DERIVED FUZZY IMPORTANCE OF ATTRIBUTES BASED ON THE WEAKEST TRIANGULAR NORM-BASED FUZZY ARITHMETIC AND APPLICATIONS TO THE HOTEL SERVICES

A. I. BAN, O. I. BAN AND D. A. TUŞE

Abstract. The correlation between the performance of attributes and the overall satisfaction such as they are perceived by the customers is often used to calculate the importance of attributes in the crisp case. Recently, the method was extended, based on the standard Zadeh extension principle, to the fuzzy case, taking into account the specificity of the human thinking. The difficulties of calculation are important and only approximations of the analytic results can be obtained. In the present paper we give a simplified and exact method to compute the derived importance of the attributes in the case of input data given by triangular fuzzy numbers. The effective calculation is based on the T_W-extension principle and it uses reasonable computer resources even if a large number of attributes and customers is considered. The proposed derived method is later on compared with other methods of calculation of the fuzzy importance of attributes. The results of a survey with respect to the quality of hotel services in Oradea (Romania) are subject to the application of the proposed method.

1. **Introduction**

The fuzzy sets, particularly the fuzzy numbers, are often used to modeling uncertain and/or incomplete information in different areas including economy, the recent literature being illustrative. As the simplest example, the classical Likert scale assumes that the responses in a survey are given according to the binary logic and the differences between the successive categories are equal (see, e.g., [37]). For instance, the linguistic scale (”strongly disagree”, ”disagree”, ”fair”, ”agree”, ”strongly agree”) is transformed into 5-point Likert scale $(1, 2, 3, 4, 5)$. Since the human thinking is subjective and ambiguous this kind of responses may lead to unreasonable results, such as fuzzy numbers are preferred instead to crisp numbers for modeling the answers (see, e.g., [4, 5, 7, 11, 14, 20, 39, 40]).

An essential step in many methods related with the decision theory is the determination of the importance of attributes. The measurement of the importance of attributes can be obtained by direct or indirect methods. The direct methods have significant disadvantages pointed out in many papers (see, e.g., [1, 2, 13]), such as

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mathematical methods to derive the importance of attributes were elaborated (see [12, 18, 19, 21, 32, 34, 35]). Even if it is difficult to establish the most appropriate method, the calculation of the importance of attributes as the correlation coefficient between the performance of attributes and the overall customer satisfaction is often preferred (see [12, 19, 32, 34, 35]). In Section 3 we recall this method in the classical case as well as its extension when the input data are expressed by fuzzy numbers ([4]). In the fuzzy case the effective calculation is based on the standard Zadeh's extension principle, but analytical solutions are very difficult to be given. Numerical solutions are obtained by solving crisp mathematical programs (see [4]) which ask huge computer resources, especially in the case of a large number of attributes and/or customers.

In the present paper we propose an indirect method for computing the fuzzy importance of attributes, having as source the correlation coefficient of fuzzy numbers under the weakest triangular norm-based fuzzy arithmetic operations (see [10, 22, 23, 24, 26, 27, 29, 30, 33]). The calculation is drastically simplified, an exact solution can be given, with reasonable computer resources, even in the case of a large volume of input data. Another advantage of the usage of the weakest triangular norm is a smaller fuzziness of the output data (see, e.g., [10, 27, 30]). The method is described in Section 4, after a preliminary section and a description of the problem in Section 3, but the algorithm of calculation is given in Appendix A. Sometimes it is very important to determine a hierarchy of the importance of attributes such that in the same Section 4 and in Appendix B we give a method based on the expected value as a suitable ranking index of fuzzy numbers (see [3]). The methods proposed in Section 4 are exemplified to the study of the quality of hotel services. A survey applied to the customers of four 4-stars hotels from Oradea, Romania, gives us the input data. The results are presented in Section 5. In Section 6 we compare our method with the direct method and the method given in [4] for calculating the fuzzy importance of attributes.

2. **Fuzzy Numbers and Arithmetical Operations**

We recall the basic notions and we fix the notations used in this paper.

Definition 2.1. [43] A fuzzy set A (fuzzy subset of X) is defined as a mapping $A : X \to [0, 1]$, where $A(x)$ is the membership degree of x to the fuzzy set A.

The fuzzy numbers generalize the real numbers and they are fuzzy subsets of the real line with some additional properties.

Definition 2.2. (see [15] or [16]) A fuzzy number A is a fuzzy subset of the real line, $A : \mathbb{R} \to [0, 1]$, satisfying the following properties:

(i) A is normal (i.e. there exists $x_0 \in \mathbb{R}$ such that $A(x_0) = 1$);

(ii) A is fuzzy convex (i.e. $A(\lambda x_1 + (1 - \lambda) x_2) \geq \min(A(x_1), A(x_2))$, for every $x_1, x_2 \in \mathbb{R}$ and $\lambda \in [0, 1]$);

(iii) A is upper semicontinuous on $\mathbb{R}$ (i.e. $\forall \varepsilon > 0, \exists \delta > 0$ such that $A(x) - A(x_0) < \varepsilon$, whenever $|x - x_0| < \delta$);

(iv) $cl\{x \in \mathbb{R} : A(x) > 0\}$ is compact, where $cl(M)$ denotes the closure of the set M.

We denote by $F(\mathbb{R})$ the set of all fuzzy numbers and we consider the following well-known description of a fuzzy number A:

$$A(x) = \begin{cases} 0, & \text{if } x \leq a_1 \\ l_A(x), & \text{if } a_1 \leq x \leq a_2 \\ 1, & \text{if } a_2 \leq x \leq a_3 \\ r_A(x), & \text{if } a_3 \leq x \leq a_4 \\ 0, & \text{if } a_4 \leq x, \end{cases}$$

where $a_1, a_2, a_3, a_4 \in \mathbb{R}, l_A : [a_1, a_2] \to [0,1]$ is a nondecreasing upper semicontinuous function, $l_A(a_1) = 0, l_A(a_2) = 1$, called the left side of A, and $r_A : [a_3, a_4] \to [0,1]$ is a nonincreasing upper semicontinuous function, $r_A(a_3) = 1, r_A(a_4) = 0$, called the right side of A.

Fuzzy numbers with simple membership functions are preferred in practice. A triangular fuzzy number (see, e.g., [22]) $\triangle = (a, \alpha, \beta)$ is defined by the membership function

$$\triangle(x) = \begin{cases} 1 - \frac{a}{\alpha} + \frac{1}{\alpha}x, & \text{if } a - \alpha \leq x \leq a \\ 1 + \frac{a}{\beta} - \frac{1}{\beta}x, & \text{if } a \leq x \leq a + \beta \\ 0, & \text{otherwise.} \end{cases} \tag{1}$$

We denote by $F^{\triangle}(\mathbb{R})$ the set of all triangular fuzzy numbers.

The expected interval EI and the expected value EV of a fuzzy number were introduced (see [9, 17]) by

$$EI(A) = \left[a_2 - \int_{a_1}^{a_2} l_A(x)\,dx, a_3 + \int_{a_3}^{a_4} r_A(x)\,dx\right] \tag{2}$$

$$EV(A) = \frac{a_2 + a_3}{2} - \frac{1}{2}\int_{a_1}^{a_2} l_A(x)\,dx + \frac{1}{2}\int_{a_3}^{a_4} r_A(x)\,dx. \tag{3}$$

It is immediate that

$$EV(\triangle) = a + \frac{\beta - \alpha}{4} \tag{4}$$

for any triangular fuzzy number $\triangle = (a, \alpha, \beta)$.

In [3] and [6] it was proved that a simple and effective ranking index on fuzzy numbers, particularly on triangular fuzzy numbers, is given by the expected value. Namely, for $A, B \in F(\mathbb{R})$ we introduce

$$A \prec B \text{ if and only if } EV(A) < EV(B), \tag{5}$$

$$A \sim B \text{ if and only if } EV(A) = EV(B) \tag{6}$$

and

$$A \preceq B \text{ if and only if } EV(A) \leq EV(B). \tag{7}$$

Definition 2.3. A triangular norm (t-norm, for short) T is an associative, commutative, non-decreasing binary operation on the unit interval such that $T(x, 1) = x$ for every $x \in [0,1]$.

From a mathematical point of view, the most important triangular norms (see, e.g., [25]) are

$$T_M(x,y) = \min(x,y), \tag{8}$$

$$T_P(x,y) = xy, \tag{9}$$

$$T_L(x,y) = \max(0, x+y-1) \tag{10}$$

and T_W, the weakest t-norm, given by

$$T_W(x,y) = \begin{cases} x, & \text{if } y = 1 \\ y, & \text{if } x = 1 \\ 0, & \text{otherwise.} \end{cases} \tag{11}$$

The Zadeh extension principle based on a triangular norm T extends an arithmetical operation $*$ on reals to an arithmetical operation $\circledast$ on fuzzy numbers by (see [22, 44]):

$$(A \circledast B)(z) = \sup_{x*y=z} T(A(x), B(y)). \tag{12}$$

The T_M-based operations are the most popular, but the T_W-based operations have some obvious advantages: the calculation is drastically simplified, the fuzziness of the results is small and the T_W-based addition and multiplication preserve the shape of fuzzy numbers, in particular they preserve the triangular fuzzy numbers ([23, 24, 26, 27, 29, 33]).

We summarize the usual T_W-based arithmetic operations on triangular fuzzy numbers as follows.

Let $A = (a, \alpha, \beta)$ and $B = (b, \gamma, \delta)$ be two triangular fuzzy numbers and $\lambda \in \mathbb{R}, \lambda > 0$. We have (see [10, 22, 23, 24, 26, 27, 29, 30, 33])

$$(a,\alpha,\beta) + (b,\gamma,\delta) = (a+b, \max(\alpha,\gamma), \max(\beta,\delta)), \tag{13}$$

$$(a,\alpha,\beta) - (b,\gamma,\delta) = (a-b, \max(\alpha,\delta), \max(\beta,\gamma)), \tag{14}$$

$$\lambda \cdot (a,\alpha,\beta) = (\lambda a, \alpha, \beta), \tag{15}$$

$$\sqrt{(a,\alpha,\beta)} = \left(\sqrt{a}, \frac{\alpha}{\sqrt{a}}, \frac{\beta}{\sqrt{a}}\right), \text{ for every } a > 0, \tag{16}$$

$$(a,\alpha,\beta) \cdot (b,\gamma,\delta) = \begin{cases} (ab, \max(\alpha b, \gamma a), \max(\beta b, \delta a)), & \text{if } a, b \geq 0 \\ (ab, -\max(\alpha b, \gamma a), -\max(\beta b, \delta a)), & \text{if } a, b \leq 0 \\ (ab, \max(\alpha b, -\delta a), \max(\beta b, -\gamma a)), & \text{if } a \leq 0, b \geq 0 \\ (ab, \max(\gamma a, -\beta b), \max(\delta a, -\alpha b)), & \text{if } a \geq 0, b \leq 0, \end{cases} \tag{17}$$

$$\frac{(a,\alpha,\beta)}{(b,\gamma,\delta)} = \\ = \begin{cases} \left(\frac{a}{b}, \max\left(\frac{\alpha}{b}, \frac{a\delta}{b(b+\delta)}\right), \max\left(\frac{\beta}{b}, \frac{a\gamma}{b(b-\gamma)}\right)\right), & \text{if } a > 0, b > 0, b-\gamma > 0 \\ \left(\frac{a}{b}, \max\left(-\frac{\beta}{b}, -\frac{a\gamma}{b(b-\gamma)}\right), \max\left(-\frac{\alpha}{b}, -\frac{a\delta}{b(b+\delta)}\right)\right), & \text{if } a < 0, b < 0, b+\delta < 0 \\ \left(0, \frac{\alpha}{b}, \frac{\beta}{b}\right), & \text{if } a = 0, b > 0, b-\gamma > 0 \\ \left(0, -\frac{\beta}{b}, -\frac{\alpha}{b}\right), & \text{if } a = 0, b < 0, b+\delta < 0 \\ \left(\frac{a}{b}, \max\left(-\frac{\beta}{b}, \frac{a\delta}{b(b+\delta)}\right), \max\left(-\frac{\alpha}{b}, \frac{a\gamma}{b(b-\gamma)}\right)\right), & \text{if } a > 0, b < 0, b+\delta < 0 \\ \left(\frac{a}{b}, \max\left(\frac{\alpha}{b}, -\frac{a\gamma}{b(b-\gamma)}\right), \max\left(\frac{\beta}{b}, -\frac{a\delta}{b(b+\delta)}\right)\right), & \text{if } a < 0, b > 0, b-\gamma > 0. \end{cases} \tag{18}$$

Remark 2.4. The definition of the division of two triangular fuzzy numbers given by (18) does not cover all the possibilities. Nevertheless, the existing cases are enough for our purposes.

In the description of the situations where the classical quantitative expressions are inadequate, we can use the linguistic variables. As example, they are used, most often, to model the answers to questions in a survey. An adequate representation of the linguistic variables is by trapezoidal or triangular fuzzy numbers. In this way, the data can be processed by mathematical methods. We do not give here an additional example, Table 3 contains two 5-level scales by linguistic variables and their representation as triangular fuzzy numbers.

3. Calculation of the Importance of Attributes by Correlation Method

Let us consider n attributes, $A_1, ..., A_n$, of a service or product and m customers, $C_1, ..., C_m$, consumers of that service or product. We denote by X_{ij} the performance of the attribute $A_j, j \in \{1, ..., n\}$ in the opinion of the customer $C_i, i \in \{1, ..., m\}$, by X_i the overall level of satisfaction of the customer $C_i, i \in \{1, ..., m\}$ and by W_{ij} the importance of the attribute $A_j, j \in \{1, ..., n\}$ in the opinion of the customer $C_i, i \in \{1, ..., m\}$. The importance of the attribute A_j can be given by a direct method, aggregating the values W_{ij}. As a simplest example, if the arithmetic mean is used, then

$$W_j = \frac{1}{m}\sum_{i=1}^{m} W_{ij}$$

can be considered as the importance of the attribute A_j. The method can be extended to the case of input data given as fuzzy numbers in an obvious way, by replacing the arithmetical operations on reals with arithmetical operations on fuzzy numbers (see (12)). Even if they are still widely used, the direct methods have some important disadvantages, pointed out in [1, 2, 4, 13].

The correlation coefficient between the performance perceived for each attribute and the overall satisfaction is already accepted as a successful indirect method to determine the importance of the attributes in the crisp case (see [12, 34, 35]). We recall, in statistical theory, the strength of the relationship between two variables $X = (X_1, ..., X_p)$ and $Y = (Y_1, ..., Y_p)$ is measured by the correlation coefficient $r_{X,Y}$ introduced as (see [38])

$$r_{X,Y} = \frac{\sum_{i=1}^{p}\left(X_i - X^M\right)\left(Y_i - Y^M\right)}{\sqrt{\sum_{i=1}^{p}\left(X_i - X^M\right)^2 \sum_{i=1}^{p}\left(Y_i - Y^M\right)^2}}, \tag{19}$$

where $X^M = \frac{1}{p}\sum_{i=1}^{p} X_i$ and $Y^M = \frac{1}{p}\sum_{i=1}^{p} Y_i$. Based on (19) and with the notations from the beginning of this section, the importance of the attribute $A_j, j \in \{1, ..., n\}$, denoted by W_j, was introduced as the correlation coefficient between $(X_{1j}, ..., X_{mj})$ and $(X_1, ..., X_m)$, therefore

$$W_j = \frac{\sum_{i=1}^{m}\left(X_{ij} - X_j^M\right)\left(X_i - X^M\right)}{\sqrt{\sum_{i=1}^{m}\left(X_{ij} - X_j^M\right)^2 \sum_{i=1}^{m}\left(X_i - X^M\right)^2}},$$

where $X_j^M = \frac{1}{m}\sum_{i=1}^{m} X_{ij}$ and $X^M = \frac{1}{m}\sum_{i=1}^{m} X_i$.

The correlation coefficient of two variables $\widetilde{X} = \left(\widetilde{X_1}, ..., \widetilde{X_p}\right)$ and $\widetilde{Y} = \left(\widetilde{Y_1}, ..., \widetilde{Y_p}\right)$, $\widetilde{X_i}, \widetilde{Y_i}, i \in \{1, ..., p\}$ being fuzzy numbers, was introduced in [31] by

$$r_{\widetilde{X},\widetilde{Y}} = \frac{\sum\limits_{i=1}^{p} \left(\widetilde{X_i} - \widetilde{X^M}\right) \cdot \left(\widetilde{Y_i} - \widetilde{Y^M}\right)}{\sqrt{\sum\limits_{i=1}^{p} \left(\widetilde{X_i} - \widetilde{X^M}\right)^2 \cdot \sum\limits_{i=1}^{p} \left(\widetilde{Y_i} - \widetilde{Y^M}\right)^2}}, \tag{20}$$

where

$$\widetilde{X^M} = \frac{1}{p} \cdot \sum_{i=1}^{p} \widetilde{X_i} \tag{21}$$

and

$$\widetilde{Y^M} = \frac{1}{p} \cdot \sum_{i=1}^{p} \widetilde{Y_i}. \tag{22}$$

In [31] the effective calculation is based on the standard Zadeh's extension principle (that is $T = T_M$, where $T_M(x, y) = \min(x, y)$ in (12)) which generates all the operations in (20)-(22). An analytical calculation is very difficult to be given and, unfortunately, a numerical solution leads to an outrunning of the computer resources in the case of a large sets of input data. Following the idea in the crisp case and the correlation of two fuzzy variables (see (20)), in [4] we introduced the fuzzy importance of attributes as the correlation coefficient between the performance of attributes and the overall satisfaction as follows.

If $\widetilde{X_{ij}} \in F(\mathbb{R})$ denotes the performance of the attribute $A_j, j \in \{1, ..., n\}$ in the opinion of the customer $C_i, i \in \{1, ..., m\}$ and $\widetilde{X_i} \in F(\mathbb{R})$ denotes the overall satisfaction in the opinion of the customer $C_i, i \in \{1, ..., m\}$, then we define the fuzzy importance of the attribute $A_j, j \in \{1, ..., n\}$, as

$$\widetilde{W_j} = \frac{\sum\limits_{i=1}^{m} \left(\widetilde{X_{ij}} - \widetilde{X_j^M}\right) \cdot \left(\widetilde{X_i} - \widetilde{X^M}\right)}{\sqrt{\sum\limits_{i=1}^{m} \left(\widetilde{X_{ij}} - \widetilde{X_j^M}\right)^2 \cdot \sum\limits_{i=1}^{m} \left(\widetilde{X_i} - \widetilde{X^M}\right)^2}}, \tag{23}$$

where

$$\widetilde{X_j^M} = \frac{1}{m} \cdot \sum_{i=1}^{m} \widetilde{X_{ij}} \tag{24}$$

and

$$\widetilde{X^M} = \frac{1}{m} \cdot \sum_{i=1}^{m} \widetilde{X_i}. \tag{25}$$

Because the arithmetical operations between fuzzy numbers are based on the triangular norm T_M too, we have similar difficulties with those already described above (see also [4]).

In the present paper we overcome these problems by using the T_W-extension principle (see (12)), that is the weakest triangular norm T_W given by (11) as a basis for the arithmetical operations on fuzzy numbers. It is well-known that the shape of fuzzy numbers is preserved by the generated addition and multiplication, the calculus is simple and, moreover, the ambiguity of the output data is preserved in reasonable limits. If the input data are triangular fuzzy numbers, that is $\widetilde{X_{ij}} \in F^{\triangle}(\mathbb{R})$ and $\widetilde{X_i} \in F^{\triangle}(\mathbb{R})$ for every $i \in \{1, ..., m\}$ and $j \in \{1, ..., n\}$, then in (23)-(25)

we can use the operations introduced by (13)-(18) to calculate the fuzzy importance of an attribute.

4. Calculating and Ordering of the Fuzzy Importance of Attributes by Correlation Method Under the Weakest t-norm Based Fuzzy Arithmetic

The triangular fuzzy numbers are most common in current applications and often they are considered to be sufficient for processing and modeling the fuzzy information (see, e.g., [36]). Therefore, with the notations in Section 3, we consider $\widetilde{X_{ij}} = (x_{ij}, \alpha_{ij}, \beta_{ij}), i \in \{1, ..., m\}, j \in \{1, ..., n\}$ be the performance with respect to the attribute $A_j, j \in \{1, ..., n\}$ in the opinion of the customer $C_i, i \in \{1, ..., m\}$ and $\widetilde{X_i} = (x_i, \gamma_i, \delta_i)$ be the overall level of satisfaction in the opinion of the customer $C_i, i \in \{1, ..., m\}$. Based on (23)-(25) and the arithmetical operations detailed in (13)-(18), in Appendix A we describe an algorithm for calculating the importance of the attributes in the case of input data as triangular fuzzy numbers (see (1)).

On the other hand, the results obtained by fuzzy methods can be easiest interpreted after defuzzification and, in addition, sometimes it is more important to obtain a hierarchy of the importance of attributes than evaluations of these. A fuzzy number can be defuzzified starting from several methods: center of area, mean of maxima, expected value etc. We choose to work with the expected value (see (3), (4)) because it is proved to be a parameter associated to fuzzy numbers which act as a suitable ranking index (see [3]), often used in applications (see, e.g., [5, 7, 14]) due to its simplicity.

We obtain the crisp values of the fuzzy importance of attributes calculated by the correlation method under the weakest t-norm based fuzzy arithmetic and, simultaneously, a hierarchy of them by applying the algorithm described in Appendix B to the input data $\widetilde{X_{ij}}$ and $\widetilde{X_i}$, $i \in \{1, ..., m\}, j \in \{1, ..., n\}$ as above.

In Appendix C we consider some theoretical examples to prove the feasibility of the method described by the algorithms in Appendix A and Appendix B.

5. Study of the Hotel Services in Oradea by the Proposed Method

The linguistic variables are useful for describing situations where the classical quantitative expressions are inadequate. The best example is the answers to surveys. Instead to express the answers to questions by real numbers, it is more suitable to attach linguistic variables which later on may be associated with fuzzy numbers. In this way, the subsequent processing starts from data according with the human thinking and the results refine those obtained by crisp methods.

We exemplify the method proposed in the previous sections to the study of the quality of hotel services. We consider the same survey as in [4].

During two weeks in June 2012 a number of 125 questionnaires was applied to customers of four 4-stars hotels from Oradea, Romania. For the establishment of the attributes the SERVQUAL scale was considered. The complete list of attributes was included in Table 1.

The value of the α-Cronbach coefficient (0.827) is a satisfactory one (see [8]) for the validity of the questionnaire. The performance of attributes and the overall

No.	Attribute
1	*The room facilities are appropriate*
2	*The room is clean enough*
3	*The hotel has sufficient restaurant facilities*
4	*The staff has an appropriate and professional look*
5	*The location of the hotel is suitable*
6	*The staff provide correct information to guests*
7	*The staff is able to offer services in a short period of time*
8	*The staff is able to resolve guests' problems*
9	*The staff is able to provide information in a short time*
10	*The availability of staff*
11	*Clients complaints are resolved quickly*
12	*Different payment facilities are available*
13	*The safety of the installations in the hotel*
14	*Service professionalism*
15	*Service customization*
16	*Friendliness of staff*
17	*Proper opening hours of hotel's facilities*
18	*The hotel has entertaining facilities*
19	*Big variety and proper quality of meals*
20	*Internet connection is available*
21	*Aesthetics of rooms and of the hotel*

Table 1. List of Attributes

customer satisfaction (OCS) measured on a five Likert scale {Very poor (VP), Poor (P), Medium (M), Good (G), Very good (VG)} for all hotels and separately for each hotel are summarized in Table 2.

Our aim is to apply the method described in Sections 3 and 4 to the set of attributes in Table 1 and their performances given in Table 2.

We consider the corresponding triangular fuzzy numbers to linguistic variables as they are indicated in Table 3.

By applying the algorithm in Appendix A to the input data in Table 2 for all hotels and separately for each hotel, we obtain the results in Table 4. We get the importance of attributes expressed by real numbers and implicitly a hierarchy of the attributes according with their importance, by applying the algorithm described in Appendix B. The results are included in Table 5.

The main aim of the present paper is to propose a new method of calculation of the importance of attributes in the fuzzy case and not to analyze or interpret the results obtained in applications. Nevertheless, we point out some immediate conclusions that can be drawn from Tables 4 and 5. The positivity of the values for all hotels in the second column of Table 5 reveal us that, by synthesizing all the answers, any improvement of a service is resulting in a higher level of satisfaction. This conclusion does not remain valid by passing to a certain hotel. For example, 14 of 21 attributes have quite close too zero but negative importance in the case of Hotel 4. This means that an improvement of the performance of an attribute from this category does not increase the level of satisfaction of the customers. There is nothing new: an overemphasis on some attributes may lead to dissatisfaction (see, e.g., [2, 8, 13]). From another point of view, from Table 5, the column corresponding to all hotels, we get that for the persons which accommodate in the 4-stars in Oradea the most important is attribute 9 ("The staff is able to provide information

		Attribute																					
Hotel		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	29	21	OCS
	VP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	P	2	1	1	2	3	2	0	2	2	2	3	2	1	2	1	2	2	4	2	2	0	0
All	M	8	9	10	5	15	11	13	13	13	10	10	13	9	10	16	14	12	13	10	15	10	9
	G	43	33	49	46	49	35	29	41	53	42	46	35	43	48	57	42	49	42	61	39	29	45
	VG	72	82	65	72	58	77	83	69	57	71	66	75	72	65	51	67	62	66	52	69	86	71
	VP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	2	0	0
Hotel 1	M	1	1	3	0	1	1	4	3	2	1	4	2	2	1	3	1	1	1	2	5	0	1
	G	7	0	16	7	13	3	5	5	11	6	9	13	10	8	17	2	15	13	25	8	6	6
	VG	31	38	20	32	25	35	30	31	26	32	26	24	27	30	19	36	23	25	12	24	33	32
	VP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hotel 2	M	0	1	0	0	2	0	0	1	1	1	0	1	0	0	1	1	0	0	1	0	0	1
	G	1	1	3	6	7	2	3	3	4	2	4	2	2	3	6	3	5	3	2	2	1	2
	VG	20	19	18	15	12	19	18	17	16	18	17	18	19	18	14	17	16	18	18	19	20	18
	VP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	P	2	1	1	2	3	2	0	2	2	2	3	2	1	2	1	2	2	4	2	0	0	0
Hotel 3	M	7	7	7	5	7	9	8	9	9	8	4	8	6	8	10	9	8	10	5	8	6	4
	G	14	18	11	13	11	12	12	11	15	13	16	6	14	12	15	17	9	9	15	13	14	21
	VG	6	3	10	9	8	6	9	7	3	6	6	13	8	7	3	1	10	6	7	8	9	4
	VP	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	P	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
Hotel 4	M	0	0	0	0	5	1	1	0	1	0	2	2	1	1	2	3	3	2	2	2	4	3
	G	21	14	19	20	18	18	9	22	23	21	17	14	17	25	19	20	20	17	19	16	8	16
	VG	15	22	17	16	13	17	26	14	12	15	17	20	18	10	15	13	13	17	15	18	24	17

TABLE 2. Performance of Attributes

Performance of attributes/ Overall satisfaction	Rating of importance	Triangular fuzzy number
Very poor (VP)	Unimportant (U)	$(1, 1, 1)$
Poor (P)	Less important (L)	$(3, 2, 1)$
Medium (M)	Medium important (M)	$(5, 2, 2)$
Good (G)	Important (I)	$(7, 1, 2)$
Very good (VG)	Very important (VI)	$(9, 1, 1)$

TABLE 3. Linguistic Variables

in a short time") and the least important is attribute 20 ("Internet connection is available").

6. Comparisons with Other Methods of Calculation

In this section we compare the method proposed in the previous sections with the direct method and with the method given in [4] for calculating the importance of attributes.

6.1. Proposed Method Versus Direct Calculation. The direct methods of determination of the importance of attributes are widely used, even if they become

At.	All	Hotel 1	Hotel 2	Hotel 3	Hotel 4
1	(0.30, 0.05, 0.05)	(-0.21, 0.21, 0.21)	(0.81, 0.43, 0.43)	(0.16, 0.14, 0.14)	(-0.16, 0.06, 0.12)
2	(0.42, 0.05, 0.05)	(-0.07, 0.34, 0.34)	(0.70, 0.37, 0.48)	(0.20, 0.18, 0.18)	(-0.05, 0.06, 0.12)
3	(0.35, 0.03, 0.03)	(0.39, 0.13, 0.13)	(0.66, 0.23, 0.35)	(0.39, 0.08, 0.08)	(-0.14, 0.06, 0.12)
4	(0.42, 0.03, 0.03)	(0.23, 0.12, 0.12)	(0.39, 0.15, 0.21)	(0.45, 0.08, 0.08)	(-0.02, 0.06, 0.12)
5	(0.36, 0.02, 0.02)	(0.10, 0.17, 0.17)	(0.27, 0.21, 0.21)	(0.41, 0.08, 0.08)	(0.06, 0.09, 0.09)
6	(0.41, 0.03, 0.03)	(-0.14, 0.26, 0.26)	(0.20, 0.29, 0.59)	(0.23, 0.10, 0.10)	(0.22, 0.05, 0.05)
7	(0.19, 0.03, 0.03)	(-0.06, 0.15, 0.15)	(0.39, 0.23, 0.23)	(0.09, 0.09, 0.09)	(-0.15, 0.14, 0.14)
8	(0.42, 0.03, 0.03)	(0.16, 0.17, 0.17)	(0.55, 0.32, 0.32)	(0.30, 0.09, 0.09)	(0.05, 0.05, 0.05)
9	(0.44, 0.04, 0.04)	(0.18, 0.15, 0.15)	(0.50, 0.30, 0.30)	(0.09, 0.14, 0.14)	(0.31, 0.06, 0.06)
10	(0.39, 0.03, 0.03)	(-0.08, 0.21, 0.21)	(0.62, 0.35, 0.35)	(0.31, 0.09, 0.09)	(-0.07, 0.06, 0.12)
11	(0.31, 0.04, 0.04)	(0.29, 0.13, 0.13)	(0.30, 0.20, 0.20)	(0.00, 0.14, 0.14)	(0.09, 0.10, 0.10)
12	(0.18, 0.03, 0.03)	(-0.14, 0.17, 0.17)	(0.43, 0.35, 0.35)	(0.00, 0.07, 0.07)	(0.00, 0.11, 0.11)
13	(0.24, 0.03, 0.03)	(-0.18, 0.17, 0.17)	(0.20, 0.29, 0.59)	(0.25, 0.08, 0.08)	(-0.05, 0.11, 0.11)
14	(0.37, 0.03, 0.03)	(0.11, 0.20, 0.20)	(0.39, 0.25, 0.49)	(0.23, 0.09, 0.09)	(-0.04, 0.12, 0.12)
15	(0.21, 0.03, 0.03)	(-0.24, 0.16, 0.16)	(0.24, 0.27, 0.27)	(0.09, 0.12, 0.12)	(0.00, 0.10, 0.10)
16	(0.37, 0.03, 0.03)	(-0.12, 0.28, 0.28)	(0.55, 0.15, 0.29)	(0.00, 0.14, 0.14)	(-0.06, 0.10, 0.10)
17	(0.34, 0.03, 0.03)	(0.05, 0.18, 0.18)	(0.01, 0.20, 0.41)	(0.28, 0.07, 0.07)	(0.22, 0.10, 0.10)
18	(0.32, 0.04, 0.04)	(0.10, 0.19, 0.19)	(0.39, 0.23, 0.23)	(0.07, 0.11, 0.11)	(-0.06, 0.10, 0.10)
19	(0.17, 0.04, 0.04)	(0.11, 0.13, 0.13)	(0.62, 0.35, 0.35)	(-0.16, 0.15, 0.15)	(-0.08, 0.10, 0.10)
20	(0.10, 0.04, 0.04)	(-0.24, 0.15, 0.15)	(0.53, 0.29, 0.53)	(0.09, 0.09, 0.09)	(-0.09, 0.11, 0.11)
21	(0.25, 0.03, 0.03)	(-0.19, 0.14, 0.28)	(-0.09, 0.43, 0.81)	(0.37, 0.09, 0.09)	(-0.18, 0.10, 0.10)

TABLE 4. Fuzzy Importance of the Attributes

All		Hotel 1		Hotel 2		Hotel 3		Hotel 4	
Attr.	Imp.	Attr.	Imp.	Attr.	Imp.	Attr.	Imp.	Attr.	Imp.
9	0.444	3	0.393	1	0.810	4	0.452	9	0.314
4	0.425	11	0.290	2	0.728	5	0.415	17	0.224
2	0.420	4	0.226	3	0.691	3	0.386	6	0.221
8	0.417	9	0.183	10	0.618	21	0.369	11	0.086
6	0.410	8	0.162	19	0.618	10	0.310	5	0.058
10	0.390	19	0.108	16	0.589	8	0.296	8	0.050
14	0.374	14	0.107	20	0.584	17	0.278	12	0.000
16	0.370	18	0.095	8	0.552	13	0.250	15	-0.004
5	0.358	5	0.095	9	0.498	6	0.230	4	-0.004
3	0.355	17	0.052	14	0.451	14	0.225	2	-0.034
17	0.339	7	-0.056	12	0.427	2	0.198	14	-0.044
18	0.316	2	-0.072	4	0.406	1	0.158	13	-0.048
11	0.315	10	-0.077	18	0.389	15	0.094	10	-0.058
1	0.301	16	-0.120	7	0.389	20	0.088	18	-0.061
21	0.248	12	-0.140	11	0.301	9	0.087	16	-0.064
13	0.244	6	-0.141	13	0.275	7	0.086	19	-0.079
15	0.210	21	-0.154	6	0.275	18	0.068	20	-0.089
7	0.194	13	-0.180	5	0.274	11	0.000	3	-0.126
12	0.175	1	-0.214	15	0.245	16	0.000	7	-0.145
19	0.171	15	-0.239	17	0.061	12	0.000	1	-0.147
20	0.104	20	-0.243	21	0.011	19	-0.159	21	-0.177

TABLE 5. Decreasing Ordering of the Importance of Attributes for Each Hotel Under Study

unfeasible when more than few attributes are implied, the scores have a very small inter-itemic variation - with scores uniformly high, they increase the dimension of the survey or are influenced by the punctual performance of the products or services, etc. (see [1, 2, 13]).

Starting from the answers of the customers given by linguistic variables, which can be easily expressed as triangular fuzzy numbers (see, e.g., Table 3), we can calculate the importance of the attributes.

Let $\widetilde{W_{ij}}^* = \left(w_{ij}^*, \alpha_{ij}^*, \beta_{ij}^*\right)$ be the importance of the attribute $A_j, j \in \{1, ..., n\}$ in the opinion of the customer $C_i, i \in \{1, ..., m\}$. We denote by $\widetilde{W_j}^*, j \in \{1, ..., n\}$, the importance of the attribute $A_j, j \in \{1, ..., n\}$. We compute $\widetilde{W_j}^*, j \in \{1, ..., n\}$, by aggregating the individual opinions $\widetilde{W_{ij}}^*, i \in \{1, ..., m\}, j \in \{1, ..., n\}$ under the arithmetic mean, with the arithmetical operations generated by the weakest triangular norm T_W (see (13)-(18)). The algorithm is described in Appendix D.

The survey applied to customers of four 4-stars hotels from Oradea (see Section 5) has questions about the importance of each attribute. We do not give here the answers of the customers, but we point out that the five Likert scale in Table 3 was considered to rating the importance and the triangular fuzzy numbers in the third column of Table 3 are associated to linguistic variables.

We apply the algorithm in Appendix D to calculate the importance of attributes. We obtain the results in Table 6, where the attributes are ranked in the decreasing ordering of their importance.

All		Hotel 1		Hotel 2		Hotel 3		Hotel 4	
Attr.	Imp.	Attr.	Imp.	Attr.	Imp.	Attr.	Imp.	Attr.	Imp.
2	8.104	2	9.096	7	9.155	4	7.207	2	8.639
7	8.056	16	8.994	2	9.060	5	7.069	4	8.361
6	8.040	11	8.891	6	8.774	6	7.069	3	8.250
11	7.880	20	8.891	10	8.774	7	6.931	13	8.250
16	7.864	6	8.590	11	8.774	17	6.862	6	8.194
21	7.832	8	8.487	13	8.774	8	6.793	21	8.167
10	7.784	10	8.436	12	8.583	21	6.793	7	8.056
8	7.768	7	8.436	21	8.583	12	6.724	1	7.972
9	7.768	19	8.385	1	8.488	16	6.724	12	7.944
20	7.752	14	8.385	9	8.429	19	6.655	11	7.889
13	7.672	9	8.333	16	8.238	18	6.655	10	7.833
12	7.576	15	8.077	20	8.048	20	6.586	9	7.778
1	7.512	21	8.026	8	7.952	9	6.517	8	7.667
5	7.496	1	7.974	17	7.857	13	6.448	16	7.611
19	7.464	5	7.923	18	7.667	15	6.448	18	7.611
15	7.464	13	7.821	3	7.667	11	6.379	15	7.611
4	7.464	17	7.718	5	7.667	14	6.310	20	7.556
14	7.432	18	7.667	15	7.476	10	6.310	14	7.444
18	7.416	3	7.513	19	7.476	2	6.241	5	7.278
17	7.400	12	7.462	14	7.190	3	6.241	17	7.222
3	7.384	4	7.256	4	7.095	1	6.103	19	7.111

Table 6. Decreasing Ordering of the Importance of Attributes by Direct Method

By analyzing the results obtained by the proposed method in the defuzzified form (see Table 5) with those obtained by the direct method in the defuzzified form too (Table 6) we find that, as we expected, there are significant differences. In fact, for all attributes except 8 ("The staff is able to resolve guests' problems") and 15 ("Service customization"), there is a difference of at least 10 positions for at least one of the four hotels or in the category that groups all hotels. A good example is attribute 2 ("The rooms is clean enough") that is on the first position of importance at Hotel 1 and Hotel 4 when the direct calculation is performed, and on positions 12 and 10, respectively, when our proposed method is applied. We obtain a similar conclusion when the attribute 7 ("The staff is able to offer services in a short period of time") is considered. By applying our method, the attribute 7 is on positions 11, 14, 16, 19, 18 at Hotel 1, 2, 3, 4 and all hotels, respectively, and on positions 8, 1, 4, 7, 2 when the direct method is performed. The conclusion is not at all surprising and it was revealed in the recent literature (see, e.g., [2, 8, 13]): an overemphasis on the attributes which have been classified as "must-be" (the above discussed attributes 2 and 7 are in this category) does not necessarily lead to an increasing satisfaction. The negative values of the importance of some attributes (see Table 5) may be justified in this way too. An analogous conclusion was obtained in [8] from the method of Kano assessment of quality of attributes applied to the same study of four 4-stars hotels in Oradea.

6.2. **Proposed Method Versus Indirect Calculation in** [4]**.** The correlation coefficient of fuzzy numbers as a fuzzy number was introduced in [31]. Starting from this idea, an indirect method for computing the fuzzy importance of attributes was given in [4]. Unlike the method described in Section 4, where the calculation is based on the triangular norm T_W (see (12) and (13)-(18)), in [4] the triangular norm $T_M, T_M(x, y) = \min(x, y)$ is used in the Zadeh extension principle (12). Analytical solutions are very difficult to be given and numerical solutions are obtained by solving crisp mathematical programs, with large computer resources at least in the case of numerous attributes and/or customers. We do not discuss here these issues, our aim is to compare the results, taking into account that the same survey, the same ratings and triangular fuzzy numbers associated to ratings were considered in [4].

In the below discussion we consider all four 4-stars hotels, that is the answers to all 125 questionnaires. In the first and second column of Table 7 we recall the decreasing ordering of the importance of attributes together with their values as they were obtained in [4]. In the third and fourth column we summarize the results obtained by the indirect method based on the correlation coefficient and the triangular norm T_W, in the decreasing ordering and by noting their values (see Table 5). In the fifth column, the decreasing ordering of the importance of attributes obtained by the direct method is presented (see Table 6).

The two indirect methods of calculation give almost the same ordering of the importance of attributes. The biggest difference in the two hierarchies is of two positions (see Table 7, first and third column). In addition, the importance of the attributes calculated by these methods are very close, as it shows the values

Method in [4]		New method		Direct method	
Attribute	Importance	Attribute	Importance	Attribute	Importance
4	0.401	9	0.444	2	8.104
9	0.390	4	0.425	7	8.056
8	0.376	2	0.420	6	8.040
2	0.369	8	0.417	11	7.880
6	0.364	6	0.410	16	7.864
10	0.355	10	0.390	21	7.832
5	0.333	14	0.374	10	7.784
14	0.329	16	0.370	8	7.768
3	0.318	5	0.358	9	7.768
16	0.311	3	0.355	20	7.752
17	0.300	17	0.339	13	7.672
18	0.277	18	0.316	12	7.576
1	0.271	11	0.315	1	7.512
11	0.269	1	0.301	5	7.496
13	0.217	21	0.248	19	7.464
21	0.213	13	0.244	15	7.464
15	0.179	15	0.210	4	7.464
7	0.168	7	0.194	14	7.432
12	0.161	12	0.175	18	7.416
19	0.153	19	0.171	17	7.400
20	0.079	20	0.104	3	7.384

TABLE 7. Comparative Results

obtained after defuzzification with the expected value (second and fourth column in Table 7).

On this example, we can conclude that in the calculation of the importance of attributes based on the correlation method, already classical arithmetical operations obtained by the standard Zadeh extension principle could be replaced by the T_W-based arithmetical operations generated by (12) (see (13)-(18)). The main advantages are related with the possibility of an analytical calculation and a less complicated calculation from the point of view of resources, both important for subsequent developments.

As we already pointed when we dealt with the direct method versus the method proposed in the present paper (see Section 6.1), the differences between them are significant. Table 7 illustrates both the similarities between the results obtained by the indirect methods and the differences between them and the direct method of calculation.

7. **Conclusion**

It is well-known that the determination of the importance of attributes is an essential step in multi-criteria decision making methods and importance-performance analysis. An indirect method for calculating the fuzzy importance of attributes as the correlation between the fuzzy performance of attributes and the fuzzy overall level of satisfaction, the arithmetical operations being generated by the weakest triangular norm T_W, is proposed in the present paper. Unlike [4], where the same idea is fructified but the arithmetical operations are generated by the minimum triangular norm T_M and the numerical results are obtained by solving crisp mathematical programs, with huge computer resources, by the method developed in Section 4 and detailed in Appendices A and B, analytical results are obtained by reasonable computer resources. Generally, the importance of attributes is not so relevant itself, but it is used for the development of decision-making methods, importance-performance analysis etc. In addition, it is well to emphasize that, for real data we have received quite similar results through two indirect methods.

To conclude with, as it was already remarked even in the crisp case (see, e.g., [18]), it is very difficult to give a final answer to the question of the best method of calculation of the importance of attributes. The choosing of a method is context dependent and it is a matter of preference. Nevertheless, in the case of a derived fuzzy importance based on the correlation coefficient, the method proposed in the present paper has some already pointed out advantages.

Future researches will be dedicated to the application of the elaborated method to different categories of customers, according with certain criteria, or to the identification of the representative hotel in an area or resort. It is worth noting that the method described in the present paper is a part of a project related with the development of a fuzzy importance-performance analysis, very important for determining critical service attributes.

8. **Appendix**

A. Algorithm of Calculation of the Fuzzy Importance of Attributes

With the notations in Section 4, the following algorithm of calculation of the fuzzy importance of attributes can be elaborated.

Step 1: Compute (see (13) and (15))

$$\widetilde{X^M} = \left(\frac{1}{m} \sum_{i=1}^{m} x_i, \max_{i \in \{1,\ldots,m\}} \gamma_i, \max_{i \in \{1,\ldots,m\}} \delta_i \right).$$

Step 2: Put $i = 1$.

Step 3: Compute (see (14))

$$\widetilde{X_i} - \widetilde{X^M} = \left(x_i - \frac{1}{m} \sum_{i=1}^{m} x_i, \max\left\{ \gamma_i, \max_{i \in \{1,\ldots,m\}} \delta_i \right\}, \max\left\{ \delta_i, \max_{i \in \{1,\ldots,m\}} \gamma_i \right\} \right).$$

Step 4: Compute $\widetilde{R_i} = \left(\widetilde{X_i} - \widetilde{X^M} \right)^2 = (c_i, \varphi_i, \psi_i)$, where (see (17)) $c_i = \left(x_i - \frac{1}{m} \sum_{i=1}^{m} x_i \right)^2$.

If $x_i \geq \frac{1}{m} \sum_{i=1}^{m} x_i$ then

$$\varphi_i = \max\left\{ \gamma_i, \max_{i \in \{1,\ldots,m\}} \delta_i \right\} \left(x_i - \frac{1}{m} \sum_{i=1}^{m} x_i \right)$$

and

$$\psi_i = \max\left\{ \delta_i, \max_{i \in \{1,\ldots,m\}} \gamma_i \right\} \left(x_i - \frac{1}{m} \sum_{i=1}^{m} x_i \right).$$

If $x_i < \frac{1}{m}\sum\limits_{i=1}^{m} x_i$ then

$$\varphi_i = -\max\left\{\gamma_i, \max_{i\in\{1,\dots,m\}}\delta_i\right\}\left(x_i - \frac{1}{m}\sum_{i=1}^{m} x_i\right)$$

and

$$\psi_i = -\max\left\{\delta_i, \max_{i\in\{1,\dots,m\}}\gamma_i\right\}\left(x_i - \frac{1}{m}\sum_{i=1}^{m} x_i\right).$$

Step 5: Put $i = i+1$. If $i \le m$ then go to Step 3, else go to Step 6.

Step 6: Compute $\widetilde{R} = \sum\limits_{i=1}^{m}\widetilde{R_i} = \sum\limits_{i=1}^{m}\left(\widetilde{X_i} - \widetilde{X^M}\right)^2 = (c, \varphi, \psi)$, where (see (13)) $c = \sum\limits_{i=1}^{m} c_i, \varphi = \max\limits_{i\in\{1,\dots,m\}}\varphi_i, \psi = \max\limits_{i\in\{1,\dots,m\}}\psi_i$.

Step 7: Put $j = 1$.

Step 8: Compute (see (13) and (15))

$$\widetilde{X_j^M} = \left(\frac{1}{m}\sum_{i=1}^{m} x_{ij}, \max_{i\in\{1,\dots,m\}}\alpha_{ij}, \max_{i\in\{1,\dots,m\}}\beta_{ij}\right).$$

Step 9: Put $i = 1$.

Step 10: Compute (see (14))

$$\widetilde{X_{ij}} - \widetilde{X_j^M} = \left(x_{ij} - \frac{1}{m}\sum_{i=1}^{m} x_{ij}, \max\left\{\alpha_{ij}, \max_{i\in\{1,\dots,m\}}\beta_{ij}\right\}, \max\left\{\beta_{ij}, \max_{i\in\{1,\dots,m\}}\alpha_{ij}\right\}\right).$$

Step 11: Compute

$$r_{ij}^1 = \max\left\{\alpha_{ij}, \max_{i\in\{1,\dots,m\}}\beta_{ij}\right\}\left(x_i - \frac{1}{m}\sum_{i=1}^{m} x_i\right),$$
$$r_{ij}^2 = \max\left\{\gamma_i, \max_{i\in\{1,\dots,m\}}\delta_i\right\}\left(x_{ij} - \frac{1}{m}\sum_{i=1}^{m} x_{ij}\right),$$
$$r_{ij}^3 = \max\left\{\beta_{ij}, \max_{i\in\{1,\dots,m\}}\alpha_{ij}\right\}\left(x_i - \frac{1}{m}\sum_{i=1}^{m} x_i\right),$$
$$r_{ij}^4 = \max\left\{\delta_i, \max_{i\in\{1,\dots,m\}}\gamma_i\right\}\left(x_{ij} - \frac{1}{m}\sum_{i=1}^{m} x_{ij}\right).$$

Step 12: Compute $\widetilde{P_{ij}} = \left(\widetilde{X_{ij}} - \widetilde{X_j^M}\right)\cdot\left(\widetilde{X_i} - \widetilde{X^M}\right) = (a_{ij}, \rho_{ij}, \sigma_{ij})$, where (see (17))

$$a_{ij} = \left(x_{ij} - \frac{1}{m}\sum_{i=1}^{m} x_{ij}\right)\left(x_i - \frac{1}{m}\sum_{i=1}^{m} x_i\right).$$

If $x_{ij} \ge \frac{1}{m}\sum\limits_{i=1}^{m} x_{ij}$ and $x_i \ge \frac{1}{m}\sum\limits_{i=1}^{m} x_i$ then $\rho_{ij} = \max\left\{r_{ij}^1, r_{ij}^2\right\}$ and $\sigma_{ij} = \max\left\{r_{ij}^3, r_{ij}^4\right\}$.

If $x_{ij} \leq \frac{1}{m}\sum_{i=1}^{m} x_{ij}$ and $x_i \leq \frac{1}{m}\sum_{i=1}^{m} x_i$ then $\rho_{ij} = -\max\left\{r_{ij}^1, r_{ij}^2\right\}$ and $\sigma_{ij} = -\max\left\{r_{ij}^3, r_{ij}^4\right\}$.

If $x_{ij} \leq \frac{1}{m}\sum_{i=1}^{m} x_{ij}$ and $x_i \geq \frac{1}{m}\sum_{i=1}^{m} x_i$ then $\rho_{ij} = \max\left\{r_{ij}^1, -r_{ij}^4\right\}$ and $\sigma_{ij} = \max\left\{r_{ij}^3, -r_{ij}^2\right\}$.

If $x_{ij} \geq \frac{1}{m}\sum_{i=1}^{m} x_{ij}$ and $x_i \leq \frac{1}{m}\sum_{i=1}^{m} x_i$ then $\rho_{ij} = \max\left\{r_{ij}^2, -r_{ij}^3\right\}$ and $\sigma_{ij} = \max\left\{r_{ij}^4, -r_{ij}^1\right\}$.

Step 13: Compute $\widetilde{Q_{ij}} = \left(\widetilde{X_{ij}} - \widetilde{X_j^M}\right)^2 = (b_{ij}, \mu_{ij}, \nu_{ij})$, where (see (17)) $b_{ij} = \left(x_{ij} - \frac{1}{m}\sum_{i=1}^{m} x_{ij}\right)^2$.

If $x_{ij} \geq \frac{1}{m}\sum_{i=1}^{m} x_{ij}$ then $\mu_{ij} = \max\left\{\alpha_{ij}, \max\limits_{i\in\{1,\ldots,m\}} \beta_{ij}\right\}\left(x_{ij} - \frac{1}{m}\sum_{i=1}^{m} x_{ij}\right)$ and $\nu_{ij} = \max\left\{\beta_{ij}, \max\limits_{i\in\{1,\ldots,m\}} \alpha_{ij}\right\}\left(x_{ij} - \frac{1}{m}\sum_{i=1}^{m} x_{ij}\right)$.

If $x_{ij} < \frac{1}{m}\sum_{i=1}^{m} x_{ij}$ then $\mu_{ij} = -\max\left\{\alpha_{ij}, \max\limits_{i\in\{1,\ldots,m\}} \beta_{ij}\right\}\left(x_{ij} - \frac{1}{m}\sum_{i=1}^{m} x_{ij}\right)$ and $\nu_{ij} = -\max\left\{\beta_{ij}, \max\limits_{i\in\{1,\ldots,m\}} \alpha_{ij}\right\}\left(x_{ij} - \frac{1}{m}\sum_{i=1}^{m} x_{ij}\right)$.

Step 14: Put $i = i + 1$. If $i \leq m$ then go to Step 10, else go to Step 15.

Step 15: Compute $\widetilde{P_j} = \sum_{i=1}^{m} \widetilde{P_{ij}} = \sum_{i=1}^{m} \left(\widetilde{X_{ij}} - \widetilde{X_j^M}\right) \cdot \left(\widetilde{X_i} - \widetilde{X^M}\right) = (a_j, \rho_j, \sigma_j)$, where (see (13)) $a_j = \sum_{i=1}^{m} a_{ij}, \rho_j = \max\limits_{i\in\{1,\ldots,m\}} \rho_{ij}$ and $\sigma_j = \max\limits_{i\in\{1,\ldots,m\}} \sigma_{ij}$.

Step 16: Compute $\widetilde{Q_j} = \sum_{i=1}^{m} \widetilde{Q_{ij}} = \sum_{i=1}^{m} \left(\widetilde{X_{ij}} - \widetilde{X_j^M}\right)^2 = (b_j, \mu_j, \nu_j)$, where (see (13)) $b_j = \sum_{i=1}^{m} b_{ij}, \mu_j = \max\limits_{i\in\{1,\ldots,m\}} \mu_{ij}$ and $\nu_j = \max\limits_{i\in\{1,\ldots,m\}} \nu_{ij}$.

Step 17: Compute $\widetilde{S_j} = \widetilde{Q_j} \cdot \widetilde{R} = \sum_{i=1}^{m} \left(\widetilde{X_{ij}} - \widetilde{X_j^M}\right)^2 \cdot \sum_{i=1}^{m} \left(\widetilde{X_i} - \widetilde{X^M}\right)^2$ $= (b_j, \mu_j, \nu_j) \cdot (c, \varphi, \psi) = (s_j, \theta_j, \omega_j)$, where (see (17)) $s_j = cb_j, \theta_j = \max\left(c\mu_j, \varphi b_j\right)$ and $\omega_j = \max\left(c\nu_j, \psi b_j\right)$.

Step 18: Compute $\widetilde{T_j} = \sqrt{\widetilde{S_j}} = \sqrt{\sum_{i=1}^{m} \left(\widetilde{X_{ij}} - \widetilde{X_j^M}\right)^2 \cdot \sum_{i=1}^{m} \left(\widetilde{X_i} - \widetilde{X^M}\right)^2}$ $= (t_j, \lambda_j, \pi_j)$, where (see (16)) $t_j = \sqrt{s_j}, \lambda_j = \frac{\theta_j}{\sqrt{s_j}}$ and $\pi_j = \frac{\omega_j}{\sqrt{s_j}}$.

Step 19: Compute

$$\widetilde{W_j} = (w_j, z_j, y_j) = \frac{\widetilde{P_j}}{\widetilde{T_j}} = \frac{(a_j, \rho_j, \sigma_j)}{(t_j, \lambda_j, \pi_j)} = \frac{\sum\limits_{i=1}^{m}\left(\widetilde{X_{ij}} - \widetilde{X_j^M}\right)\cdot\left(\widetilde{X_i} - \widetilde{X^M}\right)}{\sqrt{\sum\limits_{i=1}^{m}\left(\widetilde{X_{ij}} - \widetilde{X_j^M}\right)^2 \cdot \sum\limits_{i=1}^{m}\left(\widetilde{X_i} - \widetilde{X^M}\right)^2}},$$

as follows (see (18)):

If $a_j > 0$ then $w_j = \frac{a_j}{t_j}, z_j = \max\left(\frac{\rho_j}{t_j}, a_j \frac{\pi_j}{t_j(t_j+\pi_j)}\right)$ and $y_j = \max\left(\frac{\sigma_j}{t_j}, a_j \frac{\lambda_j}{t_j(t_j-\lambda_j)}\right)$.

If $a_j = 0$ then $w_j = 0, z_j = \frac{\rho_j}{t_j}$ and $y_j = \frac{\sigma_j}{t_j}$.

If $a_j < 0$ then $w_j = \frac{a_j}{t_j}, z_j = \max\left(\frac{\rho_j}{t_j}, -a_j \frac{\lambda_j}{t_j(t_j-\lambda_j)}\right)$ and $y_j = \max\left(\frac{\sigma_j}{t_j}, -a_j \frac{\pi_j}{t_j(t_j+\pi_j)}\right)$.

Step 20: Put $j = j + 1$. If $j \leq n$ then go to Step 8, else STOP.

B. Algorithm of Ordering of the Fuzzy Importance of Attributes

With the notations in Section 4 and Appendix A, the following algorithm of ordering of the fuzzy importance of attributes can be given.

Step 1-Step 19 in the algorithm from Appendix A.

Step 20: Compute (see (4)) $\overline{W_j} = EV\left(\widetilde{W_j}\right) = w_j + \frac{y_j - z_j}{4}$.

Step 21: Put $j = j + 1$. If $j \leq n$ then go to Step 8, else go to Step 22.

Step 22: If $\overline{W_{j_1}} = EV\left(\widetilde{W_{j_1}}\right) \geq EV\left(\widetilde{W_{j_2}}\right) = \overline{W_{j_2}}$ then $\widetilde{W_{j_1}} \geq \widetilde{W_{j_2}}$, else $\widetilde{W_{j_1}} < \widetilde{W_{j_2}}$ (see (5)-(7)).

Step 23: STOP.

C. Feasibility of the Method by Examples

We choose particular data to illustrate the method based on the algorithms described in Appendices A and B.

Let us consider four customers C_1, C_2, C_3, C_4, two attributes A_1, A_2 and the linguistic rating set $S = \{VP, P, M, G, VG\}$. Firstly, let us consider the performance of the attributes and the overall level of satisfaction given in Table 8, as they are evaluated by hypothetical customers.

	Performance		
Customer	A_1	A_2	Overall satisfaction
C_1	VG	VG	VG
C_2	VP	VG	VP
C_3	VG	VG	VP
C_4	VG	VP	VG

TABLE 8. Performance of Attributes and Overall Satisfaction

According with the data, our expectation is to obtain that A_1 is more important than A_2, that is, with the above notations, $\overline{W_1} \geq \overline{W_2}$, for any representation as triangular fuzzy numbers of the linguistic variables in the rating set S. This is important, because, at the moment the representation of the linguistic variables as triangular fuzzy numbers is subjective, even if there exists a real interest for an objective determination (see [28, 41, 42]). The results in Table 9 confirm our intuition. Moreover, the difference $\overline{W_1} - \overline{W_2}$ increases when the difference between VP and VG increases, according with our intuition too, an additional confirmation of the feasibility of our method.

	Linguistic variables		Fuzzy importance		Importance	
Case	VP	VG	$\widetilde{W_1}$	$\widetilde{W_2}$	$\overline{W_1}$	$\overline{W_2}$
1	$(3,0,2)$	$(6,4,4)$	(0.57, 0.77, 0.77)	(-0.57, 1.15, 1.15)	0.577	-0.577
2	$(3,2,2)$	$(6,1,2)$	(0.57, 0.38, 1.15)	(-0.57, 1.15, 0.57)	0.770	-0.722
3	$(3,2,1)$	$(7,4,3)$	(0.57, 0.57, 1.73)	(-0.57, 1.73, 0.86)	0.866	-0.794

Table 9. Fuzzy Importance of Attributes by Correlation Method

	Performance		
Customer	A_1	A_2	Overall satisfaction
C_1	VG	VG	G
C_2	VP	VG	P
C_3	VG	VG	P
C_4	VG	VP	G

Table 10. Performance of Attributes and Overall Satisfaction in a Modified Case

Now, we modify the overall satisfaction in Table 8 as in Table 10 and we keep the same performance of attributes. The behavior illustrated above is preserved. Indeed, keeping unchanged the triangular fuzzy numbers associated to the linguistic variables VP and VG, we obtain $\overline{W_1} > \overline{W_2}$. In addition, when the difference between P and G decreases (that is the difference between VP and P and, on the other hand, between G and VG increases), the difference $\overline{W_1} - \overline{W_2}$ increases too (see Table 11). Our expectations are confirmed, therefore the proposed method seems to be feasible.

	Linguistic variables				Fuzzy importance		Importance	
Case	VP	P	G	VG	$\widetilde{W_1}$	$\widetilde{W_2}$	$\overline{W_1}$	$\overline{W_2}$
1	$(3,2,1)$	$(4,4,0)$	$(6,2,3)$	$(7,3,3)$	(0.57, 0.57, 0.57)	(-0.57, 1.73, 1.29)	0.577	-0.686
2	$(3,2,1)$	$(3,0,2)$	$(6,4,4)$	$(7,3,3)$	(0.57, 0.43, 1.73)	(-0.57, 1.73, 1.15)	0.902	-0.722
3	$(3,2,1)$	$(4,2,4)$	$(7,4,0)$	$(7,3,3)$	(0.57, 0.43, 1.73)	(-0.57, 1.73, 1.15)	0.902	-0.722

Table 11. Fuzzy Importance of Attributes by Correlation Method in a Modified Case

D. Algorithm of Aggregating of the Fuzzy Importance of Attributes Under the Weakest t-norm Based Fuzzy Arithmetic

With the notations given in Section 6.1, the following algorithm of calculus can be elaborated to aggregate the fuzzy individual opinions and to obtain an ordering of attributes.

Step 1: Put $j = 1$.

Step 2: Compute (see (13), (15))

$$\widetilde{W_j}^* = \frac{1}{m} \cdot \sum_{i=1}^{m} \widetilde{W_{ij}}^* = \left(\frac{1}{m} \sum_{i=1}^{m} w_{ij}^*, \max_{i \in \{1,\ldots,m\}} \alpha_{ij}^*, \max_{i \in \{1,\ldots,m\}} \beta_{ij}^* \right).$$

Step 3: Compute (see (4))

$$W_j = EV\left(\widetilde{W_j}^*\right) = \frac{1}{m} \sum_{i=1}^{m} w_{ij}^* + \frac{1}{4} \max_{i \in \{1,\ldots,m\}} \beta_{ij}^* - \frac{1}{4} \max_{i \in \{1,\ldots,m\}} \alpha_{ij}^*.$$

Step 4: Put $j = j + 1$. If $j \leq n$ then go to Step 2, else go to Step 5.

Step 5: If $W_{j_1} \geq W_{j_2}$ then $\widetilde{W_{j_1}}^* \geq \widetilde{W_{j_2}}^*$, else $\widetilde{W_{j_1}}^* < \widetilde{W_{j_2}}^*$ (see (5)-(7)).

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Adrian I. Ban*, Department of Mathematics and Informatics, University of Oradea, Universităţii 1, Oradea , Romania

E-mail address: aiban@uoradea.ro

Olimpia I. Ban, Department of Economics, University of Oradea, Universităţii 1, Oradea , Romania

E-mail address: oban@uoradea.ro

Delia A. Tuşe, Department of Mathematics and Informatics, University of Oradea, Universităţii 1, Oradea , Romania

E-mail address: dmerca@uoradea.ro

*Corresponding author