ROBUST STABILITY OF FUZZY MARKOV TYPE COHEN-GROSSBERG NEURAL NETWORKS BY DELAY DECOMPOSITION APPROACH

R. SATHY, P. BALASUBRAMANIAM AND R. CHANDRAN

ABSTRACT. In this paper, we investigate the delay-dependent robust stability of fuzzy Cohen-Grossberg neural networks with Markovian jumping parameter and mixed time varying delays by delay decomposition method. A new Lyapunov-Krasovskii functional (LKF) is constructed by nonuniformly dividing discrete delay interval into multiple subinterval, and choosing proper functionals with different weighting matrices corresponding to different subintervals in the LKFs. A new delay-dependent stability condition is derived with Markovian jumping parameters by T-S fuzzy model. Based on the linear matrix inequality (LMI) technique, maximum admissible upper bound (MAUB) for the discrete and distributed delays are calculated by the LMI Toolbox in MATLAB. Numerical examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

Cohen and Grossberg [3] have initially proposed and dealt Cohen-Grossberg neural networks (CGNN), since then it has attracted increasing interest of researchers due to their potential applications in many areas such as image processing, signal processing, pattern recognition, associative memory, parallel computation and nonlinear optimization problems. These applications are built upon the stability of the equilibrium point of neural networks. In hardware implementation of a CGNN using analog electronic circuits, time delay will be inevitable and will occur in the signal transmission among the neurons, which may lead to some complex dynamic behaviors. Therefore, the study of stability analysis of delayed CGNN is of both theoretical and practical importance see e.g., [1, 9, 14, 16, 26, 29, 30]. Since a neural network usually has a spatial nature due to the presence of an amount of parallel pathways of a variety of axon sizes and lengths, it is desired to model them by introducing distributed delays see e.g., [11, 12, 18, 19, 23].

Recently, increasing attention has been devoted to the stability analysis of the Markovian jumping systems (MJS). MJS is a suitable mathematical model to represent a class of dynamic systems due to random abrupt variation in their structures,

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and has many applications such as target tracking problems, manufactoring processes and fault-tolerant systems [4, 13, 22, 27]. Stability analysis of Markovian jumping CGNN with mixed time delays was discussed by the authors in [17, 31].

Fuzzy systems in the form of the Takagi-Sugeno (T-S) model have attracted rapidly growing interest in recent years [21]. T-S fuzzy systems are nonlinear systems described by a set of IF-THEN rules. Fuzzy logic theory has been effectively developed to many applications and shown to be an effective approach to model a complex nonlinear system and deal with its stability. It is worth noting that the synaptic transmission is a process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes in real nerves systems. Therefore, it is of practical importance to study the fuzzy effects on the stability property of delayed neural networks. The concept of incorporating fuzzy logic and fuzzy logic with MJS for a delayed neural networks was proposed in some papers see e.g. [2, 10, 15, 20, 25, 32]. The sliding mode control/observer problems with network-induced phenomena have been discussed in [6, 7, 8] by employing the delay-fractioning approach.

Based on the above discussions, we aim to determine a maximum admissible upper bound (MAUB) of the time delays such that delayed uncertain fuzzy Markovian type CGNN (UNFMJCGNN) is globally asymptotically stable in the mean square with uncertain switching probabilities. Besides the existing results on robust stability of fuzzy Markovian jumping neural networks, there is no result exist to study the fuzzy Markovian jumping Cohen-Grossberg neural networks (FMJCGNN) with uncertain switching probabilities. To exhibit the contributions of the proposed method, we have listed out the following important points:

1. For the first time, uncertain switching probabilities have been considered to analyze the robust stability of FMJCGNN with distributed delays.
2. Delay decomposition approach has been adopted to obtain the less conservative maximum admissible upper bounds (MAUB) of time delays.
3. The authors in [12] have discussed the robust stability of stochastic Cohen-Grossberg neural networks with discrete and distributed delays without considering fuzzy logic and Markovian jumping. In this paper, we proposed the generalized FMJCGNN with uncertain switching probabilities and compared the obtained results with the results and example of [12] as a special case by delay decomposition approach. It is found that the proposed method leads to less conservative results (see Table 3).
4. The proposed method can be applied efficiently to derive the stability conditions for genetic regulatory networks (GRN) [24], Hopfield neural networks (HNN) [15], Bidirectional associative memory neural networks (BAMNN) [14], etc.,

It is well known that delay-dependent stability conditions are usually less conservative than delay-independent ones and the less conservatism exist for large MAUB of time delay. In order to obtain some less conservative stability conditions, we employing delay decomposition method proposed by authors in [33] and decompose the whole discrete delay interval $[-\tau, 0]$ into N nonuniform multiple subintervals that is, $[-\tau, 0] = \bigcup_{j=1}^{N}[-\tau_j, -\tau_{j-1}]$. Length of the each subinterval is denoted by $\hat{\tau}_j = (\tau_j - \tau_{j-1})$. Construct a new LKF by choosing different weighting matrices on different subintervals in the LKFs. Upon the use of new LKF for the time-varying delay $\tau(t)$, a new delay- dependent stability condition is derived for uncertain CGNN with Markovian jumping parameters by T-S fuzzy model. Numerical examples are given to illustrate the effectiveness and less conservativeness of the proposed method.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and set of all $n \times n$ real matrices, respectively. The superscript T denotes the transpose of a matrix and I is the $n \times n$ identity matrix. The notation $*$ always denotes the symmetric block in one symmetric matrix.

2. Preliminaries

Consider the uncertain CGNN with discrete and distributed delays are of the form:

$$\begin{aligned} \dot{z}(t) &= -a(z(t))\Big[b(z(t)) - (A+\Delta A)g(z(t)) - (B+\Delta B)g(z(t-\tau(t))) \\ &\quad -(C+\Delta C)\int_{t-\sigma(t)}^{t} g(z(s))ds + J\Big], \\ z(s) &= \phi(s), \quad s \in [-\tau, 0], \end{aligned} \tag{1}$$

where $z(t) = [z_1(t), z_2(t), \cdots, z_n(t)]^T \in \mathbb{R}^n$ is the state vector associated with the n neurons, $a(z(t))$ is the amplification function, $b(z(t))$ is the behaved function and $g(\cdot)$ is the activation function. The matrices $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$ and $C = (c_{ij})_{n\times n}$ are respectively denote the connection weight matrix, the discretely delayed connection weight matrix and the distributively delayed connection weight matrix with the parametric uncertainties $\Delta A, \Delta B, \Delta C$. $\tau(t)$ is the discrete time-varying delay, $\sigma(t)$ is the distributed time-varying delay and $J = [J_1, J_2, \cdots, J_n]^T$ is a constant external input vector.

It is evident that bounded activation functions always guarantee the existence of an equilibrium point for CGNN (1). For convenience, we shift the equilibrium point $z^* = [z_1^*, z_2^*, \cdots, z_n^*]^T$ to the origin by the transformation $x(t) = z(t) - z^*$, which yields the following system:

$$\begin{aligned} \dot{x}(t) &= -\alpha(x(t))\Big[\beta(x(t)) - (A+\Delta A)f(x(t)) - (B+\Delta B)f(x(t-\tau(t))) \\ &\quad -(C+\Delta C)\int_{t-\sigma(t)}^{t} f(x(s))ds\Big], \end{aligned} \tag{2}$$

where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector of the transformed system, and $\alpha(x(t)) = diag(\alpha_1(x_1(t)), \cdots, \alpha_n(x_n(t)))$, $\alpha_\imath(x_\imath(t)) = a_\imath(x_\imath(t) + z_\imath^*)$, $\imath = (1, 2, \cdots, n)$, $\beta(x(t)) = [\beta_1(x_1(t)), \cdots, \beta_n(x_n(t))]^T$, $\beta_\imath(x_\imath(t)) = b_\imath(x_\imath(t) + z_\imath^*) - b_\imath(z_\imath^*)$, $f(x(\cdot)) = [f_1(x(\cdot)), f_2(x(\cdot)), \cdots, f_n(x(\cdot))]^T$, $f_\imath(x(\cdot)) = g_\imath(x_\imath(\cdot) + z_\imath^*) - g_\imath(z_\imath^*)$.

Let $\rho(t) = \rho_t (t \geq 0)$, be a right-continuous homogeneous Markov chain on the probability space taking values in a finite state space $S = \{1, 2, \cdots, s\}$ with generator $\tilde{\Pi} = \Pi + \Delta\Pi = (\pi_{kk'})_{s\times s} + (\Delta\pi_{kk'})_{s\times s}, \forall k, k' \in S$ given by

$$Pr\{\rho_{t+\delta} = k' | \rho_t = k\} = \begin{cases} (\pi_{kk'} + \Delta\pi_{kk'})\delta + o(\delta), & k \neq k', \\ 1 + (\pi_{kk} + \Delta\pi_{kk})\delta + o(\delta), & k = k', \end{cases}$$

$\delta > 0$ and $\lim_{\delta \to 0} \frac{o(\delta)}{\delta} = 0$. Here, $\pi_{kk'} \geq 0$ is the known transition rate from k to k', if $k \neq k'$ while $\pi_{kk} = -\sum_{k'=1,k'\neq k}^{s} \pi_{kk'}, \forall k, k' \in S$ and the uncertain transition rate $\Delta\pi_{kk'}$ satisfies $| \Delta\pi_{kk'} | \leq \delta_{kk'}$ and $\Delta\pi_{kk} = -\sum_{k'=1,k'\neq k}^{s} \Delta\pi_{kk'}, \forall k, k' \in S$.

In this paper, first we consider the uncertain Markovian jumping CGNN with discrete and distributed time-varying delays as follows:

$$\begin{aligned} \dot{x}(t) &= -\alpha(x(t),\rho(t))\Big[\beta(x(t),\rho(t)) - (A(\rho(t)) + \Delta A(\rho(t)))f(x(t)) \\ &\quad -(B(\rho(t)) + \Delta B(\rho(t)))f(x(t-\tau(t))) \\ &\quad -(C(\rho(t)) + \Delta C(\rho(t))) \int_{t-\sigma(t)}^{t} f(x(s))ds\Big]. \end{aligned} \tag{3}$$

We assume that the following conditions are true throughout this paper.
(A1) Each neuron activation function $f(\cdot)$ in (2) is bounded and satisfies

$$h_\imath^- \leq [f_\imath(x_1) - f_\imath(x_2)]/(x_1 - x_2) \leq h_\imath^+, \quad \forall x_1, x_2 \in \mathbb{R}, x_1 \neq x_2, \imath = 1, 2, \cdots, n$$

and $f_\imath(0) = 0$, where $h_\imath^+, h_\imath^-$ are constants. We denote $\tilde{\Sigma} = diag\{h_1^+, \cdots, h_n^+\}$, $\Sigma = diag\{h_1^-, \cdots, h_n^-\}$, and $\Sigma_1 = diag\{h_1^+ h_1^-, \cdots, h_n^+ h_n^-\}$, $\Sigma_2 = diag\{(h_1^+ + h_1^-)/2, \cdots, (h_n^+ + h_n^-)/2\}$.

(A2) The discrete time-varying delay $\tau(t)$ is continuous and differentiable function satisfying

$$0 < \tau(t) \leq \tau, \quad \dot{\tau}(t) \leq \mu, \quad \forall t \geq 0,$$

and the distributed delay $\sigma(t)$ is a continuous function satisfying

$$0 < \sigma(t) \leq \sigma, \quad \forall t \geq 0,$$

where τ, μ and σ are constants.

(A3) The amplification function and behaved function satisfy $0 < \underline{\alpha}_{\imath k} \leq \alpha_{\imath k}(\cdot) \leq \bar{\alpha}_{\imath k}$, $\underline{\alpha}_\imath = \min_{1\leq k\leq n}(\underline{\alpha}_{\imath k})$, $\bar{\alpha}_\imath = \max_{1\leq k\leq n}(\bar{\alpha}_{\imath k})$, $x_\imath(t)\beta_{\imath k}(x_\imath(t)) \geq \mu_{\imath k} x_\imath^2(t)$, $\mu_{\imath k} > 0$, $\Delta_k = diag(\mu_{k1}, \mu_{k2}, \cdots, \mu_{kn})$.

Remark 2.1. The constants $h_\imath^+, h_\imath^-$ in Assumption (A1) are allowed to be positive, negative or zero. Further, previously used Lipschitz conditions are just the special cases of Assumption (A1). Which implies that the results obtained in this paper are more general than the existing literatures.

The continuous fuzzy system was proposed to represent a nonlinear system [21]. The system dynamics can be captured by a set of fuzzy rules which characterize local correlation in the state space. Each local dynamic described by the fuzzy **IF-THEN** rule has the property of linear input-output relation. Based on the T-S fuzzy model, i^{th} rule of T-S fuzzy model uncertain Markovian jumping CGNN (3) with time-varying delays and distributed delays is of the following form:
Plant Rule i : IF $\theta_1(t)$ is η_1^i, $\cdots$, and $\theta_v(t)$ is η_v^i, **THEN**

$$\begin{aligned} \dot{x}(t) &= -\alpha_i(x(t),\rho(t))\Big[\beta_i(x(t),\rho(t)) - (A_i(\rho(t)) + \Delta A_i(\rho(t)))f(x(t)) \\ &\quad -(B_i(\rho(t)) + \Delta B_i(\rho(t)))f(x(t-\tau(t))) \\ &\quad -(C_i(\rho(t)) + \Delta C_i(\rho(t))) \int_{t-\sigma(t)}^{t} f(x(s))ds\Big], \end{aligned} \tag{4}$$

where $(\theta_1(t), \theta_2(t), \cdots, \theta_v(t))$ are the premise variables; $x(t)$ is the state variable, $(\eta_1^i, \eta_2^i, \cdots, \eta_v^i)$,$(i = 1, 2, \cdots, r)$ are the fuzzy sets with r is the number of **IF-THEN** rules. For each possible value of $\rho_t = k$, $k \in S$ in the succeeding discussion, we will denote the matrices associated with the i^{th} rule and k^{th} mode by

$$[A_i(\rho_t), B_i(\rho_t), C_i(\rho_t)] \quad = \quad [A_{(i,k)}, B_{(i,k)}, C_{(i,k)}], \tag{5}$$

$$\begin{aligned}[\Delta A_i(\rho_t), \Delta B_i(\rho_t), \Delta C_i(\rho_t)] &= [\Delta A_{(i,k)}, \Delta B_{(i,k)}, \Delta C_{(i,k)}] \\ &= E_{(i,k)}F_{(i,k)}(t)[H_{1(i,k)}, H_{2(i,k)}, H_{3(i,k)}],\end{aligned} \tag{6}$$

where the matrices in the right hand side of (5), $E_{(i,k)}$ and $H_{(1,2,3)(i,k)}$ are real known constant matrices of appropriate dimensions and the left hand side of (6) are unknown matrices that represents time-varying parameter uncertainties. The matrix $F_{(i,k)}(t)$ is the unknown time-varying matrix function satisfies

$$F^T_{(i,k)}(t)F_{(i,k)}(t) \leq I, \quad \forall k \in S. \tag{7}$$

The defuzzified output system (4) is inferred as follows:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{r} h_i(\theta(t))\Big\{ - \alpha_{(i,k)}(x(t))\Big[\beta_{(i,k)}(x(t)) - [(A_{(i,k)} + \Delta A_{(i,k)})]f(x(t)) \\ &\quad -[(B_{(i,k)} + \Delta B_{(i,k)})]f(x(t-\tau(t))) \\ &\quad -[(C_{(i,k)} + \Delta C_{(i,k)})]\int_{t-\sigma(t)}^{t} f(x(s))ds\Big]\Big\}, \\ &= -\alpha_\ell(x(t))\Big[\beta_\ell(x(t)) - \bar{A}_\ell f(x(t)) - \bar{B}_\ell f(x(t-\tau(t))) \\ &\quad -\bar{C}_\ell \int_{t-\sigma(t)}^{t} f(x(s))ds\Big],\end{aligned} \tag{8}$$

where $\ell = (i, k)$ notation is used for our convenience,

$$\begin{aligned}\alpha_\ell(x(t))\beta_\ell(x(t)) &= \sum_{i=1}^{r} h_i(\theta(t))\alpha_{(i,k)}(x(t))\beta_{(i,k)}(x(t)), \\ [\bar{A}_\ell, \bar{B}_\ell, \bar{C}_\ell] &= [(A_\ell(t) + \Delta A_\ell(t)), (B_\ell(t) + \Delta B_\ell(t)), \\ &\quad (C_\ell(t) + \Delta C_\ell(t))], \\ [A_\ell(t), B_\ell(t), C_\ell(t)] &= \sum_{i=1}^{r} h_i(\theta(t))[A_{(i,k)}, B_{(i,k)}, C_{(i,k)}], \\ [\Delta A_\ell(t), \Delta B_\ell(t), \Delta C_\ell(t)] &= \sum_{i=1}^{r} h_i(\theta(t))[\Delta A_{(i,k)}, \Delta B_{(i,k)}, \Delta C_{(i,k)}],\end{aligned}$$

$h_i(\theta(t)) = \mathcal{M}_i(\theta(t))/\sum_{i=1}^r \mathcal{M}_i(\theta(t)), \mathcal{M}_i(\theta(t)) = \prod_{q=1}^v \eta_q^i(\theta_q(t))$, and $\eta_q^i(\theta_q(t))$ is the grade of membership of $\theta_q(t)$ in η_q^i. It is assumed that $\mathcal{M}_i(\theta(t)) \geq 0$, where $i = 1, 2, \cdots, r$ and $\sum_{i=1}^r \mathcal{M}_i(\theta(t)) > 0$ for all t. Therefore, $h_i(\theta(t)) \geq 0$ for $i = 1, 2, \cdots, r$ and $\sum_{i=1}^r h_i(\theta(t)) = 1$.

Definition 2.2. For the UNFMJCGNN (8) and every $\phi \in C([-\tau, 0]; \mathbb{R}^n)$, $\rho(0) = k_0 \in S$, the equilibrium point is globally robustly asymptotically stable in the mean square, if for every network mode the following condition holds:

$$\lim_{t\to\infty} \mathbb{E}\|x(t; \phi, \tau(t))\|^2 = 0, \quad \forall t \geq 0,$$

for all parameter uncertainties satisfying (6) and (7) with mathematical expectation $\mathbb{E}$.

Now we state the following Lemmas which will be used to derive our main result.

Lemma 2.3. [5] *For any constant matrix $X \in \mathbb{R}^{n\times n}$, $X = X^T > 0$, scalar h with $0 \le \sigma(t) \le h$ and a vector-valued function $\dot{x} : [t-h, t] \to \mathbb{R}^n$, the following integration is well defined,*

$$-\sigma(t)\int_{t-\sigma(t)}^{t} \dot{x}^T(s)X\dot{x}(s)ds \quad \le \quad \begin{bmatrix} x(t) \\ x(t-\sigma(t)) \end{bmatrix}^T \begin{bmatrix} -X & X \\ * & -X \end{bmatrix} \begin{bmatrix} x(t) \\ x(t-\sigma(t)) \end{bmatrix}.$$

Lemma 2.4. *(Schur Complement) Given constant matrices N_1, N_2 and N_3 with appropriate dimensions, where $N_1^T = N_1$ and $N_2^T = N_2 > 0$, then $N_1 + N_3^T N_2^{-1} N_3 < 0$ if and only if $\begin{bmatrix} N_1 & N_3^T \\ * & -N_2 \end{bmatrix} < 0$.*

Lemma 2.5. [28] *Assume that Q, N and E are real matrices with appropriate dimensions and $F(t)$ is a matrix function satisfying $F^T(t)F(t) \le I$. Then $Q + EF(t)N + N^T F^T(t)E^T < 0$ holds if and only if there exists an $\epsilon > 0$ satisfying $Q + \epsilon^{-1}EE^T + \epsilon N^T N < 0$.*

3. Robust Stability Analysis

In this section, we will derive the global and robust asymptotic stability of UNFMJCGNN with mixed time-varying delays and uncertain switching probabilities by delay decomposition approach proposed by [33]. Let $N > 0$ be an integer and $\tau_j (j = 0, 1, 2, \cdots, N)$ be some scalars satisfying $0 = \tau_0 < \tau_1 < \tau_2 < \cdots < \tau_N = \tau$, then the discrete delay interval $[-\tau, 0]$ is nonuniformly decomposed into N multiple subintervals that is, $[-\tau, 0] = \bigcup_{j=1}^{N}[-\tau_j, -\tau_{j-1}]$. Length of the each subinterval is denoted by $\hat{\tau}_j = (\tau_j - \tau_{j-1})$. To avoid the confusion in our stability analysis, we are not decompose the distributed delay interval $[-\sigma, 0]$.

Theorem 3.1. *Let (A1)-(A3) holds and for a given $\tau > 0$ and $\sigma > 0$, the UNFMJCGNN (8) with uncertain switching probabilities is globally robustly asymptotically stable in the mean square if there exist symmetric positive definite matrices $P_k > 0, D > 0, Q_0 > 0, X_0 > 0, S_0 > 0, R_j > 0, \begin{bmatrix} Q_j & X_j \\ * & S_j \end{bmatrix} > 0, (j = 1, 2, \cdots, N)$ of appropriate dimensions, diagonal matrices $\Lambda \ge 0, \Delta_\ell \ge 0, U > 0, V > 0$, constant matrix $M > 0$, real number sets $\{\epsilon_\kappa > 0, \kappa = 1, 2, \cdots, N, \gamma_{kk'} > 0, \forall k, k' \in S, k \ne k'\}$ such that the following LMI condition holds for $i = (1, 2, \cdots, r)$ and $k = (1, 2, \cdots, s)$,*

$$\Sigma = \begin{bmatrix} \Omega^{(i,k)} & \check{\Gamma}_1 & \check{\Gamma}_2 & \check{\Gamma}_3 \\ * & -\epsilon_\kappa I & 0 & 0 \\ * & * & -\epsilon_\kappa I & 0 \\ * & * & * & -\Xi_k \end{bmatrix} < 0, \tag{9}$$

where

$$
\begin{aligned}
\Omega^{(i,k)} &= \begin{bmatrix} \Omega_{11}^{(i,k)}+\Omega_{11}^{\kappa} & \Omega_{12}^{(i,k)} \\ * & \Omega_{22}^{(i,k)} \end{bmatrix}, \\
\Omega_{11}^{(i,k)} &= \begin{bmatrix}
-2\Lambda\Delta_{(i,k)}+\varphi_1 & R_1 & 0 & \cdots & 0 & 0 & 0 & -\underline{\alpha}\Delta_{(i,k)}M^T+P_k \\
* & \varphi_2 & R_2 & \cdots & 0 & 0 & 0 & 0 \\
* & * & \varphi_3 & \cdots & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & \cdots & \varphi_N & R_N & 0 & 0 \\
* & * & * & \cdots & * & \varphi_{N+1} & 0 & 0 \\
* & * & * & \cdots & * & * & -V\Sigma_1-(1-\mu)Q_0 & 0 \\
* & * & * & \cdots & * & * & 0 & (-2M+\mathcal{R})/\bar{\alpha}^2
\end{bmatrix}, \\
\Omega_{12}^{(i,k)} &= \begin{bmatrix}
\Lambda A_{(i,k)}+\beta_1 & 0 & 0 & \cdots & 0 & 0 & \Lambda B_{(i,k)} & \Lambda C_{(i,k)} & 0 \\
0 & \beta_2 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \beta_3 & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & 0 \\
0 & 0 & 0 & \cdots & \beta_N & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & \beta_{N+1} & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & V\Sigma_2-(1-\mu)X_0 & 0 & 0 \\
MA_{(i,k)} & 0 & 0 & \cdots & 0 & 0 & MB_{(i,k)} & MC_{(i,k)} & 0
\end{bmatrix}, \\
\Omega_{22}^{(i,k)} &= \begin{bmatrix}
\sigma^2 D+\psi_1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
* & \psi_2 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 \\
* & * & \psi_3 & \cdots & 0 & 0 & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\
* & * & * & \cdots & \psi_N & 0 & 0 & 0 & 0 \\
* & * & * & \cdots & * & \psi_{N+1} & 0 & 0 & 0 \\
* & * & * & \cdots & * & * & -V-(1-\mu)S_0 & 0 & 0 \\
* & * & * & \cdots & * & * & * & -D & 0 \\
* & * & * & \cdots & * & * & * & * & -D
\end{bmatrix}, \\
\Omega_{11}^{\kappa} &= [\Psi_{ij}^{\kappa}]_{(N+3)\times(N+3)},
\end{aligned}
$$

with,

$$
\begin{aligned}
\Psi_{ij}^{\kappa} &= \begin{cases} -R_\kappa, & i=\kappa,\ j=\kappa+1, \\ R_\kappa, & i\in\{\kappa,\kappa+1\},\ j=N+2, \\ -2R_\kappa, & i=N+2,\ j=N+2, \\ 0, & \textit{otherwise}, \end{cases} \\
\varphi_j &= \begin{cases} Q_0+Q_1-R_1-U\Sigma_1 & \\ \qquad +\sum_{k'=1}^{s}\pi_{kk'}P_{k'}+\sum_{k'=1}^{s}\gamma_{kk'}\delta_{kk'}^2/4, & j=1, \\ Q_j-Q_{j-1}-R_j-R_{j-1}-U\Sigma_1, & j=2,\cdots,N, \\ -Q_N-R_N-U\Sigma_1, & j=N+1, \end{cases} \\
\beta_j &= \begin{cases} X_0+X_1+U\Sigma_2,\ j=1, \\ X_j-X_{j-1}+U\Sigma_2,\ j=2,\cdots,N, \\ -X_N+U\Sigma_2,\ j=N+1, \end{cases} \\
\psi_j &= \begin{cases} S_0+S_1-U,\ j=1, \\ S_j-S_{j-1}-U,\ j=2,\cdots,N, \\ -S_N-U,\ j=N+1, \end{cases} \\
\breve{\Gamma}_1 &= \begin{bmatrix} \Lambda E_{(i,k)} \\ 0_{(N+1)\times 1} \\ ME_{(i,k)} \\ 0_{(N+4)\times 1} \end{bmatrix},\ \breve{\Gamma}_2 = \begin{bmatrix} 0_{(N+3)\times 1} \\ \epsilon_\kappa H_{1(i,k)}^T \\ 0_{N\times 1} \\ \epsilon_\kappa H_{2(i,k)}^T \\ \epsilon_\kappa H_{3(i,k)}^T \end{bmatrix},\ \breve{\Gamma}_3 = \begin{bmatrix} \bar{\Gamma}_k \\ 0_{(N+6)\times 1} \end{bmatrix}, \\
\bar{\Gamma}_k &= [(P_k-P_1),(P_k-P_2),\cdots,(P_k-P_s)],\ \Lambda = diag\{\lambda_1,\lambda_2,\cdots,\lambda_n\}\geq 0, \\
\Delta_\ell &= diag\{\mu_{\ell 1},\mu_{\ell 2},\cdots,\mu_{\ell n}\}\geq 0,\ \mathcal{R} = \hat{\tau}_1^2 R_1+\hat{\tau}_2^2 R_2+\cdots+\hat{\tau}_N^2 R_N, \\
\Xi_k &= diag[\gamma_{k1}I,\gamma_{k2}I,\cdots,\gamma_{ks}I].
\end{aligned}
$$

Proof. Consider the LKFs as:

$$V(i,k,t,x(t)) = V_1(i,k,t,x(t)) + V_2(i,k,t,x(t)) + V_3(i,k,t,x(t)), \tag{10}$$

where,

$$\begin{aligned}
V_1(i,k,t,x(t)) &= x^T(t)P_k x(t) + \int_{t-\tau(t)}^{t} \xi^T(s) \begin{bmatrix} Q_0 & X_0 \\ * & S_0 \end{bmatrix} \xi(s)ds, \\
V_2(i,k,t,x(t)) &= 2\sum_{i=1}^{n} \lambda_i \int_0^{x_i(t)} s/\alpha_{ik}(s)ds \\
&\quad + \sigma \int_{-\sigma}^{0} \int_{t+\theta}^{t} f^T(x(s))Df(x(s))dsd\theta, \\
V_3(i,k,t,x(t)) &= \sum_{j=1}^{N} \int_{-\tau_j}^{-\tau_{j-1}} \xi^T(t+s) \begin{bmatrix} Q_j & X_j \\ * & S_j \end{bmatrix} \xi(t+s)ds \\
&\quad + \sum_{j=1}^{N} \hat{\tau}_j \int_{-\tau_j}^{-\tau_{j-1}} \int_{t+\theta}^{t} \dot{x}^T(s)R_j\dot{x}(s)dsd\theta,
\end{aligned}$$

where $\xi(s) = [x^T(s) \quad f^T(x(s))]^T$ and $\hat{\tau}_j = (\tau_j - \tau_{j-1})$.

Taking the derivative of $V(i,k,t,x(t))$ with respect to t along the trajectory of (8) yields

$$\begin{aligned}
\dot{V}_1 &= 2x^T(t)P_k\dot{x}(t) + \sum_{k'=1}^{s} (\pi_{kk'} + \Delta\pi_{kk'})[x^T(t)P_{k'}x(t)] + x^T(t)Q_0x(t) \\
&\quad -(1-\mu)x^T(t-\tau(t))Q_0x(t-\tau(t)) + 2x^T(t)X_0f(x(t)) \\
&\quad -2(1-\mu)x^T(t-\tau(t))X_0f(x(t-\tau(t))) + f^T(x(t))S_0f(x(t)) \\
&\quad -(1-\mu)f^T(x(t-\tau(t))S_0f(x(t-\tau(t)),
\end{aligned} \tag{11}$$

$$\begin{aligned}
\dot{V}_2 &= -2x^T(t)\Lambda\Delta_\ell x(t) + 2x^T(t)\Lambda\bar{A}_\ell f(x(t)) \\
&\quad +2x^T(t)\Lambda\bar{B}_\ell f(x(t-\tau(t))) + 2x^T(t)\Lambda\bar{C}_\ell \int_{t-\sigma(t)}^{t} f(x(s))ds \\
&\quad +\sigma^2 f^T(x(s))Df(x(s)) - \Big(\int_{t-\sigma(t)}^{t} f(x(s))ds\Big)^T D\Big(\int_{t-\sigma(t)}^{t} f(x(s))ds\Big) \\
&\quad -\Big(\int_{t-\sigma}^{t-\sigma(t)} f(x(s))ds\Big)^T D\Big(\int_{t-\sigma}^{t-\sigma(t)} f(x(s))ds\Big),
\end{aligned} \tag{12}$$

$$\begin{aligned}
\dot{V}_3 &= \sum_{j=1}^{N} \Big\{\xi^T(t-\tau_{j-1}) \begin{bmatrix} Q_j & X_j \\ * & S_j \end{bmatrix} \xi(t-\tau_{j-1}) - \xi^T(t-\tau_j) \begin{bmatrix} Q_j & X_j \\ * & S_j \end{bmatrix} \xi(t-\tau_j) \\
&\quad +\dot{x}^T(t)\hat{\tau}_j^2 R_j\dot{x}(t) - \hat{\tau}_j \int_{t-\tau_j}^{t-\tau_{j-1}} \dot{x}^T(s)R_j\dot{x}(s)ds\Big\}.
\end{aligned} \tag{13}$$

Noting $\Delta\pi_{kk} = -\sum_{k'=1,k'\neq k}^{s} \Delta\pi_{kk'}, k, k' \in S$, one has

$$\begin{aligned}
\sum_{k'=1}^{s} \Delta\pi_{kk'}P_{k'} &= \sum_{k'=1,k'\neq k}^{s} \Delta\pi_{kk'}(P_{k'} - P_k) \\
&\leq \sum_{k'=1,k'\neq k}^{s} [\gamma_{kk'}\delta_{kk'}^2/4) + (P_{k'} - P_k)^2/\gamma_{kk'}].
\end{aligned} \tag{14}$$

We now disclose the inter relationship between states $x^T(t), x^T(t-\tau_1), x^T(t-\tau_2), \cdots, x^T(t-\tau_N)$ and the state $x^T(t-\tau(t))$. Since $\tau(t)$ is a continuous function satisfying (A2) $\forall t \geq 0$, there should exists a positive integer $\kappa \in \{1, 2, \cdots, N\}$ such that $\tau(t) \in [\tau_{\kappa-1}, \tau_\kappa]$. In this situation,

$$\begin{aligned} -\hat{\tau}_\kappa \int_{t-\tau_\kappa}^{t-\tau_{\kappa-1}} \dot{x}^T(s) R_\kappa \dot{x}(s) ds &= -\tau_\kappa \int_{t-\tau_\kappa}^{t-\tau(t)} \dot{x}^T(s) R_\kappa \dot{x}(s) ds \\ &\quad -\tau_\kappa \int_{t-\tau(t)}^{t-\tau_{\kappa-1}} \dot{x}^T(s) R_\kappa \dot{x}(s) ds \\ &\leq -[\tau_\kappa - \tau(t)] \int_{t-\tau_\kappa}^{t-\tau(t)} \dot{x}^T(s) R_\kappa \dot{x}(s) ds \\ &\quad -[\tau(t) - \tau_{\kappa-1}] \int_{t-\tau(t)}^{t-\tau_{\kappa-1}} \dot{x}^T(s) R_\kappa \dot{x}(s) ds. \end{aligned} \tag{15}$$

Applying Lemma 2.3 to the last two integral terms in (15) respectively and after simple manipulations, we have

$$-\hat{\tau}_\kappa \int_{t-\tau_\kappa}^{t-\tau_{\kappa-1}} \dot{x}^T(s) R_\kappa \dot{x}(s) ds \leq \zeta^T(t) \begin{bmatrix} -R_\kappa & 0 & R_\kappa \\ * & -R_\kappa & R_\kappa \\ * & * & -2R_\kappa \end{bmatrix} \zeta(t), \tag{16}$$

where $\zeta(t) = [x^T(t-\tau_{\kappa-1}) \quad x^T(t-\tau_\kappa) \quad x^T(t-\tau(t))]^T$.

For $j \neq \kappa$, Lemma 2.3 yields the following inequalities:

$$\begin{aligned} -\hat{\tau}_j \int_{t-\tau_j}^{t-\tau_{j-1}} \dot{x}^T(s) R_j \dot{x}(s) ds &\leq \begin{bmatrix} x(t-\tau_{j-1}) \\ x(t-\tau_j) \end{bmatrix}^T \\ &\quad \times \begin{bmatrix} -R_j & R_j \\ * & -R_j \end{bmatrix} \begin{bmatrix} x(t-\tau_{j-1}) \\ x(t-\tau_j) \end{bmatrix}. \end{aligned} \tag{17}$$

For any diagonal matrices $U > 0,\ V > 0$ of appropriate dimensions, it follows from assumption (A1) that

$$\begin{aligned} 0 &\leq \sum_{j=0}^{N} \xi^T(t-\tau_j) \begin{bmatrix} -U\Sigma_1 & U\Sigma_2 \\ U\Sigma_2 & -U \end{bmatrix} \xi(t-\tau_j) \\ &\quad + \xi^T(t-\tau(t)) \begin{bmatrix} -V\Sigma_1 & V\Sigma_2 \\ V\Sigma_2 & -V \end{bmatrix} \xi(t-\tau(t)). \end{aligned} \tag{18}$$

For any constant matrix $M > 0$ of appropriate dimensions, it is easy to have

$$\begin{aligned} 0 &= 2\dot{x}^T(t) M \Big\{ -\dot{x}(t) - \alpha_\ell(x(t)) \Big[\beta_\ell(x(t)) - \bar{A}_\ell f(x(t)) - \bar{B}_\ell f(x(t-\tau(t))) \\ &\quad -\bar{C}_\ell \int_{t-\sigma(t)}^{t} f(x(s)) ds \Big] \Big\}. \end{aligned} \tag{19}$$

Since $h_i(\theta(t)) \geq 0,\ (i = 1, 2, \cdots, r)$ and $\sum_{i=1}^{r} h_i(\theta(t)) = 1$, combining (10)-(19), it can be derived that

$$\dot{V}(i, k, t, x(t)) \leq \sum_{i=1}^{r} h_i(\theta(t)) \Big\{ \begin{bmatrix} \Upsilon_1(t) \\ \Upsilon_2(t) \end{bmatrix}^T \Phi^{(i,k)} \begin{bmatrix} \Upsilon_1(t) \\ \Upsilon_2(t) \end{bmatrix} \Big\}, \tag{20}$$

where

$$\Phi^{(i,k)} = \begin{bmatrix} \tilde{\Omega}_{11}^{(i,k)} + \Omega_{11}^{\kappa} & \tilde{\Omega}_{12}^{(i,k)} \\ * & \Omega_{22}^{(i,k)} \end{bmatrix} + \Gamma_1 F_{(i,k)}(t)\Gamma_2 + \Gamma_2^T F_{(i,k)}^T(t)\Gamma_1^T,$$

$$\Gamma_1 = \begin{bmatrix} \Lambda E_{(i,k)} \\ 0_{(N+1)\times 1} \\ M\alpha_{(i,k)}(x(t))E_{(i,k)} \\ 0_{(N+4)\times 1} \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0_{(N+3)\times 1} \\ \epsilon_\kappa H_{1(i,k)}^T \\ 0_{N\times 1} \\ \epsilon_\kappa H_{2(i,k)}^T \\ \epsilon_\kappa H_{3(i,k)}^T \end{bmatrix},$$

$$\Upsilon_1(t) = \begin{bmatrix} x(t) \\ x(t-\tau_1) \\ x(t-\tau_2) \\ \vdots \\ x(t-\tau_N) \\ x(t-\tau(t)) \\ \dot{x}^T(t) \end{bmatrix}, \quad \Upsilon_2(t) = \begin{bmatrix} f(x(t)) \\ f(x(t-\tau_1) \\ f(x(t-\tau_2) \\ \vdots \\ f(x(t-\tau_N) \\ f(x(t-\tau(t)) \\ \int_{t-\sigma(t)}^{t} f(x(s))ds \\ \int_{t-\sigma}^{t-\sigma(t)} f(x(s))ds \end{bmatrix},$$

$\tilde{\Omega}_{11}^{(i,k)} = \Omega_{11}^{(i,k)}$, if φ_1 is replaced by $\varphi_1 + \sum_{k'=1,k'\neq k}^{s}(P_{k'} - P_k)^2/\gamma_{kk'}$ in the diagonal element (1,1) and $(-2M + \mathcal{R})/\bar{\alpha}^2$ is replaced by $(-2M + \mathcal{R})$ in the diagonal element $(N+3, N+3)$, $\tilde{\Omega}_{12}^{(i,k)} = \Omega_{12}^{(i,k)}$, if $M(A_{(i,k)}, B_{(i,k)}, C_{(i,k)})$ is replaced by $M\alpha_{(i,k)}(x(t))(A_{(i,k)}, B_{(i,k)}, C_{(i,k)})$ in the $(N+3)^{th}$ row. We know that the sufficient condition for global and robust asymptotic stability of UNFMJCGNN (8) in the mean square (by Definition 2.2) is that there exists a scalar $\epsilon_\kappa > 0$ such that $\Phi^{(i,k)} < 0$. Since, the amplification function $\alpha_{(i,k)}(x(t))$ is nonlinear and satisfies $\alpha_{(i,k)}(x(t))\alpha_{(i,k)}(x(t)) \leq \bar{\alpha}^2 I$, pre-and post- multiplying the left-hand side of inequality $\Phi^{(i,k)} < 0$ by $diag(I,\ I,\ \cdots, I, \alpha_{(i,k)}(x(t))_{((N+3),(N+3))}, I, \cdots, I)$ and using the Lemmas 2.4 and 2.5, we get the LMI condition $\Omega^{(i,k)} + \epsilon_\kappa^{-1}\Gamma_1\Gamma_1^T + \epsilon_\kappa \Gamma_2^T\Gamma_2 < 0$. This is equivalent to (9) (i.e., $\Sigma < 0$). Considering all possibilities of κ in the set $\kappa \in \{1, 2, \cdots, N\}$, we arrive at the condition that (9) holds for any κ. This completes the proof. □

Remark 3.2. It is worth noting that the Theorem 3.1 holds for any time-varying delay functions $\tau(t)$ and $\sigma(t)$ for UNFMJCGNN (8). Theorem 3.1 does not hold for the case of time varying delay $\tau(t)$ is constant (i.e., the case $\mu = 0$ is not applicable to Theorem 3.1).

Remark 3.3. The case $\mu = 0$ is a particular case of Theorem 3.1 by replacing $\tau(t) = \tau$ in UNFMJCGNN (8) and in the LKF (10). Proof of this case follows from Theorem 3.1 and is omitted here for considering the length of the paper.

Remark 3.4. When $r = 1$ and $(\Delta A_{(i,k)}, \Delta B_{(i,k)}, \Delta C_{(i,k)}, \Delta\Pi) = 0$, then the system (8) without Markovian jumping is reduced to the CGNN equation (44) of [12] (that is, (22) in Example 4.2 of this paper) with mixed delays. Thus our results makes another novel criterion on CGNNs. The obtained less conservative results are compared with the results of Example 1 in [12]. To the best of our knowledge, it is the first time to investigate the global and robust stability of uncertain fuzzy Markov type CGNN with uncertain switching probabilities and mixed time-varying delays by delay-decomposition method. It is worth noting that the less conservative results exist for $N \geq 3$ by delay decomposition approach (see Table 3). Thus our results make another new stability criterion on CGNN.

Remark 3.5. From Table 3, it is seen that the obtained results in the case of $N = 2$ is very close to the existing result of [12] and the proposed method of this paper with delay decomposition approach significantly improves the results for the case $N = 3$. Hence it is worth noting that, if N increases in our analysis, number of variables in the LMIs are automatically increasing and the elapsed time for determining MAUB of time delay for Theorem 3.1 is much time-consuming. Hence it is well known that the result in the case of $N = 2$ is sufficient for most practical applications.

4. **Illustrative Examples**

In this section, we illustrate the effectiveness of our results with the two modes of Markovian jumping parameter and $r = 2$ for the system UNFMJCGNN (8).

Example 4.1. Consider the UFMJCGNN (8) with two modes (k=1,2) T-S fuzzy model of the form:

$$\begin{aligned}\dot{x}(t) &= \sum_{i=1}^{2} h_i(\theta(t))\Big\{ -\alpha_{(i,k)}(x(t))\Big[\beta_{(i,k)}(x(t)) - [(A_{(i,k)} + \Delta A_{(i,k)})]f(x(t)) \\ &\quad -[(B_{(i,k)} + \Delta B_{(i,k)})]f(x(t-\tau(t))) \\ &\quad -[(C_{(i,k)} + \Delta C_{(i,k)})]\int_{t-\sigma(t)}^{t} f(x(s))ds\Big]\Big\}, \qquad (21)\end{aligned}$$

where

$$\begin{aligned}
A_{11} &= \begin{bmatrix} 1 & -1.7 \\ -1.6 & 1 \end{bmatrix},\ A_{12} = \begin{bmatrix} -1 & 0 \\ -1 & -1.2 \end{bmatrix},\ A_{21} = \begin{bmatrix} 1.5 & -0.6 \\ 1 & -3 \end{bmatrix}, \\
A_{22} &= \begin{bmatrix} 0.12 & -0.13 \\ -0.1 & 0.16 \end{bmatrix},\ B_{11} = \begin{bmatrix} 1 & 0.6 \\ 0.5 & 0.8 \end{bmatrix},\ B_{12} = \begin{bmatrix} -0.2 & 0 \\ 0.1 & 0.3 \end{bmatrix}, \\
B_{21} &= \begin{bmatrix} 0.1 & 0.6 \\ 0.5 & 0.8 \end{bmatrix},\ B_{22} = \begin{bmatrix} -0.15 & -0.02 \\ -0.12 & -0.17 \end{bmatrix},\ C_{11} = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.3 \end{bmatrix}, \\
C_{12} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ C_{21} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},\ C_{22} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \\
\tilde{\Sigma} &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},\ \Sigma = \Sigma_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ \Sigma_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
\Delta_{(i,k)} &= \begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix},\ \alpha(x) = \begin{bmatrix} 1 - 0.1cos(x_1) & 0 \\ 0 & 1 + 0.2sin(x_2) \end{bmatrix}, \\
\beta(x) &= \begin{bmatrix} 0.9x_1 \\ 0.8x_2 \end{bmatrix},\ f(x) = \begin{bmatrix} 0.2tanh(x_1) \\ 0.2tanh(x_2) \end{bmatrix},\ \underline{\alpha} = 0.8,\ \bar{\alpha} = 1.2, \\
E_{(i,k)} &= \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix},\ H_{1(i,k)} = \begin{bmatrix} 0.3 & 0.1 \\ 0.2 & 0.4 \end{bmatrix},\ H_{2(i,k)} = \begin{bmatrix} 0.2 & 0.1 \\ 0.2 & 0.4 \end{bmatrix}, \\
H_{3(i,k)} &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 0.2 \end{bmatrix},\ \Pi = \begin{bmatrix} -0.4 & 0.4 \\ 0.3 & -0.3 \end{bmatrix},\ \Delta\Pi = \begin{bmatrix} 0.2 & -0.2 \\ 0.3 & -0.3 \end{bmatrix},
\end{aligned}$$

and the fuzzy membership functions are $\eta^1 = e^{2x_1(t)}$, $\eta^2 = 1 - \eta^1$. Using Theorem 3.1 in this paper, it can be verified that UFMJCGNN (21) is globally robustly asymptotically stable in the mean square with uncertain switching probabilities. The determined MAUB of $\tau = N\hat{\tau}_j$ and σ are listed in Table 1. The simulation results are given in Figure 1 and Figure 2 with $F_{(i,k)}(t) = sin(t), \tau(t) = 1.5 + 0.2|sin(t)|,\ \sigma(t) = 5.78$ and different initial conditions.

Methods	Unknown μ	$\mu = 0.5$	$\mu \geq 1$
N=2	1.5716	1.5964	1.5716
N=3	1.6890	1.6995	1.6890
N=5	1.8085	1.8298	1.8085
N=7	1.8792	1.8912	1.8792

TABLE 1. The MAUB of Time Varying Delay τ of Theorem 3.1 with $0 < \sigma(t) \leq \sigma = 10.211$

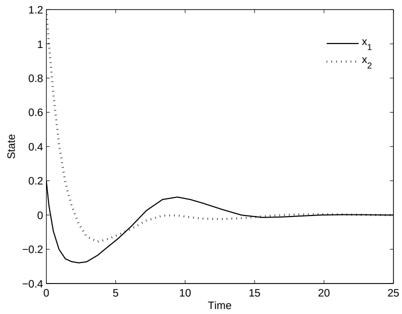

FIGURE 1. State Response of $x_1(t), x_2(t)$ for $i = 1, 2$ and $k = 1$ in Example 4.1

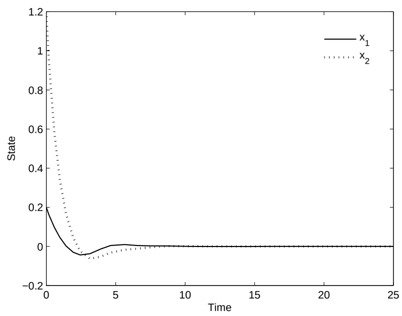

FIGURE 2. State Response of $x_1(t), x_2(t)$ for $i = 1, 2$ and $k = 2$ in Example 4.1

Example 4.2. [12] Consider the UFMJCGNNs (8) without uncertainties, fuzzy and Markovian jumping is of the form:

$$\begin{aligned} \dot{x}(t) &= -\alpha(x(t))\Big[\beta(x(t)) - Af(x(t)) - Bf(x(t-\tau(t))) \\ &\quad -C\int_{t-\sigma(t)}^{t} f(x(s))ds\Big], \end{aligned} \tag{22}$$

Methods	Unknown μ	$\mu = 0.5$	$\mu \geq 1$
N=2	2.9788	5.1250	2.9788
N=3	3.3072	5.5332	3.3072
N=5	3.6610	6.0525	3.6610
N=7	3.8514	6.1215	3.8514

Table 2. The MAUB of Time Varying Delay τ of Theorem 3.1 with $0 < \sigma(t) \leq \sigma = 12.997$ and Without Uncertainties

Methods	Unknown μ	$\mu = 0.5$	$\mu \geq 1$
T.Li et al [12]	$[11.01, 1.113]$	$[11.50, 2.011]$	$[11.11, 1.210]$
N=2	$[9.4880, 1.113]$	$[11.3259, 2.011]$	$[10.9688, 1.210]$
N=3	$[11.4528, 1.113]$	$[13.7058, 2.011]$	$[13.2843, 1.210]$
N=5	$[14.4270, 1.113]$	$[17.4177, 2.011]$	$[16.8285, 1.210]$
N=7	$[17.2107, 1.113]$	$[18.3012, 2.011]$	$[17.2107, 1.210]$

Table 3. Comparing Some Upper Bounds of Time Varying Delay τ of Theorem 3.1 with the Existing Literature

Methods	Unknown μ	$\mu = 0.5$	$\mu \geq 1$
N=2	1.5754	1.6012	1.5754
N=3	1.7043	1.7268	1.7043
N=5	1.8290	1.8485	1.8290
N=7	1.8907	1.9089	1.8907

Table 4. The MAUB of Time Varying Delay τ of Theorem 3.1 with $0 < \sigma(t) \leq \sigma = 10.211$ and with Uncertainties

with

$$
\begin{aligned}
A &= \begin{bmatrix} 1 & -1.7 \\ -1.6 & 1 \end{bmatrix}, \; B = \begin{bmatrix} 1 & 0.6 \\ 0.5 & 0.8 \end{bmatrix}, \; C = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.3 \end{bmatrix}, \\
\tilde{\Sigma} &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \; \Sigma = \Sigma_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \; \Sigma_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
\Delta &= \begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix}, \; \underline{\alpha} = 0.8, \; \bar{\alpha} = 1.2.
\end{aligned}
$$

Using Theorem 3.1 in this paper, it is found that the system (22) is globally asymptotically stable and the MAUB of $\tau(t)$ and $\sigma(t)$ are given in the following Table 2, for $N = 2, 3, \cdots$. Clearly, the results obtained in this paper are less conservative than the results obtained in [12] for $N \geq 3$. This can be verified and determined MAUB are tabulated in the Table 3. When we consider the uncertainties in (22), the system is also globally asymptotically stable and the calculated MAUB are listed in Table 4. The simulation results are given in Figure 3 and Figure 4 with $\tau(t) = 3.7 + 0.2|sin(t)|, \; \sigma(t) = 12.997$ and different initial conditions.

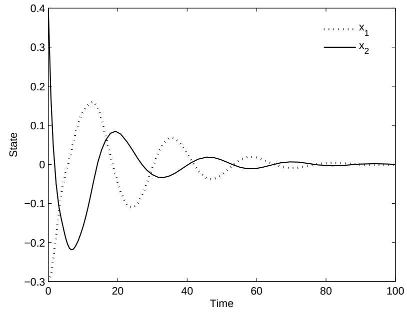

FIGURE 3. State Response of $x_1(t), x_2(t)$ in Example 4.2 Without Uncertainties

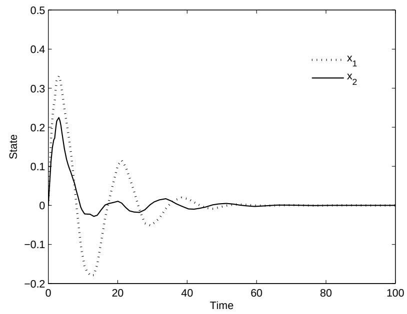

FIGURE 4. State Response of $x_1(t), x_2(t)$ in Example 4.2 with Uncertainties

5. Conclusion

The problem of global and robust asymptotic stability of UFMJCGNNs with mixed time-varying delays has been addressed by using new LKF, which has been constructed by nonuniformly dividing the discrete delay interval into multiple segments. More general activation function and uncertain switching probabilities are considered in our analysis. The restrictive condition in the derivative of discrete time-varying delay being less than 1 is released in our method. Delay-dependent stability condition has been easily solved by LMI Toolbox in Matlab. Moreover, it is found that the delay decomposition approach improves MAUB of discrete and distributed delays. Finally, numerical examples are given to illustrate the less conservatism of our proposed method.

In future, the extension of the present results to more general cases to be considered, for example, the case that the delay-probability-distribution-dependent stability and the case that the mode dependent stability [22]. Apparently the reduced conservatism of Theorems and Corollaries in this paper may be reduced from the augmented LKFs constructed along with the main idea of delay fractioning

[7, 6, 8, 24] and convex combination techniques, hence the potential conservatism of the LMI conditions in this paper may be reduced. The reduction of the conservatism with the fewer variables remains as a future work. Further, the sliding mode control/observer problems in [7, 6, 8] by employing the delay-fractioning approach can be considered for UNFMJCGNN with switching probabilities.

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R. Sathy, Department of Social Sciences, Tamil Nadu Agricultural University, Coimbatore - 641 003, Tamilnadu, India

E-mail address: maths_sathy@yahoo.co.in

P. Balasubramaniam*, Department of Mathematics, Gandhigram Rural Institute - Deemed University, Gandhigram - 624 302, Tamilnadu, India., Phone: 91-451-2452371, Fax:91-451-2453071

E-mail address: balugru@gmail.com

R. Chandran, Department of Computer Science, Government Arts College, Melur, Madurai - 625 106, Tamilnadu, India

E-mail address: rchandran62@gmail.com

*Corresponding author