# ROBUST STABILITY OF STOCHASTIC FUZZY IMPULSIVE RECURRENT NEURAL NETWORKS WITH TIME-VARYING DELAYS

M. SYED ALI

Abstract. In this paper, global robust stability of stochastic impulsive recurrent neural networks with time-varying delays which are represented by the Takagi-Sugeno (T-S) fuzzy models is considered. A novel Linear Matrix Inequality (LMI)-based stability criterion is obtained by using Lyapunov functional theory to guarantee the asymptotic stability of uncertain fuzzy stochastic impulsive recurrent neural networks with time-varying delays. The results are related to the size of delay and impulses. Finally, numerical examples and simulations are given to demonstrate the correctness of the theoretical results.

## 1. Introduction

The well known Takagi-Sugeno (T-S) fuzzy model [32] is a popular, powerful and convenient tool in functional approximations for complex nonlinear systems. Unlike conventional modelling, it combines some simple local linear dynamical systems with their linguistic distribution to represent highly nonlinear dynamical systems. The physical complex system is assumed to exhibit explicit linear or nonlinear dynamics around some operating points. These local models are smoothly aggregated via fuzzy inferences, which lead to the construction of complete system dynamics. This method is feasible since in many situations, human experts can provide linguistic descriptions of local systems in terms of IF-THEN rules. The method is quite interesting because it gives a way to smoothly connect local linear systems to form global nonlinear systems by fuzzy membership functions. So it has been widely and successfully applied to a variety of industrial processes such as batch chemical reactors, cement kilns etc. ( see [7, 24, 35]).

A recurrent neural network, which naturally involves dynamic elements in the form of feedback connections used as internal memories. Unlike the feedforward neural network whose output is a function of its current inputs only and is limited to static mapping, recurrent neural network perform dynamic mapping. Recurrent networks are needed for problems where there exist at least one system state variable which cannot be observed. Most of the existing recurrent neural networks are obtained by adding trainable temporal elements to feedforward neural networks

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(like multilayer perceptron networks [10] and radial basis function networks [4]) to make the output history sensitive. Like feedforward neural networks, these network functions as block boxes and the meaning of each weight in these nodes are not known. They play an important role in applications such as classification of patterns, associate memories and optimization etc. ( see [10], [4] and the references therein). Thus, research on the properties especial stability problem and relaxed stability problem of recurrent neural networks, has become a very active area in the fast few years (for example [1, 2, 15, 19, 22, 36]). Although discrete time delays in the delayed feedback neural networks having a small number of cells serve usually as good approximation of the prime models, a real system is usually affected by external perturbations. Therefore, it is significant and of prime importance to consider stochastic effects to the stability property of the neural networks with delays ( see [16, 18, 23, 28]).

On the other hand, impulsive effects may be unavoidable. For instance, in implementation of electronic networks, the state of the network is subject to instantaneous perturbations and experiences abrupt changes at certain instants, which may be caused by switching phenomenon, frequency change or other sudden noise, that is, it exhibits impulsive effects [20, 21, 33, 34] . Neural networks are often subject to impulsive perturbations that in turn affect dynamical behaviors of the systems. Therefore, it is necessary to take both time delays and impulsive effects into account on dynamical behaviors of neural networks. In recent years, significant progress has been made in the theory of impulsive neural networks [11, 37, 38].

The concept of incorporating T-S fuzzy logic into a neural network is proposed in some papers ([3, 5, 9, 13]). These neural fuzzy networks have so many advantages over the feedforward neural networks as mentioned above, it seems worth constructing a recurrent neural network based on a neural fuzzy network. Based on the above discussions, the ordinary fuzzy models has been extended to describe the delayed neural networks and derived the stability criterion in terms on LMIs ([14, 17, 25, 26, 27, 29, 30, 31]) which can be easily calculated by MATLAB LMI toolbox [8]. The main advantage of the LMI based approaches is that the LMI stability conditions can be solved numerically using the effective interior-point algorithms [6].

In this paper, we generalize the ordinary T-S fuzzy models to express a class of uncertain stochastic recurrent neural networks with time-varying delays and impulsive perturbations. To the best of the authors knowledge, there were no (or few) results for robust stability criterion in the form of LMI for stochastic fuzzy impulsive recurrent networks with time-varying delays in the literature. By using the Lyapunov functional technique, global robust stability conditions for the uncertain stochastic fuzzy impulsive recurrent neural networks are given in terms of LMIs. Numerical examples are provided to demonstrate the effectiveness and applicability of the proposed stability results.

**Notations:** Throughout the manuscript we will use the notation $A > 0$ $(or A < 0)$ to denote that the matrix A is a symmetric and positive definite (or negative definite) matrix. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (it is right continuous and $\mathcal{F}_0$ contains all $\mathcal{P}$-null sets); $\mathcal{L}^p_{\mathcal{F}_0}([-\tau,0];\mathbf{R}^n)$ be the family of all bounded, $\mathcal{F}_0$- measurable $\mathcal{C}([-\tau,0];\mathbf{R}^n)$-valued random variables $\xi=\{\xi(\theta):-\tau\leq\theta\leq 0\}$ such that $\sup_{-\tau\leq\theta\leq 0}\mathbf{E}|\xi(\theta)|^p<\infty$. The mathematical expectation operator with respect to the given probability measure $\mathcal{P}$ is denoted by $\mathbf{E}\{\cdot\}$. The shorthand *diag* $\{\cdots\}$ denotes the block diagonal matrix. $\|\cdot\|$ stands for the Euclidean norm. Moreover, the notation * always denotes the symmetric block in one symmetric matrix.

## 2. **System Description and Preliminaries**

Consider the following uncertain recurrent neural network with time-varying delays described by,

$$\dot{u}_i(t)=-(a_i+\Delta a_i)u_i(t)+\sum_{j=1}^{n}(w_{ij}+\Delta w_{ij})F_j(u_j(t))+\sum_{j=1}^{n}(h_{ij}+\Delta h_{ij})F_j(u_j(t$$
$$-\tau_j(t)))+I_i, t\neq t_k,\ t\geq t_0, \tilde{\Delta}u_i(t_k)=u_i(t_k)-u_i(t_k^-)=I_{ik}u_i(t_k), t=t_k, \quad (1)$$

in which $u_i(t)$ is the activation of the $i^{th}$ neuron. Positive constant $a_i$ denotes the rates with which the cell $i$ reset their potential to the resting state when isolated from the other cells and inputs. $w_{ij}$ and $h_{ij}$ are the connection weights at the time t, symbol $\Delta$ denotes the uncertain term, $I_i$ denote the external input, the impulse time $t_k$ satisfy $0\leq t_0<t_1<\ldots<t_k\ldots,$. $F_j(\cdot)$ is the neuron activation function of $j^{th}$ neuron. $\tau_j(t)$ is the bounded time varying delays in the state and satisfy $0\leq\tau_j(t)\leq\bar{\tau},\ 0\leq\dot{\tau}_j(t)\leq d<1,\ i,j=1,2,..,n$. $\Delta u_i(t_k)=u_i(t_k^+)-u_i(t_k^-)$ is the impulses at moments $t_k, t_1<t_2<\ldots$ is strictly increasing sequence such that $lim_{k\to\infty}t_k=+\infty$. As usual in the theory of impulsive differential equations, at the points of discontinuity $t_k,\ k=1,2,\ldots,m$, we assume that $u_i(t_k)=u_i(t_k^-)$ and $u_i'(t_k)=u_i'(t_k^-)$.

The initial conditions associated with system (1) are of the form $u(s)=\phi(s),\ s\in[t_0-\tau,t_0]$, where $u(s)=(u_1(s),u_2(s),\ldots,u_n(s))^T$, $\phi(s)=(\phi_1(s),\ \phi_2(s),\ \ldots,\phi_n(s))^T$ $\in PC([-\tau,0],R^n)$, $PC([-\tau,0],R^n)=\{\eta:\eta(t_k^+)=\eta(t_k)\}$. For $\eta\in PC([-\tau,0],R^n)$, the norm of $\eta$ is defined by $|||\eta|||_\tau=sup_{-\tau\leq\theta\leq 0}||\eta(\theta)||$. For any $t_0\geq 0$, let $PC_\delta(t_0)=\{\eta\in PC([-\tau,0],R^n):||\eta||_\tau<\delta\}$.

Assume that $u^*=(u_1^*,u_2^*,...,u_n^*)^T$ is the equilibrium point of the system. Impulsive operator is viewed as perturbation of the equilibrium point $u^*$ of such system without impulsive effects. Then we shift the equilibrium points to the origin by the transformation, $x_i(t)=u_i(t)-u_i^*,\ f_j(x_j(t))=F_j(u_j(t))-F_j(u_j^*)$. Then transformed system is given by,

$$\dot{x}_i(t)=-(a_i+\Delta a_i)x_i(t)+\sum_{j=1}^{n}(w_{ij}+\Delta w_{ij})f_j(x_j(t))+\sum_{j=1}^{n}(h_{ij}+\Delta h_{ij})f_j(x_j(t-\tau_j(t))),$$
$$t\neq t_k,\ \tilde{\Delta}x_i(t_k)=I_{ik}(x_i(t_k)),\ t=t_k. \quad (2)$$

Conveniently, we can write (2) in the form

$$\dot{x}(t)=-(A+\Delta A)x(t)+(W+\Delta W)f(x(t))+(H+\Delta H)f(x(t-\tau(t))),\ t\neq t_k,$$
$$\tilde{\Delta}x(t_k)=I_k(x(t_k)),\ t=t_k,$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$, $A = diag\{a_1, a_2, \ldots, a_n\}, W = [(w_{ij})_{n\times n}]^T$, $H = [(h_{ij})_{n\times n}]^T$, $f(x(t)) = [f_1(x_1(t)),\ f_2(x_2(t)), \ldots, f_n(x_n(t))]^T$ and $\tau(t) = (\tau_1(t), \tau_2(t), \ldots, \tau_n(t))^T$.
The following assumption is made on the activation function.

**(A)** The neuron activation function $f_j(\cdot)$ in (2) is monotonically increasing, continiously differentiable and satisfies the following Lipschitz condition,

$$|f_j(x) - f_j(y)| \leq |L_j(x-y)|,$$

for all $x, y \in R, j = 1, 2, ..., n$ where $L_j \in R^{n\times n}$ are known constants. Then we have

$$f^T(x(t))f(x(t)) \leq x^T(t)L^T Lx(t),$$

where $L = diag\{L_1, L_2, \ldots, L_n\}$.
As mentioned earlier, it is often the case in practice that the neural network is disturbed by environmental noises that affect the stability of the equilibrium. Motivated by this we generalize the ordinary T-S fuzzy models to express a complex stochastic system whose consequent parts are a set of stochastic impulsive recurrent neural networks with time varying delays.

The T-S fuzzy model has $r$ rules, where the $l^{th}$ rule of this T-S fuzzy model is of the following form,

**Plant Rule $l$:**

**IF** $\{u_1(t)\ is\ \eta_1^l\}$ and . . . and $\{u_\gamma(t)\ is\ \eta_\gamma^l\}$

**THEN**

$$dx(t) = [-(A_l + \Delta A_l)x(t) + (W_l + \Delta W_l)f(x(t)) + (H_l + \Delta H_l)f(x(t-\tau(t)))]dt + \sigma_l(t, x(t), x(t-\tau(t))d\omega(t),$$

where $\eta_i^l$, $(i = 1, 2, ..., \gamma, l = 1, 2, \ldots, r)$ are fuzzy sets, $(u_1(t), u_2(t), ..., u_\gamma(t))$ is the premise variable vector, $x(t)$ is the state variable and $r$ is the number of IF-THEN rules. Further $\omega(t) = (\omega_1(t), \omega_2(t), ...., \omega_m(t))^T$ is an $m-$ dimensional Brownian motion defined on $(\Omega, \mathfrak{F}, \{\mathfrak{F}_t\}_{t\geq 0}, \mathcal{P})$ and $\sigma(t, x(t), x(t-\tau(t))) : R_+ \times R^n \times R^n \to R^{n\times m}$ is locally Lipschitz continuous, satisfies the linear growth condition as well and $\sigma_l(0, 0, 0) = 0$ . We impose the following hypothesis:

**(H1)** The parameter uncertainties $\Delta A_l(t),\ \Delta W_l(t),\ \Delta H_l(t)$ are defined as follows

$$\Big[\Delta A_l(t), \quad \Delta W_l(t), \quad \Delta H_l(t)\Big] = M_l F_l(t)\Big[N_{1l}, \quad N_{2l}, \quad N_{3l}\Big] \tag{3}$$

where $N_{1l},\ N_{2l},\ N_{3l},\ M_l$ are known constant matrices of appropriate dimensions and $F_l(t)$ is an known time varying matrix with Lebegue measurable elements bounded by,

$$F_l^T(t)F_l(t) \leq I,$$

where $I$ is the identity matrix with appropriate dimension.

**(H2)** Let $\sigma(t, x, y) : R^+ \times R^n \times R^n \to R^{n\times m}$ be locally Lipchitz continuous and satisfies the linear growth condition as well. Moreover, $\sigma_l$ satisfies,

$$trace\Big[\sigma_l\big(t, x(t), x(t-\tau(t))\big)^T \sigma_l\big(t, x(t), x(t-\tau(t))\big)\Big] \leq C_1|x(t)|^2 + C_2|x(t-\tau(t))|^2,$$

where $C_1$ and $C_2$ are known constant matrices with appropriate dimensions.
**(H3)** The impulsive operator satisfy,

$$I_{ik}(u_i(t_k)) = -\gamma_{ik}(u_i(t_k) - u_i^*), 0 \leq \gamma_{ik} \leq 2, i = 1, 2, \ldots, n,$$

$\Gamma_k = diag\{\gamma_{1k}, \gamma_{2k}, \ \ldots, \gamma_{nk}\}$.

Let $\lambda_l(\xi(t))$ be the normalized membership function of the inferred fuzzy set $\beta_l(\xi(t))$. The state equation is defined as follows:

$$\left.\begin{array}{rl} dx(t) & = \sum_{l=1}^{r} \lambda_l(\xi(t))\big\{[-(A_l + \Delta A_l)x(t) + (W_l + \Delta W_l)f(x(t)) \\ & \times + (H_l + \Delta H_l)f(x(t-\tau(t)))]dt + \sigma_l(t, x(t), x(t-\tau(t)))d\omega(t) \end{array}\right\}, \quad (4)$$

where $\lambda_l(\xi(t)) = \frac{\beta_l(\xi(t))}{\sum_{l=1}^{r}\beta_l(\xi(t))}$, $\beta_l(\xi(t)) = \prod_{p=1}^{\gamma} \eta_p^l(z_p(t))$ and $\eta_p^l(z_p(t))$ is the grade of the membership function of $z_p(t)$ in $\eta_p^l$. We assume $\beta_l(\xi(t)) \geq 0, \ l = 1, 2, ..., r$ $\sum_{l=1}^{r} \beta_l(\xi(t)) > 0$, for any $\xi(t)$. Hence $\lambda_l(\xi(t))$ satisfy $\lambda_l(\xi(t)) \geq 0, \ k = 1, 2, ...., r$, $\sum_{l=1}^{r} \lambda_l(\xi(t)) = 1$, for any $\xi(t)$.

The following Lemmas will be essential for the proofs in the next section.

**Lemma 2.1.** *(Schur complement* [6]*). Let $M, P, Q$ be given matrices such that $Q > 0$, then*

$$\begin{bmatrix} P & M^T \\ M & -Q \end{bmatrix} < 0 \quad \Leftrightarrow P + M^T Q^{-1} M < 0.$$

**Lemma 2.2.** [28] *For matrices $P \in \boldsymbol{R}^{n\times n}, M \in \boldsymbol{R}^{n\times k}, \ N \in \boldsymbol{R}^{l\times n}$ and $F \in \boldsymbol{R}^{k\times l}$ with $P > 0, \|F\| \leq 1$ and scalar $\epsilon > 0$, one has the following,*

$$(MFN)^T P + P(MFN) \leq \epsilon PMM^T P + \epsilon^{-1} N^T N.$$

## 3. Robust Stochastic Stability Results

In this section, some sufficient conditions of robust stability for system (4) are obtained.

**Definition 3.1.** For system (4) and every $\phi \in \mathcal{L}_{\mathcal{F}_0}^p([-\tau, 0]; \mathbf{R}^n)$, the trivial solution is globally asymptotically stable in the mean square if

$$lim_{t\to\infty}\mathbf{E}|x(t, \phi)|^2 = 0.$$

Itô's formula plays an important role in the analysis of stochastic systems. For a general stochastic system $dx(t) = f(t, x(t))dt + g(t, x(t))d\omega(t)$ on $t \geq 0$ with initial condition $x(t_0) = x_0 \in R^n$, where $f : R^+ \times R^n \to R^n$ and $g : R^+ \times R^n \to R^{n\times m}$. Let $V \in \mathcal{C}(R^+ \times R^n; R^n)$, an operator $\mathcal{L}V$ is defined from $R^+ \times R^n$ to $R$ by

$$\mathcal{L}V(t, x(t)) = V_t(t, x(t)) + V_x(t, x(t))f(t, x(t)) + \frac{1}{2}trace[g^T(t, x(t))V_{xx}(t, x(t))g(t, x(t))],$$

where

$$V_t(t, x(t)) = \frac{\partial V(t, x(t))}{\partial t}, \ V_x(t, x(t)) = \Big(\frac{\partial V(t, x(t))}{\partial x_1}, \ldots, \frac{\partial V(t, x(t))}{\partial x_n}\Big),$$

$$V_{xx}(t, x(t)) = \Big(\frac{\partial^2 V(t, x(t))}{\partial x_i \ \partial x_j}\Big)_{n\times n}.$$

**Theorem 3.2.** *Given $\tau > 0, d > 0$, if there exist symmetric positive definite matrices $P > 0, Q > 0,\ R > 0$, positive diagonal matrices $S > 0,\ X > 0,\ Y > 0$ and positive scalars $\epsilon_j (j = 1, 2, ..., 6), \rho, \mu$ such that feasible solution exist for*

$$\Omega^l = \begin{bmatrix} \Omega^l_{11} & 0 & \Omega^l_{13} & \Omega^l_{14} & \Omega^l_{15} & 0 \\ * & \Omega^l_{22} & 0 & 0 & 0 & 0 \\ * & * & \Omega^l_{33} & \Omega^l_{34} & 0 & \Omega^l_{36} \\ * & * & * & \Omega^l_{44} & 0 & 0 \\ * & * & * & * & \Omega^l_{55} & 0 \\ * & * & * & * & * & \Omega^l_{66} \end{bmatrix} < 0, \tag{5}$$

$$\Omega^l_k = \begin{bmatrix} \Omega^{lk}_{11} & \Gamma^T_{lk} P \\ * & -P \end{bmatrix} < 0, \tag{6}$$

$$P < \rho I, \tag{7}$$

$$S < \mu I, \tag{8}$$

$\Omega^l_{11} = -A^T_l P - P^T A_l + (\rho+\mu)C_1 + Q + 2L^T XL + 2YL + (\epsilon_1 + \epsilon_4)N^T_{l1} N_{l1}, \Omega^l_{13} = PW_l - A^T_l S - Y, \Omega^l_{14} = PH_l,\ \Omega^l_{22} = -(1-d)Q + (\rho+\mu)C_2, \Omega^l_{33} = R - 2X + SW_l + W^T_l S + (\epsilon_2 + \epsilon_5)N^T_{l2} N_{l2},\ \Omega^l_{34} = SH_l,\ \Omega^l_{44} = -(1-d)R + (\epsilon_3 + \epsilon_6)N^T_{3l} N_{3l}, \Omega^{lk}_{11} = -\Gamma^T_{lk} P - P\Gamma_{lk} + \Gamma^T_{lk} SL\Gamma_{lk} - \Gamma^T_{lk} SL - SL\Gamma_{lk}, \Omega^l_{15} = [PM_l\ PM_l\ PM_l],\ \Omega^l_{36} = [SM_l\ SM_l\ SM_l],$

$$\Omega^l_{55} = diag\{-\epsilon_1 I,\ -\epsilon_2 I,\ -\epsilon_3 I\},\ \Omega^l_{66} = diag\{-\epsilon_4 I,\ -\epsilon_5 I,\ -\epsilon_6 I\},$$

*then the system* (4) *is robustly asymptotically stable in the mean square.*

*Proof.* We consider the following Lyapunov functional to derive the stability result,

$$V(t, x(t)) = V_1(t, x(t)) + V_2(t, x(t)) + V_3(t, x(t)) + V_4(t, x(t)),$$

where

$$V_1(t, x(t)) = x^T(t)\ P\ x(t),\ V_2(t, x(t)) = 2\sum_{i=1}^n s_i \int_0^{x_i} f_i(s)ds,$$

$$V_3(t, x(t)) = \int_{t-\tau(t)}^t x^T(s)Qx(s)ds,\ V_4(t, x(t)) = \int_{t-\tau(t)}^t f^T(x(s))Rf(x(s))ds.$$

For $t = t_k$, by using Lemma 2.1 we have

$$V(x(t^+_k)) = V(x(t^-_k) + I_k(x(t^-_k))) = (x(t^-_k) + I_k(x(t^-_k)))^T\ P\ (x(t^-_k) + I_k(x(t^-_k)))$$

$$+2\sum_{i=1}^n s_i \int_0^{(x_i(t^-_k)+I_k(x(t^-_k)))} f_i(s)ds, + \int_{t-\tau(t^+_k)}^{t^+_k} x^T(s)Qx(s)ds$$

$$+ \int_{t^+_k-\tau(t^+_k)}^{t^+_k} f^T(x(s))Rf(x(s))ds = (x(t^-_k) - \Gamma_{lk}(x(t^-_k)))^T\ P\ (x(t^-_k) - \Gamma_{lk}(x(t^-_k)))$$

$$+2\sum_{i=1}^n s_i \int_0^{x_i(t^-_k)} f_i(s)ds + 2\sum_{i=1}^n s_i \int_{x_i(t^-_k)}^{x_i(t^-_k)+I_k(x_i(t^-_k))} f_i(s)ds + \int_{t-\tau(t_k)}^{t_k} x^T(s)Qx(s)ds$$

$$+\int_{t_k-\tau(t_k)}^{t_k} f^T(x(s))Rf(x(s))ds = x^T(t_k^-)Px(t_k^-) + 2\sum_{i=1}^{n} s_i \int_0^{x_i(t_k^-)} f_i(s)ds$$

$$+\int_{t-\tau(t_k)}^{t_k} x^T(s)Qx(s)ds + \int_{t_k-\tau(t_k)}^{t_k} f^T(x(s))Rf(x(s))ds$$

$$+x^T(t_k^-)(\Gamma_{lk}^T P\Gamma_{lk} - \Gamma_{lk}P - P\Gamma_{lk})x(t_k^-) + 2\sum_{i=1}^{n} s_i \int_{x_i(t_k^-)}^{x_i(t_k^-)+I_k(x_i(t_k^-))} f_i(s)ds$$

$$= V(x(t_k^-)) + x^T(t_k^-)(\Gamma_{lk}^T P\Gamma_{lk} - \Gamma_{lk}P - P\Gamma_{lk} + \Gamma_{lk}^T SL\Gamma_{lk} - \Gamma_{lk}^T SL - SL\Gamma_{lk})x(t_k^-)$$

$$= V(x(t_k^-)) + x^T(t_k^-)\bar{\Omega}_k^l x(t_k^-).$$

We define

$$\bar{\Omega}_k^l = \Gamma_{lk}^T P\Gamma_{lk} - \Gamma_{lk}P - P\Gamma_{lk} + \Gamma_{lk}^T SL\Gamma_{lk} - \Gamma_{lk}^T SL - SL\Gamma_{lk}.$$

By using Schur complement Lemma $\bar{\Omega}_k^l$ can be written as $\Omega_k^l$. Thus by (6) we have

$$V(x(t_k^+)) \leq V(x(t_k^-)).$$

When $t \in [t_{k-1}, t_k)$, we can calculate $\mathcal{L}V$ along the trajectories of the system (4), then we have

$$\mathcal{L}V(t, x(t)) = \mathcal{L}V_1(t, x(t)) + \mathcal{L}V_2(t, x(t)) + \mathcal{L}V_3(t, x(t)) + \mathcal{L}V_4(t, x(t)),$$

where

$$\mathcal{L}V_1(t, x(t)) = 2x^T(t)P[-(A_l + \Delta A_l)x(t) + (W_l + \Delta W_l)f(x(t)) + (H_l + \Delta H_l)$$
$$f(x(t-\tau(t)))] + trace[\sigma_i^T(t, x(t), x(t-\tau(t)))P\sigma_i(t, x(t), x(t-\tau(t))) \quad (9)$$

,

$$\mathcal{L}V_2(t, x(t)) = 2f^T(x(t))\Big[[-S(A_l + \Delta A_l)]x(t) + S(W_l + \Delta W_l)f(x(t))$$
$$+S(H_l + \Delta H_l)f(x(t-\tau(t)))\Big] + trace[\sigma_i^T(\cdot)(S)\sigma_j(\cdot)], \quad (10)$$

$$\mathcal{L}V_3(t, x(t)) \leq x^T(t)Qx(t) - (1-d)x^T(t-\tau(t))Qx(t-\tau(t)), \quad (11)$$

$$\mathcal{L}V_4(t, x(t)) \leq f^T(x(t))Rf(x(t)) - (1-d)f^T(x(t-\tau(t)))Rf(x(t-\tau(t))), \quad (12)$$

Noting that, for any diagonal matrices $X > 0$ and $Y > 0$ using Assumption (A), we can have that,

$$0 = 2x^T(t)Yf(x(t)) - 2f^T(x(t))Xf(x(t)) - 2x^T(t)Yf(x(t)) + 2f^T(x(t))Xf(x(t)),$$
$$\leq 2x^T(t)YLx(t) - 2f^T(x(t))Xf(x(t)) - 2x^T(t)Yf(x(t)) + 2x^T(t)L^TXLx(t), \quad (13)$$

Thus from (9)-(13) we obtain,

$$\leq \sum_{l=1}^{r} \lambda_l(\xi(t))\Big\{\xi^T(t)\Sigma^l\xi(t)\Big\}, \quad (14)$$

where

$$\Sigma^l = \begin{bmatrix} \hat{\Sigma}^l_{11} & 0 & \hat{\Sigma}^l_{12} & \hat{\Sigma}^l_{13} \\ * & \Sigma^l_{22} & 0 & 0 \\ * & * & \hat{\Sigma}^l_{33} & \hat{\Sigma}^l_{34} \\ * & * & * & \Sigma^l_{44} \end{bmatrix},$$

where

$$\hat{\Sigma}^l_{11} = \Sigma^l_{11} - 2PM_lF_l(t)N_{l1},\ \hat{\Sigma}^l_{12} = \Sigma^l_{12} + PM_lF_l(t)N_{l2} - M_lF_l(t)N_{l1}S, \Sigma^l_{34} = SH_l,$$
$$\hat{\Sigma}^l_{13} = \Sigma^l_{13} + PM_lF_l(t)N_{l3},\ \hat{\Sigma}^l_{33} = \Sigma^l_{33} + 2SM_lF_l(t)N_{l2},\ \hat{\Sigma}^l_{34} = \Sigma^l_{34} + M_lF_l(t)N_{l3},$$
$$\Sigma^l_{11} = -A_l^TP - P^TA_l + (\rho+\mu)C_1 + Q + 2L^TXL + 2YL,\ \Sigma^l_{12} = PW_l - A_l^TS - Y,$$
$$\Sigma^l_{13} = PH_l,\ \Sigma^l_{22} = -(1-d)Q + (\rho+\mu)C_2,\ \Sigma^l_{33} = R - 2X + SW_l + W_l^TS,$$
$$\Sigma^l_{44} = -(1-d)R,\ \xi^T(t) = [x^T(t)\ x^T(t-\tau(t))\ f^T(x(t))\ f^T(x(t-\tau(t)))]^T.$$

It follows from Lemma 2.2 that,

$$-2x^T(t)P\Delta A_lx(t) \le \epsilon_1^{-1}x^T(t)PM_lM_l^TP^Tx(t) + \epsilon_1x^T(t)N_{l1}^TN_{l1}x(t), \quad (15)$$

$$2x^T(t)P\Delta W_lf(x(t)) \le \epsilon_2^{-1}x^T(t)PM_lM_l^TP^Tx(t) + \epsilon_2f^T(x(t))N_{l2}^TN_{l2}f(x(t)), \quad (16)$$

$$\begin{aligned} 2x^T(t)P\Delta H_lf(x(t-\tau(t))) &\le \epsilon_3^{-1}x^T(t)PM_lM_l^TP^Tx(t) \\ &+\epsilon_3f^T(x(t-\tau(t)))N_{l3}^TN_{l3}f(x(t-\tau(t))), \end{aligned} \quad (17)$$

$$-2f^T(x(t))S\Delta A_lx(t) \le \epsilon_4^{-1}f^T(x(t))SM_lM_l^TSf(x(t)) + \epsilon_4x^T(t)N_{l1}^TN_{l1}x(t), \quad (18)$$

$$\begin{aligned} 2f^T(x(t))S\Delta W_lf(x(t)) &\le \epsilon_5^{-1}f^T(x(t))SM_lM_l^TSf(x(t)) \\ &+\epsilon_5f^T(x(t))N_{l2}^TN_{l2}f(x(t)), \end{aligned} \quad (19)$$

$$\begin{aligned} 2f^T(x(t))S\Delta H_lf(x(t-\tau(t))) &\le \epsilon_6^{-1}f^T(x(t))SM_lM_l^TSf(x(t)) \\ &+\epsilon_6f^T(x(t-\tau(t)))N_{l3}^TN_{l3}f(x(t-\tau(t))). \end{aligned} \quad (20)$$

By using (15)-(20) and Schur complement (Lemma 2.1) $\Sigma^l$ can be written as $\Omega^l < 0$. Then there must exist a scalar $\alpha > 0$ such that $\Sigma^l + diag\{\alpha I, 0, 0, 0\}$. Thus, one can obtain that $\mathcal{L}V(t, x(t)) \le [\xi^T(t)\Sigma^l\xi(t)] \le \alpha|x(t)|^2$,
which indicates from the Lyapunov stability theory [12], that the dynamics of the fuzzy neural network (4) is robustly, globally, asymptotically stable in the mean square for all uncertainties, which completes the proof. □

**Case 1.** If there are no parameter uncertainties in $A$, $W$ and $H$ then the system is simplified to

$$dx(t) = [-Ax(t) + Wf(x(t)) + Hf(x(t-\tau(t)))]dt + \sigma(t, x(t), x(t-\tau(t))d\omega(t).$$

The T-S model is composed of $r$ plant rules, the $l^{th}$ rule of this model can be described as follows,

**Plant Rule $l$:**

**IF** $\{u_1(t)\ is\ \eta_1^l\}$ and . . . and $\{u_\gamma(t)\ is\ \eta_\gamma^l\}$

**THEN**

$$dx(t) = [-A_lx(t) + W_lf(x(t)) + H_lf(x(t-\tau(t)))]dt + \sigma_l(t, x(t), x(t-\tau(t))d\omega(t),$$

where $\eta_i^l$ $(i = 1, 2, ..., \gamma, l = 1, 2, \ldots, r)$ are fuzzy sets, $(u_1(t), u_2(t), \ldots, u_\gamma(t))$ is the premise variable vector, $x(t)$ is the state variable and $r$ is the number of IF - THEN rules. Further, $w(t) = (w_1(t), w_2(t), \ldots, w_m(t))^T$ is an $m-$ dimensional Brownian motion defined on $(\Omega, \mathfrak{F}, \{\mathfrak{F}_t\}_{t\geq 0}, \mathcal{P})$.
Let $\lambda_l(\xi(t))$ be the normalized membership function of the inferred fuzzy set $\beta_l(\xi(t))$. The state equation is defined as follows

$$dx(t) = \sum_{l=1}^{r} \lambda_l(\xi(t))\{\big[-A_l x(t) + W_l f(x(t)) + H_l f(x(t-\tau(t)))]dt$$
$$+\sigma_l(t, x(t), x(t-\tau(t))d\omega(t)\}. \tag{21}$$

**Corollary 3.3.** *Given $\tau > 0, d > 0$, if there exist symmetric positive definite matrices $P > 0, Q > 0,\ R > 0$, positive diagonal matrices $S > 0,\ X > 0,\ Y > 0$, positive scalars $\rho, \mu$ such that feasible solution exist for,*

$$\Sigma^l = \begin{bmatrix} \Sigma_{11}^l & 0 & \Sigma_{12}^l & \Sigma_{13}^l \\ * & \Sigma_{22}^l & 0 & 0 \\ * & * & \Sigma_{33}^l & \Sigma_{34}^l \\ * & * & * & \Sigma_{44}^l \end{bmatrix} < 0,$$

$$\Omega_k^l = \begin{bmatrix} \Omega_{11}^{lk} & \Gamma_{lk}^T P \\ * & -P \end{bmatrix} < 0,\ P < \rho I,\ S < \mu I,$$

*where $\Sigma_{11}^l, \Sigma_{12}^l,\ \Sigma_{13}^l,\ \Sigma_{22}^l,\ \Sigma_{33}^l,\ \Sigma_{44}^l,\ \Omega_{11}^{lk}$ are defined in Theorem 3.2, then the system* (21) *is asymptotically stable in the mean square.*

The proof of the above theorem is similar to that of Theorem 3.2 by choosing the functionals as described in the proof of Theorem 3.2 and it is thus omitted.

**Case 2.** If there are no parameter uncertainties, stochastic disturbance and impulsive effects then we have the following corollary.

**Corollary 3.4.** *Given $\tau > 0, d > 0$, if there exist symmetric positive definite matrices $P > 0, Q > 0,\ R > 0$, positive diagonal matrix $S > 0,\ X > 0,\ Y > 0$ such that feasible solution exist for,*

$$\Pi^l = \begin{bmatrix} \Pi_{11}^l & 0 & \Pi_{13}^l & \Pi_{14}^l \\ * & \Pi_{22}^l & 0 & 0 \\ * & * & \Pi_{33}^l & \Pi_{34}^l \\ * & * & * & \Pi_{44}^l \end{bmatrix} < 0, \tag{22}$$

*where*

$\Pi_{11}^l = -A_l^T P - P^T A_l + Q + 2L^T XL + 2YL,\ \Pi_{13}^l = PW_l - A_l^T K - Y, \Pi_{14}^l = PH_l,$

$\Pi_{22}^l = -(1-d)Q,\ \Pi_{33}^l = R - 2X + SW_l + W_l^T S, \Pi_{34}^l = SH_l,\ \Pi_{44}^l = -(1-d)R,$

*then the system* (21) *is globally asymptotically stable.*

The proof of the above theorem is similar to that of Theorem 3.2 by choosing the functionals as described in the proof of Theorem 3.2 and it is thus omitted.

**Remark 3.5.** The motivation for the system (1) containing uncertainties $\Delta A(t)$, $\Delta W(t)$ and $\Delta H(t)$ stems from the fact that, in practical , it is almost impossible to get an exact mathematical model of a dynamical system due to modelling errors, measurement errors, liberalization approximation, and so on. Indeed, it is reasonable and practical to assume that the model of the system to be controlled almost always contains some type of uncertainty.

## 4. **Numerical Examples**

**Example 4.1.** Consider the stochastic fuzzy impulsive recurrent neural network (4) with uncertainties. The T-S fuzzy model of this system is of the following form:
**Plant Rules:**
**Rule 1: IF** $\{u_1(t)\ is\ \eta^1\}$, **THEN**

$$dx(t) = [-(A_1 + \Delta A_1)x(t) + (W_1 + \Delta W_1)f(x(t)) + (H_1 + \Delta H_1)f(x(t - \tau(t)))]dt + \sigma_1(t, x(t), x(t - \tau(t))d\omega(t),$$

**Rule 2: IF** $\{u_2(t)\ is\ \eta^2\}$, **THEN**

$$dx(t) = [-(A_2 + \Delta A_2)x(t) + (W_2 + \Delta W_2)f(x(t)) + (H_2 + \Delta H_2)f(x(t - \tau(t)))]dt + \sigma_2(t, x(t), x(t - \tau(t))d\omega(t),$$

where $f(x) = tanh(x) = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. Obviously, $tanh(\cdot)$ satisfies, $tanh(x) < 1$, for every $x \in R$, further

$$|tanh(x) - tanh(y)| = \Big|\frac{d(tanh(z))}{dt}\Big|_{z=\zeta}|x - y| = \Big|\frac{4}{(e^{\zeta} + e^{-\zeta})^2}\Big||x - y| \leq |x - y|,$$

for every $x, y \in R$ and $\zeta \in (x, y)$ or $\zeta \in (y, x)$. Thus, $L = diag\{1, 1\}$. The membership functions for Rule 1 and Rule 2 are $\eta^1 = \frac{1}{e^{-2u_1(t)}}$, $\eta^2 = 1 - \eta^1$,

$$A_1 = \begin{bmatrix} 5 & 0 \\ 0 & 1 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 1 & 0.16 \\ 0.05 & 0.16 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.1 & 0 \\ 0.05 & 0.1 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 8 \end{bmatrix}, \quad W_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.2 \end{bmatrix}, \quad H_2 = \begin{bmatrix} -0.3 & -0.2 \\ 0 & 0.1 \end{bmatrix},$$

$$\Gamma_{1_k} = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.9 \end{bmatrix}, \quad \Gamma_{2_k} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$C_2 = \begin{bmatrix} 0.07 & 0 \\ 0 & 0.07 \end{bmatrix}, F_l = \begin{bmatrix} sin(t) & 0 \\ 0 & cos(t) \end{bmatrix}, \quad M_l = \begin{bmatrix} 0.08 & 0 \\ 0 & 0.08 \end{bmatrix},$$

$N_{l_i} = 0.2\, I, i = 1, 2, 3$. By using the Matlab LMI toolbox, we solve the LMI $(5) - (8)$ for $\epsilon_i > 0$, $(i = 1, 2, ..., 6)$, $\bar{\tau} = 0.5$ and $d = 0.5$ the feasible solutions are,

$$P = \begin{bmatrix} 18.4323 & -0.2007 \\ -0.2007 & 18.2516 \end{bmatrix}, \quad Q = \begin{bmatrix} 5.6137 & -0.0256 \\ -0.0256 & 7.0995 \end{bmatrix},$$

$$R = \begin{bmatrix} 8.9430 & 2.2011 \\ 2.2011 & 9.4290 \end{bmatrix}, \qquad S = \begin{bmatrix} 0.0447 & 0 \\ 0 & 4.8046 \end{bmatrix},$$
$$X = \begin{bmatrix} 8.5387 & 0 \\ 0 & 10.3753 \end{bmatrix}, \qquad Y = \begin{bmatrix} 0.0289 & 0 \\ 0 & 0.0882 \end{bmatrix}.$$

Therefore, the concerned stochastic fuzzy neural networks with time-varying delays is robustly, asymptotically stable in the mean square sense.

**Example 4.2.** Consider the two-neuron delayed neural network (Example 2 in [39]) with the following parameters:

$$A_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} -0.1 & 0.1 \\ 0.1 & -0.1 \end{bmatrix}, \qquad H_1 = \begin{bmatrix} -0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}.$$

We let $L = I,\ C_1 = C_2 = 0,\ r = 1,\ d = 0.5$ and $\bar{\tau} = 0.5$. By using the Matlab LMI toolbox, we solve the LMI (22) the feasible solutions are,

$$P = \begin{bmatrix} 34.1005 & -0.7033 \\ -0.7033 & 34.2023 \end{bmatrix}, \qquad Q = \begin{bmatrix} 15.0141 & 1.0315 \\ 1.0315 & 15.0435 \end{bmatrix},$$

$$R = \begin{bmatrix} 16.7130 & 1.1709 \\ 1.1709 & 16.9450 \end{bmatrix}, \qquad S = \begin{bmatrix} 4.4609 & 0 \\ 0 & 4.4575 \end{bmatrix},$$

$$X = \begin{bmatrix} 15.4397 & 0 \\ 0 & 15.5464 \end{bmatrix}, Y = \begin{bmatrix} 3.0558 & 0 \\ 0 & 3.0460 \end{bmatrix}.$$

If we let, $\tau(t) = 0.9 + 0.9 sint$, it is obvious that in this case $d = 0.5$ and $\bar{\tau} = 1.8$. The condition in [39] are not feasible, but using Theorem 3.4 in this paper we can find that the system in this example is globally, asymptotically stable. Therefore our method is less conservative than that in [39].

## 5. Conclusion

In this paper, we have performed the robust stability analysis for a class of uncertain stochastic fuzzy impulsive recurrent neural network with time varying delays. Some new stability criteria have been presented to guarantee the recurrent neural network to be robustly asymptotically stable in the mean square. Linear matrix inequality(LMI) approach has been used to solve the underlying problem. The applicability of the derived results have been demonstrated through the numerical examples for the effectiveness. Comparison results shows that the obtained results in the proposed method are less conservative than those derived in existing results.

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M. Syed Ali, Department of Mathematics, Thiruvalluvar University, Vellore - 632 106, Tamilnadu, India

*E-mail address*: syedgru@gmail.com