

DELAY-DEPENDENT ROBUST STABILIZATION AND H_∞ CONTROL FOR UNCERTAIN STOCHASTIC T-S FUZZY SYSTEMS WITH MULTIPLE TIME DELAYS

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ABSTRACT. In this paper, the problems of robust stabilization and H_∞ control for uncertain stochastic systems with multiple time delays represented by the Takagi-Sugeno (T-S) fuzzy model have been studied. By constructing a new Lyapunov-Krasovskii functional (LKF) and using the bounding techniques, sufficient conditions for the delay-dependent robust stabilization and H_∞ control scheme are presented in terms of linear matrix inequalities (LMIs). By solving these LMIs, a desired fuzzy controller can be obtained which can be easily calculated by Matlab LMI control toolbox. Finally, a numerical simulation is given to illustrate the effectiveness of the proposed method.

1. Introduction

During the past two decades, a great amount of effort has been devoted to the study of Takagi-Sugeno (T-S) fuzzy systems since, by using a T-S fuzzy plant model, we can describe a nonlinear system as a weighted sum of some simple linear subsystems (see [9, 23, 25]). This fuzzy model is described by a family of fuzzy IF-THEN rules that represent local linear input/output relations of the system. The overall fuzzy model of the system is achieved by smoothly blending these local linear models together with membership functions. There has been a rapidly growing interest in the study of stability analysis and controller synthesis for T-S fuzzy systems and many results have been reported, see for example [4, 5, 24, 27, 32]. So far, a great number of results have been reported for T-S fuzzy systems. Recently stability, stabilization and H_∞ control designs have been studied in many papers [8, 12, 17, 19, 22, 30, 33, 34, 39, 40].

Many practical systems can be described by differential equations with various kinds of delays which lead the modeling of aircraft systems, biology systems, electronic circuits, population ecology, chemical process, communication systems and network control systems etc. The existence of time delays are often the main sources of instability and poor performance of a control system. Further, uncertainties are also unavoidable in control systems due to modeling errors, measurement errors, and so on. Therefore, the problem of stability analysis and controller design for fuzzy dynamic time delay uncertain systems are practically important and have attracted

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considerable attention over the past years, see for example in [4, 7, 11, 30, 32, 40] and references therein. Most of the results can be classified into two types: delay-independent results as reported in [4, 32] and delay-dependent results as described in [7, 11]. Generally, delay-dependent approaches are less conservative than the delay-independent ones since the results depend explicitly on time delay. Recently, robust H_∞ fuzzy control for multiple time delays are considered in [14, 35], and fuzzy H_∞ filter design for multiple time delays has been addressed in [36].

On the other hand, modeling and control design for stochastic systems play an important role in many industrial fields. In recent years, increasing efforts have been made to study stochastic systems with time delays [1, 2, 6, 10, 13, 29, 31]. Recently, the T-S fuzzy model-based frameworks are introduced to attack nonlinear stochastic systems. The linear matrix inequalities (LMIs) approach for robust stability analysis of stochastic fuzzy systems with time delays has been proposed by [26]. Delay-dependent stabilization problems for time delay stochastic fuzzy systems have been concerned in [16, 38]. Fuzzy model-based control of nonlinear stochastic systems with time delays has been studied in [15]. A guaranteed cost control problem for a class of uncertain stochastic fuzzy systems with multiple time delays has been discussed in [37]. To the best of our knowledge, the robust stabilization and H_∞ control for uncertain stochastic systems with multiple time delays represented by the Takagi-Sugeno (T-S) fuzzy model have not been fully investigated yet, which are very important in both theories and applications, which motivates our research.

This paper is organized as follows. In section 2, the problem formulation and related preliminaries are presented. Some sufficient condition for the robust stochastic stabilization and H_∞ control problem are presented in sections 3 and 4, respectively. Section 5 demonstrates a numerical example. Finally, the paper is concluded in section 6.

Notations. The following fairly standard notations are used throughout this paper. \mathbb{R}^n and $\mathbb{R}^{n \times m}$ denote, respectively, the n dimensional Euclidean space and the set of all $n \times m$ real matrices. The superscript T denotes transpose and the notation $X \geq Y$ (respectively, $X > Y$) where X and Y are symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite), and I denotes the identity matrix with appropriate dimensions. $L_2[0, \infty)$ is the space of square integrable vector. Let $\tau > 0$ and $C([-\tau, 0]; \mathbb{R}^n)$ denotes the family of continuous functions φ from $[-\tau, 0]$ to \mathbb{R}^n with the norm $\|\varphi\| = \sup_{-\tau \leq \theta \leq 0} |\varphi(\theta)|$, where $|\cdot|$ stands for the Euclidean vector norm and $\|\cdot\|_2$ represents the $L_2[0, \infty)$ norm. Moreover, let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$ satisfying the usual conditions (i.e., the filtration contains all \mathcal{P} -null sets and is right continuous). Denote $L^2_{\mathcal{F}_0}([-\tau, 0]; \mathbb{R}^n)$ by the family of all \mathcal{F}_0 measurable $C([-\tau, 0]; \mathbb{R}^n)$ - valued random variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\sup_{-\tau \leq \theta \leq 0} \mathcal{E}|\xi(\theta)|^p < \infty$, where $\mathcal{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure \mathcal{P} . The asterisk “*” denotes a matrix that can be inferred by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem Formulation and Preliminaries

Consider the following uncertain stochastic T-S fuzzy model with multiple time delays described by

Plant rule i : IF $\theta_1(t)$ is η_{i1} and \dots and $\theta_p(t)$ is η_{ip} THEN

$$\begin{aligned} (\Sigma) : dx(t) &= \left[\sum_{k=0}^m (A_{ik} + \Delta A_{ik}(t))x(t - \tau_k) + (B_{1i} + \Delta B_{1i}(t))u(t) + B_{vi}v(t) \right] dt \\ &\quad + \left[\sum_{k=0}^m (D_{ik} + \Delta D_{ik}(t))x(t - \tau_k) + (B_{2i} + \Delta B_{2i}(t))u(t) \right. \\ &\quad \left. + D_{vi}v(t) \right] dw(t), \end{aligned} \quad (1)$$

$$z(t) = \sum_{k=0}^m C_{ik}x(t - \tau_k) + B_{3i}u(t) + C_{vi}v(t), \quad (2)$$

$$x(t) = \phi(t), \quad t \in [-\tau, 0] \quad (3)$$

where $i = 1, 2, \dots, r$; r is the number IF-THEN rules; $\theta_1(t), \dots, \theta_p(t)$ are the premise variables; η_{ij} are the fuzzy set, $j = 1, 2, \dots, p$; $x(t) \in \mathbb{R}^n$ is the state; $u(t) \in \mathbb{R}^m$ is the control input; and $v(t) \in \mathbb{R}^p$ is the disturbance input which belongs to $L_2[0, \infty)$; $z(t) \in \mathbb{R}^q$ is the controlled output vector; $\tau_0 = 0$; $\tau_k > 0$, $k = 1, \dots, m$, denote the state delay; $\tau = \max\{\tau_k, k \in [1, m]\}$; $\omega(t)$ is a scalar Brownian motion defined on the probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t \geq 0}, \mathcal{P})$ satisfying $\mathcal{E}\{dw(t)\} = 0$, $\mathcal{E}\{dw(t)^2\} = dt$. In the system (Σ) , A_{ik} , B_{1i} , B_{vi} , D_{ik} , B_{2i} , D_{vi} , C_{ik} , B_{3i} and C_{vi} are known real constant matrices with appropriate dimensions. $\Delta A_{ik}(t)$, $\Delta B_{1i}(t)$, $\Delta D_{ik}(t)$ and $\Delta B_{2i}(t)$ are uncertain matrix functions satisfy the following condition:

$$\begin{bmatrix} \Delta A_{ik}(t) & \Delta B_{1i}(t) & \Delta D_{ik}(t) & \Delta B_{2i}(t) \end{bmatrix} = E_i F_i(t) \begin{bmatrix} H_{1ik} & H_{2i} & H_{3ik} & H_{4i} \end{bmatrix}, \quad (4)$$

where E_i , H_{1ik} , H_{2i} , H_{3ik} and H_{4i} are known as real constant matrices with appropriate dimensions and $F_i(t)$ is an unknown real time-varying matrix function satisfying

$$F_i^T(t) F_i(t) \leq I. \quad (5)$$

It is assumed that all elements of $F_i(t)$ are Lebesgue measurable. $\Delta A_{ik}(t)$, $\Delta B_{1i}(t)$, $\Delta D_{ik}(t)$ and $\Delta B_{2i}(t)$ are said to be admissible if both (4) and (5) hold. By using center average defuzzifier, product inference and singleton fuzzifier, the global dynamics of the T-S fuzzy system (Σ) can be inferred as

$$\begin{aligned} (\hat{\Sigma}) : dx(t) &= \sum_{i=1}^r h_i(\theta(t)) \left\{ \left[\sum_{k=0}^m A_{ik}(t)x(t - \tau_k) + B_{1i}(t)u(t) + B_{vi}v(t) \right] dt \right. \\ &\quad \left. + \left[\sum_{k=0}^m D_{ik}(t)x(t - \tau_k) + B_{2i}(t)u(t) + D_{vi}v(t) \right] dw(t) \right\}, \end{aligned} \quad (6)$$

$$z(t) = \sum_{i=1}^r h_i(\theta(t)) \left\{ \sum_{k=0}^m C_{ik}x(t - \tau_k) + B_{3i}u(t) + C_{vi}v(t) \right\}, \quad (7)$$

$$x(t) = \phi(t), \quad t \in [-\tau, 0] \quad (8)$$

where $A_{ik}(t) = A_{ik} + \Delta A_{ik}(t)$, $B_{1i}(t) = B_{1i} + \Delta B_{1i}(t)$, $D_{ik}(t) = D_{ik} + \Delta D_{ik}(t)$, $B_{2i}(t) = B_{2i} + \Delta B_{2i}(t)$ with $h_i(\theta(t)) = \frac{\nu_i(\theta(t))}{\sum_{i=1}^r \nu_i(\theta(t))}$, $\nu_i(\theta(t)) = \prod_{j=1}^p \eta_{ij}(\theta_j(t))$, and $\eta_{ij}(\theta_j(t))$ is the grade of membership value of $\theta_j(t)$ in η_{ij} . In this paper, we assume that $\nu_i(\theta(t)) \geq 0$ for $i = 1, 2, \dots, r$ and $\sum_{i=1}^r \nu_i(\theta(t)) > 0$ for all t . Therefore, $h_i(\theta(t)) \geq 0$ (for $i = 1, 2, \dots, r$) and $\sum_{i=1}^r h_i(\theta(t)) = 1$ for all t . In the sequel, for simplicity we use h_i to represent $h_i(\theta(t))$.

Based on the parallel distributed compensation schemes, a fuzzy model of a state feedback controller for the system ($\hat{\Sigma}$) is formulated as follows:

Control rule i: IF $\theta_1(t)$ is η_{i1} and \dots and $\theta_p(t)$ is η_{ip} THEN

$$u(t) = K_i x(t), \quad i = 1, 2, \dots, r. \quad (9)$$

The over all state feedback fuzzy control law is represented by

$$u(t) = \sum_{i=1}^r h_i K_i x(t) \quad (10)$$

where K_i ($i = 1, 2, \dots, r$) are the local control gains. Under control law (10), the overall closed-loop system is obtained as follows:

$$(\tilde{\Sigma}): dx(t) = \left[\sum_{k=0}^m A_{ak} x(t - \tau_k) + B_v v(t) \right] dt + \left[\sum_{k=0}^m D_{ak} x(t - \tau_k) + D_v v(t) \right] dw(t), \quad (11)$$

$$z(t) = \sum_{k=0}^m C_{ak} x(t - \tau_k) + C_v v(t), \quad (12)$$

$$x(t) = \phi(t), \quad t \in [-\tau, 0], \quad (13)$$

where the expressions for A_{ak} , B_v , D_{ak} , D_v , C_{ak} and C_v are given by

$$\begin{aligned} A_{ak} &= \begin{cases} \sum_{i=1}^r \sum_{j=1}^r h_i h_j (A_{ik}(t) + B_{1i}(t) K_j), & \text{for } k = 0 \\ \sum_{i=1}^r \sum_{j=1}^r h_i h_j A_{ik}(t) = \sum_{i=1}^r h_i A_{ik}(t), & \text{for } k = 1, 2, \dots, m \end{cases} \\ D_{ak} &= \begin{cases} \sum_{i=1}^r \sum_{j=1}^r h_i h_j (D_{ik}(t) + B_{2i}(t) K_j), & \text{for } k = 0 \\ \sum_{i=1}^r \sum_{j=1}^r h_i h_j D_{ik}(t) = \sum_{i=1}^r h_i D_{ik}(t), & \text{for } k = 1, 2, \dots, m \end{cases} \\ C_{ak} &= \begin{cases} \sum_{i=1}^r \sum_{j=1}^r h_i h_j (C_{ik} + B_{3i} K_j), & \text{for } k = 0 \\ \sum_{i=1}^r \sum_{j=1}^r h_i h_j C_{ik} = \sum_{i=1}^r h_i C_{ik}, & \text{for } k = 1, 2, \dots, m \end{cases} \\ B_v &= \sum_{i=1}^r \sum_{j=1}^r h_i h_j B_{v_i} = \sum_{i=1}^r h_i B_{v_i}, \quad D_v = \sum_{i=1}^r \sum_{j=1}^r h_i h_j D_{v_i} = \sum_{i=1}^r h_i D_{v_i}, \\ C_v &= \sum_{i=1}^r \sum_{j=1}^r h_i h_j C_{v_i} = \sum_{i=1}^r h_i C_{v_i}. \end{aligned}$$

Throughout the paper we shall adopt the following definition.

Definition 2.1. [29] The nominal system (6) and (8) with $u(t)=0$ and $v(t)=0$ is said to be mean-square stable if for any $\varepsilon > 0$, there exists a $\delta(\varepsilon) > 0$ such that $\mathcal{E}\{|x(t)|^2\} < \varepsilon$, $t > 0$, when

$$\sup_{-\tau \leq s \leq 0} \mathcal{E}\{|\phi(s)|^2\} < \delta(\varepsilon).$$

If, in addition

$$\lim_{t \rightarrow \infty} \mathcal{E}\{|x(t)|^2\} = 0,$$

for any initial conditions, then the nominal system (6) and (8) with $u(t)=0$, $v(t)=0$ is said to be mean-square asymptotically stable. The uncertain stochastic system (6) and (8) is said to be robustly stochastically stable if the system associated to (6) and (8) with $u(t)=0$ and $v(t)=0$ is mean-square asymptotically stable for all admissible uncertainties satisfying (4)-(5).

Definition 2.2. [29] Given a scalar $\gamma > 0$, the forced stochastic system ($\tilde{\Sigma}$) with $u(t) = 0$ is said to be robustly stochastically stable with disturbance attenuation γ if it is robustly stochastically stable and under zero initial conditions, $\|z(t)\|_{\mathcal{E}_2} < \gamma \|v(t)\|_2$ is satisfied for all nonzero $v(t) \in L_2[0, \infty)$ and all admissible uncertainties satisfying (4)-(5), where

$$\|z(t)\|_{\mathcal{E}_2} = \left(\mathcal{E} \left\{ \int_0^\infty |z(t)|^2 dt \right\} \right)^{\frac{1}{2}}.$$

Remark 2.3. This paper, deals with the problems of robust stochastic stabilization and robust H_∞ control for the uncertain stochastic fuzzy system ($\tilde{\Sigma}$). More specifically, for the problem of robust stochastic stabilization, our attention is focused on the design of a memoryless state feedback fuzzy controller such that the resulting closed-loop system is robustly stochastically stable. In this case, the system (11) and (13) with $v(t) = 0$ is said to be robustly stochastically stabilizable. For the problem of robust H_∞ control, we are concerned with the design of a memoryless state feedback fuzzy controller such that the resulting closed-loop system is robustly stochastically stable with disturbance attenuation γ . In this case, system ($\tilde{\Sigma}$) is said to be robustly stochastically stabilizable with disturbance attenuation γ .

The following four lemmas are essential to prove the main results in the next sections.

Lemma 2.4. [3] Given constant matrices M , P and Q with appropriate dimensions, where $P^T = P$ and $Q^T = Q > 0$, then $P + M^T Q^{-1} M < 0$ if and only if

$$\begin{bmatrix} P & M^T \\ M & -Q \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -Q & M \\ M^T & P \end{bmatrix} < 0.$$

Lemma 2.5. [28] For any vectors $x, y \in \mathbb{R}^n$, matrices $P \in \mathbb{R}^{n \times n}$, $D \in \mathbb{R}^{n \times n_f}$, $E \in \mathbb{R}^{n_f \times n}$ and $F \in \mathbb{R}^{n_f \times n_f}$ with $P > 0$, $\|F\| \leq 1$, and scalar $\varepsilon > 0$, we have

- (a) $2x^T y \leq x^T P^{-1} x + y^T P y$,
(b) $D F E + E^T F^T D^T \leq \varepsilon^{-1} D D^T + \varepsilon E^T E$.

Lemma 2.6. [21] *Assume that $a(\cdot) \in \mathbb{R}^{n_a}$, $b(\cdot) \in \mathbb{R}^{n_b}$ and $N \in \mathbb{R}^{n_a \times n_b}$ are defined on the interval Ω . Then, for any matrices $X \in \mathbb{R}^{n_a \times n_a}$, $Y \in \mathbb{R}^{n_a \times n_b}$ and $Z \in \mathbb{R}^{n_b \times n_b}$ the following holds*

$$-2 \int_{\Omega} a^T(\alpha) N b(\alpha) d\alpha \leq \int_{\Omega} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix}^T \begin{bmatrix} X & Y - N \\ Y^T - N^T & Z \end{bmatrix} \begin{bmatrix} a(\alpha) \\ b(\alpha) \end{bmatrix} d\alpha$$

where

$$\begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix} \geq 0.$$

Lemma 2.7. [11] *For any real matrices X_{ij} for $i, j = 1, 2, \dots, r$, and $\Lambda > 0$ with appropriate dimensions, we have*

$$\sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r \sum_{l=1}^r h_i h_j h_q h_l X_{ij}^T \Lambda X_{ql} \leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j X_{ij}^T \Lambda X_{ij}$$

where h_i ($1 \leq i \leq r$) are defined as $h_i(\theta(t)) \geq 0$, $\sum_{i=1}^r h_i(\theta(t)) = 1$.

3. Robust Stochastic Stabilization

In this section, sufficient conditions for the solvability of the delay-dependent stabilization problem for stochastic fuzzy systems (11) and (13) with $v(t) = 0$ are going to be derived.

Theorem 3.1. *Assume that the controller gains $\{K_i\}_{i=1}^r$ are given. For given scalars $\tau_k > 0$, $k = 1, 2, \dots, m$, the closed-loop stochastic fuzzy system (11) and (13) with $v(t) = 0$ is said to be robustly stochastically stabilizable, if there exist matrices $P > 0$, $S_k > 0$, $R_k > 0$, W_k , M_k , $Q_k > 0$, $k = 1, 2, \dots, m$ and scalars $\varepsilon_{1ij} > 0$, $\varepsilon_{2ij} > 0$ ($1 \leq i \leq j \leq r$) such that the following LMIs hold*

$$\begin{bmatrix} W_k & M_k \\ * & Q_k \end{bmatrix} \geq 0, \quad 1 \leq k \leq m \quad (14)$$

$$\Psi^{ii} < 0, \quad 1 \leq i \leq r \quad (15)$$

$$\Psi^{ij} + \Psi^{ji} < 0, \quad 1 \leq i < j \leq r \quad (16)$$

where

$$\Psi^{ij} = \begin{bmatrix} \Psi_{11}^{ij} & \Psi_{12}^{ij} & \Psi_{13}^{ij} & \Psi_{14}^{ij} & \Psi_{15}^{ij} \\ * & \Psi_{22}^{ij} & 0 & \Psi_{24}^{ij} & 0 \\ * & * & \Psi_{33}^{ij} & \Psi_{34}^{ij} & 0 \\ * & * & * & \Psi_{44}^{ij} & 0 \\ * & * & * & * & \Psi_{55}^{ij} \end{bmatrix}$$

with

$$\begin{aligned} \Psi_{11}^{ij} &= \begin{bmatrix} \vartheta^{ij} & PA_{i1} - M_1 & PA_{i2} - M_2 & \dots & PA_{im} - M_m \\ * & -S_1 & 0 & \dots & 0 \\ * & * & -S_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \dots & -S_m \end{bmatrix}, \\ \Psi_{12}^{ij} &= [\tilde{D}_{ij}^T \quad \tau_1 \tilde{A}_{ij}^T \quad \tau_2 \tilde{A}_{ij}^T \quad \dots \quad \tau_m \tilde{A}_{ij}^T], \\ \Psi_{13}^{ij} &= [\tau_1 \tilde{D}_{ij}^T \quad \tau_2 \tilde{D}_{ij}^T \quad \dots \quad \tau_m \tilde{D}_{ij}^T \quad \tilde{M}_1 \quad \tilde{M}_2 \quad \dots \quad \tilde{M}_m], \\ \Psi_{14}^{ij} &= \begin{bmatrix} \varepsilon_{1ij} E_i^T P & \underbrace{0 \dots 0}_m \\ 0 & \underbrace{0 \dots 0}_m \end{bmatrix}^T, \\ \Psi_{15}^{ij} &= \begin{bmatrix} H_{1i0} + H_{2i} K_j & H_{1i1} & H_{1i2} & \dots & H_{1im} \\ H_{3i0} + H_{4i} K_j & H_{3i1} & H_{3i2} & \dots & H_{3im} \end{bmatrix}^T, \\ \Psi_{22} &= -\text{diag}(P^{-1}, \tau_1 Q_1^{-1}, \tau_2 Q_2^{-1}, \dots, \tau_m Q_m^{-1}), \\ \Psi_{24}^{ij} &= \begin{bmatrix} 0 & \varepsilon_{1ij} \tau_1 E_i^T & \varepsilon_{1ij} \tau_2 E_i^T & \dots & \varepsilon_{1ij} \tau_m E_i^T \\ \varepsilon_{2ij} E_i^T & 0 & 0 & \dots & 0 \end{bmatrix}^T, \\ \Psi_{33} &= -\text{diag}(\tau_1 R_1^{-1}, \tau_2 R_2^{-1}, \dots, \tau_m R_m^{-1}, R_1, R_2, \dots, R_m), \\ \Psi_{34}^{ij} &= \begin{bmatrix} 0 & 0 & \dots & 0 & \underbrace{0 \dots 0}_m \\ \varepsilon_{2ij} \tau_1 E_i^T & \varepsilon_{2ij} \tau_2 E_i^T & \dots & \varepsilon_{2ij} \tau_m E_i^T & \underbrace{0 \dots 0}_m \end{bmatrix}^T, \\ \Psi_{44}^{ij} &= \text{diag}(-\varepsilon_{1ij} I \quad -\varepsilon_{2ij} I), \quad \Psi_{55}^{ij} = \text{diag}(-\varepsilon_{1ij} I \quad -\varepsilon_{2ij} I) \end{aligned}$$

and

$$\begin{aligned} \vartheta^{ij} &= P(A_{i0} + B_{1i} K_j) + (A_{i0} + B_{1i} K_j)^T P + \sum_{k=1}^m (\tau_k W_k + M_k + M_k^T + S_k), \\ \tilde{A}_{ij}^T &= [A_{i0} + B_{1i} K_j \quad A_{i1} \quad A_{i2} \quad \dots \quad A_{im}]^T, \\ \tilde{D}_{ij}^T &= [D_{i0} + B_{2i} K_j \quad D_{i1} \quad D_{i2} \quad \dots \quad D_{im}]^T, \\ \tilde{M}_k &= [M_k^T \quad \underbrace{0 \dots 0}_m]^T, \quad k = 1, 2, \dots, m. \end{aligned}$$

Proof. First, we define two new state variables,

$$y(t) = \sum_{k=0}^m A_{ak}x(t - \tau_k), \quad (17)$$

$$g(t) = \sum_{k=0}^m D_{ak}x(t - \tau_k). \quad (18)$$

Then the closed-loop system (11) with $v(t) = 0$ can also be written as

$$dx(t) = y(t)dt + g(t)dw(t). \quad (19)$$

It follows from (19) that for $k = 1, 2, \dots, m$, we have

$$x(t - \tau_k) = x(t) - \int_{t-\tau_k}^t y(s)ds - \int_{t-\tau_k}^t g(s)dw(s).$$

Substituting the above equality into (17), we get

$$0 = \sum_{k=0}^m A_{ak}x(t) - y(t) - \sum_{k=1}^m A_{ak} \left(\int_{t-\tau_k}^t y(s)ds + \int_{t-\tau_k}^t g(s)dw(s) \right). \quad (20)$$

It is easy to see that for $t \geq \tau$, system (11) is equivalent to systems (19) and (20). Construct a Lyapunov-Krasovskii functional (LKF) for systems (19) of the form

$$V(x(t), t) = \sum_{i=1}^4 V_i(x(t), t), \quad (21)$$

where

$$\begin{aligned} V_1(x(t), t) &= x^T(t)Px(t), \\ V_2(x(t), t) &= \sum_{k=1}^m \int_{-\tau_k}^0 \int_{t+\theta}^t y^T(s)Q_k y(s)dsd\theta, \\ V_3(x(t), t) &= \sum_{k=1}^m \int_{t-\tau_k}^t x^T(s)S_k x(s)ds, \\ V_4(x(t), t) &= \sum_{k=1}^m \int_{-\tau_k}^0 \int_{t+\theta}^t g^T(s)R_k g(s)dsd\theta \end{aligned}$$

with P , Q_k , S_k and R_k , $k = 1, 2, \dots, m$ are symmetric and positive definite matrices with appropriate dimensions. Defining $x_t(s) = x(t + s)$, $-2\tau \leq s \leq 0$, the weak infinitesimal operator \mathcal{L} of the stochastic process $\{x_t, t \geq 0\}$ is given by (see, for example [20])

$$\mathcal{L}V_1(x(t), t) = 2x^T(t)Py(t) + g^T(t)Pg(t). \quad (22)$$

Thus owing to relation (20),

$$\begin{aligned} 2x^T(t)Py(t) &= 2x^T(t)P \sum_{k=0}^m A_{ak}x(t) - 2x^T(t)P \sum_{k=1}^m A_{ak} \int_{t-\tau_k}^t y(s)ds \\ &\quad - 2x^T(t)P \sum_{k=1}^m A_{ak} \int_{t-\tau_k}^t g(s)dw(s). \end{aligned}$$

It then follows new bounding techniques from Lemma 2.6, for $k = 1, 2, \dots, m$,

$$\begin{aligned} -2x^T(t)PA_{ak} \int_{t-\tau_k}^t y(s)ds &\leq \int_{t-\tau_k}^t \begin{bmatrix} x(t) \\ y(s) \end{bmatrix}^T \begin{bmatrix} W_k & M_k - PA_{ak} \\ * & Q_k \end{bmatrix} \begin{bmatrix} x(t) \\ y(s) \end{bmatrix} ds \\ &= x^T(t)\tau_k W_k x(t) + 2x^T(t)[M_k - PA_{ak}] \int_{t-\tau_k}^t y(s)ds + \int_{t-\tau_k}^t y^T(s)Q_k y(s)ds, \end{aligned}$$

where W_k, M_k, Q_k are constant matrices with appropriate dimensions, and satisfying

$$\begin{bmatrix} W_k & M_k \\ * & Q_k \end{bmatrix} \geq 0.$$

By (19), we get

$$\int_{t-\tau_k}^t y(s)ds = x(t) - x(t - \tau_k) - \int_{t-\tau_k}^t g(s)dw(s)$$

which yields

$$\begin{aligned} -2x^T(t)PA_{ak} \int_{t-\tau_k}^t y(s)ds &\leq x^T(t)\tau_k W_k x(t) + \int_{t-\tau_k}^t y^T(s)Q_k y(s)ds \\ &\quad + 2x^T(t)[M_k - PA_{ak}] \left(x(t) - x(t - \tau_k) - \int_{t-\tau_k}^t g(s)dw(s) \right). \end{aligned}$$

Therefore,

$$\begin{aligned} 2x^T(t)Py(t) &\leq x^T(t) \left(PA_{a0} + A_{a0}^T P + \sum_{k=1}^m \left[\tau_k W_k + M_k + M_k^T \right] \right) x(t) \\ &\quad + 2x^T(t) \sum_{k=1}^m [PA_{ak} - M_k] x(t - \tau_k) - 2x^T(t) \sum_{k=1}^m M_k \int_{t-\tau_k}^t g(s)dw(s) \\ &\quad + \sum_{k=1}^m \int_{t-\tau_k}^t y^T(s)Q_k y(s)ds. \end{aligned} \quad (23)$$

For symmetric positive definite matrix $R_k > 0$, $k = 1, \dots, m$, it follows from the Lemma 2.5 (a) that

$$\begin{aligned} -2x^T(t)M_k \int_{t-\tau_k}^t g(s)dw(s) &\leq x^T(t)M_k R_k^{-1} M_k^T x(t) \\ &\quad + \left(\int_{t-\tau_k}^t g(s)dw(s) \right)^T R_k \left(\int_{t-\tau_k}^t g(s)dw(s) \right). \end{aligned} \quad (24)$$

Combining (22)-(24),

$$\begin{aligned} \mathcal{L}V_1(x(t), t) &\leq x^T(t) \left(PA_{a0} + A_{a0}^T P + \sum_{k=1}^m \left(\tau_k W_k + M_k + M_k^T + M_k R_k^{-1} M_k^T \right) \right) x(t) \\ &\quad + 2x^T(t) \sum_{k=1}^m \left(PA_{ak} - M_k \right) x(t - \tau_k) + \sum_{k=1}^m \int_{t-\tau_k}^t y^T(s) Q_k y(s) ds \\ &\quad + \sum_{k=1}^m \left(\int_{t-\tau_k}^t g(s) dw(s) \right)^T R_k \left(\int_{t-\tau_k}^t g(s) dw(s) \right) + g^T(t) P g(t). \end{aligned} \quad (25)$$

Direct computation gives

$$\mathcal{L}V_2(x(t), t) = \sum_{k=1}^m \left(y^T(t) \tau_k Q_k y(t) - \int_{t-\tau_k}^t y^T(s) Q_k y(s) ds \right), \quad (26)$$

$$\mathcal{L}V_3(x(t), t) = \sum_{k=1}^m \left(x^T(t) S_k x(t) - x^T(t - \tau_k) S_k x(t - \tau_k) \right), \quad (27)$$

$$\mathcal{L}V_4(x(t), t) = \sum_{k=1}^m \left(g^T(t) \tau_k R_k g(t) - \int_{t-\tau_k}^t g^T(s) R_k g(s) ds \right). \quad (28)$$

Combining (25)-(28), we get

$$\begin{aligned} \mathcal{L}V(x(t), t) &\leq x^T(t) \left(\sum_{k=1}^m \left(\tau_k W_k + M_k + M_k^T + M_k R_k^{-1} M_k^T + S_k \right) \right. \\ &\quad \left. + PA_{a0} + A_{a0}^T P \right) x(t) + 2x^T(t) \sum_{k=1}^m \left(PA_{ak} - M_k \right) x(t - \tau_k) \\ &\quad - \sum_{k=1}^m x^T(t - \tau_k) S_k x(t - \tau_k) + y^T(t) \left(\sum_{k=1}^m \tau_k Q_k \right) y(t) \\ &\quad + g^T(t) \left(P + \sum_{k=1}^m \tau_k R_k \right) g(t) - \sum_{k=1}^m \int_{t-\tau_k}^t g^T(s) R_k g(s) ds \\ &\quad + \sum_{k=1}^m \left(\int_{t-\tau_k}^t g(s) dw(s) \right)^T R_k \left(\int_{t-\tau_k}^t g(s) dw(s) \right). \end{aligned} \quad (29)$$

Using Lemma 2.7, one can derive

$$\begin{aligned} y^T(t) \left(\sum_{k=1}^m \tau_k Q_k \right) y(t) &= \left[A_{a0} x(t) + \sum_{k=1}^m A_{ak} x(t - \tau_k) \right]^T \left(\sum_{k=1}^m \tau_k Q_k \right) \\ &\quad \times \left[A_{a0} x(t) + \sum_{k=1}^m A_{ak} x(t - \tau_k) \right], \\ &= \sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r \sum_{l=1}^r h_i h_j h_q h_l \xi^T(t) \tilde{A}_{ij}^T(t) \left(\sum_{k=1}^m \tau_k Q_k \right) \tilde{A}_{ql}(t) \xi(t), \\ &\leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j \xi^T(t) \tilde{A}_{ij}^T(t) \left(\sum_{k=1}^m \tau_k Q_k \right) \tilde{A}_{ij}(t) \xi(t). \end{aligned} \quad (30)$$

Similarly

$$g^T(t) \left(P + \sum_{k=1}^m \tau_k R_k \right) g(t) \leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j \xi^T(t) \tilde{D}_{ij}^T(t) \left(P + \sum_{k=1}^m \tau_k R_k \right) \tilde{D}_{ij}(t) \xi(t) \quad (31)$$

where

$$\begin{aligned}\tilde{A}_{ij}^T(t) &= \left[A_{i0}(t) + B_{1i}(t)K_j \quad A_{i1}(t) \quad A_{i2}(t) \quad \dots \quad A_{im}(t) \right]^T, \\ \tilde{D}_{ij}^T(t) &= \left[D_{i0}(t) + B_{2i}(t)K_j \quad D_{i1}(t) \quad D_{i2}(t) \quad \dots \quad D_{im}(t) \right]^T, \\ \xi^T(t) &= \left[x^T(t) \quad x^T(t - \tau_1) \quad x^T(t - \tau_2) \quad \dots \quad x^T(t - \tau_m) \right].\end{aligned}$$

Therefore, considering (29)-(31), we have

$$\begin{aligned}\mathcal{L}V(x(t), t) &\leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j \xi^T(t) \tilde{\Psi}^{ij} \xi(t) - \sum_{k=1}^m \int_{t-\tau_k}^t g^T(s) R_k g(s) ds \\ &\quad + \sum_{k=1}^m \left(\int_{t-\tau_k}^t g(s) dw(s) \right)^T R_k \left(\int_{t-\tau_k}^t g(s) dw(s) \right)\end{aligned}\quad (32)$$

where

$$\begin{aligned}\tilde{\Psi}^{ij} &= \psi^{ij} + \tilde{D}_{ij}^T(t) P \tilde{D}_{ij}(t) + \tilde{A}_{ij}^T(t) \left(\sum_{k=1}^m \tau_k Q_k \right) \tilde{A}_{ij}(t) + \tilde{D}_{ij}^T(t) \left(\sum_{k=1}^m \tau_k R_k \right) \tilde{D}_{ij}(t), \\ \psi^{ij} &= \begin{bmatrix} \varphi^{ij} & PA_{i1}(t) - M_1 & PA_{i2}(t) - M_2 & \dots & PA_{im}(t) - M_m \\ * & -S_1 & 0 & \dots & 0 \\ * & * & -S_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \dots & -S_m \end{bmatrix}\end{aligned}$$

with

$$\begin{aligned}\varphi^{ij} &= P(A_{i0}(t) + B_{1i}(t)K_j) + (A_{i0}(t) + B_{1i}(t)K_j)^T P \\ &\quad + \sum_{k=1}^m \left(\tau_k W_k + M_k + M_k^T + M_k R_k^{-1} M_k^T + S_k \right).\end{aligned}$$

Note that for $k = 1, 2, \dots, m$

$$\mathcal{E} \left\{ \left(\int_{t-\tau_k}^t g(s) dw(s) \right)^T R_k \left(\int_{t-\tau_k}^t g(s) dw(s) \right) \right\} = \mathcal{E} \left\{ \int_{t-\tau_k}^t g^T(s) R_k g(s) ds \right\}.\quad (33)$$

Taking the mathematical expectation on both sides of (32) and considering (33), we have $\mathcal{E} \left\{ \mathcal{L}V(x(t), t) \right\} \leq \mathcal{E} \left\{ \sum_{i=1}^r \sum_{j=1}^r h_i h_j \xi^T(t) \tilde{\Psi}^{ij} \xi(t) \right\}$.

If $\tilde{\Psi}^{ii} < 0$, and $(\tilde{\Psi}^{ij} + \tilde{\Psi}^{ji}) < 0$ hold for any $1 \leq i \leq j \leq r$, equivalently to yield $\mathcal{E} \left\{ \mathcal{L}V(x(t), t) \right\} < 0$ for every $\xi^T(t) \neq 0$. Employing the Schur complement, $\tilde{\Psi}^{ii} < 0$ and $(\tilde{\Psi}^{ij} + \tilde{\Psi}^{ji}) < 0$, which are equivalent to

$$\hat{\Psi}^{ij} + \hat{\Psi}^{ji} < 0, \quad 1 \leq i \leq j \leq r\quad (34)$$

where

$$\hat{\Psi}^{ij} = \begin{bmatrix} \tilde{\varphi}^{ij} & \bar{D}_{ij}^T(t) & \tau_1 \bar{A}_{ij}^T(t) & \tau_2 \bar{A}_{ij}^T(t) & \dots & \tau_m \bar{A}_{ij}^T(t) & \tau_1 \bar{D}_{ij}^T(t) & \tau_2 \bar{D}_{ij}^T(t) & \dots & \tau_m \bar{D}_{ij}^T(t) & \bar{M}_1 & \dots & \bar{M}_m \\ * & -P^{-1} & 0 & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ * & * & -\tau_1 Q_1^{-1} & 0 & \dots & 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ * & * & * & -\tau_2 Q_1^{-1} & \dots & 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & -\tau_m Q_m^{-1} & 0 & 0 & \dots & 0 & 0 & \dots & 0 \\ * & * & * & * & * & * & -\tau_1 R_1^{-1} & 0 & \dots & 0 & 0 & \dots & 0 \\ * & * & * & * & * & * & * & -\tau_2 R_2^{-1} & \dots & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & -\tau_m R_m^{-1} & 0 & \dots & 0 \\ * & * & * & * & * & * & * & * & \dots & * & -R_1 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & * & * & * & * & * & \dots & * & * & -R_m & * \end{bmatrix}$$

with

$$\tilde{\psi}^{ij} = \begin{bmatrix} \tilde{\varphi}^{ij} & PA_{i1}(t) - M_1 & PA_{i2}(t) - M_2 & \dots & PA_{im}(t) - M_m \\ * & -S_1 & 0 & \dots & 0 \\ * & * & -S_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \dots & -S_m \end{bmatrix},$$

$$\tilde{\varphi}^{ij} = P \left(A_{i0}(t) + B_{1i}(t)K_j \right) + \left(A_{i0}(t) + B_{1i}(t)K_j \right)^T P$$

$$+ \sum_{k=1}^m \left(\tau_k W_k + M_k + M_k^T + S_k \right)$$

and \bar{M}_k is defined in Theorem 3.1.

Define the following matrices

$$\hat{\Psi}^{ij} = \check{\Psi}^{ij} + \Lambda_{1i}^T F_i(t) \Lambda_{3ij} + \Lambda_{3ij}^T F_i^T(t) \Lambda_{1i} + \Lambda_{2i}^T F_i(t) \Lambda_{4ij} + \Lambda_{4ij}^T F_i^T(t) \Lambda_{2i}, \quad (35)$$

where

$$\check{\Psi}^{ij} = \begin{bmatrix} \Psi_{11}^{ij} & \Psi_{12}^{ij} & \Psi_{13}^{ij} \\ * & \Psi_{22} & 0 \\ * & * & \Psi_{33} \end{bmatrix},$$

$$\Lambda_{1i} = \left[E_i^T P \underbrace{0 \dots 0}_{m+1} \tau_1 E_i^T \tau_2 E_i^T \dots \tau_m E_i^T \underbrace{0 \dots 0}_{2m} \right],$$

$$\Lambda_{2i} = \left[\underbrace{0 \dots 0}_{m+1} E_i^T \underbrace{0 \dots 0}_m \tau_1 E_i^T \tau_2 E_i^T \dots \tau_m E_i^T \underbrace{0 \dots 0}_m \right],$$

$$\Lambda_{3ij} = \left[H_{1i0} + H_{2i}K_j \ H_{1i1} \ H_{1i2} \ \dots \ H_{1im} \ \underbrace{0 \dots 0}_{3m+1} \right],$$

$$\Lambda_{4ij} = \left[H_{3i0} + H_{4i}K_j \ H_{3i1} \ H_{3i2} \ \dots \ H_{3im} \ \underbrace{0 \dots 0}_{3m+1} \right]$$

further Ψ_{11}^{ij} , Ψ_{12}^{ij} , Ψ_{13}^{ij} , Ψ_{22} and Ψ_{33} are defined as in Theorem 3.1. By the Lemma 2.5 (b), we have

$$\hat{\Psi}^{ij} \leq \check{\Psi}^{ij} + \varepsilon_{1ij} \Lambda_{1i}^T \Lambda_{1i} + \varepsilon_{1ij}^{-1} \Lambda_{3ij}^T \Lambda_{3ij} + \varepsilon_{2ij} \Lambda_{2i}^T \Lambda_{2i} + \varepsilon_{2ij}^{-1} \Lambda_{4ij}^T \Lambda_{4ij}. \quad (36)$$

By using the Schur complement with (36), we obtain the LMIs (15) and (16), Definition 2.1 and [18] yield that the closed-loop stochastic fuzzy system (11) and (13) with $v(t) = 0$ is robustly stochastically stable. \square

Theorem 3.2. For given scalars $\tau_k > 0$, $k = 1, 2, \dots, m$, the closed-loop stochastic fuzzy system (11) and (13) with $v(t) = 0$ is said to be robustly stochastically stabilizable, if there exist matrices $X > 0$, $\bar{S}_k > 0$, $\tilde{R}_k > 0$, \bar{W}_k , \bar{M}_k , $\tilde{Q}_k > 0$, $k = 1, 2, \dots, m$, Y_j ($j = 1, 2, \dots, r$) and scalars $\varepsilon_{1ij} > 0$, $\varepsilon_{2ij} > 0$ ($1 \leq i \leq j \leq r$) such that the following LMIs hold

$$\begin{bmatrix} \bar{W}_k & \bar{M}_k \\ * & 2X - \tilde{Q}_k \end{bmatrix} \geq 0, \quad 1 \leq k \leq m \quad (37)$$

$$\bar{\Psi}^{ii} < 0, \quad 1 \leq i \leq r \quad (38)$$

$$\bar{\Psi}^{ij} + \bar{\Psi}^{ji} < 0, \quad 1 \leq i < j \leq r \quad (39)$$

where

$$\bar{\Psi}^{ij} = \begin{bmatrix} \bar{\Psi}_{11}^{ij} & \bar{\Psi}_{12}^{ij} & \bar{\Psi}_{13}^{ij} & \bar{\Psi}_{14}^{ij} & \bar{\Psi}_{15}^{ij} \\ * & \bar{\Psi}_{22}^{ij} & 0 & \bar{\Psi}_{24}^{ij} & 0 \\ * & * & \bar{\Psi}_{33}^{ij} & \bar{\Psi}_{34}^{ij} & 0 \\ * & * & * & \bar{\Psi}_{44}^{ij} & 0 \\ * & * & * & * & \bar{\Psi}_{55}^{ij} \end{bmatrix}$$

with

$$\bar{\Psi}_{11}^{ij} = \begin{bmatrix} \bar{\vartheta}^{ij} & A_{i1}X - \bar{M}_1 & A_{i2}X - \bar{M}_2 & \dots & A_{im}X - \bar{M}_m \\ * & -\bar{S}_1 & 0 & \dots & 0 \\ * & * & -\bar{S}_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ * & * & * & \dots & -\bar{S}_m \end{bmatrix},$$

$$\bar{\Psi}_{12}^{ij} = [\bar{D}_{ij}^T \quad \tau_1 \bar{A}_{ij}^T \quad \tau_2 \bar{A}_{ij}^T \quad \dots \quad \tau_m \bar{A}_{ij}^T],$$

$$\bar{\Psi}_{13}^{ij} = [\tau_1 \bar{D}_{ij}^T \quad \tau_2 \bar{D}_{ij}^T \quad \dots \quad \tau_m \bar{D}_{ij}^T \quad \hat{M}_1 \quad \hat{M}_2 \quad \dots \quad \hat{M}_m],$$

$$\bar{\Psi}_{14}^{ij} = \begin{bmatrix} \varepsilon_{1ij} E_i^T & \underbrace{0 \dots 0}_m \\ 0 & \underbrace{0 \dots 0}_m \end{bmatrix}^T,$$

$$\bar{\Psi}_{15}^{ij} = \begin{bmatrix} H_{1i0}X + H_{2i}Y_j & H_{1i1}X & H_{1i2}X & \dots & H_{1im}X \\ H_{3i0}X + H_{4i}Y_j & H_{3i1}X & H_{3i2}X & \dots & H_{3im}X \end{bmatrix}^T,$$

$$\bar{\Psi}_{22} = -diag(X, \tau_1 \tilde{Q}_1, \tau_2 \tilde{Q}_2, \dots, \tau_m \tilde{Q}_m),$$

$$\bar{\Psi}_{33} = diag(-\tau_1 \tilde{R}_1, -\tau_2 \tilde{R}_2 \dots - \tau_m \tilde{R}_m, -2X + \tilde{R}_1, -2X + \tilde{R}_2 \dots - 2X + \tilde{R}_m)$$

and

$$\begin{aligned}\bar{\vartheta}^{ij} &= \left(A_{i0}X + B_{1i}Y_j \right) + \left(A_{i0}X + B_{1i}Y_j \right)^T + \sum_{k=1}^m \left(\tau_k \bar{W}_k + \bar{M}_k + \bar{M}_k^T + \bar{S}_k \right), \\ \bar{A}_{ij}^T &= \left[A_{i0}X + B_{1i}Y_j \quad A_{i1}X \quad A_{i2}X \quad \dots \quad A_{im}X \right]^T, \\ \bar{D}_{ij}^T &= \left[D_{i0}X + B_{2i}Y_j \quad D_{i1}X \quad D_{i2}X \quad \dots \quad D_{im}X \right]^T, \\ \hat{M}_k &= \left[\bar{M}_k^T \underbrace{0 \dots 0}_m \right]^T, \quad k = 1, 2, \dots, m\end{aligned}$$

further Ψ_{24}^{ij} , Ψ_{34}^{ij} , Ψ_{44}^{ij} and Ψ_{55}^{ij} are defined as in Theorem 3.1. Moreover the state feedback gain can be constructed as $K_j = Y_j X^{-1}$ ($j = 1, 2, \dots, r$).

Proof. Define $P = X^{-1}$, pre- and post-multiply (15) and (16) with

$$\text{diag} \left(\underbrace{X, X, \dots, X}_{m+1}, \underbrace{I, I, \dots, I}_{2m+1}, \underbrace{X, X, \dots, X}_m, I, I, I, I \right)$$

and its transpose, pre- and post multiply (14) with

$$\text{diag}(X, X)$$

and its transpose, respectively, and define $XS_kX = \bar{S}_k$, $XM_kX = \bar{M}_k$, $XW_kX = \bar{W}_k$, $1 \leq k \leq m$ and $K_jX = Y_j$, $1 \leq j \leq r$.

Then, it follows from inequalities

$$\begin{aligned}XR_kX - 2X + R_k^{-1} &= (X - R_k^{-1})R_k(X - R_k^{-1}) \geq 0, \quad k = 1, 2, \dots, m \\ XQ_kX - 2X + Q_k^{-1} &= (X - Q_k^{-1})Q_k(X - Q_k^{-1}) \geq 0, \quad k = 1, 2, \dots, m\end{aligned}$$

that

$$-2X + R_k^{-1} \geq -XR_kX, \quad -2X + Q_k^{-1} \geq -XQ_kX, \quad k = 1, 2, \dots, m.$$

It is assumed that $R_k^{-1} = \tilde{R}_k$, $Q_k^{-1} = \tilde{Q}_k$, $k = 1, 2, \dots, m$, then the LMIs (38)-(39) and (37) can be obtained from (15)-(16) and (14) respectively. It means that the closed-loop stochastic fuzzy system (11) and (13) with $v(t) = 0$ is asymptotically stable in the mean-square. \square

Remark 3.3. Based on the LMI approach, Theorem 3.2 provides a sufficient condition for the existence of robust stabilizing state feedback fuzzy controllers for uncertain stochastic fuzzy systems with multiple time delays. The desired fuzzy controller is readily constructed by solving the LMIs in (37)-(39), which can be implemented by standard numerical algorithm [3], without requiring any tuning of parameters.

4. Robust Stochastic H_∞ Control

In this section, the sufficient condition for the solvability of robust H_∞ control problem is proposed. Further an LMI approach for designing the desired state feedback fuzzy controller is developed.

Theorem 4.1. For given scalars $\gamma > 0$ and $\tau_k > 0$, $k = 1, 2, \dots, m$, the closed-loop stochastic fuzzy system $(\tilde{\Sigma})$ is robustly stochastically stabilizable with disturbance attenuation γ , if there exist matrices $X > 0$, $\bar{S}_k > 0$, $\tilde{R}_k > 0$, \bar{W}_k , \bar{M}_k , $\tilde{Q}_k > 0$, $k = 1, 2, \dots, m$, Y_j ($j = 1, 2, \dots, r$) and scalars $\varepsilon_{1ij} > 0$, $\varepsilon_{2ij} > 0$ ($1 \leq i \leq j \leq r$) such that the following LMIs hold

$$\begin{bmatrix} \bar{W}_k & \bar{M}_k \\ * & 2X - \tilde{Q}_k \end{bmatrix} \geq 0, \quad 1 \leq k \leq m \quad (40)$$

$$\Omega^{ii} < 0, \quad 1 \leq i \leq r \quad (41)$$

$$\Omega^{ij} + \Omega^{ji} < 0, \quad 1 \leq i < j \leq r \quad (42)$$

where

$$\Omega^{ij} = \begin{bmatrix} \Omega_{11}^{ij} & \Omega_{12}^{ij} & \Omega_{13}^{ij} & \Omega_{14}^{ij} & \Omega_{15}^{ij} \\ * & \Omega_{22} & 0 & \Omega_{24}^{ij} & 0 \\ * & * & \Omega_{33} & \Omega_{34}^{ij} & 0 \\ * & * & * & \Omega_{44}^{ij} & 0 \\ * & * & * & * & \Omega_{55}^{ij} \end{bmatrix}$$

with

$$\Omega_{11}^{ij} = \begin{bmatrix} \bar{\vartheta}^{ij} & A_{i1}X - \bar{M}_1 & A_{i2}X - \bar{M}_2 & \dots & A_{im}X - \bar{M}_m & B_{vi} \\ * & -\bar{S}_1 & 0 & \dots & 0 & 0 \\ * & * & -\bar{S}_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \dots & -\bar{S}_m & 0 \\ * & * & * & \dots & 0 & -\gamma^2 I \end{bmatrix},$$

$$\Omega_{12}^{ij} = \begin{bmatrix} \hat{D}_{ij}^T & \tau_1 \hat{A}_{ij}^T & \tau_2 \hat{A}_{ij}^T & \dots & \tau_m \hat{A}_{ij}^T \end{bmatrix},$$

$$\Omega_{13}^{ij} = \begin{bmatrix} \tau_1 \hat{D}_{ij}^T & \tau_2 \hat{D}_{ij}^T & \dots & \tau_m \hat{D}_{ij}^T & \tilde{M}_1 & \tilde{M}_2 & \dots & \tilde{M}_m & \hat{C}_{ij} \end{bmatrix},$$

$$\Omega_{14}^{ij} = \begin{bmatrix} \varepsilon_{1ij} E_i^T & \underbrace{0 \dots 0}_{m+1} \\ 0 & \underbrace{0 \dots 0}_{m+1} \end{bmatrix}^T,$$

$$\Omega_{15}^{ij} = \begin{bmatrix} H_{1i0}X + H_{2i}Y_j & H_{1i1}X & H_{1i2}X & \dots & H_{1im}X & 0 \\ H_{3i0}X + H_{4i}Y_j & H_{3i1}X & H_{3i2}X & \dots & H_{3im}X & 0 \end{bmatrix}^T,$$

$$\Omega_{22} = -\text{diag}(X \tau_1 \tilde{Q}_1 \tau_2 \tilde{Q}_2 \dots \tau_m \tilde{Q}_m),$$

$$\Omega_{24}^{ij} = \begin{bmatrix} 0 & \varepsilon_{1ij} \tau_1 E_i^T & \varepsilon_{1ij} \tau_2 E_i^T & \dots & \varepsilon_{1ij} \tau_m E_i^T \\ \varepsilon_{2ij} E_i^T & 0 & 0 & \dots & 0 \end{bmatrix}^T,$$

$$\Omega_{33} = \text{diag}(-\tau_1 \tilde{R}_1, -\tau_2 \tilde{R}_2, \dots, -\tau_m \tilde{R}_m, -2X + \tilde{R}_1, -2X + \tilde{R}_2, \dots, -2X + \tilde{R}_m, -I),$$

$$\Omega_{34}^{ij} = \begin{bmatrix} 0 & 0 & \dots & 0 & \underbrace{0 \dots 0}_{m+1} \\ \varepsilon_{2ij} \tau_1 E_i^T & \varepsilon_{2ij} \tau_2 E_i^T & \dots & \varepsilon_{2ij} \tau_m E_i^T & \underbrace{0 \dots 0}_{m+1} \end{bmatrix}^T,$$

$$\Omega_{44}^{ij} = \text{diag}(-\varepsilon_{1ij} I \quad -\varepsilon_{2ij} I), \quad \Omega_{55}^{ij} = \text{diag}(-\varepsilon_{1ij} I \quad -\varepsilon_{2ij} I)$$

and

$$\begin{aligned}\tilde{\vartheta}^{ij} &= (A_{i0}X + B_{1i}Y_j) + (A_{i0}X + B_{1i}Y_j)^T + \sum_{k=1}^m \left(\tau_k \bar{W}_k + \bar{M}_k + \bar{M}_k^T + \bar{S}_k \right), \\ \hat{A}_{ij}^T &= \begin{bmatrix} A_{i0}X + B_{1i}Y_j & A_{i1}X & A_{i2}X & \dots & A_{im}X & B_{vi} \end{bmatrix}^T, \\ \hat{D}_{ij}^T &= \begin{bmatrix} D_{i0}X + B_{2i}Y_j & D_{i1}X & D_{i2}X & \dots & D_{im}X & D_{vi} \end{bmatrix}^T, \\ \hat{C}_{ij}^T &= \begin{bmatrix} C_{i0}X + B_{3i}Y_j & C_{i1}X & C_{i2}X & \dots & C_{im}X & C_{vi} \end{bmatrix}^T, \\ \tilde{M}_k &= \left[\bar{M}_k^T \underbrace{0 \dots 0}_{m+1} \right]^T, \quad k = 1, 2, \dots, m.\end{aligned}$$

Moreover the state feedback gain can be constructed as $K_j = Y_j X^{-1}$ ($j = 1, 2, \dots, r$).

Proof. For convenience, we set

$$\begin{aligned}y(t) &= \sum_{k=0}^m A_{ak}x(t - \tau_k) + B_v v(t), \\ g(t) &= \sum_{k=0}^m D_{ak}x(t - \tau_k) + D_v v(t).\end{aligned}$$

From Theorem 3.2, the closed-loop system ($\tilde{\Sigma}$) is robustly stochastically stable. Now we show that under the zero initial condition, system ($\tilde{\Sigma}$) satisfies $\|z(t)\|_{\mathcal{E}_2} < \gamma \|v(t)\|_2$ for all nonzero $v(t) \in L_2[0, \infty)$. Choose a LKF candidate as defined in (21) and utilizing Itô's formula, we have

$$\begin{aligned}\mathcal{L}V(x(t), t) &\leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j \zeta^T(t) \tilde{\Omega}^{ij} \zeta(t) - \sum_{k=1}^m \int_{t-\tau_k}^t g^T(s) R_k g(s) ds \\ &\quad + \sum_{k=1}^m \left(\int_{t-\tau_k}^t g(s) dw(s) \right)^T R_k \left(\int_{t-\tau_k}^t g(s) dw(s) \right)\end{aligned}\quad (43)$$

where

$$\begin{aligned}\tilde{\Omega}^{ij} &= \Phi^{ij} + \hat{D}_{ij}^T(t) P \hat{D}_{ij}(t) + \hat{A}_{ij}^T(t) \left(\sum_{k=1}^m \tau_k Q_k \right) \hat{A}_{ij}(t) + \hat{D}_{ij}^T(t) \left(\sum_{k=1}^m \tau_k R_k \right) \hat{D}_{ij}(t), \\ \Phi^{ij} &= \begin{bmatrix} \varphi^{ij} & PA_{i1}(t) - M_1 & PA_{i2}(t) - M_2 & \dots & PA_{im}(t) - M_m & PB_{vi} \\ * & -S_1 & 0 & \dots & 0 & 0 \\ * & * & -S_2 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ * & * & * & \dots & -S_m & 0 \\ * & * & * & * & * & 0 \end{bmatrix}\end{aligned}$$

with

$$\begin{aligned}\hat{A}_{ij}^T(t) &= \begin{bmatrix} A_{i0}(t) + B_{1i}(t)K_j & A_{i1}(t) & A_{i2}(t) & \dots & A_{im}(t) & B_{vi} \end{bmatrix}^T, \\ \hat{D}_{ij}^T(t) &= \begin{bmatrix} D_{i0}(t) + B_{2i}(t)K_j & D_{i1}(t) & D_{i2}(t) & \dots & D_{im}(t) & D_{vi} \end{bmatrix}^T, \\ \zeta^T(t) &= \begin{bmatrix} x^T(t) & x^T(t - \tau_1) & x^T(t - \tau_2) & \dots & x^T(t - \tau_m) & v^T(t) \end{bmatrix}\end{aligned}$$

and φ^{ij} are defined as in the proof of Theorem 3.1.

Note that

$$\begin{aligned}z^T(t)z(t) &= \sum_{i=1}^r \sum_{j=1}^r \sum_{q=1}^r \sum_{l=1}^r h_i h_j h_q h_l \zeta^T(t) \tilde{C}_{ij}^T \tilde{C}_{ql} \zeta(t), \\ &\leq \sum_{i=1}^r \sum_{j=1}^r h_i h_j \zeta^T(t) \tilde{C}_{ij}^T \tilde{C}_{ij} \zeta(t)\end{aligned}\quad (44)$$

where

$$\tilde{C}_{ij}^T = \begin{bmatrix} C_{i0} + B_{3i}K_j & C_{i1} & C_{i2} & \dots & C_{im} & C_{vi} \end{bmatrix}^T.$$

Now, we set

$$J(t) = \mathcal{E} \left\{ \int_0^t [z^T(s)z(s) - \gamma^2 v^T(s)v(s)] ds \right\} \quad (45)$$

where $t > 0$. Because $V(\phi(t), 0) = 0$ under the initial condition, that is, $\phi(t) = 0$ for $t \in [-\tau, 0]$, then by Itô's formula, it follows that

$$\begin{aligned}J(t) &= \mathcal{E} \left\{ \int_0^t [z^T(s)z(s) - \gamma^2 v^T(s)v(s) + \mathcal{L}V(x(s), s)] ds \right\} - \mathcal{E} \left\{ V(x(t), t) \right\}, \\ &\leq \mathcal{E} \left\{ \int_0^t [z^T(s)z(s) - \gamma^2 v^T(s)v(s) + \mathcal{L}V(x(s), s)] ds \right\}, \\ &\leq \mathcal{E} \left\{ \int_0^t \zeta^T(s) \hat{\Omega}^{ij} \zeta(s) ds \right\}\end{aligned}\quad (46)$$

where

$$\hat{\Omega}^{ij} = \tilde{\Omega}^{ij} + \tilde{C}_{ij}^T \tilde{C}_{ij} + \underbrace{\text{diag}(0, \dots, 0)}_{m+1}, -\gamma^2 I).$$

Then, considering LMI (41) and (42), using the same technique as in the proof of Theorem 3.2, we have $\hat{\Omega}^{ii} < 0$ and $\hat{\Omega}^{ij} + \hat{\Omega}^{ji} < 0$, which imply that $J(t) < 0$ for $t > 0$, therefore we have $\|z(t)\|_{\mathcal{E}_2} < \gamma \|v(t)\|_2$, which completes the proof of Theorem 4.1. \square

Remark 4.2. Note that by Theorem 4.1, the problem of finding the upper bound of delay τ_k ($1 \leq k \leq m$) for given γ or the smallest attenuation level γ for a given τ_k can be easily checked by testing the feasibility of corresponding LMIs. For instance, the smallest γ for given τ_k obtainable from Theorem 4.1 can be determined by solving the following convex optimization problem:

$$\begin{cases} \min & \chi \\ \text{subject to} & X > 0, \bar{S}_k > 0, \bar{R}_k > 0, \bar{W}_k > 0, \bar{Q}_k > 0 \quad (1 \leq k \leq m), \\ & \varepsilon_{1ij} > 0, \varepsilon_{2ij} > 0 \quad (1 \leq i \leq j \leq r) \text{ and LMIs (42) - (40) with } \chi = \gamma^2, \end{cases}$$

then, the minimum value of optimal H_∞ performance γ^* is given by $\gamma^* = \min(\chi)^{1/2}$.

5. Numerical Example

In this section, we provide an example to demonstrate the effectiveness of the proposed method.

Consider the uncertain stochastic T-S fuzzy system $(\tilde{\Sigma})$ with parameters as follows

$$\begin{aligned}
A_{10} &= \begin{bmatrix} -7.4 & 1.2 \\ 1 & -5.3 \end{bmatrix}, & A_{20} &= \begin{bmatrix} -6.2 & 2 \\ 1 & -3 \end{bmatrix}, & A_{11} &= \begin{bmatrix} 0.01 & 0.6 \\ 0.1 & 1.6 \end{bmatrix}, \\
A_{21} &= \begin{bmatrix} 0.2 & 0.5 \\ 0.3 & 0.5 \end{bmatrix}, & A_{12} &= \begin{bmatrix} 0.2 & -0.3 \\ 0.5 & -0.4 \end{bmatrix}, & A_{22} &= \begin{bmatrix} 1.2 & 1 \\ 0.2 & 0.5 \end{bmatrix}, \\
B_{11} &= \begin{bmatrix} 0.01 & 0.2 \\ 0.8 & 0.6 \end{bmatrix}, & B_{12} &= \begin{bmatrix} 2 & 1 \\ 0.5 & 2 \end{bmatrix}, & B_{v1} &= \begin{bmatrix} -0.4 & 0 \\ 0 & 0.8 \end{bmatrix}, \\
B_{v2} &= \begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix}, & D_{10} &= \begin{bmatrix} 0.01 & 0 \\ 0 & 0.2 \end{bmatrix}, & D_{20} &= \begin{bmatrix} -0.8 & 0.5 \\ 0.6 & 0.5 \end{bmatrix}, \\
D_{11} &= \begin{bmatrix} -0.1 & 0.1 \\ -0.3 & 0.6 \end{bmatrix}, & D_{21} &= \begin{bmatrix} 0.01 & 0.02 \\ 0.2 & 0.1 \end{bmatrix}, & D_{12} &= \begin{bmatrix} 0.2 & -0.3 \\ 0.1 & 0.1 \end{bmatrix}, \\
D_{22} &= \begin{bmatrix} 0.1 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}, & B_{21} &= \begin{bmatrix} 0.1 & 0.3 \\ 0.1 & 0.8 \end{bmatrix}, & B_{22} &= \begin{bmatrix} 0.1 & -0.2 \\ 0.1 & 0.1 \end{bmatrix}, \\
D_{v1} &= \begin{bmatrix} -0.9 & 0.2 \\ 0 & -0.2 \end{bmatrix}, & D_{v2} &= \begin{bmatrix} 0.1 & -0.6 \\ -0.5 & -0.1 \end{bmatrix}, & E_1 = E_2 &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
F_1(t) = F_2(t) &= \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}, & H_{110} &= \begin{bmatrix} 0.2 & -0.1 \\ 0.3 & 0.2 \end{bmatrix}, & H_{120} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
H_{111} &= \begin{bmatrix} 0.4 & 0.2 \\ 0.1 & 0.01 \end{bmatrix}, & H_{121} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, & H_{112} &= \begin{bmatrix} 0.1 & -0.3 \\ 0.3 & 0.1 \end{bmatrix}, \\
H_{122} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, & H_{21} &= \begin{bmatrix} 0.01 & 0.2 \\ 0.1 & -0.5 \end{bmatrix}, & H_{22} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \\
H_{310} &= \begin{bmatrix} 0.15 & 0.3 \\ 0.01 & 0.14 \end{bmatrix}, & H_{320} &= \begin{bmatrix} 0.01 & 0 \\ 0 & 0.01 \end{bmatrix}, & H_{311} &= \begin{bmatrix} 0.01 & -0.01 \\ 0.1 & 0.3 \end{bmatrix}, \\
H_{321} &= \begin{bmatrix} 0.01 & 0.02 \\ 0.01 & 0.1 \end{bmatrix}, & H_{312} &= \begin{bmatrix} 0.12 & 0.13 \\ -0.2 & -0.4 \end{bmatrix}, & H_{322} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.01 \end{bmatrix}, \\
H_{41} &= \begin{bmatrix} 0.02 & 0.01 \\ 0.1 & -0.14 \end{bmatrix}, & H_{42} &= \begin{bmatrix} 0.01 & 0 \\ 0.01 & 0.01 \end{bmatrix}, & C_{10} &= \begin{bmatrix} -0.2 & 0.1 \\ 0.2 & 0.1 \end{bmatrix}, \\
C_{20} &= \begin{bmatrix} -0.4 & 2 \\ -1 & 0.3 \end{bmatrix}, & C_{11} &= \begin{bmatrix} 0.2 & 0.6 \\ 0.1 & 0.2 \end{bmatrix}, & C_{21} &= \begin{bmatrix} 0.1 & 0.01 \\ 0.1 & 0.4 \end{bmatrix}, \\
C_{12} &= \begin{bmatrix} 0.1 & 0.2 \\ 0 & 0.8 \end{bmatrix}, & C_{22} &= \begin{bmatrix} 0.3 & 0.6 \\ -0.1 & 0.6 \end{bmatrix}, & B_{31} &= \begin{bmatrix} 0.5 & -0.2 \\ 0.4 & -0.5 \end{bmatrix}, \\
B_{32} &= \begin{bmatrix} 0.4 & 0.1 \\ 0.3 & -0.3 \end{bmatrix}, & C_{v1} &= \begin{bmatrix} 3 & 2 \\ -1 & 0.5 \end{bmatrix}, & C_{v2} &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}.
\end{aligned}$$

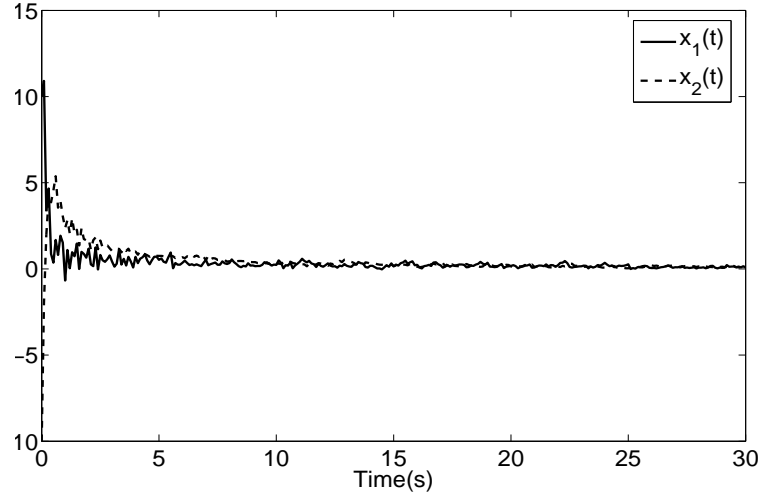


FIGURE 1. State Response of the Closed-loop System

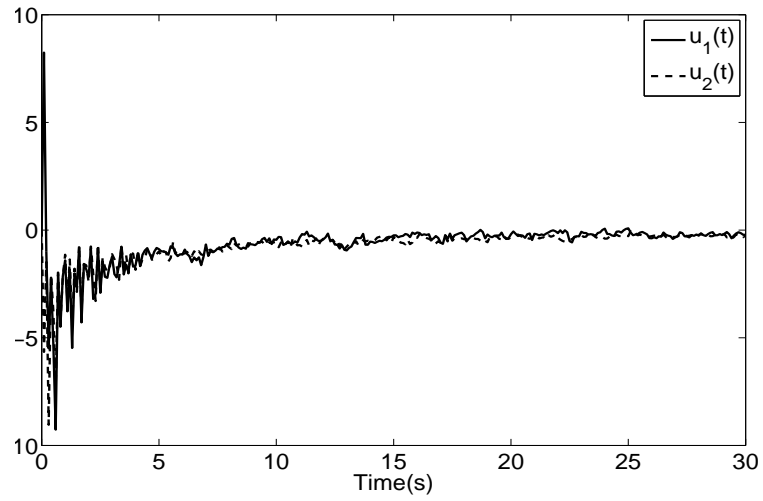


FIGURE 2. Response of the Control Input

In this example, we assume that the delay $\tau_1 = 0.1$, $\tau_2 = 0.2$ and $\gamma = 3.8$. By using Matlab LMI control toolbox to solve the LMIs (40)-(42), we obtain a feasible

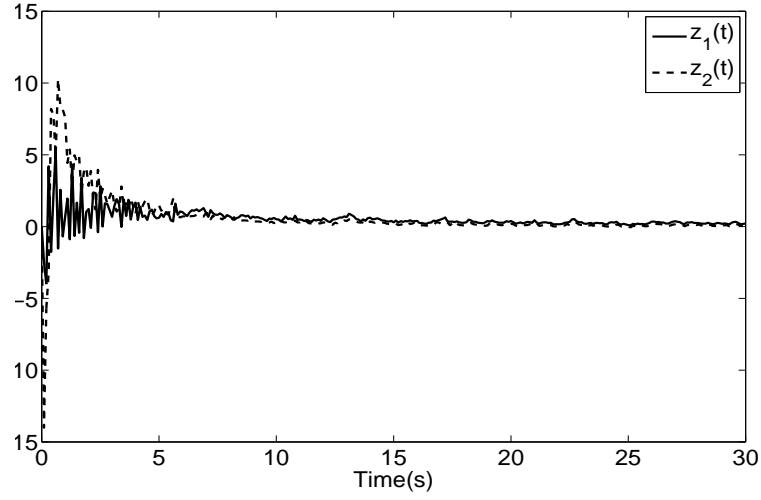


FIGURE 3. Response of the Controlled Output

set of solutions as follows

$$\begin{aligned}
 X &= \begin{bmatrix} 1.5628 & -0.3650 \\ -0.3650 & 0.5160 \end{bmatrix}, & \bar{S}_1 &= \begin{bmatrix} 3.6545 & -0.3375 \\ -0.3375 & 1.9485 \end{bmatrix}, \\
 \bar{S}_2 &= \begin{bmatrix} 2.5080 & -0.5889 \\ -0.5889 & 0.6179 \end{bmatrix}, & \tilde{R}_1 &= \begin{bmatrix} 2.1834 & -0.3808 \\ -0.3808 & 0.5170 \end{bmatrix}, \\
 \tilde{R}_2 &= \begin{bmatrix} 2.8517 & -0.6038 \\ -0.6038 & 0.9133 \end{bmatrix}, & \bar{W}_1 &= \begin{bmatrix} 1.2172 & -0.2828 \\ -0.2828 & 0.8513 \end{bmatrix}, \\
 \bar{W}_2 &= \begin{bmatrix} 0.2743 & -0.0957 \\ -0.0957 & 0.1013 \end{bmatrix}, & \bar{M}_1 &= \begin{bmatrix} -0.3934 & 0.1029 \\ 0.1897 & -0.2561 \end{bmatrix}, \\
 \bar{M}_2 &= \begin{bmatrix} -0.0668 & 0.0188 \\ 0.0391 & -0.0264 \end{bmatrix}, & \tilde{Q}_1 &= \begin{bmatrix} 2.9597 & -0.6562 \\ -0.6562 & 0.9475 \end{bmatrix}, \\
 \tilde{Q}_2 &= \begin{bmatrix} 3.0907 & -0.7141 \\ -0.7141 & 1.0206 \end{bmatrix}.
 \end{aligned}$$

Then by Theorem 4.1, gain matrices of a desired state feedback control law can be chosen as

$$K_1 = \begin{bmatrix} 0.2664 & -0.2126 \\ -0.3954 & -0.5721 \end{bmatrix}, \quad K_2 = \begin{bmatrix} 0.9785 & -2.2852 \\ -1.1681 & -0.6724 \end{bmatrix}.$$

For convenience of simulation, defining the membership functions $h_1(x_1(t)) = \frac{1-\sin(x_1(t))}{2}$ and $h_2(x_1(t)) = \frac{1+\sin(x_1(t))}{2}$; the initial function is $\phi(t) = [-10, 10]^T$

with the delay $\tau_1 = 0.1$, $\tau_2 = 0.2$ and the disturbance input is assumed to be $v_1(t) = \frac{1}{0.2+t^2}$, $v_2(t) = \frac{1}{1+50t^2}$, $t \geq 0$. Then, with the state-feedback fuzzy controller as defined earlier, the simulation results of the state response of the closed-loop system is given in Figure 1, Figure 2 shows the control input while Figure 3 shows the controlled output. It is confirmed from the simulation results that all the expected objectives are well achieved.

6. Conclusions

In this paper, problems of robust stabilization and H_∞ control for uncertain stochastic fuzzy systems with multiple time delays are studied. Based on a new LKF and using the bounding techniques, the delay-dependent robust stabilization and H_∞ control scheme is presented in terms of LMIs. It has been shown that a desired state feedback fuzzy controller can be constructed when the given LMIs are feasible. Finally a numerical example is provided to demonstrate the effectiveness of the proposed method.

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