Iranian Journal of Fuzzy Systems **Vol. 13, No. 2,** (2016) pp. 37-59

A HYBRID MULTI-ATTRIBUTE GROUP DECISION MAKING METHOD BASED ON GREY LINGUISTIC 2-TUPLE

C. RAO, J. ZHENG, C. WANG AND X. XIAO

Abstract. Because of the complexity of decision-making environment, the uncertainty of fuzziness and the uncertainty of grey maybe coexist in the problems of multi-attribute group decision making. In this paper, we study the problems of multi-attribute group decision making with hybrid grey attribute data (the precise values, interval numbers and linguistic fuzzy variables coexist, and each attribute value has a certain grey degree), and present a new grey hybrid multi-attribute group decision making method based on grey linguistic 2-tuple. Concretely, the concept of grey linguistic 2-tuple is defined based on the traditional linguistic 2-tuple, and the transformation methods of transforming the precise values, interval numbers and linguistic fuzzy variables into the grey linguistic 2-tuples are presented respectively. Further, a new grey linguistic 2-tuple weighted averaging ($GLTWA$) operator is presented to aggregate multiple decision makers' individual decision information into comprehensive decision information, and then a ranking method based on grey 2-tuple correlation degree is presented to rank all alternatives and to select the winners. An application decision making example of supplier selection is also given to validate the method developed and to highlight the implementation, practicality and effectiveness of the presented method.

1. Introduction

Multi-attribute group decision making (MAGDM) is widely spread in the fields of society, economy, management, engineering systems, and so on, which is a decision making activity where finding a desirable solution from a finite number of feasible alternatives assessed on multiple quantitative or qualitative attributes by multiple decision makers [41, 51, 53, 61, 66]. In the practical MAGDM, due to the problem's complexity and uncertainty, time pressure and people's limited expertise, sometimes decision makers not only provide their evaluation results in the form of precise values but also give their preference information in the form of interval number values, fuzzy numbers (such as triangular fuzzy number, trapezoidal fuzzy numbers, etc.), and linguistic fuzzy variables (such as "good", "poor", "fair", "high", "fast", etc.) [33-36]. Nowadays, multi-attribute group decision making under linguistic fuzzy environment is a hot research topic in the decision theory, which has received more and more attention from scholars.

Received: December 2014; Revised: August 2015; Accepted: January 2016

Key words and phrases: Hybrid multi-attribute group decision making, Grey linguistic 2-tuple, GLTWA operator, Grey 2-tuple correlation degree.

Up to now, many methods have been proposed to deal with the multi-attribute group decision making with linguistic fuzzy information. These methods mainly can be divided into four types [29, 50], i.e., (i) The methods based on extension principle, which makes operations on the fuzzy numbers that support the semantics of the linguistic terms [1, 2, 10]. (ii) The methods based on symbols, which makes computations on the indexes of the linguistic terms [3, 30]. (iii) The methods based on the linguistic aggregation operators which compute with words directly, such as extended ordered weighted averaging (EOWA) operator and extended ordered weighted geometric (EOWG) operator [57], induced uncertain linguistic OWA (IULOWA) operators [58], uncertain linguistic ordered weighted geometric (ULOWG) operator [59], uncertain linguistic hybrid geometric mean (ULHGM) operator [49], uncertain power-geometric (PG) operator and uncertain power ordered weighted geometric (UPOWG) operator [60], hesitant fuzzy prioritized weighted average (HFPWA) operator [53], weighted geometric aggregation (WGA) operator [23], uncertain pure linguistic hybrid harmonic averaging (UPLHHA) operator and a generalized interval aggregation (GIA) operator [31], interval-valued intuitionistic fuzzy Einstein hybrid weighted geometric (IVIFHWG) operator [48], induced linguistic ordered weighted geometric (ILOWG) operator and linguistic continuous ordered weighted geometric (LCOWG) operator [69], power average (PA) operator [39], interval-valued intuitionistic trapezoidal fuzzy weighted geometric (IVITFWG) operator, interval-valued intuitionistic trapezoidal fuzzy ordered weighted geometric (IVITFOWG) operator and interval-valued intuitionistic trapezoidal fuzzy hybrid geometric (IVITFHG) operator [55], uncertain linguistic ordered weighted averaging (ULOWAf) operator, induced uncertain linguistic ordered weighted averaging (IULOWAf) operator and uncertain linguistic weighted averaging (ULWAf) operator [18], cloud weighted arithmetic averaging (CWAA) operator, cloud-ordered weighted arithmetic averaging (COWA) operator, and cloud hybrid arithmetic (CHA) operator [47], interval-valued intuitionistic uncertain linguistic Choquet averaging (IVIULCA operator and the interval-valued intuitionistic uncertain linguistic Choquet geometric mean (IVIULCGM) operator [26], intuitionistic fuzzy weighted neutral averaging (IFWNA) operator and intuitionistic fuzzy ordered weighted neutral averaging (IFOWNA) operator [9], trapezoidal interval type-2 fuzzy geometric Bonferroni mean (TIT2FGBM) operator and the trapezoidal interval type-2 fuzzy weighted geometric Bonferroni mean (TIT2FWGBM) operator [8], power geometric operators of TrIFNs [43], intuitionistic linguistic power generalized weighted average (ILPGWA) operator, intuitionistic linguistic power generalized ordered weighted average (ILPGOWA) operator [24], partial binary tree DEA-DA cyclic classification model [20], hesitant fuzzy linguistic term set (HFLTS) method [46], interval-valued intuitionistic fuzzy (IVIF) mathematical programming method [42], and the method of uncertain linguistic fuzzy soft sets (ULFSSs) [37]. (iv) The methods based on 2-tuple linguistic representation model [11-13, 25].

The 2-tuple linguistic representation model was proposed by Herrera and Martinez in 2000 based on the concept of symbolic translation [11]. Practice has proven that this model can avoid information distortion and information loss in the linguistic information processing. In recent years, the 2-tuple linguistic model has been examined and widely applied in group decision making problems. Herrera and Martnez [11] first presented 2-tuple arithmetic averaging (TAA) operator, 2-tuple weighted averaging (TWA) operator, 2-tuple ordered weighted averaging (TOWA) operator and extended 2-tuple weighted averaging (ET-WA) operator. Based on these operators, many new aggregation operators and corresponding MAGDM methods have been proposed. For example, Wang [44] presented a modified linguistic OWA operator on the base of entropy maximization. Wang and Hao [45] developed the quantifier-guided OWA aggregation operator and anchoring value-based OWA aggregation operator for 2-tuples. Dong et al. [5] proposed the 2-tuple OWA operator and the extended OWA operator. Merig et al. [27] developed a new approach for decision making with Dempster-Shafer theory of evidence by using linguistic information, and suggested the use of different types of linguistic aggregation operators in the model. Wei [50] proposed the extended 2-tuple weighted geometric (ET-WG) operator and the extended 2-tuple ordered weighted geometric (ET-OWG) operator, and presented a method based on these two operators for solving the problems of multiple attribute group decision-making with linguistic information of attribute values and weight values. Xu et al. [63] used the virtual linguistic label to replace 2-tuple linguistic variable and proposed the linguistic power average operators including LPA, LPWA and LPOWA. They further developed the uncertain linguistic power average operators, such as ULPA, ULPWA and ULPOWA. Wei and Zhao [54] developed some dependent 2-tuple linguistic aggregation operators: the dependent 2-tuple ordered weighted averaging (D2TOWA) operator and the dependent 2-tuple ordered weighted geometric (D2TOWG) operator, and then applied them to develop some approaches for multiple attribute group decision making with 2-tuples linguistic information. By using the Choquet integral, Yang and Chen [65] proposed the 2-tuple correlated averaging operator, the 2-tuple correlated geometric operator and the generalized 2-tuple correlated averaging operator. Park et al. [29] presented the 2-tuple linguistic weighted harmonic (2TLWH) operator and 2-tuple linguistic hybrid harmonic (2TLHH) operator, and then presented an approach to multiple attribute group decision making with 2-tuple linguistic information based on these two operators. Wan [40] studied the multi-attribute group decision making (MAGDM) problems in which the attribute values, attribute weights, and expert weights are all in the form of 2-tuple linguistic information, and solved this kind of problems by developing a new decision method based on 2-tuple linguistic hybrid arithmetic aggregation operator. Merig and Gil-Lafuente [28] presented an induced 2-tuple linguistic generalized ordered weighted averaging (2-TILGOWA) operator, and discussed its application in multi-attribute decision-making. Zhang [68] introduced some new aggregation operators such as the interval-valued 2-tuple weighted geometric (IVTWG) operator, the interval-valued 2-tuple ordered weighted geometric (IVTOWG) operator, the generalized interval-valued 2-tuple weighted average (GIVTWA) operator and the generalized interval-valued 2-tuple ordered weighted average (GIVTOWA), and discussed their applications in multi-attribute group decision making. Xu et al. [64] investigated decision making problems with fuzzy linguistic information, and developed some new aggregation operators such as PT weighted geometric averaging operator, PT ordered weighted geometric averaging operator and PT hybrid geometric averaging operator, motivated by the idea of proportional 2-tuple (PT). Estrella et al. [7] proposed a fuzzy linguistic decision tools enhancement suite so-called Flintstones to solve linguistic decision making problems based on the 2-tuple linguistic model and its extensions. Xu et al. [62] proposed a four-way procedure to estimate missing preference values when dealing with acceptable incomplete 2-tuple fuzzy linguistic preference relations (FLPRs), and gave a simple algorithm to select the best alternative for the group decision making (GDM) problem with incomplete 2-tuple FLPRs. Truck [38] stressed on a comparison between both the 2-tuple semantic model and the 2-tuple symbolic model, and they obtained that the 2-tuple semantic model can generate a partitioning that is identical to the one that would be generated thanks to the symbolic 2-tuple model. Also, some valuable applications on 2-tuples are given so far, for example, Liu et al. [21] proposed a modified MULTIMOORA method based on interval 2-tuple linguistic variables (named ITL-MULTIMOORA) for evaluating and selecting HCW treatment technologies, Doukas et al. [6] presented a coherent and transparent methodological multi-criteria framework based on the developed 2-tuple TOPSIS method for assessing companies' energy and environmental corporate policies, Rao et al. [32] presented a fuzzy multi-attribute group decision making (FMAGDM) technique based on a linguistic 2-tuple to evaluate potential alternative CLC locations, and so on.

Combining 2-tuple linguistic representation model with other methods, many combination methods for group decision making were proposed. Ju et al. [16] presented a hybrid fuzzy method consisting fuzzy AHP and 2-tuple fuzzy linguistic approach to evaluate emergency response capacity. Ju and Wang [15] proposed a new method to solve multi-criteria group decision making problems in which both the criteria values and criteria weights take the form of linguistic information based on the traditional idea of VIKOR method and 2-tuple linguistic representation model. Wei [52] presented a combination method by combining the traditional grey relational analysis (GRA) with 2-tuple linguistic representation model to solve the 2-tuple linguistic multiple attribute group decision making problems within complete weight information. Liu [22] et al. presented a novel MAGDM approach with multi-granularity linguistic assessment information based on deviation and TOPSIS. You et al. [67] proposed an extended VIKOR method for group multi-criteria supplier selection with interval 2-tuple linguistic information. Lin et al. [19] proposed a new interval linguistic aggregation operator based on the Shapley value and 2-tuple linguistic representation.

Above literature has done valuable works to provide many valid and feasible methods to solve the multi-attribute group decision making problems with linguistic fuzzy information. These methods can well deal with the uncertainty of fuzziness in the multi-attribute group decision making problems. In fact, in addition to the uncertainty of fuzziness, the uncertainty of grey maybe also exist in the practical decision making problems. Generally, the uncertainty of grey is caused by incomplete and inadequate information due to the complexity of decision-making environment, the constraints of decision time and decision space, and so on. For example, a decision maker will evaluate the carbon emissions level for three enterprises according to the carbon emissions data in the last five years. The enterprise 1 can provide all data of last five years, but due to some objective reasons (such as machine fault, natural disasters, trade secret, etc.), the enterprise 2 can only provide the statistical data of last three years, and the enterprise 3 can only provide the statistical data of last four years. Thus, the decision maker faces a decision environment with incomplete and inadequate information. In this case, the evaluation results (high, low, average etc.) on carbon emissions level of the enterprise 2 and the enterprise 3 has a certain degree of grey uncertainty (which is called grey degree). However, up to now, few literature considered both the uncertainty of fuzziness and the uncertainty of grey in a multi-attribute group decision making problem with linguistic fuzzy information.

In this paper, we investigate a kind of multi-attribute group decision making problems with hybrid grey attribute data (the precise values, interval numbers and linguistic fuzzy variables coexist, and each attribute value has a certain grey degree), and present a new grey hybrid multi-attribute group decision making method based on grey linguistic 2-tuple. Different from other methods, our contributions are thus to consider both the uncertainty of fuzziness and the uncertainty of grey in a multi-attribute group decision making problem with hybrid grey attribute data, and to present the grey linguistic 2-tuple to express the grey fuzzy data, and to propose a ranking method of grey 2-tuple correlation degree to rank alternatives and select winners.

The rest of the paper is organized as follows. Section 2 defines the concept of grey linguistic 2-tuple based on the traditional linguistic 2-tuple, and presents a new grey linguistic 2-tuple weighted averaging ($GLTWA$) operator to aggregate multiple grey linguistic 2-tuples. Section 3 presents the transformation methods of transforming the precise values, interval numbers and linguistic fuzzy variables into the grey linguistic 2-tuples respectively, and then proposes a ranking method based on grey 2-tuple correlation degree to rank all alternatives and select winners. Section 4 gives a decision making example of supplier selection to show how to implement the proposed decision method and to demonstrate the effectiveness and feasibility of the presented method. Section 5 concludes the paper.

2. **Preliminaries**

2.1. **Linguistic 2-tuple.** The concept of linguistic 2-tuple was first presented by Herrera and Martinez [11] to deal with the decision making problems with linguistic assessment information. Concretely, a linguistic 2-tuple is expressed by a dual combination (s_k, a_k), where s_k and a_k are given as follows.

(1) s_k is the k-th element in a predefined linguistic evaluation set S, where S is formed by $t + 1$ linguistic fuzzy variables, i.e., $S = \{s_0, s_1, \ldots, s_t\}$, where s_k is the k-th element in S, and it satisfies the following characteristics. (i) Property of ordering, i.e., if $k \geq l$, then $s_k \geq s_l$. (ii) Negation operator. $Neg(s_k) = s_l$, where $l = t - k$. (iii) Max operator and Min operator. If $s_k \geq s_l$, then max(s_k, s_l)=s_k and min(s_k, s_l)=s_l.

(2) a_k is a numerical value that represents the value of the symbolic translation, such that $a_k \in [-0.5, 0.5)$, which means the deviation between the evaluation result and s_k.

For the linguistic evaluation set S, for example, a set of five terms S could be given as follows.

$$S = \{s_0 : \text{Very\ \ poor}, s_1 : \text{Poor}, s_2 : \text{Average}, s_3 : \text{Good}, s_4 : \text{Very\ \ good}\}.$$

Next some relative definitions with linguistic 2-tuple are given as follows.

Definition 2.1. Let $s_k \in S$ be a linguistic fuzzy variable, then its corresponding linguistic 2-tuple can be obtained by the following function θ.

$$\theta : S \to S \times [-0.5, 0.5),$$
$$\theta(s_k) = (s_k, 0), s_k \in S.$$

Definition 1 gives a direct transformation method between a linguistic fuzzy variable and a linguistic 2-tuple. That is, the corresponding linguistic 2-tuple for a linguistic fuzzy variable $s_k \in S$ is just $(s_k, 0)$.

The following Definition 2 is presented by Herrera and Martinez [11] to give a transformation method between a numeric value and a linguistic 2-tuple.

Definition 2.2. Let $S = \{s_0, s_1, \ldots, s_t\}$ be a known linguistic evaluation set, and $\beta \in [0, t]$ be a real number which is a value supporting the result of a symbolic aggregation operation, then $\beta \in [0, t]$ can be transformed into an equivalent linguistic 2-tuple by the function Δ:

$$\Delta : [0, t] \to S \times [-0.5, 0.5), \Delta(\beta) = (s_k, a_k),$$

where

$$k = \text{round}(\beta), \quad a_k = \beta - k, a_k \in [-0.5, 0.5)$$

and "round" is the usual rounding operation. Conversely, for a known linguistic 2-tuple (s_k, a_k), there is an inverse function Δ^{-1} such that from a 2-tuple (s_k, a_k) it returns its equivalent numerical value $\beta \in [0, t]$, i.e.,

$$\Delta^{-1} : S \times [-0.5, 0.5) \to [0, t],$$
$$\Delta^{-1}(s_k, a_k) = k + a_k = \beta.$$

Specially, Zhang [68] presented a similar transformation method based on the method in Definition 2 between a numerical value $\beta \in [0, 1]$ and a linguistic 2-tuple.

Definition 2.3. Let $S = \{s_0, s_1, \ldots, s_t\}$ be a known linguistic evaluation set, and $\beta \in [0, 1]$ be a real number which is a value supporting the result of a symbolic aggregation operation, then $\beta \in [0, 1]$ can be transformed into an equivalent linguistic 2-tuple by function Δ:

$$\Delta : [0, 1] \to S \times [-0.5, 0.5), \Delta(\beta) = (s_k, a_k),$$

where

$$k = \text{round}(\beta \cdot t), \quad a_k = \beta \cdot t - k, a_k \in [-0.5, 0.5)$$

and "round" is the usual rounding operation. Conversely, for a known linguistic 2-tuple (s_k, a_k), there exists an inverse function Δ^{-1} such that from a 2-tuple (s_k, a_k) it returns its equivalent numerical value $\beta \in [0, 1]$, i.e.,

$$\Delta^{-1} : S \times [-0.5, 0.5) \to [0, 1],$$

$$\Delta^{-1}(s_k, a_k) = \frac{k + a_k}{t} = \beta.$$

For an interval number, we can use the following Definition 4 [13] to transform it into a linguistic 2-tuple.

Definition 2.4. Let $S = \{s_0, s_1, \dots, s_t\}$ be a known linguistic evaluation set, and $I = [a, b]$ be an interval number, the membership function of $I = [a, b]$ is denoted as

$$\mu_I(x) = \begin{cases} 1, & x \in [a, b] \\ 0, & \text{else,} \end{cases}$$

and the intersection of the interval number $I = [a, b]$ and every linguistic variable s_k inside the predefined standard linguistic evaluation set $S = \{s_0, s_1, \dots, s_t\}$ is denoted as

$$r_k = \max_x \min\{\mu_I(x), \mu_{s_k}(x)\}, \quad k \in \{0, 1, \dots, t\}.$$

Then the crisp value $I_\beta = \dfrac{\sum_{k=1}^{t} k \cdot r_k}{\sum_{k=1}^{t} r_k}$ is the equivalent numerical value of a 2-tuple (s_k, a_k). Afterward we use the method given by Definition 2 to transform I_β into a linguistic 2-tuple (s_k, a_k), which is the corresponding linguistic 2-tuple for interval number $I = [a, b]$.

In addition, for any two linguistic 2-tuples (s_k, a_k) and (s_l, a_l) , the comparison rules are as follows.

(1) If $k > l$, then $(s_k, a_k) > (s_l, a_l)$.
(2) If $k = l$, three cases exist, i.e.,
(i) If $a_k = a_l$, then $(s_k, a_k) = (s_l, a_l)$.
(ii) If $a_k > a_l$, then $(s_k, a_k) > (s_l, a_l)$.
(iii) If $a_k < a_l$, then $(s_k, a_k) < (s_l, a_l)$.

2.2. **Grey Linguistic 2-tuple.** In practical decision making, due to the complexity of the decision-making environment, and the constraints of decision time and decision space, the decision makers' differences in knowledge frame, experience level, status and individual preference, the attribute values given by decision makers often have a certain grey degree. Grey degree is the measurement to measure the insufficiency, incompletion or unlikelihood of information. Generally, the information content can be divided into five levels, i.e., very sufficient, sufficient, common, incomplete and very incomplete. Now we give the definition of grey degree of information and its quantization method in the following Definition 5.

Definition 2.5. Let the set of information content be $U = \{u_1 : \text{very sufficient}, u_2 : \text{sufficient}, u_3 : \text{common}, u_4 : \text{incomplete}, u_5 : \text{very incomplete}\}$, then u_1, u_2, u_3, u_4, and u_5 are called the grey degree of information, where u_1, u_2, u_3, u_4, and u_5 are all real numbers, and their quantization methods are as follows:

$$u_1 \in [0, 0.2), u_2 \in [0.2, 0.5), u_3 \in [0.5, 0.6), u_4 \in [0.6, 0.9), u_5 \in [0.9, 1].$$

In this paper, we will consider the problem of multi-attribute group decision making with grey hybrid attribute data, i.e., the precise values, interval numbers and linguistic fuzzy variables coexist in the attribute values, and each attribute value has a certain grey degree. In order to aggregate grey hybrid attribute data to make a comprehensive evaluation for all alternatives, we will transform all hybrid attribute data into grey linguistic 2-tuples. The grey linguistic 2-tuple is defined in the following Definition 6.

Definition 2.6. Let (s_k, a_k) be a linguistic 2-tuple with a grey degree v, where $v \in U = \{u_1, u_2, \ldots, u_5\}$ and $v \in [0, 1]$, then $g = ((s_k, a_k), v)$ is called a grey linguistic 2-tuple, where v is called the grey part of g.

Especially, when $v = 0$, g is just a linguistic 2-tuple. For any two grey linguistic 2-tuples $g_1 = ((s_k, a_k), v_1)$ and $g_2 = ((s_l, a_l), v_2)$, if $(s_k, a_k) = (s_l, a_l)$ and $v_1 = v_2$, then g_1 is equal to g_2.

The operations of grey linguistic 2-tuples are defined as follows.

Definition 2.7. Let $g_1 = ((s_k, a_k), v_1)$ and $g_2 = ((s_l, a_l), v_2)$ be two grey linguistic 2-tuples, the relative operation rules for g_1 and g_2 are defined as follows:

1) $g_1 + g_2 = (\Delta(\Delta^{-1}(s_k, a_k) + \Delta^{-1}(s_l, a_l)), (v_1 + v_2) \wedge 1)$;

2) $kg_1 = (\Delta(k\Delta^{-1}(s_k, a_k)), (kv_1) \wedge 1)$, $k \geq 0$,

where "$\wedge$" is the usual minimizing operation.

Definition 2.8. Let $g_1 = ((s_k, a_k), v_1)$ and $g_2 = ((s_l, a_l), v_2)$ be two grey linguistic 2-tuples, the distance operation from g_1 and g_2 is defined by

$$d(g_1, g_2) = \frac{1}{2}(|\Delta^{-1}(s_k, a_k) - \Delta^{-1}(s_l, a_l)| + |v_1 - v_2|).$$

Theorem 2.9. *Let $g_1 = ((s_k, a_k), v_1)$, $g_2 = ((s_l, a_l), v_2)$ and $g_3 = ((s_h, a_h), v_3)$ be any three grey linguistic 2-tuples, then the distance operation $d(g_1, g_2)$ given by Definition 8 satisfies the following conditions.*

(i) $d(g_1, g_2) \geq 0$, $d(g_1, g_2) = 0 \Longleftrightarrow g_1 = g_2$;

(ii) $d(g_1, g_2) = d(g_2, g_1)$;

(iii) $d(g_1, g_3) \leq d(g_1, g_2) + d(g_2, g_3)$.

Proof. From Definition 6, (i) and (ii) are obvious. Here we merely prove condition (iii).

$$d(g_1, g_3) = \frac{1}{2}(|\Delta^{-1}(s_k, a_k) - \Delta^{-1}(s_h, a_h)| + |v_1 - v_3|)$$
$$= \frac{1}{2}(|\Delta^{-1}(s_k, a_k) - \Delta^{-1}(s_l, a_l) + \Delta^{-1}(s_l, a_l) - \Delta^{-1}(s_h, a_h)| + |v_1 - v_2 + v_2 - v_3|)$$
$$\leq \frac{1}{2}(|\Delta^{-1}(s_k, a_k) - \Delta^{-1}(s_l, a_l)| + |\Delta^{-1}(s_l, a_l) - \Delta^{-1}(s_h, a_h)| + |v_1 - v_2| + |v_2 - v_3|)$$
$$= d(g_1, g_2) + d(g_2, g_3).$$

□

Theorem 1 is proved.

Based on the above definitions and operations, we present the following operation to aggregate multiple grey linguistic 2-tuples.

Definition 2.10. Let $((r_1, a_1), v_1)$, $((r_2, a_2), v_2)$, ..., $((r_n, a_n), v_n)$ be n grey linguistic 2-tuples, and $W = \{w_1, w_2, \ldots, w_n\}$ is the weight vector of grey linguistic 2-tuples $((r_j, a_j), v_j)$, $j = 1, 2, \ldots, n$, such that $0 \leq w_j \leq 1$ and $\sum_{j=1}^{n} w_j = 1$, then the grey linguistic 2-tuple weighted averaging ($GLTWA$) operator is defined as

$$GLTWA_W(((r_1, a_1), v_1), ((r_2, a_2), v_2), \ldots, ((r_n, a_n), v_n))$$
$$= \left(\Delta \left(\sum_{j=1}^{n} w_j \Delta^{-1}(r_j, a_j) \right), (\sum_{j=1}^{n} w_j v_j) \wedge 1 \right).$$

We now discuss the properties for above $GLTWA$ operator.

Theorem 2.11. *Let $((r_1^{'}, a_1^{'}), v_1^{'})$, $((r_2^{'}, a_2^{'}), v_2^{'})$, ..., $((r_n^{'}, a_n^{'}), v_n^{'})$ be any permutation of $((r_1, a_1), v_1)$, $((r_2, a_2), v_2)$, ..., $((r_n, a_n), v_n)$, then we have*

$$GLTWA_W(((r_1, a_1), v_1), ((r_2, a_2), v_2), \ldots, ((r_n, a_n), v_n))$$
$$= GLTWA_W(((r_1^{'}, a_1^{'}), v_1^{'}), ((r_2^{'}, a_2^{'}), v_2^{'}), \ldots, ((r_n^{'}, a_n^{'}), v_n^{'})),$$

which means the GLTWA operator has the property of commutativity.

Proof. From the definition of $GLTWA$ operator given by Definition 9, Theorem 2 is obvious. □

Theorem 2.12. *If $((r_1, a_1), v_1) = ((r_2, a_2), v_2) = \ldots = ((r_n, a_n), v_n) = ((r, a), v)$, then*

$$GLTWA_W(((r_1, a_1), v_1), ((r_2, a_2), v_2), \ldots, ((r_n, a_n), v_n)) = ((r, a), v),$$

which means the GLTWA operator has the property of idempotency.

Proof. By $((r_1, a_1), v_1) = ((r_2, a_2), v_2) = \ldots = ((r_n, a_n), v_n) = ((r, a), v)$ and $\sum_{j=1}^{n} w_j = 1$, we have

$$GLTWA_W(((r_1, a_1), v_1), ((r_2, a_2), v_2), \ldots, ((r_n, a_n), v_n))$$
$$= \left(\Delta \left(\sum_{j=1}^{n} w_j \Delta^{-1}(r_j, a_j) \right), (\sum_{j=1}^{n} w_j v_j) \wedge 1 \right)$$
$$= \left(\Delta \left(\sum_{j=1}^{n} w_j \Delta^{-1}(r, a) \right), (\sum_{j=1}^{n} w_j v) \wedge 1 \right)$$
$$= \left(\Delta \left(\Delta^{-1}(r, a) \sum_{j=1}^{n} w_j \right), (v \sum_{j=1}^{n} w_j) \wedge 1 \right)$$
$$= ((r, a), v).$$

The theorem is proved. □

3. Decision Making Method Based on Grey Linguistic 2-tuple

3.1. Problem Description. The problem of grey hybrid multi-attribute group decision making (GHMAGDM) can be described as follows. Let $X = \{X_1, X_2, \dots, X_m\}$ be the set of alternatives, $A = \{A_1, A_2, \dots, A_n\}$ be the set of evaluation attributes, and $W = \{w_1, w_2, \dots, w_n\}$ be the set of attribute weights, which satisfies $0 \le w_j \le 1$ and $\sum_{j=1}^{n} w_j = 1$.

Suppose that l decision makers $M_1, M_2, \dots, M_l$ are invited to evaluate m alternatives, and the weight set of these decision makers is $\omega = (\omega_1, \omega_2, \dots, \omega_l)$, such that $0 \le \omega_j \le 1$ and $\sum_{k=1}^{l} \omega_k = 1$. The original decision matrix formed by the values of n evaluation attributes for m alternatives given by decision maker M_k is denoted as $B_k = (b_{ij}^{(k)}, v_{ij}^{(k)})_{m\times n}$, $i = 1, 2, \dots, m$, $j = 1, 2, \dots, n$, $k = 1, 2, \dots, l$, where $b_{ij}^{(k)}$ is the evaluation value for alternative X_i with respect to attribute A_j given by decision maker M_k, and $v_{ij}^{(k)}$ is the grey degree of evaluation value $b_{ij}^{(k)}$. We set $N_1 = \{1, 2, \dots, n_1\}$, $N_2 = \{n_1 + 1, n_1 + 2, \dots, n_2\}$, $N_3 = \{n_2 + 1, n_2 + 2, \dots, n\}$. If $j \in N_1$, then $b_{ij}^{(k)} \in R$. If $j \in N_2$, then $b_{ij}^{(k)}$ is an interval number $b_{ij}^{(k)} = [b_{ij}^{(k)L}, b_{ij}^{(k)U}]$. If $j \in N_3$, then $b_{ij}^{(k)}$ is a linguistic fuzzy variable (i.e., Very poor, Poor, Average, Good and Very good).

Now the decision goal is to evaluate and to rank all alternatives according to the data information in $B_1, B_2, \dots, B_l$, and then to select the winners which we need among all alternatives.

3.2. Data Processing for Original Decision Data. From Section 3.1, we know that the precise values, interval numbers and linguistic fuzzy variables coexist in matrixes $B_1, B_2, \dots, B_l$, and each attribute value has a certain grey degree. Here we transform all hybrid attribute data in $B_1, B_2, \dots, B_l$ into grey linguistic 2-tuples. The transformation methods are as follows.

1) If $j \in N_1 = \{1, 2, \dots, n_1\}$, then $b_{ij}^{k} \in R$.

(i) For the cost type attributes, the attribute value $b_{ij}^{(k)}$ can be normalized by the following equation (1).

$$y_{ij}^{(k)} = \frac{\max\limits_i b_{ij}^{(k)} - b_{ij}^{(k)}}{\max\limits_i b_{ij}^{(k)} - \min\limits_i b_{ij}^{(k)}}, i = 1, 2, \dots, m. \tag{1}$$

For the benefit type attributes, the attribute value $b_{ij}^{(k)}$ can be normalized by the following equation (2).

$$y_{ij}^{(k)} = \frac{b_{ij}^{(k)} - \min\limits_i b_{ij}^{(k)}}{\max\limits_i b_{ij}^{(k)} - \min\limits_i b_{ij}^{(k)}}, i = 1, 2, \dots, m. \tag{2}$$

2) If $j \in N_2 = \{n_1 + 1, n_1 + 2, \dots, n_2\}$, then $b_{ij}^{(k)}$ is an interval number $b_{ij}^{(k)} = [b_{ij}^{(k)L}, b_{ij}^{(k)U}]$.

(i) For the cost type attributes, the attribute value $b_{ij}^{(k)} = [b_{ij}^{(k)L}, b_{ij}^{(k)U}]$ can be normalized by the following equation (3).

$$y_{ij}^{(k)L} = \frac{\frac{1}{b_{ij}^{(k)U}}}{\sqrt{\sum_{i=1}^{m}(\frac{1}{b_{ij}^{(k)L}})^2}}, y_{ij}^{(k)U} = \frac{\frac{1}{b_{ij}^{(k)L}}}{\sqrt{\sum_{i=1}^{m}(\frac{1}{b_{ij}^{(k)U}})^2}}, i = 1, 2, \ldots, m. \tag{3}$$

(ii) For the benefit type attributes, the attribute value $b_{ij}^{(k)} = [b_{ij}^{(k)L}, b_{ij}^{(k)U}]$ can be normalized by the following equation (4).

$$y_{ij}^{(k)L} = \frac{b_{ij}^{(k)L}}{\sqrt{\sum_{i=1}^{m}(b_{ij}^{(k)U})^2}}, y_{ij}^{(k)U} = \frac{b_{ij}^{(k)U}}{\sqrt{\sum_{i=1}^{m}(b_{ij}^{(k)L})^2}}, i = 1, 2, \ldots, m. \tag{4}$$

3) If $j \in N_3$, then $b_{ij}^{(k)}$ is a linguistic fuzzy variable such as Very poor, Poor, Average, Good and Very good. The value of linguistic fuzzy variable $b_{ij}^{(k)}$ $(i = 1, 2, \ldots, m)$ can be determined by the following equation (5).

$$\text{Very poor} = s_0, \text{Poor} = s_1, \text{Average} = s_2, \text{Good} = s_3, \text{Very good} = s_4. \tag{5}$$

By the above data processing, the original evaluation matrixes $B_1, B_2, \ldots, B_l$ become l new decision matrixes $Y_1, Y_2, \ldots, Y_l$, where $Y_k = (y_{ij}^{(k)}, v_{ij}^{(k)})_{m\times n}$, and

$$y_{ij}^{(k)} = \begin{cases} \beta \in [0,1] \in R, & j \in N_1, \\ [y_{ij}^{(k)L}, y_{ij}^{(k)U}], & j \in N_2, \\ s_k \in S = \{s_0, s_1, s_2, s_3, s_4\}, & j \in N_3. \end{cases}$$

For the decision matrixes $Y_1, Y_2, \ldots, Y_l$, we use the transformation methods given in Definition 1, Definition 3 and Definition 4 to transform the linguistic fuzzy variables, precise numbers and interval numbers into linguistic 2-tuples respectively, and the decision matrix $Z_k = (((r_{ij}^{(k)}, a_{ij}^{(k)}), v_{ij}^{(k)}))_{m\times n}$, $k = 1, 2, \ldots, l$ is obtained.

In order to make a comprehensive evaluation for all alternatives, we use the $GLTWA$ operator given in Definition 9 to aggregate the overall evaluation value corresponding to decision maker M_k and get the collective overall evaluation value for all alternatives. Then the comprehensive decision matrix $C = (((r_{ij}, a_{ij})v_{ij}))_{m\times n}$ is obtained, where

$$\begin{aligned}((r_{ij}, a_{ij}), v_{ij}) &= GLTWA_w(((r_{ij}^{(1)}, a_{ij}^{(1)}), v_{ij}^{(1)}), ((r_{ij}^{(2)}, a_{ij}^{(2)}), v_{ij}^{(2)}), \ldots, ((r_{ij}^{(l)}, a_{ij}^{(l)}), v_{ij}^{(l)})) \\ &= \left(\Delta\left(\sum_{k=1}^{l}\omega_k \Delta^{-1}(r_{ij}^{(k)}, a_{ij}^{(k)})\right), (\sum_{k=1}^{l}\omega_k v_{ij}^{(k)}) \wedge 1\right),\end{aligned} \tag{6}$$

where $\omega = (\omega_1, \omega_2, \ldots, \omega_l)$ is the weight set of l decision makers.

3.3. **The ranking method based on grey 2-tuple correlation degree.** In this section, we will present a new ranking method based on grey 2-tuple correlation degree. Now we give the interrelated definitions of grey 2-tuple correlation degree.

Definition 3.1. Let $C = (((r_{ij}, a_{ij}), v_{ij}))_{m\times n}$ from equation (6) be a comprehensive decision matrix, and we set

$$\begin{aligned}h^+ &= (h_1^+, h_2^+, \ldots, h_n^+) \\ &= ((\max_i(r_{i1}, a_{i1}), \min_i v_{i1}), (\max_i(r_{i2}, a_{i2}), \min_i v_{i2}), \ldots, (\max_i(r_{in}, a_{in}), \min_i v_{in}),\end{aligned} \tag{7}$$

where "max" is the maximizing operation of linguistic 2-tuple, and "min" is the minimizing operation of grey degree. Then h^+ is called a positive ideal solution.

For the comprehensive decision matrix $C = (((r_{ij}, a_{ij}), v_{ij}))_{m\times n}$, the n elements of each row form a sequence, that is,

$$h_i = (((r_{i1}, a_{i1}), v_{i1}), ((r_{i2}, a_{i2}), v_{i2}), \dots, ((r_{in}, a_{in}), v_{in}), i = 1, 2, \dots, m. \tag{8}$$

h_i is called as alternative point i.

Based on the sequence $h_i(i = 1, 2, \dots, m)$ and the positive ideal solution h^+, we now give the definition of grey 2-tuple correlation degree as follows.

Definition 3.2. From the comprehensive decision matrix $C = (((r_{ij}, a_{ij}), v_{ij}))_{m\times n}$, let $h_1, h_2, \dots, h_m$ which are expressed by (8) be the compared sequences, and the positive ideal solution h^+ which is expressed by (7) be the reference sequence, then the grey relational coefficient between the grey linguistic 2-tuple h_j^+ and $h_i(j)$ is defined as the following equation (9).

$$r(h_j^+, h_i(j)) = \frac{0.5 \max\limits_i \max\limits_j d(h_j^+, h_i(j))}{d(h_j^+, h_i(j)) + \max\limits_i \max\limits_j d(h_j^+, h_i(j))}, \tag{9}$$

and the grey 2-tuple correlation degree between h^+ and $h_i(i = 1, 2, \dots, m)$ is defined as the following equation (10).

$$r(h^+, h_i) = \sum_{j=1}^{n} w_j r(h_j^+, h_i(j)), \tag{10}$$

where

$$d(h_j^+, h_i(j)) = \frac{1}{2}(|\Delta^{-1}[\max_i(r_{ij}, a_{ij})] - \Delta^{-1}(r_{ij}, a_{ij})| + |\min_i v_{ij} - v_{ij}|).$$

By using the similar proof process of the traditional grey correlation degree [4, 56], we can easily prove that the grey 2-tuple correlation degree which is defined by Definition 11 satisfies the four axioms of grey relational analysis (GRA), i.e., normality, symmetry, wholeness and approachability.

For the grey 2-tuple correlation degree $r(h^+, h_i)$ between h^+ and h_i(i=1,2,$\dots, m$), the greater the value of $r(h^+, h_i)$, the better is the alternative X_i. Thus we can rank the order for all alternatives according to the value of $r(h^+, h_i), i = 1, 2, \dots, m$.

3.4. **Decision Making Steps.** Synthesize the discussion and analysis of Section 3.1, Section 3.2 and Section 3.3, now we give the detailed decision making steps for solving the problems of GHMAGDM.

Step 1: For m decision alternatives, l decision makers give the original evaluation matrixes $B_1, B_2, \dots, B_l$.

Step 2: Use equations (1), (2), (3), (4) and (5) to process the data in the original evaluation matrixes $B_1, B_2, \dots, B_l$, the normalized decision matrixes $Y_1, Y_2, \dots, Y_l$ are obtained.

Step 3: Use the transformation methods given in Definition 1, Definition 3 and Definition 4 to transform the linguistic fuzzy variables, precise numbers and interval numbers in decision matrixes $Y_1, Y_2, \ldots, Y_l$ into linguistic 2-tuples respectively, and the decision matrix $Z_1, Z_2, \ldots, Z_l$ formed by grey linguistic 2-tuples are obtained.

Step 4: Use equation (6) to aggregate the evaluation matrixes $Z_1, Z_2, \ldots, Z_l$ of l decision makers to a comprehensive decision matrix. Then the comprehensive decision matrix $C = (((r_{ij}, a_{ij}), v_{ij}))_{m\times n}$ is found.

Step 5: Use equations (7) and (8) to determine the reference sequence h^+ and the compared sequences $h_1, h_2, \ldots, h_m$ from comprehensive decision matrix $C = (((r_{ij}, a_{ij}), v_{ij}))_{m\times n}$, respectively.

Step 6: Use equations (9) and (10) to calculate the grey relational coefficient $r(h_j^+, h_i(j))$ and the grey 2-tuple correlation degree $r(h^+, h_i), i = 1, 2, \ldots, m$.

Step 7: Rank all alternatives in accordance to the value of $r(h^+, h_i), i = 1, 2, \ldots, m$. The greater the value of $r(h^+, h_i)$, the better is the alternative X_i.

4. Numerical Illustration

In this section, we give a decision making example to show how to implement above GHMAGDM method and to demonstrate the effectiveness and feasibility of this method.

Now we consider a supplier selection problem. Suppose that a purchasing company wants to procure Q_0 barrels of Brent oil, and six alternative suppliers participate in the supply competition. The set of alternative suppliers is denoted as $X = \{X_1, X_2, \ldots, X_6\}$. The purchasing company invites three experts to participate in the evaluation decision, and the weight set of three experts is $\omega = (\omega_1, \omega_2, \omega_3) = (0.25, 0.35, 0.4)$. Six evaluation attributes [14, 17] are used to evaluate these six alternative suppliers, i.e., A_1 Price (\$/barrel), A_2 Delivery time (days), A_3 Quality, A_4 Supplier reputation, A_5 Carbon emissions (tonnes), A_6 Level of environmental management. Let the weight set of these six attributes be $W = (w_1, w_2, \ldots, w_6) = (0.2, 0.15, 0.2, 0.15, 0.15, 0.15)$. Each attribute is described briefly as follows.

A_1 Price, refers to the selling price of per tonne oil for a supplier. Generally, suppliers will submit the values directly to the buyer in the form of precise values in accordance to their actual total input costs.

A_2 Delivery time, refers to the time taken by a supplier to deliver Q_0 barrels of Brent oil to a buyer under contract. This time is affected by production, transportation and inventory, so it is generally given by the supplier in the form of interval numbers to the buyer in order to avoid any contractual disputes.

A_3 Quality, refers to the quality level of oil, which can be characterized by the standard of the quality system. The experts evaluate whether the supplier implements the required quality system or a related specification for the oil which he wants to procure. The attribute values of the quality level are usually given in the form of linguistic fuzzy variables such as Very poor, Poor, Average, Good and Very good.

Supplier	A_1	A_2	A_3	A_4	A_5	A_6
X_1	(118,0.2)	([20,25],0.2)	(s_3,0.1)	(s_3,0.1)	(8,0.3)	(s_2,0.3)
X_2	(105,0.2)	([18,24],0.3)	(s_4,0.2)	(s_4,0.3)	(8.5,0.2)	(s_3,0.1)
X_3	(113,0.1)	([25,30],0.2)	(s_2,0.3)	(s_2,0.3)	(9,0.4)	(s_2,0.3)
X_4	(100,0.2)	([15,20],0.1)	(s_4,0.2)	(s_3,0.1)	(8.3,0.4)	(s_4,0.1)
X_5	(116,0.1)	([25,30],0.1)	(s_3,0.1)	(s_3,0.2)	(9.2,0.2)	(s_4,0.2)
X_6	(108,0.2)	([28,33],0.2)	(s_2,0.3)	(s_2,0.3)	(8.5,0.3)	(s_2,0.3)

Table 1. Original Decision Matrix B_1

Supplier	A_1	A_2	A_3	A_4	A_5	A_6
X_1	(118,0.2)	([20,25],0.2)	(s_4,0.2)	(s_3,0.2)	(8,0.3)	(s_3,0.3)
X_2	(105,0.2)	([18,24],0.3)	(s_3,0.1)	(s_4,0.2)	(8.5,0.2)	(s_4,0.3)
X_3	(113,0.1)	([25,30],0.2)	(s_3,0.3)	(s_3,0.3)	(9,0.4)	(s_3,0.2)
X_4	(100,0.2)	([15,20],0.1)	(s_4,0.3)	(s_4,0.2)	(8.3,0.4)	(s_4,0.2)
X_5	(116,0.1)	([25,30],0.1)	(s_2,0.2)	(s_4,0.1)	(9.2,0.2)	(s_3,0.1)
X_6	(108,0.2)	([28,33],0.2)	(s_3,0.2)	(s_3,0.3)	(8.5,0.3)	(s_2,0.2)

Table 2. Original Decision Matrix B_2

A_4 Supplier reputation is an important attribute which is related to the success of fulfilling the procurement contract. A supplier with good reputation has good peer evaluation, and can always fulfill the procurement contract to provide high-quality products within a specified time. The attribute values of supplier reputation are also given in the form of linguistic fuzzy variables such as Very poor, Poor, Average, Good and Very good.

A_5 Carbon emissions, refers to carbon emissions of per unit of output, which is equal to the value of total carbon emissions divided by total production. Its values are given by the purchasing company in the form of precise values.

A_6 Level of environmental management, which can be measured by the development and implementation level of environmental management systems and environmental information management. A good supplier should have a perfect environment management system and environmental information management, and the implementation and development should be good, and the data monitoring for environment should be realized well. The attribute values of A_6 are also given in the form of linguistic fuzzy variables such as Very poor, Poor, Average, Good and Very good.

For the above 6 attributes, A_1 Price, A_2 Delivery time and A_5 Carbon emissions (tonnes) are the cost type, and A_3 Quality, A_4 Supplier reputation and A_6 Level of environmental management are the benefit type. After evaluating the six attributes for six suppliers, the three experts give the original decision matrix $B_k = (b_{ij}^k, v_{ij}^k)_{4\times 6}$, $(k = 1, 2, 3)$ (see Tables 1-3). Our decision goal is to evaluate and to rank all alternative suppliers, and then to select the best one to supply Q_0 barrels of Brent oil to the purchasing company.

4.1. **Decision Making Process of the Proposed Method.** Applying the decision method given by section 3.2, we give the decision making process for selecting the optimal supplier as follows.

Supplier	A_1	A_2	A_3	A_4	A_5	A_6
X_1	(118,0.2)	([20,25],0.2)	(s_3,0.2)	(s_4,0.3)	(8,0.3)	(s_2,0.4)
X_2	(105,0.2)	([18,24],0.3)	(s_3,0.2)	(s_4,0.2)	(8.5,0.2)	(s_4,0.3)
X_3	(113,0.1)	([25,30],0.2)	(s_4,0.2)	(s_2,0.2)	(9,0.4)	(s_2,0.3)
X_4	(100,0.2)	([15,20],0.1)	(s_4,0.3)	(s_4,0.3)	(8.3,0.4)	(s_4,0.2)
X_5	(116,0.1)	([25,30],0.1)	(s_2,0.1)	(s_3,0.2)	(9.2,0.2)	(s_4,0.3)
X_6	(108,0.2)	([28,33],0.2)	(s_2,0.3)	(s_2,0.2)	(8.5,0.3)	(s_2,0.4)

TABLE 3. Original Decision Matrix B_3

Supplier	A_1	A_2	A_3	A_4	A_5	A_6
X_1	(0,0.2)	([0.33,0.53],0.2)	(s_3,0.1)	(s_3,0.1)	(1,0.3)	(s_2,0.3)
X_2	(0.72,0.2)	([0.35,0.59],0.3)	(s_4,0.2)	(s_4,0.3)	(0.58,0.2)	(s_3,0.1)
X_3	(0.28,0.1)	([0.28,0.42],0.2)	(s_2,0.3)	(s_2,0.3)	(0.17,0.4)	(s_2,0.3)
X_4	(1,0.2)	([0.42,0.70],0.1)	(s_4,0.2)	(s_3,0.1)	(0.75,0.4)	(s_4,0.1)
X_5	(0.11,0.1)	([0.28,0.42],0.1)	(s_3,0.1)	(s_3,0.2)	(0,0.2)	(s_4,0.2)
X_6	(0.56,0.2)	([0.25,0.38],0.2)	(s_2,0.3)	(s_2,0.3)	(0.58,0.3)	(s_2,0.3)

TABLE 4. The Normalized Decision Matrix Y_1

Supplier	A_1	A_2	A_3	A_4	A_5	A_6
X_1	(0,0.2)	([0.33,0.53],0.2)	(s_4,0.2)	(s_3,0.2)	(1,0.3)	(s_3,0.3)
X_2	(0.72,0.2)	([0.35,0.59],0.3)	(s_3,0.1)	(s_4,0.2)	(0.58,0.2)	(s_4,0.3)
X_3	(0.28,0.1)	([0.28,0.42],0.2)	(s_3,0.3)	(s_3,0.3)	(0.17,0.4)	(s_3,0.2)
X_4	(1,0.2)	([0.42,0.70],0.1)	(s_4,0.3)	(s_4,0.2)	(0.75,0.4)	(s_4,0.2)
X_5	(0.11,0.1)	([0.28,0.42],0.1)	(s_2,0.2)	(s_4,0.1)	(0,0.2)	(s_3,0.1)
X_6	(0.56,0.2)	([0.25,0.38],0.2)	(s_3,0.2)	(s_3,0.3)	(0.58,0.3)	(s_2,0.2)

TABLE 5. The Normalized Decision Matrix Y_2

Supplier	A_1	A_2	A_3	A_4	A_5	A_6
X_1	(0,0.2)	([0.33,0.53],0.2)	(s_3,0.2)	(s_4,0.3)	(1,0.3)	(s_2,0.4)
X_2	(0.72,0.2)	([0.35,0.59],0.3)	(s_3,0.2)	(s_4,0.2)	(0.58,0.2)	(s_4,0.3)
X_3	(0.28,0.1)	([0.28,0.42],0.2)	(s_4,0.2)	(s_2,0.2)	(0.17,0.4)	(s_2,0.3)
X_4	(1,0.2)	([0.42,0.70],0.1)	(s_4,0.3)	(s_4,0.3)	(0.75,0.4)	(s_4,0.2)
X_5	(0.11,0.1)	([0.28,0.42],0.1)	(s_2,0.1)	(s_3,0.2)	(0,0.2)	(s_4,0.3)
X_6	(0.56,0.2)	([0.25,0.38],0.2)	(s_2,0.3)	(s_2,0.2)	(0.58,0.3)	(s_2,0.4)

TABLE 6. The Normalized Decision Matrix Y_3

1) Use equations (1), (2), (3), (4) and (5) to process the original data in the evaluation matrixes B_1, B_2 and B_3, then the normalized decision matrixes Y_1, Y_2 and Y_3 are listed in the following Tables 4-6.

2) Use the transformation methods given in Definition 1, Definition 3 and Definition 4 to transform the data in decision matrixes Y_1, Y_2 and Y_3 into grey linguistic 2-tuples, and the grey linguistic 2-tuple decision matrixes Z_1, Z_2 and Z_3 are listed in Tables 7-9.

3) Use equation (6) to aggregate the evaluation matrixes $Z_1, Z_2, \ldots, Z_l$ of l decision makers into a comprehensive decision matrix C, which is listed in Table 10.

Supplier	A_1	A_2	A_3	A_4	A_5	A_6
X_1	$((s_0,0),0.2)$	$((s_2,-0.31),0.2)$	$((s_3,0),0.1)$	$((s_3,0),0.1)$	$((s_4,0),0.3)$	$((s_2,0),0.3)$
X_2	$((s_3,-0.11),0.2)$	$((s_2,-0.12),0.3)$	$((s_4,0),0.2)$	$((s_4,0),0.3)$	$((s_2,0.33),0.2)$	$((s_3,0),0.1)$
X_3	$((s_0,0.11),0.1)$	$((s_1,0.44),0.2)$	$((s_2,0),0.3)$	$((s_2,0),0.3)$	$((s_1,-0.33),0.4)$	$((s_2,0),0.3)$
X_4	$((s_4,0),0.2)$	$((s_2,0.23),0.1)$	$((s_4,0),0.2)$	$((s_3,0),0.1)$	$((s_3,0),0.4)$	$((s_4,0),0.1)$
X_5	$((s_0,0.44),0.1)$	$((s_1,0.44),0.1)$	$((s_3,0),0.1)$	$((s_3,0),0.2)$	$((s_0,0),0.2)$	$((s_4,0),0.2)$
X_6	$((s_2,0.22),0.2)$	$((s_1,0.34),0.2)$	$((s_2,0),0.3)$	$((s_2,0),0.3)$	$((s_2,0.33),0.3)$	$((s_2,0),0.3)$

TABLE 7. The Grey Linguistic 2-tuple Decision Matrix Z_1

Supplier	A_1	A_2	A_3	A_4	A_5	A_6
X_1	$((s_0,0),0.2)$	$((s_2,-0.31),0.2)$	$((s_4,0),0.2)$	$((s_3,0),0.2)$	$((s_4,0),0.3)$	$((s_3,0),0.3)$
X_2	$((s_3,-0.11),0.2)$	$((s_2,-0.12),0.3)$	$((s_3,0),0.1)$	$((s_4,0),0.2)$	$((s_2,0.33),0.2)$	$((s_4,0),0.3)$
X_3	$((s_0,0.11),0.1)$	$((s_1,0.44),0.2)$	$((s_3,0),0.3)$	$((s_3,0),0.3)$	$((s_1,-0.33),0.4)$	$((s_3,0),0.2)$
X_4	$((s_4,0),0.2)$	$((s_2,0.23),0.1)$	$((s_4,0),0.3)$	$((s_4,0),0.2)$	$((s_3,0),0.4)$	$((s_4,0),0.2)$
X_5	$((s_0,0.44),0.1)$	$((s_1,0.44),0.1)$	$((s_2,0),0.2)$	$((s_4,0),0.1)$	$((s_0,0),0.2)$	$((s_3,0),0.1)$
X_6	$((s_2,0.22),0.2)$	$((s_1,0.34),0.2)$	$((s_3,0),0.2)$	$((s_3,0),0.3)$	$((s_2,0.33),0.3)$	$((s_2,0),0.2)$

TABLE 8. The Grey Linguistic 2-tuple Decision Matrix Z_2

Supplier	A_1	A_2	A_3	A_4	A_5	A_6
X_1	$((s_0,0),0.2)$	$((s_2,-0.31),0.2)$	$((s_3,0),0.2)$	$((s_4,0),0.3)$	$((s_4,0),0.3)$	$((s_2,0),0.4)$
X_2	$((s_3,-0.11),0.2)$	$((s_2,-0.12),0.3)$	$((s_3,0),0.2)$	$((s_4,0),0.2)$	$((s_2,0.33),0.2)$	$((s_4,0),0.3)$
X_3	$((s_0,0.11),0.1)$	$((s_1,0.44),0.2)$	$((s_4,0),0.2)$	$((s_2,0),0.2)$	$((s_1,-0.33),0.4)$	$((s_2,0),0.3)$
X_4	$((s_4,0),0.2)$	$((s_2,0.23),0.1)$	$((s_4,0),0.3)$	$((s_4,0),0.3)$	$((s_3,0),0.4)$	$((s_4,0),0.2)$
X_5	$((s_0,0.44),0.1)$	$((s_1,0.44),0.1)$	$((s_2,0),0.1)$	$((s_3,0),0.2)$	$((s_0,0),0.2)$	$((s_4,0),0.3)$
X_6	$((s_2,0.22),0.2)$	$((s_1,0.34),0.2)$	$((s_2,0),0.3)$	$((s_2,0),0.2)$	$((s_2,0.33),0.3)$	$((s_2,0),0.4)$

TABLE 9. The Grey Linguistic 2-tuple Decision Matrix Z_3

Supplier	A_1	A_2	A_3
X_1	$((s_0,0),0.2)$	$((s_2,-0.31),0.2)$	$((s_3,0.35),0.175)$
X_2	$((s_3,-0.11),0.2)$	$((s_2,-0.12),0.3)$	$((s_3,0.25),0.165)$
X_3	$((s_0,0.11),0.1)$	$((s_1,0.44),0.2)$	$((s_3,0.15),0.26)$
X_4	$((s_4,0),0.2)$	$((s_2,0.23),0.1)$	$((s_4,0),0.275)$
X_5	$((s_0,0.44),0.1)$	$((s_1,0.44),0.1)$	$((s_2,0.25),0.135)$
X_6	$((s_2,0.22),0.2)$	$((s_1,0.34),0.2)$	$((s_2,0.35),0.265)$
Supplier	A_4	A_5	A_6
X_1	$((s_3,0.4),0.215)$	$((s_4,0),0.3)$	$((s_2,0.35),0.34)$
X_2	$((s_4,0),0.225)$	$((s_2,0.33),0.2)$	$((s_4,-0.25),0.25)$
X_3	$((s_2,0.35),0.26)$	$((s_1,-0.33),0.4)$	$((s_2,0.35),0.265)$
X_4	$((s_4,-0.25),0.215)$	$((s_3,0),0.4)$	$((s_4,0),0.175)$
X_5	$((s_3,0.35),0.165)$	$((s_0,0),0.2)$	$((s_4,-0.35),0.205)$
X_6	$((s_2,0.35),0.26)$	$((s_2,0.33),0.3)$	$((s_2,0),0.305)$

TABLE 10. The Comprehensive Decision Matrix C

4) Use equation (7) to determine the reference sequence h^+ from comprehensive decision matrix C, the results are as follows.

$$h^+ = (((s_4,0),0.1),((s_2,0.23),0.1),((s_4,0),0.135),$$
$$((s_4,0),0.165),((s_4,0),0.2),((s_4,0),0.175)).$$

5) Use equations (9) and (10) to calculate the grey relational coefficient $r(h_j^+, h_i(j))$ and the grey 2-tuple correlation degree $r(h^+, h_i), i = 1, 2, \ldots, m$. The grey relational coefficient matrix is

$$G = \begin{pmatrix} 0.333 & 0.701 & 0.731 & 0.733 & 0.846 & 0.488 \\ 0.593 & 0.657 & 0.717 & 0.902 & 0.569 & 0.800 \\ 0.432 & 0.649 & 0.620 & 0.520 & 0.347 & 0.523 \\ 0.846 & 1 & 0.797 & 0.830 & 0.550 & 1 \\ 0.382 & 0.736 & 0.557 & 0.772 & 0.355 & 0.824 \\ 0.503 & 0.630 & 0.503 & 0.520 & 0.516 & 0.466 \end{pmatrix},$$

and the computation results of grey 2-tuple correlation degree $r(h^+, h_i), i = 1, 2, \ldots, 6$ are as follows.

$$r(h^+, h_1) = 0.628, r(h^+, h_2) = 0.701, r(h^+, h_3) = 0.516,$$
$$r(h^+, h_4) = 0.836, r(h^+, h_5) = 0.591, r(h^+, h_6) = 0.521.$$

6) Rank all alternatives in accordance to the value of $r(h^+, h_i), i = 1, 2, \ldots, 6$.

Since $r(h^+, h_4) > r(h^+, h_2) > r(h^+, h_1) > r(h^+, h_5) > r(h^+, h_6) > r(h^+, h_3)$, the rank order is

$$\text{Supplier}X_4 \succ \text{Supplier}X_2 \succ \text{Supplier}X_1$$
$$\succ \text{Supplier}X_5 \succ \text{Supplier}X_6 \succ \text{Supplier}X_3.$$

Thus Supplier X_4 is the final winner to supply the Q_0 barrels of Brent oil to the purchasing company.

4.2. **Discussion.** In order to further verify the validity and reasonability of the decision-making results of the ranking method based on grey 2-tuple correlation degree proposed above, we now present another ranking method which is based on the deviation degree. The decision process is described as follows.

Step 1**-Step** 4**:** They are the same as the step 1), 2), 3) and 4) in Section 4.1.

Step 5**:** Calculate the deviation degree between h_i and h^+, $i = 1, 2, \ldots, 6$. The deviation degree is a measurement to measure the approachability between h_i and h^+. The deviation degree is defined as follows:

$$\sigma(h^+, h_i) = w_1 d(h_1^+, h_i(1)) + w_2 d(h_2^+, h_i(2)) + \ldots + w_6 d(h_6^+, h_i(6))$$

where

$$d(h_j^+, h_i(j)) = \frac{1}{2}(|\Delta^{-1}[\max_{1 \le i \le 6}(r_{ij}, a_{ij})] - \Delta^{-1}(r_{ij}, a_{ij})| + |\min_{1 \le i \le 6} v_{ij} - v_{ij}|).$$

The computation results are as follows.

$$\sigma(h^+, h_1) = 0.214, \sigma(h^+, h_2) = 0.127, \sigma(h^+, h_3) = 0.282,$$
$$\sigma(h^+, h_4) = 0.066, \sigma(h^+, h_5) = 0.243, \sigma(h^+, h_6) = 0.257.$$

Step 6**:** Rank all alternatives in accordance to the value of $\sigma(h^+, h_i), i = 1, 2, \ldots, 6$. The smaller the value of $\sigma(h^+, h_i)$, the better is the alternative supplier X_i.

Since $\sigma(h^+, h_4) < \sigma(h^+, h_2) < \sigma(h^+, h_1) < \sigma(h^+, h_5) < \sigma(h^+, h_6) < \sigma(h^+, h_3)$, the rank order is

$$\text{Supplier}X_4 \succ \text{Supplier}X_2 \succ \text{Supplier}X_1$$
$$\succ \text{Supplier}X_5 \succ \text{Supplier}X_6 \succ \text{Supplier}X_3.$$

By the above decision process, we can see that the ranking result obtained by using deviation degree is the same with the result obtained by using grey 2-tuple correlation degree. Thus, these two methods demonstrate the validity and reasonability of the decision-making results with each other.

In addition, by the above decision step 5) in Section 4.1, we obtain the grey relational coefficient matrix as follows.

$$G = \begin{pmatrix} 0.333 & 0.701 & 0.731 & 0.733 & 0.846 & 0.488 \\ 0.593 & 0.657 & 0.717 & 0.902 & 0.569 & 0.800 \\ 0.432 & 0.649 & 0.620 & 0.520 & 0.347 & 0.523 \\ 0.846 & 1 & 0.797 & 0.830 & 0.550 & 1 \\ 0.382 & 0.736 & 0.557 & 0.772 & 0.355 & 0.824 \\ 0.503 & 0.630 & 0.503 & 0.520 & 0.516 & 0.466 \end{pmatrix}.$$

Based on G, we can make the grey relational advantage analysis. Concretely, by comparing the values of grey relational coefficient in each line of G, we can visually conclude that Supplier 4 has the absolute advantages than other five suppliers under the attributes of A_1 (Price), A_2 (Delivery time), A_3 (Quality) and A_6 (Level of environmental management). However, Supplier 4 has disadvantages in the aspects of A_4 (Supplier reputation) and A_5 (Carbon emissions). Though he is the final winner, he must improve his reputation, i.e., fulfill the procurement contract to provide high-quality products within a specified time. Another important striving direction, he must innovate the technologies of energy conservation and emissions reduction, and increase the environmental protection input, thus to reduce carbon dioxide emissions. In today's low carbon economy development, it is the inevitable choice of realizing the sustainable development for a good supplier. As the second place in the rank, Supplier 2 has good performance in the aspects of A_1 (Price) A_3 (Quality) and A_5 (Carbon emissions), especially in the aspect of A_4 Supplier reputation. But he must try his best to improve the performance on A_2 (Delivery time) and A_6 (Level of environmental management). Taken together, Supplier 2 can be selected as an alternative winner. This analysis is horizontal comparison, i.e., the comparison is done among all suppliers. Another comparison analysis is vertical comparison, i.e., compare different attribute values for a given supplier. For example, for the supplier 4, the values of grey relational coefficient in the fourth row of G corresponding to Supplier 4 are 0.846, 1, 0.797, 0.830, 0.55 and 1. From these six values, we can see that Supplier 4 has the best performance on A_2 and A_6 (the values of grey relational coefficient are 1). However, the performance on A_3 and A_5 leaves much to be desired (the values of grey relational coefficient are 0.797 and 0.55, which far less than 1). Especially, to make some valid measures to reduce carbon dioxide emissions is his task of top priority. Using the similar method, we can make the grey relational advantage analysis for the other suppliers.

4.3. **Comparison Analysis.** Comparing with the existing multi-attribute group decision making (MAGDM) methods such as TOPSIS [1], Fuzzy TOPSIS [66], Grey Correlation Method [33], VIKOR method [41], and so on, the GHMAGDM method based on grey 2-tuple correlation degree proposed in this paper has its own characteristics, which are listed as follows.

Firstly, in this decision method, the uncertainty of fuzziness and the uncertainty of grey are both considered at the same time, and the grey linguistic 2-tuple is presented to express this kind of grey fuzzy information. However, most of existing methods only separately considered the uncertainty of fuzziness or the uncertainty of grey, and especially few existing methods considered the uncertainty of grey caused by incomplete and inadequate information.

Secondly, our presented method can not only give a comprehensive evaluation ranking result for all alternatives, but also can make the grey relational advantage analysis for all alternatives. Through horizontal comparison and vertical comparison for all alternatives, the advantages and disadvantages of each alternative can be fully understood, which can help the decision makers to make more scientific and reasonable decisions, and also help the alternatives to find out the improvement directions and measures to overcome their disadvantages and deficiencies, thus to improve their comprehensive competitiveness.

Finally, our presented method is presented to deal with a kind of novel multi-attribute decision making problems, i.e., grey hybrid multi-attribute decision making problems with hybrid data in which the values of attributes integrate with real numbers, interval numbers and linguistic fuzzy variables. There are scarcely any methods to deal with this kind of problems among the existing decision-making methods. In the decision process of our presented method, all hybrid data of alternatives are transformed into the grey linguistic 2-tuples, and a new $GLTWA$ operator is presented to aggregate multiple grey linguistic 2-tuples. This data process can effectively avoid the loss and distortion of information in the process of information gathering comparing with some traditional methods.

5. **Conclusion**

Focusing on the problems of MAGDM which the precise values, interval numbers and linguistic fuzzy variables coexist in the attribute values and each attribute value has a certain grey degree, this paper presents a new grey hybrid multi-attribute decision making method. In our presented method, all hybrid attribute data of alternatives are transformed into grey linguistic 2-tuples, and a new *GLTWA* operator is presented to aggregate multiple decision makers' decision making matrix into comprehensive decision matrix formed by grey linguistic 2-tuples. Then a ranking method based on grey 2-tuple correlation degree is presented to evaluate and to rank all alternatives. Moreover, an application example of supplier selection is given to demonstrate the practicality and effectiveness of our decision method. Comparing with the existing MAGDM methods, the contributions in this paper are as follows. (i) To consider both the uncertainty of fuzziness and the uncertainty of grey in a multi-attribute group decision making problem with hybrid grey attribute data is our main contribution. (ii) Our presented method can not only give a comprehensive evaluation ranking result for all alternatives, but also can make the grey relational advantage analysis for all alternatives. (iii) The data process method in our presented method can effectively avoid the loss and distortion of information in the process of information gathering. In addition, the application example also shows that our method is simple in computing and easy to carry out in computers. Thus this paper provides a good way and a new idea for solving the problem of multi-attribute group decision making under the information environment with multiple uncertainties. In the future, based on the works in this paper, we will continue to do some other relative works. For example, improve the *GLTWA* operator or develop some other new aggregation operators to aggregate multiple decision makers' individual decision information into comprehensive decision information under the decision environment of multiple uncertainties, apply our proposed method to some other practical MAGDM problems, such as sustainable supply chain management, water resource schedule and so on, develop online decision making system to enhance the manipuility and practicability of our proposed method.

Acknowledgements. The authors thank the editors and anonymous reviewers for their helpful comments and suggestions. This work is supported by the National Natural Science Foundation of China (Nos. 71201064, 71371147, 71540027), and the 2014 Key Project of Hubei Provincial Department of Education (No. D20142903).

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Congjun Rao, School of Science, Wuhan University of Technology, Wuhan 430070, P. R. China
E-mail address: cjrao@163.com

Junjun Zheng*, School of Economics and Management, Wuhan University, Wuhan 430072, P. R. China
E-mail address: jjzhengwhu@foxmail.com

Cheng Wang, School of Mathematics and Economics, Hubei University of Education, Wuhan 430072, P. R. China
E-mail address: wangc80@163.com

Xinping Xiao, School of Science, Wuhan University of Technology, Wuhan 430070, P. R. China
E-mail address: xiaoxp@whut.edu.cn

*Corresponding author