}$$

2.2. **Centroid Point of a Fuzzy Number.** The centroid point $(x_0(A), y_0(A))$ of a fuzzy number A is defined by Chu and Tsao [3] as

$$x_0(A) = \frac{\int_{a_1}^{a_2} x\mu_{AL}(x)dx + \int_{a_2}^{a_3}(x)dx + \int_{a_3}^{a_4} x\mu_{AR}(x)dx}{\int_{a_1}^{a_2} \mu_{AL}(x)dx + \int_{a_2}^{a_3} dx + \int_{a_3}^{a_4} \mu_{AR}(x)dx} \tag{1}$$

$$y_0(A) = \frac{\int_0^{w_A} y\mu^{-1}{}_{AL}(y)dy + \int_0^{w_A} y\mu^{-1}{}_{AR}(y)dy}{\int_0^{w_A} \mu^{-1}{}_{AL}(y)dx + \int_0^{w_A} \mu^{-1}{}_{AR}(y)dy}. \tag{2}$$

In the case of a generalized trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4; w_A)$,

$$\mu_{AL}(x) = w_A(x - a_1)/(a_2 - a_1) \text{ and } \mu_{AR}(x) = w_A(a_4 - x)/(a_4 - a_3).$$

The inverse functions μ_{AL}^{-1} and μ_{AR}^{-1} can be analytically expressed as

$$\mu_{AL}^{-1}(y) = [a_1 + y(a_2 - a_1)/w_A] \quad \text{and} \quad \mu_{AR}^{-1}(y) = [a_4 + y(a_3 - a_4)/w_A].$$

The centroid formulae given by Chu and Tsao are incorrect since they do not satisfy the properties of a correct centroid formula have to poses. Therefore to avoid more misapplications spread in future Wang et al. proposed a correct formula which provides a very useful computational support to any ranking methods using centroid approaches. The correct centroid formula [5, 12, 14] should be as follows:

$$\begin{aligned} x_0(A) &= \frac{\int_{-\infty}^{\infty} x\mu_A(x)dx}{\int_{-\infty}^{\infty} \mu_A(x)dx} \\ &= \frac{\int_{a_1}^{a_2} x\mu_{AL}(x)dx + \int_{a_2}^{a_3} (xw_A)dx + \int_{a_3}^{a_4} x\mu_{AR}(x)dx}{\int_{a_1}^{a_2} \mu_{AL}(x)dx + \int_{a_2}^{a_3} w_A dx + \int_{a_3}^{a_4} \mu_{AR}(x)dx} \end{aligned} \tag{3}$$

$$y_0(A) = \frac{\int_0^{w_A} y\mu^{-1}_{AR}(y)dy - \int_0^{w_A} y\mu^{-1}_{AL}(y)dy}{\int_0^{w_A} \mu^{-1}_{AR}(y)dx - \int_0^{w_A} \mu^{-1}_{AL}(y)dy} \tag{4}$$

The main errors with the formula (2) is the sign and with the formula (1) is that w_A is missing which makes it wrong when $w_A \neq 1$. Consider a general trapezoidal fuzzy number $A = (a_1, a_2, a_3, a_4; w_A)$. For this trapezoidal fuzzy number the following results have been derived from formulae (1) and (2):

$$x_0(A) = \left\{ \frac{w_A(a_4^2 - 2a_3^2 + 2a_2^2 - a_1^2 + a_4a_3 - a_1a_2) + 3(a_3^2 - a_2^2)}{3w_A(a_4 - a_3 + a_2 - a_1) + 6(a_3 - a_2)} \right\} \tag{5}$$

$$y_0(A) = \frac{w_A}{3}\left[1 + \frac{(a_2 + a_3)}{(a_1 + a_2 + a_3 + a_4)}\right]. \tag{6}$$

By formulae (3) and (4) we derive the results below:

$$x_0(A) = \frac{1}{3}\left[a_1 + a_2 + a_3 + a_4 - \frac{(a_4a_3 - a_1a_2)}{(a_4 + a_3) - (a_1 + a_2)}\right] \tag{7}$$

$$y_0(A) = \frac{w_A}{3}\left[1 + \frac{(a_3 - a_2)}{(a_4 + a_3) - (a_1 + a_2)}\right]. \tag{8}$$

To justify formulae (3) and (4), we present in the following another derivatin from the point of view of analytical geometry.

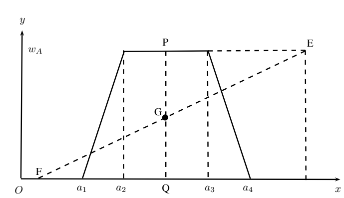

FIGURE 1. Computation of Centroid of a Trapezoid

2.3. **Geometrical Computation of Centroid of a Trapezoid.** Figure 1 shows the geometric graph of $A = (a_1,\ a_2,\ a_3,\ a_4; w_A)$ which is a trapezoid, and its extension used for determining the centroid of the trapezoid. After the extension, we have $P([a_2+a_3]/2, w_A)$, $Q([a_1+a_4]/2, 0)$, $E(a_3+a_4-a_1,\ w_A)$ and $F(a_1+a_2-a_3, 0)$. The two straight lines EF and PQ intersect at point G, which exactly represents the centre of gravity of the trapezoid A. In order to resolve the coordinates of point G, we write the equations of the two straight lines EF and PQ by using "two point" formula, as follows:

$$\text{EF} : y = [w_A(x - a_1 - a_2 + a_3)]/[(a_3 + a_4 - a_1) - (a_1 + a_2 - a_3)].$$

$$\text{PQ} : y = [w_A(a_1 + a_4 - 2x)]/[(a_1 + a_4) - (a_2 + a_3)].$$

Let $\dfrac{[w_A(x - a_1 - a_2 + a_3)]}{[(a_3 + a_4 - a_1) - (a_1 + a_2 - a_3)]} = \dfrac{[w_A(a_1 + a_4 - 2x)]}{[(a_1 + a_4) - (a_2 + a_3)]}$.
It follows that

$$x_0(A) = \frac{1}{3}\left[(a_1 + a_2 + a_3 + a_4) - \frac{(a_4a_3 - a_1a_2)}{(a_4 + a_3) - (a_1 + a_2)}\right]$$

Substituting $x = x_0(A)$ in the equation of PQ leads to the following result:

$$y_0(A) = \frac{w_A}{3}\left[1 + \frac{(a_3 - a_2)}{(a_4 + a_3) - (a_1 + a_2)}\right].$$

Define $x_0(A) = a_1$ and $y_0(A) = \frac{w_A}{2}$, if $a_1 = a_4$.

Geometrically the centroid of the trapezoid $(a_1,\ a_2,\ a_3,\ a_4; w_A)$ is given by the above equations are completely the same as equations (7) and (8).

3. **Interval-valued Fuzzy Numbers and Their Arithmetic Operations** [16]

Definition 3.1. *The interval-valued fuzzy numbers are defined by function of the form* $\mu_{\tilde{A}} : X \to \xi([0, 1])$ *where* $\xi([0, 1])$ *denotes the family of all closed intervals of the real numbers in* $[0, 1]$.

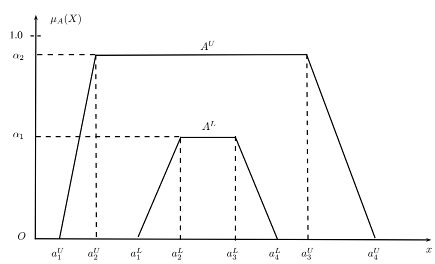

FIGURE 2. Interval-valued Trapezoidal Fuzzy Number $[A^L, A^U]$

An example of a membership function of this type is given in Figure 2. For each $x, \mu_{\tilde{A}}(x)$ is represented by the segment between the two curves, which express the identified lower and upper bounds. Thus, $\mu_{\tilde{A}}(a_2^L) = [\alpha_1, \alpha_2]$ for the example Figure 2.

An interval valued fuzzy number on R is given by $\tilde{A} = \{(x, [\mu_A^L(x), \mu_A^U(x)])\}$, $x \in R$ and $\mu_A^L(x) \leq \mu_A^U(x)$, for all $x \in R$. Denote $\tilde{A} = [A^L, A^U]$.

$$\text{Let} \quad \tilde{A} = [A^L, A^U]. = [(a_1^L, a_2^L, a_3^L, a_4^L; \lambda_A), (a_1^U, a_2^U, a_3^U, a_4^U; \rho_A)],$$
$$\text{and} \quad \tilde{B} = [B^L, B^U] = [(b_1^L, b_2^L, b_3^L, b_4^L; \lambda_B), (b_1^U, b_2^U, b_3^U, b_4^U; \rho_B)]$$

be two interval-valued generalized trapezoidal fuzzy numbers. Then define

$$\begin{aligned}
\tilde{A} + \tilde{B} &= \big[(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; \min\{\lambda_A, \lambda_B\}),\\
&\qquad (a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U; \min\{\rho_A, \rho_B\})\big]\\
\tilde{A} - \tilde{B} &= \big[(a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L; \min\{\lambda_A, \lambda_B\}),\\
&\qquad (a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U; \min\{\rho_A, \rho_B\})\big]\\
k\tilde{A} &= \big[(ka_1^L, ka_2^L, ka_3^L, ka_4^L; \lambda_A), (ka_1^U, ka_2^U, ka_3^U, ka_4^U; \rho_A)\big] \quad \text{if } k \geq 0 \text{ and } k \in R\\
k\tilde{A} &= \big[(ka_4^L, ka_3^L, ka_2^L, ka_1^L; \lambda_A), (ka_4^U, ka_3^U, ka_2^U, ka_1^U; \rho_A)\big] \quad \text{if } k < 0 \text{ and } k \in R\\
\tilde{A} \times \tilde{B} &= \big[(a_1^L b_1^L, a_2^L b_2^L, a_3^L b_3^L, a_4^L b_4^L; \min\{\lambda_A, \lambda_B\}),\\
&\qquad (a_1^U b_1^U, a_2^U b_2^U, a_3^U b_3^U, a_4^U b_4^U; \min\{\rho_A, \rho_B\})\big]\\
\tilde{A}/\tilde{B} &= \big[(a_1^L/b_4^L, a_2^L/b_3^L, a_3^L/b_2^L, a_4^L/b_1^L; \min\{\lambda_A, \lambda_B\}),\\
&\qquad (a_1^U/b_4^U, a_2^U/b_3^U, a_3^U/b_2^U, a_4^U/b_1^U; \min\{\rho_A, \rho_B\})\big]
\end{aligned}$$

where $0 \leq a_1^L \leq a_2^L \leq a_3^L \leq a_4^L \leq 1$, $0 \leq a_1^U \leq a_2^U \leq a_3^U \leq a_4^U \leq 1$, $0 \leq b_1^L \leq b_2^L \leq b_3^L \leq b_4^L \leq 1$, $0 \leq b_1^U \leq b_2^U \leq b_3^U \leq b_4^U \leq 1$.

4. **Revised Similarity Measure Between Generalized Trapezoidal Fuzzy Numbers**

Chen-and-Chen presented a similarity measure between generalized trapezoidal fuzzy numbers. It combined the concept of the geometric distance and the centre of gravity .The similarity measure $S(A, B)$ between the trapezoidal fuzzy numbers $A = (a_1, a_2, a_3, a_4; w_A)$ and $B = (b_1, b_2, b_3, b_4; w_B)$ is calculated as follows:

$$\begin{aligned}
S(A, B) &= \Big[1 - \Big(\sum |a_i - b_i|\Big)/4\Big]\\
&\qquad \times [1 - |x_A^* - x_B^*|]^{B(S(A),S(B))}[\{\min(y_A^*, y_B^*)\}/\{\max(y_A^*, y_B^*)\}]
\end{aligned}$$

where $i = 1, 2, 3, 4$; $S(A, B) \in [0, 1]$ and

$$B(S(A), S(B)) = \begin{cases} 1, & \text{if } S_A + S_B > 0,\\ 0, & \text{if } S_A + S_B = 0, \end{cases} \tag{9}$$

$$\begin{aligned}
&S_A = a_4 - a_1 \text{ and } S_B = b_4 - b_1,\\
&y_A^* = w_A(2a_4 + a_3 - a_2 - 2a_1)/6(a_4 - a_1) \quad \text{if } a_4 \neq a_1\\
&y_A^* = w_A/2 \quad \text{if } a_4 = a_1
\end{aligned} \tag{10}$$

$$\text{and } x_A^* = \{y_A^*(a_3 + a_2) + (a_4 + a_1)(w_A - y_A^*)\}/2w_A. \tag{11}$$

In the same way they defined x_B^* and y_B^*.

Since the centre of gravity formulae used by Chen-and-Chen are different from the corrected (8) and (9), the revised similarity measure is given by the following formula:

$$S(A,B) = \left[1 - \left(\sum |a_i - b_i|\right)/4\right][1 - |x_0(A) - x_0(B)|]^D$$
$$\times [\{\min(y_0(A), y_0(B)\}/\{\max(y_0(A), y_0(B)\}]$$

where $i = 1, 2, 3, 4$;

$$\begin{cases} D = 1, & \text{if } (a_4 - a_1) + (b_4 - b_1) > 0; \\ D = 0, & \text{if } (a_4 - a_1) + (b_4 - b_1) = 0 \text{ and} \end{cases}$$

$$x_0(A) = \frac{1}{3}\left[a_1 + a_2 + a_3 + a_4 - \frac{(a_4a_3 - a_1a_2)}{(a_4 + a_3) - (a_1 + a_2)}\right]$$
$$= a_1 \quad \text{if } a_1 = a_4$$
$$y_0(A) = \left(\frac{w_A}{3}\right)\left[1 + \frac{(a_3 - a_2)}{(a_4 + a_3) - (a_1 + a_2)}\right]$$
$$= \frac{w_A}{2} \quad \text{if } a_1 = a_4.$$

The larger the value of $S(A,B)$, the more the similarity between the fuzzy numbers A and B.

4.1. **A Comparison of the Similarity Measures.** In this segment, we compare fifteen sets of fuzzy numbers [17]. Figure 3 shows the fifteen sets of fuzzy numbers and Table 1 shows a comparison of calculation results of the revised method with the existing Chen-and-Chen's method.

	Set1	Set2	Set3	Set4	Set5	Set6	Set7	Set8
Chen	0.8357	1	0.42	0.49	0.8	1	0.81	0.54
Revised	0.7715	1	0.388	0.42	0.7998	1	0.9	0.765

	Set9	Set10	Set11	Set12	Set13	Set14	Set15
Chen	0.81	0.9	0.72	0.8325	0.81	0.7	0.9048
Revised	0.81	0.9	0.675	0.78	0.81	0.7	0.665

Table 1. A Comparison of the Calculated Results of the Revised Similarity Measure with the Chen-and-Chen's Method (Generalized Fuzzy Number)

From Set14 and Set15 of Figure 3, we can see that, the Set14 is more similar than Set15 by human intuition. However, from Table 1, we can see that if we apply Chen-and-Chen's method, then it gets an incorrect result. Therefore we can see that the revised method can overcome drawbacks of existing Chen-and-Chen's method.

5. Revised Similarity Measure Between Interval-valued Fuzzy Numbers

In this section we have presented the revised similarity measure between interval-valued trapezoidal fuzzy numbers based on the corrected centre of gravity formulae.

Let $\tilde{A} = [A^L, A^U] = [(a_1^L, a_2^L, a_3^L, a_4^L; \lambda_A), (a_1^U, a_2^U, a_3^U, a_4^U; \rho_A)]$,

and $\tilde{B} = [B^L, B^U] = [(b_1^L, b_2^L, b_3^L, b_4^L; \lambda_B), (b_1^U, b_2^U, b_3^U, b_4^U; \rho_B)]$

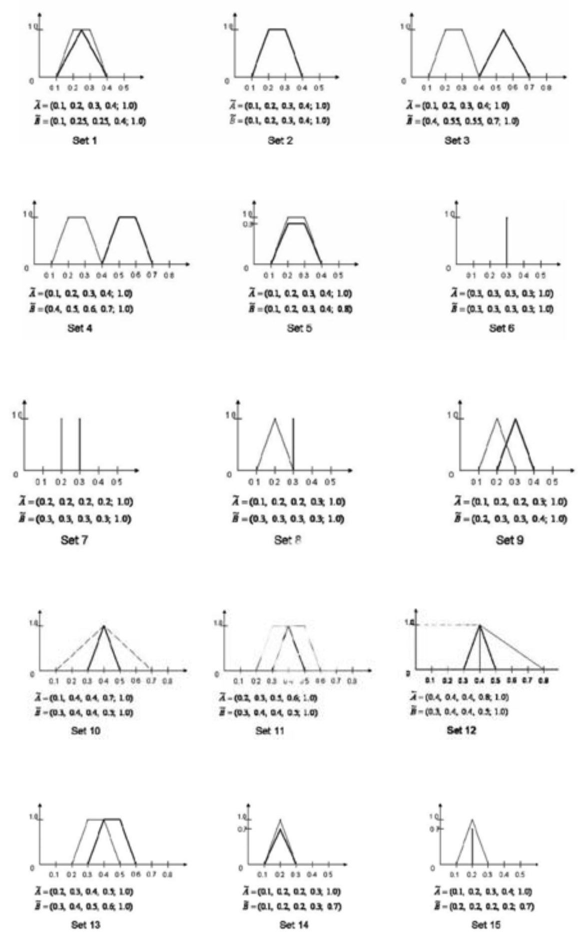

FIGURE 3. Fifteen Sets of Generalized Fuzzy Numbers

be two interval-valued generalized trapezoidal fuzzy numbers. The Chen-and-Chen's degree of similarity is

$S(\tilde{A},\tilde{B}) = [S(A^L,B^L)S(A^U,B^U)]^{1/2}$ where

$$S(A^L,B^L) = \left[1 - \left(\sum |a_i^L - b_i^L|\right)/4\right][1 - |x_A^{*L} - x_B^{*L}|]^{D(L)}$$
$$[\{\min(y_A^{*L}, y_B^{*L})\}/\{\max(y_A^{*L}, y_B^{*L})\}]$$
$$S(A^U,B^U) = \left[1 - \left(\sum |a_i^U - b_i^U|\right)/4\right][1 - |x_A^{*U} - x_B^{*U}|]^{D(U)}$$
$$[\{\min(y_A^{*U}, y_B^{*U})\}/\{\max(y_A^{*U}, y_B^{*U})\}]$$

$i = 1,2,3,4;\ x_A^{*L}, x_B^{*L}; x_A^{*U}$ and x_B^{*U} are given by the formulae (10) and (11), and

$$D(L) = \begin{cases} 1, & \text{if } (a_4^L - a_1^L) + (b_4^L - b_1^L) > 0; \\ 0, & \text{if } (a_4^L - a_1^L) + (b_4^L - b_1^L) = 0. \end{cases}$$

$$D(U) = \begin{cases} 1 & \text{if } (a_4^U - a_1^U) + (b_4^U - b_1^U) > 0; \\ 0, & \text{if } (a_4^U - a_1^U) + (b_4^U - b_1^U) = 0. \end{cases}$$

The revised formula for degree of similarity between $\tilde{A}$ and $\tilde{B}$ is

$$S(\tilde{A}, \tilde{B}) = [S(A^L, B^L) S(A^U, B^U)]^{1/2} \quad \text{where}$$

$$S(A^L, B^L) = \left[1 - \left(\sum |a_i^L - b_i^L|\right)/4\right] [1 - |x_0(A^L) - x_0(B^L)|]^{D(L)} \times [\min\{y_0(A^L), y_0(B^L)\}]/[\max\{y_0(A^L), y_0(B^L)\}], \quad i = 1, 2, 3, 4.$$

$$S(A^U, B^U) = \left[1 - \left(\sum |a_i^U - b_i^U|\right)/4\right] [1 - |x_0(A^U) - x_0(B^U)|]^{D(U)} \times [\min\{y_0(A^U), y_0(B^U)\}]/[\max\{y_0(A^U), y_0(B^U)\}], \quad i = 1, 2, 3, 4.$$

$x_0(A^L), x_0(B^L), x_0(A^U), x_0(B^U)$, $(y_0(A^L), y_0(B^L), y_0(A^U)$, and $y_0(B^U)$ are calculated from (8) and (9). And

$$D(L) = \begin{cases} 1, & \text{if } (a_4^L - a_1^L) + (b_4^L - b_1^L) > 0; \\ 0, & \text{if } (a_4^L - a_1^L) + (b_4^L - b_1^L) = 0. \end{cases}$$

$$D(U) = \begin{cases} 1, & \text{if } (a_4^U - a_1^U) + (b_4^U - b_1^U) > 0; \\ 0, & \text{if } (a_4^U - a_1^U) + (b_4^U - b_1^U) = 0. \end{cases}$$

The larger the value of $S(\tilde{A}, \tilde{B}) \in [0, 1]$, the more the similarity between the fuzzy numbers $\tilde{A}$ and $\tilde{B}$.

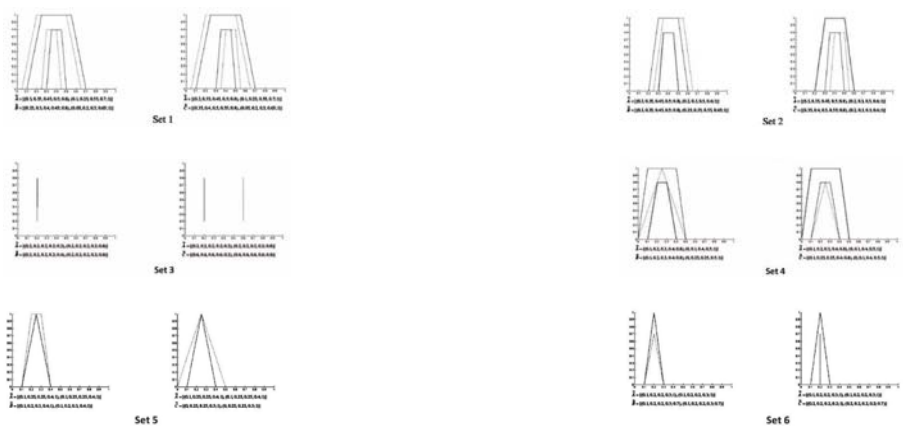

FIGURE 4. Six Sets of Interval-valued Fuzzy Numbers

5.1. **A Comparison of the Similarity Measures Between Interval-valued Fuzzy Numbers.** In this secton we use six sets of interval-valued trapezoidal fuzzy numbers [8] to compare the calculation results of the revised method with the Chen-and-Chen's method.

Set	Chen-and-Chen		Revised method	
	$S(\tilde{A},\tilde{B})$	$S(\tilde{A},\tilde{C})$	$S(\tilde{A},\tilde{B})$	$S(\tilde{A},\tilde{C})$
Set1	0.9025	0.9025	0.9025	0.74
Set2	0.95	0.95	0.8932	0.9550
Set3	0.7071	0.36	1	0.36
Set4	0.8435	0.9142	0.7867	0.8821
Set5	0.8357	0.95	0.7799	0.95
Set6	0.7	0.9048	0.7	0.7569

Table 2. Comparison of the Results of the Revised Method and the Chen-and-Chen's Method (Interval-valued Fuzzy Number)

In Set2 based on the revised method, from Table 2 we can see that $S(\tilde{A},\tilde{C})$ is larger than $S(\tilde{A},\tilde{B})$, which coincides with the human intuition. However, Chen-and-Chen‘s method does not get a reasonable result. Thus, we can see that the revised method can overcome drawbacks of Chen-and-Chen‘s method.

6. Fuzzy Risk Analysis Based on Similarity Measures of Type-2 Fuzzy Numbers

6.1. Type-2 Fuzzy Numbers.

Definition 6.1. [Zadeh] *A type-2 fuzzy set is a fuzzy set whose membership values are fuzzy sets on* $[0,1]$.

Definition 6.2. *Uncertainty in the primary membership of a type-2 fuzzy set,* $\tilde{A}$, *consists of a bounded region that we call the foot print of uncertainty (FOU). It is the union of all primary membership.*

Definition 6.3. *Let* $\tilde{A} = \bigcup_{\text{for all } \tilde{\alpha}} \tilde{\alpha}\mathrm{FOU}(\tilde{A}_{\tilde{\alpha}})$ *be a type-2 fuzzy subset of real numbers.*

Then, $\tilde{A}$ *is called a normal type-2 fuzzy number if* $\mathrm{FOU}(\tilde{A}_{\tilde{\alpha}=0})$ *is a normal interval valued fuzzy number, and* $\mathrm{FOU}(\tilde{A}_{\tilde{\alpha}=1}) = \mathrm{Pr}(\tilde{A})$ *exist.*

Definition 6.4. *A type-2 trapezoidal fuzzy number* $\tilde{A}$ *on* R *is given by* $\tilde{A} = \{x, (\mu_A^L(x), \mu_A^M(x), \mu_A^N(x), \mu_A^U(x))\}, x \in R$ *and* $\mu_A^L(x) \leq \mu_A^M(x) \leq \mu_A^N(x) \leq \mu_A^U(x)$, *for all* $x \in R$. *Denote* $\tilde{A} = (A^L, A^M, A^N, A^U)$, *where* $\tilde{A} = (A^L, A^M, A^N, A^U)$ *are the same type of fuzzy numbers.*

Definition 6.5. *If* $\tilde{A} = (A^L, A^M, A^N, A^U)$, *is called type-2 trapezoidal fuzzy number.*

Definition 6.6. *If* $A^M = A^N$ *then* $\tilde{A} = (A^L, A^M, A^U)$, *is called a type-2 triangular fuzzy number.*

Definition 6.7. *If* $A^M = A^N = A^U$, *then* $\tilde{A} = [A^L, A^U]$, *is called an interval-valued fuzzy number.*

Definition 6.8. *If* $A^L = A^M = A^N$, *then* $\tilde{A} = [A^L, A^U]$, *is called an interval-valued fuzzy number.*

Definition 6.9. *The set of elements that belong to the type-1 fuzzy set at least to the degree α is called the α-level set: $A_\alpha = \{x \in X \mid \mu_A(x) \geq \alpha\}$ where $\alpha \in [0,1]$.*

By the Decomposition Theorem [6] the fuzzy set A can be represented as

$$A = \bigcup_{\alpha \in [0,1]} \alpha A_\alpha.$$

The same analogy is used by Tahayori [16]: $\tilde{A}_{\tilde{\alpha}} = \{(x,u) \mid f_x(u) \geq \tilde{\alpha}\}$ and the T2FS can be represented as

$$\tilde{A} = \bigcup_{\tilde{\alpha} \in [0,1]} \tilde{\alpha} \tilde{A}_{\tilde{\alpha}}.$$

6.2. **Arithmetic Operation Between Type-2 Fuzzy Numbers** [13, 15].

$$\begin{aligned} \text{Let } \tilde{A} &= \bigcup_{\text{for all } \tilde{\alpha}} \tilde{\alpha}\mathrm{FOU}(\tilde{A}_{\tilde{\alpha}}) \\ &= (A^L, A^M, A^N, A^U) \\ &= \big((a_1^L, a_2^L, a_3^L, a_4^L; \lambda_A), (a_1^M, a_2^M, a_3^M, a_4^M), \\ &\quad (a_1^N, a_2^N, a_3^N, a_4^N), (a_1^U, a_2^U, a_3^U, a_4^U)\big) \\ \text{and } \tilde{B} &= \bigcup_{\text{for all } \tilde{\alpha}} \tilde{\alpha}\mathrm{FOU}(\tilde{B}_{\tilde{\alpha}}) \\ &= (B^L, B^M, B^N, B^U) \\ &= \big((b_1^L, b_2^L, b_3^L, b_4^L; \lambda_B), (b_1^M, b_2^M, b_3^M, b_4^M), \\ &\quad (b_1^N, b_2^N, b_3^N, b_4^N), (b_1^U, b_2^U, b_3^U, b_4^U)\big) \end{aligned}$$

be two type-2 trapezoidal fuzzy numbers. By using extension principle (Zadeh), we can define

$$\begin{aligned} \tilde{A} * \tilde{B} &= \left[\bigcup_{\text{for all } \tilde{\alpha}} \tilde{\alpha}\mathrm{FOU}(\tilde{A}_{\tilde{\alpha}})\right] * \left[\bigcup_{\text{for all } \tilde{\alpha}} \tilde{\alpha}\mathrm{FOU}(\tilde{B}_{\tilde{\alpha}})\right], \text{ where } * = \{+, -, \times, \div\} \\ &= \bigcup_{\text{for all } \tilde{\alpha}} \tilde{\alpha}[\mathrm{FOU}(\tilde{B}_{\tilde{\alpha}}) * \mathrm{FOU}(\tilde{B}_{\tilde{\alpha}})] \\ \tilde{A} * \tilde{B} &= (A^L * B^L, A^M * B^M, A^N * B^N, A^U * B^U) \end{aligned}$$

Then

$$\begin{aligned} \tilde{A} + \tilde{B} = \Big(&(a_1^L + b_1^L, a_2^L + b_2^L, a_3^L + b_3^L, a_4^L + b_4^L; \min\{\lambda_A, \lambda_B\}), \\ &(a_1^M + b_1^M, a_2^M + b_2^M, a_3^M + b_3^M, a_4^M + b_4^M), \\ &(a_1^N + b_1^N, a_2^N + b_2^N, a_3^N + b_3^N, a_4^N + b_4^N), \\ &(a_1^U + b_1^U, a_2^U + b_2^U, a_3^U + b_3^U, a_4^U + b_4^U)\Big) \\ \tilde{A} - \tilde{B} = \Big(&(a_1^L - b_4^L, a_2^L - b_3^L, a_3^L - b_2^L, a_4^L - b_1^L; \min\{\lambda_A, \lambda_B\}), \\ &(a_1^M - b_4^M, a_2^M - b_3^M, a_3^M - b_2^M, a_4^M - b_1^M), \\ &(a_1^N - b_4^N, a_2^N - b_3^N, a_3^N - b_2^N, a_4^N - b_1^N), \end{aligned}$$

$$
\begin{aligned}
&(a_1^U - b_4^U, a_2^U - b_3^U, a_3^U - b_2^U, a_4^U - b_1^U)\big) \\
k\tilde{A} &= \big((ka_1^L, ka_2^L, ka_3^L, ka_4^L; \lambda_A), (ka_1^M, ka_2^M, ka_3^M, ka_4^M), \\
&(ka_1^N, ka_2^N, ka_3^N, ka_4^N), (ka_1^U, ka_2^U, ka_3^U, ka_4^U)\big) \quad \text{if } k \geq 0 \text{ and } k \in R \\
k\tilde{A} &= \big((ka_4^L, ka_3^L, ka_2^L, ka_1^L; \lambda_A), (ka_4^M, ka_3^M, ka_2^M, ka_1^M), \\
&(ka_4^N, ka_3^N, ka_2^N, ka_1^N), (ka_4^U, ka_3^U, ka_2^U, ka_1^U)\big) \quad \text{if } k < 0 \text{ and } k \in R. \\
\tilde{A} \times \tilde{B} &= \Big((a_1^L b_1^L, a_2^L b_2^L, a_3^L b_3^L, a_4^L b_4^L; \min\{\lambda_A, \lambda_B\}), \\
&(a_1^M b_1^M, a_2^M b_2^M, a_3^M b_3^M, a_4^M b_4^M), \\
&(a_1^N b_1^N, a_2^N b_2^N, a_3^N b_3^N, a_4^N b_4^N), (a_1^U b_1^U, a_2^U b_2^U, a_3^U b_3^U, a_4^U b_4^U)\Big) \\
\tilde{A}/\tilde{B} &= \big(a_1^L/b_4^L, a_2^L/b_3^L, a_3^L/b_2^L, a_4^L/b_1^L; \min\{\lambda_A, \lambda_B\}, \\
&(a_1^M/b_4^M, a_2^M/b_3^M, a_3^M/b_2^M, a_4^M/b_1^M), \\
&(a_1^N/b_4^N, a_2^N/b_3^N, a_3^N/b_2^N, a_4^N/b_1^N), \\
&(a_1^U/b_4^U, a_2^U/b_3^U, a_3^U/b_2^U, a_4^U/b_1^U) \quad \text{or} \\
\tilde{A}/\tilde{B} &= \Big(\min(U^L), \min(U^L - x^L), \\
&\max(U^L - y^L), \max(U^L); \min\{\lambda_A, \lambda_B\}\big), \\
&\big(\min(U^M), \min(U^M - x^M), \max(U^M - y^M), \max(U^M)\big); \\
&\big(\min(U^N), \min(U^N - x^N), \max(U^N - y^N), \max(U^N)\big); \\
&\big(\min(U^U), \min(U^U - x^U), \max(U^U - y^U), \max(U^U)\big);
\end{aligned}
$$

where $U^L - x^L$ denotes, deleting the element x^L from the set U^L and

$$
\begin{aligned}
U^L &= \{a_1^L/b_1^L, a_2^L/b_2^L, a_3^L/b_3^L, a_4^L/b_4^L\}, \\
U^M &= \{a_1^M/b_1^M, a_2^M/b_2^M, a_3^M/b_3^M, a_4^M/b_4^M\}, \\
U^N &= \{a_1^N/b_1^N, a_2^N/b_2^N, a_3^N/b_3^N, a_4^N/b_4^N\}, \\
U^U &= \{a_1^U/b_1^U, a_2^U/b_2^U, a_3^U/b_3^U, a_4^U/b_4^U\},
\end{aligned}
$$

$$
\begin{aligned}
x^L &= \min(U^L), \quad x^M = \min(U^M), \\
x^N &= \min(U^N), \quad x^U = \min(U^U), \\
y^L &= \max(U^L), \quad y^M = \max(U^m), \\
y^N &= \max(U^N), \quad y^U = \max(U^U),
\end{aligned}
$$

where

$$
\begin{aligned}
&0 \leq a_1^L \leq a_2^L \leq a_3^L \leq a_4^L \leq 1, \qquad 0 \leq a_1^M \leq a_2^M \leq a_3^M \leq a_4^M \leq 1, \\
&0 \leq a_1^N \leq a_2^N \leq a_3^N \leq a_4^N \leq 1, \quad 0 \leq a_1^U \leq a_2^U \leq a_3^U \leq a_4^U \leq 1, \\
&0 \leq b_1^L \leq b_2^L \leq b_3^L \leq b_4^L \leq 1, \qquad 0 \leq b_1^M \leq b_2^M \leq b_3^M \leq b_4^M \leq 1,
\end{aligned}
$$

$$0 \le b_1^N \le b_2^N \le b_3^N \le b_4^N \le 1 \quad \text{and } 0 \le b_1^U \le b_2^U \le b_3^U \le b_4^U \le 1.$$

6.3. **Proposed Similarity Measure Between Type-2 Fuzzy Numbers.** Let $\tilde{A}$ and $\tilde{B}$ be two type-2 fuzzy numbers. Let us define the degree of similarity, $S(\tilde{A}, \tilde{B}) = \{S(A^L, B^L)S(A^M, B^M)S(A^N, B^N)S(A^U, B^U)^{1/4}\}$. The larger the value of $S(\tilde{A}, \tilde{B})$, the more the similarity between the fuzzy numbers $\tilde{A}$ and $\tilde{B}$.

6.4. **Some Properties of the Proposed Similarity Measures.**

Property 6.10. $S(\tilde{A}, \tilde{A}) = 1$.

Property 6.11. $S(\tilde{A}, \tilde{B}) = S(\tilde{B}, \tilde{A})$.

Proof. From the definition of $S(A^L, B^L)$ we can show that $S(A^M, B^M) = S(B^M, A^M)$; $S(A^N, B^N) = S(B^N, A^N)$ and $S(A^U, B^U) = S(B^U, A^U)$.
Therefore $S(\tilde{A}, \tilde{B}) = S(\tilde{B}, \tilde{A})$. □

Property 6.12. *$\tilde{A}$ and $\tilde{B}$ are identical if and only if*

$$S(\tilde{A}, \tilde{B}) = 1.$$

Proof. If $\tilde{A}$ and $\tilde{B}$ are identical, then A^L and B^L are identical.

$$\begin{aligned} \text{i.e. } a_1^L = b_1^L, \quad a_2^L = b_2^L, \quad a_3^L = b_3^L, \quad a_4^L = b_4^L, \\ \lambda_A = \lambda_B, \quad x_0(A^L) = x_0(B^L), \\ y_0(A^L) = y_0(B^L) \text{ and } \min(y_0(A^L), y_0(B^L)) \\ = \max(y_0(A^L), y_0(B^L)). \end{aligned}$$

Therefore

$$\begin{aligned} S(A^L, B^L) &= [1 - (\textstyle\sum |a_i^L - b_i^L|)/4][1 - |x_0(A^L) - x_0(B^L)|]^{D(L)} \\ &\quad [\{\min(y_0(A^L), y_0(B^L)\}/\{\max\{y_0(A^L), y_0(B^L)\}], \quad i = 1, 2, 3, 4 \\ &= [1 - 0][1 - 0]^{D(L)}[1]. \end{aligned}$$

i.e. $S(A^L, B^L) = 1$.

Similarly we can show that

$$\begin{aligned} S(A^M, B^M) = S(A^N, B^N) = S(A^U, B^U) = 1. \text{ Hence} \\ S(\tilde{A}, \tilde{B}) = \{S(A^L, B^L)S(A^M, B^M)S(A^N, B^N)S(A^U, B^U)\}^{1/4} \\ = \{1 \cdot 1 \cdot 1 \cdot 1 \cdot 1\}^{1/4} = 1. \end{aligned}$$

Converse:
If $S(\tilde{A}, \tilde{B}) = 1$, then $\{S(A^L, B^L)S(A^M, B^M)S(A^N, B^N)S(A^U, B^U)^{1/4}\} = 1$.
i.e. $S(A^L, B^L) = S(A^M, B^M) = S(A^N, B^N) = S(A^U, B^U) = 1$.
$[1 - (\sum |a_i^L - b_i^L|)/4][1 - |x_0(A^L) - x_0(B^L)|]^{D(L)}$
$\times[\{\min(y_0(A^L), y_0(B^L)\}/\{\max y_0(A^L), y_0(B^L)\}] = 1.\ i = 1, 2, 3, 4.$
i.e., $a_1^L = b_1^L$, $a_2^L = b_2^L$, $a_3^L = b_3^L$, $a_4^L = b_4^L$, $\lambda_A = \lambda_B$, $x_0(A^L) = x_0(B^L)$, $y_0(A^L) = y_0(B^L)$ and $\min(y_0(A^L), y_0(B^L)) = \max(y_0(A^L), y_0(B^L))$. Therefore A^L and B^L are identical. Similarly we can show that, A^M and B^M are identical; A^N and B^N are identical; A^U and B^U are identical.

Hence $\tilde{A}$ and $\tilde{B}$ are identical. □

Property 6.13. *If* $\tilde{A} = a$ *and* $\tilde{B} = b$ *are crisp numbers,* $0 \leq a,\ b \leq 1$*, then* $S(\tilde{A}, \tilde{B}) = 1 - |a - b|$.

Proof.

$$\tilde{A} = a = ((a, a, a, a; 1), (a, a, a, a; 1), (a, a, a, a; 1), (a, a, a, a; 1))$$

and

$$\tilde{B} = b = ((b, b, b, b; 1), (b, b, b, b; 1), (b, b, b, b; 1), (b, b, b, b; 1))$$

$$x_0(A^L) = a, \quad x_0(B^L) = b,$$

$$y_0(A^L) = 1/2, \quad y_0(B^L) = 1/2, \text{ and } D(L) = 0.$$

$$\begin{aligned} S(A^L, B^L) &= \left[1 - \left(\sum |a_i^L - b_i^L|\right)/4\right][1 - |x_0(A^L) - x_0(B^L)|]^{D(L)} \\ &\quad \times [\min(y_0(A^L), y_0(B^L)/\max y_0(A^L), y_0(B^L)] \\ &= [1 - (4|a - b|/4)][1 - |a - b|]^0[(1/2)/(1/2)]. \end{aligned}$$

Therefore

$$S(A^L, B^L) = 1 - |a - b|. \text{ Similarly we can show that}$$

$$S(A^M, B^M) = 1 - |a - b|,$$

$$S(A^N, B^N) = 1 - |a - b|,$$

$$S(A^U, B^U) = 1 - |a - b|.$$

Hence

$$S(\tilde{A}, \tilde{B}) = S(A^L, B^L)S(A^M, B^M)S(A^N, B^N)S(A^U, B^U)^1/4\} = 1 - |a - b|.$$ □

Numerical Example 6.14. *Let*

$$\begin{aligned} \tilde{A} &= ((.4, .5, .6, .65; .7), (.3, .5, .6, .7; .8), \\ &\qquad (.2, .5, .6, .8; .9), (.1, .5, .6, .9; 1)), \\ \tilde{B} &= ((.35, .4, .45, .5; .8), (.25, .4, .45, .55; .8), \\ &\qquad (.2, .4, .45, .6; 9), (.1, .4, .45, .7; 1)), \\ \tilde{C} &= ((.5, .55, .65, .7; .7), (.4, .55, .65, .75; .8), \\ &\qquad (.3, .55, .65, .8; .9), (.25, .55, .65, .85; 1)) \end{aligned}$$

be three type-2 fuzzy numbers. By using the proposed similarity measure, we have

$$\begin{aligned} S(A^L, B^L) &= 0.5398, \quad S(A^M, B^M) = 0.7519, \\ S(A^N, B^N) &= 0.7684, \quad S(A^U, B^U) = 0.7668, \\ S(A^L, C^L) &= 0.6523, \quad S(A^M, C^M) = 0.8605, \\ S(A^N, C^N) &= 0.8839, \quad S(A^U, C^U) = 0.8539. \\ S(A, B) &= 0.6993, \quad S(A, C) = 0.8067. \end{aligned}$$

We can see that $S(\tilde{A},\tilde{C})$ is larger than $S(\tilde{A},\tilde{B})$, which coincides with the human intuition.

Numerical Example 6.15. *Consider the structure of fuzzy risk analysis as shown in Figure 5 where the component A consists of three sub components A_1, A_2, A_3 and we want to evaluate the probability of failure $\tilde{R}$ of the component A. Table 4 shows the linguistic values $\tilde{R}_i$ and $\tilde{W}_i$ of the evaluating item "probability of failure" and "severity of loss" of the sub components A_1, A_2 and A_3 respectively, where the linguistic values are represented by type-2 trapezoidal fuzzy numbers as shown in Table 3. Let us consider the structure of fuzzy risk analysis as shown in Figure 5.*

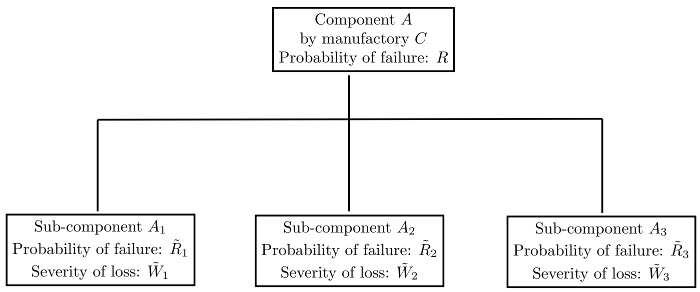

FIGURE 5. Structure of Fuzzy Risk Analysis

Linguistic Terms	Type-2 Fuzzy Numbers
Absolutely-low	((0,0,0,0; 0.7),(0,0,0,0; 0.8), (0,0,0,0; 0.9),(0,0,0,0; 1))
Very-low	((0,0,.02,.07; 0.7)(0,0,.02,.07; 0.8), (0, 0,.02,.07; 0.9),(0,0,.02,.07; 1))
Low	((.04,.1,.18,.23; 0.7),(.04,.1,.18,.23; 0.8), (.04,.1,.18,.23; 0.9),(.04,.1,.18,.23; 1))
Fairly-low	((.17,.22,.36,.42; 0.7),(.17,.22,.36,.42; 0.8), (.17,.22,.36,.42; 0.9),(.17,.22,.36,.42; 1))
Medium	((.32,.41,.58,.65; 0.7),(.32,.41,.58,.65; 0.8), (.32,.41,.58,.65; 0.9),(.32,.41,.58,.65; 1))
Fairly-high	((.58,.63,.80,.86; 0.7),(.58,.63,.80,.86; 0.8), (.58,.63,.80,.86; 0.9),(.58,.63,.80,.86; 1))
High	((.72,.78,.92,.97; 0.7),(.72,.78,.92,.97; 0.8), (.72,.78,.92,.97; 0.9),(.72,.78,.92,.97; 1))
Very-high	((.93,.98,1,1; 0.7),(.93,.98,1,1; 0.8), (.93,.98,1,1; 0.9),(.93,.98,1,1; 1))
Absolutely-high	((1,1,1,1; 0.7(1,1,1,1; 0.8), (1,1,1,1; 0.9),(1,1,1,1; 1))

TABLE 3. A Nine Member Interval Linguistic Term Set

Sub component	Linguistic Value $\tilde{W}_i$	Linguistic Value $\tilde{R}_i$
A_1	Very-low	Absolutely-high
A_2	low	Very-high
A_3	medium	Very-high

TABLE 4. Linguistic Values of the Evaluating Items $\tilde{R}_i$ and $\tilde{W}_i$ of the Three Sub Components A_1, A_2 and A_3 of the Numerical Example 6.15

Each sub-component A_i is evaluated by two evaluating items, "probability of failure" and "severity of loss", the linguistic term $\tilde{R}_i$ denotes the probability of failure of the sub component A_i and the linguistic term $\tilde{W}_i$ denotes the severity of loss of the sub-component A_i, $i = 1, 2, 3$. Each linguistic term in the nine member linguistic term set is corresponding to a type-2 trapezoidal fuzzy number, as shown in Table 3.

The proposed algorithm for dealing with fuzzy risk analysis is now presented as follows:

Step 1: Based on the type-2 fuzzy number arithmetic operations and the fuzzy weighted mean method, integrate the linguistic values $\tilde{R}_i$ and $\tilde{W}_i$ of each sub components A_i to get the total risk $\tilde{R}$ of the component A, which can be calculated as follows:

$$\tilde{R} = \left(\sum \tilde{W}_i \times \tilde{R}_i\right) / \left(\sum \tilde{W}_i + \alpha\right),$$

where α is a real value for avoiding division by zero. The closer the value of α to zero, the better the effect we get.

Step 2: Use the proposed similarity measure to evaluate the degree of similarity between the fuzzy number $\tilde{R}$ and each linguistic term shown in Table 4. The probability of failure of the component A is equal to the linguistic term with the largest degree of similarity with respect to $\tilde{R}$.

Set $\alpha = 0.01$ in formula (11). Based on (11), Tables 3 and 4, the probability of failure $\tilde{R}$ of the component A can be calculated as follows:

$$\begin{aligned}
\tilde{R} &= \{(\text{absolutely-high} \times \text{very-low}) + (\text{very-high} \times \text{low}) \\
&\qquad + (\text{very-high} \times \text{medium})\}/\{\text{very-low} + \text{low} + \text{medium} + \alpha\} \\
&= \big((.905, .926, .929, .99; .7), (.905, .926, .929, .99; .8), \\
&\qquad (.905, .926, .929, .99; .9), (.905, .926, .929, .99; 1)\big)
\end{aligned}$$

$$\begin{aligned}
&S(\tilde{R}, \text{absolutely-low}) \\
&= \big[S(\tilde{R}^L, \text{absolutely-low}^L\}) \times S(\tilde{R}^M, \text{absolutely-low}^M) \\
&\quad \times S(\tilde{R}^N, \text{absolutely-low}^N\}) \times S(\tilde{R}^U, \text{absolutely-low}^U)\big]^{1/4} \\
&= 0.0034.
\end{aligned}$$

In the same way, we can obtain the degrees of similarity between the type-2 trapezoidal fuzzy number $\tilde{R}$ and the other linguistic terms, as in Table 5.

Linguistic term X_i	$S(\tilde{R}_i, X_i)$
Absolutely-low	0.0034
Very-low	0.0052
Low	0.0330
Fairly-low	0.1061
Medium	0.2464
Fairly-high	0.4797
High	0.6381
Very-high	0.8277
Absolutely-high	0.8085

Table 5. The Degree of Similarity Between $\tilde{R}$ and Each Interval Linguistic Term

We can see that $S(\tilde{R}, \text{very-high}) = 0.8277$ has the largest value. It means that the probability of failure of the component A is very-high.

7. Conclusion

We have proposed a revised similarity measure between generalized fuzzy numbers and between interval-valued fuzzy numbers. This method combines the concept of geometric distance and corrected centre of gravity points. The revised method can overcome the drawbacks of the existing Chen-and-Chen's method. We have also presented a new similarity measure between type-2 fuzzy numbers and proved some properties of similarity measure between type-2 fuzzy numbers. We have also presented a fuzzy risk analysis algorithm based on the proposed similarity measure between type-2 fuzzy numbers.

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