EFFICIENCY IN FUZZY PRODUCTION POSSIBILITY SET

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Abstract. The existing Data Envelopment Analysis models for evaluating the relative efficiency of a set of decision making units by using various inputs to produce various outputs are limited to crisp data in crisp production possibility set. In this paper, first of all the production possibility set is extended to the fuzzy production possibility set by extension principle in constant return to scale, and then the fuzzy model of Charnes, Cooper and Rhodes in input oriented is proposed so that it satisfies the initial concepts with crisp data. Finally, the fuzzy model of Charnes, Cooper and Rhodes for evaluating decision making units is illustrated by solving two numerical examples.

1. Introduction

The Data Envelopment Analysis (DEA) was suggested by Charnes, Cooper and Rhodes (CCR) model [6], and built on the idea of Farrell [9] which is concerned with the estimation of technical efficiency and efficient frontiers. Several models were introduced for evaluating efficiency regarding to production possibility set (PPS), [6, 9, 4, 8]. The DEA requires input and output data to be precisely known. In some cases, we have to use imprecise input and output. To deal quantitatively with imprecision in decision progress, Bellman et al. [5] introduced the notion of fuzziness. Since the models basically is a fuzzy linear programming problem, there are several research about solving fuzzy linear programming problem, [15, 23, 13, 2]. Some researchers have proposed several fuzzy models to evaluate decision making units (DMUs) with fuzzy data, without access to fuzzy production possibility set (FPPS), [16, 11, 14, 18, 19, 24, 17, 12, 20]. S. Ramazanzadeh et al. [22] considered fuzzy random variables for inputs and outputs in DEA. Wang et al. [25] proposed two new fuzzy DEA models constructed from the perspective of fuzzy arithmetic to deal with fuzziness in input and output data in DEA. Wen et al. [26] presented a fuzzy DEA model based on credibility measure and also a method of ranking all the DMUs and to solve the fuzzy model, designed the hybrid algorithm combined with fuzzy simulation and genetic algorithm. Liu and Chuang [21] developed a fuzzy DEA/AR method based on Zadehs extension principle, a pair of two-level mathematical programs is formulated to calculate the lower and upper bounds of the fuzzy efficiency score. In this paper, DEA and PPS for crisp data as well as CCR model are reviewed in section 2. Fuzzy system, extension principle, fuzzy number and subsequently FPPS with its properties are introduced in section 3. The fuzzy CCR (FCCR) model is defined by triangular fuzzy numbers data on

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FPPS in section 4. Finally, two numerical examples for illustration of the model is considered in section 5 and conclusions are drawn in section 6.

2. **Data Envelopment Analysis (DEA)**

DEA utilizes a technique of mathematical programming to evaluate n DMUs as:

$$DMU_1, DMU_2, \ldots, DMU_n.$$

Let the input and output data for DMU_j be $X_j = (x_{1j}, ..., x_{mj}) \geq\neq 0$ and $Y_j = (y_{1j}, ..., y_{sj}) \geq\neq 0$, respectively, [6, 7].

2.1. **Production Possibility Set (PPS).**

We will call a pair of input $X \in R^m$ and output $Y \in R^s$ an activity and express them by the notation (X, Y). The set of feasible activities is called the PPS and is denoted by P with the following properties:

1: The observed activities $(X_j, Y_j) \in P,\ j = 1, .., n$.

2: If $(X, Y) \in P$, then $(tX, tY) \in P$ for all $t > 0$.

3: If $(X, Y) \in P$, $\overline{X} \geq X$ ($\overline{X}$ input) and $\overline{Y} \leq Y$ ($\overline{Y}$ output), then $(\overline{X}, \overline{Y}) \in P$.

4: If $(X, Y) \in P$ and $(X', Y') \in P$, then $(\lambda X + (1-\lambda)X', \lambda Y + (1-\lambda)Y') \in P$ for all $\lambda \in [0, 1]$(X' is an input and Y' is an output).

We show a set with the above properties as follows [7]:

$$P = \{(X, Y) \mid \overset{t}{X} \geq \sum_{j=1}^{n} \lambda_j \overset{t}{X}_j, \overset{t}{Y} \leq \sum_{j=1}^{n} \lambda_j \overset{t}{Y}_j, \lambda_j \geq 0, j = 1, ..., n\} \tag{1}$$

2.2. **The CCR Model.**

The CCR model proposed by Charnes et al. [6] is as follows:

$$\begin{array}{llcll} min & \theta & & & \\ s.t & \theta \overset{t}{X}_0 & \geq & \sum\limits_{j=1}^{n} \lambda_j \overset{t}{X}_j & \\ & \overset{t}{Y}_0 & \leq & \sum\limits_{j=1}^{n} \lambda_j \overset{t}{Y}_j & \\ & \lambda_j & \geq & 0 & j = 1, ..., n \end{array} \tag{2}$$

The construction of CCR model require the activity $(\theta X_0, Y_0) \in P$, when the objective is to seek the $min\ \theta$ that reduces the input vector X_0 radially to θX_0 while remaining in P. In CCR model, we are looking for an activity in P that guarantees at least the output level Y_0 of DMU_0 in all components, while reducing the input vector X_0 proportionally (radially) to a value as small as possible.

3. **Fuzzy Systems**

Let U be a nonempty set. A fuzzy set $\tilde{A}$ in U is characterized by its membership function $\mu_{\tilde{A}} : t \longrightarrow [0, 1]$ and $\mu_{\tilde{A}}(t)$ is interpreted as the degree of membership of element t in fuzzy set $\tilde{A}$ for each $t \in U$. A fuzzy set $\tilde{A}$ is completely determined by the set of tuples $\tilde{A} = \{(t, \mu_{\tilde{A}}(t)) \mid t \in U\}$, [5, 10, 27].

3.1. **Extension Principle.** Let U be a cartesian product of universes $U = U_1 \times U_2 \times ... \times U_r$ and $\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_r$ be r fuzzy sets in $U_1, U_2, ..., U_r$, respectively. f is a mapping from U to a universe W, $w = f(t_1, ..., t_r)$. Then the extension principle allows us to define a fuzzy set $\tilde{B}$ in W by:

$$\tilde{B} = \{(w, \mu_{\tilde{B}}(w)) \mid w = f(t_1, ..., t_r), (t_1, ..., t_r) \in U\} \tag{3}$$

where

$$\mu_{\tilde{B}}(w) = \begin{cases} \sup\limits_{(t_1,...,t_r)\in\ f^{-1}(w)} \min\{\mu_{\tilde{A}_1}(t_1), ..., \mu_{\tilde{A}_r}(t_r)\} & if \quad f^{-1}(w) \neq \emptyset \\ 0 & otherwise \end{cases} \tag{4}$$

f^{-1} is the inverse of f.

Remark 3.1. Let $U_1, U_2, ..., U_r \subseteq R$ and $\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_r$ be r fuzzy sets in $U_1, U_2, ..., U_r$, respectively, so that $\tilde{A}_i = \left\{(t_i, \mu_{\tilde{A}_i}(t_i)) \mid t_i \in U_i\right\}$, for $i = 1, ..., r$. Then

$$\tilde{R} = \{((t_1, ..., t_r), \mu_{\tilde{R}}(t_1, ..., t_r)) \mid (t_1, ..., t_r) \in U_1 \times U_2 \times ... \times U_r\} \tag{5}$$

is a fuzzy relation on $\tilde{A}_1, \tilde{A}_2, ..., \tilde{A}_r$ if:

$$\mu_{\tilde{R}}(t_1, ..., t_r) = \min\left\{\mu_{\tilde{A}_1}(t_1), ..., \mu_{\tilde{A}_r}(t_r)\right\} \tag{6}$$

3.2. **Fuzzy Number.** A fuzzy number $\tilde{A}$ is a fuzzy set of the real line with a normal, (fuzzy) convex and continuous membership function of bounded support.

Remark 3.2. Let $\tilde{A}$ be a fuzzy number. The $\alpha-$level set of $\tilde{A}$ is the set $[\tilde{A}]^\alpha$ as $[\tilde{A}]^\alpha = \{t \in R : \mu_{\tilde{A}}(t) \geq \alpha\}$.

Remark 3.3. Let$\tilde{A} = (a^l, a^m, a^r)$ be a triangular fuzzy number so that a^l, a^m, a^r are the left, the mean and the right values on $\tilde{A}$, respectively. If $\mu_{\tilde{A}}(t) = \alpha$, then

$$t = \begin{cases} \alpha a^m + (1-\alpha)a^l & if \quad a^l \leq t \leq a^m \\ \alpha a^m + (1-\alpha)a^r & if \quad a^m \leq t \leq a^r \end{cases} \tag{7}$$

3.3. **Comparison of Two Fuzzy Numbers.** Regarding this property in PPS, where "if $(X, Y) \in P$, then $(\overline{X}, \overline{Y}) \in P$ if $\overline{X} \geq X$ and $\overline{Y} \leq Y$" we need some ranking function about fuzzy numbers for comparison. It is evident that there are many different methods for comparison of fuzzy numbers to apply in PPS by fuzzy data. One of the easiest ranking function that satisfies in PPS properties is as follows. Adamo [1] used the concept of $\alpha-$level set to obtain an $\alpha-$preference index which is given by

$$F_\alpha(\tilde{A}_i) = max\{t \mid \mu_{\tilde{A}_i}(t) \geq \alpha\} \tag{8}$$

Now we define

$$G_\alpha(\tilde{A}_i) = min\{t \mid \mu_{\tilde{A}_i}(t) \geq \alpha\} \tag{9}$$

for a given threshold $\alpha \in [0,1]$ and then use the following ranking method for this task.

$$\tilde{A}_i \preceq \tilde{A}_j \Leftrightarrow \begin{cases} F_\alpha(\tilde{A}_i) \leq F_\alpha(\tilde{A}_j) \\ G_\alpha(\tilde{A}_i) \leq G_\alpha(\tilde{A}_j) \end{cases} \tag{10}$$

for all $\alpha \in [0,1]$.

4. Fuzzy Production Possibility Set and Fuzzy CCR Model

In this section, we are going to define the FPPS by using Zadeh's extension principle in constant return to scale. Also, the FCCR model is introduced in input oriented.

4.1. **FPPS.** Let the input and output data for DMU_j $(j = 1, ..., n)$ are $\tilde{X}_j = (\tilde{x}_{1j}, ..., \tilde{x}_{mj})$ and $\tilde{Y}_j = (\tilde{y}_{1j}, ..., \tilde{y}_{sj})$, respectively, where $\tilde{x}_{ij}$ $(i = 1, ..., m)$ and $\tilde{y}_{rj}$ $(r = 1, ..., s)$, for $j = 1, ..., n$ are $m + s$ fuzzy numbers. We call the set of feasible activities with fuzzy data FPPS and denote it by $\tilde{P}$.
We define the FPPS as follows:

$$\tilde{P} = \{((X,Y), \mu_{\tilde{P}}(X,Y)) \mid X \in R^m, Y \in R^s\} \tag{11}$$

Where

$$X = (x_1, ..., x_m)\ ,\ (x_i, \mu_{\tilde{x_i}}(x_i)) \in \tilde{x}_i\ i = 1, ..., m \tag{12}$$

$$Y = (y_1, ..., y_s)\ ,\ (y_r, \mu_{\tilde{y_r}}(y_r)) \in \tilde{y}_r\ r = 1, ..., s \tag{13}$$

and

$$\begin{aligned} \mu_{\tilde{P}}(X,Y)) = \quad & max \quad \min_{\lambda_j > 0} \ \left\{\mu_{\tilde{x_{1j}}}(x_{1j}), ..., \mu_{\tilde{x_{mj}}}(x_{mj}), \mu_{\tilde{y_{1j}}}(y_{1j}), ..., \mu_{\tilde{y_{sj}}}(y_{sj})\right\} \\ s.t \quad & \overset{t}{X} \geq \sum_{j=1}^{n} \lambda_j \overset{t}{X_j} \\ & \overset{t}{Y} \leq \sum_{j=1}^{n} \lambda_j \overset{t}{Y_j} \\ & \lambda_j \geq 0, j = 1, ..., n \end{aligned} \tag{14}$$

such as $(\tilde{X}_j, \tilde{Y}_j)$ $(j = 1, 2, ..., n)$ is considered to be activities.

We postulate that the claimed properties of P must be hold for $\tilde{P}^{\cdot}$

Claimed Properties :
1: The observed activities $(\tilde{X}_j, \tilde{Y}_j) \in \tilde{P}$, $(j = 1, 2, ..., n)$.

Proof. It is sufficient to show that $\mu_{\tilde{P}}(X_j, Y_j) > 0$, for any $((X_j, Y_j), \mu_{(\tilde{X}_j, \tilde{Y}_j)}(X_j, Y_j)) \in (\tilde{X}_j, \tilde{Y}_j)$, where $X_j = (x_{1j}, x_{2j}, ..., x_{mj})$ and

$Y_j = (y_{1j}, y_{2j}, ..., y_{sj})$.

$$\begin{array}{lll} \mu_{\tilde{P}}(X_j, Y_j) = & max \quad \underset{\lambda_k>0}{min} & \{\mu_{\tilde{x_{1k}}}(x_{1k}), ..., \mu_{\tilde{x_{mk}}}(x_{mk}), \mu_{\tilde{y_{1k}}}(y_{1k}), ..., \mu_{\tilde{y_{sk}}}(y_{sk})\} \\ s.t & & \overset{t}{X_j} \geq \sum_{k=1}^{n} \lambda_k \overset{t}{X_k} \\ & & \overset{t}{Y_j} \leq \sum_{k=1}^{n} \lambda_k \overset{t}{Y_k} \\ & & \lambda_k \geq 0, \ k = 1, ..., n \end{array}$$

$\mu_{\tilde{P}}(X_j, Y_j)$ is positive, because $(\tilde{X}_j, \tilde{Y}_j)$ is the observed activities and if $\lambda_k = 1$ $(j = k)$ and $\lambda_k = 0$ $(j \neq k)$, then $\lambda = (\lambda_1, ..., \lambda_n)$ is a feasible. □

2: If an activity$(\tilde{X}\ , \tilde{Y}) \in \tilde{P}$, then the activity $(t\tilde{X}, t\tilde{Y}) \in \tilde{P}$ for all $t > 0$.

Proof. It is sufficient to show that $\mu_{\tilde{P}}(tX, tY) > 0$ for any $((X, Y), \mu_{(\tilde{X},\tilde{Y})}(X, Y)) \in (\tilde{X}, \tilde{Y})$.

$$\begin{array}{lll} \mu_{\tilde{P}}(tX, tY)) = & max \quad \underset{\lambda_j>0}{min} & \left\{\mu_{\tilde{x_{1j}}}(x_{1j}), ..., \mu_{\tilde{x_{mj}}}(x_{mj}), \mu_{\tilde{y_{1j}}}(y_{1j}), ..., \mu_{\tilde{y_{sj}}}(y_{sj})\right\} \\ s.t. & & t\overset{t}{X} \geq \sum_{j=1}^{n} \lambda_j \overset{t}{X_j} \\ & & t\overset{t}{Y} \leq \sum_{j=1}^{n} \lambda_j \overset{t}{Y_j} \\ & & \lambda_j \geq 0, j = 1, ..., n \end{array}$$

$$\begin{array}{lll} \mu_{\tilde{P}}(X, Y)) = & max \quad \underset{\lambda_j>0}{min} & \left\{\mu_{\tilde{x_{1j}}}(x_{1j}), ..., \mu_{\tilde{x_{mj}}}(x_{mj}), \mu_{\tilde{y_{1j}}}(y_{1j}), ..., \mu_{\tilde{y_{sj}}}(y_{sj})\right\} \\ s.t & & \overset{t}{X} \geq \sum_{j=1}^{n} \lambda_j' \overset{t}{X_j} \\ & & \overset{t}{Y} \leq \sum_{j=1}^{n} \lambda_j' \overset{t}{Y_j} \\ & & \lambda_j' \geq 0, \lambda_j' = \frac{\lambda_j}{t}, j = 1, ..., n \end{array}$$

$\mu_{\tilde{P}}(tX, tY) > 0$ because $\lambda_j > 0$ if and only if $\lambda_j' > 0$. □

3: For any $(\tilde{X}, \tilde{Y}) \in \tilde{P}$, $(\tilde{\bar{X}}, \tilde{\bar{Y}}) \in \tilde{P}$ if $\tilde{\bar{X}} \succeq \tilde{X}$ and $\tilde{\bar{Y}} \preceq \tilde{Y}$.

Proof. It is sufficient to show that $\mu_{\tilde{P}}(\bar{X}, \bar{Y}) > 0$, for any $((\bar{X}, \bar{Y}), \mu_{(\tilde{\bar{X}},\tilde{\bar{Y}})}(\bar{X}, \bar{Y})) \in (\tilde{\bar{X}}, \tilde{\bar{Y}})$.

$$\begin{array}{lll} \mu_{\tilde{P}}(\bar{X}, \bar{Y})) = & max \quad \underset{\lambda_j>0}{min} & \left\{\mu_{\tilde{x_{1j}}}(x_{1j}), ..., \mu_{\tilde{x_{mj}}}(x_{mj}), \mu_{\tilde{y_{1j}}}(y_{1j}), ..., \mu_{\tilde{y_{sj}}}(y_{sj})\right\} \\ s.t & & \overset{t}{\bar{X}} \geq \sum_{j=1}^{n} \lambda_j \overset{t}{X_j} \\ & & \overset{t}{\bar{Y}} \leq \sum_{j=1}^{n} \lambda_j \overset{t}{Y_j} \\ & & \lambda_j \geq 0, j = 1, ..., n \end{array} \qquad (15)$$

But, regarding (10), for any $(\bar{X}, \bar{Y}) \in supp(\tilde{\bar{X}}, \tilde{\bar{Y}})$ there is $(X, Y) \in supp(\tilde{X}, \tilde{Y})$ such that $\bar{X} \geq X$ and $\bar{Y} \leq Y$ therefor with $\mu_{\tilde{P}}(X, Y) > 0$, where

$$\begin{array}{lll} \mu_{\tilde{P}}(X,Y)) = & max \ \min\limits_{\lambda_j>0} & \{\mu_{\tilde{x_{1j}}}(x_{1j}), ..., \mu_{\tilde{x_{mj}}}(x_{mj}), \mu_{\tilde{y_{1j}}}(y_{1j}), ..., \mu_{\tilde{y_{sj}}}(y_{sj})\} \\ s.t & & \overset{t}{X} \geq \sum\limits_{j=1}^{n} \lambda_j^{'} \overset{t}{X_j} \\ & & \overset{t}{Y} \leq \sum\limits_{j=1}^{n} \lambda_j^{'} \overset{t}{Y_j} \\ & & \lambda_j^{'} \geq 0, j = 1, ..., n \end{array} \qquad (16)$$

$\mu_{\tilde{P}}(\bar{X}, \bar{Y}) > 0$, because if $\lambda^{'} = (\lambda_1^{'}, ..., \lambda_n^{'})$ is one of the feasible solutions of (16), then $\lambda^{'} = (\lambda_1^{'}, ..., \lambda_n^{'})$ is a feasible solution of (15). □

4: If activities $(\tilde{X}\ , \tilde{Y})$, $(\tilde{X}^{'}\ , \tilde{Y}^{'}) \in \tilde{P}$, then $(\delta\tilde{X} + (1-\delta)\tilde{X}^{'}, \delta\tilde{Y} + (1-\delta)\tilde{Y}^{'}) \in \tilde{P}$ for any $\delta \in [0, 1]$.

Proof. It is sufficient to show that
$\mu_{\tilde{P}}(\delta X + (1-\delta)X^{'}, \delta Y + (1-\delta)Y^{'}) > 0$.

$$\begin{array}{lll} & & \mu_{\tilde{P}}(\delta X + (1-\delta)X^{'}, \delta Y + (1-\delta)Y^{'})) = \\ & max \ \min\limits_{\lambda_j>0} & \{\mu_{\tilde{x_{1j}}}(x_{1j}), ..., \mu_{\tilde{x_{mj}}}(x_{mj}), \mu_{\tilde{y_{1j}}}(y_{1j}), ..., \mu_{\tilde{y_{sj}}}(y_{sj})\} \\ s.t & & \delta\overset{t}{X} + (1-\delta)\overset{t}{X^{'}} \geq \sum\limits_{j=1}^{n} \lambda_j \overset{t}{X_j} \\ & & \delta\overset{t}{Y} + (1-\delta)\overset{t}{Y^{'}} \leq \sum\limits_{j=1}^{n} \lambda_j \overset{t}{Y_j} \\ & & \lambda_j \geq 0, j = 1, ..., n \end{array} \qquad (17)$$

$$\begin{array}{lll} \mu_{\tilde{P}}(X,Y)) = & max \ \min\limits_{\lambda_j>0} & \{\mu_{\tilde{x_{1j}}}(x_{1j}), ..., \mu_{\tilde{x_{mj}}}(x_{mj}), \mu_{\tilde{y_{1j}}}(y_{1j}), ..., \mu_{\tilde{y_{sj}}}(y_{sj})\} \\ s.t & & \overset{t}{X} \geq \sum\limits_{j=1}^{n} \lambda_j \overset{t}{X_j} \\ & & \overset{t}{Y} \leq \sum\limits_{j=1}^{n} \lambda_j \overset{t}{Y_j} \\ & & \lambda_j \geq 0, j = 1, ..., n \end{array} \qquad (18)$$

$$\begin{array}{lll} \mu_{\tilde{P}}(X^{'},Y^{'})) = & max \ \min\limits_{\lambda_j>0} & \{\mu_{\tilde{x_{1j}}}(x_{1j}), ..., \mu_{\tilde{x_{mj}}}(x_{mj}), \mu_{\tilde{y_{1j}}}(y_{1j}), ..., \mu_{\tilde{y_{sj}}}(y_{sj})\} \\ s.t & & \overset{t}{X^{'}} \geq \sum\limits_{j=1}^{n} \lambda_j \overset{t}{X_j} \\ & & \overset{t}{Y^{'}} \leq \sum\limits_{j=1}^{n} \lambda_j \overset{t}{Y_j} \\ & & \lambda_j \geq 0, j = 1, ..., n \end{array} \qquad (19)$$

Let $\overset{t}{X} \geq \sum_{j=1}^{n} \lambda_j \overset{t}{X}_j \quad \overset{t}{Y} \leq \sum_{j=1}^{n} \lambda_j \overset{t}{Y}_j$ and $\overset{t}{X}' \geq \sum_{j=1}^{n} \lambda_j' \overset{t}{X}_j$, $\overset{t}{Y}' \leq \sum_{j=1}^{n} \lambda_j' \overset{t}{Y}_j$, then

$$\delta \overset{t}{X} + (1-\delta)\overset{t}{X}' \geq \sum_{j=1}^{n} (\delta\lambda_j + (1-\delta)\lambda_j')\overset{t}{X}_j$$

similarly,

$$\delta \overset{t}{Y} + (1-\delta)\overset{t}{Y}' \leq \sum_{j=1}^{n} (\delta\lambda_j + (1-\delta)\lambda_j')\overset{t}{Y}_j$$

$\lambda_j + \lambda_j' > 0$ if and only if $\delta\lambda_j + (1-\delta)\lambda_j' > 0$. Hence, for any feasible $\lambda' = (\lambda_1', ..., \lambda_n')$ and $\lambda = (\lambda_1, ..., \lambda_n)$ from (18) and (19) then $\delta\lambda_j + (1-\delta)\lambda_j'$ is a feasible λ on (17), hence the proof is completed. □

4.2. **Fuzzy CCR Model (FCCR).** Let us consider the DMU_0. In CCR model with fuzzy data, the objective function seeks the minimum value of θ when the activity $(\theta\tilde{X}_0, \tilde{Y}_0)$ is belong to $\tilde{P}$.

While $(X_0, \mu_{\tilde{X}_0}(X_0)) \in \tilde{X}_0$, any input vector X_0 reduces radially to θX_0. Hence FCCR model proposes the following model:

$$\begin{array}{ccc} max & min & \theta \\ (\tilde{X}_0, \tilde{Y}_0) & s.t & \mu_{\tilde{P}}(\theta X_0, Y_0) > 0 \end{array} \tag{20}$$

But

$$\begin{array}{cccc} \mu_{\tilde{P}}(\theta X_0, Y_0) & & > & 0 \\ s.t & \theta \overset{t}{X}_0 & \geq & \sum_{j=1}^{n} \lambda_j \overset{t}{X}_j \\ & \overset{t}{Y}_0 & \leq & \sum_{j=1}^{n} \lambda_j \overset{t}{Y}_j \\ & \lambda_j & \geq & 0 \\ j = 1, ..., n & & & (X_j, Y_j) \in supp(\tilde{X}_j, \tilde{Y}_j) \end{array} \tag{21}$$

and FCCR can be extended as follows:

$$\begin{array}{cccccc} \theta^* = & max & min & \theta & & \\ & (\tilde{X}_0, \tilde{Y}_0) & s.t & \theta \overset{t}{X}_0 & \geq & \sum_{j=1}^{n} \lambda_j \overset{t}{X}_j \\ & & & \overset{t}{Y}_0 & \leq & \sum_{j=1}^{n} \lambda_j \overset{t}{Y}_j \\ & & & \lambda_j & \geq & 0 \\ & & j = 1, ..., n & & & (X_j, Y_j) \in supp(\tilde{X}_j, \tilde{Y}_j) \end{array} \tag{22}$$

We introduce the following LP model for triangular fuzzy data:

$$\begin{array}{ccc} min & \theta & \\ s.t & \theta \overset{t}{X}{}_0^l \geq & \sum_{j=1}^{n} \lambda_j \overset{t}{X}{}_j^l \\ & \theta \overset{t}{X}{}_0^m \geq & \sum_{j=1}^{n} \lambda_j \overset{t}{X}{}_j^m \\ & \theta \overset{t}{X}{}_0^r \geq & \sum_{j=1}^{n} \lambda_j \overset{t}{X}{}_j^r \\ & \overset{t}{Y}{}_0^l \leq & \sum_{j=1}^{n} \lambda_j \overset{t}{Y}{}_j^l \\ & \overset{t}{Y}{}_0^m \leq & \sum_{j=1}^{n} \lambda_j \overset{t}{Y}{}_j^m \\ & \overset{t}{Y}{}_0^r \leq & \sum_{j=1}^{n} \lambda_j \overset{t}{Y}{}_j^r \\ & 0 \leq & \lambda_j \\ & & j = 1, ..., n \end{array} \tag{23}$$

where $X_j^l, X_j^m, X_j^r, Y_j^l, Y_j^m$ and Y_j^r $(j = 1, ..., n)$ are the left, the mean and the right values of triangular fuzzy numbers $\tilde{X}_j$ and $\tilde{Y}_j$, respectively.

Theorem 4.1. *θ^* is an optimal solution of model (23), if and only if θ^* is an optimal solution of model (22).*

Proof. If θ^* is an optimal solution of model (22), then it is evident that θ^* is a feasible solution of model (23). Hence, it is sufficient to show that the optimal solution of model (23) is a feasible solution of model (22).
Let, $\lambda_j (j = 1, ..., n)$ and θ^* be the optimal solution of (23). For any $(X_0, Y_0) \in supp(\tilde{X}_0, \tilde{Y}_0)$ so that $X_0 = (x_{10}, ..., x_{m0}) \in [X_0^l, X_0^r]$ and $Y_0 = (y_{10}, ..., y_{s0}) \in [Y_0^l, Y_0^r]$ we can consider:

$$\overset{t}{X}{}_0^{\,l} = (x_{10}^l, ..., x_{m0}^l), \quad \overset{t}{X}{}_0^{\,m} = (x_{10}^m, ..., x_{m0}^m), \quad \overset{t}{X}{}_0^{\,r} = (x_{10}^r, ..., x_{m0}^r),$$

$$\exists \Gamma; \overset{t}{X}_0 = \Gamma \overset{t}{X}{}_0^m + (I_m - \Gamma) \overset{t}{X}{}_0^l \quad or \quad \overset{t}{X}_0 = \Gamma \overset{t}{X}{}_0^m + (I_m - \Gamma) \overset{t}{X}{}_0^r$$

so that

$$\Gamma = \begin{pmatrix} \gamma_1 & 0 & \dots & 0 \\ 0 & \gamma_2 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & \gamma_m \end{pmatrix}, \; I_m = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

$$\gamma_i \in (0, 1] \; for \; i = 1, ..., m \quad , \quad \mu_{\tilde{X}_0}(X_0) = \min\{\gamma_1, \dots, \gamma_m\}$$

also,

$$\overset{t}{Y}{}_0^{\,l} = (y_{10}^l, ..., y_{s0}^l), \quad \overset{t}{Y}{}_0^{\,m} = (y_{10}^m, ..., y_{s0}^m), \quad \overset{t}{Y}{}_0^{\,r} = (y_{10}^r, ..., y_{s0}^r),$$

$$\exists \Delta; \overset{t}{Y}_0 = \Delta \overset{t}{Y}{}_0^m + (I_s - \Delta) \overset{t}{Y}{}_0^l \quad or \quad \overset{t}{Y}_0 = \Delta \overset{t}{Y}{}_0^m + (I_s - \Delta) \overset{t}{Y}{}_0^r$$

so that

$$\Delta = \begin{pmatrix} \delta_1 & 0 & \dots & 0 \\ 0 & \delta_2 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & \delta_s \end{pmatrix}, \; I_s = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

$$\delta_r \in (0, 1] \; for \; r = 1, ..., s \quad , \quad \mu_{\tilde{Y}_0}(Y_0) = \min\{\delta_1, \dots, \delta_s\}$$

hence, $\mu_{\tilde{P}}(X_0, Y_0) = \min\{\gamma_1, \dots, \gamma_m, \delta_1, \dots, \delta_s\} > 0$.

Let

$$\overset{t}{X}_0 = \Gamma \overset{t}{X}{}_0^m + (I_m - \Gamma) \overset{t}{X}{}_0^l \quad , \quad \overset{t}{Y}_0 = \Delta \overset{t}{Y}{}_0^m + (I_s - \Delta) \overset{t}{Y}{}_0^l$$

Since

$$\theta^* \overset{t}{X}{}_0^l \geq \sum_{j=1}^{n} \lambda_j \overset{t}{X}{}_j^l \quad , \quad \theta^* \overset{t}{X}{}_0^m \geq \sum_{j=1}^{n} \lambda_j \overset{t}{X}{}_j^m$$
$$\overset{t}{Y}{}_0^l \leq \sum_{j=1}^{n} \lambda_j \overset{t}{Y}{}_j^l \quad , \quad \overset{t}{Y}{}_0^m \leq \sum_{j=1}^{n} \lambda_j \overset{t}{Y}{}_j^m$$

then

$$\theta^*(\Gamma \overset{t}{X}{}_0^m + (I_m - \Gamma)\overset{t}{X}{}_0^l) \geq \sum_{j=1}^{n} \lambda_j (\Gamma \overset{t}{X}{}_j^m + (I_m - \Gamma)\overset{t}{X}{}_j^l)$$

Similarly,

$$(\Delta \overset{t}{Y}{}_0^m + (I_s - \Delta)\overset{t}{Y}{}_0^l) \leq \sum_{j=1}^{n} \lambda_j (\Delta \overset{t}{Y}{}_j^m + (I_s - \Delta)\overset{t}{Y}{}_j^l)$$

Now, it is sufficient to show that:

$$(\Gamma \overset{t}{X}{}_j^m + (I_m - \Gamma)\overset{t}{X}{}_j^l, \Delta \overset{t}{Y}{}_j^m + (I_s - \Delta)\overset{t}{Y}{}_j^l) \in supp(\tilde{X}_j, \tilde{Y}_j) \ , \ j = 1, ..., n.$$

Since

$$\mu_{\tilde{X}_j}(\Gamma \overset{t}{X}{}_j^m + (I_m - \Gamma)\overset{t}{X}{}_j^l) = \min\{\gamma_1, \ldots, \gamma_m\}$$

and

$$\mu_{\tilde{Y}_j}(\Delta \overset{t}{Y}{}_j^m + (I_s - \Delta)\overset{t}{Y}{}_j^l) = \min\{\delta_1, \ldots, \delta_m\}$$

hence,

$$\mu_{\tilde{P}}(X_j, Y_j) = \min\{\gamma_1, \ldots, \gamma_m, \delta_1, \ldots, \delta_s\} > 0$$

□

The FCCR model (23) with crisp data and CCR model with exact data are the same. Moreover, the model (23) is feasible and the optimal value is $0 < \theta^* \leq 1$.

5. **Examples**

Here we present two examples to illustrate the proposed method.

Example 5.1. A simple example with fuzzy single input and single output was introduced by Guo and Tanaka [11]. G.R. Jahanshahloo et al. [14] considered this example for their method. We will consider this example with its data listed in Table 1.

In the method proposed by Guo et al. the fuzzy efficiencies of DMUs are triangular fuzzy numbers such as, $(\theta - \xi, \theta, \theta + \eta)$ where θ is mean and $\theta - \xi$ and $\theta + \eta$ are the left and right values, respectively. In their paper a DMU is called as an efficient DMU if $\xi + \eta \geq 1$. The obtained results with the Guo et al. method are listed in Table 2 where the ranking of these DMUs is as $B > D > C > A > E$.

For DMU_p Jahanshahloo et al. defined fuzzy profit as $(\theta - \Phi, \theta, \theta + \Psi)$, and DMU_p is named a fuzzy efficient (very efficient) if $|\theta| < \varepsilon_2$ $(\theta = 0)$ where ε_2 is user-specified as an acceptable tolerance. The results obtained by the method of Jahanshahloo et. al are listed in Table 3 where the ranking of these DMUs is as

$DMUs$	$input(left, mean, right)$	$output(left, mean, right)$
A	$(1.5, 2, 2.5)$	$(o.7, 1, 1.3)$
B	$(2.5, 3, 3.5)$	$(2.3, 3, 3.7)$
C	$(2.4, 3, 3.6)$	$(1.6, 2, 2.4)$
D	$(4, 5, 6)$	$(3, 4, 5)$
E	$(4.5, 5, 5.5)$	$(1.8, 2, 2.2)$

Table 1. The Triangular Fuzzy Numbers of Single

Input-Single Output

α	A (θ, ξ, η)	B (θ, ξ, η)	C (θ, ξ, η)	D (θ, ξ, η)	E (θ, ξ, η)
0	$(0.21, 0.47, 0.35)$	$(0.32, 0.95, 0.45)$	$(0.21, 0.63, 0.32)$	$(0.28, 0.76, 0.43)$	$(0.07, 0.38, 0.08)$
0.5	$(0.12, 0.49, 0.15)$	$(0.18, 0.97, 0.21)$	$(0.12, 0.65, 0.14)$	$(0.16, 0.78, 0.19)$	$(0.04, 0.39, 0.04)$
0.75	$(0.06, 0.49, 0.07)$	$(0.09, 0.98, 0.10)$	$(0.06, 0.66, 0.07)$	$(0.08, 0.79, 0.09)$	$(0.02, 0.39, 0.02)$
1	$(0.0, 0.5, 0.0)$	$(0.0, 1.0, 0.0)$	$(0.0, 0.67, 0.0)$	$(0.0, 0.8, 0.0)$	$(0.0, 0.4, 0.0)$

Table 2. Guo et al. Results

	A	B	C	D	E
θ	-0.45	0.0	-0.31	-0.20	-0.55

Table 3. Jahanshahloo et al. Results

	A	B	C	D	E
θ	0.5856	1.0	0.7246	0.8446	0.498

Table 4. Results of Proposed Model(FCCR)

$B > D > C > A > E$. If we use our proposed model (FCCR) for these DMUs, Table 4 shows the relative efficiency of all DMUs where the ranking of these DMUs is as $B > D > C > A > E$. For instance, FCCR model for DMU_A is reduced to the following model:

$$
\begin{array}{llcccccccccc}
min & \theta & & & & & & & & & & \\
s.t & 1.5\theta & \geq & 1.5\lambda_1 & + & 2.5\lambda_2 & + & 2.4\lambda_3 & + & 4\lambda_4 & + & 4.5\lambda_5 \\
 & 2\theta & \geq & 2\lambda_1 & + & 3\lambda_2 & + & 3\lambda_3 & + & 5\lambda_4 & + & 5\lambda_5 \\
 & 2.5\theta & \geq & 2.5\lambda_1 & + & 3.5\lambda_2 & + & 3.6\lambda_3 & + & 6\lambda_4 & + & 5.5\lambda_5 \\
 & 0.7 & \leq & 0.7\lambda_1 & + & 2.3\lambda_2 & + & 1.6\lambda_3 & + & 3\lambda_4 & + & 1.8\lambda_5 \\
 & 1 & \leq & 1\lambda_1 & + & 3\lambda_2 & + & 2\lambda_3 & + & 4\lambda_4 & + & 2\lambda_5 \\
 & 1.3 & \leq & 1.3\lambda_1 & + & 3.7\lambda_2 & + & 2.4\lambda_3 & + & 5\lambda_4 & + & 2.2\lambda_5 \\
 & 0 & \leq & \lambda_1, & & \lambda_2, & & \lambda_3, & & \lambda_4, & & \lambda_5
\end{array}
$$

Example 5.2. We evaluate 27 branches of Tehran Social Security Insurance Organization in this section. Each branch has three inputs in order to produce three outputs. The labels of inputs and outputs are presented in the table below.

The total triangular Fuzzy date can be seen in Tables 6, 7. "L" is considered as left number, "M" as middle number ,"U" as right number. We execute the model $FCCR$ on the 27 DMUs and show it in Table 8.

	Input	Output
1	The number of personals	The total number of insured persons
2	The total number of computers	The number of insured persons'agreements
3	The area of the branch	The total number of life-pension receivers

TABLE 5. The Labels of Inputs and Outputs

	$(L_{I_1}, M_{I_1}, U_{I_1})$	$(L_{I_2}, M_{I_2}, U_{I_2})$	$(L_{I_3}, M_{I_3}, U_{I_3})$
$DMU1$	(59153.05,73504.25,86523)	(84,86,87)	(3998.34,4000,4000)
$DMU2$	(36216,36582.75,37112)	(88,89.78,92)	(2565,2565,2565)
$DMU3$	(24566,25002.25,25369)	(85,87,89)	(1343,1344,1345)
$DMU4$	(36078,36684.75,37722)	(93,94,96)	(1499.7,1500,1500)
$DMU5$	(34734,36834.5,39445)	(83,83.44,87)	(1680,1681,1682)
$DMU6$	(58344,62611.63,73005)	(97,97,97)	(3748,3750,3750)
$DMU7$	(38849.02,41572.77,42573.02)	(90,90.67,92)	(3312.78,3313,3313)
$DMU8$	(51410,55949.63,63341)	(92,92,92)	(1500,1500,1500)
$DMU9$	(91930,95522.75,100220)	(84,89.56,92)	(1600,1601,1603.33)
$DMU10$	(52695,59080.63,61767)	(95,95.22,97)	(1725,1725,1725)
$DMU11$	(35985,40736.12,54521)	(78,78.33,79)	(1919.78,1920,1920)
$DMU12$	(26687,27300.63,27712)	(89,89.67,91)	(4430,4430,4430)
$DMU13$	(61716,63295,65026)	(103,106.33,111)	(2500,2500,2500)
$DMU14$	(93116,94969,96821)	(92,93,95)	(2800,2801,2802)
$DMU15$	(44305,50062,52856)	(92,93.44,95)	(1628.67,1630,1632)
$DMU16$	(41418,45926.25,48429)	(85,85,85)	(1127,1127,1127)
$DMU17$	(81588,82202.5,82923)	(104,104,104)	(3399,3400,3400)
$DMU18$	(72553,88782.5,98678)	(91,92.33,95)	(1304,1304,1304)
$DMU19$	(84905,87247.25,898844)	(95,96.11,98)	(4206,4206,4206)
$DMU20$	(26927,33196.5,37990)	(100,100.44,101)	(1340,1340,1343)
$DMU21$	(27744,28402.75,29058)	(88,89.33,90)	(1392.78,1393,1393)
$DMU22$	(107513,122897.88,129100)	(120,121.89,123)	(2191,2191,2191)
$DMU23$	(27704,32587.75,35194)	(100,100,100)	(2140,2140,2142.25)
$DMU24$	(60140,60866.38,61760)	(91,92,93)	(1231,1231,1231)
$DMU25$	(83940,86429.88,88038)	(90,91,92)	(1960,1960,1960)
$DMU26$	(58765,66803.13,69733)	(87,93.56,98)	(1603,1603,1603)
$DMU27$	(39142,40156.13,40860)	(81,82.33,86)	(2300,2300,2300)

TABLE 6. The Triangular Fuzzy Inputs for 27 Branches of Insurance Organization

Table 8 shows the relative efficiency of all DMUs. In this table $\theta^* \in (0, 1]$ is the value of relative efficiency and the greater, the better. If $\theta^* = 1$, then the corresponding DMU is efficient and if $\theta^* < 1$, then the DMU is inefficient. It means that $1 - \theta^*$ percentage of input in DMU can be reduced and the lesser, the worse because to produce output, input needs to have more potential and this is the concept of inefficiency. For instance, the DMU_1 θ^* is 0.8318, which means the mentioned DMU to produce the needed output has to reduce 17 percent of its input.

	$(L_{O_1}, M_{O_1}, U_{O_1})$	$(L_{O_2}, M_{O_2}, U_{O_2})$	$(L_{O_3}, M_{O_3}, U_{O_3})$
$DMU1$	(55830,57029.56,58487)	(30,40.89,58)	(1117,1269,1350)
$DMU2$	(36740,36872,37110)	(5,18.56,35)	(8385,8543.33,8776)
$DMU3$	(38004,38680,39449)	(11,20.22,32)	(6588,6594.78,6603)
$DMU4$	(35469,35933,36651)	(10,32.44,59)	(8083,10516.56,10821)
$DMU5$	(52927,54457.78,56082)	(9,30.11,44)	(9493,9684.67,9955)
$DMU6$	(70253.89,72277,78573.89)	(1,11.78,19)	(7536,8022,8752)
$DMU7$	(32585,36625,39539)	(47,101.22,129)	(13121,14513.33,15264)
$DMU8$	(42900,46360.33,50028)	(11,17.11,27)	(1563,1622.56,1661)
$DMU9$	(84531,86063.33,87858)	(43,71.44,111)	(10206,10645,11080)
$DMU10$	(46924,47242.11,47800)	(9,26.22,36)	(6608,6824.33,7472)
$DMU11$	(31554,38977.78,44298)	(81,184.11,242)	(11996,12226,12582)
$DMU12$	(27169,38214.89,39620)	(11,21.78,31)	(7422,7561.78,7731)
$DMU13$	(56144,58340.44,60545)	(30,45.22,77)	(7380,7584,7936)
$DMU14$	(80425,88472,91461)	(28,40,66)	(630,661,707)
$DMU15$	(50210,50499,50728)	(6,14.33,24)	(10247,10264.11,10293)
$DMU16$	(40166,47907.22,49855)	(15,25,37)	(7302,7491.56,7786)
$DMU17$	(52923,59579,82222)	(14,19.89,29)	(4740,4952.8,5205)
$DMU18$	(77340,83075.11,89111)	(13,23.44,33)	(4745,4917,5151)
$DMU19$	(46154,51026.56,86330)	(13,17.56,25)	(825,1528.33,1636)
$DMU20$	(27978,29658.11,33038)	(23,77.33,325)	(14473,14766.33,15125)
$DMU21$	(27128,27735,28297)	(0.1,19.22,36)	(921,940.67,973)
$DMU22$	(102175,102855,103641)	(31,47.56,77)	(252,2510.44,3577)
$DMU23$	(31819,34063.67,36205)	(12,23.11,32)	(1963,2110.89,2257)
$DMU24$	(51345,53731.33,56514)	(35,63.33,73)	(10157,10219.56,10344)
$DMU25$	(72915,75776,85431)	(40,53.89,96)	(4193,4480.33,4705)
$DMU26$	(71743,72552.67,74218)	(50,75.89,96)	(8762,12091.22,13015)
$DMU27$	(38054,38630.78,39065)	(13,24.89,39)	(1405,1460.56,1516)

Table 7. The Triangular Fuzzy Outputs for 27 Branches of Insurance Organization

	DMU_1	DMU_2	DMU_3	DMU_4	DMU_5	DMU_6	DMU_7	DMU_8
θ^*	0.8318	0.8180	1.0000	0.8877	1.0000	1.0000	1.0000	0.7485
	DMU_9	DMU_{10}	DMU_{11}	DMU_{12}	DMU_{13}	DMU_{14}	DMU_{15}	DMU_{16}
θ^*	1.0000	0.6963	1.0000	0.9875	0.8158	1.0000	0.9206	1.0000
	DMU_{17}	DMU_{18}	DMU_{19}	DMU_{20}	DMU_{21}	DMU_{22}	DMU_{23}	DMU_{24}
θ^*	1.0000	0.7978	0.9597	1.0000	0.6816	0.9664	0.8251	1.0000
	DMU_{25}	DMU_{26}	DMU_{27}					
θ^*	1.0000	1.0000	0.7451					

Table 8. The Efficiency for 27 Branches of Insurance Organization

It means that DMU_1 uses 83 percent of the input. Regarding this table, DMU_i for $i = 3, 5, 6, 7, 9, 11, 14, 16, 17, 20, 24, 25, 26$ are relative efficient and the others are inefficient.

6. **Conclusions**

In the real world, there are many problems which have fuzzy parameters such as DEA models for evaluating the relative efficiency of a set of DMUs by fuzzy data. Since the DEA models basically proposed by PPS, in this paper, the PPS with crisp data is extended to the FPPS by extension principle in constant return to scale with its properties, and subsequently the fuzzy CCR model on this FPPS were introduced so that it satisfies in the initial concepts with crisp data. In other methods the crisp model in DEA was considered with fuzzy parameters and the solution was a fuzzy number. But there was not any guaranty to the efficiency of the branch mark, non-negativity of efficiency and relative efficiency. One of the most important advantages of this work is constructing the FCCR by fuzzy principle in FPPS. We hope that the other topics of DEA with fuzzy data be used through using FPPS.

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