A NOTE TO INTERPRETABLE FUZZY MODELS AND THEIR LEARNING

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This paper is dedicated to Professor L. A. Zadeh on the occasion of his 95th birthday and the 50th year of the birth of fuzzy logic

Abstract. In this paper we turn the attention to a well developed theory of fuzzy/linguistic models that are interpretable and, moreover, can be learned from the data. We present four different situations demonstrating both interpretability as well as learning abilities of these models.

1. Introduction

This paper was inspired by paper [3] written by Hüllermeier, which contains discussion about the role of fuzzy logic in machine learning. The author's attitude is critical and he demonstrates on various examples that fuzzy logic contributes only little to machine learning. We will not discuss the whole paper, but focus only on Sections 3 ("Fuzzification of models") and 4 ("The myth of interpretability") where the author casts doubts on the way how fuzzy models are learned from data and on their interpretability. One can agree with his arguments when he speaks about fuzzy models being special fuzzy relations constructed from fuzzy rules, each of which being formed of fuzzy sets with triangular or trapezoidal shape, and Mamdani method for deriving a conclusion. Such fuzzy rules can indeed be hardly interpretable. The reason is that they are applied to approximation of a function and, therefore, the fuzzy sets forming them must necessarily be deformed to obtain as best approximation as possible. The used fuzzy sets represent certain fuzzy numbers or intervals — they do not have linguistic meaning such as "small, very big", etc. — cf. [10]. Their deformation, however, even prevents from their interpretability.

Hüllermeier in his paper makes general statements concerning fuzzy models and, unfortunately, disregards the fact that there does exist a class of models that are well interpretable and for which also a well working learning method from data exists. Moreover, these models were already many times successfully applied in various areas. We are speaking about, the so called, *fuzzy/linguistic models*. This class of models is based on the formal theory of *fuzzy natural logic* and its characteristic feature is the use of the sophisticated model of the semantics of special natural language expressions. The fuzzy/linguistic models are also composed of

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IF-THEN rules. Unlike the rules forming Mamdani fuzzy models, however, they are taken as genuine conditional clauses of natural language and interpreted accordingly. Therefore, we call them fuzzy/linguistic. Sets of such rules are called *linguistic descriptions* and can be taken as a specifically structured text in natural language.

The work with linguistic descriptions requires a special inference method called *perception-based logical deduction* (PbLD). Its specific feature is that the given linguistic description is processed locally. This means that though the rules forming the linguistic description are vague, they are at the same time distinguished from each other. For example, if we know that for small values of x, values of y must be big and for big values of x values of y must be small then it would be great surprise if for some apparently small value of x we would obtain values of y other than big. As we will see below, the PbLD method meets this requirement.

The fuzzy/linguistic model, the PbLD method and learning of linguistic descriptions were in detail described in many papers and recently also in the book [19]. Its idea roots already to 1992 [4]. It raised when testing Mamdani fuzzy control method which gave counterintuitive results when applied to fuzzy sets modeling extensions of the linguistic expressions such as "small, very big", etc. Much later, it has been even proved [5] that Mamdani method in this case cannot work in principle.

The fuzzy/linguistic model was from the very beginning developed using formal logic. However, the original formulation based on the predicate version of the, so called, fuzzy logic with evaluated syntax turned out to be not satisfactorily neat. The new formulation that uses the language of higher-order mathematical fuzzy logic is much more fitting and transparent. The theory now falls within a wider program of *fuzzy natural logic* (FNL) described, e.g., in [11]. The full formalization of fuzzy/linguistic IF-THEN rules and linguistic descriptions can be found in [8, 14]. Less formal explanation can be found in [7]. The PbLD inference method has been described formally in [6] and less formally in [2, 17, 18].

Let us emphasize that the fuzzy/linguistic model is not only theoretical concept, but it has a lot of various kinds of applications in control [13], decision-making [20], forecasting of time series [12, 16, 21]. The method and the corresponding methodology has been implemented in a software system LFL Controller*) [1, 9, 15]. A comprehensive and not too formal explanation of the methodology of fuzzy/linguistic models is presented in the above mentioned book [19].

It is necessary to emphasize that the linguistic fuzzy models follow and implement the original Zadeh's ideas concerning modeling of natural language semantics using fuzzy sets and their applications that was introduced in many seminal papers [23, 24, 25, 26, 27, 28, 29, 30].

Because the theory and detailed description of the fuzzy/linguistic models and the PbLD method has been presented in the numerous papers, we will below give only few examples giving, in our opinion, convincing arguments in favor of their interpretability and leaning abilities. We will argue that interpretability of fuzzy systems is not myth and can be reached if we base their construction on well

*)The short "LFL" means "Linguistic Fuzzy Logic".

established theory of the semantics of natural language and, of course, we well pose the goal that we want to reach. As the systems considered above consist of genuine linguistic expressions, they are well interpretable. On the other hand, they cannot be too precise — we face here the celebrated incompatibility principle formulated by Zadeh in [25] saying that precision and relevancy of information are mutually excluding characteristics.

2. Interpretable Fuzzy/linguistic Systems and Their Learning

In this section, we will give several examples of linguistic description characterizing linguistically the course of some function or a decision situation. In parallel, we will also introduce the corresponding data and show how a linguistic description characterizing the data can be learned. We will compare and discuss the results.

2.1. **Linguistic Context.** Before we start, let us recall that the crucial concept in modeling semantics of natural language is that of *possible world*. As we will deal with a special class of linguistic expressions, namely the *evaluative* ones, we will introduce a simplified concept of *linguistic context*.

Let $(U, \leq)U$ be a linearly ordered set. This can be arbitrary set but we will usually assume that it is a subset of real numbers $U \subseteq \mathbb{R}$. The *context* is a union of intervals

$$w = [v_L, v_S] \cup [v_S, v_R] \tag{1}$$

determined by three points: the *left bound* v_L, *right bound* v_R and a *middle value* v_S where $v_L < v_S < v_R$. These numbers have the following meaning: v_L denotes *the least* value that makes sense in a given situation; the v_R is *the greatest* value that makes sense. Finally, the value v_S is *the common middle* value, which is neither small not big. The latter is a typical middle value that need not lay in the precise middle of the interval $[v_L, v_R]$. We can alternatively take the context also as a triple of numbers

$$w = \langle v_L, v_S, v_R \rangle.$$

A typical example of the context can be heights of people (in cm) in middle Europe where, e.g., we can consider $w = \langle 145, 175, 220 \rangle$. These numbers can be different in north Europe or in Africa.

Below, we will consider the, so called, *simple evaluative expressions* and use for them the following shorts:

short	meaning	short	meaning	short	meaning
ze	*zero*	si	*significantly*	qr	*quite roughly*
sm	*small*	ve	*very*	vR	*very roughly*
me	*medium*	ty	typically	vv	very very roughly
bi	*big*	ml	*more or less*	no	*not*
ex	*extremely*	ro	*roughly*		

Thus, for example, "ve sm" means "very small", "ml bi" means "more or less big", etc.

Let us also recall the extensions, i.e., fuzzy sets representing collections of numbers characterized by the given evaluative expression in the specified context have shapes obtained by modification of the basic three shapes from Figure 1. Let us recall that learning is based on using a special function of *local perception* $\mathrm{LPerc}^K(x, w)$ that assigns to a given value $x \in w$ in a given context w an evaluative linguistic expression taken from a set K of available expressions. For the details and justification see the cited references.

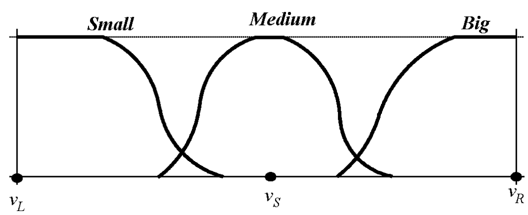

FIGURE 1. Shapes of Extensions in a Given Context of the Three Canonical Evaluative Expressions "Small", "Medium" and "Big" Forming the, So Called, *Fundamental Evaluative Trichotomy*

Let us also recall that the PbLD method provides output either in the linguistic form or by a precise value. In the latter case, a special defuzzification method denoted as DEE[†)] must be used. Because of locality of the method, the output is, in general, a partially continuous function. To make it continuous, a smooth version of PbLD has been introduced in [17]. It is based on combination of PbLD with the fuzzy transform (see [22]). The details can be found in the book [19].

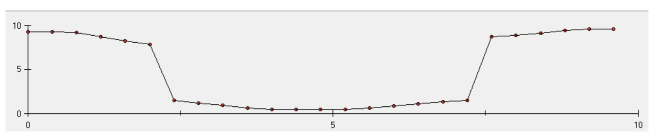

FIGURE 2. Function Obtained Using PbLD and DEE Defuzzification from the Simple Linguistic Description Consisting of Only 3 Rules

2.2. **Linguistic Description of a Simple Function.** Let us consider a situation when we have in mind a function $f : X \longrightarrow Y$ that takes *big values for small arguments, very small values for medium arguments and very big values for big arguments*. This is a linguistic description of the course of a function that can be characterized using the following linguistic description:

	X	$\Rightarrow$	Y
1.	sm		bi
2.	me		ve sm
3.	bi		ve bi

[†)]Defuzzification of Evaluative Expressions. The method first classifies the output fuzzy set and then assigns it a value using one of the methods LOM, FOM, MOM or COG.

X	Y	X	Y	X	Y	X	Y	X	Y
0	9.27	2	7.78	4	0.42	6	0.83	8	9.1
0.4	9.27	2.4	1.45	4.4	0.42	6.4	1.08	8.4	9.35
0.8	9.12	2.8	1.18	4.8	0.42	6.8	1.3	8.8	9.58
1.2	8.68	3.2	0.9	5.2	0.42	7.2	1.52	9.2	9.58
1.6	8.23	3.6	0.58	5.6	0.58	7.6	8.87	10	9.58

Table 1. Data of the Function from Figure 2

This description characterizes the shape of a considered function and gives us rough idea about its values, of course, w.r.t. some context. If, for example, we will set contexts for the arguments X and functional values Y as $w_X = \langle 0, 4.5, 10\rangle$ and $w_Y = \langle 0, 5, 10\rangle$ then after application of the PbLD inference method combined with the DEE defuzzification, we obtain the shape depicted in Figure 2. One can see that the shape fits the meaning of the linguistic description of the function above. We argue that this demonstrates that the linguistic description above is well interpretable. Even better result is obtained using smooth PbLD — see Figure 3.

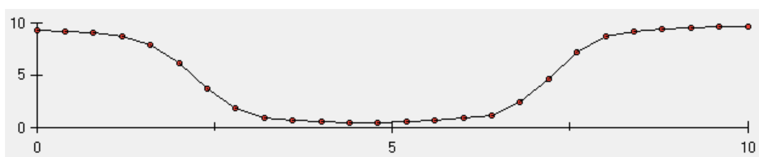

Figure 3. Function Obtained Using Smooth PbLD from the Simple Linguistic Description Consisting of Only 3 Rules Using

To demonstrate learning abilities of the fuzzy/linguistic models, let us consider the data in Table 1 obtained from the graph of Figure 2.

After learning, we obtain the following linguistic description:

	X	$\Rightarrow$	Y		X	$\Rightarrow$	Y
1.	ra sm		ra bi	5.	ra me		ve sm
2.	ml sm		ml bi	6.	ml bi		ve bi
3.	ro sm		ro bi	7.	ra bi		si bi
4.	ml me		ra sm				

Using PbLD, we obtain from this linguistic description the function depicted in Figure 4.

On can see that the results correspond to the idea of the shape of the resulting function characterized above. It is important to note, however, that the learned linguistic description provides a function that fits the *intended shape* of the resulting function described *linguistically*. Therefore, we cannot expect that such description will be a good approximation of the original function f. Though the linguistic description is learned w.r.t. the data, we cannot expect that the result will be a

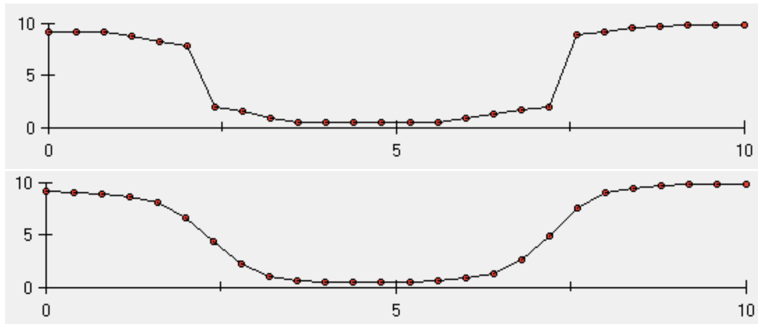

FIGURE 4. Function Obtained from the Linguistic Description Learned from the Data Determining Function from Figure 2. Up: PbLD with DEE Defuzzification; Down: Smooth PbLD

good approximation when compared with other methods, such as neural networks, Mamdani fuzzy system of F-transform. In fact, the maximal absolute error of the function obtained using PbLD (with DEE defuzzification) w.r.t. the given data above is 0.4. This is not bad but any of the other mentioned methods can surely give better approximation result. The price we have to pay, however, is interpretability provided by expressions of natural language. Recalling the Zadeh's incompatibility principle, we face the following decision: either we obtain linguistic description that is relevant but describes the function imprecisely, or we obtain a precise approximation of a function that is, however, too detailed to give relevant information about the considered function.

2.3. **Linguistic Description of a Slightly More Complex Function.** Let us now consider a slightly more complex function $g : X \longrightarrow Y$ that takes *big values for very small arguments, small values for small arguments, medium values for more or less small arguments, big values for medium arguments, very small values for big arguments and extremely big values for very big arguments.* This describes the course of a function that can be characterized using the following linguistic description:

	X	$\Rightarrow$	Y
1.	ve sm		bi
2.	sm		sm
3.	ml sm		me
4.	me		bi
5.	bi		ve sm
6.	ve bi		ex bi

If we set the following contexts for the arguments and functional values as $w_X = w_Y = \langle 0, 0.4, 1\rangle$ then, after application of the PbLD inference method, we obtain the shape depicted in Figure 5. One can see that the shape fits the meaning of the linguistic description of the function above. We argue that this demonstrates that the linguistic description above is well interpretable.

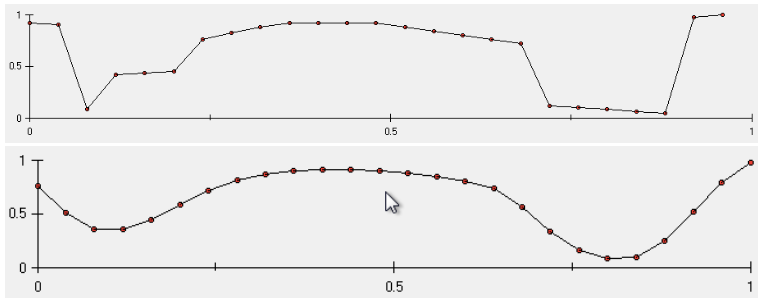

Figure 5. Function Obtained from the Simple Linguistic Description Consisting of 6 Rules. Up: PbLD with DEE Defuzzification; Down: Smooth PbLD

If, similarly as above, we prepare data obtained from the graph of Figure 5 the, after learning, we obtain the following linguistic description:

	X	$\Rightarrow$ Y		X	$\Rightarrow$ Y
1.	ze	si bi	6.	ml me	ml bi
2.	ve sm	ra bi	7.	ml bi	ra sm
3.	ra sm	ra sm	8.	ra bi	ve sm
4.	qr sm	ra me	9.	ve bi	ex bi
5.	ra me	ra bi			

Using PbLD, we obtain from this linguistic description the function depicted in Figure 5.

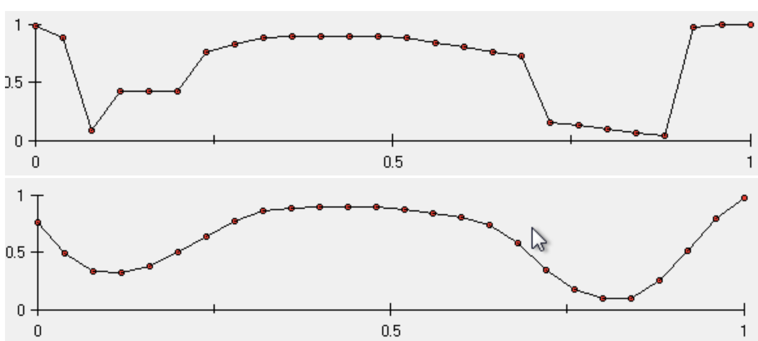

Figure 6. Function Obtained from the Linguistic Description Learned from the Data Determining Function from Figure 5. Up: PbLD with DEE Defuzzification; Down: Smooth PbLD

2.4. **Linguistic Description of a Simple Decision Situation.** Imagine the quite usual situation when approaching a traffic intersection on which the green light is on but suddenly changes to red. We now face the following decision situation: *if we are sufficiently far then we do nothing. If we are near then we break and stop. But if we are very near then it is safer to speed up a little and pass through the intersection as fast as possible.* Such a situation can be easily modeled using fuzzy/linguistic model and the PbLD method. Even more, we can simply monitor the drive's actions and then learn the linguistic description on the basis of the obtained data.

Let us imagine that we obtained data having the form as depicted in Table 2 where the distance is in m, break is represented by negative fraction numbers (0 means no action and 1 full break) and acceleration by positive ones. If we set the linguistic contexts as $w_{dist} = \langle 2, 20, 100\rangle$ and $w_{break} = \langle 0, -0.4, -1\rangle$ and $w_{accel} = \langle 0, 0.4, 1\rangle$ then the resulting learned linguistic description is the following:

	X	$\Rightarrow$	Y	*Comment*
1.	ra sm		ml bi	"accelerate moderately"
2.	qr sm		ra bi	"accelerate"
3.	vr sm		ex bi	"accelerate very much"
4.	ra me		$-$ml bi	"break moderately"
5.	ml me		ze	"do nothing"
6.	vr bi		ze	"do nothing"

(rules 5. and 6. can obviously be joined into one). The driver's behavior according to this linguistic description is depicted in Figure 6. One can see that up to the distance of about 35m no action is taken. Then up to the distance of about 17m, the instruction is "to break". In case that the car is nearer, the instruction changes into "accelerate" with various intensity (the acceleration is higher when being farther from the intersection and is lower in close vicinity of it). This example demonstrates both interpretability of the linguistic description as well as the ability to learn it from data.

2.5. **Linguistic Description of a Multicriteria Decision Situation.** The last example is a simplified version of multicriteria decision-making. Let us suppose that

distance	break/accel.	distance	break/accel.
5	0.6	28	-0.5
10	0.9	30	-0.3
12	1	40	0
18	-1	50	0
22	-0.8	60	0

Table 2. Data Characterizing Driver's Behavior When the Green Changes to Red. Such Data Can be Obtained by Monitoring the Driver's Actions

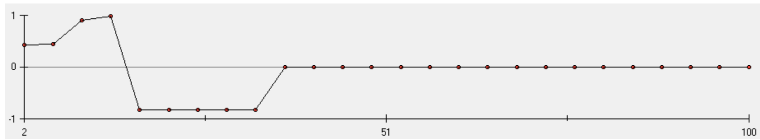

FIGURE 7. Decision Realized on the Basis of the Linguistic Description Learned from the Data Obtained by Monitoring Driver's Behavior When Approaching Intersection with Green Light on That Suddenly Change Into Red. On x-axis is Distance from the Intersection and on y-axis Break (Negative Numbers) or Acceleration (Positive Numbers)

we are to choose from several products, e.g., mobile phones[‡)] on the basis of three criteria: size (cm^2), price ($) and appearance (dimensionless units from $[0, 1]$ where 0 means "ugly" and 1 "beautiful"). These three parameters determine an overall evaluation on a dimensionless scale $[0, 1]$ of the given product. The contexts of the used variables are the following: $w_{size} = \langle 30, 60, 150\rangle$, $w_{price} = \langle 100, 450, 800\rangle$, $w_{appear} = w_{eval} = \langle 0, 0.5, 1\rangle$.

The general evaluation strategy is the following: *we prefer a product that is well looking, i.e., appearance is more important than the other two properties. The second important property is price. Too big product is not really attractive.*

Linguistic description characterizing this strategy can be obtained in two ways. Either it is formed by expert (or the user) explicitly, or we have data at disposal on the basis of which we can learn it. The data should be given in such a way that each principal part of the decision space has a representative and so, the resulting linguistic description can essentially cover the whole space (at the same time, of course, some combinations of values may have no sense and so, they need not be present). Example of such data is in Table 3. These data give rise to the following linguistic description:

No.	size	price	appear	⇒	eval.	No.	size	price	appear	⇒	eval.
1.	ex bi	qr sm	ra sm	⇒	ve sm	12.	vr bi	ra me	ra me	⇒	ra me
2.	ex bi	ra me	ra sm	⇒	ro sm	13.	ml me	ra me	ty me	⇒	ra me
3.	vr bi	ra bi	ra sm	⇒	ro sm	14.	ml me	ra me	ro sm	⇒	ra me
4.	vr bi	ra bi	ty me	⇒	vr sm	15.	ml sm	ra me	ty me	⇒	ty me
5.	ml sm	ml bi	ty me	⇒	vr sm	16.	vr bi	ty me	ra bi	⇒	ra bi
6.	ex bi	ra bi	ra sm	⇒	ra sm	17.	ex bi	ra bi	ra bi	⇒	vr bi
7.	ml me	vr bi	ra sm	⇒	ra sm	18.	vr bi	ra bi	ra bi	⇒	vr bi
8.	ex bi	ra bi	ty me	⇒	ra me	19.	ex bi	ra me	ra bi	⇒	ro bi
9.	ex bi	qr sm	ty me	⇒	ra me	20.	ml me	ml bi	ra bi	⇒	ro bi
10.	vr bi	ty me	ty me	⇒	ra me	21.	ml me	ra me	ex bi	⇒	ex bi
11.	vr bi	vr sm	ro sm	⇒	ra me						

[‡)]Though we consider real products, in our example we abstracted the parameters to no-name phones to avoid reference to real manufacturers.

After more careful inspection of this description, one can see that it indeed fits the above introduced evaluation strategy. For example, according to rules 17, 18 even higher price is not obstacle to evaluation of the given product as good, mainly due to evaluation saying that the product is rather nice-looking. For example, the product with values *size=112, price=720* and *appearance=0.9* would be evaluated by *overall evaluation=0.68* which is *very roughly big*. On the other hand, when considering *price=450*, the *overall evaluation=0.9* which is *rather big*. When changing *appearance=0.48* then the overall evaluation drops to 0.64 which is *more or less medium*.

Of course, the linguistic description should be further tuned to express better the intended evaluation. Example of such a description that has been obtained from the previous one by introducing compound evaluative expressions and modification of some rules is the following:

No.	size	price	appear	⇒ eval.
1.	ex bi	qr sm	ra sm	⇒ ve sm
2.	vr bi	bi or me	sm	⇒ sm
3.	ml sm	ml bi	ty me	⇒ vr sm
4.	vr bi	ml me	vr sm	⇒ vr sm
5.	me or bi	vr bi	sm or me	⇒ ra sm
6.	ml bi or me	me	me	⇒ ty me
7.	vr bi	ty me or ra bi	ty me or ra bi	⇒ ra bi
8.	ml me	ml bi	ra bi	⇒ ro bi
9.	ex bi	ra me	ra bi	⇒ vr bi
10.	ml me	ra me	ex bi	⇒ ex bi

It can be demonstrated that using PbLD, we can obtain results quite well fitting the above outlined evaluation strategy. Note one specific feature: though the decision is influenced by different importance of the respective criteria, unlike most of the standard decision-making methods we do not need to introduce special weights to

size	price	appear.	eval.	size	price	appear.	eval.
150	750	0.1	0.1	100	450	0.9	0.9
150	750	0.5	0.4	100	300	0.2	0.4
150	750	0.9	0.7	100	500	0.6	0.6
150	500	0.1	0.2	100	750	0.9	0.7
150	500	0.5	0.5	50	600	0.1	0.1
150	500	0.9	0.8	50	400	0.5	0.6
150	250	0.1	0.05	50	700	0.9	0.8
150	250	0.5	0.4	50	500	1.0	1.0
100	750	0.1	0.2	50	500	0.2	0.6
100	750	0.5	0.3	40	400	0.5	0.5
100	450	0.5	0.6	40	700	0.5	0.3

TABLE 3. Data for Basic Evaluation of Products. Each Line in a Given Column Corresponds to One Product

the criteria because their importance can be well expressed in the way how the fuzzy/linguistic rules are formed. We thus avoid using an external method for assigning weights. We are convinced that this is a very desirable feature of our approach because any (known to us) method for assigning weights is more or less dubious.

3. Discussion and Conclusion

The aim of this paper was to demonstrate that there exists a well developed theory of fuzzy/linguistic models that are interpretable and can also be learned from the data. We presented four different situations which can be characterized using a linguistic description. The first two cases describe situation in which the course of a function with specific properties is characterized in natural language. Next is a simple decision situation faced by a driver before a traffic intersection with green lights suddenly switching to red. The driver must decide between breaking or passing the intersection through. The last is a multicriteria decision problem in which we are to choose the best product fitting characteristics that are described using natural language. In all examples we demonstrated interpretability of the used linguistic description as well as possibility to learn it from data.

As already noted, the concept of fuzzy/linguistic model has many kinds of applications. The effectiveness of learning was demonstrated in control and forecasting of time series. In the former application, the control actions of a human operator are monitored and then the linguistic description is learned. It was demonstrated on many simulations that the PbLD inference applied to the latter controls the system equally well as the human operator. The interested reader is referred to the book [19] where many examples are given and specific details are explained.

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