FUZZY SOFT MATRIX THEORY AND ITS APPLICATION IN DECISION MAKING

N. ÇAĞMAN AND S. ENGİNOĞLU

Abstract. In this work, we define fuzzy soft (fs) matrices and their operations which are more functional to make theoretical studies in the fs-set theory. We then define products of fs-matrices and study their properties. We finally construct a fs-max-min decision making method which can be successfully applied to the problems that contain uncertainties.

1. Introduction

Soft set theory [33] was firstly introduced by Molodtsov in 1999 as a general mathematical tool for dealing with uncertain, fuzzy, not clearly defined objects. After presentation of the operations of soft sets [29], the properties and applications on the soft set theory have been studied increasingly [4, 12, 13, 25, 26, 30, 31, 35, 37, 38, 40, 46, 47, 48, 51]. The algebraic structure of soft set theory has also been studied in more detail [1, 3, 5, 11, 14, 15, 16, 17, 18, 19, 20, 22, 36, 41, 44]. In recent years, by embedding the ideas of fuzzy sets [52] many interesting applications of soft set theory have been done [2, 8, 9, 10, 21, 23, 24, 27, 28, 32, 34, 39, 42, 43, 45, 49, 50].

To develop the soft set theory, operations of the soft sets are redefined to improve several new results and uni-int decision making method is constructed by using these new operations [6]. To make easy computation with the operations of soft sets, the soft matrix theory is presented and soft max-min decision making method is set up [7]. These decision making methods are more practical and can be successfully applied to many problems that contain uncertainties.

In [9], a fuzzy soft (fs) set theory is defined. It allows constructing more efficient decision making method. In this paper, we first define fs-matrices which are representation of the fs-sets. This representation has several advantages. It is easy to store and manipulate matrices and hence the fs-sets represented by them in a computer. Here, we also construct a fs-decision making method which is more practical and can be successfully applied to many problems. We finally give an example which shows that the method successfully works.

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2. Fuzzy Soft Matrices

In this section, we define $fs$-matrices which are representations of the $fs$-sets. This style of representation is useful for storing a soft set in a computer memory. The operations can be presented by the matrices which are very useful and convenient for application.

From now on, a set of all fuzzy sets over $U$ will be denoted by $F(U)$. $\Gamma_A$, $\Gamma_B$, $\Gamma_C$, etc. and $\gamma_A$, $\gamma_B$, $\gamma_C$, etc. will be used for $fs$-sets and their fuzzy approximate functions, respectively.

**Definition 2.1.** [9] Let $U$ be an initial universe, $E$ be the set of all parameters, $A \subseteq E$ and $\gamma_A(x)$ be a fuzzy set over $U$ for all $x \in E$. Then, an $fs$-set $\Gamma_A$ over $U$ is a set defined by a function $\gamma_A$ representing a mapping

$$\gamma_A : E \rightarrow F(U) \text{ such that } \gamma_A(x) = \emptyset \text{ if } x \notin A$$

Here, $\gamma_A$ is called fuzzy approximate function of the $fs$-set $\Gamma_A$, the value $\gamma_A(x)$ is a fuzzy set called $x$-element of the $fs$-set for all $x \in E$, and $\emptyset$ is the null fuzzy set. Thus, an $fs$-set $\Gamma_A$ over $U$ can be represented by the set of ordered pairs

$$\Gamma_A = \{(x, \gamma_A(x)) : x \in E, \gamma_A(x) \in F(U)\}.$$  

Note that from now on, the sets of all $fs$-sets over $U$ will be denoted by $FS(U)$.

**Example 2.2.** Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set and $E = \{x_1, x_2, x_3, x_4\}$ is a set of all parameters.

If $A = \{x_2, x_3, x_4\}$, $\gamma_A(x_2) = \{0.5/\text{u}_2, 0.8/\text{u}_4\}$, $\gamma_A(x_3) = \emptyset$ and $\gamma_A(x_4) = U$, then the $fs$-set $\Gamma_A$ is written by $\Gamma_A = \{(x_2, \{0.5/\text{u}_2, 0.8/\text{u}_4\}), (x_4, U)\}$.

**Definition 2.3.** Let $\Gamma_A \in FS(U)$. Then a fuzzy relation form of $\Gamma_A$ is defined by

$$R_A = \{((\mu_{R_A}(u, x) / (u, x)) : (u, x) \in U \times E\},$$

where the membership function of $\mu_{R_A}$ is written by

$$\mu_{R_A} : U \times E \rightarrow [0, 1], \quad \mu_{R_A}(u, x) = \mu_{\gamma_A(x)}(u).$$

If $U = \{u_1, u_2, ..., u_m\}$, $E = \{x_1, x_2, ..., x_n\}$ and $A \subseteq E$, then the $R_A$ can be presented by a table as in the following form

<table>
<thead>
<tr>
<th>$R_A$</th>
<th>$x_1$</th>
<th>$x_2$</th>
<th>...</th>
<th>$x_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u_1$</td>
<td>$\mu_{R_A}(u_1, x_1)$</td>
<td>$\mu_{R_A}(u_1, x_2)$</td>
<td>...</td>
<td>$\mu_{R_A}(u_1, x_n)$</td>
</tr>
<tr>
<td>$u_2$</td>
<td>$\mu_{R_A}(u_2, x_1)$</td>
<td>$\mu_{R_A}(u_2, x_2)$</td>
<td>...</td>
<td>$\mu_{R_A}(u_2, x_n)$</td>
</tr>
<tr>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>$\vdots$</td>
<td>...</td>
<td>$\vdots$</td>
</tr>
<tr>
<td>$u_m$</td>
<td>$\mu_{R_A}(u_m, x_1)$</td>
<td>$\mu_{R_A}(u_m, x_2)$</td>
<td>...</td>
<td>$\mu_{R_A}(u_m, x_n)$</td>
</tr>
</tbody>
</table>
If $a_{ij} = \mu_{R_A}(u_i, x_j)$, we can define a matrix

$$[a_{ij}]_{m \times n} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}$$

which is called an $m \times n$ $fs$-matrix of the $fs$-set $\Gamma_A$ over $U$.

According to this definition, an $fs$-set $\Gamma_A$ is uniquely characterized by the matrix $[a_{ij}]_{m \times n}$. It means that an $fs$-set $\Gamma_A$ is formally equal to its soft matrix $[a_{ij}]_{m \times n}$. Therefore, we shall identify any $fs$-set with its $fs$-matrix and use these two concepts as interchangeable.

The set of all $m \times n$ $fs$-matrices over $U$ will be denoted by $FSM_{m \times n}$. From now on we shall delete the subscript $m \times n$ of $[a_{ij}]_{m \times n}$, we use $[a_{ij}]$ instead of $[a_{ij}]_{m \times n}$, since $[a_{ij}] \in FSM_{m \times n}$ means that $[a_{ij}]$ is an $m \times n$ $fs$-matrix for $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$.

**Example 2.4.** Let us consider Example 2.2. Then the relation form of $\Gamma_A$ is written by

$$R_A = \{0.5/(u_2, x_2), 0.8/(u_4, x_2), 1/(u_1, x_4), 1/(u_2, x_4), 1/(u_3, x_4), 1/(u_4, x_4), 1/(u_5, x_4)\}$$

Hence, the $fs$-matrix $[a_{ij}]$ is written by

$$[a_{ij}] = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0.5 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0.8 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Definition 2.5.** Let $[a_{ij}] \in FSM_{m \times n}$. Then $[a_{ij}]$ is called

1. a zero $fs$-matrix, denoted by $[0]$, if $a_{ij} = 0$ for all $i$ and $j$.
2. an $A$-universal $fs$-matrix, denoted by $[\bar{a}_{ij}]$, if $a_{ij} = 1$ for all $j \in I_A = \{j : x_j \in A\}$ and $i$.
3. a universal $fs$-matrix, denoted by $[1]$, if $a_{ij} = 1$ for all $i$ and $j$.

**Example 2.6.** Assume that $U = \{u_1, u_2, u_3, u_4, u_5\}$ is a universal set, $E = \{x_1, x_2, x_3, x_4\}$ is a set of parameters, $A \subseteq E$, $\gamma_A(x)$ is a fuzzy set over $U$ for all $x \in E$ and $[a_{ij}], [b_{ij}], [c_{ij}] \in FSM_{5 \times 4}$.

If $A = \{x_1, x_3\}$ and $\gamma_A(x_1) = \emptyset$, $\gamma_A(x_3) = \emptyset$, then $[a_{ij}] = [0]$ is a zero $fs$-matrix written by
\[ [0] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \]

If \( B = \{x_1, x_2\} \) and \( \gamma_B(x_1) = U, \gamma_B(x_2) = U \), then \([b_{ij}] = [\bar{b}_{ij}]\) is a \( B \)-universal \( fs \)-matrix written by

\[
[\bar{b}_{ij}] = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}
\]

If \( C = E \), and \( \gamma_C(x_i) = U \) for all \( x_i \in C, i = 1, 2, 3, 4 \), then \([c_{ij}] = [1]\) is a universal \( fs \)-matrix written by

\[
[1] = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}
\]

**Definition 2.7.** Let \([a_{ij}], [b_{ij}] \in FSM_{m \times n}\). Then

1. \([a_{ij}]\) is a \( fs \)-submatrix of \([b_{ij}]\), denoted by \([a_{ij}] \subseteq [b_{ij}]\), if \( a_{ij} \leq b_{ij} \) for all \( i \) and \( j \).
2. \([a_{ij}]\) is a proper \( fs \)-submatrix of \([b_{ij}]\), denoted by \([a_{ij}] \subset [b_{ij}]\), if \( a_{ij} \leq b_{ij} \) for all \( i \) and \( j \) and for at least one term \( a_{ij} < b_{ij} \).
3. \([a_{ij}]\) and \([b_{ij}]\) are \( fs \)-equal matrices, denoted by \([a_{ij}] = [b_{ij}]\), if \( a_{ij} = b_{ij} \) for all \( i \) and \( j \).

**Definition 2.8.** Let \([a_{ij}], [b_{ij}] \in FSM_{m \times n}\). Then the \( fs \)-matrix \([c_{ij}]\) is called

1. union of \([a_{ij}]\) and \([b_{ij}]\), denoted by \([a_{ij}] \cup [b_{ij}]\), if \( c_{ij} = \max\{a_{ij}, b_{ij}\} \) for all \( i \) and \( j \).
2. intersection of \([a_{ij}]\) and \([b_{ij}]\), denoted by \([a_{ij}] \cap [b_{ij}]\), if \( c_{ij} = \min\{a_{ij}, b_{ij}\} \) for all \( i \) and \( j \).
3. complement of \([a_{ij}]\), denoted by \([a_{ij}]^\circ\), if \( c_{ij} = 1 - a_{ij} \) for all \( i \) and \( j \).

**Definition 2.9.** Let \([a_{ij}], [b_{ij}] \in FSM_{m \times n}\). Then \([a_{ij}]\) and \([b_{ij}]\) are disjoint, if \([a_{ij}] \cap [b_{ij}] = [0]\) for all \( i \) and \( j \).

**Example 2.10.** Assume that

\[
[a_{ij}] = \begin{bmatrix} 0 & 0.6 & 0 & 0 \\ 0.1 & 0 & 1 & 0 \\ 0 & 0.3 & 0.8 & 0 \\ 0.7 & 0 & 0 & 0 \end{bmatrix}, \quad [b_{ij}] = \begin{bmatrix} 0 & 0 & 0.7 & 0.4 \\ 0 & 0.2 & 0 & 1 \\ 0 & 0 & 0 & 0.9 \\ 0 & 0 & 0.5 & 1 \end{bmatrix}
\]
Then, \([a_{ij}]^\cap [b_{ij}] = [0]\) and

\[
[a_{ij}]^\cup [b_{ij}] = \begin{bmatrix}
0 & 0.6 & 0.7 & 0.4 \\
0.1 & 0.2 & 1 & 1 \\
0 & 0.3 & 0.8 & 0.9 \\
0.7 & 0 & 0.5 & 1 \\
0 & 1 & 0 & 0.3
\end{bmatrix},
\]

\([a_{ij}]^\circ = \begin{bmatrix}
1 & 0.4 & 1 & 1 \\
0.9 & 1 & 0 & 1 \\
1 & 0.7 & 0.2 & 1 \\
0.3 & 1 & 1 & 1 \\
1 & 0 & 1 & 1
\end{bmatrix}\]

**Proposition 2.11.** Let \([a_{ij}] \in \text{FSM}_{m \times n}\). Then

1. \([a_{ij}]^\circ \circ = [a_{ij}]^\circ\]
2. \([0]^\circ = [1]\)

**Proposition 2.12.** Let \([a_{ij}], [b_{ij}] \in \text{FSM}_{m \times n}\). Then

1. \([a_{ij}] \subseteq [1]\]
2. \([0] \subseteq [a_{ij}]\]
3. \([a_{ij}] \subseteq [a_{ij}]\]
4. \([a_{ij}] \subseteq [b_{ij}] \text{ and } [b_{ij}] \subseteq [c_{ij}] \Rightarrow [a_{ij}] \subseteq [c_{ij}]\]
5. \([a_{ij}] \subseteq [b_{ij}] \equiv [a_{ij}]^\cap [b_{ij}] = [a_{ij}] \equiv [a_{ij}]^\cup [b_{ij}] = [b_{ij}]\]

**Proposition 2.13.** Let \([a_{ij}], [b_{ij}], [c_{ij}] \in \text{FSM}_{m \times n}\). Then

1. \([a_{ij}] = [b_{ij}] \text{ and } [b_{ij}] = [c_{ij}] \Rightarrow [a_{ij}] = [c_{ij}]\]
2. \([a_{ij}] \subseteq [b_{ij}] \text{ and } [b_{ij}] \subseteq [a_{ij}] \Rightarrow [a_{ij}] = [b_{ij}]\)

**Proposition 2.14.** Let \([a_{ij}], [b_{ij}], [c_{ij}] \in \text{FSM}_{m \times n}\). Then \(\text{De Morgan’s laws are valid}\)

1. \(([a_{ij}]^\cup [b_{ij}])^\circ = [a_{ij}]^\circ \cap [b_{ij}]^\circ\)
2. \(([a_{ij}]^\cap [b_{ij}])^\circ = [a_{ij}]^\circ \cup [b_{ij}]^\circ\)

**Proof.** For all \(i\) and \(j\),

1. \(([a_{ij}]^\cup [b_{ij}])^\circ = \max\{a_{ij}, b_{ij}\}^\circ = [1 - \max\{a_{ij}, b_{ij}\}] = \min\{1 - a_{ij}, 1 - b_{ij}\} = [a_{ij}]^\circ \cap [b_{ij}]^\circ\)
2. It can be proved similarly.
3. Products of fs-Matrices

In this section, we define four special products of fs-matrices to construct fs-
decision making methods.

**Definition 3.1.** Let \([a_{ij}], [b_{ik}] \in FSM_{m \times n}\). Then And-product of \([a_{ij}]\) and \([b_{ik}]\) is defined by

\[
\land : FSM_{m \times n} \times FSM_{m \times n} \rightarrow FSM_{m \times n^2}, \quad [a_{ij}] \land [b_{ik}] = [c_{ip}]
\]

where \(c_{ip} = \min\{a_{ij}, b_{ik}\}\) such that \(p = n(j - 1) + k\).

**Definition 3.2.** Let \([a_{ij}], [b_{ik}] \in FSM_{m \times n}\). Then Or-product of \([a_{ij}]\) and \([b_{ik}]\) is defined by

\[
\lor : FSM_{m \times n} \times FSM_{m \times n} \rightarrow FSM_{m \times n^2}, \quad [a_{ij}] \lor [b_{ik}] = [c_{ip}]
\]

where \(c_{ip} = \max\{a_{ij}, b_{ik}\}\) such that \(p = n(j - 1) + k\).

**Definition 3.3.** Let \([a_{ij}], [b_{ik}] \in FSM_{m \times n}\). Then And-Not-product of \([a_{ij}]\) and \([b_{ik}]\) is defined by

\[
\overline{\land} : FSM_{m \times n} \times FSM_{m \times n} \rightarrow FSM_{m \times n^2}, \quad [a_{ij}] \overline{\land} [b_{ik}] = [c_{ip}]
\]

where \(c_{ip} = \min\{a_{ij}, 1 - b_{ik}\}\) such that \(p = n(j - 1) + k\).

**Definition 3.4.** Let \([a_{ij}], [b_{ik}] \in FSM_{m \times n}\). Then Or-Not-product of \([a_{ij}]\) and \([b_{ik}]\) is defined by

\[
\overline{\lor} : FSM_{m \times n} \times FSM_{m \times n} \rightarrow FSM_{m \times n^2}, \quad [a_{ij}] \overline{\lor} [b_{ik}] = [c_{ip}]
\]

where \(c_{ip} = \max\{a_{ij}, 1 - b_{ik}\}\) such that \(p = n(j - 1) + k\).

**Example 3.5.** Assume that \([a_{ij}], [b_{ik}] \in FSM_{2 \times 3}\) are given as follows

\[
[a_{ij}] = \begin{bmatrix} 0 & 0 & 0.3 \\ 0 & 1 & 0.7 \end{bmatrix}, \quad [b_{ik}] = \begin{bmatrix} 0.5 & 0 & 0.2 \\ 0.2 & 0 & 0 \end{bmatrix}.
\]

To calculate \([a_{ij}] \land [b_{ik}] = [c_{ip}]\), we have to find \(c_{ip}\) for all \(i = 1, 2\) and \(p = 1, 2, \ldots, 9\). Let us find \(c_{17}\). Since \(n = 3\), \(i = 1\) and \(p = 7\), we get \(j = 3\) and \(k = 1\) from \(7 = 3(j - 1) + k\). Hence \(c_{17} = \min\{a_{13}, b_{11}\} = \min\{0.3, 0.5\} = 0.3\). If the other entries of \([c_{ip}]\) can be found similarly, then we can obtain the matrix as follows;

\[
[a_{ij}] \land [b_{ik}] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0.3 & 0 & 0.2 \\ 0 & 0 & 0 & 0.2 & 0 & 0.2 & 0 & 0 \end{bmatrix}.
\]

Similarly, we can also find products \([a_{ij}] \lor [b_{ik}], [a_{ij}] \overline{\land} [b_{ik}]\) and \([a_{ij}] \overline{\lor} [b_{ik}]\). Note that the commutativity is not valid for the products of fs-matrices.

**Proposition 3.6.** Let \([a_{ij}], [b_{ik}] \in FSM_{m \times n}\). Then the following De Morgan’s types of results are true.
(1) \( ([a_{ij}] \lor [b_{ik}])^o = [a_{ij}]^o \land [b_{ik}]^o \)

(2) \( ([a_{ij}] \land [b_{ik}])^o = [a_{ij}]^o \lor [b_{ik}]^o \)

(3) \( ([a_{ij}] \lor [b_{ik}])^c = [a_{ij}]^c \land [b_{ik}]^c \)

(4) \( ([a_{ij}] \land [b_{ik}])^c = [a_{ij}]^c \lor [b_{ik}]^c \)

4. \textit{fs-Max-Min Decision Making}

In this section, we construct an \( \text{fs-max-min decision making (FSMmDM)} \) method by using \( \text{fs-max-min decision function} \) which is also defined here. The method selects optimum alternatives from the set of the alternatives.

**Definition 4.1.** Let \( [c_{ip}] \in \text{FSM}_{m \times n^2} \), \( I_k = \{ p : \exists i, c_{ip} \neq 0, (k-1)n < p \leq kn \} \) for all \( k \in I = \{1, 2, ..., n\} \). Then \( \text{fs-max-min decision function}, \text{denoted } Mm \), is defined as follows

\[
Mm : \text{FSM}_{m \times n^2} \to \text{FSM}_{m \times 1}, \quad Mm[c_{ip}] = [d_{i1}] = \max_k \{ t_{ik} \}
\]

where

\[
t_{ik} = \begin{cases} 
\min_{p \in I_k} \{ c_{ip} \}, & \text{if } I_k \neq \emptyset, \\
0, & \text{if } I_k = \emptyset.
\end{cases}
\]

The one column \( \text{fs-matrix} \) \( Mm[c_{ip}] \) is called \( \text{max-min decision fs-matrix} \).

**Definition 4.2.** Let \( U = \{ u_1, u_2, ..., u_m \} \) be an initial universe and \( Mm[c_{ip}] = [d_{i1}] \). Then a subset of \( U \) can be obtained by using \( [d_{i1}] \) as in the following way

\[
\text{opt}_{[d_{i1}]}(U) = \{ d_{i1}/u_i : u_i \in U, d_{i1} \neq 0 \}
\]

which is called an optimum fuzzy set on \( U \).

Now, using definitions we can construct a \( \text{FSMmDM} \) method by the following algorithm.

**Step 1:** choose feasible subsets of the set of parameters,

**Step 2:** construct the \( \text{fs-matrix} \) for each set of parameters,

**Step 3:** find a convenient product of the \( \text{fs-matrices} \),

**Step 4:** find a max-min decision \( \text{fs-matrix} \),

**Step 5:** find an optimum fuzzy set on \( U \).

Note that, by the similar way, we can define \( \text{fs-min-max, fs-min-min and fs-max-max decision making methods} \) which may be denoted by \( (FSmMDM), (FSmMMDM), (FSMMDM) \), respectively. One of them may be useful than the others according to the type of the problems.

5. **Applications**

Assume that a real estate agent has a set of different types of houses \( U = \{ u_1, u_2, u_3, u_4, u_5 \} \) which may be characterized by a set of parameters \( E = \{ x_1, x_2, x_3, x_4 \} \). For \( j = 1, 2, 3, 4 \) the parameters \( x_j \) stand for “in good location”, “cheap”, “modern”, “large”, respectively. Then we can give the following examples.
Example 5.1. Suppose that a married couple, Mr. X and Mrs. X, come to the real estate agent to buy a house. If each partner has to consider their own set of parameters, then we select a house on the basis of the sets of partners’ parameters by using the FSMmDM as follows.

Assume that \( U = \{u_1, u_2, u_3, u_4, u_5\} \) is a universal set and \( E = \{x_1, x_2, x_3, x_4\} \) is a set of all parameters.

**Step 1:** First, Mr. X and Mrs. X have to choose the sets of their parameters, \( A = \{x_2, x_3, x_4\} \) and \( B = \{x_1, x_3, x_4\} \), respectively.

**Step 2:** Then we can write the following \( fs \)-matrices which are constructed according to their parameters.

\[
\begin{bmatrix}
0 & 0 & 0.2 & 0.4 \\
0 & 0.6 & 0.9 & 0.4 \\
0.8 & 0.7 & 0.5 & 0.2 \\
0.5 & 0 & 0 & 0.8 \\
0 & 1 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 0 & 0.9 & 0.7 \\
0.2 & 0 & 0 & 0.9 \\
0.7 & 0 & 0.4 & 0.3 \\
0 & 0.5 & 0.6 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

**Step 3:** Now, we can find a product of the \( fs \)-matrices \([a_{ij}]\) and \([b_{ik}]\) by using And-product as follows

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.2 & 0 & 0.2 & 0.2 & 0.4 & 0 & 0.4 & 0.4 \\
0 & 0 & 0 & 0 & 0.2 & 0 & 0 & 0.6 & 0.2 & 0 & 0.9 & 0.2 & 0 & 0 & 0.4 & 0.4 \\
0 & 0 & 0 & 0.7 & 0 & 0 & 0.4 & 0.3 & 0.7 & 0 & 0.4 & 0.3 & 0.5 & 0 & 0.4 & 0.3 \\
0 & 0 & 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.8
\end{bmatrix}
\]

Here, we use And-product since both Mr. X and Mrs. X’s choices have to be considered.

**Step 4:** To calculate \( Mm([a_{ij}] \land [b_{ik}]) = [d_{ij}] \), we have to find \( d_{ij} \) for all \( i \in \{1, 2, 3, 4, 5\} \). To demonstrate, let us find \( d_{31} \). Since \( i = 1 \) and \( k \in \{1, 2, 3, 4\} \),

\[
d_{31} = \max_k \{t_{3k}\} = \max\{t_{31}, t_{32}, t_{33}, t_{34}\}
\]

Here, we have to find \( t_{3k} \) for all \( k \in \{1, 2, 3, 4\} \). To demonstrate, let us find \( t_{31} \) and \( t_{32} \). \( I_1 = \{p: c_{ip} \neq 0, 0 < p \leq 4\} = \emptyset \) for \( k = 1 \) and \( n = 4 \) and \( I_2 = \{p: c_{ip} \neq 0, 4 < p \leq 8\} = \{5, 7, 8\} \) for \( k = 2 \) and \( n = 4 \). Hence \( t_{31} = 0 \) and \( t_{32} = \min\{c_{35}, c_{37}, c_{38}\} = \min\{0.7, 0.4, 0.3\} = 0.3 \)

Similarly, we can find as \( t_{33} = 0.3 \) and \( t_{34} = 0.3 \). Thus,

\[
d_{31} = \max\{0.0, 0.3, 0.3, 0.3\} = 0.3
\]

Similarly we can find \( d_{11} = 0.4, d_{21} = 0.0, d_{41} = 0.0 \) and \( d_{51} = 0.0 \). Finally, we can obtain the \( fs \)-max-min decision \( fs \)-matrix as
Step 5: Finally, we can find an optimum fuzzy set on $U$ according to $Mm([a_{ij}] \land [b_{ik}])$

$$\text{opt}_{Mm([a_{ij}] \land [b_{ik}])}(U) = \{0.4/ u_1, 0.3/ u_3\}$$

where $u_1$ is an optimum house to buy for Mr. X and Mrs. X.

Similarly, we can also use products $[a_{ij}] \lor [b_{ik}]$, $[a_{ij}] \triangledown [b_{ik}]$ and $[a_{ij}] \triangledown [b_{ik}]$ for the other convenient problems.

6. Conclusion

The $fs$-set theory is being applied to many fields varying from theoretical to practical. In this paper, we define $fs$-matrices which are matrix representation of the $fs$-sets. We then define the set-theoretic operations of $fs$-matrices which are more functional to improve several new results. Afterwards, we construct a $fs$-decision making model on the $fs$-set theory. This new decision making method depends on the ideas of fuzzy and soft sets. Its main idea is similar to the decision making method, given in [7], which only depends on soft sets. Therefore, this method is more feasible than the others, because of its fuzziness. Finally, we give an application for a real estate agent to choose an optimal house.

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