MEASURING STUDENTS MODELING CAPACITIES: A FUZZY APPROACH

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Abstract. A central aim of educational research in the area of mathematical modeling and applications is to recognize the attainment level of students at defined states of the modeling process. In this paper, we introduce principles of fuzzy sets theory and possibility theory to describe the process of mathematical modeling in the classroom. The main stages of the modeling process are represented as fuzzy sets in a set of linguistic labels indicating the degree of a student's success in each of these stages. We use the total possibilistic uncertainty on the ordered possibility distribution of all student profiles as a measure of the students' modeling capacities and illustrate our results by application to a classroom experiment.

1. Introduction

Mathematical modeling is today a dynamic tool for teaching mathematics, because it connects mathematics with the real world and everyday life, thus giving students the opportunity to realize its usefulness in practical applications [21]. Since Pollak [18] represented mathematical modeling as a cyclic interaction between mathematics and the real world in ICME-3, Kalrsruhe, 1976, much effort has been made by authors and researchers to analyze its process in detail. For a brief, but comprehensive account of the models developed along these lines, we refer the reader to [6], or [15]. It is accepted in general, that the main stages of the mathematical modeling process involve (with minor variations):

- *Analysis* of the given real world problem, i.e. understanding the statement and recognizing limitations, restrictions and requirements of the real system.
- *Mathematizing*, i.e. formulation of the real situation in so that it is ready for mathematical treatment, and construction of the model.
- *Solution* of the model, achieved by proper mathematical manipulation.
- *Validation* (control) of the model, usually achieved by simulation of the real system under the conditions existing before the solution of the model (empirical results, special cases etc).
- *Implementation* of the final mathematical results i.e. "translation" of the mathematical solution obtained in terms of the corresponding real situation , thus obtaining the solution of the given real problem.

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It is now evident that mathematization, solution and validation are the most important stages of the modeling process. In fact, although the analysis of the problem deserves attention as being a prerequisite to mathematization, it may be considered as an introductory stage of the process. Further, the stage of implementation is not expected to hide any "surprises", at least for the type of modeling problems usually solved by students in school. In other words, a student who obtains a correct mathematical solution is normally expected to be able to "translate" it correctly in terms of the corresponding real situation.

A central aim of educational research in the area of mathematical modeling and applications today, is to recognize the attainment level of students at defined stages of the modeling process. In an earlier paper [22] we introduced a Markov model for the description of the process of mathematical modeling in situations where the teacher provides such modeling problems to students for solution and succeeded in obtaining a measure of students' mathematical model building abilities. An improved version of the above model has been presented in [27].

Models for the mathematical modeling process like the above and those described in [6] and [15], are helpful in understanding *"ideal behavior"* [6], in which modelers proceed from real world problems to acceptable solutions via a mathematical model and report on them. However, life in the classroom is very differnt. Recent research, ([2, 3] and [5]), reports that when tackling mathematical modeling problems, students in school take *individual modeling routes* associated with their individual learning styles. In general, students' cognition utilizes concepts that are inherently graded and therefore fuzzy. On the other hand, from the teacher's point of view there usually exists vagueness about the degree of success of students at each stage of the modeling process. Hence it is reasonable to use the principles of fuzzy sets theory to describe the process of mathematical modeling in classroom effectively. The concept of uncertainty, which emerges naturally within the broad framework of fuzzy sets theory, is involved in any problem-solving situation, especially when dealing with real-world problems. Uncertainty is a result of information deficiency. In fact, information pertaining to the model within which a real situation is conceptualized may be incomplete, fragmentary, not fully reliable, vague, contradictory, or deficient in some other way. Thus the amount of information obtained by an action can, in general, be measured by the reduction of uncertainty resulting from the action. In other words, the amount of uncertainty regarding represents the total amount of potential information. Accordingly, a student's uncertainty during the modeling process is connected to his/her capacity to obtain relevant information. Therefore a measure of uncertainty may be adopted as a measure of a student's modeling capacities. For special facts on fuzzy sets and uncertainty theory we refer to [12] and [13].

2. **The Fuzzy Model**

Let us consider a group of n students, $n \geq 2$, while modeling a process in a classroom. Denote by A_i, $i = 1, 2, 3$, the stages of mathematization, solution and validation of the model respectively, and by a, b, c, d, and e the linguistic labels of negligible, low, intermediate, high and complete success respectively of a student in each of the A_i's. Set $U = a, b, c, d, e$. We shall represent the A_i's as fuzzy sets in U. If $n_{ia}, n_{ib}, n_{ic}, n_{id}$ and n_{ie} denote the number of students that respectively achieved negligible, low, intermediate, high and complete success at state A_i, $(i = 1, 2, 3)$, we define the membership functions m_{A_i} for each x in U, as follows:

$$m_{A_i}(x) = \begin{cases} 1, & \text{if } \frac{4n}{5} < n_{ix} \leq n \\ 0.75, & \text{if } \frac{3n}{5} < n_{ix} \leq \frac{4n}{5} \\ 0.5, & \text{if } \frac{2n}{5} < n_{ix} \leq \frac{3n}{5} \\ 0.25, & \text{if } \frac{n}{5} < n_{ix} \leq \frac{2n}{5} \\ 0, & \text{if } 0 < n_{ix} \leq \frac{n}{5} \end{cases}$$

We now write A_i as fuzzy set in U in the form:

$$A_i = \{(x, m_{A_i}(x)) : x \in U\}, i = 1, 2, 3.$$

We may similarly represent the stages of analysis and implementation as fuzzy sets in U. However this makes the presentation of our fuzzy model technically much more complicated, and therefore we will not attempt it. This manipulation is actually a general technique applied frequently during the modeling process of a real-world problem where we eliminate the variables of the real system that are not necessary for its study or solution. In this way we go from the real system to the so called *"assumed real system"*, which helps towards the formulation of the problem in a form ready for mathematical treatment [27].

We represent all possible student *profiles (overall states)* during the modeling process, by a *fuzzy relation*, say R, in U^3 of the form

$$R = \left\{(s, m_R(s)) : s = (x, y, z) \in U^3\right\}.$$

The membership function m_R of a profile is defined as follows:

Definition 2.1. A profile $s = (x, y, z)$, with x, y, z in U, is said to be *well ordered* if x corresponds to a degree of success equal or greater than y, and y corresponds to a degree of success equal or greater than z. For example, (c, c, a) is well ordered profile, while (b, a, c) is not. We now define the membership degree of a profile s as

$$m_R(s) = m_{A_i}(x) m_{A_i}(y) m_{A_i}(z)$$

if s is well ordered, and zero otherwise. If, for example, profile (b, a, c) possesses a nonzero membership degree, how is it possible for a student who has failed during the solution, to validate the model satisfactorily?

In what follows, for reasons of brevity, we shall write m_s instead of $m_R(s)$. The *possibility* r_s of profile s is defined by

$$r_s = \frac{m_s}{\max\{m_s\}}$$

where $\max\{m_s\}$ denotes the maximal value of m_s , for all s in U^3. In other words, r_s expresses the "relative membership degree" of s with respect to $\max\{m_s\}$.

As we have already explained at the end of section 2, a measure of the student group uncertainty during the modeling process may be adopted as a measure of the students' modeling capacities. Within the domain of possibility theory (cf. [13]), uncertainty consists of *strife (or discord)* among the sizes (cardinalities) of the various sets of alternatives, and *non-specificity (or imprecision)*. In other words, some alternatives are left unspecified. Strife is measured by the function $ST(r)$ on the ordered possibility distribution $r : r_1 = 1 \geq r_2 \geq \cdots \geq r_n \geq r_{n+1}$ of the student group defined by

$$ST(r) = \frac{1}{\log 2}\left[\sum_{i=2}^{n}(r_i - r_{i+1})\log\frac{i}{\sum_{j=1}^{i} r_j}\right]$$

while non-specificity is measured by

$$N(r) = \frac{1}{\log 2}\left[\sum_{i=2}^{n}(r_i - r_{i+1})\log i\right]$$

The sum $T(r) = ST(r) + N(r)$ is a measure of the *total possibilistic uncertainty* $T(r)$ for ordered possibility distributions. The total possibilistic uncertainty $T(r)$ of the student group during the modeling process can be adopted as a measure for the students' modeling capacities. This is reinforced by Shackle [19], who argues that human reasoning can be more adequately formalized by possibility theory rather than by probability theory. The performance of the student group during the modeling process improves as the value of $T(r)$ decreases.

During the modeling process, students may use reasoning that involves amplified inferences, whose content is beyond the available evidence, and hence obtain conclusions not entailed in the given premises. These conclusions may produce a generalization whose amount of information exceeds the amount of information at the level of functioning, i.e. an overgeneralization. For mathematical calculations for example, such conclusion could be the illusion that $\sqrt{a+b} = \sqrt{a} + \sqrt{b}$, or that $\log(a+b) = \log a + \log b$, etc. The appearance of conflict in conclusions requires that they be appropriately adjusted so that the resulting generalization is free of conflict.

Assume finally that we want to study the combined results of the behaviour of k different student groups ($k \geq 2$) during the process of modeling the same real situation. For this, we introduce *fuzzy variables* $A_1(t), A_2(t)$ and $A_3(t)$ with $t = 1, 2, \ldots, k$. The values of the above variables represent the states of the modeling process for each of the k student groups as fuzzy sets in U: e.g. $A_1(2)$ represents the state of mathematizing for the second group ($t = 2$). In order to measure the degree of evidence of combined results of the k groups, it is necessary to define the possibility $r(s)$ of each student profile s with respect to the membership degrees of s for all groups. We hence introduce *pseudo-frequencies*

$$f(s) = \sum_{t=1}^{k} m_s(t)$$

and define

$$r(s) = \frac{f(s)}{\max(f(s))}$$

where $\max\{f(s)\}$ denotes the maximal pseudo-frequency. Obviously the same method may be applied when studying the behaviour of a student group during the modeling process of k different real problems.

3. **A Classroom Experiment**

In order to illustrate the results obtained in the previous section we recently performed the following experiment at the Graduate Technological Educational Institute (T.E.I.) of Patras, Greece. Our subjects were 35 students of the School of Technological Applications (i.e. future engineers), and our basic tool was a list of 10 problems involving mathematical modeling which the students had to solve within 2 hours. A student's performance at each stage of the modeling process was characterized as follows:

- Negligible success, if he/she obtained positive results for less than 2 problems.
- Low success, if he/she obtained positive results for 2, 3, or 4 problems.
- Intermediate success, if he/she obtained positive results for 5, 6, or 7 problems.
- High success, if he/she obtained positive results for 8, or 9 problems.
- Complete success, if he/she obtained positive results for all problems.

15, 12 and 8 students had intermediate, high and very high success respectively at stage of mathematizing. Therefore $n_{1a} = n_{1b} = 0, n_{1c} = 15, n_{1d} = 12$ and $n_{1e} = 8$. Thus, by the definition of $m_{A_i}(x)$, mathematizing is represented as a fuzzy set in U in the form:

$$A_1 = \{(a, 0), (b, 0), (c, 0.5), (d, 0.25), (e, 0.25)\}.$$

Similarly for the solution and validation of the model we found that

$$A_2 = \{(a, 0), (b, 0), (c, 0.5), (d, 0.25), (e, 0)\}$$

and

$$A_3 = \{(a, 0.25), (b, 0.25), (c, 0.25), (d, 0), (e, 0)\}$$

respectively.

Using the definition given in the previous section we calculated the membership degrees of the 5^3 (ordered samples with replacement of 3 objects taken from 5) in all possible students' profiles (see column of $m_s(1)$ in Table 1). For example, for $s = (c, c, a)$ we have

$$m_s = m_{A_1}(c).m_{A_2}(c).m_{A_3}(a) = (0.5).(0.5).(0.25) = 0.06225$$

It turned out that (c, c, a) was one of the profiles of maximal membership degree and therefore the possibility of each s in U^3 is given by

$$r_s = \frac{m_s}{0.06225}.$$

Calculating the possibilities of all profiles (see column of $r_s(1)$ in Table 1) we obtain that the ordered possibility distribution for the student group is:

$$r_1 = r_2 = 1, r_3 = \cdots = r_8 = 0.5, r_9 = \cdots = r_{14} = 0.258, r_{15} = \cdots = r_{125} = 0$$

Thus,

$$\begin{aligned} ST(r) &= \frac{1}{\log 2}\left[\sum_{i=2}^{n}(r_i - r_{i+1})\log\frac{i}{\sum_{j=1}^{i} r_j}\right] \\ &\approx \frac{1}{0.301}\left[0.5\log\frac{2}{2} + 0.242\log\frac{8}{5} + 0.258\log\frac{14}{6.548}\right] \\ &\approx 3.32\,[(0,242).(0.204) + (0.258).(0.33)] \approx 0.445 \end{aligned}$$

and

$$\begin{aligned} N(r) = \frac{1}{\log 2}\left[\sum_{i=2}^{n}(r_i - r_{i+1})\log i\right] &= \frac{1}{\log 2}\,[0.5\log 2 + 0.24\log 8 + 0.258\log 14] \\ \approx 0.5 + 3.(0.242) + (0.857).1.146 \approx 2.208 \end{aligned}$$

Finally, $T(r) \approx 2.653$.

A few days later we performed the same experiment on a group of 30 students of the School of Management and Economics. These students study the modeling processes in detail during the course of Operations' Research, while the students of School of Technological Applications study mathematical modeling only through working examples during 2 or 3 mathematics courses. This time we found that

$$\begin{aligned} A_1 &= \{(a,0),(b,0.25),(c,0.5),(d,0.25),(e,0)\} \\ A_2 &= \{(a,0.25),(b,0.25),(c,0.5),(d,0),(e,0)\} \quad \text{and} \\ A_3 &= \{(a,0.25),(b,0.25),(c,0.25),(d,0),(e,0)\} \end{aligned}$$

When we calculated the membership degrees of all possible profiles of the student group (see column of $m_s(2)$ in Table 1), the maximal membership degree was again 0.06225. In other words, the possibility of each s is given by the same formula as for the first group. Calculating the possibilities of all profiles (see column of $r_s(2)$ in Table 1), the ordered possibility distribution of the second group was obtained as:

$$r_1 = r_2 = 1, r_3 = \cdots = r_8 = 0.5, r_9 = \cdots = r_{13} = 0.258, r_{14} = \cdots = r_{125} = 0.$$

Finally, $T(r) = 0.432 + 2.179 = 2.611$.

Since $2.611 < 2.653$, we conclude that the second group performed slightly better than the first. This result, combined with the fact that the students of School of Management and Economics attend only one course of general Mathematics (they also attend Mathematics of Finance and Statistics) is an indication that a detailed study of the modeling process possibly helps students to perform better when solving problems that involve mathematical modeling. Of course, further research and experiments are needed to validate this conjecture statistically.

In order to study the combined results of the behaviour of the two groups, we introduced the fuzzy variables $A_i(t), i = 1, 2, 3$ and $t = 1, 2. Then the pseudo - frequency of each student profile sis given by$ f(s) = m$_s(1) + m_s(2)$ (see corresponding column in Table 1). The highest pseudo-frequency is 0.124 and therefore the possibility of each student's profile is given by

$$r(s) = \frac{f(s)}{0.124}.$$

The possibilities of all profiles having non-zero pseudo-frequencies are presented in the last column of Table 1.

A_1	A_2	A_3	$m_s(1)$	$r_s(1)$	$m_s(2)$	$r_s(2)$	$f(s)$	$r(s)$
b	b	b	0	0	0.016	0.258	0.016	0.129
b	b	a	0	0	0.016	0.258	0.016	0.129
b	a	a	0	0	0.016	0.258	0.016	0.129
c	c	c	0.062	1	0.062	1	0.124	1
c	c	a	0.062	1	0.062	1	0.124	1
c	c	b	0	0	0.031	0.5	0.031	0.25
c	a	a	0	0	0,031	0.5	0.031	0.25
c	b	a	0	0	0.031	0.5	0.031	0.25
c	b	b	0	0	0.031	0.5	0.031	0.25
d	d	a	0.016	0.258	0	0	0.016	0.129
d	d	b	0.016	0.258	0	0	0.016	0.129
d	d	c	0.016	0.258	0	0	0.016	0.129
d	a	a	0	0	0.016	0.258	0.016	0.129
d	b	a	0	0	0.016	0.258	0.016	0.129
d	b	b	0	0	0.016	0.258	0.016	0.129
d	c	a	0.031	0.5	0.031	0.5	0.062	0.5
d	c	b	0.031	0.5	0.031	0.5	0.062	0.5
d	c	c	0.031	0.5	0.031	05	0.062	0.5
e	c	a	0.031	0.5	0	0	0.031	0.25
e	c	b	0.031	0.5	0	0	0.031	0.25
e	c	c	0.031	0.5	0	0	0.031	0.25
e	d	a	0.016	0.258	0	0	0.016	0.129
e	d	b	0.016	0.258	0	0	0.016	0.129
e	d	c	0.016	0.258	0	0	0.016	0.129

TABLE 1. Profiles with Non Zero Pseudo-frequencies
(The Outcomes are with Accuracy Up to the Third Decimal Point)

4. **Discussion and Conclusions**

Our fuzzy model provides useful *quantitative information* (e.g. possibilities and $T(r)$) for the process of mathematical modeling in a classroom, as well as a *qualitative view* of the behaviour of student groups (students' profiles that give a comprehensive idea of the degree of students' success at the successive stages of the modeling process in terms of the linguistic labels,). All these enable the instructor to get a concentrated view of his (her) students' cognitive status, and helps him (her) to adapt teaching methods, plans and targets to each class indivdually.

A great amount of research has been conducted in the area of student modeling in general and student diagnosis in particular and our fuzzy model for the process of mathematical modeling, combined with analogous models presented in earlier papers for the processes of learning the subject matter ([24, 28]) and Case-Based Reasoning ([29, 31]) gives a new approach for a deeper study of this area. In fact, there are many research opportunities for extending these models to cover all the analogous situations appearing in mathematics education. In this way, a fuzzy systems framework could be formed for a better understanding and an effective diagnosis of student capacities for learning and problem-solving. Notice that, analogous efforts using fuzzy logic in education have been attempted by other researchers as well; e.g. [1, 4, 10, 17, 14, 20], etc. In general, it is well known that nowadays fuzzy sets theory finds a lot of important applications in a wide variety of human activities (e.g. [7, 8, 9, 11, 16], etc).

We must finally underline the importance of *stochastic methods (Markov chain models)* as an alternative approach for the same purposes; e.g. [21, 22, 23, 25, 26, 27, 30, 31], etc. Nevertheless Markov models, although sometimes easier for a non expert (e.g. the teacher) to apply in practice, are self-restricted to provide *quantitative information* only for the situations that they represent, e.g. measures for the problem-solving, or model-building abilities of student groups, short and long-run forecasts (probabilities) for the evolution of various phenomena, etc. Therefore, one may claim that a fuzzy model, like the one presented in this paper, is more useful to the education researcher for a deeper study of the corresponding real situation, because, apart from the quantitative information, it gives also the possibility of a qualitative analysis of the problems involved. In particular our fuzzy model for modeling has also the extra advantage of giving the opportunity for a combined study of the modeling performance of several student groups, or of the same group during the modeling process of different real problems.

5. Appendix: List of the Problems Given for Solution to Students in Our Classroom Experiment

Problem 1: We want to construct a channel to run water by folding the two edges of an orthogonal metallic leaf having sides of length 20cm and 32 cm, in such a way that they will be perpendicular to the other parts of the leaf. Assuming that the flow of the water is constant, how we can run the maximum possible quantity of the water?

Remark 5.1. The correct solution is obtained by folding the edges of the longer side of the leaf

Problem 2: A car dealer has a mean annual demand of 250 cars, while he receives 30 new cars per month. The annual cost of storing a car is 100 euros and each time he places a new order, he pays an extra 2200 euros for general expenses (transportation, insurance etc). The first car of a new order arrives just when the last car of the previous order has been sold. How many cars must he order in order to achieve the minimum total cost?

Problem 3: An importation company codes the messages for the arrivals of its orders in terms of characters consisting of a combination of the binary elements 0 and 1. If it is known that the arrival of a certain order will take place from 1st until the 16^{th} of March, find the minimal number of the binary elements of each character required for coding this message.

Problem 4: Let us assign to each letter the number showing its order in the alphabet ($A = 1$, $B = 2$, $C = 3$ etc). Let us also assign to each word of 4 letters a 2×2 matrix in the obvious way; e.g. the matrix $\left[\begin{smallmatrix} 19 & 15 \\ 13 & 5 \end{smallmatrix}\right]$ corresponds to the word SOME. Using the matrix $E = \left[\begin{smallmatrix} 8 & 5 \\ 11 & 7 \end{smallmatrix}\right]$ as an encoding matrix how would you send the message LATE in the form of a camouflaged matrix to a receiver who knows the above process and how would he (she) decode your message?

Problem 5: The demand function $P(Q_d) = 25 - Q_d^2$ represents the different prices that consumers are willing to pay for different quantities Q_d of a good. On the other hand the supply function $P(Q_s) = 2Q_s + 1$ represents the prices at which different quantities Q_s of the same good will be supplied. If the market equilibrium occurs at (Q_0, P_0) producers who would supply at lower price than P_0 benefit. Find the total gain to the producers.

Problem 6: A ballot box contains 8 balls numbered from 1 to 8. One makes 3 successive drawings of a lottery with replacement. Find the probability that the balls that he draws out of the box have different numbers.

Problem 7: A box contains 3 white, 4 blue and 6 black balls. If we draw 2 balls, what is the probability that they are of the same colour?

Problem 8: The population of a country increases proportionally. If the population doubles in 50 years, in how many years will it triple?

Problem 9: A wine producer has a stock of wine weighing more than 500 and less than 750 kilos. He has calculated that, if he had the double quantity of wine and transferred it to bottles of 12, 25, or 40 kilos, 6 kilos would be left over each time. Find the quantity of stock.

Problem 10: Among all cylindrical towers having a total surface of $180\pi\ m^2$, which one has the maximal volume?

Remark 5.2. Some students did not include one base (ground-floor) in the total surface the and they found another solution, while some others did not include either base (roof and ground-floor) and they found no solution, since we cannot construct cylinder with maximal volume from its surrounding surface.

References

[1] M. Ajello and F. Spagnolo, *Some experimental observations on common sense and fuzzy logic*, Proceedings of International Conference on Mathematics Education into the 21st Century, Napoli, (2002), 35-39.

[2] R. Borroneo Ferri, *Modeling problems from a cognitive perspective*, In: C. R. Haines, P. Galbraith, W. Bloom, S. Khan, eds., Mathematical Modeling: Education, Engineering and Economics, (ICTMA 12), Horwood Publishing, Chichester, (2007), 260-270.

[3] H. M. Doer, *What knowledge do teachers need for teaching mathematics through applications and modeling?*, In: W. Blum, P. L. Galbraith, H. W. Henn and M. Niss, eds., Modeling and Applications in Mathematics Education, Springer, NY, (2007), 69-78.

[4] E. A. Espin and C. M. L. Oliveras, *Introduction to the use of fuzzy logic in the assessment of mathematics teachers'*, In: A. Gagatsis, ed., Proceedings of the 1^{st} Mediterranean Conference on Mathematics Education, Nicosia, Cyprus, (1997), 107-113.

[5] P. L. Galbraith and G. Stillman, *Assumptions and context: pursuing their role in modeling activity*, In: J. F. Matos, W. Blum, K. Houston and S. P. Carreira, eds., Modeling and Mathematics Education: Applications in Science and Technology (ICTMA 9), Chichester, (2001), 300-310.

[6] C. R. Haines and R. Crouch, *Remarks on modeling cycle and interpretation of behaviours*, In: R. A. Lesh, P. L. Galbraith, C. R. Haines and A. Harford, eds., Modeling Students' Mathematical Modeling Competencies, (ICTMA13), London, (2010), 145-154.

[7] H. L. Huang and G. Shi, *Robust H1 control for T-S time-varying delay systems with norm bounded uncertainty based on LMI approach*, Iranian Journal of Fuzzy Systems, **6(1)** (2009), 1-14.

[8] L. Iliadis and S. Spartalis, *An intelligent information system for fuzzy additive modeling (hydrological risk application)*, Iranian Journal of Fuzzy Systems, **7(1)** (2010), 1-14.

[9] N. Javiadian, Y. Maali and N. Mahadavi-Amiri, *Fuzzy linear programming with grades of satisfaction in constraints*, Iranian Journal of Fuzzy Systems, **6(3)** (2009), 17-35.

[10] I. Kaya and J. Kahraman, *Fuzzy process capability analyses: an application to teaching processes*, Journal of Intelligent and Fuzzy Systems, **19(4-5)** (2008), 259-272.

[11] C. Kezi Selva Vijila and P. Kanagasalarathy, *Intelligent technique of cancelling maternal ECG in FECG extraction*, Iranian Journal of Fuzzy Systems, **5(1)** (2008), 27-45.

[12] G. J. Klir and T. A. Folger, *Fuzzy sets, uncertainty and information*, Prentice Hall, London, 1988.

[13] G. J. Klir and M. J. Wierman, *Uncertainty-based information: elements of generalized information theory*, Physika-Verlag, Heidelberg, 1998.

[14] M. Mashinchi, A. Parchami and H. R. Maleki, *Application of fuzzy capability indices in educational comparison*, Proceedings 3d Int. Research Convention 2005, **42** (2005).

[15] S. Meier, *Identifying modeling tasks*, In: L. Paditz and A. Rogerson, eds., Models in Developing Mathematics Education (MEC21), Dresden, University of Applied Sciences, (2009), 399-403.

[16] P. K. Nayak and M. Pal, *Bi-matrix games with intuitionistic fuzzy goals*, In: L. Paditz and A. Rogerson, eds., Iranian Journal of Fuzzy Systems, **7(1)** (2010), 65-79.

[17] S. Perdikaris, *The problem of transition across levels in the van Hiele theory of geometric reasoning*, Philosophy of Mathematics Education Journal, **18** (2004).

[18] H. O. Pollak, *New trends in mathematics teaching*, Unesko, Paris, **IV** (1979).

[19] G. L. S. Shackle, *Decision, order and time in human affairs, Cambridge*, Cambridge University Press, 1961.

[20] I. Y. Subbotin, H. Badkoodehi and N. N. Bilotskii, *Fuzzy logic and iterative assessment*, Didactics of Mathematics: Problems and Investigations (Ukraine), **25** (2006), 221-227.

[21] M. G. Voskoglou and S. Perdikaris, *Measuring problem-solving skills*, International Journal of Mathematical Education in Science and Technology, **24(3)** (1993), 443-447.

[22] M. G. Voskoglou, *Measuring mathematical model building abilities*, International Journal of Mathematical Education in Science and Technology, **26(1)** (1995), 29-35.

[23] M. G. Voskoglou, *An application of ergodic Markov chains to analogical problem solving*, The Mathematics Education (India), **XXX(2)** (1996), 95-108.

[24] M. G. Voskoglou, *The process of learning mathematics: a fuzzy set approach*, Heuristics and Didactics of Exact Sciences (Ukraine), **10** (1999), 9-13.

[25] M. G. Voskoglou, *An application of Markov chains to decision making*, Studia Kupieckie (University of Lodz, Poland), **6** (2000), 69-76.
[26] M. G. Voskoglou, *The use of mathematical modeling as a tool for learning mathematics*, Quaderni di Ricerca in Didattica (Scienze Mathematiche), **16** (2006), 53-60.
[27] M. G. Voskoglou, *A stochastic model for the modeling process*, In: C. R. Haines, P. Galbraith, W. Bloom, S. Khan, eds., Mathematical Modeling: Education, Engineering and Economics, (2007), 149-157.
[28] M. G. Voskoglou, *Fuzziness or probability in the process of learning? a general question illustrated by examples from teaching mathematics*, The Journal of Fuzzy Mathematics, International Fuzzy Mathematics Institute (Los Angeles), **17(3)** (2009), 697-686.
[29] M. G. Voskoglou, *Fuzzy sets in case-based reasoning*, In: Y. Chen, H. Deng, D. Zhang and Y. Xiao, eds., Fuzzy Systems and Knowledge Discovery (FSKD 2009), **6** (2009), 252-256.
[30] M. G. Voskoglou, *A stochastic model for the process of learning*, In: L. Paditz and A. Rogerson, eds., Models in Developing Mathematics Education (MEC21) Dresden, (2009), 565-569.
[31] M. G. Voskoglou, *Mathematizing the case-based reasoning process*, In: A. M. Leeland, ed., Case-Based Reasoning: Processes, Suitability and Applications, in press, **6** (2010).

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