FUZZY AHP METHOD FOR PLANT SPECIES SELECTION IN MINE RECLAMATION PLANS: CASE STUDY SUNGUN COPPER MINE

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Abstract. All steps of a mining project generally disturb the environment’s original condition during construction. Therefore, it is crucial to implement an appropriate mine reclamation plan throughout all mine planning stages from sustainable and environmental point of view. Planting the suitable plant species in each step of any reclamation plan and each area is one of the necessities in this respect. Selecting of plant species is carried out on the basis of the primary factors. After that priorities between the selected species are defined on account of the secondary factors by a MCDM model. In this regard, a Fuzzy AHP approach was used. This method was applied to Sungun open pit copper mine in Iran as a case study. Decision making was conducted on the basis of oral judgments and group expertise in the case study. The results achieved from the analyses showed that the priorities of alternatives are as Maple, Ash, Oak, Barberry, Paliurus spina-Christi, Sloe, respectively.

1. Introduction

Today, one of the highly important issues for developing and even industrialized countries, is environmental considerations. In addition progress and prosperity, in new established mines areas will necessarily exist environmental effects. A mine reclamation plan is started from the beginning of the preliminary exploration steps and continues through other mining activities until the final phase of mine life namely “mine closure”. Consequently unique part of a mining project which continues the activity after mine closure is reclamation of mined areas. In this plan all areas which have been affected by mining activities need to be renewed properly. So all the disturbed areas are considered as mined lands. Whole mined lands have to be rehabilitated to pre-mining condition or providing conditions for other post mining land uses. Mine Reclamation, is a necessary step, in order to post mining land use, planting and create green space for the region. Thus selection of plant types, is one of the major steps to achieve the goals of a reclamation program. Superior Plant Type Selection in every reclamation program, have many benefits, including: health protection and environment restoring, preparing suitable perspective for the region, economic benefits, welfare for local people, pollution reduction of soil, water and air, protection of underground water supply, prevention of soil erosion. In
all types of post mining land use such as agriculture, Pasture, Forestry, Tourist attraction, Residency Facilities and Wild life creation, selection and planting the appropriate plant species is one the most essential requirements to implement a successful mined land reclamation plan. An up-to-date approach considers the mine reclamation plan as a multi-criteria decision-making (MCDM) problem with a number of alternatives that have to be ranked considering many different and conflicting criteria. The advantage of these methods is that they can account for both financial and non-financial impacts. Among these methods, the most popular ones are scoring models, analytic hierarchy process - AHP, analytic network process - ANP, axiomatic design - AD, utility models, TOPSIS, ELECTRE and PROMETHEE. It is essential to develop all the elements related to the situation of MCDM in detail before selecting an appropriate MCDM method to solve the problem under consideration [8, 22]. People often use knowledge that is imprecise rather than precise. The fuzzy set theory approaches could resemble human reasoning in using approximate information and uncertainty to generate decisions. It was specifically designed to mathematically represent uncertainty and vagueness and provide formalized tools for dealing with the imprecision intrinsic to many problems [18]. Multi-criteria decision making (MCDM) methods, such as AHP and fuzzy AHP, which are used for mine reclamation method selection problems in the literature, make the evaluations using the same evaluation scale and preference functions on the criteria basis. There are several examples in the literature that have assessed suitability of mined lands for post-mining land-uses, and mentioned the effective criteria in the evaluations. For instance, Coppin and Bradshaw (1982); Askenasy et al. (1998); Howat (2000); Tafi et al. (2006) and Carrick and Kruger (2007) have evaluated the factors limiting plant growth on mined soils and mentioned the most serious soil limitations [12, 10]. Soltanmohammadi et al., (2010) have used a combination of AHP and TOPSIS techniques to determine a preference ranking list for possible post-mining land uses of a hypothetical mined land based on the mined land suitability analysis framework [26]. Alavi et al., in 2012 have selected, proper plant species for Sarcheshmeh Copper Mine Reclamation plan [2]. Bangian and Osanloo have selected proper plant species for Sungun Copper Mine reclamation by traditional AHP method [5].

In this research study, the proper plant selection was carrying out for sungun mine of Iran using Fuzzy-AHP methods. In this respect the proper alternatives which have been selected on the basis of primary factors have to be assessed based upon some criteria as secondary factors. Finally the priorities of the alternatives will be defined by a Multi Criteria Decision Making model due to secondary factors. In this method the criteria which effect on the definition of the priorities are considered as attributive indexes.

2. Material and Method

2.1. Case Study. Sungun copper mine is located in northern east of Iran (East Azerbaijan province). The proven reserve of ore deposit is about 380 and total probable and possible reserves is about 1000 million tons. Average grade of the deposit is 0.67 percent and overall Stripping Ratio (OSR) is found to be 1.63 : 1.
The planned ore production rate at the end year 2010 is 7 million tons per year, and from year 2011 it has increased to 14 million tons per year. The mined land area of Sungun copper mine during mining operations is estimated about 38 square kilometers. The mine is situated in mountainous territory. The highest and lowest altitudes from sea level are 2460m and 1700, respectively. For that reason there are big differences in height (about 750 m) and topography Sungun is an open pit mine with severe climate condition. Temperature is low with moderate humidity. There are various flora and fairly compact natural vegetation [5].

2.2. Effective Factors on Selecting of Plant Species in Mine Reclamation Plans. Effective factors on selecting of plant species include two groups of considerations. The first group which is called primary factor play the main role in selecting the consistent species with the condition of application. The second group of the considerations which is called secondary factor play the side role to evaluate and define priorities of the alternatives which have consistency with primary factors [24].

2.2.1. Primary Factors. Primary factors are as follow:
- Type of post mining land use
- Regional climate condition
- Local soil nature

These considerations mainly play the fundamental role of selecting the proper alternatives. All species which have consistency with the entire three primary factors can be viable as appropriate alternatives.

Post mining land use. Appropriate post mining land use of Sungun copper mine has been suggested as forestry in another research [1]. In the present research because of the forestry post mining land use, plant species would be chosen just those trees species which are fitted with the forestry.

Climate condition. As above mentioned Sungun copper mine’s region has mountainous climate condition. The temperature in the warmest season is nearly 15°C and in the coldest season is about −20°C with relative humidity about 80 to 85 percent. Furthermore, the region’s annual rain fall is about 350 to 400 mm. Thereupon choosing of tree species in the project would be met the basis of climate condition include regularity with longtime wintriness period, strong stability in glacial, high rate relative humidity, relatively low raining and high rate snowing.

Soil nature. Soil nature in all areas of the mine include as follow:
(1) Mined lands, Waste dumps and tailing disposal dams: The amount of pyrite presence and acidic nature in mined land in order to actual considerations is about 0.5 percent. Main minerals of the ore body are formed by Copper and Molybdenum. The soil of the mined lands contains high grade of these metals. Waste rocks which are hauled to waste dumps include copper with grade lower than 0.1 percent. The main metallic minerals in this ore body are Chalcopyrite, Bournite, Malachite, Pyrite, Molybdenite, Magnetite, Gallen and Sphalerite.
(2) Borders of the project, mined land in equipment’s repairing area, depreciate and
used metals storage region, Equipment parking, access roads and the foundation area of the destroyed constructions:
The parts nature, may be neutral or alkaline [5]. With regard to the mentioned properties of the soil, plant species is proposed to have consistency and viability against the special conditions. With regard to Primary factors the available and possible plat types selected as maple, Ash, Oak, Barberry, Paliurus spina -Christi, Sloe.

2.2.2. **Secondary Factors.** The alternatives selected on the basis of the primary factors should be evaluated by the secondary factors in the next step. Therefore in this procedure in order to use secondary factors criteria to assess the alternatives, a model of Multi Criteria Decision Making (MCDM) is created [23]. By [3], the Criteria are as follow:

C1) Perspective of the region
C2) resistance against disease and insects
C3) strength and method of growth
C4) availability to plant type
C5) economic efficiency
C6) Protection of soil and storing water
C7) prevention from pollution.

Considering the listed factors and expert opinions, questionnaires were prepared such that, for example, criteria questionnaire toward goal and alternatives importance questionnaire than the sixth criteria (Protection of soil and storing water) are shown in Table 1 and Table 2. Below, the important coefficient of questionnaire for both qualitative and quantitative is given. The importance of qualitative is displaced by quantitative. Fuzzy numbers are classified in five classes; the very low [1, 2, 3], low [2, 3, 5], medium [3, 5, 7], high [5, 7, 9], very high [7, 9, 9].

<table>
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<th>Importance /criteria</th>
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**Table 1.** Criteria Questionnaire Respect to Goal

With respect to the above description, proper alternatives were defined according to the primary factors and original properties of the types of the species. Then secondary factors as attributes are applied to rank the priorities between alternatives. In this procedure fuzzy analytical hierarchy processing (F.AHP) is utilized to classify more appropriate alternatives.

2.3. **Fuzzy Sets.** In order to deal with vagueness of human thought, Zadeh [31] first introduced the fuzzy set theory. A fuzzy set is a class of objects with a continuum of grades of membership. Such a set is characterized by a membership
function which assigns to each object a grade of membership ranging between zero and one [31]. A fuzzy set is an extension of a crisp set. Crisp sets only allow full membership or non-membership at all, whereas fuzzy sets allow partial membership. In other words, an element may partially belong to a fuzzy set [14]. Fuzzy sets and fuzzy logic are powerful mathematical tools for modeling: uncertain systems in industry, nature and humanity, and facilitators for commonsense reasoning in decision-making in the absence of complete and precise information. Their role is significant when applied to complex phenomena not easily described by traditional mathematical methods, especially when the goal is to find a good approximate solution [7]. Fuzzy sets theory providing a more widely frame than classic sets theory, has been contributing to capability of reflecting real world [16]. Modeling using fuzzy sets has proven to be an effective way for formulating decision problems where the information available is subjective and imprecise [33]. Triangular fuzzy numbers can be defined as a triplet \((l, m, u)\). The parameters \(l, m\), and \(u\), respectively, indicate the smallest possible value, the most promising value, and the largest possible value that describe a fuzzy event [15].

### 2.4. Linguistic Variable

A linguistic variable is a variable whose values are words or sentences in a natural or artificial language [32]. As an illustration, age is a linguistic variable if its values are assumed to be the fuzzy variables labeled young, not young, very young, not very young, etc. rather than the numbers 0, 1, 2, 3, ... \([6]\). The concept of a linguistic variable provides a mean of approximate characterization of phenomena which is too complex or too ill-defined to be amenable to description in conventional quantitative terms. The main applications of the linguistic approach lie in the realm of humanistic systems-especially in the fields of artificial intelligence, linguistics, human decision processes, pattern recognition, psychology, law, medical diagnosis, information retrieval, economics and related areas [32].

### 2.5. Fuzzy Numbers

A fuzzy number \([33]\) \(\tilde{M}\) is a convex normalized fuzzy set \(\mu\) of the real line \(\mathbb{R}\) such that:

\(i\) There exists \(x_0 \in \mathbb{R}\) with \(\mu_{\tilde{M}}(x_0) = 1\) \((x_0\) is called mean value of \(\tilde{M}\));

\(ii\) \(\mu_{\tilde{M}}(x)\) is piecewise continuous.

It is possible to use different fuzzy numbers according to the situation. In applications it is often convenient to work with triangular fuzzy numbers (TFNs) because of their computational simplicity, and also they are useful in promoting

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<td>paliurus spina-Christi</td>
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<td>sloe</td>
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Table 2. Alternatives Questionnaire Respect to C3
representation and information processing in a fuzzy environment. In this study TFNs are adopted in the fuzzy AHP and fuzzy TOPSIS methods. There are various operations on triangular fuzzy numbers. In this paper, for two positive triangular fuzzy numbers \((l_1, m_1, u_1)\) and \((l_2, m_2, u_2)\), we define:

\[
(l_1, m_1, u_1) + (l_2, m_2, u_2) = (l_1 + l_2, m_1 + m_2, u_1 + u_2)
\]

\[
(l_1, m_1, u_1) \cdot (l_2, m_2, u_2) = (l_1 \cdot l_2, m_1 \cdot m_2, u_1 \cdot u_2)
\]

\[
(l_1, m_1, u_1)^{-1} = (1/u_1, 1/m_1, 1/l_1)
\]

\[
(l_1, m_1, u_1) \cdot K = (l_1 \cdot k, m_1 \cdot k, u_1 \cdot k)
\]

which \(K\) is a positive real number.

2.6. Fuzzy AHP Method. The Analytic Hierarchy Process (AHP) is a method developed by Saaty in 1980 to support multi-criteria decision making [25]. This method is used in the decision making process to help people setting priorities among the alternatives and making better decisions by taking into account qualitative and quantitative aspects of the decision. In 2001, Lee et al. defined the AHP as a quantitative technique that facilitates the structuring of a complex multi-attribute problem and provides an objective methodology that is applied to a wide variety of decisions in the human judgment process [21]. The AHP involves decomposing a complex MCDM problem into a multi-level hierarchical structure of objectives, criteria and alternatives. Decomposition into a hierarchy is based on previous studies, research and empirical experiences. Once the hierarchy has been developed, one moves to assess the relative importance of decision criteria, then compare the decision alternatives with respect to each criterion, and finally determine the overall priority for each decision alternative and the overall ranking of the decision alternatives. The assessment of the relative importance of decision criteria and the comparison of decision alternatives with respect to each criterion is done by a pair-wise comparison, which involves the following three tasks:

- developing a comparison matrix at each level of the hierarchy, starting from the second level and going down;
- computing the relative weights for each element of the hierarchy;
- estimating the consistency ratio to check the consistency of the judgment.

Let \(\{A_1, A_2, ..., A_n\}\) be \(n\) alternatives, and \(\{W_1, W_2, ..., W_n\}\) be their current weights. The pair-wise comparison is conducted by asking a decision-maker or experts questions, such as which criterion is more important with regard to the decision goal and by what scale (1 – 9). The answers to these questions are forming a pair wise comparison matrix that can be defined as follows:

\[
W = [W_i/W_j] = \begin{bmatrix}
W_1 & W_2 & \cdots & W_1 \\
W_2 & W_2 & \cdots & W_2 \\
\vdots & \vdots & \ddots & \vdots \\
W_n & W_n & \cdots & W_n
\end{bmatrix}
\]
The matrix of the pair wise comparison \( A = [a_{ij}] \) represents the value of the expert’s preference among individual pairs of alternatives \( (A_i \text{ versus } A_j \text{ for all } i,j = 1,2,...,n) \). After this, the decision-maker compares pairs of alternatives for all possible pairs, and the comparison matrix \( A \) is obtained, where the element \( a_{ij} \) shows the preference weight of \( A_i \) obtained by comparison with \( A_j \).

\[
A = [a_{ij}] = \begin{bmatrix}
1 & a_{12} & \cdots & a_{1n} \\
\frac{1}{a_{12}} & 1 & \cdots & a_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{1}{a_{1n}} & \frac{1}{a_{2n}} & \cdots & 1
\end{bmatrix}
\] (6)

\( a_{ij} \) estimate the ratios \( w_i/w_j \), where \( w \) is the vector of current weights of the alternative (which is our goal). The matrix has reciprocal properties, which are \( a_{ji} = \frac{1}{a_{ij}} \). After all pair-wise comparison matrices are formed, the vector of weights \( w = [w_1, w_2, ..., w_n] \) is computed on the basis of Satty’s eigenvector procedure in two steps. First, the pair-wise comparison matrix, \( A = [a_{ij}]_{n \times n} \), is normalized, and then the weights are computed.

Normalization: \( a_{ij}^n = a_{ij}/\sum_{i=1}^{n} a_{ij} \), for all \( j = 1, ..., n \). (7)

Weight calculation: \( w_i = \left( \sum_{i=1}^{n} a_{ij}^n \right)/n \), for all \( j = 1, ..., n \). (8)

For a valid comparison, we need to check the consistency of the pair-wise matrix (CI):

\[
CI = (\lambda_{\text{max}} - n)/(n - 1)
\] (9)

Where \( \lambda_{\text{max}} \) is an important validating parameter in AHP and is used as a reference index to screen information by calculating the Consistency Ratio (CR) of the estimated vector. \( CR \) can be calculated using the following equation:

\[
CR = CI/RI
\] (10)

where \( RI \) is the random consistency index obtained from a randomly generated pair-wise comparison matrix. If \( CR < 0.1 \), then the comparisons are acceptable. If \( CR > 0.1 \), the values of the ratio are indicative of inconsistent judgments. In this case, the original values in the pair-wise comparison matrix \( A \) should be reconsidered and revised.

Determining the overall priority for each decision alternative and the overall ranking of decision alternatives is done by synthesizing the results over all levels. The weighted priorities of the decision alternatives are added components in order to obtain an overall weight \( (w Ai) \) or priority of each alternative over the entire hierarchy.

Fuzzy AHP In the conventional AHP, the pair wise comparisons for each level with respect to the goal of the best alternative selection are conducted using a nine-point scale. So, by [19], the application of Saaty’s AHP has some shortcomings as follows:

1. The AHP method is mainly used in nearly crisp decision applications;
2. The AHP method creates and deals with a very unbalanced scale of judgment;
The AHP method does not take into account the uncertainty associated with the mapping of one’s judgment to a number;
(4) Ranking of the AHP method is rather imprecise;
(5) The subjective judgment, selection and preference of decision-makers have evaluating alternatives always contain ambiguity and multiplicity of meaning.

Furthermore, it is also recognized that human assessment on qualitative attributes is always subjective and thus imprecise. Therefore, conventional AHP seems inadequate to capture decision maker’s requirements explicitly [19]. In order to model this kind of uncertainty in human preference, fuzzy sets could be incorporated with the pair wise comparison as an extension of AHP. A variant of AHP, called Fuzzy AHP, comes into implementation in order to overcome the compensatory approach and the inability of the AHP in handling linguistic variables. The fuzzy AHP approach allows a more accurate description of the decision making process. The analytic hierarchy process, since its invention, has been a tool at the hands of decision-makers and researchers, becoming one of the most widely used multiple criteria decision-making tools [28]. Although the purpose of AHP is to capture the expert’s knowledge, the traditional AHP still cannot really reflect the human thinking style [20]. The traditional AHP method is problematic in that it uses an exact value to express the decision-makers opinion in a comparison of alternatives [30]. AHP method is often criticized, due to its use of unbalanced scale of judgments and its inability to adequately handle the inherent uncertainty and imprecision in the pair-wise comparison process [13]. To overcome all these shortcomings, fuzzy analytical hierarchy process was developed for solving the hierarchical problems. Decision-makers usually find that it is more accurate to give interval judgments than fixed value judgments. This is because usually he/she is unable to make his/her preference explicitly about the fuzzy nature of the comparison process [20]. The first study of fuzzy AHP is proposed by Van Laarhoven and Pedrycz (1983) which compared fuzzy ratios described by triangular fuzzy numbers that is shown Figure 1 [30]. The linguistic scale of traditional AHP method could express the fuzzy uncertainty when a decision maker is making a decision. Therefore, FAHP converts the opinions of experts from previous definite values to fuzzy numbers and membership functions, presents triangular fuzzy numbers in paired comparison of matrices to develop FAHP, thus the opinions of experts approach human thinking model, so as to achieve more reasonable evaluation criteria. Buckley (1985) initiated trapezoidal fuzzy numbers to express the decision-makers evaluation on alternatives with respect to each criterion [29]. Chang (1996) introduced a new approach for handling fuzzy AHP, with the use of triangular fuzzy numbers for pair-wise comparison scale of fuzzy AHP, and the use of the extent analysis method for the synthetic extent values of the pair-wise comparisons [11].

In this study the extent fuzzy AHP is utilized, which was originally introduced by Chang. Let \( X = \{x_1, x_2, x_3, ..., x_n\} \) be an object set, and \( G = \{g_1, g_2, g_3, ..., g_m\} \) be a goal set. Then, each object is taken and extent analysis for each goal is performed, respectively. Therefore, in extent analysis values for each object can be obtained, with the following signs the algorithm of this method can be described as follows [23]:

\[
M_{gi}^1, M_{gi}^2, ..., M_{gi}^m \quad (i = 1, ..., n),
\]
where $M_{gj} (j = 1, \ldots, m)$ all are TFNs. The steps of Chang extent analysis can be given as in the following \[3\]:

**Step 1:** Making hierarchy

**Step 2:** Making comparison dual matrix

$$A = \begin{bmatrix}
\tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\
\tilde{a}_{21} & \tilde{a}_{22} & \cdots & \tilde{a}_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
\tilde{a}_{m1} & \tilde{a}_{m2} & \cdots & \tilde{a}_{mn}
\end{bmatrix}$$

**Step 3:** Determination any matrix relative weight: The value of fuzzy synthetic extent with respect to the object is defined as:

$$S_i = \sum_{j=1}^{n} M_{gj} \otimes \left[ \sum_{i=1}^{m} \sum_{j=1}^{m} M_{gj} \right]^{-1}. \quad (11)$$

To obtain $\sum_{j=1}^{m} M_{gj}^j$ (Fuzzy Summation of Row), the fuzzy addition operation of $m$ extent analysis values for a particular matrix is performed such as:

$$\sum_{j=1}^{m} M_{gj}^j = \left( \sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \quad (12)$$

and to obtain $\left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M_{gj}^j \right]^{-1}$, the fuzzy addition operation of $M_{gj}^j$ ($j = 1, \ldots, m$) values is performed such as: Summation of Column.

$$\sum_{i=1}^{n} \sum_{j=1}^{m} M_{gj}^j = \left( \sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i \right). \quad (13)$$

And then the inverse of the vector above is computed, such as:

$$\left[ \sum_{i=1}^{n} \sum_{j=1}^{m} M_{gj}^j \right]^{-1} = \left( \frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right). \quad (14)
Step 4: As $M_1 = (l_1, m_1, u_1)$ and $M_2 = (l_2, m_2, u_2)$ are two triangular fuzzy numbers, the degree of possibility of $M_2 = (l_2, m_2, u_2) \geq M_1 = (l_1, m_1, u_1)$ is defined as:

$$V(M_2 \geq M_1) = \sup_{y \geq x} \{ \min \{ u_{M_2(x)}, u_{M_2(y)} \} \}.$$  \hspace{1cm} (15)

and can be expressed as follows:

$$V(M_2 \geq M_1) = \text{hg}(M_1 \cap M_2) = u_{M_2(d)}$$  \hspace{1cm} (16)

that $V(M_2 \geq M_1) = \text{Bigness degree}$, $M_2 = \text{First S}$, $M_1 = \text{secondary S}$.

Figure 2 illustrates Eq. (16) where $D$ is the ordinate of the highest intersection point $D$ between $M_1$ and $M_2$ to compare $M_1$ and $M_2$, we need both the values of $V(M_1 \geq M_2) \ldots V(M_k \geq M_1)$.

**Figure 2.** The Intersection Between $M_1$ and $M_2$ (Buckley. 1985)

Step 5: The degree possibility for a convex fuzzy number to be greater than $k$ convex fuzzy $M_i$ ($i = 1, 2, \ldots, k$) numbers can be defined by

$$V(M \geq M_1, M_2, \ldots, M_k) = V[(M \geq M_1) \text{ and } (M \geq M_2) \text{ and } \ldots \text{ and } (M \geq M_k)]$$

$$= \min(M \geq M_i), \quad i = 1, 2, 3, \ldots, k.$$  \hspace{1cm} (18)

For $k = 1, 2, \ldots, n$ such that $k \neq i$, the weight vector is given by

$$d'(A_i) = \min V(S_i \geq S_k)$$  \hspace{1cm} (19)

where $A_i$ are $n$ elements ($i = 1, \ldots, n$).

Step 6: Via normalization, the normalized weight vectors are:

$$W' = (d'(A_1), d'(A_2), \ldots, d'(A_n))^T$$  \hspace{1cm} (20)

where $W'$ is a non-fuzzy number.

Step 7: Determination alternatives final weight

$$A_1 = (A_1 \text{ to } C_1) \times (C_1 \text{ to GOAL})$$

$$+ (A_1 \text{ to } C_2) \times (C_2 \text{ to GOAL}) + \cdots + (A_1 \text{ to } C_n) \times (C_n \text{ to GOAL})$$

where $n =$ number of criteria.  \hspace{1cm} (21)
3. Results

According to the outcomes of the research, Multi Criteria Decision Making (MCDM) is an appropriate model and Fuzzy Analytical Hierarchy Processing (FAHP) is a proper technique for selection of plant species in mine reclamation plans. Oral judgment of experts group for the preferences of the alternatives is an effective tool to define the priorities. The First step in the procedure is related to selecting of proper plant species in a mine reclamation plan in accordance with the primary factors. Second step consists of providing oral judgments of experts group, deal with preferences, calculation of local priorities and after, overall priorities of the plant species. Finally, the priorities are created by the reason of oral judgments and group consensus, relating to the secondary factors. With regard to the selection of plant species in sungun copper mine reclamation plan as a case study, results are presented as follows: Comments on the questionnaire are averaged and 8 matrixes were developed through the expert opinion which two matrixes as an example is shown in Tables 3 and 4.

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**Table 3. Matrix of Criteria Respect to Goal**

From Table 4, according to extent analysis synthesis values respect to main goal are calculated like in Eq. (11): \( SA_1 = 0.095, 0.250, 0.505, \) \( SA_2 = 0.080, 0.194, 0.456, \) \( SA_3 = 0.068, 0.139, 0.252, \) \( SA_4 = 0.068, 0.139, 0.252, \) \( SA_5 = 0.068, 0.139, 0.252, \) \( SA_6 = 0.080, 0.139, 0.456. \)
These fuzzy values are compared by using Eq.(17), and next values are obtained: Then priority weights (Min) are calculated by using Eq.(19,20) that is shown in Table 5 and Table 6. Note that every $S_i$ denoted by $i$ in Table 5.

$$V(1 \geq 2) = 1$$
$$V(2 \geq 1) = 0.736$$
$$V(3 \geq 1) = 0.736$$
$$V(4 \geq 1) = 0.424$$
$$V(5 \geq 1) = 0.424$$
$$V(6 \geq 1) = 0.424$$

**Table 5. The Degree of Possibility in Table 4**

After normalizing these values, priority weight respect to main goal is calculated in Table 6.

$$d'(A_i)$$

<table>
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<tr>
<th></th>
<th>$A_1$</th>
<th>$A_2$</th>
<th>$A_3$</th>
<th>$A_4$</th>
<th>$A_5$</th>
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**Table 6. Un-normalized Weight and Normalized Weight**

To assure a certain quality level of a decision, we have to analyze the consistency of an evaluation. To do so, we calculate the consistency ratio ($CR$) confirming Saaty, which is defined as a ratio between the consistency of a given evaluation
matrix (consistency index $CI$) and the consistency of a random matrix (for Saaty’s scale using numerical judgments from $1/9, 1/8, \ldots, 1, 2, \ldots, 9$, Saaty quotes for a 55-reciprocal matrix a random consistency $RC$ of 1.11). The $CR$ of a decision should not exceed a certain level, namely 0.1 for a matrix larger than 4 by 4 (Saaty, 1995, 81). Therefore, we decided to include only evaluations which fulfill the condition $CR \leq 0.1$. If the $CR$ exceeded the tolerable level of 0.1, we excluded the pairwise comparison matrix of this respondent for further analysis, because this could affect the overall results negatively. Inconsistency ratio can be checked and the consistency ratio ($CR$) has to be calculated. The results obtained are: largest Eigen value of matrix, $\lambda_{max} = 5.323$; Consistency Index ($C.I.$) = 0.08075; Randomly Generated Consistency Index ($R.I.$) = 1.12 and Consistency Ratio ($C.R.$) = 0.0721. As $CR < 0.1$ the level of inconsistency present in the information stored in comparison matrix is satisfactory.

According to the fuzzy AHP, the best alternative is $A_1$ and the ranking order of the alternatives is $A_1 > A_3 > A_2 > A_4 > A_5 > A_6$. Finally general results are shown in Figure 3.

**Figure 3. Priority of Plants Types in Sungun Mine Reclamation Plan**

## 4. Conclusion

Analytical Hierarchy Processing is a powerful and comprehensive methodology designed to facilitate sound decision making by using empirical data as well as subjective group and oral judgments of the experts and decision maker. Fuzzy AHP method assists with the decision making process by providing decision makers with a structure to organize and evaluate the importance of various criteria and the preferences of alternatives solutions to a decision. The traditional AHP method cannot really reflect the human thinking style and is problematic in that it uses an exact value to express the decision maker opinion in a comparison of alternatives. Therefore, fuzzy AHP, a fuzzy extension of AHP, was developed to solve the hierarchical fuzzy problems. In the fuzzy-AHP procedure, the pair wise comparisons in the judgment matrix are fuzzy numbers that are modified by the designer’s emphasis. All alternatives are pair wise compared on the basis of each attribute separately. The comparisons are carried out by reason of experts’ judgments and oral group decision making. After all pair wise comparisons between alternatives due to all 8 criteria the criteria are pairing wise compared by them. As a result in the first stage of all pair wise comparisons, each alternative and each criterion obtains a Local priority. In the next step of the method by Synthesis
of local priorities of the alternatives and criteria. Overall priority of each alternative is generated. Finally the overall priorities identify the preferences between proper plant species for reclamation plan of each area of mined land. According to the outcomes of the research, Multi criteria Decision Making (MCDM) is an appropriate model and Fuzzy Analytical Hierarchy Processing (F.AHP) is a proper technique to select plant species in mines’ reclamation plans. Decision-makers face up to the uncertainty and vagueness from subjective perceptions and experiences in the decision making process. By using fuzzy AHP, uncertainty and vagueness from subjective perception and the experiences of decision-maker can be effectively represented and reached to a more effective decision. Firstly around the Sungun copper mine area, are surveyed, for choosing the best plant type. Then a series of tests, including testing of soil, water and native plants growing in the area are performed. Samplings in this study were several cresive plant types near the Sungun copper mine, which have been evaluated. Oral judgment of group expertise which deals with the preferences of the alternatives is an effective tool to define the priorities. In fuzzy AHP, decision-makers make pair-wise comparisons for the criteria and alternatives under each criterion. Then these comparisons integrated and decision-makers’ pair-wise comparison values are transformed into triangular fuzzy numbers. The priority weights of criteria and alternatives are determined by Chang extent analysis. According to the combination of the priority weights of criteria and alternatives, the best alternative is determined. The first step of this procedure includes selecting proper plant species relating to the primary factors of plant species selection in a mine reclamation plan. The second step consists of providing oral judgments of group expertise deal with the preferences, calculate local priorities and then overall priorities of the plant species. Finally the priorities are created by the reason of oral judgments and group consensus relating to the secondary factors. According to opinions of the natural resources and environmental administration experts and Sungun mining engineers’ expertise in questionnaire form and by using fuzzy AHP method, results show that the best plant types according to the region conditions and criteria, is maple. However, Ash, Oak, Barberry, Paliurus spina-Christi, Sloe, are also relatively suitable. However, due to the dire environmental situation around the mine, planting the plant types should be placed in Sungun mine according to priority, to absorb different pollutions and harmful elements. It also gives certain beauty to the surrounding landscape. Technicality Notice that, all selective plants are native and have consistency with sungun local environment, soil and climate condition.

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References


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