STABILITY ANALYSIS AND FEEDBACK CONTROL OF T-S FUZZY HYPERBOLIC DELAY MODEL FOR A CLASS OF NONLINEAR SYSTEMS WITH TIME-VARYING DELAY

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Abstract. In this paper, a new T-S fuzzy hyperbolic delay model for a class of nonlinear systems with time-varying delay, is presented to address the problems of stability analysis and feedback control. Fuzzy controller is designed based on the parallel distributed compensation (PDC), and with a new Lyapunov function, delay dependent asymptotic stability conditions of the closed-loop system are derived via linear matrix inequalities (LMIs). Besides, considering the differences between the model and the real system, we extent the model to uncertain T-S fuzzy hyperbolic delay model. Based on the uncertain model, a robust H_∞ fuzzy controller is obtained and stability conditions are developed in terms of LMIs. The main advantage of the control based on T-S fuzzy hyperbolic delay model is that it can achieve small control amplitude via "soft" constraint approach. Finally, a numerical example and the Van de Vusse example are given to validate the advantages of the proposed method.

1. Introduction

As is well known, fuzzy technique is recognized as a popular and powerful tool in approximating a complex nonlinear system for its universal approximation property. In particular, the fuzzy control technique based on the Takagi-Sugeno (T-S) fuzzy model [17, 19] is an effective way to handle complex nonlinear systems, which has been widely and successfully used in nonlinear system modeling and control [8]. The main idea of T-S fuzzy model is to construct a series of linear models to represent each of the subsystems. Based on the classical linear system theory, the problem of the analysis and controller synthesis of nonlinear systems can be solved. Thus, T-S fuzzy model has attracted a lot of attention in the past decades. There has been many results available for T-S fuzzy control model, including stability analysis [2, 15, 18, 25, 26, 27, 33], filter design [16, 31] and tracking control [21] and so on [8].

T-S fuzzy models have been extended to the analysis and controller synthesis of nonlinear time-delay systems. In practical applications, delays often occur in many dynamic systems, such as biological systems, network systems, and so on. The presented results indicate that the existence of delay tends to become the

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main cause of poor performance and instability of system. Therefore, the modeling and stability analysis of time-delay systems have emerged as a topic of significant interest in the control community [1, 13]. In recent years, some authors have paid their attention to control of nonlinear systems with time-delay by using T-S fuzzy models [3, 6, 7, 10, 14, 20, 23, 24, 36]. The existing results can be divided into two categories: delay-independent results [6] and delay-dependent results [3, 7, 10, 14, 20, 23, 24, 36]. Totally, the size of the delay isn't taken into account in the former, whereas the latter usually contains the delay information. Generally speaking, when the values of time delays are relatively small, delay-dependent T-S fuzzy systems are less conservative than the delay-independent ones [14]. According to the existing delay-independent results, it can be seen that there are many literature based on the T-S fuzzy delay model focused on the stability analysis of the system [3, 10, 20], robust control [7, 14], fuzzy predictive control [23], event-triggered H_∞ control [24] and so on. Very recently, new stability and stabilization conditions for T-S fuzzy systems with time delay was studied in [36], by chosen a new Lyapunov-Krasovskii functional containing the fuzzy line-integral Lyapunov function and the simple functional. With a recently developed Wirtinger-based integral inequality and introducing slack variables, less conservative conditions in terms of LMIs are successfully obtained. New sufficient conditions for delay-dependent robust H_∞ control of nonlinear system based on fuzzy hyperbolic model with time-varying delays is studied in [22, 29, 30]. However, up to our knowledge, the issue of constraint on control input for T-S fuzzy systems with time delay is rarely mentioned.

In fact, the practical application controller should be designed to stabilize the dynamic systems with requiring permissible magnitude of control inputs [37]. However, the main existing approaches on constraint control have their own drawbacks [4]. In [4], a new T-S fuzzy hyperbolic model based on [28, 32, 34] has been proposed for the stability of nonlinear systems under small control inputs, and this fuzzy model is described by fuzzy IF-THEN rules which expresses local dynamics using a hyperbolic tangent model. The T-S fuzzy hyperbolic model is a kind of T-S fuzzy system with local nonlinear model. Compared with the linear T-S fuzzy model, T-S fuzzy nonlinear models control can achieve fuzzy rules and calculation for complex nonlinear systems. The controller in [4] is designed as $u = H\tanh(Kx)$ for constant matrix H. It is easy to see that the controller is bounded for all x, since the range of each component in vector $\tanh(Kx)$ belongs to $[-1, 1]$, and $\|H\tanh(Kx)\| \leq \|H\|$ for all x. Thus, the controller design approach based on the model [4] has some reference for dealing with the issue of constraint control. And based on the model, non-fragile guaranteed cost control [5] was studied, which has also achieved good results.

Motivated by the above concerns, in this paper, the problem of constraint control is studied for a class of nonlinear system with time-varying state delay. First, a new T-S fuzzy hyperbolic delay model whose consequence is a hyperbolic tangent dynamic delay model is presented to represent nonlinear delay system. Second, owing to some good properties of hyperbolic tangent function, a fuzzy hyperbolic controller is designed via the PDC [19] method. And the fuzzy hyperbolic controller is bounded and it can achieve lower control input, which can be considered as a "soft" constraint control method. Then, a new Lyapunov function is constructed. Thus, the sufficient conditions of asymptotic stability for the nonlinear delay system are given in the form of delay-dependent LMIs. Important of all, when compared with the linear delay model, our model can achieve smaller control input. Moreover, considering the differences of the model and the real system, a new T-S fuzzy uncertain hyperbolic delay model is established, and the robust H_∞ constraint control of the uncertain T-S fuzzy hyperbolic delay system is addressed. The main contributions of this paper are concluded as follows:

1) With a new T-S fuzzy hyperbolic delay model and a Lyapunov function, the constraint control strategy is designed for a class of nonlinear system with time-varying state delay.

2) Considering the differences of the model and the real system, the robust constraint control of the uncertain T-S fuzzy hyperbolic delay system is addressed.

The proposed controller is bounded and can achieve lower control input. Three simulation examples are provided to show the effectiveness of our controller compared with the controller designed based on T-S linear model.

2. Modeling and Analysis of T-S Fuzzy Hyperbolic Delay Model

In this section, a T-S fuzzy hyperbolic delay model will be presented to represent the nonlinear systems. And based on the model, the stabilization of a class of nonlinear delay systems will also be studied.

Based on the T-S fuzzy hyperbolic model [4], a T-S fuzzy hyperbolic delay model will be presented to represent the nonlinear systems with time-varying delay. The following fuzzy dynamic model is described by fuzzy IF-THEN rules and the i-th rule of the fuzzy system is expressed as the following form:

Plant Rule i:

IF $s_1(t)$ is $F_{i1}(t)$ and and $s_g(t)$ is $F_{ig}(t)$, then

$$\begin{aligned}&\dot{x}(t) = A_i \tanh(Kx(t)) + B_i u(t) + A_{di} \tanh(Kx(t-d(t))),\\ &x(t) = \phi(t), t \in [-\tau, 0], i \in S \triangleq \{1, 2, ..., r\}\end{aligned} \tag{1}$$

where $s_1(t)$, $\cdots$, $s_g(t)$ are premise variables, F_{il} is the fuzzy set, $l = 1, 2, ..., g$, and r is the number of fuzzy rules. $x(t) \in \mathbb{R}^n$ is the state vector and $u(t) \in \mathbb{R}$ is the control input. $A_i \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^n$, $A_{di} \in \mathbb{R}^{n\times n}$. $d(t)$ is a time varying differentiable function, satisfying $0 \le d(t) \le \tau$ and $\dot{d}(t) \le \sigma$, where τ and σ are known constants. The initial condition $\phi(t)$ is a continuous function of t.

It is assumed that the premise variables do not depend on the control input $u(t)$ or disturbance $\omega(t)$. $\tanh(k_i x_i) = \frac{e^{k_i x_i} - e^{-k_i x_i}}{e^{k_i x_i} + e^{-k_i x_i}}$, $\tanh(Kx(t)) \triangleq [\tanh(k_1 x_1(t), \ldots, \tanh(k_n x_n(t))]^T$, and k_i is a positive constant. Using the fuzzy inference with a singleton fuzzifier and product inference and center average defuzzifies to the above system, we obtain the overall T-S fuzzy hyperbolic delay system:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(s(t))\big(A_i \tanh(Kx(t)) + B_i u(t) + A_{id} \tanh(Kx(t-d(t)))\big) \tag{2}$$

where $s(t) = [s_1(t) \ldots s_g(t)]^T$, $h_i(s(t)) = \frac{\mu_i(s(t))}{\sum_{i=1}^{r} \mu_i(s(t))}$ and $\mu_i(s(t)) = \prod_{l=1}^{g} F_{il}(s(t))$, $i \in S$. $F_{il}(s(t))$ is the membership of $s_l(t)$ in F_{il}, and $\mu_i(s(t))$ is assumed that

$\mu_i(s(t)) \geq 0$ and $\sum_{i=1}^{r} \mu_i(s(t)) > 0$, $\forall i \in S$. It is clear that $h_i(s(t)) \geq 0$ and $\sum_{i=1}^{r} h_i(s(t)) = 1$, $\forall i \in S$.

The following lemmas will be required in the sequel.

Lemma 2.1. [35] *Given symmetric matrix* $S = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}$, *where* S_{ii} *are symmetric matrices with dimensions of* $r_i \times r_i$, *the following inequalities are equivalent to each other.*

1)$S < 0$;
2)$S_{11} < 0$ and $S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;
3)$S_{22} < 0$ and $S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$; $(i = 1, 2)$.

Lemma 2.2. [9] *Given constant matrices* G, E *and symmetric constant matrix* S *of appropriate dimensions. If inequality* $S + GFE + E^T F^T G^T < 0$ *holds, satisfying* $F^T(t)F(t) \leq I$, *then for some* $\varepsilon > 0$, *the following inequality holds:* $S + \varepsilon GG^T + \varepsilon^{-1} E^T E < 0$.

Lemma 2.3. [28] *For any* $x \in \mathbb{R}^n$ *and a diagonal matrix* $V > 0$, *the following inequality holds:* $\big(\tanh^T(x)\big)' V \big(\tanh(x)\big)' \leq \dot{x}^T V \dot{x}$.

Utilizing the PDC method, we design the following fuzzy controller to stabilize the nonlinear time-varying delay system.

Plant rule j:

IF $s_1(t)$ is $F_{j1}(t)$ and and $s_g(t)$ is $F_{jg}(t)$, then

$$u_j(t) = H_j \tanh(Kx(t)), j \in S \tag{3}$$

where $H_j \in \mathbb{R}^{1\times n}$ is controller gain matrix to be determined in later.

Thence, the overall fuzzy controller is designed as following:

$$u_j(t) = \sum_{j=1}^{r} h_j H_j \tanh(Kx(t)) \tag{4}$$

Remark 2.4. As each component in vector $\tanh(Kx(t))$ is bounded and belongs to $[-1, 1]$, then $\|H \tanh(Kx)\| \leq \|H\|$ holds, which shows that controller (4) is bounded for all $x(t)$ and it can achieve small control amplitude.

Based on the above-designed controller, the overall closed-loop system can be expressed as:

$$\dot{x}(t) = \sum_{i,j}^{r} h_i h_j \left((A_i + B_i H_j) \tanh(Kx) + A_{di} \tanh(Kx(t - d(t)))\right) \tag{5}$$

Under the fuzzy hyperbolic control law (4), the delay-dependent asymptotic stability conditions of system (5) are summarized as follows.

Theorem 2.5. *Consider the system (5), for given upper bound of the time-varying delay* $\tau > 0$ *and upper bound of its derivative* $\sigma > 0$ *and the factor of free weighting matrices* $\lambda > 0$, *if there exist diagonal matrices* $\bar{P} > 0$, $\bar{R} > 0$, $Z > 0$, *a symmetric matrix* $\bar{Q} > 0$, *matrices* $\bar{X}_1$, $\bar{X}_2$, $\bar{X}_3$, *and* $M_i \in \mathbb{R}^{1\times n}$ *such that LMIs (6) holds,* *then the closed-loop system (5) is globally asymptotically stable, and controller gain matrix satisfies* $H_i = M_i Z^{-1}$.

$$\begin{bmatrix} \bar{\Phi}_{2ij} & \bar{X} \\ * & -\frac{1}{\tau}\bar{R} \end{bmatrix} < 0, i, j \in S \tag{6}$$

where

$$\bar{\Phi}_{2ij} = \begin{bmatrix} \bar{\Phi}_{11,ij} & \bar{\Phi}_{12,ij} & \bar{\Phi}_{13,ij} \\ * & \bar{\Phi}_{22,ij} & \bar{\Phi}_{23,ij} \\ * & * & \bar{\Phi}_{33,ij} \end{bmatrix},$$

$$\bar{\Phi}_{11,ij} = \bar{Q} + \bar{X}_1 + \bar{X}_1^T + \lambda A_i Z + \lambda B_i M_j + \lambda Z A_i^T + \lambda M_j^T B_i^T,$$

$$\bar{\Phi}_{12,ij} = -\bar{X}_1 + \bar{X}_2^T + \lambda A_{di} Z + \lambda Z A_i^T + \lambda M_j^T B_i^T,$$

$$\bar{\Phi}_{13,ij} = \bar{P} + \bar{X}_3^T - \lambda Z + \lambda Z A_i^T + \lambda M_j^T B_i^T,$$

$$\bar{\Phi}_{22,ij} = (\sigma - 1)\bar{Q} - \bar{X}_2 - \bar{X}_2^T + \lambda A_{di} Z + \lambda Z A_{di}^T,$$

$$\bar{\Phi}_{23,ij} = -\bar{X}_3^T - \lambda Z + \lambda Z A_{di}^T,$$

$$\bar{\Phi}_{33,ij} = \tau K \bar{R} K - 2\lambda Z.$$

$$\bar{X}^T = \begin{bmatrix} \bar{X}_1^T & \bar{X}_2^T & \bar{X}_3^T \end{bmatrix}.$$

Proof. The system (5) can be expressed as

$$\dot{x}(t) = \sum_{i,j}^{r} h_i h_j \left(A_{ij} \tanh(Kx(t)) + A_{di} \tanh(Kx(t - d(t))) \right) \tag{7}$$

where $A_{ij} = A_i + B_i H_j$.

Consider the following Lyapunov function:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{8}$$

where

$$V_1(t) = 2 \sum_{i=1}^{n} \frac{p_i}{k_i} \ln(\cosh(k_i x_i)),$$

$$V_2(t) = \int_{t-d(t)}^{t} \tanh^T(Kx) Q \tanh(Kx) ds,$$

$$V_3(t) = \int_{-\tau}^{0} \int_{t+\theta}^{t} \big(\tanh^T(Kx(s))\big)' R \big(\tanh(Kx(s))\big)' ds d\theta.$$

where $P = \mathrm{diag}\{p_1 \dots p_n\}$ with $p_i > 0$. Because of $\cosh(k_i x_i) = \frac{1}{2}(e^{k_i} x_i + e^{-k_i} x_i) \geq (e^{k_i x_i})^{1/2} \cdot (e^{-k_i x_i})^{1/2} = 1$, $Q > 0$, $R > 0$, then $V(t) > 0$, when $x(t) \neq 0$.

According to Lemma 2.3, $0 \leq d(t) \leq \tau$ and $\dot{d}(t) \leq \sigma$, set $K = \mathrm{diag}\{k_1, ..., k_n\}$, we have the following inequalities:

$$\dot{V}_1(t) = 2 \sum_{i=1}^{n} \tanh(k_i x_i) p_i \dot{x}_i = 2 \tanh^T(Kx) P \dot{x} \tag{9}$$

$$\dot{V}_2(t) \leq \tanh^T(Kx) Q \tanh(Kx) - (1 - \sigma) \tanh^T(Kx_d) Q \tanh(Kx_d) \tag{10}$$

$$\dot{V}_3(t) \leq \tau \dot{x}^T K R K \dot{x} - \int_{t-d(t)}^{t} \big(\tanh^T(Kx(s))\big)' R \big(\tanh(Kx(s))\big)' ds \tag{11}$$

Define free weighting matrices as $X^T = \begin{bmatrix} X_1^T & X_2^T & X_3^T \end{bmatrix}$, $X_k \in \mathbb{R}^{n\times n}$, $Y^T = \begin{bmatrix} Y_1^T & Y_2^T & Y_3^T \end{bmatrix}$, $Y_k \in \mathbb{R}^{n\times n}$, $k = 1, 2, 3$, X_k, Y_k will be determined in later.

Using the Leibniz-Newton formula and system equation (7), we have the following identical equations:

$$\begin{aligned}&\Big[\tanh^T(Kx)X_1 + \tanh^T(Kx_d)X_2 + \dot{x}^T X_3\Big]\\ &\times \Big[\tanh(Kx) - \tanh(Kx_d) - \int_{t-d(t)}^{t} \big(\tanh(Kx(s))\big)' ds\Big] = 0\end{aligned} \tag{12}$$

$$\begin{aligned}&\sum_{i,j}^{r}\Big[\tanh^T(Kx)Y_1 + \tanh^T(Kx_d)Y_2 + \dot{x}Y_3\Big]\\ &\quad\times \Big[A_{ij}\tanh(Kx) + A_{di}\tanh(Kx_d) - \dot{x}\Big] = 0\end{aligned} \tag{13}$$

Due to (9), (10), (11), (12) and (13), the time derivative of $V(x(t))$ is computed as:

$$\begin{aligned}\dot{V}(t) \leq& 2\tanh^T(Kx)P\dot{x} - (1-\sigma)\tanh^T(Kx_d)Q\tanh(Kx_d) + \tau\dot{x}^T KRK\dot{x}\\ &+ \tanh^T(Kx)Q\tanh(Kx) - \int_{t-d(t)}^{t}\big(\tanh^T(Kx(s))\big)' R\big(\tanh(Kx(s))\big) ds\\ &+ 2\Big[\tanh^T(Kx)X_1 + \tanh^T(Kx_d)X_2 + \dot{x}^T X_3\Big]\\ &\times \Big[\tanh(Kx) - \tanh(Kx_d) - \int_{t-d(t)}^{t}\big(\tanh(Kx(s))\big)' ds\Big]\\ &+ \sum_{i,j}^{r}\Big[\tanh^T(Kx)Y_1 + \tanh^T(Kx_d)Y_2 + \dot{x}Y_3\Big]\\ &\times \Big[A_{ij}\tanh(Kx) + A_{di}\tanh(Kx_d) - \dot{x}\Big]\end{aligned}$$

thus,

$$\begin{aligned}\dot{V}(t) \leq& \sum_{i=1}^{r} h_i\eta_1^T\Phi_{2ij}\eta_1 - 2\eta_1^T X\int_{t-d(t)}^{t}\big(\tanh(Kx(s))\big)' ds\\ &- \int_{t-d(t)}^{t}\big(\tanh^T(Kx(s))\big)' R\big(\tanh(Kx(s))\big)' ds\\ =& \sum_{i=1}^{r} h_i\eta_1^T\big(\Phi_{2ij} + \tau XR^{-1}X^T\big)\eta_1 - \sum_{i=1}^{r} h_i\tau\eta_1^T XR^{-1}X^T\eta_1\\ &- 2\eta_1^T X\int_{t-d(t)}^{t}\xi ds - \int_{t-d(t)}^{t}\xi^T R\xi ds\\ \leq& \sum_{i=1}^{r} h_i\eta_1^T\big(\Phi_{2ij} + \tau XR^{-1}X^T\big)\eta_1\\ &- \int_{t-d(t)}^{t}\big(\xi^T R + \eta_1^T X\big)R^{-1}\big(\xi^T R + \eta_1^T X\big)^T ds\end{aligned}$$

where $\eta_1^T = \begin{bmatrix}\tanh^T(Kx) & \tanh^T(Kx_d) & \dot{x}^T\end{bmatrix}$, $\xi = (\tanh(Kx(s)))'$, and

$$\Phi_{2ij} = \begin{bmatrix} \Phi_{11,ij} & \Phi_{12,ij} & \Phi_{13,ij} \\ * & \Phi_{22,ij} & \Phi_{23,ij} \\ * & * & \Phi_{33,ij} \end{bmatrix},$$
$$\Phi_{11,ij} = Q + X_1 + X_1^T + Y_1 A_{ij} + A_{ij}^T Y_1^T, \Phi_{12,ij} = -X_1 + X_2^T + Y_1 A_{di} + A_{ij}^T Y_2^T,$$
$$\Phi_{13,ij} = P + X_3^T - Y_1 + A_{ij}^T Y_3^T, \Phi_{22,ij} = (\sigma - 1)Q - X_2 - X_2^T + Y_2 A_{di} + A_{di}^T Y_2^T,$$
$$\Phi_{23,ij} = -X_3^T - Y_2 + A_{di}^T Y_3^T, \Phi_{33,ij} = \tau KRK - Y_3 - Y_3^T.$$

It is assumed that:

$$\Phi_{2ij} + \tau X R^{-1} X < 0 \tag{14}$$

Applying Lemma 2.1 to (14) yields:

$$\begin{bmatrix} \Phi_{2ij} & X \\ * & -\frac{1}{\tau} R \end{bmatrix} < 0, i, j \in S \tag{15}$$

In order to transform (15) into LMIs, we assume Y_1, Y_2, Y_3 are nonsingular, Z is a positive definite diagonal matrix and define $Y_k^{-T} = \lambda Z$, $\lambda > 0$, $\Theta_1 = \mathrm{diag}\{Y_1^{-1}, Y_2^{-1}, Y_3^{-1}, Y_3^{-1}\}$, $M_j = H_j Z$, $\bar{Q} = Y_1^{-1} Q Y_1^{-T}$, $\bar{P} = Y_1^{-1} P Y_1^{-T}$, $\bar{R} = Y_3^{-1} R Y_3^{-T}$, $\bar{X}_k = Y_k^{-1} X_k Y_k^{-T}$, $k = 1, 2, 3$. Pre- and post-multiplying both sides of (15) by Θ_1, we get LMIs (6) and $R^{-1} > 0$, then $\dot{V}(t) < 0$, It completes the proof. □

Remark 2.6. In Theorem 2.5, with the fuzzy hyperbolic control law (4), the stability condition of the system under small control amplitude has been developed in terms of delay-dependent LMIs. This method can be referred to as "soft" constraint control, for the boundedness of the control law.

Remark 2.7. In the prove of Theorem 2.5, based on the T-S fuzzy hyperbolic delay model, the designed Lyapunov function is different from the Lyapunov function based on T-S linear model and is also different from Lyapunov function designed in [21] for the existence of delay.

3. Robust H_∞ Control for Nonlinear System Based on T-S Fuzzy Hyperbolic Delay Model

Considering the differences between our model and the real nonlinear system, we extent the above model to discuss robust H_∞ control for nonlinear time-varying delay system with parameter uncertainties.

The uncertain T-S fuzzy hyperbolic delay model is proposed in following form:

Plant Rule i:

IF $s_1(t)$ is $F_{i1}(t)$ and $s_g(t)$ is $F_{ig}(t)$, then

$$\begin{aligned} \dot{x}(t) =& (A_i + \Delta A_i) \tanh(Kx(t)) + (B_i + \Delta B_i) u(t) \\ & + (A_{di} + \Delta A_{di}) \tanh(Kx(t - d(t))) + (D_{1i} + \Delta D_{1i}) \omega(t) \\ x(t) =& \phi(t), t \in [-\tau, 0] \end{aligned} \tag{16}$$
$$z(t) = C_i \tanh(Kx(t)) + D_{2i} \omega(t), i \in S \tag{17}$$

where $z(t) \in \mathbb{R}^q$ is control output, and $\omega(t) \in \mathbb{R}^m$ is the exogenous disturbance which belongs to $L_2[0,\infty]$. $A_i \in \mathbb{R}^{n\times n}$, $A_{di} \in \mathbb{R}^{n\times n}$, $B_i \in \mathbb{R}^n$, $C_i \in \mathbb{R}^{q\times n}$, $D_{1i} \in \mathbb{R}^{n\times m}$, $D_{2i} \in \mathbb{R}^{q\times m}$. In addition, $\Delta A_i \in \mathbb{R}^{n\times n}$, $\Delta B_i \in \mathbb{R}^n$, $\Delta A_{di} \in \mathbb{R}^{n\times n}$, $\Delta D_{1i} \in \mathbb{R}^{n\times m}$ are time varying matrices with parameter uncertainties considered to be norm-bounded and have the following form:

$$\begin{bmatrix} \Delta A_i & \Delta A_{di} & \Delta B_i & \Delta D_{1i} \end{bmatrix} = G_i F_i(t) \begin{bmatrix} E_{1i} & E_{2i} & E_{3i} & E_{4i} \end{bmatrix} \tag{18}$$

where G_i, E_{1i}, E_{2i}, E_{3i} and E_{4i} are known real constant matrices with appropriate dimensions. $F_i(t)$ are unknown matrix functions satisfying $F_i^T(t)F_i(t) \le I$, $i \in S$. Thus, the overall uncertain T-S fuzzy hyperbolic delay system can be represented as follow:

$$\begin{aligned} \dot{x}(t) = &\sum_{i=1}^{r} h_i \big[(A_i + \Delta A_i)\tanh(Kx(t)) + (B_i + \Delta B_i)u(t)\big] \\ &+ \sum_{i=1}^{r} h_i \big[(A_{di} + \Delta A_{di})\tanh(Kx(t-d(t))) + (D_{1i} + \Delta D_{1i})\omega(t)\big] \end{aligned} \tag{19}$$

We utilize the controller (4) as robust H_∞ controller for uncertain T-S fuzzy hyperbolic delay system. Therefore, the robust H_∞ controller keeps the good properties of the above-designed controller. We have:

$$\begin{aligned} \dot{x} = &\sum_{i,j}^{r} h_i h_j \big[(A_i + \Delta A_i)\tanh(Kx) + (B_i + \Delta B_i)H_j \tanh(Kx)\big] \\ &+ \sum_{i,j}^{r} h_i h_j \big[(A_{di} + \Delta A_{di})\tanh(Kx(t-d(t))) + (D_{1i} + \Delta D_{1i})\omega(t)\big] \end{aligned} \tag{20}$$

$$z(t) = \sum_{i,j}^{r} h_i h_j \big[C_i \tanh(Kx(t)) + D_{2i}\omega(t)\big] \tag{21}$$

Definition 3.1. [12] If the system (16), (17) and scalar $\gamma > 0$ satisfy the following two conditions:

(1) When $\omega(t) = 0$ the system (20) and (21) are asymptotically stable.

(2) Under the zero initial condition, for any nonzero $\omega(t) \in L_2[0,\infty]$, the control output $z(t)$ satisfies

$$\|z(t)\|_{L_2} \le \gamma \|\omega(t)\|_{L_2} \tag{22}$$

where $\|z(t)\|_{L_2} = \big(\int_0^\infty z^T(t)z(t)dt\big)^{1/2}$, then the uncertain system is robust asymptotically stable with the H_∞ performance index γ.

Then, we have the following results.

Theorem 3.2. *Consider the system (20) and (21), for given H_∞ performance index $\gamma > 0$, upper bound of the time-varying delay $\tau > 0$ and upper bound of its derivative $\sigma > 0$ and the factor of free weighting matrices $\lambda > 0$, scalar constants $\varepsilon_{ij} > 0$, $i, j \in S$, if there exist diagonal matrices $\bar{P} > 0$, $\bar{R} > 0$, $Z > 0$, a symmetric matrix $\bar{Q} > 0$, matrices $\bar{X}_1$, $\bar{X}_2$, $\bar{X}_3$, $\bar{X}_4$, and $M_i \in \mathbb{R}^{1\times n}$ such that LMIs (23)* *holds, then the closed-loop system (20) and (21) are robust asymptotically stable with the H_∞ performance index γ, and controller gain matrix satisfy $H_i = M_i Z^{-1}$.*

$$\begin{bmatrix} \bar{\Pi}' & \bar{\Gamma}'^T & \bar{M}'^T & \bar{\Delta}^T \\ * & -\varepsilon_{ij} I & 0 & 0 \\ * & * & -\varepsilon_{ij}^{-1} I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{23}$$

where

$$\bar{\Pi}' = \begin{bmatrix} \bar{\Phi}_{11,ij} & \bar{\Phi}_{12,ij} & \bar{\Phi}_{13,ij} & \bar{\Phi}''_{14,ij} & \bar{X}_1 \\ * & \bar{\Phi}_{22,ij} & \bar{\Phi}_{23,ij} & \bar{\Phi}''_{24,ij} & \bar{X}_2 \\ * & * & \bar{\Phi}_{33,ij} & \bar{\Phi}''_{34,ij} & \bar{X}_3 \\ * & * & * & \bar{\Phi}''_{44,ij} & \bar{X}_4 \\ * & * & * & * & -\frac{1}{\tau}\bar{R} \end{bmatrix},$$

$$\bar{\Phi}''_{14,ij} = \bar{X}_4^T + \lambda D_{1i} Z + \lambda Z A_i^T + \lambda M_j^T B_i^T, \bar{\Phi}''_{24,ij} = -\bar{X}_4^T + \lambda D_{1i} Z + \lambda Z A_{di}^T,$$

$$\bar{\Phi}''_{34,ij} = \lambda D_{1i} Z - \lambda Z, \bar{\Phi}''_{44,ij} = \gamma^2 (I - 2\lambda Z) + \lambda D_{1i} Z + \lambda Z D_{1i}^T,$$

$$\bar{\Gamma}'^T = \begin{bmatrix} G_i & G_i \\ G_i & G_i \\ G_i & G_i \\ G_i & G_i \\ 0 & 0 \end{bmatrix}, \bar{M}'^T = \begin{bmatrix} Z E_{1i}^T & M_j^T E_{3i}^T \\ Z E_{2i}^T & 0 \\ 0 & 0 \\ Z E_{4i}^T & 0 \\ 0 & 0 \end{bmatrix},$$

$$\bar{\Delta} = \begin{bmatrix} \frac{C_i + C_j}{2} Z & 0 & 0 & \frac{D_{2i} + D_{2j}}{2} Z & 0 \end{bmatrix},$$

Proof. (1) when $\omega(t) = 0$, (20) can be written as:

$$\dot{x} = \sum_{i,j}^{r} h_i h_j \left[\bar{A}_{ij} \tanh(Kx) + \bar{A}_{di} \tanh(Kx(t - d(t))) \right] \tag{24}$$

where $A_{ij} = A_i + B_i H_j$, $\Delta A_{ij} = \Delta A_i + \Delta B_i H_j$, $\bar{A}_{ij} = A_{ij} + \Delta A_{ij}$, $\bar{A}_{di} = A_{di} + \Delta A_{di}$.

Considering system equation (24), we have the following identical equations:

$$\sum_{i,j}^{r} \left[\tanh^T(Kx) Y_1 + \tanh^T(Kx_d) Y_2 + \dot{x}^T Y_3 \right] \times \left[\bar{A}_{ij} \tanh(Kx) + \bar{A}_{di} \tanh(Kx_d) - \dot{x} \right] = 0 \tag{25}$$

Considering Lyapunov function (8), and (12), (25), we have:

$$\dot{V}(t) \le \sum_{i,j}^{r} h_i h_j \eta_1^T \left(\Lambda_{ij} + \tau X R^{-1} X \right) \eta_1 - \int_{t-d(t)}^{t} (\xi^T R + \eta_1^T X) R^{-1} (\xi^T R + \eta_1^T X)^T ds$$

where $\eta_1^T = \begin{bmatrix} \tanh^T(Kx) & \tanh^T(Kx_d) & \dot{x}^T \end{bmatrix}$, $\xi = (\tanh(Kx(s)))'$, and

$$\Lambda_{ij} = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ * & \Lambda_{22} & \Lambda_{23} \\ * & * & \Lambda_{33} \end{bmatrix},$$

$$\Lambda_{11} = Q + X_1 + X_1^T + Y_1 \bar{A}_{ij} + \bar{A}_{ij}^T Y_1^T, \Lambda_{12} = -X_1 + X_2^T + Y_1 \bar{A}_{di} + \bar{A}_{ij}^T Y_2^T,$$

$$\Lambda_{13} = P + X_3^T - Y_1 + \bar{A}_{ij}^T Y_3^T, \Lambda_{22} = (\sigma - 1) Q - X_2 - X_2^T + Y_2 \bar{A}_{di} + \bar{A}_{di}^T Y_2^T,$$

$$\Lambda_{23} = -X_3^T - Y_2 + \bar{A}_{di}^T Y_3^T, \Lambda_{33} = \tau K R K - Y_3 - Y_3^T.$$

It is assumed that:

$$\Lambda_{ij} + \tau X R^{-1} X < 0 \tag{26}$$

Applying Lemma 2.1 to (26), we have: $\begin{bmatrix} \Lambda_{ij} & X \\ * & -\frac{1}{\tau}R \end{bmatrix} < 0, i, j \in S$, that is

$$\Psi + Sym\left(\begin{bmatrix} Y_1\Delta A_{ij} & Y_1\Delta A_{di} & 0 & 0 \\ Y_2\Delta A_{ij} & Y_2\Delta A_{di} & 0 & 0 \\ Y_3\Delta A_{ij} & Y_3\Delta A_{di} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\right) < 0 \tag{27}$$

where $\Psi = \begin{bmatrix} \Phi_{2ij} & X \\ * & -\frac{1}{\tau}R \end{bmatrix}$. Substituting (18) into (27), we have:

$$\Psi + \Gamma^T F_{ii} M + M^T F_{ii}^T \Gamma < 0 \tag{28}$$

where $\Gamma^T = \begin{bmatrix} Y_1G_i & Y_1G_i \\ Y_2G_i & Y_2G_i \\ Y_3G_i & Y_3G_i \\ 0 & 0 \end{bmatrix}$, $M = \begin{bmatrix} E_{1i} & E_{2i} & 0 & 0 \\ E_{3i}H_j & 0 & 0 & 0 \end{bmatrix}$, $F_{ii} = \begin{bmatrix} F_i & 0 \\ 0 & F_i \end{bmatrix}$, and $F_{ii}^T(t)F_{ii}(t) \le I$.

Considering (28) with Lemma 2.2, we have:

$$\Psi + \varepsilon\Gamma^T\Gamma + \varepsilon^{-1}M^TM < 0 \tag{29}$$

Applying Lemma 2.1 to (29), we have:

$$\begin{bmatrix} \Psi & \Gamma^T & M^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon^{-1}I \end{bmatrix} < 0 \tag{30}$$

Define $\Theta_2 = \mathrm{diag}\{Y_1^{-1}, Y_2^{-1}, Y_3^{-1}, Y_3^{-1}, I, I, I, I\}$, $\bar{P} = Y_1^{-1}PY_1^{-T}$, $\bar{Q} = Y_1^{-1}QY_1^{-T}$, $\bar{R} = Y_3^{-1}RY_3^{-T}$, $\bar{X}_k = Y_k^{-1}X_kY_k^{-T}$, $M_j = H_jZ$. Pre- and post-multiplying both sides of (30) by Θ_2, we have LMIs (23). Then $\Lambda_{ij} + \tau XR^{-1}X < 0$ and $R^{-1} > 0$, which shows $\dot{V}(t) < 0$. Thus, when $\omega(t) = 0$, It completes the proof.

(2) When $\omega(t) \neq 0$, considering (21), we have:

$$\begin{aligned} & z^T(t)z(t) \\ =& \sum_{i,j}^{r} h_ih_j\big[C_i\tanh(Kx) + D_{2i}\omega(t)\big]^T \times \sum_{l,m}^{r} h_lh_m\big[C_l\tanh(Kx) + D_{2l}\omega(t)\big] \\ \le& \frac{1}{4}\sum_{i,j}^{r} h_ih_j\big[(C_i + C_j)\tanh(Kx) + (D_{2i} + D_{2j})\omega(t)\big]^T \\ & \times \big[(C_i + C_j)\tanh(Kx) + (D_{2i} + D_{2j})\omega(t)\big] \end{aligned} \tag{31}$$

Substituting $\bar{D}_{1i} = D_{1i} + \Delta D_{1i}$ into (20), we have:

$$\dot{x}(t) = \sum_{i,j}^{r} h_ih_j\big[\bar{A}_{ij}\tanh(Kx(t)) + \bar{A}_{di}\tanh(Kx(t - d(t))) + \bar{D}_{1i}\omega(t)\big] \tag{32}$$

Define the free weighting matrices as $X' = \begin{bmatrix} X^T & X_4^T \end{bmatrix}^T$, $Y' = \begin{bmatrix} Y^T & Y_4^T \end{bmatrix}^T$, where $X_l \in \mathbb{R}^{n\times n}$, $Y_l \in \mathbb{R}^{n\times n}$, $l = 1, 2, 3, 4$, X_l, Y_l will be determined in later. And we have:

$$\begin{aligned}&\Big[\tanh^T(Kx)X_1 + \tanh^T(Kx_d)X_2 + \dot{x}^T X_3 + \omega^T(t)X_4\Big]\\ &\times \left[\tanh(Kx) - \tanh(Kx_d) - \int_{t-d(t)}^{t} \Big(\tanh^T(Kx(s))\Big)' ds\right] = 0\end{aligned} \tag{33}$$

$$\begin{aligned}&\sum_{i,j}^{r}\big[\tanh^T(Kx)Y_1 + \tanh^T(Kx_d)Y_2 + \dot{x}^T Y_3 + \omega^T(t)Y_4\big]\\ &\quad\times \big[\bar{A}_{ij}\tanh(Kx) + \bar{A}_{di}\tanh(Kx_d) + \bar{D}_{1i}\omega(t) - \dot{x}\big] = 0\end{aligned} \tag{34}$$

We introduce the following inequality for our proof. We assume that L is a positive definite matrix, then L^{-1} is a positive definite matrix as well. We have: $(L - K^{-1})^T L^{-1}(L - K^{-1}) \geq 0$, that is:

$$(L^T - 2K^{-1})^T \geq -K^{-T}L^{-1}K^{-1} \tag{35}$$

Utilizing (35), we have:

$$-\gamma^2 I = -\gamma^2 Z^{-T}Z^T I Z Z^{-1} \leq \gamma^2(Z^{-1}Z^{-T} - 2Z^{-T}) \tag{36}$$

Considering Lyapunov function (8), and (31), (33), (34), (36), we have:

$$\begin{aligned}&z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(t)\\ \leq\ & 2\tanh(Kx)P\dot{x} + \tanh^T(Kx)Q\tanh(Kx) - (1-\sigma)\tanh^T(Kx_d)Q\tanh(Kx_d)\\ &+\tau\dot{x}^T KRK\dot{x} - \int_{t-d(t)}^{t}\Big(\tanh^T(Kx(s))\Big)' R(\tanh(Kx(s)))' ds\\ &+2\Big[\tanh^T(Kx)X_1 + \tanh^T(Kx_d)X_2 + \dot{x}^T X_3 + \omega^T(t)X_4\Big]\\ &\quad\times\left[\tanh(Kx) - \tanh(Kx_d) - \int_{t-d(t)}^{t}\Big(\tanh^T(Kx(s))\Big)' ds\right]\\ &+\sum_{i,j}^{r}\big[\tanh^T(Kx)Y_1 + \tanh^T(Kx_d)Y_2 + \dot{x}^T Y_3 + \omega^T(t)X_4\big]\\ &\quad\times\big[\bar{A}_{ij}\tanh(Kx) + \bar{A}_{di}\tanh(Kx_d) + \bar{D}_{1i}\omega(t) - \dot{x}\big]\\ &+\frac{1}{4}\sum_{i,j}^{r} h_i h_j\big[(C_i + C_j)\tanh(Kx) + (D_{2i} + D_{2j})\omega(t)\big]^T\\ &\quad\times\big[(C_i + C_j)\tanh(Kx) + (D_{2i} + D_{2j})\omega(t)\big]\\ &+\gamma^2(Z^{-1}Z^{-T} - 2Z^{-T})\omega^T(t)\omega(t)\end{aligned}$$

Thus,

$$\begin{aligned}&z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(t)\\ \le &\sum_{i,j}^{r} h_i h_j \eta_2^T\big(\Omega_{ij} + \tau X' R^{-1} X'\big)\eta_2\\ &- \int_{t-d(t)}^{t} \big(\xi^T R + \eta_2^T X'\big) R^{-1} \big(\xi^T R + \eta_2^T X'\big)^T ds\end{aligned}$$

where $\eta_2^T = \begin{bmatrix}\tanh(Kx) & \tanh(Kx_d) & \dot{x} & \omega(t)\end{bmatrix}$, $\xi = \Big(\tanh^T(Kx(s))\Big)'$, and

$$\Omega_{ij} = \begin{bmatrix}\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14}\\ * & \Omega_{22} & \Omega_{23} & \Omega_{24}\\ * & * & \Omega_{33} & \Omega_{34}\\ * & * & * & \Omega_{44}\end{bmatrix},$$

$\Omega_{11} = \Lambda_{11} + \frac{1}{4}(C_i + C_j)^T(C_i + C_j), \Omega_{12} = \Lambda_{12}, \Omega_{13} = \Lambda_{13},$
$\Omega_{22} = \Lambda_{22}, \Omega_{23} = \Lambda_{23}, \Omega_{33} = \Lambda_{33},$
$\Omega_{14} = \frac{1}{4}(C_i + C_j)^T(D_{2i} + D_{2j}) + X_4^T + Y_1\bar{D}_{1i} + \bar{A}_{ij}^T Y_4^T,$
$\Omega_{24} = -X_4^T + Y_2\bar{D}_{1i} + \bar{A}_{di}^T Y_4^T, \Omega_{34} = Y_3\bar{D}_{1i} - Y_4^T,$
$\Omega_{44} = \frac{1}{4}(D_{2i} + D_{2j})^T(D_{2i} + D_{2j}) + \gamma^2(Z^{-1}Z^{-T} - 2Z^{-T}) + Y_4\bar{D}_{1i} + \bar{D}_{1i}^T Y_4^T.$

It is assumed that:

$$\Omega_{ij} + \tau X' R^{-1} X' < 0 \tag{37}$$

Applying Lemma 2.1 to (37) we have:

$$\begin{bmatrix}\Omega_{ij} & X'\\ * & -\frac{1}{\tau}R\end{bmatrix} < 0, i, j \in S$$

Thus,

$$\Pi + Sym\left(\begin{bmatrix}Y_1\Delta A_{ij} & Y_1\Delta A_{di} & 0 & Y_1\Delta D_{1i} & 0\\ Y_2\Delta A_{ij} & Y_2\Delta A_{di} & 0 & Y_2\Delta D_{1i} & 0\\ Y_3\Delta A_{ij} & Y_3\Delta A_{di} & 0 & Y_3\Delta D_{1i} & 0\\ Y_4\Delta A_{ij} & Y_4\Delta A_{di} & 0 & Y_4\Delta D_{1i} & 0\\ 0 & 0 & 0 & 0 & 0\end{bmatrix}\right) < 0 \tag{38}$$

where

$$\Pi = \begin{bmatrix}\Phi'_{2ij} & X'\\ * & -\frac{1}{\tau}R\end{bmatrix}$$

and

$$\Phi'_{2ij} = \begin{bmatrix}\Phi'_{11,ij} & \Phi'_{12,ij} & \Phi'_{13,ij} & \Phi'_{14,ij}\\ * & \Phi'_{22,ij} & \Phi'_{23,ij} & \Phi'_{24,ij}\\ * & * & \Phi'_{33,ij} & \Phi'_{34,ij}\\ * & * & * & \Phi'_{44,ij}\end{bmatrix},$$

$\Phi'_{11,ij} = \Phi_{11,ij} + \frac{1}{4}(C_i + C_j)^T(C_i + C_j), \Phi'_{12,ij} = \Phi_{12,ij}, \Phi'_{13,ij} = \Phi_{13,ij},$
$\Phi'_{22,ij} = \Phi_{22,ij}, \Phi'_{23,ij} = \Phi_{23,ij}, \Phi'_{33,ij} = \Phi_{33,ij},$
$\Phi'_{14,ij} = \frac{1}{4}(C_i + C_j)^T(D_{2i} + D_{2j}) + X_4^T + Y_1 D_{1i} + A_{ij}^T Y_4^T,$
$\Phi'_{24,ij} = -X_4^T + Y_2 D_{1i} + A_{di}^T Y_4^T, \Phi'_{34,ij} = Y_3 D_{1i} - Y_4^T,$

$\Phi'_{44,ij} = \frac{1}{4}(D_{2i}+D_{2j})^T(D_{2i}+D_{2j})+\gamma^2(Z^{-1}Z^{-T}-2Z^{-T})+Y_4D_{1i}+D_{1i}^TY_4^T$. According to (18), we have

$$\Pi + {\Gamma'}^T F_{ii} M' + {M'}^T F_{ii}^T \Gamma' < 0 \tag{39}$$

where ${\Gamma'}^T = \begin{bmatrix} Y_1G_i & Y_1G_i \\ Y_2G_i & Y_2G_i \\ Y_3G_i & Y_3G_i \\ Y_4G_i & Y_4G_i \\ 0 & 0 \end{bmatrix}$, $M' = \begin{bmatrix} E_{1i} & E_{2i} & 0 & E_{4i} & 0 \\ E_{3i}H_j & 0 & 0 & 0 & 0 \end{bmatrix}$, $F_{ii} = \begin{bmatrix} F_i & 0 \\ 0 & F_i \end{bmatrix}$, and $F_{ii}^T(t)F_{ii}(t) \le I$.

Considering Lemma 2.2, we have:

$$\Pi + \varepsilon {\Gamma'}^T \Gamma' + \varepsilon^{-1} {M'}^T M' < 0 \tag{40}$$

Applying Lemma 2.1 to (40), we have:

$$\begin{bmatrix} \Pi & {\Gamma'}^T & {M'}^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon^{-1} I \end{bmatrix} < 0 \tag{41}$$

In order to transform (41) into LMIs, we make a decomposition of (41)

$$\begin{bmatrix} \Pi' & {\Gamma'}^T{}_i & {M'}^T \\ * & -\varepsilon I & 0 \\ * & * & -\varepsilon^{-1} I \end{bmatrix} + \begin{bmatrix} \Delta^T \\ 0 \\ 0 \end{bmatrix} I \begin{bmatrix} \Delta & 0 & 0 \end{bmatrix} < 0 \tag{42}$$

where

$$\Pi' = \begin{bmatrix} \Phi''_{2ij} & X' \\ * & -\frac{1}{\tau}R \end{bmatrix}, \Delta = \begin{bmatrix} \frac{C_i+C_j}{2} & 0 & 0 & \frac{D_{2i}+D_{2j}}{2} & 0 \end{bmatrix},$$

and

$$\Phi''_{2ij} = \begin{bmatrix} \Phi_{11,ij} & \Phi_{12,ij} & \Phi_{13,ij} & \Phi''_{14,ij} \\ * & \Phi_{22,ij} & \Phi_{23,ij} & \Phi''_{24,ij} \\ * & * & \Phi_{33,ij} & \Phi''_{34,ij} \\ * & * & * & \Phi''_{44,ij} \end{bmatrix},$$

$$\Phi''_{14,ij} = X_4^T + Y_1D_{1i} + A_{ij}^TY_4^T, \Phi''_{24,ij} = \Phi'_{24,ij}, \Phi''_{34,ij} = \Phi'_{34,ij},$$

$$\Phi''_{44,ij} = \gamma^2(Z^{-1}Z^{-T} - 2Z^{-T}) + Y_4D_{1i} + D_{1i}^TY_4^T.$$

Applying Lemma 2.1 to (42), we have:

$$\begin{bmatrix} \Pi' & {\Gamma'}^T & {M'}^T & \Delta^T \\ * & -\varepsilon I & 0 & 0 \\ * & * & -\varepsilon^{-1} I & 0 \\ * & * & * & -I \end{bmatrix} < 0 \tag{43}$$

Assume Y_l are nonsingular, $l = 1,2,3,4$, and define $Y_l^{-T} = \lambda Z$, $\bar{P} = Y_1^{-1}PY_1^{-T}$, $\bar{Q} = Y_1^{-1}QY_1^{-T}$, $\bar{R} = Y_4^{-1}RY_4^{-T}$, $\bar{X}_l = Y_l^{-1}X_lY_l^{-T}$, $M_j = H_jZ$, $\lambda > 0$. Pre- and post-multiplying both sides of (43) by Θ_3, $\Theta_3 = diag\{Y_1^{-1}, Y_2^{-1}, Y_3^{-1}, Y_4^{-1}, Y_4^{-1}, I, I, I, I, I\}$, we have LMIs (23) . Then $\Omega_{ij} + \tau X'R^{-1}X' < 0$ and $R^{-1} > 0$, which shows

$$z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t) + \dot{V}(t) < 0 \tag{44}$$

For any $T > 0$, integrating (44) on t from 0 to T, we have:

$$\int_0^T \left[z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\right] dt + V(T) - V(0) < 0$$

Due to the initial condition $x(0) = 0$, we get $V(0) = 0$. For any $x(t)$, $V(T) \geq 0$, and $\int_0^T \left[z^T(t)z(t) - \gamma^2\omega^T(t)\omega(t)\right] dt < 0$, thus $\int_0^T z^T(t)z(t)dt < \gamma^2 \int_0^T \omega^T(t)\omega(t)dt$. Since $\gamma > 0$ and $\omega(t) \in L_2[0,\infty]$, then $\int_0^\infty \omega^T(t)\omega(t)dt$ exists, which derives that $\int_0^\infty z^T(t)z(t)dt$ exists. Thus, $\int_0^\infty z^T(t)z(t)dt \leq \gamma^2 \int_0^\infty \omega^T(t)\omega(t)dt$, that is $\|z(t)\|_{L_2} \leq \gamma\|\omega(t)\|_{L_2}$.

Thus, systems (20) and (21) are robust asymptotically stable with the H_∞ performance index γ. It completes the proof. □

Remark 3.3. In Theorem 2, the robust fuzzy hyperbolic control law (4) can not only guarantee the asymptotically stability of the uncertain delay nonlinear systems (20) and (21) with the H_∞ performance index γ, but also achieve small amplitude control. That is the advantage of the control approach based on our model.

4. **Simulation Examples**

In this section, the proposed approach is applied to the following three examples to verify its effectiveness. In the first example, a pure numerical example is given to show the implement of the proposed method. The second and the third examples are practical applications of the Van de Vusse system.

Example 4.1. Consider the following uncertain T-S fuzzy hyperbolic delay system:

R^i: IF $x_1(t)$ is L_i, then

$$\begin{aligned}\dot{x}(t) =& (A_i + \Delta A_i)\tanh(Kx(t)) + (A_{di} + \Delta A_{di})\tanh(Kx(t-d(t))) \\ &+ (B_i + \Delta B_i)u(t) + (D_{1i} + \Delta D_{1i})\omega(t), \\ z(t) =& C_i \tanh(Kx(t)) + D_{2i}\omega(t), \\ u_i(t) =& H_i \tanh(Kx(t)), i = 1, 2.\end{aligned}$$

where $A_1 = \begin{bmatrix}-0.4 & 0.4 \\ 0.4 & -0.8\end{bmatrix}, A_2 = \begin{bmatrix}-0.6 & 0.4 \\ 0.6 & -0.8\end{bmatrix}, B_1 = \begin{bmatrix}-0.6 \\ -0.4\end{bmatrix}, B_2 = \begin{bmatrix}-0.4 \\ -0.4\end{bmatrix},$

$C_1 = C_2 = \begin{bmatrix}0.1 \\ 0.1\end{bmatrix}, A_{d1} = \begin{bmatrix}-0.2 & 0.1 \\ 0.2 & -0.1\end{bmatrix}, A_{d2} = \begin{bmatrix}-0.6 & 0.4 \\ 0.6 & -0.8\end{bmatrix},$

$D_{11} = D_{12} = \begin{bmatrix}-0.2 & 0.1 \\ 0 & 0.1\end{bmatrix}, D_{21} = D_{22} = \begin{bmatrix}0.1 & 0.1\end{bmatrix}, G_1 = G_2 = \begin{bmatrix}-0.2 & 0 \\ 0.1 & 0.1\end{bmatrix},$

$E_{11} = E_{12} = \begin{bmatrix}0.2 & 0 \\ 0.1 & -0.1\end{bmatrix}, E_{21} = E_{22} = \begin{bmatrix}0.1 & 0 \\ 0 & 0.1\end{bmatrix}, E_{31} = E_{32} = \begin{bmatrix}0.1 \\ 0.1\end{bmatrix},$

$E_{41} = E_{42} = \begin{bmatrix}0.1 & 0 \\ 0 & -0.2\end{bmatrix}, F_1 = \begin{bmatrix}\sin t & 0 \\ 0 & \cos t\end{bmatrix}, F_2 = \begin{bmatrix}\cos t & 0 \\ 0 & \sin t\end{bmatrix}.$

Solving LMIs (23), we obtain the following matrices:

$P = \mathrm{diag}(0.4133, 2.2391)$, $R = \mathrm{diag}(0.2053, 0.3843)$, $Z = \mathrm{diag}(2.9362, 0.4382)$, $Q = \begin{bmatrix}0.4593 & 0.0161 \\ 0.0161 & 2.6969\end{bmatrix}$, $H_1 = \begin{bmatrix}1.1238 & 0.6781\end{bmatrix}$, $H_2 = \begin{bmatrix}1.1238 & 0.6781\end{bmatrix}$.

Define $\tanh(Kx) = \begin{bmatrix}\tanh(x_1) & \tanh(2x_2)\end{bmatrix}^T$ and membership functions as follows:

$$h_{L_1}(x_1) = \sin^2(x_1), h_{L_2}(x_1) = \cos^2(x_1)$$

For better explanation, we choose uncertain T-S fuzzy hyperbolic delay model and the following uncertain T-S fuzzy linear delay model:

$$\begin{aligned}\dot{x}(t) =&(A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t - d(t))\\ &+ (B_i + \Delta B_i)u(t) + (D_{1i} + \Delta D_{1i})\omega(t),\\ u_i(t) =&F_i x(t), i = 1, 2\end{aligned}$$

with the same system matrices to make a contrast.

According to the analysis of the stable fuzzy controller design approach for uncertain T-S fuzzy hyperbolic delay system, we can also figure out controller gain matrices for the uncertain T-S fuzzy linear delay system as following:

$$F_1 = \begin{bmatrix}1.2235 & 0.6960\end{bmatrix}, F_2 = \begin{bmatrix}1.2235 & 0.6960\end{bmatrix}$$

Let initial function $\varphi(t) = \begin{bmatrix}4 & -2\end{bmatrix}^T$ and $d(t) = 1 + \frac{1}{5}\sin(t)$, $t \in [-1.2, 0]$, $\omega(t) = 0.5\begin{bmatrix}e^{-2t} & e^{-2t}\end{bmatrix}^T$, $\gamma = 1$, $\varepsilon_{ij} = 1$, $\tau = 1.2$, $\sigma = 0.2$. The state responses and control curve of closed-loop system are shown in Figure 1 and Figure 2, respectively.

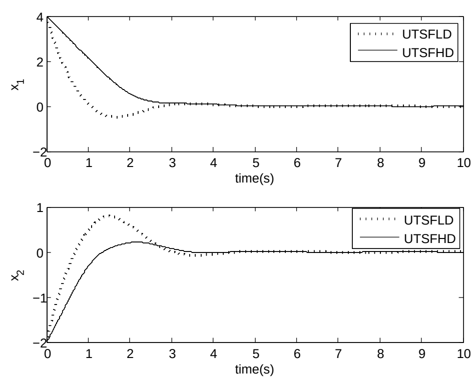

Figure 1. The State Responses of closed-loop System and Comparison (UTSFLD: Uncertain T-S Fuzzy Linear Delay System; UTSFHD: Uncertain T-S Fuzzy Hyperbolic Delay System)

From the simulation results, we can see that the uncertain T-S fuzzy hyperbolic delay system can achieve much smaller amplitude control in almost same stabilization time, which validates the advantages of our model.

Example 4.2. Consider the dynamics of an isothermal continuous stirred tank reactor (CSTR) for the Van de Vusse of the following form:

$$\begin{aligned}&\dot{x}_1 = -k_1x_1 - k_3x_1^2 - k_4x_{d1} - k_6x_{d1}^2 + u(C_{A0} - x_1),\\ &\dot{x}_2 = k_1x_1 - k_2x_2 + k_4x_{d1} - k_5x_{d2} + u(-x_2),\\ &y = x_2\end{aligned} \tag{45}$$

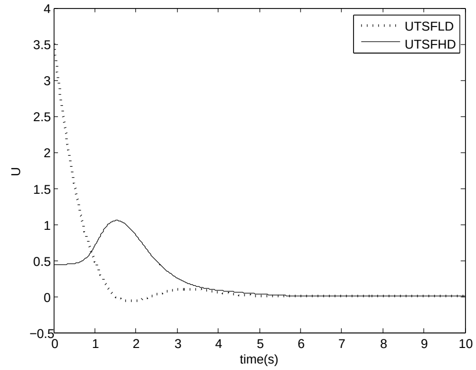

Figure 2. The Control Curve of Closed-loop System and Comparison

where the state x_1 (mol/L) is the concentration of the reactant inside the reactor, the state x_2 (mol/L) is the concentration of the product in the output stream of the CSTR, the output $y = x_2$ determines the grade of the final product, the input feed stream to the CSTR consists of a reactant with concentration C_{A0}, and the controlled input is the dilution rate $u = F/V(h^{-1})$, where F is the input flow rate to the reactor in L/h and V is the constant volume of the CSTR in liters.

In this paper, the kinetic parameters are chosen as: $k_1 = 52h^{-1}$, $k_2 = 101h^{-1}$, $k_3 = 9L/(\text{molh})$, $k_4 = 50 - k_1$, $k_5 = 100 - k_2$, $C_{A0} = 10\text{mol/L}$, and $V = 1\text{L}$. From the above system equation, some equilibrium points of above system are tabulated in Table 1.

x_e^T	x_{de}^T	u_e
[2.1789 0.9079]	[2.1789 0.9079]	20
[2.9368 1.1041]	[2.9368 1.1041]	33
[4.0623 1.2538]	[4.0623 1.2538]	62

Table 1. Equilibrium Points

Under these equilibrium points, $[x_e\ u_e]$, which are also chosen as the desired operating points $[x'_e\ u'_e]$. According to [11], we obtained the system matrices A_i, B_i, $i = 1, 2, 3$ for the T-S fuzzy linear model [17, 19]. For comparison, the system matrices A_i, B_i, $i = 1, 2, 3$, in T-S fuzzy hyperbolic delay model are the same as those in T-S fuzzy linear model. Thus, the fuzzy hyperbolic laws can be represented by:

R^1: IF $x_1(t)$ is 2.1789, then

$$\dot{x}_\delta = A_1 \tanh(Kx_\delta) + A_{d1} \tanh(Kx_{d\delta}) + B_1 u_\delta,$$
$$u_\delta = H_1 \tanh(Kx_\delta).$$

R^2: IF $x_1(t)$ is 2.9368, then

$$\dot{x}_\delta = A_2 \tanh(Kx_\delta) + A_{d2} \tanh(Kx_{d\delta}) + B_2 u_\delta,$$
$$u_\delta = H_2 \tanh(Kx_\delta).$$

R^3: IF $x_1(t)$ is 4.0623, then

$$\dot{x}_\delta = A_3 \tanh(Kx_\delta) + A_{d3} \tanh(Kx_{d\delta}) + B_3 u_\delta,$$
$$u_\delta = H_3 \tanh(Kx_\delta).$$

where $A_1 = \begin{bmatrix} -77.4697 & 14.0628 \\ 59.1006 & -118.0414 \end{bmatrix}$, $A_2 = \begin{bmatrix} -85.7904 & 19.5758 \\ 62.8698 & -129.9136 \end{bmatrix}$,
$A_3 = \begin{bmatrix} -97.1330 & 27.7742 \\ 69.4715 & -157.6076 \end{bmatrix}$, $B_1 = \begin{bmatrix} 7.8211 \\ -0.9079 \end{bmatrix}$, $B_2 = \begin{bmatrix} 7.0632 \\ -1.1041 \end{bmatrix}$,
$B_3 = \begin{bmatrix} 5.9377 \\ -1.2538 \end{bmatrix}$, $A_{d1} = \begin{bmatrix} -0.5012 & 0.7736 \\ -2 & 1 \end{bmatrix}$, $A_{d2} = \begin{bmatrix} -1.3004 & 0.9673 \\ -2 & 1 \end{bmatrix}$,
$A_{d3} = \begin{bmatrix} -2.4156 & 0.8289 \\ -2 & 1 \end{bmatrix}$, $x_\delta = x - x'_e$, $u_\delta = u - u'_e$, $x_{d\delta} = x_d - x'_{de}$, $i = 1, 2, 3$.

By solving LMIs (6), we get the following matrices: $P = \begin{bmatrix} 504.7297 & 0 \\ 0 & 58.8279 \end{bmatrix}$,
$R = \begin{bmatrix} 179.2325 & 0 \\ 0 & 7.3775 \end{bmatrix}$, $Z = \begin{bmatrix} 0.3479 & 0 \\ 0 & 2.2838 \end{bmatrix}$, $Q = \begin{bmatrix} 66.8814 & 0.9665 \\ 0.9665 & 1.9633 \end{bmatrix}$, and
$H_1 = \begin{bmatrix} -9.9615 & -3.0193 \end{bmatrix}$, $H_2 = \begin{bmatrix} -9.9615 & 3.0193 \end{bmatrix}$, $H_3 = \begin{bmatrix} 9.9615 & -3.0193 \end{bmatrix}$.

According to the analysis of the stable fuzzy controller design approach for continuous T-S fuzzy hyperbolic delay system, we can figure out controller gain matrices for the T-S fuzzy linear delay system as following:

$$F_1 = \begin{bmatrix} -2.4391 & 8.7793 \end{bmatrix}, F_2 = \begin{bmatrix} -2.4391 & 8.7793 \end{bmatrix}, F_3 = \begin{bmatrix} -2.4391 & 8.7793 \end{bmatrix}.$$

Define $\tanh(Kx) = \begin{bmatrix} \tanh(0.05x_1) & \tanh(0.05x_2) \end{bmatrix}^T$, and the membership functions of state x_1 shown in Figure 3.

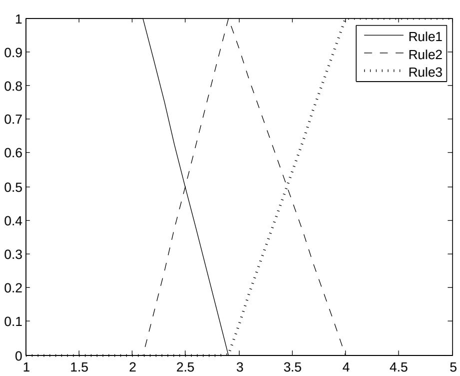

FIGURE 3. Membership functions of x_1

Thus, the whole fuzzy hyperbolic control law is:

$$u = (h_1 H_1 + h_2 H_2 + h_3 H_3) \tanh(Kx_\delta) + u'_e$$

and the whole fuzzy linear control law is:

$$u = (h_1 F_1 + h_2 F_2 + h_3 F_3) x_\delta + u'_e$$

where h_1, h_2 and h_3 satisfy $h_1 + h_2 + h_3 = 1$.

Let initial function $\varphi(t) = \begin{bmatrix} 2 & 4 \end{bmatrix}^T$, $t \in [-0.02, 0]$, and $d(t) = \frac{1}{100}(\sin(t) + 1)$, $\gamma = 1$, $\varepsilon_{ij} = 1$, $\tau = 0.02$, $\sigma = 0.01$. We choose the operating points as $x_e'^T = \begin{bmatrix} 2.9368 & 1.1041 \end{bmatrix}$ and $u_e' = 33$. The state responses and control curve of closed-loop system are showed in the Figure 4 and Figure 5, respectively.

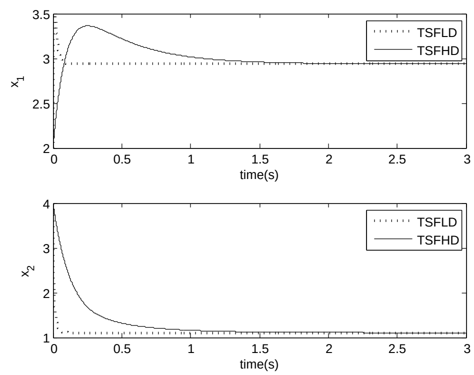

Figure 4. The State Responses of Van de Vusse and Comparison (TSFLD: T-S Fuzzy Linear Delay System; TSFHD: T-S Fuzzy Hyperbolic Delay System)

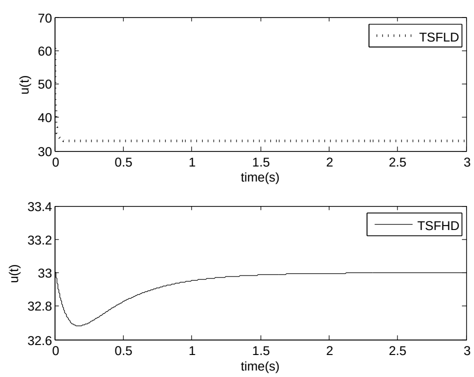

Figure 5. The Control Curve of Van de Vusse and Comparison

From the simulations, we can see that the state of nonlinear system can converge to the operating points under the two different fuzzy controllers almost in the same stabilization time. However, the amplitude of our controller is much smaller than that of the fuzzy linear controller, which validates the advantages of the control design method based on T-S fuzzy hyperbolic model.

Example 4.3. Consider the dynamics of an isothermal continuous stirred tank reactor (CSTR) for the Van de Vusse (45). In order to illustrate the robustness of the proposed method, we choose another equilibrium points of the above system tabulated in Table 2.

x_e^T	x_{de}^T	u_e
[1.3589 0.6177]	[1.3589 0.6177]	10
[2.1789 0.9079]	[2.1789 0.9079]	20
[2.8864 1.0933]	[2.8864 1.0933]	32

Table 2. Equilibrium Points

Under these equilibrium points, $[x_e\ u_e]$, which are also chosen as the desired operating points $[x'_e\ u'_e]$. Thus, the fuzzy hyperbolic laws can be represented by:

R^1: IF x_1 is about 1.3589, then

$$\dot{x}_\delta = (A_1+\Delta A_1)\tanh(Kx_\delta) + (A_{d1}+\Delta A_{d1})\tanh(Kx_{d\delta}) + (B_1+\Delta B_1)\,u_\delta,$$
$$u_\delta = H_1\tanh(Kx_\delta).$$

R^2: IF x_1 is about 2.1789, then

$$\dot{x}_\delta = (A_2+\Delta A_2)\tanh(Kx_\delta) + (A_{d2}+\Delta A_{d2})\tanh(Kx_{d\delta}) + (B_2+\Delta B_2)\,u_\delta,$$
$$u_\delta = H_2\tanh(Kx_\delta).$$

R^3: IF x_1 is about 2.8864, then

$$\dot{x}_\delta = (A_3+\Delta A_3)\tanh(Kx_\delta) + (A_{d3}+\Delta A_{d3})\tanh(Kx_{d\delta}) + (B_3+\Delta B_3)\,u_\delta,$$
$$u_\delta = H_3\tanh(Kx_\delta).$$

According to [11], the system matrices A_i, B_i, $i = 1, 2, 3$ are obtained as following: $A_1 = \begin{bmatrix} -68.0366 & 8.3743 \\ 55.7671 & -109.2877 \end{bmatrix}, A_2 = \begin{bmatrix} -77.4697 & 14.0628 \\ 59.1006 & -118.0414 \end{bmatrix}$,
$A_3 = \begin{bmatrix} -85.2526 & 19.2056 \\ 62.6003 & -128.9847 \end{bmatrix}, B_1 = \begin{bmatrix} 8.6411 \\ -0.6177 \end{bmatrix}, B_2 = \begin{bmatrix} 7.8211 \\ -0.9079 \end{bmatrix}$,
$B_3 = \begin{bmatrix} 7.1136 \\ -1.0933 \end{bmatrix}, A_{d1} = \begin{bmatrix} -0.4084 & 0.5119 \\ -2 & 1 \end{bmatrix}, A_{d2} = \begin{bmatrix} -0.5012 & 0.7736 \\ -2 & 1 \end{bmatrix}$,
$A_{d3} = \begin{bmatrix} -1.2486 & 0.5402 \\ -2 & 1 \end{bmatrix}, C_1 = C_2 = C_3 = \begin{bmatrix} 0 & 1 \end{bmatrix}$,
$D_{11} = D_{12} = D_{13} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, D_{21} = D_{22} = D_{23} = \begin{bmatrix} 1 & 0 \end{bmatrix}$,
$G_1 = G_2 = G_3 = \begin{bmatrix} 0.05 & 0 \\ 0 & -0.05 \end{bmatrix}, E_{11} = E_{12} = E_{13} = \begin{bmatrix} 0.05 & 0.05 \\ 0 & 0.05 \end{bmatrix}$,
$E_{21} = E_{22} = E_{23} = \begin{bmatrix} 0.05 & 0.05 \\ 0 & 0.05 \end{bmatrix}, E_{31} = E_{32} = E_{32} = \begin{bmatrix} 0.05 \\ 0.05 \end{bmatrix}$,
$E_{41} = E_{42} = E_{43} = \begin{bmatrix} 0.05 & 0.05 \\ 0 & 0.05 \end{bmatrix}, F_1 = \begin{bmatrix} \sin t & 0 \\ 0 & \cos t \end{bmatrix}, F_2 = \begin{bmatrix} \cos t & 0 \\ 0 & \sin t \end{bmatrix}$,
$F_3 = \begin{bmatrix} \sin t & 0 \\ 0 & \cos t \end{bmatrix}, x_\delta = x - x'_e, u_\delta = u - u'_e, x_{d\delta} = x_d - x'_{de}, i = 1, 2, 3.$

By solving LMIs (23), we obtain the following matrices: $P = \begin{bmatrix} 39279 & 0 \\ 0 & 4454 \end{bmatrix}$, $Q = \begin{bmatrix} 83087 & 0 \\ 0 & 12982 \end{bmatrix}$, $Z = \begin{bmatrix} 0.005 & 0 \\ 0 & 0.0256 \end{bmatrix}$, $Q = \begin{bmatrix} 1947.9 & -17.3 \\ -17.3 & 281.5 \end{bmatrix}$, and $H_1 = \begin{bmatrix} 14.7911 & -1.8124 \end{bmatrix}$, $H_2 = \begin{bmatrix} -14.3768 & -1.5609 \end{bmatrix}$, $H_3 = \begin{bmatrix} -14.5998 & -1.8045 \end{bmatrix}$.

According to the analysis of the stable fuzzy controller design approach for continuous T-S fuzzy hyperbolic delay system, we can figure out controller gain matrices for the T-S fuzzy linear delay system as following:

$$F_1 = \begin{bmatrix} -6.3218 & 2.9289 \end{bmatrix}, F_2 = \begin{bmatrix} -6.3517 & 3.0158 \end{bmatrix}, F_3 = \begin{bmatrix} -6.3334 & 2.9067 \end{bmatrix}.$$

Define $\tanh(Kx) = \begin{bmatrix} \tanh(0.05x_1) & \tanh(0.05x_2) \end{bmatrix}^T$, and the membership functions of state x_1 shown in Figure 6.

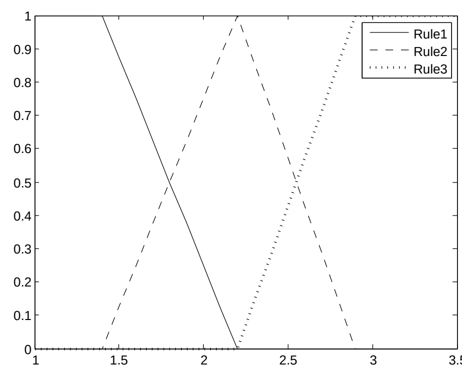

Figure 6. Membership Functions of x_1

Thus, the whole fuzzy hyperbolic control law is:

$$u = (h_1H_1 + h_2H_2 + h_3H_3)\tanh(Kx_\delta) + u'_e$$

and the whole fuzzy linear control law is:

$$u = (h_1F_1 + h_2F_2 + h_3F_3)x_\delta + u'_e$$

where h_1, h_2 and h_3 satisfy $h_1 + h_2 + h_3 = 1$.

Let initial function $\varphi(t) = \begin{bmatrix} 2 & 4 \end{bmatrix}^T$, $t \in [-0.02, 0]$, and $d(t) = \frac{1}{100}(\sin(t) + 1)$, $\gamma = 1$, $\varepsilon_{ij} = 1$, $\tau = 0.02$, $\sigma = 0.01$. We choose the operating points as $x_e'^T = \begin{bmatrix} 2.1789 & 0.9079 \end{bmatrix}$ and $u'_e = 20$. The state responses and control curve of closed-loop system are showed in the Figure 7 and Figure 8, respectively.

From the simulations, we can see that the state of nonlinear system can converge the operating points under the two different fuzzy controllers at almost the same stabilization time. However, the amplitude of our controller is much smaller than that of the fuzzy linear controller, which validates the advantages of the control design method based on uncertain T-S fuzzy hyperbolic model.

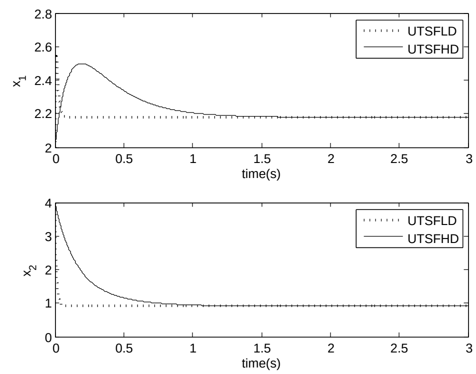

FIGURE 7. The State Responses of Van de Vusse and Comparison

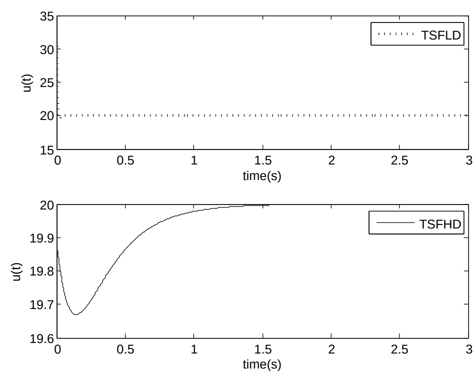

FIGURE 8. The Control Curve of Van de Vusse and Comparison

5. Conclusion

In this paper, we have built a new T-S fuzzy hyperbolic delay model to the stability analysis and robust H_∞ control for a class of nonlinear delay systems with small control amplitude. Using the PDC method, we designed the fuzzy hyperbolic controller for the T-S fuzzy hyperbolic delay system. With the designed controller and a new Lyapunov function, sufficient conditions are derived via LMIs to guarantee the asymptotic stability of the closed-loop delay system. Moreover, considering the difference between the model and the real system, we extent our model to uncertain T-S fuzzy hyperbolic delay model with disturbances. Similarly, robust H_∞ controller is designed, which can not only guarantee the asymptotically stability of the uncertain delay nonlinear system with the H_∞ performance index γ, but also achieve small amplitude control. Finally, a numerical example and the Van de Vusse example are given to validate the effectiveness of the proposed design method based on our models. Compared with control method based on the T-S fuzzy linear model, our controller can achieve smaller control amplitude in almost the same state stabilization time. In the future study, the approach can be extended to the distributed time-delay systems.

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