STRATIFIED (L, M)-FUZZY Q-CONVERGENCE SPACES

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Abstract. This paper presents the concepts of (L, M)-fuzzy Q-convergence spaces and stratified (L, M)-fuzzy Q-convergence spaces. It is shown that the category of stratified (L, M)-fuzzy Q-convergence spaces is a bireflective subcategory of the category of (L, M)-fuzzy Q-convergence spaces, and the former is a Cartesian-closed topological category. Also, it is proved that the category of stratified (L, M)-fuzzy topological spaces can be embedded in the category of stratified (L, M)-fuzzy Q-convergence spaces as a reflective subcategory, and the former is isomorphic to the category of topological stratified (L, M)-fuzzy Q-convergence spaces.

1. Introduction

Fuzzy convergence theory takes an important part in the development of fuzzy topology. In recent years, fuzzy convergence theory deserves wide attention. With respect to different kinds of fuzzy topological spaces, there have been different kinds of fuzzy convergence structures. In the situation of L-topology, Lowen [17] defined the concept of a prefilter as a subset of I^X in order to study the theory of fuzzy topology. Min [18] proposed fuzzy limit structures by means of prefilters. Using stratified L-filters, Jäger [8] introduced stratified L-fuzzy convergence spaces (which are called stratified L-generalized convergence spaces in [9]). There are some following works to study the properties of this kind of fuzzy convergence structures [4, 5, 10, 11, 12, 15, 16, 27]. In the framework of L-fuzzifying topology, Xu [26] introduced fuzzifying convergence structures by means of classical filters. Yao [28] introduced L-fuzzifying convergence structures using L-filters of ordinary subsets. In (L, M)-fuzzy setting, Güloğu et al. [6] introduced the concept of I-fuzzy convergence structures by means of I-filters. Pang and Fang [19, 20] used L-filters to define L-fuzzy Q-convergence structures and studied its separation properties. Later, Pang proposed the concepts of (L, M)-fuzzy convergence spaces [21, 22] and enriched (L, M)-fuzzy convergence spaces [23], and studied their categorical properties and their categorical relations with (L, M)-fuzzy topological spaces.

Compared with L-topological spaces [2], stratified L-topological spaces [7] have better properties in some aspects. In a natural way, the stratification condition is generalized to (L, M)-fuzzy topological spaces, generating stratified (L, M)-fuzzy topological spaces (called weakly enriched (L, M)-fuzzy topological spaces in [7]). From the viewpoint of convergence theory, stratified L-generalized convergence

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structures in the sense of Jäger provide a good tool for interpreting stratified L-topology. In [13] introduced stratified LMN-convergence tower spaces, which can induce stratified (L, M)-fuzzy convergence spaces. Actually, the "stratification" of L-topology and (L, M)-fuzzy topology depends on the "stratification" of stratified L-filters. This motivates us to how to define "stratified (L, M)-fuzzy convergence structures" by using L-filters or (L, M)-fuzzy filters without the "stratification" condition.

In this paper, we will focus on fuzzy convergence structures in the framework of stratified (L, M)-fuzzy topological spaces. We will first generalize L-fuzzy Q-convergence structures to (L, M)-fuzzy Q-convergence structures. Based on this new concept, we will introduce stratified (L, M)-fuzzy Q-convergence structures and study its relations with (L, M)-fuzzy Q-convergence structures. Further, we will investigate the categorical properties of stratified (L, M)-fuzzy Q-convergence structures and study its relations with stratified (L, M)-fuzzy topologies in a categorical sense.

2. **Preliminaries**

Throughout this paper, both L and M denote completely distributive lattices and $'$ is an order-reversing involution on L. The smallest element and the largest element in L (M) are denoted by $\perp_L$ $(\perp_M)$ and $\top_L$ $(\top_M)$, respectively. For $a, b \in L$, we say that a is wedge below b in L, in symbols $a \prec b$, if for every subset $D \subseteq L$, $\bigvee D \geqslant b$ implies $d \geqslant a$ for some $d \in D$. A complete lattice L is completely distributive if and only if $b = \bigvee\{a \in L \mid a \prec b\}$ for each $b \in L$. An element a in L is called coprime if $a \leqslant b \vee c$ implies $a \leqslant b$ or $a \leqslant c$. The set of nonzero coprime elements in L (M) is denoted by $J(L)$ $(J(M))$, respectively.

For a nonempty set X, L^X denotes the set of all L-subsets on X. The smallest element and the largest element in L^X are denoted by $\perp_L^X$ and $\top_L^X$, respectively. L^X is also a completely distributive De Morgan algebra when it inherits the structure of the lattice L in a natural way, by defining $\bigvee$, $\bigwedge$, $\leqslant$ and $'$ pointwisely. For each $x \in X$ and $a \in L$, the L-subset x_a, defined by $x_a(y) = a$ if $y = x$, and $x_a(y) = \perp_L$ if $y \neq x$, is called a fuzzy point. The set of nonzero coprime elements in L^X is denoted by $J(L^X)$. It is easy to see that $J(L^X) = \{x_\lambda \mid x \in X, \lambda \in J(L)\}$. We say that a fuzzy point $x_\lambda \in J(L^X)$ quasi-coincides with A, denoted by $x_\lambda \hat{q} A$, if $\lambda \nleqslant A'(x)$ or equivalently $x_\lambda \nleqslant A'$. The relation "does not quasi-coincide with" is denoted by $\neg\hat{q}$. For $a \in L$, $\underline{a}$ denotes the constant mapping $X \longrightarrow L$, $x \longmapsto a$. Let $\varphi : X \longrightarrow Y$ be a mapping. Define $\varphi^{\rightarrow} : L^X \longrightarrow L^Y$ and $\varphi^{\leftarrow} : L^Y \longrightarrow L^X$ by $\varphi^{\rightarrow}(A)(y) = \bigvee_{\varphi(x)=y} A(x)$ for $A \in L^X$ and $y \in Y$, and $\varphi^{\leftarrow}(B) = B \circ \varphi$ for $B \in L^Y$, respectively.

Definition 2.1. [29] A mapping $\mathcal{F} : L^X \longrightarrow M$ is called an (L, M)-fuzzy filter on X if it satisfies:

(LMF1) $\mathcal{F}(\perp_L^X) = \perp_M, \mathcal{F}(\top_L^X) = \top_M$;
(LMF2) $\mathcal{F}(A \wedge B) = \mathcal{F}(A) \wedge \mathcal{F}(B)$.

The family of all (L, M)-fuzzy filters on X is denoted by $\mathcal{F}_{LM}(X)$. On the set $\mathcal{F}_{LM}(X)$ of all (L, M)-fuzzy filters on X, we define an order by $\mathcal{F} \leqslant \mathcal{G}$ if $\mathcal{F}(A) \leqslant \mathcal{G}(A)$ for all $A \in L^X$. Obviously, $(\mathcal{F}_{LM}(X), \leqslant)$ is a partially ordered set.

Example 2.2. [21] For each $x_\lambda \in J(L^X)$, we define $\hat{q}(x_\lambda) : L^X \longrightarrow M$ as follows:

$$\forall A \in L^X, \quad \hat{q}(x_\lambda)(A) = \begin{cases} \top_M, & x_\lambda \hat{q} A, \\ \bot_M, & x_\lambda \neg\hat{q} A. \end{cases}$$

Then $\hat{q}(x_\lambda)$ is an (L, M)-fuzzy filter.

Definition 2.3. [29] Let $\mathcal{F} \in \mathcal{F}_{LM}(X)$ and $\varphi : X \longrightarrow Y$ be a mapping. Then $\varphi^{\Rightarrow}(\mathcal{F}) : L^Y \longrightarrow M,\ A \longmapsto \mathcal{F}(\varphi^{\leftarrow}(A))$ is an (L, M)-fuzzy filter on Y and is called the image of $\mathcal{F}$ under φ.

Definition 2.4. [14, 25] An (L, M)-fuzzy topology on X is a mapping $\tau : L^X \longrightarrow M$ which satisfies:

(LFT1) $\tau(\bot_L^X) = \tau(\top_L^X) = \top_M$;
(LFT2) $\tau(A \wedge B) \geqslant \tau(A) \wedge \tau(B)$;
(LFT3) $\tau(\bigvee_{j \in J} A_j) \geqslant \bigwedge_{j \in J} \tau(A_j)$.

For an (L, M)-fuzzy topology τ on X, the pair (X, τ) is called an (L, M)-fuzzy topological space.

A continuous mapping between (L, M)-fuzzy topological spaces (X, τ_X) and (Y, τ_Y) is a mapping $\varphi : X \longrightarrow Y$ such that $\tau_X(\varphi^{\leftarrow}(A)) \geqslant \tau_Y(A)$ for each $A \in L^Y$. The category of (L, M)-fuzzy topological spaces with continuous mappings as morphisms will be denoted by (L, M)-**FTop**.

Definition 2.5. [7] An (L, M)-fuzzy topology τ on X is called stratified if it satisfies:

(SLFT) $\forall a \in L,\ \tau(\underline{a}) = \top_M$.

For a stratified (L, M)-fuzzy topology τ on X, the pair (X, τ) is called a stratified (L, M)-fuzzy topological space. The full subcategory of (L, M)-**FTop**, consisting of stratified (L, M)-fuzzy topological spaces, is denoted by $S(L, M)$-**FTop**.

Definition 2.6. [3] An (L, M)-fuzzy quasi-coincident neighborhood system on X is defined to be a set $\mathcal{Q} = \{\mathcal{Q}_{x_\lambda} \mid x_\lambda \in J(L^X)\}$ of mappings $\mathcal{Q}_{x_\lambda} : L^X \longrightarrow M$ satisfying the following conditions:

(LFQ1) $\mathcal{Q}_{x_\lambda}(\top_L^X) = \top_M,\ \mathcal{Q}_{x_\lambda}(\bot_L^X) = \bot_M$;
(LFQ2) $\mathcal{Q}_{x_\lambda}(A) \neq \bot_M$ implies $x_\lambda \hat{q} A$;
(LFQ3) $\mathcal{Q}_{x_\lambda}(A \wedge B) = \mathcal{Q}_{x_\lambda}(A) \wedge \mathcal{Q}_{x_\lambda}(B)$.

The pair $(X, \mathcal{Q})$ is called an (L, M)-fuzzy quasi-coincident neighborhood space, and it will be called topological if it satisfies moreover,

(LFQ4) $\mathcal{Q}_{x_\lambda}(A) = \bigvee_{x_\lambda \hat{q} B \leqslant A} \bigwedge_{y_\mu \hat{q} B} \mathcal{Q}_{y_\mu}(B)$.

A continuous mapping between two (L, M)-fuzzy quasi-coincident neighborhood spaces $(X, \mathcal{Q}^X)$ and $(Y, \mathcal{Q}^Y)$ is a mapping $\varphi : X \longrightarrow Y$ such that $\mathcal{Q}^X_{x_\lambda}(\varphi^{\leftarrow}(A)) \geqslant \mathcal{Q}^Y_{\varphi(x)_\lambda}(A)$ for each $x_\lambda \in J(L^X)$ and $A \in L^Y$.

Proposition 2.7. [3] *Let $\mathcal{Q} = \{\mathcal{Q}_{x_\lambda} \mid x_\lambda \in J(L^X)\}$ be an (L, M)-fuzzy quasi-coincident neighborhood system on X and define $\tau^{\mathcal{Q}} : L^X \longrightarrow M$ by*

$$\forall A \in L^X, \quad \tau^{\mathcal{Q}}(A) = \bigwedge_{x_\lambda \hat{q} A} \mathcal{Q}_{x_\lambda}(A).$$

Then $\tau^{\mathcal{Q}}$ is an (L,M)-fuzzy topology on X.

Proposition 2.8. [3] *Let τ be an (L,M)-fuzzy topology on X and define $\mathcal{Q}^{\tau}_{x_\lambda}: L^X \longrightarrow M$ by*

$$\forall A \in L^X,\ \mathcal{Q}^{\tau}_{x_\lambda}(A) = \bigvee_{x_\lambda \hat{q} B \leqslant A} \tau(B).$$

Then $\mathcal{Q}^{\tau} = \{\mathcal{Q}^{\tau}_{x_\lambda} \mid x_\lambda \in J(L^X)\}$ is an (L,M)-fuzzy quasi-coincident neighborhood system on X. Moreover, $\tau(A) = \bigwedge_{x_\lambda \hat{q} A} \mathcal{Q}^{\tau}_{x_\lambda}(A)$.

Definition 2.9 (Adámek et al. [1])**.** A category **A** is called a topological category over **Set** provided that for any set X, any class J, any family $((X_j,\xi_j))_{j\in J}$ of **A**-objects and any family $(\varphi_j : X \longrightarrow X_j)_{j\in J}$ of mappings, there exists a unique **A**-structure ξ on X which is initial with respect to the source $(\varphi_j : X \longrightarrow (X_j,\xi_j))_{j\in J}$. This means that for an **A**-object (Y,η), a mapping $\psi : (Y,\eta) \longrightarrow (X,\xi)$ is an **A**-morphism iff for all $j \in J$, $\varphi_j \circ \psi : (Y,\eta) \longrightarrow (X_j,\xi_j)$ is an **A**-morphism.

The class of objects of a category **A** is denoted by $|\mathbf{A}|$. For more notions related to category theory we refer to [1, 24].

3. Stratified (L,M)-fuzzy Q-convergence Spaces

In this section, we will introduce the concepts of (L,M)-fuzzy Q-convergence spaces and stratified (L,M)-fuzzy Q-convergence spaces. Also, we will study their relations in a categorical sense.

Definition 3.1. An (L,M)-fuzzy Q-convergence structure on X is a mapping $qc : \mathcal{F}_{LM}(X) \longrightarrow L^X$ which satisfies:

(LFQC1) $x_\lambda \leqslant qc(\hat{q}(x_\lambda))$;
(LFQC2) $\mathcal{F} \leqslant \mathcal{G}$ implies $qc(\mathcal{F}) \leqslant qc(\mathcal{G})$.

For an (L,M)-fuzzy Q-convergence structure qc on X, the pair (X,qc) is called an (L,M)-fuzzy Q-convergence space.

Definition 3.2. A mapping $\varphi : (X,qc_X) \longrightarrow (Y,qc_Y)$ between (L,M)-fuzzy Q-convergence spaces is called continuous provided that $x_\lambda \leqslant qc_X(\mathcal{F})$ implies $\varphi(x)_\lambda \leqslant qc_Y(\varphi^{\Rightarrow}(\mathcal{F}))$ for each $\mathcal{F} \in \mathcal{F}_{LM}(X)$ and $x_\lambda \in J(L^X)$. That is, $\varphi^{\rightarrow}(qc_X(\mathcal{F})) \leqslant qc_Y(\varphi^{\Rightarrow}(\mathcal{F}))$ for each $\mathcal{F} \in \mathcal{F}_{LM}(X)$.

It is easy to check that all (L,M)-fuzzy Q-convergence spaces and their continuous mappings form a category, denoted by (L,M)-**FQC**.

Remark 3.3. When $L = M$, the concept of (L,M)-fuzzy Q-convergence structures in Definition 3.1 is exactly the concept of L-fuzzy Q-convergence structures in [19].

Sometimes, we need to compare more than one (L,M)-fuzzy Q-convergence structure on a common domain X, so we define as follows:

Definition 3.4. For two (L,M)-fuzzy Q-convergence spaces (X,qc_1) and (X,qc_2), we say that (X,qc_2) is finer than (X,qc_1), or (X,qc_1) is coarser than (X,qc_2), denoted by $(X,qc_2) \leqslant (X,qc_1)$, if the identity mapping $id_X : (X,qc_2) \longrightarrow (X,qc_1)$ is continuous, that is, we have $qc_2(\mathcal{F}) \leqslant qc_1(\mathcal{F})$ for each $\mathcal{F} \in \mathcal{F}_{LM}(X)$. We also write $qc_2 \leqslant qc_1$ in this case.

Example 3.5. Let X be a nonempty set.

(1) Define $qc_* : \mathcal{F}_{LM}(X) \longrightarrow L^X$ as follows:

$$\forall \mathcal{F} \in \mathcal{F}_{LM}(X), \quad qc_*(\mathcal{F}) = \begin{cases} \top_L^X, & \exists\, x_\lambda \in J(L^X) \text{ s.t. } \hat{q}(x_\lambda) \leqslant \mathcal{F}; \\ \bot_L^X, & \text{otherwise.} \end{cases}$$

It is easy to verify that qc_* is the finest (L, M)-fuzzy Q-convergence structure on X.

(2) Define $qc^* : \mathcal{F}_{LM}(X) \longrightarrow L^X$ as follows:

$$\forall \mathcal{F} \in \mathcal{F}_{LM}(X), \quad qc^*(\mathcal{F}) = \top_L^X.$$

It is easy to check that qc^* is the coarsest (L, M)-fuzzy Q-convergence structure on X.

Next we introduce the notion of stratified (L, M)-fuzzy Q-convergence spaces and study its relations with (L, M)-fuzzy Q-convergence spaces.

Definition 3.6. An (L, M)-fuzzy Q-convergence structure qc on X is called stratified provided that

(SLFQC) If $x_\lambda \leqslant qc(\mathcal{F})$ and $\lambda \nleqslant a'$, then $\mathcal{F}(\underline{a}) = \top_M$.

The pair (X, qc) is called a stratified (L, M)-fuzzy Q-convergence space.

The full subcategory of (L, M)-**FQC**, consisting of stratified (L, M)-fuzzy Q-convergence spaces, is denoted by $S(L, M)$-**FQC**.

Remark 3.7. In the axiom (SLFQC), we call an (L, M)-fuzzy filter $\mathcal{F}$ λ-stratified provided that $\lambda \nleqslant a'$ implies $\mathcal{F}(\underline{a}) = \top_M$. In this case, (SLFQC) can be interpreted as "$x_\lambda \leqslant qc(\mathcal{F})$ implies that $\mathcal{F}$ is λ-stratified". This means that only the λ-stratified (L, M)-fuzzy filter can converge to x_λ.

Theorem 3.8. *The category $S(L, M)$-**FQC** is a topological category over **Set**.*

Proof. We need only prove the existence of initial structures. Let $\{(X_j, qc_j)\}_{j \in J}$ be a family of stratified (L, M)-fuzzy Q-convergence spaces and X be a nonempty set. If $\{\varphi_j : X \longrightarrow (X_j, qc_j)\}_{j \in J}$ is a source, then $qc^X : \mathcal{F}_{LM}(X) \longrightarrow L^X$ defined by

$$\forall \mathcal{F} \in \mathcal{F}_{LM}(X),\ qc^X(\mathcal{F}) = \bigwedge_{j \in J} \varphi_j^{\leftarrow}(qc_j(\varphi_j^{\Rightarrow}(\mathcal{F})))$$

is a stratified (L, M)-fuzzy Q-convergence structure on X. In fact,

(LFQC1) it suffices to remark that for a mapping $\varphi : X \longrightarrow Y$ and the fuzzy point filter $\hat{q}(x_\lambda)$, we have for $A \in L^Y$, $\varphi^{\Rightarrow}(\hat{q}(x_\lambda))(A) = \hat{q}(x_\lambda)(\varphi^{\leftarrow}(A)) = \hat{q}(\varphi(x)_\lambda)(A)$, i.e., $\varphi^{\Rightarrow}(\hat{q}(x_\lambda)) = \hat{q}(\varphi(x)_\lambda)$. This implies that

$$qc^X(\hat{q}(x_\lambda)) = \bigwedge_{j \in J} \varphi_j^{\leftarrow}(qc_j(\hat{q}(\varphi_j(x)_\lambda)) \geqslant \bigwedge_{j \in J} \varphi_j^{\leftarrow}(\varphi_j(x)_\lambda) \geqslant x_\lambda.$$

(LFQC2) Obviously.

(SLFQC) If $x_\lambda \leqslant qc^X(\mathcal{F})$ and $\lambda \nleqslant a'$, then $\varphi_j(x)_\lambda \leqslant qc_j(\varphi_j^{\Rightarrow}(\mathcal{F}))$ for each $j \in J$. Since qc_j satisfies (SLFQC), we know

$$\mathcal{F}(\underline{a}) = \mathcal{F}(\varphi_j^{\leftarrow}(\underline{a})) = \varphi_j^{\Rightarrow}(\mathcal{F})(\underline{a}) = \top_M.$$

Let further $(Y, qc^Y) \in |S(L,M)\text{-}\mathbf{FQC}|$ and $\varphi : Y \longrightarrow X$ be a mapping. Assume that $\varphi_j \circ \varphi$ is continuous for every $j \in J$. Then for each $\mathcal{G} \in \mathcal{F}_{LM}(Y)$, we have

$$\begin{aligned} y_\mu \leqslant qc^Y(\mathcal{G}) &\Longrightarrow \varphi_j \circ \varphi(y)_\mu \leqslant qc_j((\varphi_j \circ \varphi)^{\Rightarrow}(\mathcal{G}))\ for\ each\ j \in J \\ &\Longrightarrow \varphi_j(\varphi(y))_\mu \leqslant qc_j(\varphi_j^{\Rightarrow}(\varphi^{\Rightarrow}(\mathcal{G})))\ for\ each\ j \in J \\ &\Longrightarrow \varphi(y)_\mu \leqslant \bigwedge_{j \in J} \varphi_j^{\leftarrow}(qc_j(\varphi_j^{\Rightarrow}(\varphi^{\Rightarrow}(\mathcal{G})))) \\ &\Longrightarrow \varphi(y)_\mu \leqslant qc^X(\varphi^{\Rightarrow}(\mathcal{G})). \end{aligned}$$

This shows $\varphi : (Y, qc^Y) \longrightarrow (X, qc^X)$ is continuous. Therefore, φ is continuous if and only if $\varphi_j \circ \varphi$ is continuous for each $j \in J$, as desired. □

Example 3.9 (Product space)**.** Let $\{(X_j, qc_j)\}_{j \in J}$ be a family of stratified (L, M)-fuzzy Q-convergence spaces and $\{p_k : X := \prod_{j \in J} X_j \longrightarrow (X_k, qc_k)\}_{k \in J}$ be the source formed by the family of the projection mappings $\{p_k : \prod_{j \in J} X_j \longrightarrow X_k\}_{k \in J}$. The stratified (L, M)-fuzzy Q-convergence structure on X, denoted by $\prod_{j \in J} qc_j$, that is initial with respect to $\{p_j : X \longrightarrow (X_j, c_j)\}_{j \in J}$, is called the product stratified (L, M)-fuzzy Q-convergence structure and the pair $(X, \prod_{j \in J} qc_j)$ is called the product space. For the product of two stratified (L, M)-fuzzy Q-convergence spaces (X, qc_X) and (Y, qc_Y), we usually write $(X \times Y, qc_X \times qc_Y)$.

For a nonempty set X, let $QC_s(X)$ denote the fibre

$$\{(X, qc) : qc \text{ is a stratified } (L, M)\text{-fuzzy Q-convergence structure on } X\}$$

of X. By Theorem 3.8, we obtain

Corollary 3.10. $(QC_s(X), \leqslant)$ *is a complete lattice.*

Corollary 3.11. *Let* (Y, qc_Y) *be a stratified* (L, M)*-fuzzy Q-convergence space and* $\varphi : X \longrightarrow Y$ *be a mapping. Define* $qc_X : \mathcal{F}_{LM}(X) \longrightarrow L^X$ *by*

$$\forall \mathcal{F} \in \mathcal{F}_{LM}(X),\ qc_X(\mathcal{F}) = \varphi^{\leftarrow}(qc_Y(\varphi^{\Rightarrow}(\mathcal{F}))).$$

Then qc_X *is a stratified* (L, M)*-fuzzy Q-convergence structure on* X*.*

Theorem 3.12. $S(L,M)$-$\mathbf{FQC}$ *is a bireflective subcategory of* (L,M)-$\mathbf{FQC}$*.*

Proof. Suppose that (X, qc) is an (L, M)-fuzzy Q-convergence space and

$$E_{qc} = \{c \mid (X, c) \in \mid S(L,M)\text{-}\mathbf{FQC} \mid \ and\ qc \leqslant c\}.$$

Let $qc_r = \bigwedge E_{qc}$. By Corollary 3.10, we know qc_r is a stratified (L, M)-fuzzy Q-convergence structure on X. Then $id_X : (X, qc) \longrightarrow (X, qc_r)$ is the $S(L,M)$-$\mathbf{FQC}$-bireflection. Next we prove it by two steps.

Step 1: $id_X : (X, qc) \longrightarrow (X, qc_r)$ is continuous. By the definition of qc_r, it is easy to see that $qc \leqslant qc_r$, which shows the continuity of $id_X : (X, qc) \longrightarrow (X, qc_r)$.

Step 2: For a stratified (L, M)-fuzzy Q-convergence space (Y, qc_Y) and a mapping $\varphi : X \longrightarrow Y$, the continuity of $\varphi : (X, qc) \longrightarrow (Y, qc_Y)$ implies the continuity of $\varphi : (X, qc_r) \longrightarrow (Y, qc_Y)$. Take any $x_\lambda \in J(L^X)$ and $\mathcal{F} \in \mathcal{F}_{LM}(X)$ such that $x_\lambda \leqslant qc_r(\mathcal{F})$. Then for each stratified (L, M)-fuzzy Q-convergence space (X, c) satisfying $qc \leqslant c$, it follows that $x_\lambda \leqslant qc(\mathcal{F})$. By Corollary 3.11, we know that qc_X is a stratified (L, M)-fuzzy Q-convergence structure on X. Since $\varphi : (X, qc) \longrightarrow (Y, qc_Y)$ is continuous, it is easy to verify that $qc \leqslant qc_X$. Hence, we obtain $x_\lambda \leqslant qc_X(\mathcal{F}) = \varphi^{\leftarrow}(qc_Y(\varphi^{\Rightarrow}(\mathcal{F})))$, that is, $\varphi(x)_\lambda \leqslant qc_Y(\varphi^{\Rightarrow}(\mathcal{F}))$. This implies the continuity of $\varphi : (X, qc_r) \longrightarrow (Y, qc_Y)$. □

4. Relations Between $S(L, M)$-FQC and $S(L, M)$-FTop

In this section, we will study the relations between stratified (L, M)-fuzzy Q-convergence spaces and stratified (L, M)-fuzzy topological spaces from a categorical aspect.

Definition 4.1. Let (X, qc) be a stratified (L, M)-fuzzy Q-convergence space and $x_\lambda \in J(L^X)$. Define $\mathcal{F}^{qc}_{x_\lambda} : L^X \longrightarrow M$ by $\mathcal{F}^{qc}_{x_\lambda}(A) = \bigwedge_{x_\lambda \leqslant qc(\mathcal{F})} \mathcal{F}(A)$ for each $A \in L^X$. Then $\mathcal{F}^{qc}_{x_\lambda}$ is an (L, M)-fuzzy filter, which is called the quasi-coincident neighborhood filter at x_λ.

Proposition 4.2. *Let (X, qc) be a stratified (L, M)-fuzzy Q-convergence space and define $\tau^{qc} : L^X \longrightarrow M$ by*

$$\forall A \in L^X,\ \tau^{qc}(A) = \bigwedge_{x_\lambda \hat{q} A} \mathcal{F}^{qc}_{x_\lambda}(A).$$

Then τ^{qc} is a stratified (L, M)-fuzzy topology on X.

Proof. It suffices to verify that τ^{qc} satisfies (LFT1)–(LFT3) and (SLFT). In fact,

(LFT1) $\tau^{qc}(\perp^X_L) = \bigwedge_{x_\lambda \hat{q} \perp^X_L} \mathcal{F}^{qc}_{x_\lambda}(\perp^X_L) = \bigwedge \emptyset = \top_M$,
$\tau^{qc}(\top^X_L) = \bigwedge_{x_\lambda \hat{q} \top^X_L} \mathcal{F}^{qc}_{x_\lambda}(\top^X_L) = \bigwedge \top_M = \top_M$.

(LFT2) Take any $A, B \in L^X$. Then

$$\begin{aligned}
\tau^{qc}(A) \wedge \tau^{qc}(B) &= \bigwedge_{x_\lambda \hat{q} A} \mathcal{F}^{qc}_{x_\lambda}(A) \wedge \bigwedge_{y_\mu \hat{q} B} \mathcal{F}^{qc}_{y_\mu}(B) \\
&= \bigwedge_{x_\lambda \hat{q} A} \bigwedge_{y_\mu \hat{q} B} \mathcal{F}^{qc}_{x_\lambda}(A) \wedge \mathcal{F}^{qc}_{y_\mu}(B) \\
&\leqslant \bigwedge_{x_\lambda \hat{q} (A \wedge B)} \bigwedge_{y_\mu \hat{q} (A \wedge B)} \mathcal{F}^{qc}_{x_\lambda}(A) \wedge \mathcal{F}^{qc}_{y_\mu}(B) \\
&\leqslant \bigwedge_{x_\lambda \hat{q} (A \wedge B)} \mathcal{F}^{qc}_{x_\lambda}(A) \wedge \mathcal{F}^{qc}_{x_\lambda}(B) \\
&= \bigwedge_{x_\lambda \hat{q} (A \wedge B)} \mathcal{F}^{qc}_{x_\lambda}(A \wedge B) \\
&= \tau^{qc}(A \wedge B).
\end{aligned}$$

(LFT3) For $\{A_j \mid j \in J\} \subseteq L^X$, it follows that

$$\begin{aligned}
\tau^{qc}\left(\bigvee_{j \in J} A_j\right) &= \bigwedge_{x_\lambda \hat{q} \bigvee_{j \in J} A_j} \mathcal{F}^{qc}_{x_\lambda}\left(\bigvee_{j \in J} A_j\right) \\
&= \bigwedge_{j \in J} \bigwedge_{x_\lambda \hat{q} A_j} \mathcal{F}^{qc}_{x_\lambda}\left(\bigvee_{j \in J} A_j\right) \\
&\geqslant \bigwedge_{j \in J} \bigwedge_{x_\lambda \hat{q} A_j} \mathcal{F}^{qc}_{x_\lambda}(A_j) \\
&= \bigwedge_{j \in J} \tau^{qc}(A_j).
\end{aligned}$$

(SLFT) For each $a \in L$, it follows that

$$\begin{aligned}\tau^{qc}(\underline{a}) &= \bigwedge_{x_\lambda \hat{q} \underline{a}} \bigwedge_{x_\lambda \leqslant qc(\mathcal{F})} \mathcal{F}(\underline{a}) \\ &= \bigwedge_{\lambda \nleqslant a'} \bigwedge_{x_\lambda \leqslant qc(\mathcal{F})} \mathcal{F}(\underline{a}) \\ &= \top_M. \quad \text{(by (SLFQC))}\end{aligned}$$

This shows that τ^{qc} is a stratified (L,M)-fuzzy topology on X. $\square$

Proposition 4.3. *If $\varphi : (X, qc_X) \longrightarrow (Y, qc_Y)$ is continuous, then so is $\varphi : (X, \tau^{qc_X}) \longrightarrow (Y, \tau^{qc_Y})$.*

Proof. Since $\varphi : (X, qc_X) \longrightarrow (X, qc_X)$ is continuous, it follows that for each $\mathcal{F} \in \mathcal{F}_{LM}(X)$,

$$\varphi^{\rightarrow}(qc_X(\mathcal{F})) \leqslant qc_Y(\varphi^{\Rightarrow}(\mathcal{F})).$$

Then for each $B \in L^Y$, we have

$$\begin{aligned}\tau^{qc_X}(\varphi^{\leftarrow}(B)) &= \bigwedge_{x_\lambda \hat{q} \varphi^{\leftarrow}(B)} \mathcal{F}^{qc_X}_{x_\lambda}(\varphi^{\leftarrow}(B)) \\ &= \bigwedge_{x_\lambda \hat{q} \varphi^{\leftarrow}(B)} \varphi^{\Rightarrow}(\mathcal{F}^{qc_X}_{x_\lambda})(B) \\ &= \bigwedge_{\varphi(x)_\lambda \hat{q} B} \bigwedge_{x_\lambda \leqslant qc_X(\mathcal{F})} \varphi^{\Rightarrow}(\mathcal{F})(B) \\ &\geqslant \bigwedge_{\varphi(x)_\lambda \hat{q} B} \bigwedge_{\varphi(x)_\lambda \leqslant qc_Y(\varphi^{\Rightarrow}(\mathcal{F}))} \varphi^{\Rightarrow}(\mathcal{F})(B) \\ &\geqslant \bigwedge_{y_\mu \hat{q} B} \bigwedge_{y_\mu \leqslant qc_Y(\mathcal{G})} \mathcal{G}(B) \\ &= \bigwedge_{y_\mu \hat{q} B} \mathcal{F}^{qc_Y}_{y_\mu}(B) \\ &= \tau^{qc_Y}(B).\end{aligned}$$

Thus, $\varphi : (X, \tau^{qc_X}) \longrightarrow (Y, \tau^{qc_Y})$ is continuous. $\square$

By Propositions 4.2 and 4.3, we obtain a functor $\mathbb{F} : S(L,M)$-**FTop**$\longrightarrow S(L,M)$-**FQC**, defined by

$$\mathbb{F}(X,\tau) = (X, qc^\tau) \text{ and } \mathbb{F}(\varphi) = \varphi.$$

Proposition 4.4. *Let (X,τ) be an (L,M)-fuzzy topological space and define $qc^\tau : \mathcal{F}_{LM}(X) \longrightarrow L^X$ as follows:*

$$\forall \mathcal{F} \in \mathcal{F}_{LM}(X),\ qc^\tau(\mathcal{F}) = \bigvee_{\mathcal{Q}^\tau_{y_\mu} \leqslant \mathcal{F}} y_\mu.$$

Then qc^τ is an (L,M)-fuzzy Q-convergence structure on X.

Proof. It is enough to show that qc^{τ} satisfies (LFQC1) and (LFQC2).

(LFQC1) Take any $x_{\lambda} \in J(L^X)$. Then $\mathcal{Q}^{\tau}_{x_{\lambda}} \in \mathcal{F}_{LM}(X)$ and $\mathcal{Q}^{\tau}_{x_{\lambda}} \leqslant \hat{q}(x_{\lambda})$. Further, it follows that

$$qc^{\tau}(\hat{q}(x_{\lambda})) = \bigvee_{\mathcal{Q}^{\tau}_{y_{\mu}} \leqslant \hat{q}(x_{\lambda})} y_{\mu} \geqslant x_{\lambda}.$$

(LFQC2) Obviously. □

Lemma 4.5. *Let (X, τ) be an (L, M)-fuzzy topological space. Then for each $x_{\lambda} \in J(L^X)$ and $\mathcal{F} \in \mathcal{F}_{LM}(X)$,*

$$x_{\lambda} \leqslant qc^{\tau}(\mathcal{F}) \text{ if and only if } \mathcal{Q}^{\tau}_{x_{\lambda}} \leqslant \mathcal{F}.$$

Proof. The sufficiency is obvious. It suffices to show the necessity. Suppose that $x_{\lambda} \leqslant qc^{\tau}(\mathcal{F})$. Take any $A \in L^X$ and $\alpha \in J(M)$ such that

$$\alpha \prec \mathcal{Q}^{\tau}_{x_{\lambda}}(A) = \bigvee_{x_{\lambda}\hat{q}B \leqslant A} \bigwedge_{y_{\mu}\hat{q}B} \mathcal{Q}^{\tau}_{y_{\mu}}(B).$$

Then there exists $B_{\alpha} \in L^X$ such that $x_{\lambda}\hat{q}B_{\alpha} \leqslant A$ and for each $y_{\mu}\hat{q}B_{\alpha}$, it follows that $\alpha \leqslant \mathcal{Q}^{\tau}_{y_{\mu}}(B_{\alpha})$. Then we have $qc^{\tau}(\mathcal{F}) \geqslant x_{\lambda} \nleqslant B'_{\alpha}$. By the definition of qc^{τ}, there exists $z_{\nu} \in J(L^X)$ such that $\mathcal{Q}^{\tau}_{z_{\nu}} \leqslant \mathcal{F}$ and $z_{\nu} \nleqslant B'_{\alpha}$. Thus,

$$\alpha \leqslant \mathcal{Q}^{\tau}_{z_{\nu}}(B_{\alpha}) \leqslant \mathcal{F}(B_{\alpha}) \leqslant \mathcal{F}(A).$$

By the arbitrariness of α, we get

$$\mathcal{F}(A) \geqslant \{\alpha \in J(M) \mid \alpha \prec \mathcal{Q}^{\tau}_{x_{\lambda}}(A)\} = \mathcal{Q}^{\tau}_{x_{\lambda}}(A),$$

as desired. □

Proposition 4.6. *Let (X, τ) be a stratified (L, M)-fuzzy topological space. Then qc^{τ} is a stratified (L, M)-fuzzy Q-convergence structure on X.*

Proof. By Proposition 4.4, it is enough to show that qc^{τ} satisfies (SLFQC). For each $x_{\lambda} \leqslant qc^{\tau}(\mathcal{F})$ and $\lambda \nleqslant a'$, by Lemma 4.5, we know $\mathcal{Q}^{\tau}_{x_{\lambda}} \leqslant \mathcal{F}$. Then it follows that

$$\mathcal{F}(\underline{a}) \geqslant \mathcal{Q}^{\tau}_{x_{\lambda}}(\underline{a}) = \bigvee_{x_{\lambda}\hat{q}B \leqslant \underline{a}} \tau(B) \geqslant \tau(\underline{a}) = \top_M.$$

□

Proposition 4.7. *If $\varphi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ is continuous, then so is $\varphi : (X, qc^{\tau_X}) \longrightarrow (Y, qc^{\tau_Y})$.*

Proof. Since $\varphi : (X, \tau_X) \longrightarrow (Y, \tau_Y)$ is continuous, it follows that $\varphi : (X, \mathcal{Q}^{\tau_X}) \longrightarrow (Y, \mathcal{Q}^{\tau_Y})$ is continuous, that is,

$$\forall x_{\lambda} \in J(L^X),\ \varphi^{\Rightarrow}(\mathcal{Q}^{\tau_X}_{x_{\lambda}}) \geqslant \mathcal{Q}^{\tau_Y}_{\varphi(x)_{\lambda}}.$$

Then for each $\mathcal{F} \in \mathcal{F}_{LM}(X)$, we have

$$\begin{aligned}
x_{\lambda} \leqslant qc^{\tau_X}(\mathcal{F}) &\iff \mathcal{Q}^{\tau_X}_{x_{\lambda}} \leqslant \mathcal{F} \qquad \text{(by Lemma 4.5)} \\
&\implies \varphi^{\Rightarrow}(\mathcal{Q}^{\tau_X}_{x_{\lambda}}) \leqslant \varphi^{\Rightarrow}(\mathcal{F}) \\
&\implies \mathcal{Q}^{\tau_Y}_{\varphi(x)_{\lambda}} \leqslant \varphi^{\Rightarrow}(\mathcal{F}) \\
&\iff \varphi(x)_{\lambda} \leqslant qc^{\tau_Y}(\varphi^{\Rightarrow}(\mathcal{F})) \quad \text{(by Lemma 4.5)} \\
&\iff x_{\lambda} \leqslant \varphi^{\leftarrow}(qc^{\tau_Y}(\varphi^{\Rightarrow}(\mathcal{F}))).
\end{aligned}$$

By the arbitrariness of x_λ, we obtain $qc^{\tau_X}(\mathcal{F}) \leqslant \varphi^{\leftarrow}(qc^{\tau_Y}(\varphi^{\Rightarrow}(\mathcal{F})))$, that is, $\varphi^{\rightarrow}(qc^{\tau_X}(\mathcal{F})) \leqslant qc^{\tau_Y}(\varphi^{\Rightarrow}(\mathcal{F}))$. This shows that $\varphi : (X, qc^{\tau_X}) \longrightarrow (Y, qc^{\tau_Y})$ is continuous. $\square$

By Propositions 4.6 and 4.7, we obtain a functor $\mathbb{G} : S(L,M)$-**FQC**$\longrightarrow S(L,M)$-**FTop**, defined by

$$\mathbb{G}(X, qc) = (X, \tau^{qc}) \text{ and } \mathbb{G}(\varphi) = \varphi.$$

Theorem 4.8. $(\mathbb{G}, \mathbb{F})$ *is a Galois correspondence. Moreover,* $\mathbb{G}$ *is a left inverse of* $\mathbb{F}$.

Proof. We need only show that $\mathbb{F} \circ \mathbb{G} \geqslant \mathbb{I}_{S(L,M)-\mathbf{FQC}}$ and $\mathbb{G} \circ \mathbb{F} = \mathbb{I}_{S(L,M)-\mathbf{FTop}}$. That is, for a stratified (L,M)-fuzzy Q-convergence structure qc on X and a stratified (L,M)-fuzzy topology τ on X, it follows that $qc^{\tau^{qc}} \geqslant_{qc} qc$ and $\tau^{qc^{\tau}} = \tau$.

For $qc^{\tau^{qc}} \geqslant_{qc} qc$, take any $x_\lambda \in J(L^X)$ and $A \in L^X$. It follows that

$$\mathcal{Q}^{\tau^{qc}}_{x_\lambda}(A) = \bigvee_{x_\lambda \hat{q} B \leqslant A} \bigwedge_{y_\mu \hat{q} B} \mathcal{F}^{qc}_{y_\mu}(B) \leqslant \mathcal{F}^{qc}_{x_\lambda}(A).$$

Then $\mathcal{Q}^{\tau^{qc}}_{x_\lambda} \leqslant \mathcal{F}^{qc}_{x_\lambda}$. Further, take any $\mathcal{F} \in \mathcal{F}_{LM}(X)$. Then

$$\begin{aligned} x_\lambda \leqslant qc(\mathcal{F}) &\implies \mathcal{F}^{qc}_{x_\lambda} \leqslant \mathcal{F} \\ &\implies \mathcal{Q}^{\tau^{qc}}_{x_\lambda} \leqslant \mathcal{F} \\ &\iff x_\lambda \leqslant qc^{\tau^{qc}}(\mathcal{F}). \quad \text{(by Lemma 4.5)} \end{aligned}$$

By the arbitrariness of x_λ, we obtain $qc \leqslant qc^{\tau^{qc}}$.

For $\tau^{qc^{\tau}} = \tau$, by Propositions 2.7 and 4.2, it suffices to show that $\mathcal{Q}^{\tau}_{x_\lambda} = \mathcal{F}^{qc^{\tau}}_{x_\lambda}$ for each $x_\lambda \in J(L^X)$. By Lemma 4.5, for each $A \in L^X$, we have

$$\mathcal{F}^{qc^{\tau}}_{x_\lambda}(A) = \bigwedge_{x_\lambda \leqslant qc^{\tau}(\mathcal{F})} \mathcal{F}(A) = \bigwedge_{\mathcal{Q}^{\tau}_{x_\lambda} \leqslant \mathcal{F}} \mathcal{F}(A) = \mathcal{Q}^{\tau}_{x_\lambda}(A),$$

as desired. $\square$

Corollary 4.9. *The category* $S(L,M)$-**FTop** *can be embedded in the category* $S(L,M)$-**FQC** *as a reflective subcategory.*

In general topology, we all know that the category of topological convergence spaces, which is a special kind of convergence spaces, is isomorphic to the category of topological spaces. In the framework of stratified (L,M)-fuzzy topology, we have the similar conclusion.

Definition 4.10. A stratified (L,M)-fuzzy Q-convergence structure on X is called topological if it satisfies

(LFPQC) $x_\lambda \leqslant qc(\mathcal{F}^{qc}_{x_\lambda})$;
(LFTQC) $\mathcal{F}^{qc}_{x_\lambda}(A) = \bigvee_{x_\lambda \hat{q} B \leqslant A} \bigwedge_{y_\mu \hat{q} B} \mathcal{F}^{qc}_{y_\mu}(B)$.

For a topological stratified (L,M)-fuzzy Q-convergence structure c on X, the pair (X,c) is called a topological stratified (L,M)-fuzzy Q-convergence space.

The full subcategory of $S(L,M)$-**FQC**, consisting of topological stratified (L,M)-fuzzy Q-convergence spaces, is denoted by $S(L,M)$-**FTQC**.

Theorem 4.11. *$S(L,M)$-**FTQC** is isomorphic to $S(L,M)$-**FTop**.*

Proof. By Theorem 4.8, it suffices to show that for each topological stratified (L,M)-fuzzy Q-convergence space (X,qc), $qc^{\tau^{qc}} = qc$.

Take any $x_\lambda \in J(L^X)$ and $A \in L^X$. By (LFTQC),

$$\begin{aligned} \mathcal{Q}^{\tau^{qc}}_{x_\lambda}(A) &= \bigvee_{x_\lambda \hat{q} B \leqslant A} \tau^{qc}(B) \\ &= \bigvee_{x_\lambda \hat{q} B \leqslant A} \bigwedge_{y_\mu \hat{q} B} \mathcal{F}^{qc}_{y_\mu}(B) \\ &= \mathcal{F}^{qc}_{x_\lambda}(A). \end{aligned}$$

Then for each $\mathcal{F} \in \mathcal{F}_{LM}(X)$, by (LFPQC),

$$\begin{aligned} x_\lambda \leqslant qc(\mathcal{F}) &\iff \mathcal{F}^{qc}_{x_\lambda} \leqslant \mathcal{F} \\ &\iff \mathcal{Q}^{\tau^{qc}}_{x_\lambda} \leqslant \mathcal{F} \\ &\iff x_\lambda \leqslant qc^{\tau^{qc}}(\mathcal{F}). \quad \text{(by Lemma 4.5)} \end{aligned}$$

By the arbitrariness of x_λ, we get $qc = qc^{\tau^{qc}}$, as desired. □

5. Cartesian-closedness of $S(L,M)$-FQC

Compared with topological spaces, convergence spaces possess better categorical properties, such as Cartesian-closedness. In the section, we will discuss the Cartesian-closedness of $S(L,M)$-**FQC**.

There are several ways to show that a category is Cartesian closed. Here we adopt the following way.

Lemma 5.1. [1] *Let **A** be a well-fibred topological category. Then the following statements are equivalent.*

(1) ***A** is Cartesian closed.*

(2) *For each $A \in \mid \mathbf{A} \mid$, the functor $(A\times -)$ preserves final epi-sinks.*

In Theorem 3.8, we showed that $S(L,M)$-**FQC** is a topological category. Next we show it is well-fibred.

Proposition 5.2. *$S(L,M)$-**FQC** is fibre small.*

Proof. For a fixed set X, the class of all stratified (L,M)-fuzzy Q-convergence structures on X is a subset of $(L^X)^{\mathcal{F}_{LM}(X)}$. This shows the fibre-smallness of $S(L,M)$-**FQC**. □

Lemma 5.3. *Let $X = \{x\}$ and let qc be an (L,M)-fuzzy Q-convergence structure on X. Then the following statements are equivalent.*

(1) *qc is stratified, that is, qc satisfies* (SLFQC).

(2) *$\forall x_\lambda \in J(L^X)$, $\mathcal{F} \in \mathcal{F}_{LM}(X)$, $x_\lambda \leqslant qc(\mathcal{F})$ iff $\mathcal{F} \geqslant \hat{q}(x_\lambda)$.*

(3) *$\forall \mathcal{F} \in \mathcal{F}_{LM}(X)$, $qc(\mathcal{F}) = \bigvee_{\mathcal{F} \geqslant \hat{q}(x_\lambda)} x_\lambda$.*

Proof. Since X is a singleton, each L-subset on X is actually a constant L-subset and $J(L^X) = \{x_\lambda \mid \lambda \in J(L)\}$. Next we show the equivalence of these conditions.

(1)$\Longrightarrow$(2): *Sufficiency.* If $\mathcal{F} \geqslant \hat{q}(x_\lambda)$, then by (LFQC1) and (LFQC2), we have

$$x_\lambda \leqslant qc(\hat{q}(x_\lambda)) \leqslant qc(\mathcal{F}).$$

Necessity. Take any $\underline{a} \in L^X$ with $x_\lambda \hat{q} \underline{a}$, i.e., $\lambda \nleqslant a'$. Since $x_\lambda \leqslant qc(\mathcal{F})$, by (1), we obtain that $\mathcal{F}(\underline{a}) = \top_M$. Thus, $\mathcal{F}(\underline{a}) \geqslant \hat{q}(x_\lambda)(\underline{a})$. If $x_\lambda \neg \hat{q} \underline{a}$, then $\mathcal{F}(\underline{a}) \geqslant \bot_M = \hat{q}(x_\lambda)(\underline{a})$. This shows $\mathcal{F} \geqslant \hat{q}(x_\lambda)$.

(2)$\Longrightarrow$(3): Straightforward.

(3)$\Longrightarrow$(1): Take any $\lambda \in J(L)$ and $a \in L$ such that $\lambda \nleqslant a'$ and $x_\lambda \leqslant qc(\mathcal{F})$. Then

$$\lambda \leqslant qc(\mathcal{F})(x) = \bigvee_{\mathcal{F} \geqslant \hat{q}(x_\mu)} \mu.$$

Since $\lambda \nleqslant a'$, we know that $\bigvee_{\mathcal{F} \geqslant \hat{q}(x_\mu)} \mu \nleqslant a'$. Then there exists $\mu \in J(L)$ such that $\mathcal{F} \geqslant \hat{q}(x_\mu)$ and $\mu \nleqslant a'$, i.e., $x_\mu \hat{q} \underline{a}$. This implies that

$$\mathcal{F}(\underline{a}) \geqslant \hat{q}(x_\mu)(\underline{a}) = \top_M,$$

as desired. □

By Lemma 5.3, we can obtain the following result.

Proposition 5.4. *For a singleton $X = \{x\}$, there is a unique stratified (L, M)-fuzzy Q-convergence structure on X. That is, $S(L, M)$-**FQC** satisfies the terminal separator property.*

By Theorem 3.8, Propositions 5.2 and 5.4, we have

Theorem 5.5. *$S(L, M)$-**FQC** is a well-fibred topological category.*

In order to show the Cartesian-closedness of $S(L, M)$-**FQC**, we will list some conclusions with respect to (L, M)-fuzzy filters in [21].

Lemma 5.6. [21] *For a nonempty family $\{\mathcal{F}_j\}_{j \in J}$ of (L, M)-fuzzy filters, the following statements are equivalent:*

(1) *There exists an (L, M)-fuzzy filter $\mathcal{F}$ such that $\mathcal{F} \geqslant \mathcal{F}_j$ for all $j \in J$.*

(2) *$\mathcal{F}_{j_1}(A_1) \wedge \cdots \wedge \mathcal{F}_{j_n}(A_n) = \bot_M$ if $A_1 \wedge \cdots \wedge A_n = \bot_L^X$ $(n \in \mathbf{N}$, $A_1, \cdots, A_n \in L^X$, $\{j_1, \cdots, j_n\} \subseteq J)$.*

In the case of existence, we find

$$\left(\bigvee_{j \in J} \mathcal{F}_j\right)(A) = \bigvee_{n \in \mathbf{N}} \bigvee \{\mathcal{F}_{j_1}(A_1) \wedge \cdots \wedge \mathcal{F}_{j_n}(A_n) \mid A_1 \wedge \cdots \wedge A_n \leqslant A\}$$

as the supremum of $\{\mathcal{F}_j\}_{j \in J}$ in $(\mathcal{F}_{LM}(X), \leqslant)$. We denote it by $\bigvee_{j \in J} \mathcal{F}_j$. For two (L, M)-fuzzy filters $\mathcal{F}$ and $\mathcal{G}$, we write $\mathcal{F} \vee \mathcal{G} \in \mathcal{F}_{LM}(X)$ if $\mathcal{F} \vee \mathcal{G}$ exists.

Lemma 5.7. [21] *Let $\mathcal{F} \in \mathcal{F}_{LM}(Y)$ and $\varphi : X \longrightarrow Y$ be a mapping. Then the following statements are equivalent:*

(1) *$\varphi^{\Leftarrow}(\mathcal{F}) : L^X \longrightarrow M$ defined by $\varphi^{\Leftarrow}(\mathcal{F})(A) = \bigvee_{\varphi^{\leftarrow}(B) \leqslant A} \mathcal{F}(B)$ is an (L, M)-fuzzy filter on X.*

(2) *$\forall B \in L^Y$, $\varphi^{\leftarrow}(B) = \bot_L^X$ implies $\mathcal{F}(B) = \bot_M$.*

In case $\varphi^{\Leftarrow}(\mathcal{F}) \in \mathcal{F}_{LM}(X)$, we call $\varphi^{\Leftarrow}(\mathcal{F})$ the inverse image of $\mathcal{F}$ under φ. Obviously, $\varphi^{\Leftarrow}(\mathcal{F})$ exists if φ is surjective.

Lemma 5.8. [21] *Suppose that $\perp_L$ is prime. Let $\{X_j\}_{j\in J}$ be a family of nonempty sets, $p_k : \prod_{j\in J} X_j \longrightarrow X_k$ the projection mapping and $\mathcal{F}_j \in \mathcal{F}_{LM}(X_j)$ $(\forall j \in J)$. Then $\bigvee_{j\in J} p_j^{\Leftarrow}(\mathcal{F}_j) \in \mathcal{F}_{LM}\left(\prod_{j\in J} X_j\right)$, which is called the product (L, M)-fuzzy filter and denoted by $\prod_{j\in J} \mathcal{F}_j$. For two (L, M)-fuzzy filters $\mathcal{F}$ and $\mathcal{G}$, we write $\mathcal{F} \times \mathcal{G}$.*

Lemma 5.9. [21] *Suppose that $\perp_L$ is prime. Let $\{X_j\}_{j\in J}$ be a family of nonempty sets, $p_k : \prod_{j\in J} X_j \longrightarrow X_k$ the projection mapping, $\mathcal{F}_j \in \mathcal{F}_{LM}(X_j)$ $(\forall j \in J)$ and $\mathcal{F} \in \mathcal{F}_{LM}\left(\prod_{j\in J} X_j\right)$. Then the following statements hold:*

(1) $\prod_{j\in J} p_j^{\Rightarrow}(\mathcal{F}) \leqslant \mathcal{F}$.

(2) $p_k^{\Rightarrow}\left(\prod_{j\in J} \mathcal{F}_j\right) \geqslant \mathcal{F}_k$, $\forall k \in J$.

Lemma 5.10. [21] *Suppose that $\perp_L$ is prime. Let $\mathcal{F} \in \mathcal{F}_{LM}(X_1)$, $\mathcal{G} \in \mathcal{F}_{LM}(X_2)$ and let $\varphi : X_1 \longrightarrow Y_1$ and $\psi : X_2 \longrightarrow Y_2$ be mappings. Then $(\varphi \times \psi)^{\Rightarrow}(\mathcal{F} \times \mathcal{G}) = \varphi^{\Rightarrow}(\mathcal{F}) \times \psi^{\Rightarrow}(\mathcal{G})$.*

Definition 5.11. [1] Let **A** be a category. A family $\{\varphi_j : A_j \longrightarrow A\}_{j\in J}$ of **A**-morphisms indexed by some class J, is called an epi-sink in **A** provided that for any pair $\alpha, \beta : A \longrightarrow D$ of **A**-morphisms such that $\alpha \circ \varphi_j = \beta \circ \varphi_j$ for each $j \in J$, it follows that $\alpha = \beta$.

Lemma 5.12. *Let $\{(X_j, qc_j)\}_{j\in J}$ and (X, qc_X) be a family of stratified (L, M)-fuzzy Q-convergence spaces. Then $\{\varphi_j : (X_j, qc_j) \longrightarrow (X, qc_X)\}_{j\in J}$ is an epi-sink if and only if $X = \bigcup_{j\in J} \varphi_j[X_j]$ $(\varphi_j[X_j] = \{\varphi_j(x^j) \mid x^j \in X_j\})$.*

Proof. The proof of Lemma 6.8 in [21] can be adopted. □

Since the category $S(L, M)$-**FQC** is a topological category, there exists a unique final structure with respect to a sink $\{\varphi_j : (X_j, qc_j) \longrightarrow X\}_{j\in J}$ in $S(L, M)$-**FQC**. However, here we only explore the concrete form of the final structure with respect to the epi-sink.

Theorem 5.13. *Let $\{(X_j, qc_j)\}_{j\in J}$ be a family of stratified (L, M)-fuzzy Q-convergence spaces and X be a nonempty set. If $\{\varphi_j : (X_j, qc_j) \longrightarrow X\}_{j\in J}$ is an epi-sink, then $qc_X : \mathcal{F}_{LM}(X) \longrightarrow L^X$ defined by*

$$\begin{aligned}\forall \mathcal{F} \in \mathcal{F}_{LM}(X),\ qc_X(\mathcal{F}) \quad = \quad & \bigvee\{x_\lambda \in J(L^X) \mid \exists x^j \in X_j, \mathcal{F}_j \in \mathcal{F}_{LM}(X_j) \\ & \text{s.t. } \varphi_j(x^j) = x, \varphi_j^{\Rightarrow}(\mathcal{F}_j) \leqslant \mathcal{F} \text{ and } x_\lambda^j \leqslant qc_j(\mathcal{F}_j)\}\end{aligned}$$

is the unique final structure with respect to the epi-sink $\{\varphi_j : (X_j, qc_j) \longrightarrow X\}_{j\in J}$.

Proof. Firstly, we show that qc_X is a stratified (L, M)-fuzzy Q-convergence structure on X. That is, qc_X satisfies (LFQC1), (LFQC2) and (SLFQC). In fact,

(LFQC1) Take any $x_\lambda \in J(L^X)$. By Lemma 5.12, we obtain $x \in \bigcup \varphi_j[X_j]$. Then there exist $j \in J$ and $x^j \in X_j$ such that $\varphi_j(x^j) = x$. Obviously, $\hat{q}(x_\lambda^j) \in \mathcal{F}_{LM}(X_j)$,

$\varphi_j^{\Rightarrow}(\hat{q}(x_\lambda^j)) = \hat{q}(\varphi_j(x^j)_\lambda) = \hat{q}(x_\lambda)$ and $x_\lambda^j \leqslant qc_j(\hat{q}(x_\lambda^j))$. This implies that $x_\lambda \leqslant qc_X(\hat{q}(x_\lambda))$.

(LFQC2) Straightforward.

(SLFQC) Take any $\lambda \in J(M)$ and $a \in L$ such that $\lambda \nleqslant a'$ and $x_\lambda \leqslant qc_X(\mathcal{F})$. Then

$$\lambda \leqslant qc_X(\mathcal{F})(x) = \bigvee\{\mu \in J(L) \mid \exists x^j \in X_j, \mathcal{F}_j \in \mathcal{F}_{LM}(X_j) \; s.t.\; \varphi_j(x^j) = x, \varphi_j^{\Rightarrow}(\mathcal{F}_j) \leqslant \mathcal{F} \text{ and } x_\mu^j \leqslant qc_j(\mathcal{F}_j)\}$$

Since $\lambda \nleqslant a'$, it follows that $qc_X(\mathcal{F})(x) \nleqslant a'$. This means that there exist $\mu \in J(L)$, $j \in J$, $x^j \in X_j$ and $\mathcal{F}_j \in \mathcal{F}_{LM}(X_j)$ such that $\varphi_j(x^j) = x$, $\varphi_j^{\Rightarrow}(\mathcal{F}_j) \leqslant \mathcal{F}$, $x_\mu^j \leqslant qc_j(\mathcal{F}_j)$ and $\mu \nleqslant a'$. Since qc_j satisfies (SLFQC), we obtain $\mathcal{F}_j(\underline{a}) = \top_M$. This implies that

$$\mathcal{F}(\underline{a}) \geqslant \varphi_j^{\Rightarrow}(\mathcal{F}_j)(\underline{a}) = \mathcal{F}_j(\varphi_j^{\leftarrow}(\underline{a})) = \mathcal{F}_j(\underline{a}) = \top_M.$$

Secondly, let (Y, qc_Y) be a stratified (L,M)-fuzzy Q-convergence space and $\varphi : X \longrightarrow Y$ be a mapping satisfying that $\varphi \circ \varphi_j : (X_j, qc_j) \longrightarrow (Y, qc_Y)$ is continuous for all $j \in J$. Take any $\mathcal{F} \in \mathcal{F}_{LM}(X)$ and $x_\lambda \in J(L^X)$ such that $x_\lambda \leqslant qc_X(\mathcal{F})$, that is, $\lambda \leqslant qc_X(\mathcal{F})(x)$. Let $\mu \in J(L)$ such that $\mu \prec \lambda$. Then there exist $\nu \in J(L)$, $j \in J$, $x^j \in X^j$, $\mathcal{F}_j \in \mathcal{F}_{LM}(X_j)$ such that $\varphi_j(x^j) = x$, $\varphi_j^{\Rightarrow}(\mathcal{F}_j) \leqslant \mathcal{F}$, $x_\nu^j \leqslant qc_j(\mathcal{F}_j)$ and $\mu \leqslant \nu$. Obviously, $x_\mu^j \leqslant qc_j(\mathcal{F}_j)$. By the continuity of $\varphi \circ \varphi_j$, we obtain $\varphi(\varphi_j(x^j))_\mu \leqslant qc_Y(\varphi^{\Rightarrow}(\varphi_j^{\Rightarrow}(\mathcal{F}_j)))$. This implies $\varphi(x)_\mu \leqslant qc_Y(\varphi^{\Rightarrow}(\mathcal{F}))$. By the arbitrariness of μ, we obtain $\varphi(x)_\lambda \leqslant qc_Y(\varphi^{\Rightarrow}(\mathcal{F}))$. This proves the continuity of $\varphi : (X, qc_X) \longrightarrow (Y, qc_Y)$.

As a consequence, the continuity of $\varphi \circ \varphi_j$ for all $j \in J$ implies the continuity of φ. Hence, we obtain that qc_X is the final structure on X with respect to the sink $\{\varphi_j : (X_j, qc_j) \longrightarrow X\}_{j \in J}$. □

Theorem 5.14. *Suppose that $\perp_L$ is prime. Then $S(L,M)$-**FQC** is Cartesian closed.*

Proof. Suppose that $\{\varphi_j : (X_j, qc_j) \longrightarrow (X, qc_X)\}_{j \in J}$ is a final epi-sink in $S(L,M)$-**FQC**. By Theorem 5.13, we know that

$$\forall \mathcal{F} \in \mathcal{F}_{LM}(X), qc_X(\mathcal{F}) = \bigvee\{x_\lambda \in J(L^X) \mid \exists x^j \in X_j, \mathcal{F}_j \in \mathcal{F}_{LM}(X_j) \; s.t.\; \varphi_j(x^j) = x, \varphi_j^{\Rightarrow}(\mathcal{F}_j) \leqslant \mathcal{F} \text{ and } x_\lambda^j \leqslant qc_j(\mathcal{F}_j)\}$$

Next we show that for each stratified (L,M)-fuzzy Q-convergence space (Y, qc_Y), the family $\{1_Y \times \varphi_j : (Y \times X_j, qc_Y \times qc_j) \longrightarrow (Y \times X, qc_Y \times qc_X)\}_{j \in J}$ is a final epi-sink in $S(L,M)$-**FQC**.

Let (Z, qc_Z) be a stratified (L,M)-fuzzy Q-convergence space and $\alpha,\ \beta : (Y \times X, qc_Y \times qc_X) \longrightarrow (Z, qc_Z)$ be continuous such that $\alpha \circ (1_Y \times \varphi_j) = \beta \circ (1_Y \times \varphi_j)$ for each $j \in J$. Take each $(y, x) \in Y \times X$. Since $X = \bigcup_{j \in J} \varphi_j[X_j]$, there exist $j \in J$ and $x^j \in X_j$ such that $\varphi_j(x^j) = x$. Hence, $\alpha((y,x)) = \alpha((y, \varphi_j(x^j))) = \alpha((1_Y \times \varphi_j)((y, x^j))) = \beta((1_Y \times \varphi_j)((y, x^j))) = \beta((y, \varphi_j(x^j))) = \beta((y,x))$, i.e., $\alpha = \beta$. This means $\{1_Y \times \varphi_j : (Y \times X_j, qc_Y \times qc_j) \longrightarrow Y \times X\}_{j \in J}$ is an epi-sink in $S(L,M)$-**FQC**.

In order to show that $qc_Y \times qc_X$ is the final structure with respect to the sink $\{1_Y \times \varphi_j : (Y \times X_j, qc_Y \times qc_j) \longrightarrow Y \times X\}_{j \in J}$, we need only show that for each stratified (L, M)-fuzzy Q-convergence space (Z, qc_Z) and each mapping $\varphi : Y \times X \longrightarrow Z$, the continuity of $\varphi \circ (1_Y \times \varphi_j)$ for each $j \in J$ implies the continuity of φ. Take any $\mathscr{H} \in \mathcal{F}_{LM}(Y \times X)$ and $(y, x)_\lambda \in J(L^{Y \times X})$ such that $(y, x)_\lambda \leqslant (qc_Y \times qc_X)(\mathscr{H})$. By Example 3.10, it follows that $\lambda \leqslant qc_Y(p_Y^{\Rightarrow}(\mathscr{H}))(y)$ and $\lambda \leqslant qc_X(p_X^{\Rightarrow}(\mathscr{H}))(x)$. Take any $\mu \in J(L)$ such that $\mu \prec \lambda \leqslant qc_X(p_X^{\Rightarrow}(\mathscr{H}))(x)$. Then there exist $\nu \in J(L)$, $j \in J$, $x^j \in X_j$, $\mathcal{F}_j \in \mathcal{F}_{LM}(X_j)$ such that $\varphi_j(x^j) = x$, $\varphi_j^{\Rightarrow}(\mathcal{F}_j) \leqslant p_X^{\Rightarrow}(\mathscr{H})$, $x^j_\nu \leqslant qc_j(\mathcal{F}_j)$ and $\mu \leqslant \nu$. Thus, we get $\mu \leqslant \lambda \leqslant qc_Y(p_Y^{\Rightarrow}(\mathscr{H}))(y)$ and $\mu \leqslant \nu \leqslant qc_j(\mathcal{F}_j)(x^j)$. By Lemma 5.9 (2), we obtain

$$\mu \leqslant qc_Y(p_Y^{\Rightarrow}(\mathscr{H}))(y) \wedge qc_j(\mathcal{F}_j)(x^j) \leqslant (qc_Y \times qc_j)(p_Y^{\Rightarrow}(\mathscr{H}) \times \mathcal{F}_j)((y, x^j)).$$

That is, $(y, x^j)_\mu \leqslant (qc_Y \times qc_j)(p_Y^{\Rightarrow}(\mathscr{H}) \times \mathcal{F}_j)$. By the continuity of $\varphi \circ (1_Y \times \varphi_j)$, we obtain

$$\begin{aligned}
\varphi(y, x)_\mu &= (\varphi \circ (1_Y \times \varphi_j))(y, x^j)_\mu \\
&\leqslant qc_Z((\varphi \circ (1_Y \times \varphi_j))^{\Rightarrow}(p_Y^{\Rightarrow}(\mathscr{H}) \times \mathcal{F}_j)) \\
&= qc_Z(\varphi^{\Rightarrow}(p_Y^{\Rightarrow}(\mathscr{H}) \times \varphi_j^{\Rightarrow}(\mathcal{F}_j))) \qquad \text{(by Lemma 5.10)} \\
&\leqslant qc_Z(\varphi^{\Rightarrow}(p_Y^{\Rightarrow}(\mathscr{H}) \times p_X^{\Rightarrow}(\mathscr{H}))) \\
&\leqslant qc_Z(\varphi^{\Rightarrow}(\mathscr{H})). \qquad \text{(by Lemma 5.9 (2))}
\end{aligned}$$

By the arbitrariness of μ, we obtain $\varphi(y, x)_\lambda \leqslant qc_Z(\varphi^{\Rightarrow}(\mathscr{H}))$. This proves that $\varphi : (Y \times X, c_Y \times c_X) \longrightarrow (Z, c_Z)$ is continuous. As a consequence, we obtain that the family $\{1_Y \times \varphi_j : (Y \times X_j, qc_Y \times qc_j) \longrightarrow (Y \times X, qc_Y \times qc_X)\}_{j \in J}$ is a final episink in $S(L, M)$-**FQC**. By Lemma 5.1, we know that the category $S(L, M)$-**FQC** is Cartesian closed. □

6. **Conclusions**

In this paper, we generalized L-fuzzy Q-convergence structures to (L, M)-fuzzy Q-convergence structures and proposed the concept of stratified (L, M)-fuzzy Q-convergence structures. This new concept not only provides a good approach to describe stratified (L, M)-fuzzy topology, but also possesses nice categorical properties. For example, the resulting category of stratified (L, M)-fuzzy Q-convergence spaces is a Cartesian closed topological category. As we all know, stratified (L, M)-fuzzy topological spaces have better properties than (L, M)-fuzzy topological spaces in some aspects. This motivates us to consider topological properties that stratified (L, M)-fuzzy Q-convergence spaces possess. In the future, we will consider topological properties of stratified (L, M)-fuzzy topological spaces, such as compactness and separation properties.

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