FUZZY SOFT MATRIX THEORY AND ITS APPLICATION IN DECISION MAKING

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Abstract. In this work, we define fuzzy soft (fs) matrices and their operations which are more functional to make theoretical studies in the fs-set theory. We then define products of fs-matrices and study their properties. We finally construct a fs-*max-min* decision making method which can be successfully applied to the problems that contain uncertainties.

1. Introduction

Soft set theory [33] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. After presentation of the operations of soft sets [29], the properties and applications on the soft set theory have been studied increasingly [4, 12, 13, 25, 26, 30, 31, 35, 37, 38, 40, 46, 47, 48, 51]. The algebraic structure of soft set theory has also been studied in more detail [1, 3, 5, 11, 14, 15, 16, 17, 18, 19, 20, 22, 36, 41, 44]. In recent years, by embedding the ideas of fuzzy sets [52] many interesting applications of soft set theory have been done [2, 8, 9, 10, 21, 23, 24, 27, 28, 32, 34, 39, 42, 43, 45, 49, 50].

To develop the soft set theory, operations of the soft sets are redefined to improve several new results and *uni-int* decision making method is constructed by using these new operations [6]. To make easy computation with the operations of soft sets, the soft matrix theory is presented and soft *max-min* decision making method is set up [7]. These decision making methods are more practical and can be successfully applied to many problems that contain uncertainties.

In [9], a fuzzy soft (fs) set theory is defined. It allows constructing more efficient decision making method. In this paper, we first define fs-matrices which are representation of the fs-sets. This representation has several advantages. It is easy to store and manipulate matrices and hence the fs-sets represented by them in a computer. Here, we also construct a fs-decision making method which is more practical and can be successfully applied to many problems. We finally give an example which shows that the method successfully works.

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2. **Fuzzy Soft Matrices**

In this section, we define fs-matrices which are representations of the fs-sets. This style of representation is useful for storing a soft set in a computer memory. The operations can be presented by the matrices which are very useful and convenient for application.

From now on, a set of all fuzzy sets over U will be denoted by $F(U)$. Γ_A, Γ_B, Γ_C,..., etc. and γ_A, γ_B, γ_C,..., etc. will be used for fs-sets and their fuzzy approximate functions, respectively.

Definition 2.1. [9] Let U be an initial universe, E be the set of all parameters, $A \subseteq E$ and $\gamma_A(x)$ be a fuzzy set over U for all $x \in E$. Then, an fs-set Γ_A over U is a set defined by a function γ_A representing a mapping

$$\gamma_A : E \to F(U) \text{ such that } \gamma_A(x) = \emptyset \text{ if } x \notin A$$

Here, γ_A is called fuzzy approximate function of the fs-set Γ_A, the value $\gamma_A(x)$ is a fuzzy set called x-element of the fs-set for all $x \in E$, and $\emptyset$ is the null fuzzy set. Thus, an fs-set Γ_A over U can be represented by the set of ordered pairs

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in F(U)\}.$$

Note that from now on, the sets of all fs-sets over U will be denoted by $FS(U)$.

Example 2.2. Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4\}$ is a set of all parameters.

If $A = \{x_2, x_3, x_4\}$, $\gamma_A(x_2) = \{0.5/u_2, 0.8/u_4\}$, $\gamma_A(x_3) = \emptyset$ and $\gamma_A(x_4) = U$, then the fs-set Γ_A is written by $\Gamma_A = \{(x_2, \{0.5/u_2, 0.8/u_4\}), (x_4, U)\}$.

Definition 2.3. Let $\Gamma_A \in FS(U)$. Then a *fuzzy relation form* of Γ_A is defined by

$$R_A = \{(\mu_{R_A}(u, x)/(u, x)) : (u, x) \in U \times E\},$$

where the membership function of μ_{R_A} is written by

$$\mu_{R_A} : U \times E \to [0, 1], \quad \mu_{R_A}(u, x) = \mu_{\gamma_A(x)}(u).$$

If $U = \{u_1, u_2, ..., u_m\}$, $E = \{x_1, x_2, ..., x_n\}$ and $A \subseteq E$, then the R_A can be presented by a table as in the following form

R_A	x_1	x_2	...	x_n
u_1	$\mu_{R_A}(u_1, x_1)$	$\mu_{R_A}(u_1, x_2)$	...	$\mu_{R_A}(u_1, x_n)$
u_2	$\mu_{R_A}(u_2, x_1)$	$\mu_{R_A}(u_2, x_2)$	...	$\mu_{R_A}(u_2, x_n)$
$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
u_m	$\mu_{R_A}(u_m, x_1)$	$\mu_{R_A}(u_m, x_2)$	...	$\mu_{R_A}(u_m, x_n)$

If $a_{ij} = \mu_{R_A}(u_i, x_j)$, we can define a matrix

$$[a_{ij}]_{m\times n} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

which is called an $m \times n$ fs-matrix of the fs-set Γ_A over U.

According to this definition, an fs-set Γ_A is uniquely characterized by the matrix $[a_{ij}]_{m\times n}$. It means that an fs-set Γ_A is formally equal to its soft matrix $[a_{ij}]_{m\times n}$. Therefore, we shall identify any fs-set with its fs-matrix and use these two concepts as interchangeable.

The set of all $m\times n$ fs-matrices over U will be denoted by $FSM_{m\times n}$. From now on we shall delete the subscript $m \times n$ of $[a_{ij}]_{m\times n}$, we use $[a_{ij}]$ instead of $[a_{ij}]_{m\times n}$, since $[a_{ij}] \in FSM_{m\times n}$ means that $[a_{ij}]$ is an $m \times n$ fs-matrix for $i = 1, 2, ...m$ and $j = 1, 2, ...n$.

Example 2.4. Let us consider Example 2.2. Then the relation form of Γ_A is written by

$$\begin{aligned} R_A \quad = \quad & \{0.5/(u_2, x_2), 0.8/(u_4, x_2), 1/(u_1, x_4) \\ & 1/(u_2, x_4), 1/(u_3, x_4), 1/(u_4, x_4), 1/(u_5, x_4)\} \end{aligned}$$

Hence, the fs-matrix $[a_{ij}]$ is written by

$$[a_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0.5 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0.8 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Definition 2.5. Let $[a_{ij}] \in FSM_{m\times n}$. Then $[a_{ij}]$ is called

(1) a zero fs-matrix, denoted by $[0]$, if $a_{ij} = 0$ for all i and j.
(2) an A-universal fs-matrix, denoted by $[\tilde{a}_{ij}]$, if $a_{ij} = 1$ for all $j \in I_A = \{j : x_j \in A\}$ and i.
(3) a universal fs-matrix, denoted by $[1]$, if $a_{ij} = 1$ for all i and j.

Example 2.6. Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set, $E = \{x_1, x_2, x_3, x_4\}$ is a set of parameters, $A \subseteq E$, $\gamma_A(x)$ is a fuzzy set over U for all $x \in E$ and $[a_{ij}], [b_{ij}], [c_{ij}] \in FSM_{5\times 4}$.

If $A = \{x_1, x_3\}$ and $\gamma_A(x_1) = \emptyset$, $\gamma_A(x_3) = \emptyset$, then $[a_{ij}] = [0]$ is a zero fs-matrix written by

$$[0] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

If $B = \{x_1, x_2\}$ and $\gamma_B(x_1) = U$, $\gamma_B(x_2) = U$, then $[b_{ij}] = [\tilde{b}_{ij}]$ is a B-universal fs-matrix written by

$$[\tilde{b}_{ij}] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

If $C = E$, and $\gamma_C(x_i) = U$ for all $x_i \in C$, $i = 1, 2, 3, 4$, then $[c_{ij}] = [1]$ is a universal fs-matrix written by

$$[1] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

Definition 2.7. Let $[a_{ij}], [b_{ij}] \in FSM_{m\times n}$. Then

(1) $[a_{ij}]$ is a fs-submatrix of $[b_{ij}]$, denoted by $[a_{ij}]\tilde{\subseteq}[b_{ij}]$, if $a_{ij} \leqslant b_{ij}$ for all i and j.
(2) $[a_{ij}]$ is a proper fs-submatrix of $[b_{ij}]$, denoted by $[a_{ij}]\tilde{\subset}[b_{ij}]$, if $a_{ij} \leqslant b_{ij}$ for all i and j and for at least one term $a_{ij} < b_{ij}$.
(3) $[a_{ij}]$ and $[b_{ij}]$ are fs-equal matrices, denoted by $[a_{ij}] = [b_{ij}]$, if $a_{ij} = b_{ij}$ for all i and j.

Definition 2.8. Let $[a_{ij}], [b_{ij}] \in FSM_{m\times n}$. Then the fs-matrix $[c_{ij}]$ is called

(1) union of $[a_{ij}]$ and $[b_{ij}]$, denoted $[a_{ij}]\tilde{\cup}[b_{ij}]$, if $c_{ij} = \max\{a_{ij}, b_{ij}\}$ for all i and j.
(2) intersection of $[a_{ij}]$ and $[b_{ij}]$, denoted $[a_{ij}]\tilde{\cap}[b_{ij}]$, if $c_{ij} = \min\{a_{ij}, b_{ij}\}$ for all i and j.
(3) complement of $[a_{ij}]$, denoted by $[a_{ij}]^{\circ}$, if $c_{ij} = 1 - a_{ij}$ for all i and j.

Definition 2.9. Let $[a_{ij}], [b_{ij}] \in FSM_{m\times n}$. Then $[a_{ij}]$ and $[b_{ij}]$ are disjoint, if $[a_{ij}]\tilde{\cap}[b_{ij}] = [0]$ for all i and j.

Example 2.10. Assume that

$$[a_{ij}] = \begin{bmatrix} 0 & 0.6 & 0 & 0 \\ 0.1 & 0 & 1 & 0 \\ 0 & 0.3 & 0.8 & 0 \\ 0.7 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}, \ [b_{ij}] = \begin{bmatrix} 0 & 0 & 0.7 & 0.4 \\ 0 & 0.2 & 0 & 1 \\ 0 & 0 & 0 & 0.9 \\ 0 & 0 & 0.5 & 1 \\ 0 & 0 & 0 & 0.3 \end{bmatrix}$$

Then, $[a_{ij}]\tilde{\cap}[b_{ij}] = [0]$ and

$$[a_{ij}]\tilde{\cup}[b_{ij}] = \begin{bmatrix} 0 & 0.6 & 0.7 & 0.4 \\ 0.1 & 0.2 & 1 & 1 \\ 0 & 0.3 & 0.8 & 0.9 \\ 0.7 & 0 & 0.5 & 1 \\ 0 & 1 & 0 & 0.3 \end{bmatrix}, \ [a_{ij}]^\circ = \begin{bmatrix} 1 & 0.4 & 1 & 1 \\ 0.9 & 1 & 0 & 1 \\ 1 & 0.7 & 0.2 & 1 \\ 0.3 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

Proposition 2.11. *Let* $[a_{ij}] \in FSM_{m\times n}$. *Then*

(1) $[[a_{ij}]^\circ]^\circ = [a_{ij}]$
(2) $[0]^\circ = [1]$

Proposition 2.12. *Let* $[a_{ij}], [b_{ij}] \in FSM_{m\times n}$. *Then*

(1) $[a_{ij}]\tilde{\subseteq}[1]$
(2) $[0]\tilde{\subseteq}[a_{ij}]$
(3) $[a_{ij}]\tilde{\subseteq}[a_{ij}]$
(4) $[a_{ij}]\tilde{\subseteq}[b_{ij}]$ *and* $[b_{ij}]\tilde{\subseteq}[c_{ij}] \Rightarrow [a_{ij}]\tilde{\subseteq}[c_{ij}]$
(5) $[a_{ij}]\tilde{\subseteq}[b_{ij}] \Leftrightarrow [a_{ij}]\tilde{\cap}[b_{ij}] = [a_{ij}] \Leftrightarrow [a_{ij}]\tilde{\cup}[b_{ij}] = [b_{ij}]$

Proposition 2.13. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FSM_{m\times n}$. *Then*

(1) $[a_{ij}] = [b_{ij}]$ *and* $[b_{ij}] = [c_{ij}] \Leftrightarrow [a_{ij}] = [c_{ij}]$
(2) $[a_{ij}]\tilde{\subseteq}[b_{ij}]$ *and* $[b_{ij}]\tilde{\subseteq}[a_{ij}] \Leftrightarrow [a_{ij}] = [b_{ij}]$

Proposition 2.14. *Let* $[a_{ij}], [b_{ij}], [c_{ij}] \in FSM_{m\times n}$. *Then*

(1) $[a_{ij}]\tilde{\cap}[b_{ij}] = [b_{ij}]\tilde{\cap}[a_{ij}]$
(2) $[a_{ij}]\tilde{\cup}[b_{ij}] = [b_{ij}]\tilde{\cup}[a_{ij}]$
(3) $([a_{ij}]\tilde{\cap}[b_{ij}])\tilde{\cap}[c_{ij}] = [a_{ij}]\tilde{\cap}([b_{ij}]\tilde{\cap}[c_{ij}])$
(4) $([a_{ij}]\tilde{\cup}[b_{ij}])\tilde{\cup}[c_{ij}] = [a_{ij}]\tilde{\cup}([b_{ij}]\tilde{\cup}[c_{ij}])$
(5) $[a_{ij}]\tilde{\cap}([b_{ij}]\tilde{\cup}[c_{ij}]) = ([a_{ij}]\tilde{\cap}[b_{ij}])\tilde{\cup}([a_{ij}]\tilde{\cap}[c_{ij}])$
(6) $[a_{ij}]\tilde{\cup}([b_{ij}]\tilde{\cap}[c_{ij}]) = ([a_{ij}]\tilde{\cup}[b_{ij}])\tilde{\cap}([a_{ij}]\tilde{\cup}[c_{ij}])$

Note that, $[a_{ij}]\tilde{\cap}[a_{ij}]^\circ \neq [0]$ and $[a_{ij}]\tilde{\cup}[a_{ij}]^\circ \neq [1]$

Proposition 2.15. *Let* $[a_{ij}], [b_{ij}] \in FSM_{m\times n}$. *Then De Morgan's laws are valid*

(1) $([a_{ij}]\tilde{\cup}[b_{ij}])^\circ = [a_{ij}]^\circ\tilde{\cap}[b_{ij}]^\circ$
(2) $([a_{ij}]\tilde{\cap}[b_{ij}])^\circ = [a_{ij}]^\circ\tilde{\cup}[b_{ij}]^\circ$

Proof. For all i and j,

$$\begin{aligned} (1)\ ([a_{ij}]\tilde{\cup}[b_{ij}])^\circ &= [\max\{a_{ij}, b_{ij}\}]^\circ \\ &= [1-\max\{a_{ij}, b_{ij}\}] \\ &= [\min\{1-a_{ij}, 1-b_{ij}\}] \\ &= [a_{ij}]^\circ\tilde{\cap}[b_{ij}]^\circ \end{aligned}$$

(2) It can be proved similarly.

□

3. Products of fs-Matrices

In this section, we define four special products of fs-matrices to construct fs-decision making methods.

Definition 3.1. Let $[a_{ij}], [b_{ik}] \in FSM_{m\times n}$. Then *And*-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by

$$\wedge : FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, \quad [a_{ij}] \wedge [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \min\{a_{ij}, b_{ik}\}$ such that $p = n(j-1)+k$.

Definition 3.2. Let $[a_{ij}], [b_{ik}] \in FSM_{m\times n}$. Then *Or*-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by

$$\vee : FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, \quad [a_{ij}] \vee [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \max\{a_{ij}, b_{ik}\}$ such that $p = n(j-1)+k$.

Definition 3.3. Let $[a_{ij}], [b_{ik}] \in FSM_{m\times n}$. Then *And-Not*-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by

$$\overline{\wedge} : FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, \quad [a_{ij}] \overline{\wedge} [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \min\{a_{ij}, 1-b_{ik}\}$ such that $p = n(j-1)+k$.

Definition 3.4. Let $[a_{ij}], [b_{ik}] \in FSM_{m\times n}$. Then *Or-Not*-product of $[a_{ij}]$ and $[b_{ik}]$ is defined by

$$\underline{\vee} : FSM_{m\times n} \times FSM_{m\times n} \to FSM_{m\times n^2}, \quad [a_{ij}] \underline{\vee} [b_{ik}] = [c_{ip}]$$

where $c_{ip} = \max\{a_{ij}, 1-b_{ik}\}$ such that $p = n(j-1)+k$.

Example 3.5. Assume that $[a_{ij}], [b_{ik}] \in FSM_{2\times 3}$ are given as follows

$$[a_{ij}] = \begin{bmatrix} 0 & 0 & 0.3 \\ 0 & 1 & 0.7 \end{bmatrix}, \ [b_{ik}] = \begin{bmatrix} 0.5 & 0 & 0.2 \\ 0.2 & 0 & 0 \end{bmatrix}.$$

To calculate $[a_{ij}] \wedge [b_{ik}] = [c_{ip}]$, we have to find c_{ip} for all $i = 1,2$ and $p = 1,2,...,9$. Let us find c_{17}. Since $n = 3$, $i = 1$ and $p = 7$, we get $j = 3$ and $k = 1$ from $7 = 3(j-1)+k$. Hence $c_{17} = \min\{a_{13}, b_{11}\} = \min\{0.3, 0.5\} = 0.3$. If the other entries of $[c_{ip}]$ can be found similarly, then we can obtain the matrix as follows;

$$[a_{ij}] \wedge [b_{ik}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0.2 \\ 0 & 0 & 0 & 0.2 & 0 & 0 & 0.2 & 0 & 0 \end{bmatrix}$$

Similarly, we can also find products $[a_{ij}] \vee [b_{ik}]$, $[a_{ij}] \overline{\wedge} [b_{ik}]$ and $[a_{ij}] \underline{\vee} [b_{ik}]$.
Note that the commutativity is not valid for the products of fs-matrices.

Proposition 3.6. *Let $[a_{ij}], [b_{ik}] \in FSM_{m\times n}$. Then the following De Morgan's types of results are true.*

(1) $([a_{ij}] \vee [b_{ik}])^\circ = [a_{ij}]^\circ \wedge [b_{ik}]^\circ$
(2) $([a_{ij}] \wedge [b_{ik}])^\circ = [a_{ij}]^\circ \vee [b_{ik}]^\circ$
(3) $([a_{ij}] \veebar [b_{ik}])^\circ = [a_{ij}]^\circ \barwedge [b_{ik}]^\circ$
(4) $([a_{ij}] \barwedge [b_{ik}])^\circ = [a_{ij}]^\circ \veebar [b_{ik}]^\circ$

4. fs-Max-Min Decision Making

In this section, we construct an fs-max-min decision making ($FSMmDM$) method by using fs-max-min decision function which is also defined here. The method selects optimum alternatives from the set of the alternatives.

Definition 4.1. Let $[c_{ip}] \in FSM_{m\times n^2}$, $I_k = \{p : \exists i, c_{ip} \neq 0, (k-1)n < p \leqslant kn\}$ for all $k \in I = \{1, 2, ..., n\}$. Then fs-max-min decision function, denoted Mm, is defined as follows

$$Mm : FSM_{m\times n^2} \to FSM_{m\times 1}, \quad Mm[c_{ip}] = [d_{i1}] = [\max_k\{t_{ik}\}]$$

where

$$t_{ik} = \begin{cases} \min_{p\in I_k}\{c_{ip}\}, & \text{if } I_k \neq \emptyset, \\ 0, & \text{if } I_k = \emptyset. \end{cases}$$

The one column fs-matrix $Mm[c_{ip}]$ is called max-min decision fs-matrix.

Definition 4.2. Let $U = \{u_1, u_2, ..., u_m\}$ be an initial universe and $Mm[c_{ip}] = [d_{i1}]$. Then a subset of U can be obtained by using $[d_{i1}]$ as in the following way

$$opt_{[d_{i1}]}(U) = \{d_{i1}/u_i : u_i \in U, d_{i1} \neq 0\}$$

which is called an optimum fuzzy set on U.

Now, using definitions we can construct a $FSMmDM$ method by the following algorithm.

Step 1: choose feasible subsets of the set of parameters,
Step 2: construct the fs-matrix for each set of parameters,
Step 3: find a convenient product of the fs-matrices,
Step 4: find a max-min decision fs-matrix,
Step 5: find an optimum fuzzy set on U.

Note that, by the similar way, we can define fs-min-max, fs-min-min and fs-max-max decision making methods which may be denoted by ($FSmMDM$), ($FSmmDM$), ($FSMMDM$), respectively. One of them may be useful than the others according to the type of the problems.

5. Applications

Assume that a real estate agent has a set of different types of houses $U = \{u_1, u_2, u_3, u_4, u_5\}$ which may be characterized by a set of parameters $E = \{x_1, x_2, x_3, x_4\}$. For $j = 1, 2, 3, 4$ the parameters x_j stand for "in good location", "cheap", "modern", "large", respectively. Then we can give the following examples.

Example 5.1. Suppose that a married couple, Mr. X and Mrs. X, come to the real estate agent to buy a house. If each partner has to consider their own set of parameters, then we select a house on the basis of the sets of partners' parameters by using the $FSMmDM$ as follows.

Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4\}$ is a set of all parameters.

Step 1: First, Mr. X and Mrs. X have to choose the sets of their parameters, $A = \{x_2, x_3, x_4\}$ and $B = \{x_1, x_3, x_4\}$, respectively.

Step 2: Then we can write the following fs-matrices which are constructed according to their parameters.

$$[a_{ij}] = \begin{bmatrix} 0 & 0 & 0.2 & 0.4 \\ 0 & 0.6 & 0.9 & 0.4 \\ 0 & 0.8 & 0.7 & 0.5 \\ 0 & 0.5 & 0 & 0 \\ 0 & 1 & 0 & 0.8 \end{bmatrix} \qquad [b_{ik}] = \begin{bmatrix} 1 & 0 & 0.9 & 0.7 \\ 0.2 & 0 & 0 & 0.9 \\ 0.7 & 0 & 0.4 & 0.3 \\ 0 & 0 & 0.5 & 0.6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Step 3: Now, we can find a product of the fs-matrices $[a_{ij}]$ and $[b_{ik}]$ by using *And*-product as follows

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0.2 & 0.2 & 0.4 & 0 & 0.4 & 0.4 \\ 0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0.6 & 0.2 & 0 & 0 & 0.9 & 0.2 & 0 & 0 & 0.4 \\ 0 & 0 & 0 & 0 & 0.7 & 0 & 0.4 & 0.3 & 0.7 & 0 & 0.4 & 0.3 & 0.5 & 0 & 0.4 & 0.3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8 \end{bmatrix}$$

Here, we use And-product since both Mr. X and Mrs. X's choices have to be considered.

Step 4: To calculate $Mm([a_{ij}] \wedge [b_{ik}]) = [d_{i1}]$, we have to find d_{i1} for all $i \in \{1, 2, 3, 4, 5\}$. To demonstrate, let us find d_{31}. Since $i = 1$ and $k \in \{1, 2, 3, 4\}$,

$$d_{31} = max_k\{t_{3k}\} = max\{t_{31}, t_{32}, t_{33}, t_{34}\}$$

Here, we have to find t_{3k} for all $k \in \{1, 2, 3, 4\}$. To demonstrate, let us find t_{31} and t_{32}. $I_1 = \{p : c_{ip} \neq 0,\ 0 < p \leq 4\} = \emptyset$ for $k = 1$ and $n = 4$ and $I_2 = \{p : c_{ip} \neq 0,\ 4 < p \leq 8\} = \{5, 7, 8\}$ for $k = 2$ and $n = 4$. Hence $t_{31} = 0$ and

$$t_{32} = \min\{c_{35}, c_{37}, c_{38}\} = \min\{0.7, 0.4, 0.3\} = 0.3$$

Similarly, we can find as $t_{33} = 0.3$ and $t_{34} = 0.3$. Thus,

$$d_{31} = max\{0.0, 0.3, 0.3, 0.3\} = 0.3$$

Similarly we can find $d_{11} = 0.4$, $d_{21} = 0.0$, $d_{41} = 0.0$ and $d_{51} = 0.0$. Finally, we can obtain the fs-max-min decision fs-matrix as

$$Mm([a_{ij}] \wedge [b_{ik}]) = [d_{i1}] = \begin{bmatrix} 0.4 \\ 0 \\ 0.3 \\ 0 \\ 0 \end{bmatrix}$$

Step 5: Finally, we can find an optimum fuzzy set on U according to $Mm([a_{ij}] \wedge [b_{ik}])$

$$opt_{Mm([a_{ij}] \wedge [b_{ik}])}(U) = \{0.4/u_1, 0.3/u_3\}$$

where u_1 is an optimum house to buy for Mr. X and Mrs. X.

Similarly, we can also use products $[a_{ij}] \vee [b_{ik}]$, $[a_{ij}] \barwedge [b_{ik}]$ and $[a_{ij}] \veebar [b_{ik}]$ for the other convenient problems.

6. **Conclusion**

The fs-set theory is being applied to many fields varying from theoretical to practical. In this paper, we define fs-matrices which are matrix representation of the fs-sets. We then define the set-theoretic operations of fs-matrices which are more functional to improve several new results. Afterwards, we construct a fs-decision making model on the fs-set theory. This new decision making method depends on the ideas of fuzzy and soft sets. Its main idea is similar to the decision making method, given in [7], which only depends on soft sets. Therefore, this method is more feasible than the others, because of its fuzziness. Finally, we give an application for a real estate agent to choose an optimal house.

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References

[1] U. Acar, F. Koyuncu and B. Tanay, *Soft sets and soft rings*, Computers and Mathematics with Applications, **59** (2010), 3458-3463.

[2] B. Ahmad and A. Kharal, *On fuzzy soft sets*, Advances in Fuzzy Systems, (2009), 1-6.

[3] H. Aktaş and N. Çağman, *Soft sets and soft groups*, Information Sciences, **1(77)** (2007), 2726-2735.

[4] M. I. Ali, F. Feng, X. Liu, W. K. Min and M. Shabir, *On some new operations in soft set theory*, Computers and Mathematics with Applications, **57**(2009), 1547-1553.

[5] A. O. Atagün and A. Sezgin, *Soft substructures of rings, fields and modules*, Comput. Math. Appl., **61(3)** (2011), 592-601.

[6] N. Çağman and S. Enginoğlu, *Soft set theory and uni-int decision making*, European Journal of Operational Research, **207** (2010), 848-855.

[7] N. Çağman and S. Enginoğlu, *Soft matrix theory and its decision making*, Computers and Mathematics with Applications, **59** (2010), 3308-3314.

[8] N. Çağman, F. Cıtak and S. Enginoğlu, *Fuzzy parameterized fuzzy soft set theory and its applications*, Turkish Journal of Fuzzy Systems, **1(1)** (2010), 21-35.

[9] N. Çağman, S. Enginoğlu and F. Cıtak, *Fuzzy soft set theory and its applications*, Iranian Journal of Fuzzy Systems, **8(3)** (2011), 137-147.

[10] F. Feng, Y. B. Jun, X. Liu and L. Li, *An adjustable approach to fuzzy soft set based decision making*, Journal of Computational and Applied Mathematics, **234** (2010), 10-20.

[11] F. Feng, Y. B. Jun and X. Zhao, *Soft semirings*, Fuzzy Sets and Systems: Theory and Applications, **56(10)** (2008), 2621-2628.

[12] T. Herawan, A. N. M. Rose and M. M. Deris, *Soft set theoretic approach for dimensionality reduction*, In Database Theory and Application, Springer Berlin Heidelberg, **64** (2009), 171-178.

[13] Y. Jiang, Y. Tang, Q. Chen, J. Wang and S. Tang, *Extending soft sets with description logics*, Computers and Mathematics with Applications, **59** (2010), 2087-2096.

[14] Y. B. Jun, *Soft BCK/BCI-algebras*, Computers and Mathematics with Applications, **56** (2008), 1408-1413.

[15] Y. B. Jun and C. H. Park, *Applications of soft sets in ideal theory of BCK/BCI-algebras*, Information Sciences, **178** (2008), 2466-2475.

[16] Y. B. Jun and C. H. Park, *Applications of soft sets in Hilbert algebras*, Iranian Journal of Fuzzy Systems, **6(2)** (2009), 55-86.

[17] Y. B. Jun, H. S. Kim and J. Neggers, *Pseudo d-algebras*, Information Sciences, **179** (2009), 1751-1759.

[18] Y. B. Jun, K. J. Lee and A. Khan, *Soft ordered semigroups*, Mathematical Logic Quarterly, **56(1)** (2010), 42-50.

[19] Y. B. Jun, K. J. Lee and C. H. Park, *Soft set theory applied to commutative ideals, in BCK–algebras*, Journal of Applied Mathematics Informatics, **26(3-4)** (2008), 707-720.

[20] Y. B. Jun, K. J. Lee and C. H. Park, *Soft set theory applied to ideals in d-algebras*, Computers and Mathematics with Applications, **57** (2009), 367-378.

[21] Y. B. Jun, K. J. Lee and C. H. Park, *Fuzzy soft set theory applied to BCK/BCI-algebras*, Computers and Mathematics with Applications, **59** (2010), 3180-3192.

[22] Y. B. Jun, K. J. Lee and J. Zhan, *Soft p-ideals of soft BCI-algebras*, Computers and Mathematics with Applications, **58** (2009), 2060-2068.

[23] S. J. Kalayathankal and G. S. Singh, *A fuzzy soft flood alarm model*, Mathematics and Computers in Simulation, **80** (2010), 887-893.

[24] A. Kharal and B. Ahmad, *Mappings on fuzzy soft classes*, Advances in Fuzzy Systems, (2009), 1-6.

[25] Z. Kong, L. Gao, L. Wang and S. Li, *The normal parameter reduction of soft sets and its algorithm*, Computers and Mathematics with Applications, **56** (2008), 3029-3037.

[26] D. V. Kovkov, V. M. Kolbanov and D. A. Molodtsov, *Soft sets theory-based optimization*, Journal of Computer and Systems Sciences International, **46(6)** (2007), 872-880.

[27] P. K. Maji, R. Biswas and A. R. Roy, *Fuzzy soft sets*, Journal of Fuzzy Mathematics, **9(3)** (2001), 589-602.

[28] P. K. Maji, R. Biswas and A. R. Roy, *Intuitionistic fuzzy soft sets*, Journal of Fuzzy Mathematics, **9(3)** (2001), 677-691.

[29] P. K. Maji, R. Biswas and A. R. Roy, *Soft set theory*, Computers and Mathematics with Applications, **45** (2003), 555-562.

[30] P. K. Maji, A. R. Roy and R. Biswas, *An application of soft sets in a decision making problem*, Computers and Mathematics with Applications, **44** (2002), 1077-1083.

[31] P. Majumdar and S. K. Samanta, *Similarity measure of soft sets*, New Mathematics and Natural Computation, **4(1)** (2008), 1-12.

[32] P. Majumdar and S. K. Samanta, *Generalised fuzzy soft sets*, Computers and Mathematics with Applications, **59** (2010), 1425-1432.

[33] D. A. Molodtsov, *Soft set theory-first results*, Computers and Mathematics with Applications, **37** (1999), 19-31.

[34] A. Mukherjee and S. B. Chakraborty, *On intuitionistic fuzzy soft relations*, Bulletin of Kerala Mathematics Association, **5(1)** (2008), 35-42.

[35] M. M. Mushrif, S. Sengupta and A. K. Ray, *Texture classification using a novel*, Soft-Set Theory Based Classification, Algorithm. Lecture Notes in Computer Science., **3851** (2006), 246-254.

[36] C. H. Park, Y. B. Jun and M. A. Öztürk, *Soft WS-algebras*, Communications of the Korean Mathematical Society, **23(3)** (2008), 313-324.
[37] D. Pei and D. Miao, *From soft sets to information systems*, In: Proceedings of Granular Computing (eds: X. Hu, Q. Liu, A. Skowron, T. Y. Lin, R. R. Yager, B. Zhang) IEEE 2005, **2** (2005), 617- 621.
[38] K. Qin and Z. Hong, *On soft equality*, Journal of Computational and Applied Mathematics, **234** (2010), 1347-1355.
[39] A. R. Roy and P. K. Maji, *A fuzzy soft set theoretic approach to decision making problems*, Journal of Computational and Applied Mathematics, **203** (2007), 412-418.
[40] A. Sezgin and A. O. Atagün, *On operations of soft sets*, Comput. Math. Appl., **61(5)** (2011), 1457-1467.
[41] A. Sezgin, A. O. Atagün and E. Aygün, *A note on soft near-rings and idealistic soft near-rings*, Filomat, **25** (2011), 53-68.
[42] T. Som, *On the theory of soft sets soft relation and fuzzy soft relation*, Proc. of the National Conference on Uncertainty: A Mathematical Approach, UAMA-06, Burdwan, (2006), 1-9.
[43] M. J. Son, *Interval-valued fuzzy soft sets*, Journal of Fuzzy Logic and Intelligent Systems, **17(4)** (2007), 557-562.
[44] Q. M. Sun, Z. L. Zhang and J. Liu, *Soft sets and soft modules*, Proceedings of Rough Sets and Knowledge Technology, Third International Conference, RSKT 2008, 17-19 May, Chengdu, China, (2008), 403-409.
[45] Z. Xiao, K. Gong and Y. Zou, *A combined forecasting approach based on fuzzy soft sets*, Journal of Computational and Applied Mathematics, **228** (2009), 326-333.
[46] Z. Xiao, K. Gong, S. Xia and Y. Zou, *Exclusive disjunctive soft sets*, Computers and Mathematics with Applications, **59** (2010), 2128-2137.
[47] Z. Xiao, Y. Li, B. Zhong and X. Yang, *Research on synthetically evaluating method for business competitive capacity based on soft set*, Statistical Research, (2003), 52-54.
[48] W. Xu, W. J. Ma, S. Wang and G. Hao, *Vague soft sets and their properties*, Computers and Mathematics with Applications, **59** (2010), 787-794.
[49] X. Yang, T. Y. Lin, J. Yang, Y. Li and D. Yu, *Combination of interval-valued fuzzy set and soft set*, Computers and Mathematics with Applications, **58** (2009), 521-527.
[50] J. Zhan and Y. B. Jun, *Soft BL-algebras based on fuzzy sets*, Computers and Mathematics with Applications, **59** (2010), 2037-2046.
[51] Y. Zou and Z. Xiao, *Data analysis approaches of soft sets under incomplete information*, Knowledge-Based Systems, **21** (2008), 941-945.
[52] L. A. Zadeh, *Fuzzy sets*, Information and Control, **8** (1965), 338-353.

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