

## FUZZY SOFT MATRIX THEORY AND ITS APPLICATION IN DECISION MAKING

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ABSTRACT. In this work, we define fuzzy soft ( $fs$ ) matrices and their operations which are more functional to make theoretical studies in the  $fs$ -set theory. We then define products of  $fs$ -matrices and study their properties. We finally construct a  $fs$ -max-min decision making method which can be successfully applied to the problems that contain uncertainties.

### 1. Introduction

Soft set theory [33] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. After presentation of the operations of soft sets [29], the properties and applications on the soft set theory have been studied increasingly [4, 12, 13, 25, 26, 30, 31, 35, 37, 38, 40, 46, 47, 48, 51]. The algebraic structure of soft set theory has also been studied in more detail [1, 3, 5, 11, 14, 15, 16, 17, 18, 19, 20, 22, 36, 41, 44]. In recent years, by embedding the ideas of fuzzy sets [52] many interesting applications of soft set theory have been done [2, 8, 9, 10, 21, 23, 24, 27, 28, 32, 34, 39, 42, 43, 45, 49, 50].

To develop the soft set theory, operations of the soft sets are redefined to improve several new results and *uni-int* decision making method is constructed by using these new operations [6]. To make easy computation with the operations of soft sets, the soft matrix theory is presented and soft *max-min* decision making method is set up [7]. These decision making methods are more practical and can be successfully applied to many problems that contain uncertainties.

In [9], a fuzzy soft ( $fs$ ) set theory is defined. It allows constructing more efficient decision making method. In this paper, we first define  $fs$ -matrices which are representation of the  $fs$ -sets. This representation has several advantages. It is easy to store and manipulate matrices and hence the  $fs$ -sets represented by them in a computer. Here, we also construct a  $fs$ -decision making method which is more practical and can be successfully applied to many problems. We finally give an example which shows that the method successfully works.

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## 2. Fuzzy Soft Matrices

In this section, we define *fs*-matrices which are representations of the *fs*-sets. This style of representation is useful for storing a soft set in a computer memory. The operations can be presented by the matrices which are very useful and convenient for application.

From now on, a set of all fuzzy sets over  $U$  will be denoted by  $F(U)$ .  $\Gamma_A$ ,  $\Gamma_B$ ,  $\Gamma_C, \dots$ , etc. and  $\gamma_A$ ,  $\gamma_B$ ,  $\gamma_C, \dots$ , etc. will be used for *fs*-sets and their fuzzy approximate functions, respectively.

**Definition 2.1.** [9] Let  $U$  be an initial universe,  $E$  be the set of all parameters,  $A \subseteq E$  and  $\gamma_A(x)$  be a fuzzy set over  $U$  for all  $x \in E$ . Then, an *fs*-set  $\Gamma_A$  over  $U$  is a set defined by a function  $\gamma_A$  representing a mapping

$$\gamma_A : E \rightarrow F(U) \text{ such that } \gamma_A(x) = \emptyset \text{ if } x \notin A$$

Here,  $\gamma_A$  is called fuzzy approximate function of the *fs*-set  $\Gamma_A$ , the value  $\gamma_A(x)$  is a fuzzy set called  $x$ -element of the *fs*-set for all  $x \in E$ , and  $\emptyset$  is the null fuzzy set. Thus, an *fs*-set  $\Gamma_A$  over  $U$  can be represented by the set of ordered pairs

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in F(U)\}.$$

Note that from now on, the sets of all *fs*-sets over  $U$  will be denoted by  $FS(U)$ .

**Example 2.2.** Assume that  $U = \{u_1, u_2, u_3, u_4, u_5\}$  is a universal set and  $E = \{x_1, x_2, x_3, x_4\}$  is a set of all parameters.

If  $A = \{x_2, x_3, x_4\}$ ,  $\gamma_A(x_2) = \{0.5/u_2, 0.8/u_4\}$ ,  $\gamma_A(x_3) = \emptyset$  and  $\gamma_A(x_4) = U$ , then the *fs*-set  $\Gamma_A$  is written by  $\Gamma_A = \{(x_2, \{0.5/u_2, 0.8/u_4\}), (x_4, U)\}$ .

**Definition 2.3.** Let  $\Gamma_A \in FS(U)$ . Then a *fuzzy relation form* of  $\Gamma_A$  is defined by

$$R_A = \{(\mu_{R_A}(u, x)/(u, x)) : (u, x) \in U \times E\},$$

where the membership function of  $\mu_{R_A}$  is written by

$$\mu_{R_A} : U \times E \rightarrow [0, 1], \quad \mu_{R_A}(u, x) = \mu_{\gamma_A(x)}(u).$$

If  $U = \{u_1, u_2, \dots, u_m\}$ ,  $E = \{x_1, x_2, \dots, x_n\}$  and  $A \subseteq E$ , then the  $R_A$  can be presented by a table as in the following form

$R_A$	$x_1$	$x_2$	$\dots$	$x_n$
$u_1$	$\mu_{R_A}(u_1, x_1)$	$\mu_{R_A}(u_1, x_2)$	$\dots$	$\mu_{R_A}(u_1, x_n)$
$u_2$	$\mu_{R_A}(u_2, x_1)$	$\mu_{R_A}(u_2, x_2)$	$\dots$	$\mu_{R_A}(u_2, x_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$u_m$	$\mu_{R_A}(u_m, x_1)$	$\mu_{R_A}(u_m, x_2)$	$\dots$	$\mu_{R_A}(u_m, x_n)$

If  $a_{ij} = \mu_{R_A}(u_i, x_j)$ , we can define a matrix

$$[a_{ij}]_{m \times n} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

which is called an  $m \times n$   $f$ s-matrix of the  $f$ s-set  $\Gamma_A$  over  $U$ .

According to this definition, an  $f$ s-set  $\Gamma_A$  is uniquely characterized by the matrix  $[a_{ij}]_{m \times n}$ . It means that an  $f$ s-set  $\Gamma_A$  is formally equal to its soft matrix  $[a_{ij}]_{m \times n}$ . Therefore, we shall identify any  $f$ s-set with its  $f$ s-matrix and use these two concepts as interchangeable.

The set of all  $m \times n$   $f$ s-matrices over  $U$  will be denoted by  $FSM_{m \times n}$ . From now on we shall delete the subscript  $m \times n$  of  $[a_{ij}]_{m \times n}$ , we use  $[a_{ij}]$  instead of  $[a_{ij}]_{m \times n}$ , since  $[a_{ij}] \in FSM_{m \times n}$  means that  $[a_{ij}]$  is an  $m \times n$   $f$ s-matrix for  $i = 1, 2, \dots, m$  and  $j = 1, 2, \dots, n$ .

**Example 2.4.** Let us consider Example 2.2. Then the relation form of  $\Gamma_A$  is written by

$$R_A = \{0.5/(u_2, x_2), 0.8/(u_4, x_2), 1/(u_1, x_4), 1/(u_2, x_4), 1/(u_3, x_4), 1/(u_4, x_4), 1/(u_5, x_4)\}$$

Hence, the  $f$ s-matrix  $[a_{ij}]$  is written by

$$[a_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0.5 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0.8 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Definition 2.5.** Let  $[a_{ij}] \in FSM_{m \times n}$ . Then  $[a_{ij}]$  is called

- (1) a zero  $f$ s-matrix, denoted by  $[0]$ , if  $a_{ij} = 0$  for all  $i$  and  $j$ .
- (2) an  $A$ -universal  $f$ s-matrix, denoted by  $[\tilde{a}_{ij}]$ , if  $a_{ij} = 1$  for all  $j \in I_A = \{j : x_j \in A\}$  and  $i$ .
- (3) a universal  $f$ s-matrix, denoted by  $[1]$ , if  $a_{ij} = 1$  for all  $i$  and  $j$ .

**Example 2.6.** Assume that  $U = \{u_1, u_2, u_3, u_4, u_5\}$  is a universal set,  $E = \{x_1, x_2, x_3, x_4\}$  is a set of parameters,  $A \subseteq E$ ,  $\gamma_A(x)$  is a fuzzy set over  $U$  for all  $x \in E$  and  $[a_{ij}], [b_{ij}], [c_{ij}] \in FSM_{5 \times 4}$ .

If  $A = \{x_1, x_3\}$  and  $\gamma_A(x_1) = \emptyset$ ,  $\gamma_A(x_3) = \emptyset$ , then  $[a_{ij}] = [0]$  is a zero  $f$ s-matrix written by

$$[0] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

If  $B = \{x_1, x_2\}$  and  $\gamma_B(x_1) = U$ ,  $\gamma_B(x_2) = U$ , then  $[b_{ij}] = [\tilde{b}_{ij}]$  is a  $B$ -universal  $fs$ -matrix written by

$$[\tilde{b}_{ij}] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

If  $C = E$ , and  $\gamma_C(x_i) = U$  for all  $x_i \in C$ ,  $i = 1, 2, 3, 4$ , then  $[c_{ij}] = [1]$  is a universal  $fs$ -matrix written by

$$[1] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

**Definition 2.7.** Let  $[a_{ij}], [b_{ij}] \in FSM_{m \times n}$ . Then

- (1)  $[a_{ij}]$  is a  $fs$ -submatrix of  $[b_{ij}]$ , denoted by  $[a_{ij}] \tilde{\subseteq} [b_{ij}]$ , if  $a_{ij} \leq b_{ij}$  for all  $i$  and  $j$ .
- (2)  $[a_{ij}]$  is a proper  $fs$ -submatrix of  $[b_{ij}]$ , denoted by  $[a_{ij}] \tilde{\subset} [b_{ij}]$ , if  $a_{ij} \leq b_{ij}$  for all  $i$  and  $j$  and for at least one term  $a_{ij} < b_{ij}$ .
- (3)  $[a_{ij}]$  and  $[b_{ij}]$  are  $fs$ -equal matrices, denoted by  $[a_{ij}] = [b_{ij}]$ , if  $a_{ij} = b_{ij}$  for all  $i$  and  $j$ .

**Definition 2.8.** Let  $[a_{ij}], [b_{ij}] \in FSM_{m \times n}$ . Then the  $fs$ -matrix  $[c_{ij}]$  is called

- (1) union of  $[a_{ij}]$  and  $[b_{ij}]$ , denoted  $[a_{ij}] \tilde{\cup} [b_{ij}]$ , if  $c_{ij} = \max\{a_{ij}, b_{ij}\}$  for all  $i$  and  $j$ .
- (2) intersection of  $[a_{ij}]$  and  $[b_{ij}]$ , denoted  $[a_{ij}] \tilde{\cap} [b_{ij}]$ , if  $c_{ij} = \min\{a_{ij}, b_{ij}\}$  for all  $i$  and  $j$ .
- (3) complement of  $[a_{ij}]$ , denoted by  $[a_{ij}]^\circ$ , if  $c_{ij} = 1 - a_{ij}$  for all  $i$  and  $j$ .

**Definition 2.9.** Let  $[a_{ij}], [b_{ij}] \in FSM_{m \times n}$ . Then  $[a_{ij}]$  and  $[b_{ij}]$  are disjoint, if  $[a_{ij}] \tilde{\cap} [b_{ij}] = [0]$  for all  $i$  and  $j$ .

**Example 2.10.** Assume that

$$[a_{ij}] = \begin{bmatrix} 0 & 0.6 & 0 & 0 \\ 0.1 & 0 & 1 & 0 \\ 0 & 0.3 & 0.8 & 0 \\ 0.7 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \quad [b_{ij}] = \begin{bmatrix} 0 & 0 & 0.7 & 0.4 \\ 0 & 0.2 & 0 & 1 \\ 0 & 0 & 0 & 0.9 \\ 0 & 0 & 0.5 & 1 \\ 0 & 0 & 0 & 0.3 \end{bmatrix}$$

Then,  $[a_{ij}] \tilde{\cap} [b_{ij}] = [0]$  and

$$[a_{ij}] \tilde{\cup} [b_{ij}] = \begin{bmatrix} 0 & 0.6 & 0.7 & 0.4 \\ 0.1 & 0.2 & 1 & 1 \\ 0 & 0.3 & 0.8 & 0.9 \\ 0.7 & 0 & 0.5 & 1 \\ 0 & 1 & 0 & 0.3 \end{bmatrix}, [a_{ij}]^\circ = \begin{bmatrix} 1 & 0.4 & 1 & 1 \\ 0.9 & 1 & 0 & 1 \\ 1 & 0.7 & 0.2 & 1 \\ 0.3 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

**Proposition 2.11.** *Let  $[a_{ij}] \in FSM_{m \times n}$ . Then*

- (1)  $[[a_{ij}]^\circ]^\circ = [a_{ij}]$
- (2)  $[0]^\circ = [1]$

**Proposition 2.12.** *Let  $[a_{ij}], [b_{ij}] \in FSM_{m \times n}$ . Then*

- (1)  $[a_{ij}] \tilde{\subseteq} [1]$
- (2)  $[0] \tilde{\subseteq} [a_{ij}]$
- (3)  $[a_{ij}] \tilde{\subseteq} [a_{ij}]$
- (4)  $[a_{ij}] \tilde{\subseteq} [b_{ij}]$  and  $[b_{ij}] \tilde{\subseteq} [c_{ij}] \Rightarrow [a_{ij}] \tilde{\subseteq} [c_{ij}]$
- (5)  $[a_{ij}] \tilde{\subseteq} [b_{ij}] \Leftrightarrow [a_{ij}] \tilde{\cap} [b_{ij}] = [a_{ij}] \Leftrightarrow [a_{ij}] \tilde{\cup} [b_{ij}] = [b_{ij}]$

**Proposition 2.13.** *Let  $[a_{ij}], [b_{ij}], [c_{ij}] \in FSM_{m \times n}$ . Then*

- (1)  $[a_{ij}] = [b_{ij}]$  and  $[b_{ij}] = [c_{ij}] \Leftrightarrow [a_{ij}] = [c_{ij}]$
- (2)  $[a_{ij}] \tilde{\subseteq} [b_{ij}]$  and  $[b_{ij}] \tilde{\subseteq} [a_{ij}] \Leftrightarrow [a_{ij}] = [b_{ij}]$

**Proposition 2.14.** *Let  $[a_{ij}], [b_{ij}], [c_{ij}] \in FSM_{m \times n}$ . Then*

- (1)  $[a_{ij}] \tilde{\cap} [b_{ij}] = [b_{ij}] \tilde{\cap} [a_{ij}]$
- (2)  $[a_{ij}] \tilde{\cup} [b_{ij}] = [b_{ij}] \tilde{\cup} [a_{ij}]$
- (3)  $([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cap} [c_{ij}] = [a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cap} [c_{ij}])$
- (4)  $([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cup} [c_{ij}] = [a_{ij}] \tilde{\cup} ([b_{ij}] \tilde{\cup} [c_{ij}])$
- (5)  $[a_{ij}] \tilde{\cap} ([b_{ij}] \tilde{\cup} [c_{ij}]) = ([a_{ij}] \tilde{\cap} [b_{ij}]) \tilde{\cup} ([a_{ij}] \tilde{\cap} [c_{ij}])$
- (6)  $[a_{ij}] \tilde{\cup} ([b_{ij}] \tilde{\cap} [c_{ij}]) = ([a_{ij}] \tilde{\cup} [b_{ij}]) \tilde{\cap} ([a_{ij}] \tilde{\cup} [c_{ij}])$

Note that,  $[a_{ij}] \tilde{\cap} [a_{ij}]^\circ \neq [0]$  and  $[a_{ij}] \tilde{\cup} [a_{ij}]^\circ \neq [1]$

**Proposition 2.15.** *Let  $[a_{ij}], [b_{ij}] \in FSM_{m \times n}$ . Then De Morgan's laws are valid*

- (1)  $([a_{ij}] \tilde{\cup} [b_{ij}])^\circ = [a_{ij}]^\circ \tilde{\cap} [b_{ij}]^\circ$
- (2)  $([a_{ij}] \tilde{\cap} [b_{ij}])^\circ = [a_{ij}]^\circ \tilde{\cup} [b_{ij}]^\circ$

*Proof.* For all  $i$  and  $j$ ,

$$\begin{aligned} (1) ([a_{ij}] \tilde{\cup} [b_{ij}])^\circ &= [\max\{a_{ij}, b_{ij}\}]^\circ \\ &= [1 - \max\{a_{ij}, b_{ij}\}] \\ &= [\min\{1 - a_{ij}, 1 - b_{ij}\}] \\ &= [a_{ij}]^\circ \tilde{\cap} [b_{ij}]^\circ \end{aligned}$$

(2) It can be proved similarly.

□

### 3. Products of $f$ s-Matrices

In this section, we define four special products of  $f$ s-matrices to construct  $f$ s-decision making methods.

**Definition 3.1.** Let  $[a_{ij}], [b_{ik}] \in FSM_{m \times n}$ . Then *And*-product of  $[a_{ij}]$  and  $[b_{ik}]$  is defined by

$$\wedge : FSM_{m \times n} \times FSM_{m \times n} \rightarrow FSM_{m \times n^2}, \quad [a_{ij}] \wedge [b_{ik}] = [c_{ip}]$$

where  $c_{ip} = \min\{a_{ij}, b_{ik}\}$  such that  $p = n(j-1) + k$ .

**Definition 3.2.** Let  $[a_{ij}], [b_{ik}] \in FSM_{m \times n}$ . Then *Or*-product of  $[a_{ij}]$  and  $[b_{ik}]$  is defined by

$$\vee : FSM_{m \times n} \times FSM_{m \times n} \rightarrow FSM_{m \times n^2}, \quad [a_{ij}] \vee [b_{ik}] = [c_{ip}]$$

where  $c_{ip} = \max\{a_{ij}, b_{ik}\}$  such that  $p = n(j-1) + k$ .

**Definition 3.3.** Let  $[a_{ij}], [b_{ik}] \in FSM_{m \times n}$ . Then *And-Not*-product of  $[a_{ij}]$  and  $[b_{ik}]$  is defined by

$$\bar{\wedge} : FSM_{m \times n} \times FSM_{m \times n} \rightarrow FSM_{m \times n^2}, \quad [a_{ij}] \bar{\wedge} [b_{ik}] = [c_{ip}]$$

where  $c_{ip} = \min\{a_{ij}, 1 - b_{ik}\}$  such that  $p = n(j-1) + k$ .

**Definition 3.4.** Let  $[a_{ij}], [b_{ik}] \in FSM_{m \times n}$ . Then *Or-Not*-product of  $[a_{ij}]$  and  $[b_{ik}]$  is defined by

$$\bar{\vee} : FSM_{m \times n} \times FSM_{m \times n} \rightarrow FSM_{m \times n^2}, \quad [a_{ij}] \bar{\vee} [b_{ik}] = [c_{ip}]$$

where  $c_{ip} = \max\{a_{ij}, 1 - b_{ik}\}$  such that  $p = n(j-1) + k$ .

**Example 3.5.** Assume that  $[a_{ij}], [b_{ik}] \in FSM_{2 \times 3}$  are given as follows

$$[a_{ij}] = \begin{bmatrix} 0 & 0 & 0.3 \\ 0 & 1 & 0.7 \end{bmatrix}, \quad [b_{ik}] = \begin{bmatrix} 0.5 & 0 & 0.2 \\ 0.2 & 0 & 0 \end{bmatrix}.$$

To calculate  $[a_{ij}] \wedge [b_{ik}] = [c_{ip}]$ , we have to find  $c_{ip}$  for all  $i = 1, 2$  and  $p = 1, 2, \dots, 9$ . Let us find  $c_{17}$ . Since  $n = 3$ ,  $i = 1$  and  $p = 7$ , we get  $j = 3$  and  $k = 1$  from  $7 = 3(j-1) + k$ . Hence  $c_{17} = \min\{a_{13}, b_{11}\} = \min\{0.3, 0.5\} = 0.3$ . If the other entries of  $[c_{ip}]$  can be found similarly, then we can obtain the matrix as follows;

$$[a_{ij}] \wedge [b_{ik}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0.2 \\ 0 & 0 & 0 & 0.2 & 0 & 0 & 0.2 & 0 & 0 \end{bmatrix}$$

Similarly, we can also find products  $[a_{ij}] \vee [b_{ik}]$ ,  $[a_{ij}] \bar{\wedge} [b_{ik}]$  and  $[a_{ij}] \bar{\vee} [b_{ik}]$ .

Note that the commutativity is not valid for the products of  $f$ s-matrices.

**Proposition 3.6.** Let  $[a_{ij}], [b_{ik}] \in FSM_{m \times n}$ . Then the following De Morgan's types of results are true.

- (1)  $([a_{ij}] \vee [b_{ik}])^\circ = [a_{ij}]^\circ \wedge [b_{ik}]^\circ$
- (2)  $([a_{ij}] \wedge [b_{ik}])^\circ = [a_{ij}]^\circ \vee [b_{ik}]^\circ$
- (3)  $([a_{ij}] \underline{\vee} [b_{ik}])^\circ = [a_{ij}]^\circ \overline{\wedge} [b_{ik}]^\circ$
- (4)  $([a_{ij}] \overline{\wedge} [b_{ik}])^\circ = [a_{ij}]^\circ \underline{\vee} [b_{ik}]^\circ$

#### 4. *fs*-Max-Min Decision Making

In this section, we construct an *fs*-max-min decision making (*FSMmDM*) method by using *fs*-max-min decision function which is also defined here. The method selects optimum alternatives from the set of the alternatives.

**Definition 4.1.** Let  $[c_{ip}] \in FSM_{m \times n^2}$ ,  $I_k = \{p : \exists i, c_{ip} \neq 0, (k-1)n < p \leq kn\}$  for all  $k \in I = \{1, 2, \dots, n\}$ . Then *fs*-max-min decision function, denoted *Mm*, is defined as follows

$$Mm : FSM_{m \times n^2} \rightarrow FSM_{m \times 1}, \quad Mm[c_{ip}] = [d_{i1}] = [\max_k \{t_{ik}\}]$$

where

$$t_{ik} = \begin{cases} \min_{p \in I_k} \{c_{ip}\}, & \text{if } I_k \neq \emptyset, \\ 0, & \text{if } I_k = \emptyset. \end{cases}$$

The one column *fs*-matrix  $Mm[c_{ip}]$  is called max-min decision *fs*-matrix.

**Definition 4.2.** Let  $U = \{u_1, u_2, \dots, u_m\}$  be an initial universe and  $Mm[c_{ip}] = [d_{i1}]$ . Then a subset of  $U$  can be obtained by using  $[d_{i1}]$  as in the following way

$$opt_{[d_{i1}]}(U) = \{d_{i1}/u_i : u_i \in U, d_{i1} \neq 0\}$$

which is called an optimum fuzzy set on  $U$ .

Now, using definitions we can construct a *FSMmDM* method by the following algorithm.

- Step 1:** choose feasible subsets of the set of parameters,
- Step 2:** construct the *fs*-matrix for each set of parameters,
- Step 3:** find a convenient product of the *fs*-matrices,
- Step 4:** find a max-min decision *fs*-matrix,
- Step 5:** find an optimum fuzzy set on  $U$ .

Note that, by the similar way, we can define *fs*-min-max, *fs*-min-min and *fs*-max-max decision making methods which may be denoted by (*FSmMDM*), (*FSmmDM*), (*FSMMDM*), respectively. One of them may be useful than the others according to the type of the problems.

#### 5. Applications

Assume that a real estate agent has a set of different types of houses  $U = \{u_1, u_2, u_3, u_4, u_5\}$  which may be characterized by a set of parameters  $E = \{x_1, x_2, x_3, x_4\}$ . For  $j = 1, 2, 3, 4$  the parameters  $x_j$  stand for “in good location”, “cheap”, “modern”, “large”, respectively. Then we can give the following examples.

**Example 5.1.** Suppose that a married couple, Mr. X and Mrs. X, come to the real estate agent to buy a house. If each partner has to consider their own set of parameters, then we select a house on the basis of the sets of partners' parameters by using the *FSMmDM* as follows.

Assume that  $U = \{u_1, u_2, u_3, u_4, u_5\}$  is a universal set and  $E = \{x_1, x_2, x_3, x_4\}$  is a set of all parameters.

**Step 1:** First, Mr. X and Mrs. X have to choose the sets of their parameters,  $A = \{x_2, x_3, x_4\}$  and  $B = \{x_1, x_3, x_4\}$ , respectively.

**Step 2:** Then we can write the following *fs*-matrices which are constructed according to their parameters.

$$[a_{ij}] = \begin{bmatrix} 0 & 0 & 0.2 & 0.4 \\ 0 & 0.6 & 0.9 & 0.4 \\ 0 & 0.8 & 0.7 & 0.5 \\ 0 & 0.5 & 0 & 0 \\ 0 & 1 & 0 & 0.8 \end{bmatrix} \quad [b_{ik}] = \begin{bmatrix} 1 & 0 & 0.9 & 0.7 \\ 0.2 & 0 & 0 & 0.9 \\ 0.7 & 0 & 0.4 & 0.3 \\ 0 & 0 & 0.5 & 0.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Step 3:** Now, we can find a product of the *fs*-matrices  $[a_{ij}]$  and  $[b_{ik}]$  by using *And*-product as follows

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0.2 & 0.2 & 0.4 & 0 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0.6 & 0.2 & 0 & 0 & 0.9 & 0.2 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 0 & 0.7 & 0 & 0.4 & 0.3 & 0.7 & 0 & 0.4 & 0.3 & 0.5 & 0 & 0.4 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8 \end{bmatrix}$$

Here, we use *And*-product since both Mr. X and Mrs. X's choices have to be considered.

**Step 4:** To calculate  $Mm([a_{ij}] \wedge [b_{ik}]) = [d_{i1}]$ , we have to find  $d_{i1}$  for all  $i \in \{1, 2, 3, 4, 5\}$ . To demonstrate, let us find  $d_{31}$ . Since  $i = 1$  and  $k \in \{1, 2, 3, 4\}$ ,

$$d_{31} = \max_k \{t_{3k}\} = \max\{t_{31}, t_{32}, t_{33}, t_{34}\}$$

Here, we have to find  $t_{3k}$  for all  $k \in \{1, 2, 3, 4\}$ . To demonstrate, let us find  $t_{31}$  and  $t_{32}$ .  $I_1 = \{p : c_{ip} \neq 0, 0 < p \leq 4\} = \emptyset$  for  $k = 1$  and  $n = 4$  and  $I_2 = \{p : c_{ip} \neq 0, 4 < p \leq 8\} = \{5, 7, 8\}$  for  $k = 2$  and  $n = 4$ . Hence  $t_{31} = 0$  and

$$t_{32} = \min\{c_{35}, c_{37}, c_{38}\} = \min\{0.7, 0.4, 0.3\} = 0.3$$

Similarly, we can find as  $t_{33} = 0.3$  and  $t_{34} = 0.3$ . Thus,

$$d_{31} = \max\{0.0, 0.3, 0.3, 0.3\} = 0.3$$

Similarly we can find  $d_{11} = 0.4$ ,  $d_{21} = 0.0$ ,  $d_{41} = 0.0$  and  $d_{51} = 0.0$ . Finally, we can obtain the *fs*-max-min decision *fs*-matrix as



$$Mm([a_{ij}] \wedge [b_{ik}]) = [d_{i1}] = \begin{bmatrix} 0.4 \\ 0 \\ 0.3 \\ 0 \\ 0 \end{bmatrix}$$

**Step 5:** Finally, we can find an optimum fuzzy set on  $U$  according to  $Mm([a_{ij}] \wedge [b_{ik}])$

$$opt_{Mm([a_{ij}] \wedge [b_{ik}])}(U) = \{0.4/u_1, 0.3/u_3\}$$

where  $u_1$  is an optimum house to buy for Mr. X and Mrs. X.

Similarly, we can also use products  $[a_{ij}] \vee [b_{ik}]$ ,  $[a_{ij}] \bar{\wedge} [b_{ik}]$  and  $[a_{ij}] \bar{\vee} [b_{ik}]$  for the other convenient problems.

## 6. Conclusion

The  $fs$ -set theory is being applied to many fields varying from theoretical to practical. In this paper, we define  $fs$ -matrices which are matrix representation of the  $fs$ -sets. We then define the set-theoretic operations of  $fs$ -matrices which are more functional to improve several new results. Afterwards, we construct a  $fs$ -decision making model on the  $fs$ -set theory. This new decision making method depends on the ideas of fuzzy and soft sets. Its main idea is similar to the decision making method, given in [7], which only depends on soft sets. Therefore, this method is more feasible than the others, because of its fuzziness. Finally, we give an application for a real estate agent to choose an optimal house.

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