AN OPTIMIZATION MODEL FOR MULTI-OBJECTIVE CLOSED-LOOP SUPPLY CHAIN NETWORK UNDER UNCERTAINTY: A HYBRID FUZZY-STOCHASTIC PROGRAMMING METHOD

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Abstract. In this research, we address the application of uncertainty programming to design a multi-site, multi-product, multi-period, closed-loop supply chain (CLSC) network. In order to make the results of this article more realistic, a CLSC for a case study in the iron and steel industry has been explored. The presented supply chain covers three objective functions: maximization of profit, minimization of new products' delivery time, collection time and disposal time of used products, and maximizing flexibility. To solve the proposed model, an interactive hybrid solution methodology is adopted through combining a hybrid fuzzy-stochastic programming method and a fuzzy multi-objective approach. Finally, the numerical experiments are given to demonstrate the significance of the proposed model and the solution approach.

1. Introduction

The study of closed-loop supply chains, which take account of product returns, along with the downstream flow of materials, has recently become overwhelmingly popular [32, 28, 33]. Reverse operations supply chains management enables companies to have used products recycled, which then re-enter the forward supply chain [42]. Closed-loop supply chains focus on reducing resource consumption and waste generation, therefore supporting the industrial companies’ vital role in achieving international sustainability [34]. However, the configuration of the reverse logistics network influences sharply the running capacity of the forward logistics network and vice versa as they have a number of resources such as, transport and warehouse capacity in common. As long as separately designing the forward and reverse logistics leads to sub-optimal designs regarding costs, service levels, and responsiveness, designing the forward and reverse logistics networks has to be integrated [8, 18, 38].

This kind of integration is referred to as ”horizontal integration”, since it deals with the same decision level of integration of related optimization problems (i.e. strategic, tactical, or operational). Two instances of horizontal integration at strategic level can be numerated as, integrating supplier selection with network design and/or integrating the forward and reverse supply chain designs. ”Vertical integration” is defined as integrating decision- making processes across decision levels, for example

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by considering tactical or operational inventory levels when attending to strategic network design issues. Problems of real world network design are usually identified by multiple objectives. Minimizing total costs and maximizing network responsiveness are the major single objectives used in forward logistics network design; however, these objectives are typically conflicting and concurrently taking them into account is the most favorable alternative for a large number of decision makers [1, 25]. Network responsiveness is also a critical issue in reverse logistics [5], because it is undesirable for customers or retailers to keep used products for a long period of time because of their costly retention. Companies who set to collect more used products from customers should also pay attention to network responsiveness when minimizing costs, considering the fact that customers tend to discard the used products as soon as possible. Uncertainty is considered as one of the main focuses of CLSC management due to the demand, land filling, and recovery rates, the three important factors related to the uncertainty [43]. In other words, the important aspects of RL and CLSC network design problems are uncertainty in timing, quality, and quantity of product returns. Wang and Hsu [43] also pointed out that still appropriate models which handle the uncertainty in CLSCs are lacking. Ilgin and Gupta [14] report that this uncertainty can be handled by vigorous optimization techniques in stochastic RL network design models. Moreover, fuzzy mathematical programming can also be applied since the deterministic models consider certainty in all aspects of the problem even though some of the parameters cannot be precisely handled like capacities and aim for objective function achievement [2]. The articles dealing with uncertainty issue and the related methodologies can be outlined as follows: Salema et al. [35] redeveloped the generic RL network design model assuming the production or storage capacities of opened facilities, multiple product production, and uncertainty in relation to the customers’ demand and product returns utilizing a multiple-scenario based approach. Chung et al. [4] proposed a multi-echelon CLSC inventory model to obtain optimal inventory policy to maximize the common profits of the supplier, manufacturer, and third-party recycle dealer and retailer. In order to maximize revenue, a two-stage stochastic programming model was presented by Kara and Onut [17] for a single product, two-echelon capacitated RL network design in waste paper recycling industry; demand and amounts of collected products were supposed to be uncertain and in accordance with the normal distribution while generating alternative scenarios. El-Sayed et al. [7] prepared a multi-period, multi-echelon, and multi-stage stochastic program for an integrated forward-reverse logistics network design, assuming the demands of customer zones stochastic. Lee et al. [19] put forward a two-stage stochastic programming model for sustainable logistics network design. Stochastic parameters are demands of forward products and the supply of returned products with known distributions. Pishvaee and Torabi [29] took into consideration uncertain demands, returns, delivery times, costs, and capacities employing possibilistic programming approach for a CLSC network design problem which integrates strategic and tactical planning decisions. Due to pitfalls of the stochastic programming such as difficult availability of historical data and its complex modeling, uncertain demand and purchasing cost in strategic agile CLSC network design problem of perishable
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goods was dealt with through a powerful interval optimization technique offered by Hasani et al. [13]. Uncertainties in demand and yield rate were modeled by Qiang et al. [31] for a CLSC network along with decentralized decision makers consisting of raw material suppliers, retailers, and manufacturers. Subulan et al. [36] implemented a fuzzy mixed integer programming model with non-linear constraints for medium-term planning with remanufacturing option in a CLSC. Storage capacities, retailers and wholesalers’ demands, return rates, acceptance ratios, weekly on sale production or remanufacturing times, transportation upper bounds, and objective function values are considered fuzzy in this model. Olugu and Wong [27] designed a fuzzy rule-based system for performance evaluation of a CLSC in automotive industry, and Pishvaee and Razmi [30] posited a multi-objective possibilistic mixed integer programming model for an environmental supply chain network design using a possibilistic programming approach for controlling the imprecise parameters such as lower cost items, demands, return quantities, and the facilities’ capacities. Based on the above discussions, this paper presents a multi-objective model for designing a multi-site, multi-product, multi-period, closed-loop logistic network in a fuzzy environment. The model considers three objective functions: minimization of delivery time of new products, collection time and disposal time of used products, and maximization of flexibility. To solve the proposed model, an interactive hybrid solution methodology is adopted by combining hybrid fuzzy-stochastic programming method and fuzzy multi-objective approach. The structure of this paper is as follows. In Section 2, we present a multi-objective model for multi-site, multi-period, multi-product, closed-loop supply chain network under uncertainty. The solution methodology of the proposed model is explained in Section 3. Section 4 presents a numerical example and discusses the computational results. Finally, we give the conclusions of this paper in Section 5.

2. Problem Description and the Proposed Model

2.1. Modeling Framework. In this study, a CLSC network is investigated considering the iron and steel industry. The structure of the CLSC network is depicted in Figure 1. We dealt with capacitated remanufacturing network design problem settings by determining the locations of the forward and reverse channel facilities, i.e., we determined the optimal locations of the manufacturing/remanufacturing facilities, distribution centers, collection centers, and disposal centers. This setting is applicable for a company who is to develop a new CLSC network to manage multiple products. Thus we coordinated the forward and reverse flows via capacitated bidirectional facilities, product-specific hybrid metal manufacturing facilities, and disposal centers leading to a common infrastructure to manage the forward and reverse flows. The bidirectional facilities represent aggregate capacities that are shared by all products. Therefore, for integrating the non-uniformity and capacity usage, we use product-specific coefficients as modifiers to one capacity use unit. It is noteworthy that the inclusion of capacities on bidirectional facilities leads to stronger relation among the forward and reverse flows related to different product types. Also, capacities of disposal centers are limited. In the CLSC setting,
determining the best locations for the hybrid metal manufacturing facilities, bidirectional facilities, and the disposal centers with respect to the known customer zone locations and the best flow of products in the CLSC network encompasses a great importance because the total cost of location, processing, and transportation can be minimized.

![Diagram](image)

**Figure 1.** Closed-Loop Supply Chain Process in The Studied Network

### 2.2. Sets and Indices.

**Sets**
- \( P \) : Number of products \((p = 1, 2, ..., P)\)
- \( I \) : Number of customer zones \((i = 1, 2, ..., I)\)
- \( B \) : Number of candidate bidirectional facilities \((b = 1, 2, ..., B)\)
- \( L \) : Number of candidate hybrid metal manufacturing facilities \((l = 1, 2, ..., L)\)
- \( D \) : Number of potential disposal centers \((d = 1, 2, ..., D)\)
- \( T \) : Number of time periods \((t = 1, 2, ..., T)\)
- \( Q \) : Number of transportation mode \((q = 1, 2, ..., Q)\)

**Parameters**
- \( DE_{ipt} \) : Demand of customer zone \(i\) for product \(p\) in time period \(t\)
- \( \delta_{ipt} \) : Return fraction at customer zone \(i\) for product \(p\) in time period \(t\)
- \( \xi_{ipt} \) : Recycling rate of product \(p\) return of customer zone \(i\) in time period \(t\)
- \( F_{bt} \) : fixed cost of opening a bidirectional facility at location \(b\) in time \(t\)
- \( F_{dpt} \) : fixed cost of opening a disposal center for product \(p\) at location \(d\) in time \(t\)
- \( F_{lpt} \) : fixed cost of opening a hybrid metal facility for product \(p\) at location \(l\) in time \(t\)
- \( C_{mnt} \) : Unit transportation cost from a location \(m\) to a location \(n\) for \(m, n \in I, B\) in time \(t\)
- \( C_{mntq} \) : Unit transportation cost from a location \(m\) to a location \(n\) for \(m, n \in L, D, B\) in time \(t\) using transportation mode \(q\)
- \( CD \) : Cost of delivery delay per product per unit of time
- \( CC \) : Cost of collection delay per product per unit of time
- \( CF \) : Cost of disposal delay per product per unit of time
- \( \gamma_{pt} \) : Storage capacity coefficient at the bidirectional facility for distribution processing for product \(p\) in time \(t\)
- \( \gamma'_{pt} \) : Storage capacity coefficient at the bidirectional facility for collection processing for product \(p\) in time \(t\)
- \( \eta_{bpt} \) : Unit distribution processing cost of product \(p\) at bidirectional facility \(b\) in time \(t\)
\( \eta_{dpt} \): Unit obliterate cost of product \( p \) at disposal center \( d \) in time \( t \)
\( \rho_{bpt} \): Unit collection processing cost of product \( p \) at bidirectional facility \( b \) in time \( t \)
\( \varepsilon_{ipt} \): Unit cost of product \( p \) shipped out at hybrid metal facility \( l \) in time \( t \)
\( \beta_{lpt} \): Unit remanufacturing cost of product \( p \) shipped out hybrid metal facility \( l \) in time \( t \)
\( \alpha_{ipt} \): Recovery fraction for product \( p \) at hybrid metal facility \( l \) in time \( t \)
\( CAP_{bl} \): Storage capacity at bidirectional facility \( b \)
\( CAP_{dl} \): Storage capacity for disposal product \( p \) at disposal center \( d \)
\( CAP_{hl} \): Production capacity of products at hybrid metal facility \( l \)
\( ir \): Interest rate
\( PR_{ipt} \): Unit sale price of product \( p \) at customer zone \( i \) in time \( t \)
\( W_{1} \): Weight factor for capacity utilization of hybrid metal manufacturing facilities
\( W_{2} \): Weight factor for capacity utilization of bidirectional facilities
\( W_{3} \): Weight factor for capacity utilization of disposal center
\( TD_{bip} \): Delivery time of product \( p \) from bidirectional facility \( b \) to customer zone \( i \)
\( ED_{ipt} \): Expected delivery time of product \( p \) for customer zone \( i \) in time \( t \)
\( TC_{lbp} \): Collection time of product \( p \) from customer zone \( i \) by bidirectional facility \( b \)
\( EC_{ipt} \): Expected collection time of product \( p \) for customer zone \( i \) in time \( t \)
\( TF_{dbp} \): Disposal time of product \( p \) from bidirectional facility \( b \) by disposal center \( d \)
\( EF_{pdt} \): Expected disposal time of product \( p \) for disposal center \( d \) in time \( t \)
\( D^{i} \{ b \mid TD_{b lp} \geq ED_{ipt} \} \): in period \( t \)
\( C^{i} \{ b \mid TC_{lbp} \geq EC_{ipt} \} \): in period \( t \)
\( P^{i} \{ d \mid TF_{d bp} \geq EF_{p dt} \} \): in period \( t \)
\( M \): A sufficient large number

**Decision variables**

\[ w_{ipt} = \begin{cases} 1 & \text{if customer zone } i \text{ is assigned to facility } b \text{ for the forward flow of products in time period } t \\ 0 & \text{otherwise} \end{cases} \]
\[ z_{ipt} = \begin{cases} 1 & \text{if customer zone } i \text{ is assigned to facility } b \text{ for the reverse flow of products in time period } t \\ 0 & \text{otherwise} \end{cases} \]
\[ V_{ib} = \begin{cases} 1 & \text{if bidirectional facility } b \text{ is open in time period } t \\ 0 & \text{otherwise} \end{cases} \]
\[ s_{ipt} = \begin{cases} 1 & \text{if hybrid metal manufacturing facility } l \text{ is used for product } p \text{ in time period } t \\ 0 & \text{otherwise} \end{cases} \]
\[ d_{dpt} = \begin{cases} 1 & \text{if disposal center } d \text{ is used for product } p \text{ in time period } t \\ 0 & \text{otherwise} \end{cases} \]
\[ H_{blt} = \begin{cases} 1 & \text{if transportation mode } q \text{ is used for connecting metal facility } l \text{ and bidirectional facility } b \text{ in time } t \\ 0 & \text{otherwise} \end{cases} \]
\[ H_{lbt} = \begin{cases} 1 & \text{if transportation mode } q \text{ is used for connecting bidirectional facility } b \text{ and metal facility } l \text{ in time } t \\ 0 & \text{otherwise} \end{cases} \]
\[ g_{dptq} = \begin{cases} 1 & \text{if transportation mode } q \text{ is used for connecting bidirectional facility } b \text{ and disposal center } d \text{ in time } t \\ 0 & \text{otherwise} \end{cases} \]

\( y_{dptql} \): Total quantity of product \( p \) transported from bidirectional facility \( b \) to hybrid metal manufacturing facility \( l \) in time period \( t \) using transportation mode \( q \)
\( x_{lbpqt} \): Total quantity of new and remanufactured product \( p \) transported from hybrid metal manufacturing facility \( l \) to bidirectional facility \( b \) in time period \( t \) using transportation mode \( q \)
\( U_{dptq} \): Total quantity of waste product \( p \) transported from bidirectional facility \( b \) to disposal center \( d \) in time period \( t \) using transportation mode \( q \)
\( Q_{il} \): Production quantity of products in metal facility \( l \) in time period \( t \)
\( \tau_{ib} \): Quantity of products received by bidirectional facility \( b \) in time period \( t \)
\( L_{dpt} \): Disposal quantity of product \( p \) at disposal center \( d \) in time period \( t \)

2.3. Model Formulation.

\[
\max \ Z_1(NPV) = \sum_{t=1}^{T} \frac{\text{Incomess}_t - \text{Costs}_t}{(1+ir)^t} 
\]

\( \text{max} \ Z_2 = W_1(\sum_{p=1}^{P} \sum_{i=1}^{I} (CAP_{pt} - Q_{it})) + W_2(\sum_{b=1}^{B} \sum_{t=1}^{T} (CAP_{bt} - \tau_{bt})) + W_3(\sum_{d=1}^{D} \sum_{p=1}^{P} \sum_{t=1}^{T} (CAP_{dpt} - L_{dpt})) \)

\( \text{max} \ Z_3 = CD \sum_{t=1}^{T} \sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{b=1}^{B} \sum_{d \in D} (TD_{bip} - ED_{ipt})DE_{ipt}w_{bit} + \)
\( CC \sum_{t=1}^{T} \sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{b \in C_i} (TC_{btp} - EC_{ipt})\delta_{ipt}DE_{ipt}z_{ibt} + \)
\( CF \sum_{q=1}^{Q} \sum_{t=1}^{T} \sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{b=1}^{B} \sum_{d \in D} (TF_{dtp} - EF_{pdt})U_{bdptq} \)

\( \text{Incomess}_t = \sum_{q=1}^{Q} \sum_{l=1}^{L} \sum_{i=1}^{I} \sum_{p=1}^{P} \sum_{b=1}^{B} PR_{pit}DE_{ipt}w_{bit} \)

\( \text{Costs}_t = \sum_{b=1}^{B} F_{bt}(V_{b,t} - V_{b,t-1}) + \sum_{p=1}^{P} \sum_{d=1}^{D} F'_{dpt}(f_{dpt,t} - f_{dpt,t-1}) + \)
\( \sum_{p=1}^{P} \sum_{b=1}^{B} F_{bp}(S_{pt} - S_{pt-1}) + \)
\( \sum_{p=1}^{P} \sum_{b=1}^{B} \sum_{l=1}^{L} (\rho_{bpt} + C_{ibt})\delta_{ipt}DE_{ipt}z_{ibt} + \)
\( \sum_{q=1}^{Q} \sum_{p=1}^{P} \sum_{b=1}^{B} \sum_{l=1}^{L} (\alpha_{ipt}\beta_{ipt} + C_{ibtq})y_{bdptq} + \)
\( \sum_{q=1}^{Q} \sum_{p=1}^{P} \sum_{b=1}^{B} \sum_{l=1}^{L} \xi_{ipt}(\chi_{bdptq} - \chi_{bdptq}) + \)
\( \sum_{p=1}^{P} \sum_{b=1}^{B} \sum_{l=1}^{L} (\eta_{dpt} + C_{bd})DE_{ipt}w_{bit} + \)
\( \sum_{q=1}^{Q} \sum_{p=1}^{P} \sum_{b=1}^{B} \sum_{l=1}^{L} \eta_{bdptq} + \)
\( \sum_{q=1}^{Q} \sum_{p=1}^{P} \sum_{b=1}^{B} \sum_{l=1}^{L} \eta_{bdptq} + \)

S.t:

\( \sum_{b=1}^{B} w_{bit} = 1, \forall i \in (1,2,...,I), \forall t \in (1,2,...,T) \)

\( \sum_{b=1}^{B} z_{ibt} = 1, \forall i \in (1,2,...,I), \forall t \in (1,2,...,T) \)

\( \sum_{d=1}^{D} f_{dpt} = 1, \forall p \in (1,2,...,P), \forall t \in (1,2,...,T) \)

\( \sum_{l=1}^{L} s_{ipt} = 1, \forall p \in (1,2,...,P), \forall t \in (1,2,...,T) \)

\( \sum_{q=1}^{Q} \sum_{l=1}^{L} y_{bdptq} = \sum_{q=1}^{Q} \sum_{l=1}^{L} z_{ibt} = \sum_{q=1}^{Q} \sum_{l=1}^{L} y_{bdptq} + \sum_{q=1}^{Q} \sum_{l=1}^{L} U_{bdptq} = \sum_{q=1}^{Q} \sum_{l=1}^{L} U_{bdptq} + \)
\( \forall p \in (1,2,...,P), \forall t \in (1,2,...,T), \forall b \in (1,2,...,B) \)

\( \sum_{q=1}^{Q} \sum_{d=1}^{D} U_{bdptq} = \sum_{q=1}^{Q} \sum_{d=1}^{D} U_{bdptq} + \)
\( \forall p \in (1,2,...,P), \forall t \in (1,2,...,T), \forall d \in (1,2,...,D) \)

\( \sum_{q=1}^{Q} \sum_{b=1}^{B} y_{bdptq} = \sum_{q=1}^{Q} \sum_{b=1}^{B} y_{bdptq} + \)
\( \forall p \in (1,2,...,P), \forall t \in (1,2,...,T), \forall d \in (1,2,...,D) \)
\[
\sum_{q=1}^{Q} \sum_{b=1}^{B} \chi_{lbptq} = \sum_{i=1}^{I} DE_{ipt} s_{ipt}
\]
\[
\forall p \in (1, 2, \ldots, P), \quad \forall t \in (1, 2, \ldots, T), \quad \forall l \in (1, 2, \ldots, L)
\]

\[
\sum_{q=1}^{Q} \sum_{l=1}^{L} \chi_{lbptq} = \sum_{i=1}^{I} DE_{ipt} w_{bit}
\]
\[
\forall p \in (1, 2, \ldots, P), \quad \forall t \in (1, 2, \ldots, T), \quad \forall b \in (1, 2, \ldots, B)
\]

\[
\sum_{p=1}^{P} \sum_{i=1}^{I} \gamma_{pt} DE_{ipt} w_{bit} + \sum_{p=1}^{P} \sum_{i=1}^{I} \gamma_{pt} \delta_{ipt} DE_{ipt} z_{ibt} \leq \tau_{bt},
\]
\[
\forall b \in (1, 2, \ldots, B), \quad \forall t \in (1, 2, \ldots, T)
\]

\[
\tau_{bt} \leq CAP_{bt} V_{bt},
\]
\[
\forall b \in (1, 2, \ldots, B), \quad \forall t \in (1, 2, \ldots, T)
\]

\[
\sum_{q=1}^{Q} \sum_{b=1}^{B} U_{bdptq} \leq L_{dpt},
\]
\[
\forall p \in (1, 2, \ldots, P), \quad \forall d \in (1, 2, \ldots, D), \quad \forall t \in (1, 2, \ldots, T)
\]

\[
L_{dpt} \leq CAP_{dpt} f_{dpt},
\]
\[
\forall p \in (1, 2, \ldots, P), \quad \forall d \in (1, 2, \ldots, D), \quad \forall t \in (1, 2, \ldots, T)
\]

\[
\sum_{b=1}^{B} \sum_{q=1}^{Q} \sum_{p=1}^{P} \chi_{lbptq} \leq Q_{lt},
\]
\[
\forall t \in (1, 2, \ldots, T), \quad \forall l \in (1, 2, \ldots, L)
\]

\[
Q_{lt} \leq CAP_{lt},
\]
\[
\forall l \in (1, 2, \ldots, L), \quad \forall t \in (1, 2, \ldots, T)
\]

\[
w_{bit} \leq V_{bt},
\]
\[
\forall b \in (1, 2, \ldots, B), \quad \forall t \in (1, 2, \ldots, T), \quad \forall i \in (1, 2, \ldots, I)
\]

\[
z_{ibt} \leq V_{bt},
\]
\[
\forall b \in (1, 2, \ldots, B), \quad \forall t \in (1, 2, \ldots, T), \quad \forall i \in (1, 2, \ldots, I)
\]

\[
\sum_{q=1}^{Q} R_{lbqt} \leq 1,
\]
\[
\forall b \in (1, 2, \ldots, B), \quad \forall t \in (1, 2, \ldots, T), \quad \forall l \in (1, 2, \ldots, L)
\]

\[
\sum_{q=1}^{Q} H_{blqt} \leq 1,
\]
\[
\forall b \in (1, 2, \ldots, B), \quad \forall t \in (1, 2, \ldots, T), \quad \forall l \in (1, 2, \ldots, L)
\]

\[
\sum_{q=1}^{Q} g_{bdqt} \leq 1,
\]
\[
\forall b \in (1, 2, \ldots, B), \quad \forall t \in (1, 2, \ldots, T), \quad \forall d \in (1, 2, \ldots, D)
\]

\[
R_{lbqt} \leq \sum_{p=1}^{P} \chi_{lbptq}
\]
\[
\forall b \in (1, 2, \ldots, B), \quad \forall t \in (1, 2, \ldots, T), \quad \forall d \in (1, 2, \ldots, D), \quad \forall q \in (1, 2, \ldots, Q)
\]

\[
H_{blqt} \leq \sum_{p=1}^{P} y_{blptq}
\]
\[
\forall b \in (1, 2, \ldots, B), \quad \forall t \in (1, 2, \ldots, T), \quad \forall d \in (1, 2, \ldots, D), \quad \forall q \in (1, 2, \ldots, Q)
\]

\[
g_{bdqt} \leq \sum_{p=1}^{P} U_{bdptq}
\]
\[
\forall b \in (1, 2, \ldots, B), \quad \forall t \in (1, 2, \ldots, T), \quad \forall d \in (1, 2, \ldots, D), \quad \forall q \in (1, 2, \ldots, Q)
\]
The first objective was to maximize Net Present Value (NPV) by calculating the total income minus the total cost with respect to the interest rate Eq.(1). NPV includes the present value of money in the future which is a central tool in the analysis of discounted cash flow. The total income was obtained from multiplying the number of products shipped to customer zones by the unit price of the product Eq.(4). The costs of the closed-loop logistic network calculated by Eq.(5) and the first three terms in Eq.(5) represent the fixed costs related to locating the product-specific bidirectional facilities, disposal centers, and hybrid metal manufacturing facilities. The fourth term is the transportation costs from the customer zones and collection processing costs at the bidirectional facilities. The fifth term is the transportation costs from the bidirectional facilities to the hybrid metal manufacturing facilities, the sixth term is the transportation costs from the bidirectional facilities to the customer zones, and the distribution processing costs at the bidirectional facilities, and the seventh term is the transportation costs from the bidirectional facilities to the disposal centers together with the obliterate costs at the bidirectional facilities. The second objective function was to maximize the total volume flexibility consisting of hybrid metal manufacturing facilities, bidirectional facilities volume flexibility, and disposal volume flexibility Eq.(2). These volumes' flexibility is calculated as the difference between capacity and capacity utilization [38]. The third objective function Eq.(3) was to minimize the delivery time of new products, collection time, and disposal time of used products. The following constraint sets ensure a specific aspect for each period: In constraint set (6) a customer zone i is designated to only one bidirectional facility for the forward flow of products; in constraint set (7) a customer zone i is assigned to one bidirectional facility
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for the reverse flow of products; Constraint set (8) guarantees that, for each product p, a single dedicated disposal center location t is established; Constraint set (2.3) guarantees that, for each product p, a single dedicated hybrid metal manufacturing facility location l is developed; Constraint sets (10) to (15) represent the flow conservation for each product type at the hybrid metal manufacturing facilities, bidirectional facilities, customer zones and disposal centers; Constraint sets (16) and (17) ensure that the total forward and reverse shipment at any bidirectional facilities does not exceed its aggregate processing capacity; Constraint sets (18) and (19) enforce the capacity restrictions at the disposal centers; Constraints (20) and (21) state that the sum of the flow exiting from each hybrid metal manufacturing facility does not exceed the production capacity of this hybrid metal manufacturing facility; Constraints (22) and (23) assure the movement of the products from bidirectional facilities to customer zones in forward flow and from customer zones to bidirectional facilities in reverse flow to be conducted exactly when the corresponding bidirectional facility is open. In order to guarantee the single sourcing of transportation mode, one transportation option can be selected at most between nodes l-b, nodes -l , and nodes b -d ; therefore, we impose constraints (24),(25) and (26) . Constraints (27), (28) and (29) establish no links between any locations without actual shipments during all periods. Constraints (30), (31) and (32) indicate no shipping between the non-linked locations. These constraints demonstrate that the flow is allowed specifically via active transportation modes chosen for the network design; constraint sets (33) and (34) are the restrictions on the decision variables.

3. Solution Methodology

3.1. The Equivalent Auxiliary Crisp Model. Signifying uncertainties as probability distributions while making the decisions to be established periodically over time, the study problem can be formulated as a two-phase stochastic programming (TSP) with recourse model. In TSP, decision variables are divided into two subsets: those to be determined before the random variables are disclosed, and those to be determined after the availability of the random variables’ realized values [3, 11, 20]. Therefore, the TSP methods make decision makers to assign a cost to recourse activities that are to ensure feasibility of the second-stage problem i.e., in TSP, infeasibilities in the second stage are allowed at a certain penalty. A TSP model can be formulated as follows [21]:

$$\max Z = cx - E[Q(x, w)]$$  (35a)

$$S.t:$$

$$Ax \leq b$$  (35b)

$$x \geq 0$$  (35c)

Where x is the first-stage anticipated decisions made before observing the random variables, and Q(x, w)is the optimal value, for any given Ω, of the following nonlinear program [21]:


\[
\min q(y, w) \tag{36a}
\]
\[
S.t:
W(w)y = h(w) - T(w)x \tag{36b}
\]
\[
y \geq 0 \tag{36c}
\]

Where \( y \) is the second-stage decision variable that depends on the realization of the first-stage random vector; \( q(y, w) \) denotes the second-stage cost function; \( T(w), W(w), h(w) \mid w \in \Omega \) are model parameters with reasonable dimensions, and functions of the random vector \((w)\). For values of the first-stage variables\((x)\), the second-stage problem can be decomposed into independent linear sub-problems, with one sub-problem for every realization of the uncertain parameters. Then, model (35) can be reformulated as [29]:

\[
\begin{align*}
\max Z &= cx - \mathbb{E}[\min q(y, w) \mid T(w)x + W(w)y = h(w)]_{y \geq 0} \tag{37a} \\
\quad S.t: \\
\quad &Ax \leq b \tag{37b} \\
\quad &x \geq 0 \tag{37c}
\end{align*}
\]

As a general rule, the above TSP problem is nonlinear, and the set of feasible constraints is convex solely under some specific distributions. Nevertheless, the problem can be equally formulated as a linear programming model by considering discrete distributions for the uncertain parameters [3, 11]. Put random vector \( w \) as a discrete and finite distribution, with support \( \Omega = \{w_1, w_2, ..., w_s\} \), and mark \( p_h \) as the probability of realization of scenario \( w_h \), with \( p_h > 0 \) and \( \sum_{h=1}^{s} p_h = 1 \). The expected value of the second-stage optimization problem can be expressed as [21]:

\[
\mathbb{E}[Q(x, w)] = \sum_{h=1}^{s} p_h Q(x, w_h) \tag{38}
\]

Then, according to the discrete distributions assumption for the uncertain parameters, model (35) can be equivalently modified into the following linear program [21]:

\[
\begin{align*}
\max Z &= cx - \sum_{h=1}^{s} p_h q(y_n, w_h) \tag{39a} \\
\quad S.t: \\
\quad &Ax \leq b \tag{39b} \\
\quad &T(w_h)x + W(w_h)y_{wh} = h(w_h), \quad w_h \in \Omega \tag{39c} \\
\quad &x \geq 0 \tag{39d} \\
\quad &y_h \geq 0 \tag{39e}
\end{align*}
\]
The TSP model can develop a relation between the pre-regulated policies and the associated economic implications. Chance-constrained programming (CCP), in comparison, can reflect the reliability of satisfying system constraints under uncertainty in an effective way [10]. In CCP, the constraints should be satisfied under given probabilities. Based on the CCP concept, a TSP model with chance constraints can be formulated as follows [21]:

\[
\begin{align*}
\text{max } Z &= \sum_{j=1}^{n_1} c_j x_j - \sum_{j=1}^{n_2} \sum_{h=1}^{s} p_h d_j y_{jh} \\
\text{s.t.} & \\
\sum_{j=1}^{n_1} a_{rj} x_j &\leq b_r, \quad \forall r = 1, 2, ..., m_1 \\
Pr\{\sum_{j=1}^{n_1} a_{ij} x_j + \sum_{j=1}^{n_2} a_{ij} y_{jh} \leq w_{ih}\} &\geq \gamma_i, \quad \forall i = 1, 2, ..., m_2; h = 1, 2, ..., s \\
x_j &\geq 0, \quad \forall j = 1, 2, ..., n_1 \\
y_{jh} &\geq 0, \quad \forall j = 1, 2, ..., n_2; h = 1, 2, ..., s
\end{align*}
\]
Therefore, one potential assumption to take such complex uncertainties into account is to introduce FPP into the model (40), giving a hybrid fuzzy stochastic programming model as follows [21]:

\[
\text{max } Z = \sum_{j=1}^{n_1} \tilde{c}_j x_j - \sum_{j=1}^{n_2} \tilde{p}_h \tilde{d}_j y_{jh} \tag{41a}
\]

s.t.:

\[
\sum_{j=1}^{n_1} \tilde{a}_{ij} x_j \leq \tilde{b}_r, \quad \forall r = 1, 2, \ldots, m_1 \tag{41b}
\]

\[
\sum_{j=1}^{n_1} \tilde{a}_{ij} x_j + \sum_{j=1}^{n_2} \tilde{a}_{ij} \tilde{y}_{jh} \leq \tilde{w}_{ih}, \quad \forall i = 1, 2, \ldots, m_2; h = 1, 2, \ldots, s \tag{41c}
\]

\[
x_j \geq 0, \quad \forall j = 1, 2, \ldots, n_1
\]

\[
y_{jh} \geq 0, \quad \forall j = 1, 2, \ldots, n_2; \quad h = 1, 2, \ldots, s \tag{41e}
\]

Where \(x_j (j = 1, 2, \ldots, n_1)\) are first-stage decision variables; \(y_{jh} (\forall j = 1, 2, \ldots, n_2; \ h = 1, 2, \ldots, s)\) are second-stage decision variables; \(\tilde{c}_j (j = 1, 2, \ldots, n_1)\) and \(\tilde{d}_j (j = 1, 2, \ldots, n_2)\) are fuzzy coefficients in the objective function; \(\tilde{a}_{ij} (i = 1, 2, \ldots, m_2; j = 1, 2, \ldots, n_1)\) and \(\tilde{a}_{ij} (i = 1, 2, \ldots, m_2; j = 1, 2, \ldots, n_2)\) are fuzzy coefficients in constraints; \(\tilde{w}_{ih} (i = 1, 2, \ldots, m_2; h = 1, 2, \ldots, s)\) are independent random variables with known probability distributions. To solve model (41), an “equivalent” deterministic version can be defined, realized by employing fuzzy tolerance measure and CCP approach, which consist of fixing a probability \(\tilde{q}_i (\tilde{q}_i \in [0, 1])\) for uncertain constraint \(i\) and imposing the constraint to be satisfied with a probability level of \(\gamma_i = 1 - \tilde{q}_i (i = 1, 2, \ldots, m_2)\) and \(0 \leq \gamma_i \leq 1\) at any rate. So, the above fuzzy-stochastic constraint can be converted as [21]:

\[
P_r \{ \sum_{j=1}^{n_1} \tilde{a}_{ij} x_j + \sum_{j=1}^{n_2} \tilde{a}_{ij} \tilde{y}_{jh} \leq \tilde{w}_{ih} \} \geq \gamma_i, \quad \forall i = 1, 2, \ldots, m_2; h = 1, 2, \ldots, s \tag{42}
\]

Constraint (42) is generally assumed as nonlinear, and the set of feasible constraints is convex only for some certain distributions and levels of \(\tilde{q}_i\). Based on the CCP method, the fuzzy-stochastic constraint can be converted to the following deterministic fuzzy equivalent [39]:

\[
\sum_{j=1}^{n_1} \tilde{a}_{ij} x_j + \sum_{j=1}^{n_2} \tilde{a}_{ij} \tilde{y}_{jh} \leq \tilde{w}_{ih}, \quad \forall i = 1, 2, \ldots, m_2; h = 1, 2, \ldots, s \tag{43}
\]

Where \(\tilde{w}_{ih} = F_{\tilde{w}_{ih}}^{-1}(\tilde{q}_i)\), considering the cumulative distribution function of \(\tilde{w}_{ih}\) [i.e. \(F_{\tilde{w}_{ih}}(\tilde{w}_{ih})\)] and the probability of violating constraint \(i(\tilde{q}_i)\). Then, model (41) can be reformulated as follows [21]:

\[
\text{max } Z = \sum_{j=1}^{n_1} \tilde{c}_j x_j - \sum_{j=1}^{n_2} \tilde{p}_h \tilde{d}_j y_{jh} \tag{44a}
\]

s.t.:

\[
\sum_{j=1}^{n_1} a_{ij} x_j \leq \tilde{b}_r, \quad \forall r = 1, 2, \ldots, m_1
\]

\[
\sum_{j=1}^{n_1} a_{ij} x_j \leq \tilde{b}_r, \quad \forall r = 1, 2, \ldots, m_1
\]
\[
\sum_{j=1}^{n_1} a_{ij} x_j + \sum_{j=1}^{n_2} \tilde{a}_{ij} y_{jhi} \leq \tilde{w}_{ih}, \\
\forall i = 1, 2, ..., m_2; h = 1, 2, ..., s
\]  
(44c)

\[
x_j \geq 0, \quad \forall j = 1, 2, ..., n_1
\]  
(44d)

\[
y_{jhi} \geq 0, \quad \forall j = 1, 2, ..., n_2; h = 1, 2, ..., s
\]  
(44e)

A two-step solution method is put forth to solve the mentioned model. The solutions are combinations of probabilistic and possibilistic data, thus offering flexibility in result interpretation and decision-alternative generation. The possibility distributions of fuzzy parameters can be configured as fuzzy sets. For example, fuzzy parameter \( \tilde{b}_r = (\tilde{b}_r, b_{r1}, b_{r2}, \tilde{b}_r) \) can be presented as a triangular fuzzy set assuming \( b_{r1} = b_{r2} \), or a trapezoidal fuzzy set when \( b_{r1} < b_{r2} \). Parameter \( \tilde{a}_{ij} \) under each \( \alpha \)-cut level can be introduced into a closed interval: \([ (1 - \alpha)(\tilde{b}_r) + \alpha(b_{r1}), (1 - \alpha)(\tilde{b}_r) + \alpha(b_{r2}) ] \). An \( \alpha \)-cut can be defined as “a set of elements that belong to a fuzzy set at least to a membership grade of \( \alpha \); this grade is also called the degree of confidence or the degree of plausibility” [45]. It ranges from 0 to 1, and capable of being pre-regulated. The most valid value is assigned grade 1 membership, hence any number which is below the lowest possible value or more than the highest possible value is assigned grade 0 membership. Then, model (44) can be directly converted into two deterministic sub models that are in accordance with the lower and upper bounds of the objective-function value [21].

**Lower Sub Model**

\[
Z^l = \sum_{j=1}^{n_1} (1 - \alpha)\tilde{c}_j + \alpha c_{j1} x_j - \sum_{j=1}^{n_2} \sum_{h=1}^{s} p_h [(1 - \alpha)\tilde{d}_j + \alpha d_{j1}] y_{jhi} 
\]  
(45a)

\[
S.t:  \\
\sum_{j=1}^{n_1} (1 - \alpha)\tilde{p}_{rj} + \alpha a_{rj} x_j \leq [(1 - \alpha)\tilde{b}_r + \alpha b_{r1}], \\
\forall r = 1, 2, ..., m_1
\]  
(45b)

\[
\sum_{j=1}^{n_1} (1 - \alpha)\tilde{p}_{rj} + \alpha a_{rj} x_j + \sum_{j=1}^{n_2} [(1 - \alpha)\tilde{q}_{ij} + \alpha a_{ij1}] y_{jhi} \leq w_r^{[1 - \alpha]q_{r1} + \alpha a_{r1}}, \\
\forall i = 1, 2, ..., m_2; h = 1, 2, ..., s
\]  
(45c)

\[
x_j \geq 0, \quad \forall j = 1, 2, ..., n_1
\]  
(45d)

\[
y_{jhi} \geq 0, \quad \forall j = 1, 2, ..., n_2; h = 1, 2, ..., s
\]  
(45e)

**Upper Sub Model**

\[
Z^u = \sum_{j=1}^{n_1} (1 - \alpha)\tilde{c}_j + \alpha c_{j2} x_j - \sum_{j=1}^{n_2} \sum_{h=1}^{s} p_h [(1 - \alpha)\tilde{d}_j + \alpha d_{j1}] y_{jhi} 
\]  
(46a)

\[
S.t:  \\
\sum_{j=1}^{n_1} (1 - \alpha)\tilde{p}_{rj} + \alpha a_{rj1} x_j \leq [(1 - \alpha)\tilde{b}_r + \alpha b_{r2}], \\
\forall r = 1, 2, ..., m_1
\]  
(46b)
\[
\sum_{j=1}^{n_1} [(1-\alpha)q_{ij} + \alpha a_{ij}] x_j + \sum_{j=1}^{n_2} [(1-\alpha)\pi'_{ij} + \alpha a'_{ij}] y_{jh} \leq w_{ih}, \quad \forall i = 1, 2, ..., m_2; h = 1, 2, ..., s
\] (46c)

\[x_j \geq 0, \quad \forall j = 1, 2, ..., n_1\] (46d)

\[y_{jh} \geq 0, \quad \forall j = 1, 2, ..., n_2; h = 1, 2, ..., s\] (46e)

Then, solving (45) and (46) sub models with various \(\alpha\)-cut levels (i.e. \(\alpha \in [0, 1]\)), a set of associated solutions with probabilistic and possibilistic information for the objective and decision variables are obtained as follows [21]:

\[x_{jopt} = [x^l_{jopt}, x^u_{jopt}], \quad \forall j\] (47a)

\[y_{jhopt} = [y^l_{jhopt}, y^u_{jhopt}], \quad \forall j, h\] (47b)

\[Z_{opt} = [Z^l_{opt}, Z^u_{opt}]\] (47a)

### 3.2. The Proposed Fuzzy Solution Approach.

Multiple approaches have been proposed to challenge the multi-objective linear programming (MOLP) models in the literature, fuzzy programming approaches (FPAs) being used more widely. This extensive use of FPAs, in recent years is because the FPAs can directly measure the satisfaction level of each objective function. The first fuzzy solution method for MOLP problems was developed by Zimmermann [46], ”min-max approach”; however, this technique sometimes gives inefficient solutions [24]. Torabi and Has-sini [37] proposed a new method for solving MOLP problems, ”TH method”. This paper proposes an interactive fuzzy solution approach by combining the presented method in previous section and TH method. The steps of the proposed hybrid method can be summarized as the following algorithm:

**Algorithm:**

**Step 1:** Identifying all uncertain parameters and decision variables

**Step 2:** Formulating a specific fuzzy-stochastic multi-objective mathematical model

**Step 3:** Converting the fuzzy-stochastic mathematical model to equivalent auxiliary crisp model through subsection 3.1

**Step 4:** Reformulating and solving sub-model one which corresponds to \(Z^l\) and go to the step 6

**Step 5:** Reformulating and solving sub-model two which corresponds to \(Z^u\) and go to the step 6

**Step 6:** Determining the \(\alpha\)-positive ideal solution (\(\alpha\)-PIS) and \(\alpha\)-negative ideal solution (\(\alpha\)-NIS) for each objective function to reach the \(\alpha\)-positive ideal solutions, i.e.,
\[
\begin{align*}
Z_H^{\alpha-PISu} &= x_H^{\alpha-PISu}, y_H^{\alpha-PISu}, \quad Z_H^{\alpha-NISu} = z_h(x_H^{\alpha-PISu}, y_H^{\alpha-PISu}), \quad Z_H^{\alpha-NISl} = z_h(x_H^{\alpha-PISl}, y_H^{\alpha-PISl})
\end{align*}
\]

**Step 7:** Determining a linear membership function for each objective function as follows:

\[
\begin{align*}
\mu^1_H(x, y) &= \begin{cases} 
1 & \text{if } Z_H^u > Z_H^{\alpha-PISu} \\
\frac{Z_H^u - Z_H^{\alpha-NISu}}{Z_H^{\alpha-PISu} - Z_H^{\alpha-NISu}} & \text{if } Z_H^{\alpha-NISu} \leq Z_H^u \leq Z_H^{\alpha-PISu} \\
0 & \text{if } Z_H^u < Z_H^{\alpha-NISu}
\end{cases} \\
\mu^2_H(x, y) &= \begin{cases} 
1 & \text{if } Z_H^u < Z_H^{\alpha-PISu} \\
\frac{Z_H^u - Z_H^{\alpha-NISu}}{Z_H^{\alpha-PISu} - Z_H^{\alpha-NISu}} & \text{if } Z_H^{\alpha-PISu} \leq Z_H^u \leq Z_H^{\alpha-NISu} \\
0 & \text{if } Z_H^u > Z_H^{\alpha-NISu}
\end{cases} \\
\mu^1_H(x, y) &= \begin{cases} 
1 & \text{if } Z_H^l > Z_H^{\alpha-PISl} \\
\frac{Z_H^l - Z_H^{\alpha-NISl}}{Z_H^{\alpha-PISl} - Z_H^{\alpha-NISl}} & \text{if } Z_H^{\alpha-NISl} \leq Z_H^l \leq Z_H^{\alpha-PISl} \\
0 & \text{if } Z_H^l < Z_H^{\alpha-NISl}
\end{cases} \\
\mu^2_H(x, y) &= \begin{cases} 
1 & \text{if } Z_H^l < Z_H^{\alpha-PISl} \\
\frac{Z_H^l - Z_H^{\alpha-NISl}}{Z_H^{\alpha-PISl} - Z_H^{\alpha-NISl}} & \text{if } Z_H^{\alpha-PISl} \leq Z_H^l \leq Z_H^{\alpha-NISl} \\
0 & \text{if } Z_H^l > Z_H^{\alpha-NISl}
\end{cases}
\]

In fact, \( \mu^H_{l,u}(x, y) \) identifies the satisfaction degree of the Hth objective function for solution vector \((x, y)\) for each of the sub-models. Notably, \( \mu^1_H(x, y) \) and \( \mu^2_H(x, y) \) are utilized for maximizing and minimizing objective function, respectively.

**Step 8:** Transforming the equivalent crisp models into a single-objective model using the Torabi and Hassini [37] aggregation function. It is noteworthy that this method makes obtaining of efficient solutions possible. The TH aggregation function is calculated by:

\[
\text{max } \lambda(x, y) = \psi \lambda_0 + (1 - \psi) \sum_H \theta_H \mu_H(x, y)
\]

Subject to:

\[
\begin{align*}
\lambda_0 &\leq \mu_H(x, y), \quad \forall H \\
x, y &\in F(x, y), \quad \lambda, \lambda_0 \in [0, 1]
\end{align*}
\]

Where \( F(x, y) \) is the feasible region, comprising some constraints of the parallel crisp model. Also the importance of the Hth objective function and the coefficient of compensation (\( \sum_H \theta_H = 1, \theta_h > 0 \)) are denoted by \( \theta_H \), respectively. Notably, the optimal value of variable \( \lambda_0 = \min_H \{ \mu_H(x, y) \} \) shows the minimum satisfaction degree of objective functions. The TH aggregation function seeks for a compromise value between the minimum operator and the weighted sum operator according to value of \( \psi \). That is the decision makers can obtain balanced and unbalanced compromised solution by manipulating the value of \( \theta_H \) and \( \psi \) parameters assuming their preferences.
Step 9: Determining the value of the coefficient of compensation $\psi$ and the relative importance of the fuzzy objectives ($\theta_H$), and solve the related single-objective models. If the decision maker is satisfied with the proposed solution, stop; otherwise find another compromised solution by manipulating the values of $\psi$ and $\alpha$ (and if required, the value of ($\theta_H$) ).

Step 10: Generating the final solutions of $Z_H^l = [Z_H^l, Z_H^u]$.

Computational Complexity:
The computational complexity (CC) of Algorithm of the algorithm can be calculated as follows. The CC for $Z^l$ is equal to $Z^u$ and obtaining as below: CC for Eq.(1) is $5T$, CC for Eq.(2) is $3DPT + 2BT + 2LT$, CC for Eq.(3) is $4TIP + 5T^2IIP + 3QT^2BP$, CC for Eq.(4) is $3QLPIB$, CC for Eq.(5) is $2B + 2PD + 2PL + 3PBI + 3PUBL + 2QPLB + 2PBI + QLPB + 2QPBD$ CC for constraints is $2B + LD + QL + QD + 3I + QB + 3I + QB + 2I + QL + 3I + QB + 2I + QL + 2I + 2P + 2PI + QB + BPQ + 3Q + 6P, CC$ for Eq.(49) and (53) is 24. We assumed that maximum the upper bound of all sets is $n$, hence, we have CC for Eq.(1) is $5n$, CC for Eq.(2) is $3n^3 + 4n^2 = O(n^3), CC$ for Eq.(3) is $4n^3 + 5n^4 + 3n^5 = O(n^5), CC$ for Eq.(4) is $O(n^5), CC$ for Eq.(5) is $O(n^5) CC$ for constraints is $O(n^3), CC$ for Eq.(49) and (53) is 24. Therefore, the CC for the $Z^l$ and $Z^u$ is equal to $O(n^5)$. Therefore, the complexity of the proposed algorithm is $2(O(n^5) + 24)$, we can summarize it $2(O(n^5))$.

Numerical Example

Step 1: Deterministic form of multi-objective mathematical model is as follows:

$$\begin{align*}
\min Z_1 &= C_{1x}x + C_{1y}y + C_{1z}z + C_{1v}v + C_{1w}w \\
\min Z_2 &= C_{2x}x + C_{2y}y + C_{2z}z + C_{2v}v + C_{2w}w \\
\text{S.t:} & \\
& \quad a_{1x}x + a_{1y}y + a_{1z}z + a_{1v}v + a_{1w}w \leq b_1 \\
& \quad a_{2x}x + a_{2y}y + a_{2z}z + a_{2v}v + a_{2w}w \leq b_2 \\
& \quad x, y, z, v, w \geq 0
\end{align*}$$

Step 2: Specifying uncertain parameters in above mentioned multi-objective mathematical model: To display the details of the proposed algorithm, we assumed that coefficients of the objective functions ($C_{(1,2)x}, C_{(1,2)y}, C_{(1,2)z}, C_{(1,2)v}, C_{(1,2)w}$), technological coefficients ($a_{(1,2)x}, a_{(1,2)y}, a_{(1,2)z}, a_{(1,2)v}, a_{(1,2)w}$), the first right hand side ($b_1$) and probability ($q$) are trapezoidal fuzzy numbers. But, the second right hand side is independent random variable with uniform probability distribution $[b_2 \sim \text{uniform}(l, k)]$. Moreover, the first three decision variables ($x, y, z$) are first-stage decision variables and ($v_b, w_b$) are second-stage decision variables under scenario. It should be noted that the cumulative uniform distribution function is $\frac{b_2 - l}{k - l}$.

Step 3: Uncertain parameters and decision variables are as Tables 1 and 2:

- $CFOF$: Coefficients of first objective function
- $CSOF$: Coefficients of second objective function
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DV: Decision variables
RHS: Right hand sides
PRS: Probability of realization of scenario
TC: Technological coefficients

\[ \text{Step 4:} \text{ Fuzzy-stochastic multi-objective mathematical model are as follows:} \]
\[ \begin{align*}
\min Z_1 &= \tilde{C}_{1x} x + \tilde{C}_{1y} y + \tilde{C}_{1z} z + \sum_{h=1}^{2} p_h \tilde{C}_{1v} v_h + \sum_{h=1}^{2} p_h \tilde{C}_{1w} w_h \\
\min Z_2 &= \tilde{C}_{2x} x + \tilde{C}_{2y} y + \tilde{C}_{2z} z + \sum_{h=1}^{2} p_h \tilde{C}_{2v} v_h + \sum_{h=1}^{2} p_h \tilde{C}_{2w} w_h \\
\text{s.t.:} & \quad \tilde{a}_{1x} x + \tilde{a}_{1y} y + \tilde{a}_{1z} z + \tilde{a}_{1v} v + \tilde{a}_{1w} w \leq \tilde{b}_1 \\
& \quad \tilde{a}_{2x} x + \tilde{a}_{2y} y + \tilde{a}_{2z} z + \tilde{a}_{2v} v + \tilde{a}_{2w} w \leq \tilde{b}_2 \\
& \quad x, y, z, v_h, w_h \geq 0, \quad \forall h = 1, 2
\end{align*} \]

It should be noted that the equivalent auxiliary of the second right hand side parameter (\(\tilde{b}_2\)) can be calculated as follows: \(\tilde{b}_2 = F^{-1}(\tilde{q}) = \tilde{q}(k - 1) + l\).

\[ \text{Step 5:} \text{ Replacing the values of parameters in fuzzy-stochastic multi-objective mathematical model are as follows:} \]
\[ \begin{align*}
\min Z_1 &= (1, 1.5, 1.75, 2) x + (3, 3.5, 4, 4.25) y + (1, 1.25, 2, 2.25) z + \\
&\quad [0.2(4, 4.5, 5, 5.5)v_1] + [0.8(4, 4.5, 5, 5.5)v_2] + [0.2(0.5, 1, 1.5, 1.75)w_1] \\
&\quad + [0.8(0.5, 1, 1.5, 1.75)w_2] \\
&\quad + [0.8(0.5, 1, 1.5, 1.75)w_2] (a) \\
\min Z_2 &= (1, 1.5, 2, 2.5) x + (0.5, 0.75, 1, 1.25) y + (-2, -1.5, -1, -0.75) z \\
&\quad [0.2(2, 3, 4, 4.5)v_1] + [0.8(2, 3, 4, 4.5)v_2] + [0.2(-3, -2.25, -2, -1)w_1]
\end{align*} \]
Step 6-8: Equivalent auxiliary crisp model based on \((\alpha = 0.5)\) are as follows:

\[
\begin{align*}
\text{min } Z'_1 &= \left[ (1 - \alpha)1 + \alpha1.5 \right] x + \left[ (1 - \alpha)3 + \alpha3.5 \right] y \\
&\quad + \left[ (1 - \alpha)1 + \alpha1.25 \right] z + \left[ (1 - \alpha)0.8 \right] \\
&\quad + \alpha0.9v_1 + \left[ (1 - \alpha)3.2 + \alpha3.6 \right] v_2 + \left[ (1 - \alpha)0.1 + \alpha0.2 \right] w_1 \\
&\quad + \left[ (1 - \alpha)0.4 + \alpha0.8 \right] w_2 \\
&= 1.25x + 3.25y + 1.125z + 0.85v_1 + 3.4v_2 + 0.15w_1 + 0.6w_2
\end{align*}
\]

S.t:

\[
\begin{align*}
&(1 - \alpha)1.5 + \alpha2] x + [(1 - \alpha)0.5 + \alpha1] y \\
&\quad + [(1 - \alpha)0.5 + \alpha0.75] z \\
&\quad \geq [(1 - \alpha)7 + \alpha6.5] \rightarrow 1.75x + 0.75y + 0.625z \geq 6.75 \\
&(1 - \alpha)(-2) + \alpha(-2.25)] x + [(1 - \alpha)(-0.5) + \alpha(-1)] y \\
&\quad + [(1 - \alpha)0.5 + \alpha1] z + \\
&\quad [(1 - \alpha)1 + \alpha1.25] v_1 + [(1 - \alpha)1.75 + \alpha2] w_1 \\
&\quad \geq \{9 - 7\}[(1 - \alpha)0.25 + \alpha0.4] + 7 \rightarrow \\
&\quad -1.125x - 0.75y + 0.75z + 1.125v_1 + 1.875w_1 \geq 7.65 \\
&(1 - \alpha)(-2) + \alpha(-2.25)] x + [(1 - \alpha)(-0.5) + \alpha(-1)] y \\
&\quad + [(1 - \alpha)0.5 + \alpha1] z \\
&\quad + [(1 - \alpha)1 + \alpha1.25] v_2 + [(1 - \alpha)1.75 + \alpha2] w_2 \\
&\quad \geq \{10 - 6\}[(1 - \alpha)0.25 + \alpha0.4] + 6 \\
&\rightarrow -1.125x - 0.75y + 0.75z + 1.125v_2 + 1.875w_2 \geq 7.3
\end{align*}
\]
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\[ \begin{align*}
&x, y, z, v_1, v_2, w_1, w_2 \geq 0 \\
&\text{min} Z_1^\alpha = (1 - \alpha)2 + \alpha 1.75 |x| + (1 - \alpha)4.25 + \alpha 4|y| \\
&\quad + [(1 - \alpha)2.25 + \alpha 2]z \\
&\quad + [(1 - \alpha)1.1 + \alpha 1]v_1 + [(1 - \alpha)4.4 + \alpha 4]v_2 \\
&\quad + [(1 - \alpha)0.35 + \alpha 0.3]w_1 \\
&\quad + [(1 - \alpha)1.4 + \alpha 1.2]w_2 = 1.875x + 4.125y + 2.125z + 1.05v_1 + 4.2v_2 \\
&\quad + 0.325w_1 + 1.3w_2 \\
\text{S.t:} \\
&[(1 - \alpha)3 + \alpha 2.5]x + [(1 - \alpha)2 + \alpha 1.5]y + [(1 - \alpha)1.75 + \alpha 1.25]z \\
&\quad \geq [(1 - \alpha)5 + \alpha 6] \\
&\quad \rightarrow 2.75x + 1.75y + 1.5z \geq 5.5 \\
&\quad + [(1 - \alpha)(-3) + \alpha (-2.5)]x + [(1 - \alpha)(-2) + \alpha (-1.5)]y \\
&\quad + [(1 - \alpha)1.75 + \alpha 1.25]z \\
&\quad + [(1 - \alpha)1.75 + \alpha 1.5]v_1 + [(1 - \alpha)2.75 + \alpha 2.25]w_1 \\
&\geq \{(10 - 6)(1 - \alpha)0.65 + \alpha 0.5 + 6\} - 2.75x - 1.75y + 1.5z + 1.625v_1 \\
&\quad + 2.5w_1 \geq 7.825 \\
&\quad + [(1 - \beta)(-3) + \alpha (-2.5)]x + [(1 - \alpha)(-2) + \alpha (-1.5)]y \\
&\quad + [(1 - \alpha)1.75 + \alpha 1.25]z \\
&\quad + [(1 - \alpha)1.75 + \alpha 1.5]v_1 + [(1 - \alpha)2.75 + \alpha 2.25]w_1 \\
&\geq \{(10 - 6)(1 - \alpha)0.65 + \alpha 0.5 + 6\} - 2.75x - 1.75y + 1.5z + 1.625v_2 \\
&\quad + 2.5w_2 \geq 8.3 \\
&x, y, z, v_1, v_2, w_1, w_2 \geq 0 \\
&\text{min} Z_2^\alpha = (1 - \alpha)1 + \alpha 1.5 |x| + (1 - \alpha)0.5 + \alpha 0.75|y| \\
&\quad + [(1 - \alpha)0.4 + \alpha 0.6]v_1 + [(1 - \alpha)1.6 + \alpha 2.4]v_2 \\
&\quad + [(1 - \alpha)(-0.2) + \alpha (-0.4)]w_1 + +[(1 - \alpha)(-0.8) + \alpha (-1.6)]w_2 \\
&\quad = 1.25x + 0.625y - 0.875z + 0.5v_1 + 2v_2 - 0.3w_1 - 1.2w_2 \\
\text{S.t:} \\
&1.75x + 0.75y + 0.625z \geq 6.75 \\
&-1.125x - 0.75y + 0.75z + 1.125v_1 + 1.875w_1 \geq 7.65 \\
&-1.125x - 0.75y + 0.75z + 1.125v_2 + 1.875w_2 \geq 7.3 \\
&x, y, z, v_1, v_2, w_1, w_2 \geq 0
\[
\begin{align*}
\text{min} Z_2^\alpha &= [(1-\alpha)2.5 + \alpha 2]x + [(1-\alpha)1.25 + \alpha 1]y \\
&+ [(1-\alpha)(-2) + \alpha (-1.5)]z \\
&+ [(1-\alpha)0.9 + \alpha 0.8]v_1 + [(1-\alpha)3.6 + \alpha 3.2]v_2 \\
&+ [(1-\alpha)(-0.6) + \alpha (-0.45)]w_1 \\
&+ [(1-\alpha)(-2.4) + \alpha (-1.8)]w_2 = 2.25x + 1.125y - 1.75z + 0.85v_1 \\
&+ 3.4v_2 - 0.525w_1 - 2.1w_2
\end{align*}
\]

S.t:
\[
\begin{align*}
2.75x + 1.75y + 1.5z &\geq 5.5 \\
-2.75x - 1.75y + 1.5z + 1.625v_1 + 2.5w_1 &\geq 7.825 \\
-2.75x - 1.75y + 1.5z + 1.625v_2 + 2.5w_2 &\geq 8.3
\end{align*}
\]

\[
x, y, z, v_1, v_2, w_1, w_2 \geq 0
\]

**Step 9:** \(\alpha\)-positive ideal solution (\(\alpha\)-PIS) and \(\alpha\)-negative ideal solution (\(\alpha\)-NIS) for each objective function are as Table 3:

<table>
<thead>
<tr>
<th>Positive ideal solution</th>
<th>Negative ideal solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Z_1^{\alpha\text{PIS}} = 9.505, w_1 = 6.394, w_2 = 6.20, x = 3.857)</td>
<td>(Z_1^{\alpha\text{NIS}} = 12.138, Z_2^{\alpha\text{NIS}} = -4.545)</td>
</tr>
<tr>
<td>(Z_1^{\alpha\text{PIS}} = -9.45, z = 10.79)</td>
<td>(Z_1^{\alpha\text{NIS}} = 9.549, z = 3.66, w_1 = 9.90, w_2 = 1.12)</td>
</tr>
<tr>
<td>(Z_1^{\alpha\text{NIS}} = -9.683, z = 5.53)</td>
<td>(Z_1^{\alpha\text{NIS}} = 11.751, Z_2^{\alpha\text{NIS}} = -9.245)</td>
</tr>
</tbody>
</table>

**Step 10:** Linear membership function for each objective function are as follows:

\[
\begin{align*}
\mu_1^\alpha &= \begin{cases} 
1 & Z_1^\alpha \leq 9.505 \\
\frac{12.138 - Z_1^\alpha}{12.138 - 9.505} & 9.505 < Z_1^\alpha < 12.138 \\
0 & Z_1^\alpha > 12.138
\end{cases}
\]

\[
\begin{align*}
\mu_2^\alpha &= \begin{cases} 
1 & Z_2^\alpha \leq -9.45 \\
\frac{-4.545 - Z_2^\alpha}{-4.545 - (-9.45)} & -9.45 < Z_2^\alpha < -4.545 \\
0 & Z_2^\alpha > -4.545
\end{cases}
\]

**Step 11 and 12:** The TH aggregation function based on \(\psi = 0.5\) are as follows:

\[
\max \lambda^\alpha = \psi \lambda_0 + (1 - \psi)\left[\theta_1 \frac{12.138 - Z_1^\alpha}{12.138 - 9.505} + \theta_2 \frac{-4.545 - Z_2^\alpha}{-4.545 - (-9.45)}\right]
\]
S.t:

\[
\begin{align*}
\lambda_0 & \leq \frac{12.138 - Z_l^1}{12.138 - 9.505} \\
\lambda_0 & \leq \frac{-4.545 - Z_l^2}{-4.545 - (-9.45)} \\
1.75x + 0.75y + 0.625z & \geq 6.75 \\
-1.125x - 0.75y + 0.75z + 1.125v_1 + 1.875w_1 & \geq 7.65 \\
-1.125x - 0.75y + 0.75z + 1.125v_2 + 1.875w_2 & \geq 7.3 \\
x, y, z, v_1, v_2, w_1, w_2 & \geq 0, \lambda_0, \lambda^l \in [0, 1] \\
\max \lambda^u = 0.5\lambda_0 + 0.5[(0.4 - 11.751 - Z_u^1) + (0.6 - 9.245 - Z_u^2)]
\end{align*}
\]

S.t:

\[
\begin{align*}
\lambda_0 & \leq \frac{11.751 - Z_u^1}{11.751 - 9.549} \\
\lambda_0 & \leq \frac{-9.245 - Z_u^2}{-9.245 - (-9.683)} \\
2.75x + 1.75y + 1.5z & \geq 5.5 \\
\rightarrow -2.75x - 1.75y + 1.5z & \geq 7.825 \\
-2.75x - 1.75y + 1.5z & \geq 8.3 \\
x, y, z, v_1, v_2, w_1, w_2 & \geq 0, \lambda_0, \lambda^l \in [0, 1] \\
\end{align*}
\]

**Step13:** By solving \(\lambda^l\) and \(\lambda^u\), the final solutions are as follows:

\[
Z_l^{l,u} = [10.84, 11.32], Z_l^{u,u} = [-7.45, -5.71],
\]

4. **Computational Results**

To demonstrate the validity and the applicability of the proposed model, the numerical examples, from the literature \[39, 40, 41\] are given and the related results are reported. Consider a multi-echelon closed-loop supply chain network similar to the one shown in Figure 1. The scale of each is shown in Table 4. The detailed uncertain parameters are given in Table 5. All the mathematical models are codified in the optimization software (i.e. GAMS). All positive variables are stochastic variables. In this paper we considered three scenarios for these variables and capacity parameters. The summaries of test results on the basis of \(\psi = 0.5\) and \((\theta_1 = \theta_2 = \theta_3), (p_1 = 0.2, p_2 = 0.5, p_3 = 0.3)\) are provided in Tables 6-8. The results of our computations show that the proposed model and presented solution methodology can effectively support the development of a reliable CLSC model for the companies in a supply chain environment under uncertainty.
Table 4. Size of Test Problems

<table>
<thead>
<tr>
<th>Problem no.</th>
<th>P</th>
<th>L</th>
<th>B</th>
<th>D</th>
<th>I</th>
<th>Q</th>
<th>T</th>
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<td>4</td>
<td>6</td>
<td>4</td>
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<td>8</td>
<td>4</td>
<td>10</td>
<td>4</td>
<td>3</td>
</tr>
</tbody>
</table>

Table 5. Sources of Parameters for Three Test Problems

5. Conclusion

In this paper, a closed-loop supply chain design problem has been evaluated that incorporates both strategic and tactical decisions under a fuzzy-stochastic environment. The model consisted of three objective functions: maximization of profit, minimization of delivery time of new products, collection time and disposal time of used products and maximization of flexibility. To solve the model, an interactive hybrid solution methodology was adopted by integrating hybrid fuzzy-stochastic programming method with fuzzy multi-objective approach. The framework is not only a tool for researchers to investigate interesting research issues,
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References


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