

## MINIMUM TIME SWING UP AND STABILIZATION OF ROTARY INVERTED PENDULUM USING PULSE STEP CONTROL

P. MELBA MARY AND N. S. MARIMUTHU

**ABSTRACT.** This paper proposes an approach for the minimum time swing up of a rotary inverted pendulum. Our rotary inverted pendulum is supported by a pivot arm. The pivot arm rotates in a horizontal plane by means of a servo motor. The opposite end of the arm is instrumented with a joint whose axis is along the radial direction of the motor. A pendulum is suspended at the joint. The task is to design a controller that swings up the pendulum, keeps it upright and maintains the arm position. In the general intelligent hybrid controller, a PD controller with a positive feedback is designed for swinging up and a fuzzy balance controller is designed for stabilization. In order to achieve the swing up in a minimal time, a controller named Minimum Time Intelligent Hybrid Controller is proposed which is precisely a PD controller together with a pulse step controller for swinging up along with the fuzzy balance controller for stabilization. The impulsive control action is tuned by trial and error to achieve the minimum-time swing-up. An energy based switching control method is proposed to switch over from swing up mode to stabilization mode. Extensive computer simulation results demonstrate that the swing up time of the proposed minimum-time controller is significantly less than that of the existing general hybrid nonlinear controller.

### 1. Introduction

The Rotary Inverted Pendulum (RIP) system is a complex, multivariable, non-minimum phase and unstable, electromechanical system with severe nonlinearity and obtaining its stabilizing control is a representative problem in the application of the control theory. The control process can reflect many problems, such as stabilization problems, nonlinear problems and robust problems.

The control strategy of an RIP system is composed of the swing up control of the pendulum and stabilizing control of the whole system, comprising the angular control of the pendulum in the upright position and the position control of the rotating arm to its initial position while balancing. First, the swing up control brings the pendulum from the downward position to the upright position. This is achieved by repeatedly giving a voltage of appropriate magnitude and direction to the motor to drive the arm clockwise or anticlockwise until the pendulum is close to the upright position. Thereafter, the stabilizing control balances the pendulum in the upright position.

---

Received: March 2008; Revised: February 2009; Accepted: May 2009

*Key words and phrases:* Fuzzy logic control (FLC), Rotary inverted pendulum (RIP), Swing up control, PD control, Energy based switching control, Hybrid control and Minimum-time control.

In theoretical study, a swing up inverted pendulum has many advantages such as simple structure, nonlinear, and uncertain characteristics. Many control methods, for example, swing up using fuzzy control algorithm [16], robust swing up control [14], nonlinear control [10], and energy based methods [19], [1], have been reported for the swing up of an inverted pendulum. Astrom and Furuta showed that the global behavior is characterized by the ratio of the maximum acceleration of the pivot to the acceleration of gravity. Though good performance can be obtained by using these algorithms, they are very complicated to implement and their parameters are not easy to design. In [18] and [17] a fuzzy controller based on single input rule modules (SIRMs) was presented, in which each input term was assigned a SIRM and a dynamic importance degree. In [6] a fuzzy swing up control combined with an LQR stabilizing control was proposed, where the influence of disturbance was discussed with an adaptive state controller. Lin and Mon proposed a hierarchical fuzzy sliding controller to achieve decoupling performance [5]. Lam etc. investigated the stability with TSK fuzzy logic associated with feedback gains tuned by a genetic algorithm [3]. An observer-based hybrid adaptive fuzzy neural network controller combined with a supervisory controller was presented in [12]. Recently, hybrid-control, an integration of several methods, has attracted attention. In [7], a hybrid PD servo feedback controller designed by pole placement method has been reported. It is known that a control system assigned by the pole placement method may not be optimal. In another paper[8], a hybrid controller is proposed where the part of servo state feedback controller designed by linear quadratic regulator(LQR) problem for swing up and the inverted pendulum is stabilized on cart. Though it is considered good, optimal and robust, the inverted pendulum poses serious problems for qualitative modeling methods, and hence is a good benchmark to test their performance. Our approach to this task consists of deriving control rules from the actions of a human operator stabilizing the pendulum and subsequently using them for automatic control by means of fuzzy intelligent techniques. Rule derivation is based on the “learning from examples” principle and does not require knowledge of a quantitative model of the system. Next, we seek to apply PD controller for swing up. Based on our simulation experience, we found that it is hard to achieve the minimum time swing up control by only tuning PD parameters, so to achieve minimum time swing up, we propose to add a pulse-step feed forward control which is tuned by several trial and error tuning methods. During our simulation, we also applied a simple energy mode switch control to change over from swing up mode to stabilization mode. This paper is organized as follows. The mathematical modeling of the Rotary Pendulum is presented in section 2. The positive feedback PD algorithm for swing up process is described in section 3. Section 4 is devoted to the design of stabilizing controller using fuzzy techniques. Switching control is discussed in section 5. Section 6 is allocated to the design of swing up process using PD controller along with pulse step control. Simulation results are presented in section 7 and, finally, section 8 contains a conclusion.

## 2. Dynamic Model of the Rotary Pendulum

**2.1. Description of the System.** The rotary pendulum system consists of a rotary servo motor system which drives an independent output gear. A rotary pendulum arm of radius 'r' is mounted to the output gear and a pendulum of length 'L' and mass 'm' is attached to the hinge. Clearly, this is an under-actuated mechanical system. The pivot arm of radius 'r' rotates in a horizontal plane by means of a servo motor and must be moved in such a way that the pendulum, which rotates in a plane that is always perpendicular to the rotating arm (Figure 1), is in the upright position.

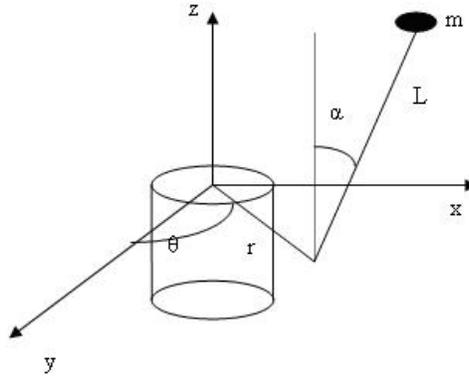


FIGURE 1. Simplified Schematic Picture of Rotary Inverted Pendulum

In this figure,  $\alpha$  denotes the angle of the pendulum to the upright position and  $\theta$  denotes the angle of the rotor arm. The angle of the pendulum  $\alpha$  is defined to be zero when in the upright position and positive when the pendulum is moving clockwise. Similarly, the angle of the arm  $\theta$  is positive when the arm is moving in the clockwise direction. The purpose is to design a controller that starts with the pendulum in the “down”hanging position, swings it “up” and maintains it upright.

**2.2. Derivation of Servo Motor Model.** The transfer function of the servo motor plant is developed as in the block diagram as shown in Figure 2.

$$\frac{\theta_0(S)}{V_i(S)} = \frac{\frac{\eta K_m K_g}{R_a J_{eq}}}{s \left( s + \frac{B_{eq}}{J_{eq}} + \frac{\eta K_m^2 K_g^2}{R_a J_{eq}} \right)} \quad (1)$$

$$\frac{\theta_0(S)}{V_i(S)} = \frac{a_m}{s(s + b_m)} \quad (2)$$

where

$$a_m = \frac{\eta K_m K_g}{R_a J_{eq}} \text{ and} \quad (3)$$

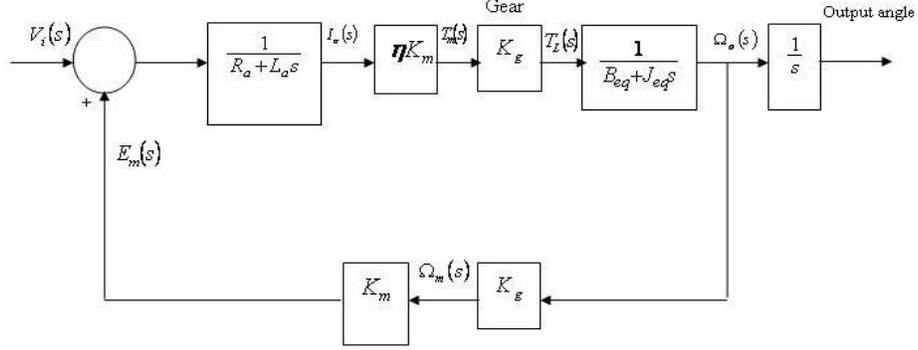


FIGURE 2. Block Diagram of Servo Motor

$$b_m = \frac{B_{eq}}{J_{eq}} + \frac{\eta K_m^2 K_g^2}{R_a J_{eq}} \quad (4)$$

Also from the block diagram, we see that the output torque  $T_L$ , in the s-domain is

$$T_L(s) = \frac{\eta K_m K_g}{R_a} [V_i(s) - K_m K_g \Omega_o(s)], \quad (5)$$

where  $\Omega_o(s)$  is the angular velocity of the arm in the s-domain and

$$T_L(t) = K_1 v_i(t) - K_2 \omega(t), \quad (6)$$

where

$$K_1 = \frac{\eta K_m K_g}{R_a} \text{ and} \quad (7)$$

$$K_2 = \frac{\eta K_m^2 K_g^2}{R_a} \quad (8)$$

The parameters used for servo motor model [2] are defined in Table 1.

**2.3. Derivation of Inverted Pendulum Model.** To derive a dynamic system model, coordinate frame systems are introduced as shown in Figure 3. With standard assumptions such as zero friction, rigid objects etc, the dynamic model is as follows:

$$(J_{eq} + mr^2) \dot{\omega} - \frac{1}{2} m L r \cos \alpha \dot{v} + \frac{1}{2} m L r \sin \alpha v^2 + B_{eq} \omega = T_L \quad (9)$$

$$\frac{1}{3} m L^2 \dot{v} - \frac{1}{2} m L r \cos \alpha \dot{\omega} - \frac{1}{2} m g L \sin \alpha = 0 \quad (10)$$

The above equations which describe the dynamics of the model are quite non linear.

Symbol	Description	value	unit
$R_a$	Armature resistance	2.6	$\Omega$
$L_a$	Armature inductance	negligible	
$K_m$	Motor voltage constant	0.00767	V-s/rad
$K_\tau$	Motor torque constant	0.00767	N-m/A
$J_m$	Armature inertia	$3.87 \times 10^{-7}$	Kg m <sup>2</sup>
$J_{tach}$	Tachometer inertia	$0.7 \times 10^{-7}$	Kg m <sup>2</sup>
$K_g$	High gear ratio	(14)(5)	
$B_{eq}$	Equivalent viscous friction referred to the secondary gear	$4 \times 10^{-3}$	Nm/(rad/s)
$\eta_{mr}$	Motor efficiency due to rotational loss	0.87	
$\eta_{gb}$	Gearbox efficiency	0.85	
$J_L$	Load inertia	$5.2823 \times 10^{-5}$	Kg m <sup>2</sup>
$V_i$	Motor input voltage	6	volts
$J_{eq}$	Equivalent inertia	0.0023	Kg m <sup>2</sup>

TABLE 1. Servo Motor Model Parameters

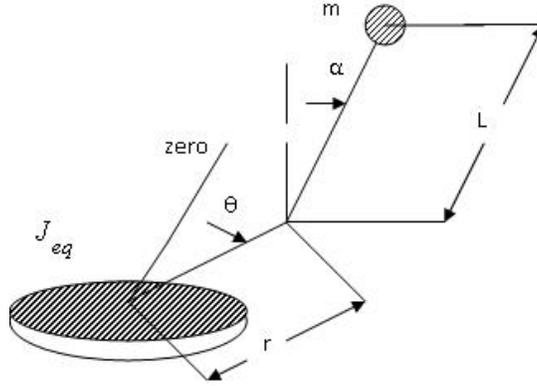


FIGURE 3. An Illustrative Configuration of Swing up Controller

The parameters of the pendulum [2] are defined in Table 2.

**2.4. Linearized Inverted Pendulum Model.** A linear approximation to the non-linear system equations is obtained using the small angle formula.

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (11)$$

$$y(t) = Cx(t) \quad (12)$$

where

$$x(t) = [ \theta \quad \alpha \quad \omega \quad v ]^T \text{ and} \quad (13)$$

**Fixed Parameters**

Symbol	Description	value	unit
m	mass of the pendulum	0.125	Kg
L	pendulum length	16.75	cm
r	length of the arm	21.5	cm
g	Gravitational acceleration	9.8	m/ s <sup>2</sup>

**Variable Parameters**

Symbol	Description
$\alpha$	Pendulum angle
$\theta$	Servo gear angular displacement
$\omega$	Servo gear angular Velocity
$v$	Pendulum angular velocity

TABLE 2. Inverted Pendulum Model Parameters

$$\begin{bmatrix} \dot{\theta} \\ \dot{\alpha} \\ \dot{\omega} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 52.8058 & -22.9637 & 0 \\ 0 & 189.5215 & -44.2137 & 0 \end{bmatrix} \begin{bmatrix} \theta \\ \alpha \\ \omega \\ v \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 40.7813 \\ 78.5192 \end{bmatrix} v_i(t) \quad (14)$$

**3. Swing up Using a Simple PD Positive Feedback Controller**

As stated, the goal of the controller is to swing the pendulum from stable “down” position to the unstable equilibrium “up” position and be balance it there. The overall controller can be divided into 1) the swing up controller, 2) the balancing /stabilizing controller and 3) the catching controller/mode switching controller.

Many different control algorithms ( trajectory tracking, rectangular reference input swing up type, Pulse Width Modulation (PWM) etc.) can be used to perform the swing up control. Energy is gradually added to the system to bring the pendulum to the inverted position. Here, a positive feedback PD controller is proposed because of its simple structure, effectiveness and easy tuning. The block diagram representation of this swing up controller is as shown in Figure 4. We see that it consists of two loops. The outer loop specifies the trajectory for the arm angle and at the same time excites the internal dynamics to swing the pendulum to the balancing position. By moving the arm back and forth, one can eventually bring up the pendulum.

**3.1. Outer loop PD Controller Design.** It is fairly intuitive to design the outer loop as follows:

$$\theta_d = P\alpha + D\dot{\alpha}, \quad (15)$$

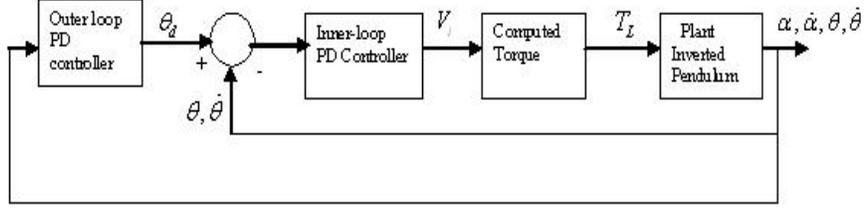


FIGURE 4. Block Diagram Representation of Swing up Controller

where  $\theta_d$  is the given trajectory of the arm and  $\alpha$  is the pendulum angle from the “down position ( positive in the clockwise direction and negative in the counter clockwise direction). Note that  $\alpha$  takes values in the interval  $\pm 180$ (wrapped around).

The values of the two parameters P and D play a key role in bringing up the pendulum smoothly. To prevent the pendulum from colliding with the other components, we need to limit  $\theta$  to the interval  $\pm 90$ . Initially, P is chosen as 0.02 and D is set as 0.015(sec.). P and D can be tuned to adjust the “positive damping” in the system and meet the experiment criteria.

**3.2. Inner loop PD Controller Design.** The inner loop performs the position control of the arm. For the servo arm to track the desired position, a feedback PD controller is designed as follows:

$$V_i(s) = K_p [\theta_d(s) - \theta_o(s)] - K_d \Omega_o(s), \quad (16)$$

where  $K_p$  and  $K_d$  are the parameters to be tuned. First, we must obtain the closed-loop transfer function between the input and the output of the arm angle to satisfy the following performance requirements:

- (1) The percent overshoot must be less than 10%
- (2) The time to first peak  $t_p = 0.115$ sec,

where  $\theta_d$  is position to be tracked. The closed loop transfer function can be expressed as

$$\frac{\theta_o(s)}{\theta_d(s)} = \frac{K_p a_m}{s^2 + (K_d a_m + b_m) s + K_p a_m} \quad (17)$$

According to performance requirements, a natural frequency,  $\omega_n = 45.53$  rad/sec and damping ratio  $\zeta = 0.8$  can be obtained. The closed loop response of the arm is be considerably faster than that of the pendulum and a better compromise between overshoot and transient time can be achieved.

Comparing the coefficients of the closed loop system with a standard second-order characteristic polynomial

$$s^2 + 2\zeta\omega_n s + \omega_n^2, \quad (18)$$

the controller gains are obtained as given below:

$$K_p = \frac{\omega_n^2}{a_m} = 99.0983 \quad (19)$$

and

$$K_d = \frac{2\zeta\omega_n - b_m}{a_m} = 2.8624, \quad (20)$$

where  $a_m$  and  $b_m$  are calculated as in equations 3 and 4.

#### 4. Balance Controller

When the pendulum is almost upright, a stabilizing controller is implemented to maintain it in the upright position and to reject possible external disturbances. Traditional methods such as pole placement and LQR, employ state feed back control to stabilize. We have designed the balance controller using fuzzy logic, because of its robustness and its ability to represent linguistic language in rule form.

**4.1. Fuzzy Balance Controller Design.** For the fuzzy balance controller, the two input variables are alpha and alphasdot and the output variable is the servo motor voltage. Each variable is decomposed into a set of fuzzy regions called labels. The most popular labels are Negative Big (NB), Negative Small (NS), Zero (Z), Positive Small (PS) and Positive Big (PB). Based on experience and understanding of the system characteristics, the membership functions of the premise and consequent parts are assumed to be triangular. Equivalently, when  $\alpha$  and  $\dot{\alpha}$  are NB(PB) and the voltage  $V_i$  needed for the pendulum to rotate clockwise (counterclockwise) to the upright position is PB (NB),  $\alpha$  and  $\dot{\alpha}$  will converge toward Z. The fuzzy inference rules are summarized in Table 3.

$\dot{\alpha}$ \ $\alpha$	<b>NB</b>	<b>NS</b>	<b>Z</b>	<b>PS</b>	<b>PB</b>
<b>NB</b>	PB	PB	PB	PS	Z
<b>NS</b>	PB	PB	PS	Z	NS
<b>Z</b>	PB	PS	Z	NS	NB
<b>PS</b>	PS	Z	NS	NB	NB
<b>PB</b>	Z	NS	NB	NB	NB

TABLE 3. Inference Rules for Fuzzy Balance Controller

## 5. Switching Controller

With both the swing up controller and balance controller complete, a transition algorithm is needed to connect them. The switching/catching controller determines when to switch between the two controllers. It is clear that energy is the right criterion for the mode switching control [13]. Moreover, this energy – based mode switching control is simple to implement and can achieve a much better performance. Hence the switching function for the mode changeover is accomplished by checking the absolute values of alpha less than or equal to zero and the energy E of the system when the pendulum is at its strictly “up position greater than or equal to 0.1(J). If the condition is satisfied balance control will be selected, otherwise the controller will remain in the swing up mode.

## 6. PD Controller and Impulse Step - Controller for Swing Up

Several different strategies can be combined to swing up the pendulum. It follows from the Pontryagin’s maximum principle that to achieve a minimal time swing up the minimal time strategy for the swinging up the pendulum is of the bang-bang type. The complexity of the minimal-time control strategy increases with the order of the system. For a second order plant, a simple pulse-step control can be used to give fast-set point changes and sub- optimal results. This impulsive control is inspired by optimal control theory, but is also deduced from our observations. In other words, if we apply a pulse torque at one end of the pendulum, so that the direction of the torque is the same as the velocity of the pendulum, the pendulum can be expected to swing up more aggressively. Pulse-step control gives good results for the system using simple controllers as illustrated in [11]. The motivation for using impulsive control is well explained in [15]. The analysis and design methods for impulsive control systems can be found in [9] and [4]. Note that the pulse-step control method proposed in this paper is actually an open loop strategy. To make the control strategy more robust, a feedback-feed forward structure may be considered, where the system uncertainty in the system model and the disturbance is compensated by the feedback controller. In our experiment the pulse-step control signal is a step type function of the following form:

$$u_{ff}(t) = \begin{cases} 0 & : \quad \dot{\alpha} = 0 \\ \bar{u} & : \quad \dot{\alpha} < 0 \\ \underline{u} & : \quad \dot{\alpha} > 0, \end{cases} \quad (21)$$

where  $\bar{u}$  and  $\underline{u}$  are the constant amplitude to be further tuned. To make the control stable, the pulse step control signal is added before the inner-loop feedback (Figure 5). We can achieve the optimal objective by selecting the parameters appropriately. Pulse-step control is important to the minimal-time swing up. At the beginning, the amplitude of the pendulum movement is small and so is the amplitude of the arm. At the outset, the energy of the pendulum increases slowly. Applying a pulse control, that is, an additional torque independent of the initial states of the pendulum, the energy accumulating process is accelerated and consequently the swinging up time is reduced. To speed up the swing up, the proposed pulse step

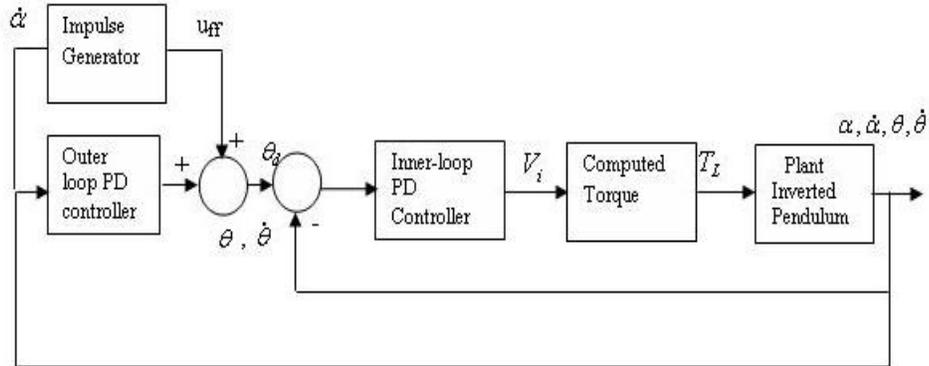


FIGURE 5. Block Diagram of PD Positive Feedback Swing up Control with Step Pulse

control signal  $u_{ff}$  is added to the whole swing up process. This is expected to increase the control effort and pump energy to the pendulum more quickly. The open-loop feed forward control is quite simple. Referring to (21), we use  $\underline{u} = -\bar{u}$ .

The optimal value of the control signal  $\bar{u}$  is decided by the observations in an iterative way. If  $\bar{u}$  is too large, the arm may move too much in one direction and actually reduce the amplitude of oscillations of the pendulum. For our simulation study, the initial value of  $\bar{u}$  was -1 rad/sec and then incrementally increased.

To further explore the possibility of reducing the swing up time, we observe that there is interference within the control signal when the velocity of the pendulum crosses zero. So it is reasonable to add a dead zone block after the velocity signal of the pendulum's arm. When the output of the dead zone block is zero, no pulse control is activated.

## 7. Simulation Results

The simulation study was conducted for the inverted pendulum using General Intelligent Hybrid Controller (GIHC) and Minimum Time Intelligent Hybrid controller (MTIHC) for the dynamic process of swing up; the results obtained by SIMULINK in MATLAB 7.0 are shown in Figures 6 and 7. It can be seen from Figure 6a that the pendulum can be swung up from the natural pendant position to upright position in about 5.2 seconds by PD controller alone. When the upright position is reached, the system is switched to stabilizing control mode to stabilize the pendulum. Figure 7a shows that stabilization is achieved with a minimal time of 2.65 seconds. In Figure 6(a), we observe that the system employing GIHC requires thirteen swings to stabilize, whereas the proposed system with MTIHC does it in five. Figures 6(b) and 7(b) show the response of the angular velocity of the

pendulum using GIHC and the proposed MTIHC. Figures 6(c) and 7(c) show the movement of the arm back and forth with respect to the movement of the pendulum for the two controllers such as GIHC and MTIHC. According to the design, the arm angle is restricted to move within 120 degrees in order to avoid any damage. This restricted travel of the arm is observed in both controllers. From swing up to stabilization, the movement of the arm is smooth in 6(c), whereas in 7(c) since the arm initially acquires high impulse energy in the MTIHC, the movement of the arm is abrupt.

Figures 6(d) and 7(d) illustrate the angular velocity of the arm in both modes. The responses shown in Figures 6(e) and 7(e) are respectively the control signal of the GIHC and proposed MTIHC system for swing up and stabilizing control. We see that during the swing up state the control signal is larger than the control signal in the stabilizing state. This is relevant because a large amount of energy is required to swing the pendulum up to its upright position and only a small amount energy is required to stabilize it. Figures 6(f) and 7(f) respectively show the energy profile of the pendulum in the general case and when the minimal time controller with the pulse is applied. Figures 6(g) and 7(g) show the moment at which the pendulum will shift to linear/balance mode from the swing up mode. Figure 8 shows the pattern of pulse control signal generated to achieve minimum time swing up.

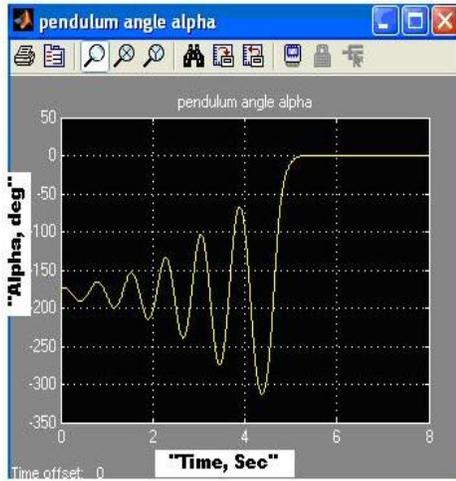


FIGURE 6(a) Pendulum Angle Alpha for GIHC

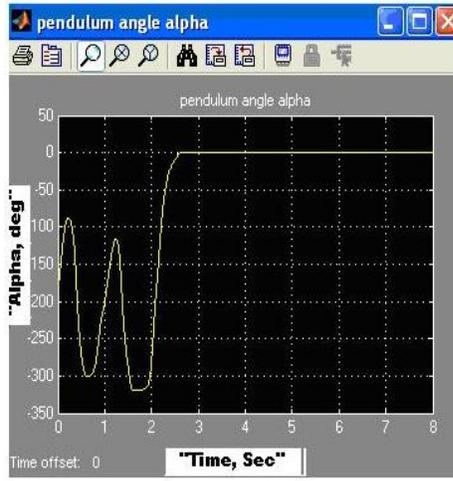


FIGURE 7(a) Pendulum Angle Alpha for MTIHC Fg6b

The control strategy of MTIHC can be summarized as follows: When  $P = 0.03$ ,  $D=0.015$  (sec),  $K_p= 99.99$  (V/rad),  $K_d = (2.107(V / rad / sec))$ , and  $\bar{u}= 1.9138$

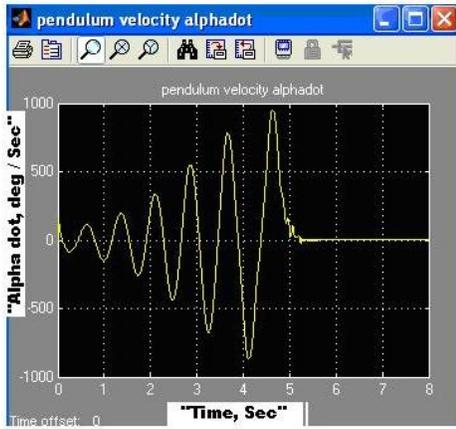


FIGURE 6(b) Arm angle Theta for GIHC

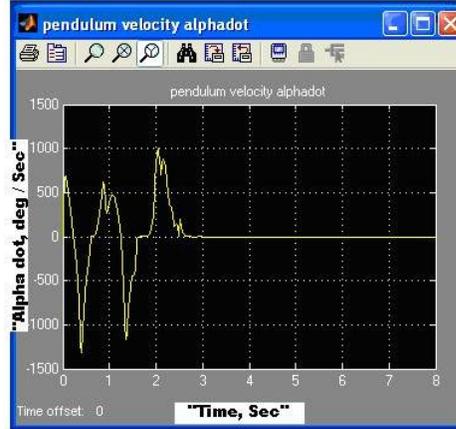


FIGURE 7(b) Pendulum Velocity Alphadot for MTIHC

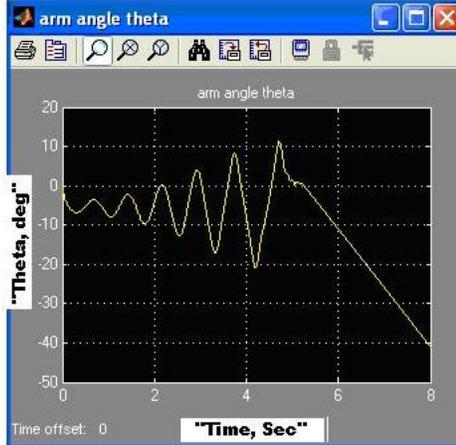


FIGURE 6(c) Arm Angle Theta for GIHC

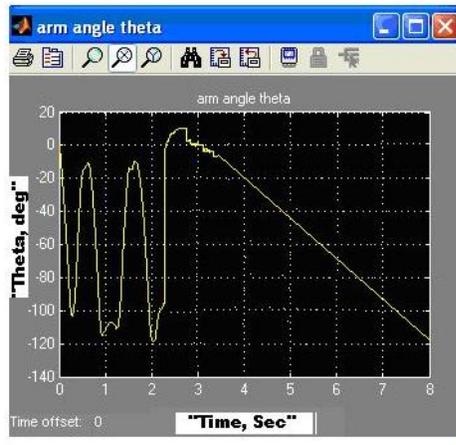


FIGURE 7(c) Arm Angle Theta for MTIHC

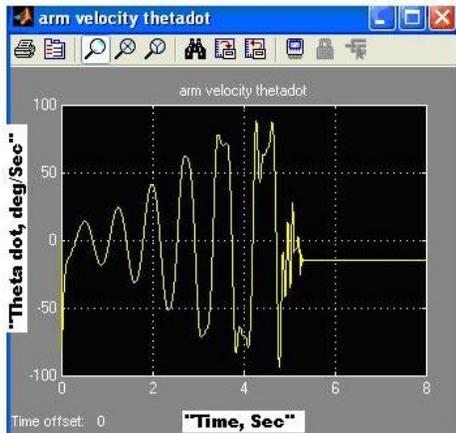


FIGURE 6(d) Arm Velocity Thetadot for GIHC

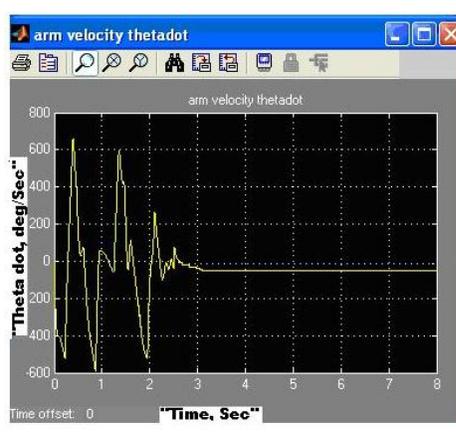


FIGURE 7(d) Arm Velocity Thetadot for MTIHC

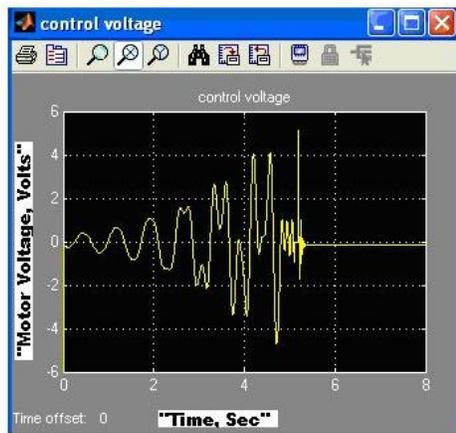


FIGURE 6(e) Voltage Input to Arm Motor for GIHC

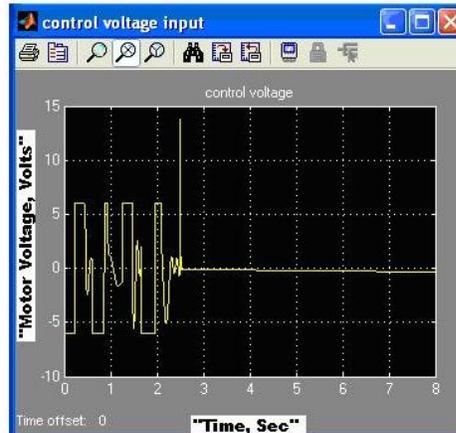


FIGURE 7(e) Voltage Input to Arm Motor for MTIHC

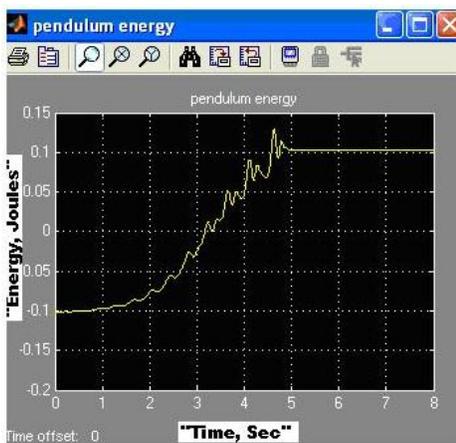


FIGURE 6(f) Energy of Pendulum for GIHC

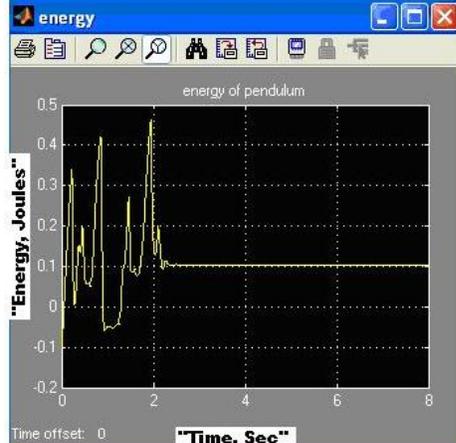


FIGURE 7(f) Energy of Pendulum for MTIHC

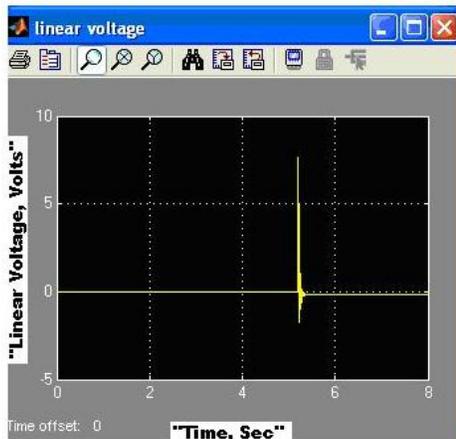


FIGURE 6(g) Balance Controller Voltage for GIHC

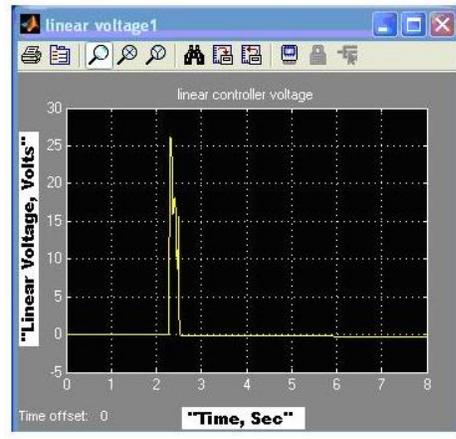


FIGURE 7(g) Balance Controller Voltage for MTIHC

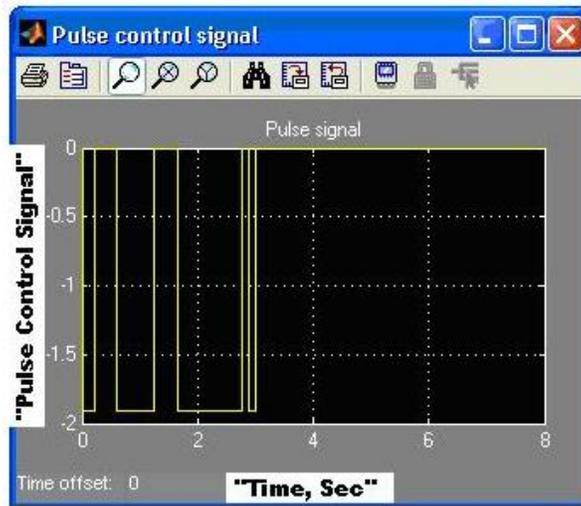


FIGURE 8. Pulse Control Signal in the MTIHC

rad /sec, the achieved minimal swing up time is  $t = 2.65$  seconds, which is approximately half of the swing up time achieved by PID controller alone, that is by the GIHC.

## 8. Conclusion

In this paper, the Rotary Inverted Pendulum problem has been extensively studied. The control of an inverted pendulum on arm was simulated with the cooperative tasks of PD positive feed back position controller and Fuzzy balance controller for stabilization. Similarly, the minimum time swing up problem of RIP using the proposed method with the cooperative tasks of a PD position controller and a feed – forward pulse step controller was investigated. To make the switch mode control simpler, an energy based switching method was also considered. A simulation study showed that the proposed method is more effective.

## REFERENCES

- [1] K. J. Astrom and K. Furuta, *Swinging up a pendulum by energy control*, Automatica, **36** (2000), 287-295.
- [2] [http://people.msoe.edu/~saadat/download/3\\_Inverted\\_Pendulum\\_Project.pdf](http://people.msoe.edu/~saadat/download/3_Inverted_Pendulum_Project.pdf)
- [3] H. K. Lam, F. H. Leung and P. K. S. Tam, *Design and stability analysis of fuzzy model-based nonlinear controller for nonlinear systems using genetic algorithm*, IEEE Trans. on Systems, Man and Cybernetics, Part B, **33**(2) (2003), 250-257.
- [4] Z. G. Li, C. Y. Wen and Y. C. Soh, *Analysis and design of impulsive control systems*, IEEE Trans. on Automatic Control, **46**(6) (2001), 894-897.
- [5] C. M. Lin and Y. J. Mon, *Decoupling control by hierarchical fuzzy sliding-mode controller*, IEEE Trans. on Control Systems Technology, **13**(4) (2005), 593-598.

- [6] N. Muskinja and B. Tovornik, *Swinging up and Stabilization of a real inverted pendulum*, IEEE Trans. on Industry electronics, **53**(2) (2006), 631-639.
- [7] S. Nudrakwang, T. Benjanarasuth, J. Ngamwiwit and N. Komine, *Hybrid PD-servo state feedback control algorithm for swing up inverted pendulum system*, Proceedings of the ICCAS, **2**(5) (2005), 690-693.
- [8] S. Nudrakwang, T. Benjanarasuth, J. Ngamwiwit and N. Komine, *Hybrid controller for Swinging up Inverted Pendulum System*, Proceedings of the IEEE, ICICS (2005), 488-492.
- [9] G. N. Silva and R. B. Vinter, *Necessary conditions for optimal impulsive control problems*, SIAM Journal on Control and Optimization, **35**(6) (1997), 1829-1846.
- [10] W. Torres-Pomales and O. R. Gonzalez, *Nonlinear control of swing-up inverted pendulum*, Proceedings of the IEEE International Conference on Control Application, (1996), 259-264.
- [11] A. Wallen and K. J. Astrom, *Pulse -step control*, Proceedings of the 15th IFAC Triennial World Congress, Barcelona, Spain, 2002.
- [12] C. H. Wang, T. C. Lin, T. T. Lee and H. L. Liu, *Adaptive hybrid intelligent control for uncertain nonlinear dynamical systems*, IEEE Trans. on Systems, Man and Cybernetics, Part B, **32** (5) (2002), 583-597.
- [13] S. Weibel, G. W. Howell and J. Baillieul, *Control of single-degree-of-freedom Hamiltonian systems with impulsive inputs*, Proceedings of the 35th IEEE Conference on Decision and Control, Kobe, Japan, (1996), 4661-4666.
- [14] M. Yamakita, et.al., *Robust swing up control of double pendulum*, Proceedings of the American Control Conference, **1** (1995), 290-295.
- [15] T. Yang, *Impulsive control*, IEEE Trans. Autom. Contr, **44** (1999), 1081-1083.
- [16] S. Yasunobu and M. Mori, *Swing up fuzzy controller for inverted pendulum based on a human control strategy*, Proceedings of the Sixth IEEE International Conference on Fuzzy Systems, **3** (1997), 1621-1625.
- [17] J. Yi, N. Yubazaki and K. Hirota, *A new fuzzy controller for stabilization of parallel-type double inverted pendulum system*, Fuzzy Sets and Systems, **126**(1) (2002), 105-119.
- [18] J. Yi, N. Yubazaki and K. Hirota, *Upswing and stabilization control of inverted pendulum system based on the SIRMs dynamically connected fuzzy inference model*, Fuzzy Sets and Systems, **122** (2001), 139-152.
- [19] K. Yoshida, *Swing up control of an inverted pendulum by energy based methods*, Proceedings of the American Control Conference, **6** (1999), 4045- 4047.

P. MELBA MARY\*, ANNA UNIVERSITY, DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING, TAMILNADU, INDIA

*E-mail address:* melbance\_2k4@yahoo.co.in

N. S. MARIMUTHU, ANNA UNIVERSITY, DEPARTMENT OF ELECTRICAL AND ELECTRONICS ENGINEERING, TAMILNADU, INDIA

*E-mail address:* drnsmphd@yahoo.co.in

\*CORRESPONDING AUTHOR