APPLICATION OF TABU SEARCH FOR SOLVING THE BI-OBJECTIVE WAREHOUSE PROBLEM IN A FUZZY ENVIRONMENT

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Abstract. The bi-objective warehouse problem in a crisp environment is often not effective in dealing with the imprecision or vagueness in the values of the problem parameters. To deal with such situations, several researchers have proposed that the parameters be represented as fuzzy numbers. We describe a new algorithm for fuzzy bi-objective warehouse problem using a ranking function followed by an application of tabu search. The method is illustrated on a numerical example, demonstrating the effectiveness of the tabu search method. Numerical results are compared for both fuzzy and crisp versions of the problem.

1. Introduction

Facility location is a well established research area within operations research. The facility location-allocation was initially studied by Cooper [13] in 1963. Afterwards, many authors considered various types of the facility location problems and proposed several approaches for solving these problems. A comprehensive survey of existing methods, extensions and applications in the area of facility location, multi-criteria facility location and supply chain management in crisp environment can be found in [15, 20, 31]

Any facility location model representing real-world situations involves a lot of parameters whose values are assigned by experts and in the conventional approach, they are required to fix an exact value to the aforementioned parameters. However, both experts and the decision maker frequently do not precisely know the value of those parameters. If exact values are suggested, these are only statistical inference from past data and their stability is doubtful, so the parameters of the problem are usually defined by the decision maker in an uncertain way or by means of language statement parameters. Therefore, it is useful to consider the knowledge of experts about the parameters as fuzzy data.

Zadeh [40] introduced the concepts of fuzzy sets to deal with real life situations. Fuzzy optimization problems play a very important role in many fuzzy systems. Bellman and Zadeh [3] first introduced the fuzzy sets theory into multicriteria analysis for effectively dealing with the imprecision, vagueness and subjectiveness

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of the human decision making. Since then, tremendous efforts have been spent; significant advances have been made on the development of numerous methodologies and their applications to various decision problems.

Narasimhan [33] presented an application of fuzzy set theory to the problem of locating gas stations. Darzentas [14] formulated the facility location problem as a fuzzy set partitioning model using integer programming. Bhattacharya et al. [4] presented a fuzzy goal programming model for locating a single facility within a given convex region subject to the simultaneous consideration of three criteria. Bhattacharya et al. [5] formulated a fuzzy goal programming model for locating a single facility within a given convex region subject to the simultaneous consideration of two criteria. Chu [12] presented a fuzzy TOPSIS model under group decisions for solving the facility location selection problem where the ratings of various alternative locations under different subjective attributes and the importance weights of all attributes are assessed in linguistic values, represented by fuzzy numbers. Kahraman et al. [21] solved facility location problems using different solution approaches of fuzzy multi-attribute group decision-making. Gen and Syarif [17] dealt with a production/distribution problem to determine an efficient integration of production, distribution and inventory system so that products are produced and distributed in the right quantities, to the right customers and at the right time, in order to minimize system wide costs while satisfying all demands required. Lin and Kwok [24] applied two other classes of meta-heuristics: tabu search and simulated annealing and compared their performances through a new statistical procedure. Caballero et al. [7] used the multi-objective meta-heuristic using an adaptative memory procedure method which is based on tabu search. Uno and Katagiri [38] proposed to apply the interactive fuzzy satisfying method with a tabu search algorithm in order to find several M-Pareto optimal solutions in their location problem.

Real numbers can be linearly ordered by $\geq$ or $\leq$; however, this type of inequality does not exist in fuzzy numbers. Since fuzzy numbers are represented by possibility distribution, they can overlap with each other and it is difficult to determine clearly whether one fuzzy number is larger or smaller than other. Some authors [9, 10, 23, 26, 30, 32] proposed different ranking functions for the comparison of fuzzy numbers. Ranking function is used in different areas of fuzzy optimization [1, 2, 6, 8, 11, 16, 25, 27, 28, 29, 34, 35, 36, 39].

Tabu search is a meta-heuristic method originally developed for combinatorial optimizations by Glover [18, 19]. By means of the use of tabu search, one may escape from local optimal solution and seek improved global solutions. Generally speaking, tabu search requires some memory, known as tabu list, to trace the recent states which have been investigated. The next potential solution is chosen from neighbours of current solution. The solution can be accepted as the next move if it is not tabued, or if it does so, it should satisfy the aspiration criterion. The existence of the tabu list makes tabu search a powerful technique to conduct global search rather than searching on a small portion of the search space. The important characteristics of the tabu search are following:

(1) It temporarily prevents cycling i.e., revisiting the solutions already examined by using a tabu list. The tabu list contains forbidden moves which prohibit revisiting the solution that have been examined recently. The tabu list changes as we move to the next solution whether it is better or worse.
(2) It uses the concept of an incumbent solution which is the best solution obtained so far. The incumbent solution changes to a better current solution as and when we arrive at it.
(3) The procedure terminates when either we revisit an already examined solution or we are not able to find a new current solution.

Till now, to the best of our knowledge, no one has used the ranking function with tabu search for solving the bi-objective warehouse problems in fuzzy environment. In this paper, a new algorithm consisting of a combination of add and drop rule, incorporating tabu search and ranking function, is proposed for selecting a fixed number of warehouse sites among a given number of potential warehouse sites for clustering a given number of ration shops to them. The problem is bi-objective fuzzy non-linear programming problem in which all the parameters are represented by trapezoidal fuzzy numbers.

This paper is organized as follows: In section 2, advantages of proposed approach are discussed. In section 3, some basic definitions, arithmetic operations and ranking function are reviewed. In section 4, the formulation of bi-objective warehouse problem in fuzzy environment is discussed. In section 5, a new algorithm consisting of a combination of add and drop rule, incorporating tabu search and ranking function, is proposed. In section 6, to illustrate the proposed algorithm a numerical example is solved. In section 7, results in crisp and fuzzy versions are compared.

2. Advantages of Proposed Approach

In the existing approaches to solve the bi-objective facility location problems under fuzzy environment, using Zimmermann approach [41] the fuzzy linear programming problem is converted into crisp linear programming problem and then obtained crisp linear programming problem is solved to find the fuzzy optimal solutions. For solving, the fuzzy bi-objective facility location problems using the existing approaches, a decision maker should have good knowledge of fuzzy linear programming problems, Zimmermann approach and methods to solve crisp linear programming problem. It is very difficult to implement the existing algorithms to programming language. By using the proposed algorithms for finding the optimal solutions for fuzzy bi-objective facility location problems, we have the following advantages:

(1) We do not use linear programming techniques.
(2) We do not use goal and parametric programming techniques.
(3) The optimal solution is a fuzzy number.
(4) The proposed method is very easy to understand and to apply.
(5) There is no need of much knowledge of fuzzy linear programming, Zimmermann approach and crisp linear programming.

(6) The decision maker should have only the knowledge of ranking function and arithmetic operations of fuzzy numbers which is very easy to learn for a new decision maker.
(7) The proposed algorithms can be easily implemented into a programming language.

3. Preliminaries

In this section, some basic definitions, arithmetic operations and ranking functions are reviewed.

3.1. **Basic Definitions.** In this section, some basic definitions are reviewed [22].

Definition 3.1. The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range $[0, 1]$ i.e., $\mu_{\tilde{A}} : X \to [0, 1]$. The assigned values indicate the membership grade of the element in the set A. The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set.

Definition 3.2. A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:

(1) $\mu_{\tilde{A}} : R \to [0, 1]$ is continuous.
(2) $\mu_{\tilde{A}}(x) = 0 \; \forall \; x \in (-\infty, c] \cup (d, \infty]$.
(3) It is strictly increasing on $[c, a]$ and strictly decreasing on $[b, d]$.
(4) $\mu_{\tilde{A}}(x) = 1$ for all $x \in [a, b]$.

Definition 3.3. A fuzzy number $\tilde{A} = (a, b, c, d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)} & a \leq x \leq b \\ 1 & b \leq x \leq c \\ \frac{(d-x)}{(d-c)} & c \leq x \leq d \\ 0 & \text{otherwise} \end{cases}$$

where $a, b, c, d \in R$.

3.2. **Arithmetic Operations.** In this section, arithmetic operations between two trapezoidal fuzzy numbers, defined on universal set of real numbers R, are reviewed [22].
Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2)$ be two trapezoidal fuzzy numbers, then

(1) $\tilde{A}_1 + \tilde{A}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$
(2) $\tilde{A}_1 - \tilde{A}_2 = (a_1 - d_2, b_1 - c_2, c_1 - b_2, d_1 - a_2)$
(3) $\lambda \tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1) & \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1) & \lambda < 0 \end{cases}$

Remark 3.4. In this paper, at all places $\sum_{i=1}^{m} \lambda_i \tilde{A}_i$ and $\sum_{i=1}^{m} \lambda_i A_i$ represent the fuzzy and crisp additions respectively i.e. $\sum_{i=1}^{m} \lambda_i \tilde{A}_i = \lambda_1 \tilde{A}_1 + \ldots + \lambda_m \tilde{A}_m$ and $\sum_{i=1}^{m} \lambda_i A_i = \lambda_1 A_1 + \ldots + \lambda_m A_m$, where $\tilde{A}_i$ and A_i are fuzzy number and real number respectively.

3.3. **Ranking Function.** A convenient method for comparing fuzzy numbers is by the use of ranking function [26]. A ranking function $\Re : F(R) \to R$, where $F(R)$ the set of fuzzy numbres defined on the set of real numbers, maps each fuzzy number onto a real number. Let $\tilde{A}$ and $\tilde{B}$ be two fuzzy numbers, then

$$\text{(i) } \tilde{A} \succeq \tilde{B} \text{ if } \Re(\tilde{A}) \geq \Re(\tilde{B})$$
$$\text{(ii) } \tilde{A} \succ \tilde{B} \text{ if } \Re(\tilde{A}) > \Re(\tilde{B})$$
$$\text{(iii) } \tilde{A} \simeq \tilde{B} \text{ if } \Re(\tilde{A}) = \Re(\tilde{B})$$

e.g.,

Let $\tilde{A} = (1, 2, 4, 5)$ and $\tilde{B} = (2, 5, 7, 9)$, then $\tilde{B} \succ \tilde{A}$ as $\Re(\tilde{B}) > \Re(\tilde{A})$

Similarly

Let $\tilde{A} = (1, 2, 4, 5)$ and $\tilde{B} = (0, 1, 5, 6)$, then $\tilde{B} \simeq \tilde{A}$ as $\Re(\tilde{B}) = \Re(\tilde{A})$

Remark 3.5. Let $\{\tilde{a}_i : i = 1, 2, \ldots, n\}$ be a set of trapezoidal fuzzy numbers. If $\Re(\tilde{a}_k) \leq \Re(\tilde{a}_i)$ for all i then the fuzzy number $\tilde{a}_k$ is the minimum of $\{\tilde{a}_i : i = 1, 2, \ldots, n\}$ and if $\Re(\tilde{a}_k) \geq \Re(\tilde{a}_i)$ for all i, then the fuzzy number $\tilde{a}_k$ is the maximum of $\{\tilde{a}_i : i = 1, 2, \ldots, n\}$

4. Formulation of Bi-objective Warehouse Problem in Fuzzy Environment

Prakash, Sharma and Singh [37] formulated and solved the problem "Selection of warehouse sites for clustering ration shops to them with two objectives" in crisp environment by assuming that there is no uncertainty about the values of parameters (cost, time etc.), but in real life problems there always exists uncertainty about parameters. So to overcome this shortcoming, in this section, the problem mentioned formulated in fuzzy environment where all the parameters are represented by trapezoidal fuzzy numbers.

Let there exists M ration shops and N potential warehouse sites. A decision maker wants to select k, a maximum number of warehouse sites from N potential warehouse sites and then wants to cluster M ration shops to k selected warehouse sites in order to minimize the total fuzzy cost and maximum fuzzy time of meeting all requirements. It is assumed that each site selected to locate warehouse must have at least one ration shop attached to it. Now, our objective is to formulate the problem and to propose a new algorithm to find fuzzy optimal solutions of such type of problems, occurring in real life. Mathematically, the above problem may be formulated as follows.

$$\text{Minimize } \tilde{C} \simeq \sum_{i=1}^{M} \sum_{j=1}^{N} \tilde{c}_{ij} x_{ij} \tag{1}$$

Minimize $\tilde{T}$, where $\tilde{T} \simeq \text{maximum}\{\tilde{t}_{ij} : x_{ij} = 1, i = 1, 2, \ldots, M;\ j = 1, 2, \ldots, N\}$ (2)

subject to

$$\sum_{j=1}^{N} y_j \leq k \tag{3}$$

$$\sum_{j=1}^{N} x_{ij} = 1, \quad i = 1, 2, \ldots, M \tag{4}$$

$$x_{ij} - y_j \leq 0, \quad i = 1, 2, \ldots, M;\ j = 1, 2, \ldots, N \tag{5}$$

$$\sum_{j=1}^{N} \tilde{b}_j y_j \preceq \tilde{B} \tag{6}$$

$$x_{ij}, y_j = 0 \text{ or } 1, \quad i = 1, 2, \ldots, M;\ j = 1, 2, \ldots, N \tag{7}$$

$$\sum_{i=1}^{M} x_{ij} \geq y_j, \quad j = 1, 2, \ldots, N \tag{8}$$

where

M: Total number of ration shops.
N: Total number of potential warehouse sites.
k: Maximum number of warehouse sites selected for locating warehouse at them.
$\tilde{c}_{ij}$: Fuzzy cost of meeting requirement from j^{th} site to i^{th} ration shop.
$\tilde{t}_{ij}$: Fuzzy time of meeting requirement from j^{th} site to i^{th} ration shop.
$\tilde{b}_j$: Fuzzy setup cost of setting up of a warehouse at the j^{th} site.
$\tilde{B}$: Total fuzzy budgetary amount allocated for setting up of warehouses.

$$x_{ij} = \begin{cases} 1 & \text{if ration shop } i^{th} \text{ is served from } j^{th} \text{ site} \\ 0 & \text{otherwise} \end{cases}$$

$$y_j = \begin{cases} 1 & \text{if a warehouse is established at } j^{th} \text{ site} \\ 0 & \text{otherwise} \end{cases}$$

$\tilde{C}$: Total fuzzy cost of meeting requirements.
$\tilde{T}$: Maximum fuzzy time of meeting requirements.
X^l: The combination of x_{ij}'s during l^{th} non-dominated solution.
Y^l: The combination of y_j's during l^{th} non-dominated solution.

(1) and (2) are the objectives of the problem in which we want to minimize the total fuzzy cost and maximum fuzzy time of meeting requirements of all ration shops from their assigned warehouses at the selected sites. The constraint (3) ensures that up to a maximum of k warehouse sites can be selected from the N potential warehouse sites and the constraint (6) ensures that the total fuzzy setup cost of warehouses does not exceed the allotted total fuzzy budgetary amount $\tilde{B}$. Constraints (4) and (5) ensure that each ration shop is assigned to a unique site

which is selected for locating a warehouse. Constraint (8) ensures that each site selected to locate a warehouse must have at least one ration shop assigned to it. There is no restriction on number of ration shops to be clustered to a selected site. So, the problem is uncapacitated one. The problem is bi-objective fuzzy non-linear programming problem.

Remark 4.1. It is required to obtain the set of fuzzy efficient solutions of the problem given by (1) to (8). For the purpose of listing the fuzzy efficient solutions, a solution (X^1, Y^1) shall be called the 1^{st} fuzzy efficient solution if it is the optimal solution of the problem with the minimization of $\tilde{C}$ and $\tilde{T}$ as the first and second prioritized objectives respectively. A solution (X^2, Y^2) shall be called the 2^{nd} fuzzy efficient solution if efficient solution of the problem exists satisfying the conditions (i) $\tilde{C}(X^1) \prec \tilde{C}(X^2)$ and (ii) $\tilde{T}(X^1) \succ \tilde{T}(X^2)$ and there does not exist any efficient solution (X, Y) satisfying the conditions (i) $\tilde{C}(X^1) \prec \tilde{C}(X) \prec \tilde{C}(X^2)$ and (ii) $\tilde{T}(X^1) \succ \tilde{T}(X) \succ \tilde{T}(X^2)$. 3^{rd} and the subsequent efficient solutions are found in the same way as is found for the 2^{nd} efficient solution.

5. **Proposed Algorithm**

Till now, to the best of our knowledge, no one has used the ranking function with tabu search for solving the bi-objective warehouse problems in fuzzy environment. In this section, a new algorithm consisting of a combination of add and drop rule, incorporating tabu search and ranking function, is proposed for selecting a fixed number of warehouse sites among a given number of potential warehouse sites for clustering a given number of ration shops to them. The problem is fuzzy bi-objective non-linear programming problem in which all the parameters are represented by trapezoidal fuzzy numbers.

The fuzzy cost of meeting requirements from j^{th} site to i^{th} ration shop $(\tilde{c}_{ij})$, fuzzy time of meeting requirements from j^{th} site to i^{th} ration shop $(\tilde{t}_{ij})$ and the fuzzy setup cost for setting up a warehouse at the j^{th} site $(\tilde{b}_j)$ are shown in Table 1.

Warehouse sites (j) → Ration shops (i) ↓	1		q_1				N
1	$\tilde{c}_{11}$ $\tilde{t}_{11}$						$\tilde{c}_{1N}$ $\tilde{t}_{1N}$
i			$\tilde{c}_{iq_1}$ $\tilde{t}_{iq_1}$				
M	$\tilde{c}_{M1}$ $\tilde{t}_{M1}$						$\tilde{c}_{MN}$ $\tilde{t}_{MN}$
Setup cost $(\tilde{b}_j)$	$\tilde{b}_1$		$\tilde{b}_{q_1}$				$\tilde{b}_N$

Table 1. Tableau Representation of Bi-objective Warehouse Problem

Step 1: Let $\tilde{a} \simeq$ minimum$\{\sum\limits_{i=1}^{M} \tilde{c}_{ij} : j = 1, 2, \ldots, N\}$ and $\tilde{T}_j \simeq$ maximum$\{\tilde{t}_{ij} : i = 1, 2, \ldots, M\}$ for $j = 1, 2, \ldots, N$

There are two cases:

Case 1: If $\tilde{a}$ occurs corresponding to unique value of j i.e. $j = t$, where $t \in \{1, 2, \ldots, N\}$, then t is the first potential site for the first warehouse.

Case 2: If $\tilde{a}$ occurs corresponding to more than one values of j i.e. $j = p_1, p_2, \ldots, p_m$ then find minimum$\{\tilde{T}_{p_1}, \tilde{T}_{p_2}, \ldots, \tilde{T}_{p_m}\}$.

Case 2a: If minimum is unique and it is corresponding to p_l, then p_l is first potential site for first warehouse.

Case 2b: If minimum occurs corresponding to more than one values of j i.e. $j = p_{s_1}, p_{s_2}, \ldots, p_{s_q}$, then choose any j from $p_{s_1}, p_{s_2}, \ldots, p_{s_q}$ for the first potential site for the first warehouse.

Step 2: Let the first potential site selected for the first warehouse be q_1 where $q_1 \in \{1, 2, \ldots, N\}$. Now for selecting the next warehouse site q_2 where $q_2 \neq q_1$, construct the new table as follows.

Warehouse sites $(j) \rightarrow$ Ration shops $(i) \downarrow$	$(q_1, 1)$	$(q_1, 2)$		(q_1, j)				(q_1, N)
1								
2								
i				$\tilde{c}^1_{ij}$ $\tilde{t}^1_{ij}$				
M								

Table 2. Tableau Representation of Bi-objective Warehouse Problem

Let the cost and time of $(i, j)^{th}$ cell, $j \neq q_1$ in new table be $\tilde{c}^1_{ij}$ and $\tilde{t}^1_{ij}$, where

$$\tilde{c}^1_{ij} \simeq \text{minimum } \{\tilde{c}_{iq_1}, \tilde{c}_{ij}\} \text{ and } \tilde{t}^1_{ij} \simeq \begin{cases} \tilde{t}_{ij} & \text{if minimum } (\tilde{c}_{iq_1}, \tilde{c}_{ij}) \simeq \tilde{c}_{ij} \\ \tilde{t}_{iq_1} & \text{if minimum } (\tilde{c}_{iq_1}, \tilde{c}_{ij}) \simeq \tilde{c}_{iq_1} \end{cases}$$

Now the newly constructed Table 2 is the same as Table 1 except that it has $(N-1)$ columns. Now apply Step 1 and find new combination of warehouses say (q_1, q_2) where $q_2 \in \{1, 2, \ldots, N\}$ and $q_2 \neq q_1$.

Step 3: Let the pair (q_1, q_2) represent the two potential sites for the two warehouses. Similarly find k-potential sites for k-warehouses by applying Step 2. Let the combination of k-potential sites for k-warehouses be $(q_1, q_2, \ldots, q_k)$. Selection of sites using add rule in the way explained above, yields the 0^{th} iterative solution (X^1_0, Y^1_0) for the 1^{st} efficient solution (X^1, Y^1) and is also the incumbent solution for it. The incumbent solution for the 1^{st} efficient solution (X^1, Y^1) is the best solution obtained so far. The total fuzzy cost $\tilde{C}(X_0^1)$ and fuzzy time $\tilde{T}(X_0^1)$ of 0^{th} iterative solution (X_0^1, Y_0^1) can be obtained using (1) and (2).

Step 4: We invoke drop and add rules with tabu search for further improving the solution. Drop the site which was first selected i.e. q_1, the site dropped recently is included in the tabu list for add which will be prohibited from being selected in the next 1^{st} iterative solution (X_1^1, Y_1^1) and $k-1$ most recently added warehouses i.e. $(q_2, q_3, \ldots, q_k)$ appearing in the order of their selection in the 0^{th} iterative solution are included in the tabu list for drop. Find all the entries of the column corresponding to combination $(q_2, q_3, \ldots, q_k)$ using Steps 1 and 2. Then add the new site by replacing q_1 by $(q_2, q_3, \ldots, q_k)$ in Step 2. Suppose the new combination of k-potential sites for the k-warehouses are $(q_2, q_3, \ldots, q_k, q_h)$ where $q_h \neq q_1, q_2, \ldots, q_k$. Selection of sites $(q_2, q_3, \ldots, q_k, q_h)$ in the way explained above yields the 1^{st} iterative solution (X_1^1, Y_1^1) for the 1^{st} efficient solution (X^1, Y^1). Find the cost $\tilde{C}(X_1^1)$ and time $\tilde{T}(X_1^1)$ for $(q_2, q_3, \ldots, q_k, q_h)$. If $\tilde{C}(X_1^1) \prec \tilde{C}(X_0^1)$, then the current iterative solution will be treated as the incumbent solution for the next iteration, otherwise there will be no change in incumbent solution.

Step 5: The 2^{nd} iterative solution (X_2^1, Y_2^1) is obtained in similar manner as explained in Step 4 by changing the both tabu lists. The tabu list for the drop rule used for obtaining 2^{nd} iterative solution (X_2^1, Y_2^1) will now include the $k-1$ most recent selected sites included in the 1^{st} iterative solution (X_1^1, Y_1^1) i.e. $(q_3, \ldots, q_k, q_h)$ appearing in the order of their selection. Similarly, the tabu list for the add rule used for obtaining the 2^{nd} iterative solution (X_2^1, Y_2^1) will now include the 1^{st} site selected in the 1^{st} iterative solution (X_1^1, Y_1^1) i.e. q_2. If the 2^{nd} iterative solution (X_2^1, Y_2^1) is better than the incumbent solution, then the current iterative solution will be treated as incumbent solution for the next iteration otherwise the incumbent solution does not change. The 3^{rd} iterative solution (X_3^1, Y_3^1) and subsequent ones will be obtained in the same way as done in obtaining the 1^{st} and 2^{nd} iterative solutions. We change the incumbent solution to the current iterative solution if it is better than the incumbent solution. We terminate this process of obtaining newer iterative solutions when either we revisit an already visited iterative solution or there are no sites left that will serve to improve the solution. When this happens, then the most recent incumbent solution provides the 1^{st} efficient solution (X^1, Y^1) of the formulated problem.

Step 6: To find the 2^{nd} fuzzy efficient solution (X^2, Y^2) of the formulated problem, replace $\tilde{t}_{ij}$'s $\succeq \tilde{T}(X^1)$ by an arbitrary large fuzzy number (L, L, L, L) and repeat Steps 1-5.

Step 7: The incumbent solution at the end of iterative process yields 2^{nd} fuzzy efficient solution (X^2, Y^2) of formulated problem. 3^{rd} and the subsequent solutions are obtained in the same way as is done to obtain the 2^{nd} fuzzy efficient solution. The process of obtaining the fuzzy efficient solutions is terminated when it is no longer possible to obtain a new fuzzy efficient solution with lesser duration.

6. **Numerical Example**

Now we illustrate the algorithm, proposed in section 5, by applying it to following numerical problem to obtain the set of fuzzy efficient solutions which are obtained by taking $M = 5, N = 7, k = 3, \tilde{B} = (1380, 1390, 1410, 1420)$ and assigning numerical values to all $\tilde{b}_j$'s, $\tilde{c}_{ij}$'s, and $\tilde{t}_{ij}$'s in the problem formulated in section 4. Table 3 provides tableau representation of the numerical problem. In this table, rows 1-5 correspond to ration shops and columns 1-7 correspond to potential warehouse sites. Row 6 corresponds to approximate setup costs $\tilde{b}_j$'s of warehouses at the respective potential sites. Upper and lower entries of $(i, j)^{th}$ cell represent the fuzzy cost $\tilde{c}_{ij}$ and fuzzy time $\tilde{t}_{ij}$ of transportation, respectively from j^{th} potential warehouse site to i^{th} ration shop.

Warehouse sites $(j) \rightarrow$ Ration shops $(i) \downarrow$	1	2	3	4	5	6	7
1	(36, 38, 42, 44) (5, 8, 9, 10)	(28, 29, 31, 32) (1, 2, 2, 3)	(44, 49, 53, 54) (9, 10, 11, 14)	(117, 118, 121, 124) (3, 6, 7, 8)	(175, 177, 183, 185) (8, 9, 11, 12)	(165, 167, 169, 179) (6, 8, 9, 13)	(143, 145, 153, 157) (5, 6, 8, 9)
2	(68, 69, 70, 73) (3, 6, 7, 8)	(73, 80, 82, 85) (3, 6, 7, 8)	(128, 129, 130, 133) (6, 8, 9, 13)	(128, 129, 130, 133) (5, 6, 8, 9)	(165, 167, 169, 179) (5, 8, 9, 10)	(133, 136, 144, 147) (8, 9, 11, 12)	(11, 15, 25, 29) (9, 10, 11, 14)
3	(73, 80, 82, 85) (5, 8, 9, 10)	(11, 15, 25, 29) (6, 8, 9, 13)	(186, 190, 205, 209) (3, 4, 5, 8)	(68, 69, 70, 73) (5, 8, 9, 10)	(133, 136, 144, 147) (9, 10, 11, 14)	(128, 129, 130, 133) (9, 10, 14, 15)	(56, 58, 62, 64) (8, 9, 11, 12)
4	(56, 58, 62, 64) (9, 10, 11, 14)	(8, 9, 11, 12) (3, 6, 7, 8)	(68, 69, 70, 73) (8, 12, 14, 18)	(73, 80, 82, 85) (9, 10, 11, 14)	(28, 29, 31, 32) (6, 8, 9, 13)	(156, 159, 160, 165) (5, 8, 9, 10)	(199, 203, 217, 221) (3, 6, 7, 8)
5	(81, 89, 93, 97) (8, 9, 11, 12)	(89, 98, 104, 109) (10, 12, 13, 17)	(16, 18, 22, 24) (5, 8, 9, 10)	(56, 58, 62, 64) (9, 10, 14, 15)	(36, 38, 42, 44) (3, 6 7, 8)	(44, 49, 53, 54) (2, 6, 7, 9)	(197, 205, 230, 248) (8, 14, 16, 18)
Setup cost $(\tilde{b}_j)$	(89, 98, 104, 109)	(275, 288, 309, 328)	(680, 690, 700, 730)	(712, 784, 832, 872)	(380, 392, 410, 418)	(178, 196, 208, 218)	(445, 490, 520, 545)

TABLE 3. Tableau Representation of the Numerical Problem

The total fuzzy cost $(\sum_{i=1}^{5} \tilde{c}_{ij})$ and fuzzy time $\tilde{T}_j$ obtained by Step 1, is shown in Table 4.

Since the minimum value of total fuzzy cost $(\tilde{a})$ is corresponding to $j = 2$. So, $q_1 = 2$ is the first potential site for the first warehouse site. Now applying Step 2, for selecting second warehouse site, Table 5 is obtained.

Since the minimum value of total fuzzy cost $(\tilde{a})$ is corresponding to $(2, 3)$. So the pair $(2, 3)$ represents the two potential sites for two warehouses. Again, applying Step 2, for next warehouse site, Table 6 is obtained.

Warehouse sites $(j) \rightarrow$	1	2	3	4	5	6	7
Total fuzzy cost $(\sum_{i=1}^{5} \tilde{c}_{ij})$	$(314, 334, 349, 363)$	$(209, 231, 253, 267)$	$(442, 455, 480, 503)$	$(442, 454, 465, 479)$	$(537, 547, 569, 587)$	$(626, 640, 656, 678)$	$(606, 626, 689, 719)$
$\tilde{T}_j$	$(9, 10, 11, 14)$	$(10, 12, 13, 17)$	$(8, 12, 14, 18)$	$(9, 10, 14, 15)$	$(9, 10, 11, 14)$	$(9, 10, 14, 15)$	$(8, 14, 16, 18)$

TABLE 4. Total Fuzzy Cost and Maximum Fuzzy Time for 1^{st} Warehouse

Warehouse sites $(j) \rightarrow$ Ration shops $(i) \downarrow$	$(2, 1)$	$(2, 3)$	$(2, 4)$	$(2, 5)$	$(2, 6)$	$(2, 7)$
1	$(28, 29, 31, 32)$ $(1, 2, 2, 3)$	$(28, 29, 31, 32)$ $(1, 2, 2, 3)$	$(28, 29, 31, 32)$ $(1, 2, 2, 3)$	$(28, 29, 31, 32)$ $(1, 2, 2, 3)$	$(28, 29, 31, 32)$ $(1, 2, 2, 3)$	$(28, 29, 31, 32)$ $(1, 2, 2, 3)$
2	$(68, 69, 70, 73)$ $(3, 6, 7, 8)$	$(73, 80, 82, 85)$ $(3, 6, 7, 8)$	$(73, 80, 82, 85)$ $(3, 6, 7, 8)$	$(73, 80, 82, 85)$ $(3, 6, 7, 8)$	$(73, 80, 82, 85)$ $(3, 6, 7, 8)$	$(11, 15, 25, 29)$ $(9, 10, 11, 14)$
3	$(11, 15, 25, 29)$ $(6, 8, 9, 13)$	$(11, 15, 25, 29)$ $(6, 8, 9, 13)$	$(11, 15, 25, 29)$ $(6, 8, 9, 13)$	$(11, 15, 25, 29)$ $(6, 8, 9, 13)$	$(11, 15, 25, 29)$ $(6, 8, 9, 13)$	$(11, 15, 25, 29)$ $(6, 8, 9, 13)$
4	$(8, 9, 11, 12)$ $(3, 6, 7, 8)$	$(8, 9, 11, 12)$ $(3, 6, 7, 8)$	$(8, 9, 11, 12)$ $(3, 6, 7, 8)$	$(8, 9, 11, 12)$ $(3, 6, 7, 8)$	$(8, 9, 11, 12)$ $(3, 6, 7, 8)$	$(8, 9, 11, 12)$ $(3, 6, 7, 8)$
5	$(81, 89, 93, 97)$ $(8, 9, 11, 12)$	$(16, 18, 22, 24)$ $(5, 8, 9, 10)$	$(56, 58, 62, 64)$ $(9, 10, 14, 15)$	$(36, 38, 42, 44)$ $(3, 6 7, 8)$	$(44, 49, 53, 54)$ $(2, 6, 7, 9)$	$(89, 98 104, 109)$ $(10, 12, 13, 17)$
Total fuzzy cost $(\sum_{i=1}^{5} \tilde{c}_{ij})$	$(196, 211, 230, 243)$	$(136, 151, 171, 182)$	$(176, 191, 211, 222)$	$(156, 171, 191, 202)$	$(164, 182, 202, 212)$	$(147, 166, 196, 211)$
$\tilde{T}_j$	$(8, 9, 11, 12)$	$(6, 8, 9, 13)$	$(9, 10, 14, 15)$	$(6, 8, 9, 13)$	$(6, 8, 9, 13)$	$(10, 12, 13, 17)$

TABLE 5. Total Fuzzy Cost and Maximum Fuzzy Time for Two Warehouses

Combination of sites $(2, 3, 4)$ and $(2, 3, 7)$ are not chosen in Table 6 as they do not satisfy the budgetary constraint (6). Sites $(2, 3, 5)$ and $(2, 3, 7)$ do not satisfy constraint (8). Since the minimum value of total fuzzy cost $(\tilde{a})$ is corresponding to $(2, 3, 1)$, so the combination $(2, 3, 1)$ represent the three potential sites for three warehouses. Hence the current and incumbent solutions of 0^{th} iteration are given in Table 7.

Now, we use tabu search method to improve the solution with respect to the first objective function.

Warehouse sites (j) $\rightarrow$ Ration shops (i) $\downarrow$	$(2,3,1)$	$(2,3,5)$	$(2,3,6)$
1	$(28,29,31,32)$ $(1,2,2,3)$	$(28,29,31,32)$ $(1,2,2,3)$	$(28,29,31,32)$ $(1,2,2,3)$
2	$(68,69,70,73)$ $(3,6,7,8)$	$(73,80,82,85)$ $(3,6,7,8)$	$(73,80,82,85)$ $(3,6,7,8)$
3	$(11,15,25,29)$ $(6,8,9,13)$	$(11,15,25,29)$ $(6,8,9,13)$	$(11,15,25,29)$ $(6,8,9,13)$
4	$(8,9,11,12)$ $(3,6,7,8)$	$(8,9,11,12)$ $(3,6,7,8)$	$(8,9,11,12)$ $(3,6,7,8)$
5	$(16,18,22,24)$ $(5,8,9,10)$	$(16,18,22,24)$ $(5,8,9,10)$	$(16,18,22,24)$ $(5,8,9,10)$
Total fuzzy cost ($\sum_{i=1}^{5} \tilde{c}_{ij}$)	$(131,140,159,170)$	$(136,151,171,182)$	$(136,151,171,182)$
$\tilde{T}_j$	$(6,8,9,13)$	$(6,8,9,13)$	$(6,8,9,13)$

Table 6. Total Fuzzy Cost and Maximum Fuzzy Time for Three Warehouses

Current Solution (X_0^1, Y_0^1)			Incumbent Solution (X_0^1, Y_0^1)		
Variables x_{ij}'s and y_j's	$\tilde{C}(X_0^1)$	$\tilde{T}(X_0^1)$	Variables x_{ij}'s and y_j's	$\tilde{C}(X_0^1)$	$\tilde{T}(X_0^1)$
$y_2, y_3, y_1, x_{12}, x_{21}, x_{32}, x_{42}, x_{53}$	$(131,140$ $159,170)$	$(6,8,9,13)$	$y_2, y_3, y_1, x_{12}, x_{21}, x_{32}, x_{42}, x_{53}$	$(131,140$ $159,170)$	$(6,8,9,13)$

Table 7. Current and Incumbent Solutions of 0^{th} Iteration for 1^{st} Non-dominated Solution

Iteration No. 1. Since the combination of sites selected in 0^{th} iterative solution for setting up of warehouses are $(2,3,1)$. Thus, the two tabu lists are given in Table 8.

Using Step 2, add the new site to the combination of sites $(3,1)$. Hence the solutions at this stage are:
Combination of sites $(3,1,4)$ is not chosen in Table 9 as it does not satisfy budgetary constraint (6). Combination of sites $(3,1,6)$ does not satisfy constraint (8). The value of $\tilde{a}$ is corresponding to $(3,1,7)$. The total fuzzy cost of 1^{st} iteration

Tabu list for drop	Tabu list for add
$\{y_3, y_1\}$	$\{y_2\}$

TABLE 8. Two Tabu Lists in 1^{st} Iteration for 1^{st} Non-dominated Solution

Warehouse sites (j) $\rightarrow$ Ration shops (i) $\downarrow$	$(3,1,5)$	$(3,1,6)$	$(3,1,7)$
1	$(36,38,42,44)$ $(5,8,9,10)$	$(36,38,42,44)$ $(5,8,9,10)$	$(36,38,42,44)$ $(5,8,9,10)$
2	$(68,69,70,73)$ $(3,6,7,8)$	$(68,69,70,73)$ $(3,6,7,8)$	$(11,15,25,29)$ $(9,10,11,14)$
3	$(73,80,82,85)$ $(5,8,9,10)$	$(73,80,82,85)$ $(5,8,9,10)$	$(56,58,62,64)$ $(8,9,11,12)$
4	$(28,29,31,32)$ $(6,8,9,13)$	$(56,58,62,64)$ $(9,10,11,14)$	$(56,58,62,64)$ $(9,10,11,14)$
5	$(16,18,22,24)$ $(5,8,9,10)$	$(16,18,22,24)$ $(5,8,9,10)$	$(16,18,22,24)$ $(5,8,9,10)$
Total fuzzy cost ($\sum\limits_{i=1}^{5} \tilde{c}_{ij}$)	$(221,234,247,258)$	$(249,263,278,290)$	$(175,187,213,225)$
$\tilde{T}_j$	$(6,8,9,13)$	$(9,10,11,14)$	$(9,10,11,14)$

TABLE 9. Total Fuzzy Cost and Maximum Fuzzy Time for 1^{st} Iteration

$\tilde{C}(X_1^1) = (175, 187, 213, 225)$ is greater than total fuzzy cost $\tilde{C}(X_0^1)$ in incumbent solution. So, the incumbent solution will remain the same. The current and incumbent solutions are given in Table 10.

Current Solution (X_1^1, Y_1^1)			Incumbent Solution (X_0^1, Y_0^1)		
Variables x_{ij}'s and y_j's	$\tilde{C}(X_1^1)$	$\tilde{T}(X_1^1)$	Variables x_{ij}'s and y_j's	$\tilde{C}(X_0^1)$	$\tilde{T}(X_0^1)$
$y_3, y_1, y_7,$ $x_{11}, x_{27}, x_{37},$ x_{41}, x_{53}	$(175, 187$ $213, 225)$	$(9, 10,$ $11, 14)$	$y_2, y_3, y_1,$ $x_{12}, x_{21}, x_{32},$ x_{42}, x_{53}	$(131, 140$ $159, 170)$	$(6, 8,$ $9, 13)$

TABLE 10. Current and Incumbent Solutions During 1^{st} Iteration

Iteration No. 2. Since the combination of sites selected in 1^{st} iterative solution for setting up of warehouses are $(3, 1, 7)$. Thus, the two tabu lists are given in Table 11.

Tabu list for drop	Tabu list for add
$\{y_1, y_7\}$	$\{y_3\}$

TABLE 11. Two Tabu Lists in 2^{nd} Iteration of 1^{st} Non-dominated Solution

Similarly proceeding in the same manner as explained above, we have Table 12 showing all the iterative solutions for 1^{st} non-dominated solution. Since the iterative solutions (X_6^1, Y_6^1) and (X_2^1, Y_2^1) are identical, the 6^{th} iterative solution revisits the 2^{nd} iterative solution indicating that the computation for obtaining the 1^{st} efficient solution should be stopped. Note that (X_3^1, Y_3^1) being the most recent incumbent solution and is the best solution among all iterative solutions, so provides the 1^{st} efficient or non-dominated solution (X^1, Y^1) of the numerical problem.

Iterative solutions	Variables x_{ij}'s and y_j's in the order of appearance	Total fuzzy cost $\tilde{C}(X_r^1)$	Fuzzy time $\tilde{T}(X_r^1)$	Tabu list for drop rule	Tabu list for add rule	Incumbent solution
(X_0^1, Y_0^1) 0^{th} iterative solution	$x_{12}, x_{21}, x_{32},$ x_{42}, x_{53} y_2, y_3, y_1	$(131, 140,$ $159, 170)$	$(6, 8,$ $9, 13)$	$\{y_3, y_1\}$	$\{y_2\}$	(X_0^1, Y_0^1)
(X_1^1, Y_1^1) 1^{st} iterative solution	$x_{11}, x_{27}, x_{37},$ x_{41}, x_{53} y_3, y_1, y_7	$(175, 187,$ $213, 225)$	$(9, 10,$ $11, 14)$	$\{y_1, y_7\}$	$\{y_3\}$	(X_0^1, Y_0^1)
(X_2^1, Y_2^1) 2^{nd} iterative solution	$x_{12}, x_{27}, x_{32},$ x_{42}, x_{51} y_1, y_7, y_2	$(139, 157,$ $185, 199)$	$(9, 10,$ $11, 14)$	$\{y_7, y_2\}$	$\{y_1\}$	(X_0^1, Y_0^1)
(X_3^1, Y_3^1) 3^{rd} iterative solution	$x_{12}, x_{27}, x_{32},$ x_{42}, x_{55} y_7, y_2, y_5	$(94, 106,$ $134, 146)$	$(9, 10,$ $11, 14)$	$\{y_2, y_5\}$	$\{y_7\}$	(X_3^1, Y_3^1)
(X_4^1, Y_4^1) 4^{th} iterative solution	$x_{12}, x_{21}, x_{32},$ x_{42}, x_{55} y_2, y_5, y_1	$(151, 160,$ $179, 190)$	$(6, 8,$ $9, 13)$	$\{y_5, y_1\}$	$\{y_2\}$	(X_3^1, Y_3^1)
(X_5^1, Y_5^1) 5^{th} iterative solution	$x_{11}, x_{27}, x_{37},$ x_{45}, x_{55} y_5, y_1, y_7	$(167, 178,$ $202, 213)$	$(9, 10,$ $11, 14)$	$\{y_1, y_7\}$	$\{y_5\}$	(X_3^1, Y_3^1)
(X_6^1, Y_6^1) 6^{th} iterative solution	$x_{12}, x_{27}, x_{32},$ x_{42}, x_{51} y_1, y_7, y_2	$(139, 157,$ $185, 199)$	$(9, 10,$ $11, 14)$	$\{y_7, y_2\}$	$\{y_1\}$	(X_3^1, Y_3^1)

TABLE 12. The Iterative Solutions $(X_r^1, Y_r^1)(r = 0, 1, \dots, 6)$ for 1^{st} Non-dominated Solution

2^{nd} **Non-dominated Solution.** Since rank of $\tilde{T}(X^1)$ is 11. So, replace all $\tilde{t}_{ij}$'s by (L, L, L, L) (large positive fuzzy number) whose rank is ≥ 11.

Warehouse sites $(j) \rightarrow$ Ration shops $(i) \downarrow$	1	2	3	4	5	6	7
1	$(36, 38, 42, 44)$ $(5, 8, 9, 10)$	$(28, 29, 31, 32)$ $(1, 2, 2, 3)$	$(44, 49, 53, 54)$ (L, L, L, L)	$(117, 118, 121, 124)$ $(3, 6, 7, 8)$	$(175, 177, 183, 185)$ $(8, 9, 11, 12)$	$(165, 167, 169, 179)$ $(6, 8, 9, 13)$	$(143, 145, 153, 157)$ $(5, 6, 8, 9)$
2	$(68, 69, 70, 73)$ $(3, 6, 7, 8)$	$(73, 80, 82, 85)$ $(3, 6, 7, 8)$	$(128, 129, 130, 133)$ $(6, 8, 9, 13)$	$(128, 129, 130, 133)$ $(5, 6, 8, 9)$	$(165, 167, 169, 179)$ $(5, 8, 9, 10)$	$(133, 136, 144, 147)$ $(8, 9, 11, 12)$	$(11, 15, 25, 29)$ (L, L, L, L)
3	$(73, 80, 82, 85)$ $(5, 8, 9, 10)$	$(11, 15, 25, 29)$ $(6, 8, 9, 13)$	$(186, 190, 205, 209)$ $(3, 4, 5, 8)$	$(68, 69, 70, 73)$ $(5, 8, 9, 10)$	$(133, 136, 144, 147)$ (L, L, L, L)	$(128, 129, 130, 133)$ (L, L, L, L)	$(56, 58, 62, 64)$ $(8, 9, 11, 12)$
4	$(56, 58, 62, 64)$ (L, L, L, L)	$(8, 9, 11, 12)$ $(3, 6, 7, 8)$	$(68, 69, 70, 73)$ (L, L, L, L)	$(73, 80, 82, 85)$ (L, L, L, L)	$(28, 29, 31, 32)$ $(6, 8, 9, 13)$	$(156, 159, 160, 165)$ $(5, 8, 9, 10)$	$(199, 203, 217, 221)$ $(3, 6, 7, 8)$
5	$(81, 89, 93, 97)$ $(8, 9, 11, 12)$	$(89, 98, 104, 109)$ (L, L, L, L)	$(16, 18, 22, 24)$ $(5, 8, 9, 10)$	$(56, 58, 62, 64)$ (L, L, L, L)	$(36, 38, 42, 44)$ $(3, 6\ 7, 8)$	$(44, 49, 53, 54)$ $(2, 6, 7, 9)$	$(197, 205, 230, 248)$ (L, L, L, L)
Setup cost $(\tilde{b}_j)$	$(89, 98, 104, 109)$	$(275, 288, 309, 328)$	$(680, 690, 700, 730)$	$(712, 784, 832, 872)$	$(380, 392, 410, 418)$	$(178, 196, 208, 218)$	$(445, 490, 520, 545)$

TABLE 13. Tableau Representation of the Numerical Problem for 2^{nd} Non-dominated Solution

Proceeding in the same manner as done for 1^{st} non-dominated solution, we will get 2^{nd} non-dominated solution, as shown in Table 14.

2^{nd} Efficient solution	Variables (X^2, Y^2)	Total fuzzy cost of meeting requirement $(\tilde{C}(X^2))$	Total fuzzy time $(\tilde{T}(X^2))$
(X^2, Y^2)	$y_2, y_3, y_1,$ $x_{12}, x_{21}, x_{32},$ x_{42}, x_{53}	$(131, 140, 159, 170)$	$(6, 8, 9, 13)$

TABLE 14. 2^{nd} Non-dominated Solution of the Numerical Problem

As solved for 2^{nd} non-dominated solution, similarly solving for 3^{rd} and 4^{th} non-dominated solutions, we have the following non-dominated solutions shown in Table 15. It is found that when we solve for 5^{th} non-dominated solution, no improvement in fuzzy time is found. So, the procedure of finding efficient solutions stops here. So, the four fuzzy efficient solutions of our numerical example are given in the following Table 15.

Efficient solutions	Variables x_{ij}'s and y_j's	Total fuzzy cost of meeting requirement	Total fuzzy time
(X^1, Y^1)	$y_7, y_2, y_5,$ $x_{12}, x_{27}, x_{32},$ x_{42}, x_{55}	$(94, 106,$ $134, 146)$	$(9, 10,$ $11, 14)$
(X^2, Y^2)	$y_2, y_3, y_1,$ $x_{12}, x_{21}, x_{32},$ x_{42}, x_{53}	$(131, 140,$ $159, 170)$	$(6, 8,$ $9, 13)$
(X^3, Y^3)	$y_2, y_3, y_1,$ $x_{12}, x_{21}, x_{31},$ x_{42}, x_{53}	$(193, 205,$ $216, 226)$	$(5, 8,$ $9, 10)$
(X^4, Y^4)	$y_2, y_5, y_3,$ $x_{12}, x_{22}, x_{33},$ x_{42}, x_{55}	$(331, 346,$ $371, 392)$	$(3, 6,$ $7, 8)$

Table 15. Efficient Solutions of Numerical Problem

7. **Results and Discussions**

1. The proposed algorithm with tabu search gives four efficient solutions, but without tabu search gives three efficient solutions (last three solutions in Table 15). Therefore providing more choices for decision making.

Solutions obtained without Tabu search			Solutions obtained with Tabu search		
Variables x_{ij}'s and y_j's	Total fuzzy cost ($\tilde{C}$)	Fuzzy time ($\tilde{T}$)	Variables x_{ij}'s and y_j's	Total fuzzy cost ($\tilde{C}$)	Fuzzy time ($\tilde{T}$)
$y_2, y_3, y_1,$ $x_{12}, x_{21}, x_{32},$ x_{42}, x_{53}	$(131, 140,$ $159, 170)$	$(6, 8,$ $9, 13)$	$y_7, y_2, y_5,$ $x_{12}, x_{27}, x_{32},$ x_{42}, x_{55}	$(94, 106,$ $134, 146)$	$(9, 10,$ $11, 14)$
$y_2, y_3, y_1,$ $x_{12}, x_{21}, x_{31},$ x_{42}, x_{53}	$(193, 205,$ $216, 226)$	$(5, 8,$ $9, 10)$	$y_2, y_3, y_1,$ $x_{12}, x_{21}, x_{31},$ x_{42}, x_{53}	$(193, 205,$ $216, 226)$	$(5, 8,$ $9, 10)$
$y_2, y_5, y_3,$ $x_{12}, x_{22}, x_{33},$ x_{42}, x_{55}	$(331, 346,$ $371, 392)$	$(3, 6,$ $7, 8)$	$y_2, y_5, y_3,$ $x_{12}, x_{22}, x_{33},$ x_{42}, x_{55}	$(331, 346,$ $371, 392)$	$(3, 6,$ $7, 8)$

Table 16. Comparison Among Efficient Solutions with and without Tabu Search

2. From Table 16, it is clear that the incorporation of tabu search leads to better results than those obtained earlier without its incorporation.

Solutions obtained in crisp environment			Solutions obtained in fuzzy environment		
Variables x_{ij}'s and y_j's	Total cost (C)	Time (T)	Variables x_{ij}'s and y_j's	Total fuzzy cost ($\tilde{C}$)	Fuzzy time ($\tilde{T}$)
$y_7, y_2, y_5, x_{12}, x_{27}, x_{32}, x_{42}, x_{55}$	120	11	$y_7, y_2, y_5, x_{12}, x_{27}, x_{32}, x_{42}, x_{55}$	$(94, 106, 134, 146)$	$(9, 10, 11, 14)$
$y_2, y_3, y_1, x_{12}, x_{21}, x_{32}, x_{42}, x_{53}$	150	9	$y_2, y_3, y_1, x_{12}, x_{21}, x_{32}, x_{42}, x_{53}$	$(131, 140, 159, 170)$	$(6, 8, 9, 13)$
$y_2, y_3, y_1, x_{12}, x_{21}, x_{31}, x_{42}, x_{53}$	210	8	$y_2, y_3, y_1, x_{12}, x_{21}, x_{31}, x_{42}, x_{53}$	$(193, 205, 216, 226)$	$(5, 8, 9, 10)$
$y_2, y_5, y_3, x_{12}, x_{22}, x_{33}, x_{42}, x_{55}$	360	6	$y_2, y_5, y_3, x_{12}, x_{22}, x_{33}, x_{42}, x_{55}$	$(331, 346, 371, 392)$	$(3, 6, 7, 8)$

Table 17. Efficient Solutions of the Numerical Problem in Crisp and Fuzzy Environment

3. It is obvious from Table 17 that the results obtained by using the proposed approach can represent the solution in more realistic manner i.e. if there is no uncertainty about any parameter then the results of proposed approach will be the same as obtained in crisp environment. Thus, the algorithm proposed in fuzzy environment is a generalization of the existing algorithm in crisp environment [37].

8. **Conclusions**

In this paper a new algorithm is proposed for finding fuzzy efficient solutions of bi-objective warehouse problems occurring in real life situations. The algorithm is very easy to understand and simple to apply on real life problems and the obtained results are more flexible than existing approach [37].

References

[1] S. Abbasbandy and M. Alavi, *A method for solving fuzzy linear systems*, Iranian Journal of Fuzzy Systems, **2(2)** (2005), 37-43.

[2] T. Allahviranloo, F. H. Lotfi, M. K. Kiasary, N. A. Kiani and L. Alizadeh, *Solving fully fuzzy linear programming problem by the ranking function*, Applied Mathematical Sciences, **2(1)** (2008), 19-32.

[3] R. E. Bellman and L. A. Zadeh, *Decision making in a fuzzy environment*, Management Science, **17(4)** (1970), 141-164.

[4] U. Bhattacharya, J. R. Rao and R. N. Tiwari, *Fuzzy multi-criteria facility location problem*, Fuzzy Sets and Systems, **51(3)** (1992), 277-287.

[5] U. Bhattacharya, J. R. Rao and R. N. Tiwari, *Bi-criteria multi facility location problem in Fuzzy environment*, Fuzzy Sets and Systems, **56(2)** (1993), 145-153.

[6] J. Buckley and T. Feuring, *Evolutionary algorithm solution to fuzzy problems: fuzzy linear programming*, Fuzzy Sets and Systems, **109(1)** (2000), 35-53.

[7] R. Caballero, M. Gonzalez, F. M. Guerrero, J. Molina and C. Paralera, *Solving a multiobjective location routing problem with a metaheuristic based on tabu search: application to a real case in Andalusia*, European Journal of Operational Research, **177(3)** (2007), 1751-1763.

[8] J. M. Cadenas and J. L. Verdegay, *A primer on fuzzy optimization models and methods*, Iranian Journal of Fuzzy Systems, **3(1)** (2006), 1-21.
[9] L. M. Campos Ibanez and A. Gonzalez Munoz, *A subjective approach for ranking fuzzy number*, Fuzzy Sets and Systems, **29(2)** (1989), 145-153.
[10] L. Campos and J. L. Verdegay, *Linear programming problems and ranking of fuzzy numbers*, Fuzzy Sets and Systems, **32(1)** (1989), 1-11.
[11] S. P. Chen and Y. J. Hsueh, *A simple approach to fuzzy critical path analysis in project networks*, Applied Mathematical Modelling, **32(7)** (2008), 1289-1297.
[12] T. C. Chu, *Facility location selection using fuzzy TOPSIS under group decisions*, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, **10(6)** (2002), 687-702.
[13] L. Cooper, *Location-allocation problems*, Operations Research, **11(3)** (1963), 331-343.
[14] J. Darzentas, *A discrete location model with fuzzy accessibility measures*, Fuzzy Sets and Systems, **23(1)** (1987), 149-154.
[15] R. Z. Farahani, M. SteadieSeifi and N. Asgari, *Multiple criteria facility location problems: a survey*, Applied Mathematical Modelling, **34(7)** (2010), 1689-1709.
[16] K. Ganesan and P. Veeramani, *Fuzzy linear programs with trapezoidal fuzzy Numbers*, Annals of Operations Research, **143(1)** (2006), 305-315.
[17] M. Gen and A. Syarif, *Hybrid genetic algorithm for multi-time period production/distribution planning*, Computers and Industrial Engineering, **48(4)** (2005), 799-809.
[18] F. Glover, *Tabu search-part I*, ORSA Journal on Computing, **1(3)** (1989), 190-206.
[19] F. Glover, *Tabu search-part II*, ORSA Journal on Computing, **2(1)** (1990), 4-32.
[20] T. S. Hale and C. R. Moberg, *Location science research: a review*, Annals of Operations Research, **123(1-4)** (2003), 21-35.
[21] C. Kahraman, D. Ruan and I. Dogan, *Fuzzy group decision-making for facility location selection*, Information Sciences, **157** (2003), 135-153.
[22] A. Kaufmann and M. M. Gupta, *Introduction to fuzzy arithmetics: theory and applications*, New York, Van Nostrand Reinhold, 1991.
[23] K. Kim and K. S. Park, *Ranking fuzzy number with index of optimism*, Fuzzy Sets and Systems, **35(2)** (1990), 143-150.
[24] C. K. Y. Lin and R. C. W. Kwok, *Multi-objective metaheuristics for a location-routing Problem with multiple use of vehicles on real data and simulated data*, European Journal of Operational Research, **175(3)** (2006), 1833-1849.
[25] F. T. Lin, *Time-cost tradeoff in fuzzy critical path analysis based on* $(1-\alpha)\times 100\%$ *confidence-interval estimates*, IEEE International Conference on Systems, Man and Cybernetics, (2008), 601-606.
[26] T. S. Liou and M. J. Wang, *Ranking fuzzy numbers with integral value*, Fuzzy Sets and Systems, **50(3)** (1992), 247-255.
[27] N. Mahdavi-Amiri and S. H. Nasseri, *Duality in fuzzy number linear programming by the use of a certain linear ranking function*, Applied Mathematics and Computation, **180(1)** (2006), 206-216.
[28] G. S. Mahapatra and T. K. Roy, *Fuzzy multi-objective mathematical programming on reliability optimization model*, Applied Mathematics and Computation, **174(1)** (2006), 643-659.
[29] H. R. Maleki and M. Mashinchi, *Multiobjective geometric programming with fuzzy parameters*, International Journal of Information Science and Technology, **5(2)** (2007), 35-45.
[30] H. R. Maleki, M. Tata and M. Mashinchi, *Linear programming with fuzzy variables*, Fuzzy Sets and Systems, **109(1)** (2000), 21-33.
[31] M. T. Melo, S. Nickel and F. Saldanha-da-Gama, *Facility location and supply chain management-a review*, European Journal of Operational Research, **196(2)** (2009), 401-412.
[32] H. Mishmast Nehi, H. R. Maleki and M. Mashinchi, *Solving fuzzy number linear programming problem by lexicographic ranking function*, Italian J. Pure Appl. Math., **16** (2004), 9-20.
[33] R. Narasimhan, *A fuzzy subset characterization of a site-selection problem*, Decision Sciences, **10(4)** (1979), 618-628.

[34] A. A. Noora and P. Karami, *Ranking functions and its application to fuzzy DEA*, International Mathematical Forum, **3(30)** (2008), 1469-1480.
[35] S. Okada and T. Soper, *A shortest path problem on a network with fuzzy arc lengths*, Fuzzy Sets and Systems, **109(1)** (2000), 129-140.
[36] P. Pandian and G. Natarajan, *A new algorithm for finding a fuzzy optimal solution for fuzzy transportation problems*, Applied Mathematical Sciences, **4(2)** (2010), 79-90.
[37] S. Prakash, M. K. Sharma and A. Singh, *Selection of warehouse sites for clustering ration shops to them with two objectives through a heuristic algorithm incorporating tabu search*, Opsearch, **46(4)** (2010), 449-460.
[38] T. Uno and H. Katagiri, *Single and multi-objective defensive location problems on a network*, European Journal of Operational Research, **188(1)** (2008), 76-84.
[39] J. R. Yu and T. H. Wei, *Solving the fuzzy shortest path problem by using a linear programming*, Journal of the Chinese Institute of Industrial Engineers, **24(5)** (2007), 360-365.
[40] L. A. Zadeh, *Fuzzy sets*, Information and Control, **8(3)** (1965), 338-353.
[41] H. J. Zimmermann, *Fuzzy programming and linear programming with several objective functions*, Fuzzy Sets and Systems, **1(1)** (1978), 45-55.

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