AN OBSERVER-BASED INTELLIGENT DECENTRALIZED VARIABLE STRUCTURE CONTROLLER FOR NONLINEAR NON-CANONICAL NON-AFFINE LARGE SCALE SYSTEMS

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This paper is dedicated to Professor L. A. Zadeh on the occasion of his 95th birthday and the 50th year of the birth of fuzzy logic

ABSTRACT. In this paper, an observer based fuzzy adaptive controller (FAC) is designed for a class of large scale systems with non-canonical non-affine nonlinear subsystems. It is assumed that functions of the subsystems and the interactions among subsystems are unknown. By constructing a new class of state observer for each follower, the proposed consensus control method solves the problem of unmeasured states of nonlinear non-canonical non-affine subsystems. The main characteristics of the proposed observer-based intelligent controller are: 1) on-line adaptation of the controller and the observer parameters, 2) ultimate boundedness of both the output and the observer errors, 3) boundedness of all signals involved, 4) employing experts' knowledge in the controller design procedure and 5) chattering avoidance. The simulation results are further carried out to demonstrate better the effectiveness of the proposed fuzzy based consensus controller method.

1. Introduction

To control large scale systems, one usually faces poor knowledge of the plant parameters and interconnections among subsystems. An adaptive control technique can then serve as an appropriate candidate for such applications to be employed. Large scale interconnected systems appear in a variety of engineering applications such as power systems, manufacturing processes, and communication systems.

As a result of both tunable structure of the Fuzzy Adaptive Controller (FAC) and using the experts' knowledge in controller design procedure, FAC attracted many researchers to develop appropriate controllers for nonlinear systems especially for large scale systems (LSS).

In the recent years, FAC has been fully studied. Initially, Takagi-Sugeno (TS) fuzzy systems have been used to model nonlinear systems and then TS based controllers have been designed with guaranteed stability [6,7]. Modeling affine nonlinear systems and designing stable TS fuzzy based controllers have been employed in [14]. Designing of a sliding mode fuzzy adaptive controller for a class of multivariable TS fuzzy systems was presented in [1]. In [11, 25], the non-affine nonlinear function were first approximated by the TS fuzzy systems, and then a stable TS fuzzy controller as well as observer have been designed for the obtained model. In these papers, due to the

Invited Paper: August 2016
Keywords and phrases: Lyapunov stability, Adaptive control, Non-affine nonlinear system, large scale systems, Fuzzy systems, Nonlinear observer.

assumption that the systems should be linearizable around some operating points modeling and designing of appropriate controllers could then be simply done.

The linguistic fuzzy systems have also been used to design controllers for nonlinear systems. [40,34,16,32,44] have considered linguistic fuzzy systems to design stable adaptive controller for affine systems based on feedback linearization and furthermore in [32,44], it has been considered that the zero dynamics should be stable. Stable FAC based on sliding mode was designed for affine systems in [18]. Designing FAC for affine chaotic systems was presented in [29,2]. Designing stable FAC and linear observers for a class of affine nonlinear systems was fully discussed in [35,28,43,13]. Fuzzy adaptive sliding mode controllers were presented for a class of affine nonlinear time delay systems in [42,17,3]. The output feedback FAC for a class of affine nonlinear MIMO systems was suggested in [41].

A robust adaptive fuzzy controller, based on a linear state observer, for a class of affine nonlinear systems has been presented in [12]. In [31], direct and indirect adaptive output-feedback fuzzy decentralized controllers for a class of large-scale affine nonlinear systems have been developed based on linear observers. [36] presented fuzzy adaptive controllers for a class of affine nonlinear systems. This method guarantied ultimately boundedness of tracking error. A direct adaptive fuzzy controller for a non-minimum phase two-axis inverted pendulum servomechanism has been presented based on real-time implementation in [37]. The main drawbacks of these methods are the restricted conditions imposed on the system dynamics. For example, it is assumed that the control gain is bounded to some known functions or constant values.

[19,21] developed stable FAC for a class of non-affine nonlinear systems. The main limitation of these methods is that convergence of tracking errors to zero was not guaranteed. [8,9] proposed a decentralized fuzzy model reference state tracking controller for a class of canonical nonlinear large scale systems. The main limitations of these references are both considering the interaction as a bounded disturbance and availability of all states. [10] dealt with designing FAC based on sliding mode for a class of large scale affine nonlinear systems. [39] designed FAC for a class of affine nonlinear time delayed systems. In none of these studies, fuzzy adaptive controllers were developed for nonlinear non-affine systems.

The basic idea of consensus control is that all subsystems are driven to an agreement by a consensus protocol, which is designed based on local information. In consensus control, two control strategies, leaderless consensus and leader-following consensus, have been extensively developed. The most of the research results were limited to first-order or second-order multi-agent systems [4,24,15,26,46].

Recently, the high-order consensus problem has received obviously increasing attention, and several novel consensus design methods for high-order linear multi-agent systems have been developed [27]. In [27], the authors proposed a class of l-order ($l > 3$) consensus control approaches by generalizing the first-order and second order consensus algorithms. In recent years, the challenging high-order consensus control has been extended to nonlinear multi-agent systems [45]. Intelligent adaptive backstepping technique for a class of nonlinear strict-feedback or semi-strict feedback systems was discussed in [5,38].The observer-based adaptive control has been well developed for a class of both SISO and MIMO nonlinear systems in [33,22,34,23].

In this paper, we propose a new method to design a stable observer based decentralized adaptive controller based on fuzzy systems for a class of large scale non-canonical non-affine nonlinear systems. The main contributions of this paper are:1) on-line adaptation of both the controller and the observer parameters is possible, 2) ultimate boundedness of both the output and the observer error and 3) boundedness of all signals involved are guaranteed, 4) employing experts' knowledge in the controller design procedure and 5) chattering avoidance are fully provided. Compared with the previous studies, which are mainly concentrated on observer-based affine SISO subsystems and observer based affine subsystems, the proposed method does indeed represent an observer based non-canonical non-affine nonlinear subsystem.

The remainder of the paper is organized as follows. Section 2 gives the problem statement. General concepts of the fuzzy system are formulated in Section 3. Design of the proposed fuzzy adaptive controllers and nonlinear observers are fully formulated and discussed in Section 4. Section 5 presents simulation results of the proposed controller and finally, Section 5 concludes the paper.

2.**Preliminaries**

Consider a nonlinear large scale system that consists of N interconnected subsystems with uncertain dynamics. The system model of every subsystem can be described by the following non-affine non-canonical nonlinear dynamics:

$$\begin{cases} \dot{z}_{i,l} = g_{i,l}(\underline{z}_i) \quad l = 1,2,...,n_i - 1 \quad , i = 1,2,...,N \\ \dot{z}_{i,n_i} = g_i(\underline{z}_i, u_i) + \Delta_i(\underline{z}_1, \underline{z}_2, ..., \underline{z}_N) + d_i'(t) \\ y_i = h_i(\underline{z}_i) = h_{i,1}(z_{i,1}) + ... + h_{i,n_i}(z_{i,n_i}) \end{cases} \tag{1}$$

where $z_{i,l}$ declares l^{th} state of the i^{th} subsystem, n_i is number of the states in the i^{th} subsystem, N is the number of the subsystems, $\underline{z}_i = [z_{i,1}, . . . , z_{i,n_i}]^T \in R^{n_i}$ is the state vector of the system which is assumed unavailable for measurement, $u_i \in R$ is the control input, $y_i \in R$ is the subsystem output, $g_{i,l}(\underline{z}_i)$'s and $g_i(\underline{z}_i, u_i)$ are unknown smooth nonlinear functions, $h_{i,1}(z_{i,l})$'s are known, one-to-one and onto, $\Delta_i(\underline{z}_1, \underline{z}_2, ..., \underline{z}_N)$ is an unknown nonlinear interconnection term between subsystems, and $d_i'(t)$ represent bounded disturbances.

To linearize the output mapping of the system given in equation (1), the following transformation is considered.

$$\underline{x}_i = T_i(\underline{z}_i) \equiv [h_{i,1}(z_{i,1}),\ldots,h_{i,n_i}(z_{i,n_i})]^T \tag{2}$$

Use the above transformation and rewrite equation (1) as:

$$\begin{cases} \dot{x}_{i,l} = \left.\dfrac{\partial h_{i,l}(z_{i,l})}{\partial z_{i,l}} g_{i,l}(\underline{z}_i)\right|_{z_i=T_i^{-1}(\underline{x}_i)} \qquad l=1,2,\ldots,n-1 \\ \dot{x}_{i,n_i} = \left.\begin{array}{l} \dfrac{\partial h_{i,n_i}(z_{i,n_i})}{\partial z_{i,n_i}} g_n(\underline{z}_i,u_i) + \dfrac{\partial h_{i,n_i}(z_{i,n_i})}{\partial z_{i,n_i}} \Delta_i(\underline{z}_1,\underline{z}_2,\ldots,\underline{z}_N) \\ + \dfrac{\partial h_{i,n_i}(z_{i,n_i})}{\partial z_{i,n_i}} d_i'(t) \end{array}\right|_{z_i=T_i^{-1}(\underline{x}_i)} \\ y = C_i^T \underline{x}_i \end{cases} \tag{3}$$

This can be further rewritten to the following compact form:

$$\begin{cases} \dot{x}_{i,l} = f_{i,l}(\underline{x}_i) \quad l=1,2,\ldots,n_i-1 \quad ,i=1,2,\ldots,N \\ \dot{x}_{i,n_i} = f_i(\underline{x}_i,u_i) + m_i(\underline{x}_1,\underline{x}_2,\ldots,\underline{x}_N) + d_i(t) \\ y_i = C_i^T \underline{x}_i \end{cases} \tag{4}$$

where

$$\left.\frac{\partial h_{i,l}(z_{i,l})}{\partial z_{i,l}} g_{i,l}(\underline{z}_i)\right|_{z_i=T_i^{-1}(\underline{x}_i)} = f_{i,l}(z),$$

$$\left.\frac{\partial h_{i,n_i}(z_{i,n_i})}{\partial z_{i,n_i}} g_n(\underline{z}_i,u_i)\right|_{z_i=T_i^{-1}(\underline{x}_i)} = f_i(\underline{x}_i,u_i),$$

$$\left.\frac{\partial h_{i,n_i}(z_{i,n_i})}{\partial z_{i,n_i}} d_i'(t)\right|_{z_i=T_i^{-1}(\underline{x}_i)} = d_i(t)\text{, and}$$

$$\left.\frac{\partial h_{i,n_i}(z_{i,n_i})}{\partial z_{i,n_i}} \Delta_i(\underline{z}_1,\underline{z}_2,\ldots,\underline{z}_N)\right|_{z_i=T_i^{-1}(\underline{x}_i)} = m_i(x_1,x_2,\ldots,x_N).$$

The above equation can be revised as

$$\begin{cases} \dot{\underline{x}}_i = A_{i0}\underline{x}_i + \underbrace{(-A_{i0}\underline{x}_i + f_i(\underline{x}))}_{f_i'(\underline{x})} \\ \quad + b_i\left(f_i(\underline{x}_i, u_i) + m_i(\underline{x}_1, \underline{x}_2, ..., \underline{x}_N) + d_i(t)\right) \\ y_i = C_i^T \underline{x}_i \end{cases} \tag{5}$$

where $f_i(\underline{x}_i)$, A_{i0} and b_i are defined below.

$$\mathrm{A}_{\mathrm{i0}} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix} \in R^{n\times n}$$

$$\mathrm{b}_{\mathrm{i}} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix} \in R^{n} \tag{6}$$

$$and \;\; f_i(\underline{x}) = \begin{bmatrix} f_{i,1}(\underline{x}_i) \\ \vdots \\ f_{i,n_i-1}(\underline{x}_i) \\ 0 \end{bmatrix}$$

The control objective is to design an observer based adaptive fuzzy controller for system (1) such that both the tracking error between the leader and the follower and the observer error are to be ultimately bounded while all signals in the closed-loop system remain bounded and all subsystems track the leader.

In this paper, we will make the following assumptions concerning system (1) as well as the desired trajectory (leader) $x_{im}(\mathrm{t})$ stated below.

Assumption 1: without loss of generality, it is assumed that the nonzero function $\mathrm{f_u}(\underline{\mathrm{x}}_\mathrm{i}, \mathrm{u_i}) = \partial \mathrm{f_i}\ (\underline{\mathrm{x}}_\mathrm{i}, \mathrm{u_i}) / \partial \mathrm{u_i}$ satisfies the following conditions:

$$\begin{aligned} & f_u(\underline{x}_i, u_i) \geq f_{min} > 0 \quad \forall(\underline{x}_i, u_i) \in R^{n_i} \times R \\ & \frac{d f_u(\underline{x}_i, u_i)}{dt} \geq f_{dm} \end{aligned} \tag{7}$$

$f_{dm} \in R$ is a known constant parameter and will be defined later. Furthermore, the following controller and observer design can be reconstructed for $f_{min} < 0$ in the same way.

Assumption 2: The desired (leader) trajectory $x_{im}(t)$ is generated by the following desired reference system.

$$\begin{cases} \dot{\underline{x}}_{im} = A_{i0}\underline{x}_{im} + b_i r_i(t) \\ y_{im} = C_i^T \underline{x}_{im} \end{cases} \tag{8}$$

where $r_i(t)$ is the external reference input.

The interactions can be considered as external inputs as functions of the states of the subsystems; thus, they will be bounded by some constant time varying signal, which is in general a function of all the states. To make it more suitable for the proposed controller derivation, the following assumption is also used.

Assumption 3: the interconnection term satisfies the following:

$$\left| m_i(\underline{x}_1, \underline{x}_2, ..., \underline{x}_N) \right| \leq \xi_{i0} + \sum_{j=1}^{N} \xi_{ij} \left(\left| C_i^T \tilde{\underline{e}}_i \right| \right) \left\| \hat{\underline{x}}_j \right\| \tag{9}$$

$\xi_{i0} + \sum_{j=1}^{N} \xi_{ij} \left(\left| C_i^T \tilde{\underline{e}}_i \right| \right) \left\| \hat{\underline{x}}_j \right\|$ is an unknown upper bound of the interaction terms. To use this upper bound in the controller design procedure, we denote $\hat{\zeta}_{ij}$'s as estimations of ζ_{ij} 's that are adaptively tuned.

Assumption 4: the external disturbance satisfies the following property.

$$\left\| d_i(t) \right\|_\infty \leq d_{max} \tag{10}$$

Now consider $\hat{\underline{x}}_i(t)$ as an estimation of $\underline{x}_i(t)$ and write.

$$\begin{aligned} & \underline{e}_i = \underline{x}_{im} - \underline{x}_i = [e_i \;\; \dot{e}_i \;\; ...e_i^{(n-1)}]^T \\ & \hat{\underline{e}}_i = \underline{x}_{im} - \hat{\underline{x}}_i = [\hat{e}_i \;\; \dot{\hat{e}}_i \;\; ...\hat{e}_i^{(n-1)}]^T \\ & \tilde{\underline{e}}_i = \underline{e}_i - \hat{\underline{e}}_i \end{aligned} \tag{11}$$

where $\underline{e}_i$ stands for the tracking error, $\hat{\underline{e}}_i$ presents the observer error and $\tilde{\underline{e}}_i$ stands for the observation error.

Consider the following tracking error vector.

$$\underline{\mathrm{e}}_{\mathrm{i}} = [\mathrm{e}_{\mathrm{i},1}, \mathrm{e}_{\mathrm{i},2}, \ldots, \mathrm{e}_{\mathrm{i},\mathrm{n}_\mathrm{i}}]^{\mathrm{T}} \in R^{\mathrm{n_i}} \tag{12}$$

Taking the derivative of both sides of equation (7) we have

$$\begin{cases} \dot{\underline{\mathrm{e}}}_{\mathrm{i}} = \dot{\underline{\mathrm{x}}}_{\mathrm{im}} - \dot{\underline{\mathrm{x}}}_{\mathrm{i}} = A_{i0}\underline{x}_{im} + b_i r_i(t) - A_{i0}\underline{x}_i - f_i'(\underline{x}) \\ \qquad - b_i\left(f_i(\underline{x}_i, u_i) + m_i(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N) + d_i(t)\right) \\ \underline{\mathrm{e}}_{\mathrm{iy}} = C_i^T \underline{\mathrm{e}}_{\mathrm{i}} \end{cases} \tag{13}$$

Use equation (6) and rewrite the above equation as:

$$\begin{cases} \dot{\underline{\mathrm{e}}}_{\mathrm{i}} = \mathrm{A}_{\mathrm{i0}}\underline{\mathrm{e}}_{\mathrm{i}} - f_i'(\underline{x}) + \mathrm{b}_{\mathrm{i}}\{r_i(t) - f_i(\underline{x}_i, u_i) \\ \qquad - m_i(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N) - d_i(t)\} \\ \underline{\mathrm{e}}_{\mathrm{iy}} = C_i^T \underline{\mathrm{e}}_{\mathrm{i}} \end{cases} \tag{14}$$

To construct the controller, the dummy variable v_i is defined as:

$$\mathrm{v}_{\mathrm{i}} = r_i(t) + \mathrm{k}_{\mathrm{ic}}^{\mathrm{T}}\hat{\underline{\mathrm{e}}}_{\mathrm{i}} + v_i' \tag{16 }$$

The term v_i' is defined below. Consider the vector $\mathrm{k}_{\mathrm{ic}} = [\mathrm{k}_{\mathrm{i},1}, \mathrm{k}_{\mathrm{i},2}, \ldots, \mathrm{k}_{\mathrm{i},\mathrm{n}_\mathrm{i}}]^{\mathrm{T}}$ as the coefficients of the polynomial $\psi(s) = s^{n_i} + \mathrm{k}_{\mathrm{i},\mathrm{n}_\mathrm{i}}\mathrm{s}^{\mathrm{n_i}-1} + \ldots + \mathrm{k}_{\mathrm{i},1}$, which are chosen so that the roots of this polynomial are appropriately located in the open left-half plane. This makes the matrix $\mathrm{A}_{\mathrm{ioc}} = \mathrm{A}_{\mathrm{i0}} - \mathrm{b}_{\mathrm{i}}\mathrm{k}_{\mathrm{ic}}^{\mathrm{T}}$ Hurwitz.

By adding and subtracting the term $\left(\mathrm{k}_{\mathrm{ic}}^{\mathrm{T}}\hat{\underline{\mathrm{e}}}_{\mathrm{i}} + v_i'\right)$ from the right-hand side of equation (14), we obtain

$$\begin{cases} \dot{\underline{\mathrm{e}}}_{i} = \mathrm{A}_{\mathrm{i0}}\underline{\mathrm{e}}_{i} - \mathrm{b}_i\mathrm{k}_{ic}^{\mathrm{T}}\hat{\underline{\mathrm{e}}}_{i} - f_i'(\underline{x}) - \mathrm{b}_i\{f_i(\underline{x}_i, u_i) \\ \qquad - v_i + m_i(\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_N) + d_i(t) + v_i'\} \\ \mathrm{e}_{\mathrm{iy}} = \mathrm{C}_{\mathrm{i}}^{\mathrm{T}}\underline{\mathrm{e}}_{i} \end{cases} \tag{17 }$$

Using assumption (1), equation (16) and the signal v_i, which is not explicitly dependent on the control input u_i, the following inequality is easily satisfied:

$$\frac{\partial(f_i(\underline{x}_i,u_i)-v_i)}{\partial u_i}=\frac{\partial f_i(\underline{x}_i,u_i)}{\partial u_i}>0 \tag{18}$$

Invoking the implicit function theorem, it is obvious that the nonlinear algebraic equation $f_i(\underline{x}_i,u_i)-v_i=0$ is locally soluble for the input u_i for an arbitrary $(\underline{x}_i, v_i)$. Thus, there exists some ideal controller $u_i^*(\underline{x}_i, v_i)$ satisfying the following equality for a given $(\underline{x}_i, v_i)\in R^{n_i}\times R$:

$$f_i(\underline{x}_i,u_i^*)-v_i=0 \tag{19}$$

As a result of the mean value theorem, there exists a constant λ in the range $0<\lambda<1$, such that the nonlinear function $f_i(\underline{x}_i, u_i)$ can be expressed around u_i^* as:

$$\begin{aligned} f_i(\underline{x}_i, u_i) &= f_i(\underline{x}_i, u_i^*) + (u_i-u_i^*)f_{iu_\lambda} \\ &= f_i(\underline{x}_i, u_i^*) + e_{u_i} f_{iu_\lambda} \end{aligned} \tag{20}$$

where $f_{iu_\lambda}=\partial f_i\ (\underline{x}_i, u_i)/\partial u_i|_{u_i=u_{i\lambda}}$ and $u_{i\lambda}=\lambda u_i+(1-\lambda)u_i^*$.

Substituting equation (20) into the error equation (17) and using (19), we get

$$\begin{cases} \dot{\underline{e}}_i = A_{i0}\underline{e}_i - b_i k_{ic}^T \hat{\underline{e}}_i - f_i'(\underline{x}) - b_i\{e_{iu} f_{iu_\lambda} \\ \quad + m_i(\underline{x}_1,\underline{x}_2,\dots,\underline{x}_N)+d_i(t)+v_i'\} \\ e_{iy}=C_i^T\underline{e}_i \end{cases} \tag{21}$$

However, the implicit function theory only guarantees the existence of the ideal controller $u_i^*(\underline{x}_i,v_i)$ for system (19), and does not provided a technique for constructing any solution even if the dynamics of the system are well known. In the following, a fuzzy system and a classical controller will be used to represent the unknown ideal controller.

3.**Fuzzy Systems**

Figure 1 shows the basic configuration of the fuzzy system considered in this paper. Here, we consider a multi-input, single-output fuzzy systems: $x\in U\subset R^n \rightarrow y\in V\subset R$. Knowing that a multi-output system can be viewed as a group of single-output systems, from now on we consider single output fuzzy systems.

The fuzzifier performs a mapping from a crisp input vector $x=[x_1,x_2,\dots,x_n]^T$ to a fuzzy set, where the label of the fuzzy sets are such as "small", "medium", "large", etc.

The fuzzy rule base is consisted of a collection of fuzzy IF-THEN rules. Assume that there are M rules, and the lth rule is

$$R^l(u): \; if\,(x_1\; is\; A_1^l\; \dots\; x_n\; is\; A_1^l)\; then\,(y\; is\; B^l)\;\; l=1,2,\dots,M \quad (22)$$

where $x=[x_1,x_2,\dots,x_n]^T$ and y are respectively the crisp input and output of the fuzzy system, A_j^l and B^l are fuzzy membership functions in U_j and V, respectively.

The fuzzy inference performs a mapping from fuzzy sets in U to fuzzy sets in V, based on the fuzzy IF-THEN rules in the fuzzy rule base.

The defuzzifier maps fuzzy sets in V to a crisp value in V. The configuration of Figure 1 represents a general framework of fuzzy systems noting that many different choices are allowed for each block in Figure 1, and various combinations of these choices will construct different fuzzy systems [4]. Here, we use the sum-product inference and the center-average defuzzifier. Therefore, the fuzzy system output can be expressed as

$$y(x)=\frac{\sum_{l=1}^{M} y^l \prod_{i=1}^{n} \mu_{A_i^l}(x_i)}{\sum_{l=1}^{M}\prod_{i=1}^{n}\mu_{A_i^l}(x_i)} \quad (23)$$

where $\mu_{A_i^l}(x_i)$ is the membership degree of the input x_i to the fuzzy set A_j^l and y^l is the point at which the membership function of fuzzy set B^l achieves its maximum value.

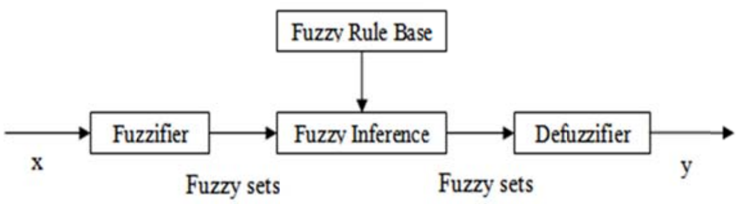

FIGURE 1. Configuration of Fuzzy System

The fuzzy systems in the form of (23) are proven in [24] to be a universal approximator if their parameters are properly chosen.

Theorem 3.1.[4] Suppose $f(x)$ is a continuous function on a compact set U. Then, for any $\varepsilon>0$, there exists a fuzzy system like (23) satisfying:

$$\sup_{x \in U} \left| f(x) - y(x) \right| \le \varepsilon \tag{24}$$

The output given by (23) can be rewritten in the following compact form:

$$y(x) = w(x)^T \theta \tag{25}$$

where $\theta = \left[y^1 \; y^2 \; \dots \; y^M \right]$ is a vector grouping all consequent parameters, and $w(x) = [w_1(x) \; w_2(x) \; \dots \; w_M(x)]^T$ is a set of normalized basis functions defined as:

$$w_i(x) = \frac{\prod_{i=1}^{n} \mu_{A_i^l}(x_i)}{\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{A_i^l}(x_i)} \tag{26}$$

Obviously the denominator term $\sum_{l=1}^{M} \prod_{i=1}^{n} \mu_{A_i^l}(x_i)$ must different from zero for all $x \in U$ in order to make the fuzzy system (23) well defined.

4.**Observer Based Fuzzy Adaptive Controller Design**

In Section 2, it has been shown that there exists an ideal control signal for achieving the control objectives. In this section, we show how to develop a fuzzy system to adaptively approximate the unknown ideal controller. Indeed, this section deals with the observer and controller design procedure.

To design a proper observer for the subsystem given in equation (21), this paper proposes the following observer error system:

$$\begin{cases} \dot{\underline{\hat{e}}}_i = \underbrace{(A_{i0} - b_i k_{ic}^T)}_{A_{ioc}} \underline{\hat{e}}_i + K_{i0} C_i^T \underline{\tilde{e}} + b_i k_{ino}(\underline{\tilde{e}}_i, \underline{\hat{e}}_i) \left| C_i^T \underline{\tilde{e}}_i \right| \\ \hat{e}_{iy} = C_i^T \underline{\hat{e}}_i \end{cases} \tag{27}$$

where K_{io}, k_{ino} are the linear observer gain and the nonlinear observer gain, respectively. K_o is selected to make sure that the characteristic polynomial of $(A_{ioo} = A_{io} - K_{i0} C_i^T)$ is Hurwitz.

Defining the observation error $\underline{\tilde{e}}_i = \underline{e}_i - \underline{\hat{e}}_i$ and subtract (16) from (22) to obtain

$$\begin{cases} \dot{\underline{\tilde{e}}}_i = \underbrace{(A_{i0}\text{-}K_{i0}C_i^T)}_{A_{oo}}\underline{\tilde{e}}_i - f_i'(\underline{x}) - b_i\{e_{iu}f_{iu_\lambda} + m_i(\underline{x}_1,\underline{x}_2,...,\underline{x}_N) \\ \quad + d_i(t) + v_i' + k_{ino}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)\left|C_i^T\underline{\tilde{e}}_i\right|\} \\ \tilde{e}_{iy} = C_i^T\underline{\tilde{e}}_i \end{cases} \tag{28}$$

The output error dynamics of the above equation can be given as:

$$\begin{aligned} \tilde{e}_{iy} = H_i(s)\{f_i'(\underline{x}) + b_i\{e_{iu}f_{iu_\lambda} + m_i(\underline{x}_1,\underline{x}_2,...,\underline{x}_N) \\ + d_i(t) + v_i' + k_{ino}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)\left|C_i^T\underline{\tilde{e}}_i\right|\}\} \end{aligned} \tag{29}$$

where

$$H_i(s) = -C_i^T\left(sI\text{-}(A_{i0}\text{-}K_{i0}C_i^T)\right)^{-1}B_i \tag{30}$$

and B_i is the identity matrix, $H_i(s)$ is a known stable transfer function. In order to use the SPR-Lyapunov design approach, equation (29) is rewritten as

$$\begin{aligned} \tilde{e}_{iy} = H_i(s)L_i(s)\{f_{if}'(\underline{x}) + b_i\{e_{iu}f_{iu_\lambda} + m_i(\underline{x}_1,\underline{x}_2,...,\underline{x}_N) \\ + d_{if}(t) + v_{if}' + k_{inof}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)\left|C_i^T\underline{\tilde{e}}_i\right|\}\} \end{aligned} \tag{31}$$

where $f_{if}'(\underline{x}_i) = f_i'(\underline{x}_i)$, $f_{iu_\lambda f} = L_i(s)^{-1}f_{iu_\lambda}$, $k_{inof} = L_i^{-1}(s)k_{ino}$, $v_{if}' = L_i^{-1}(s)v_i'$, $d_{if}(t) = L_i(s)^{-1}d_i(t)$. $L_i(s)$ is chosen so that $L_i^{-1}(s)$ is a proper stable transfer function and $H_i(s)L_i(s)$ is a proper strictly-positive-real (SPR) transfer function. Let $L_i(s) = s^m + b_{i1}s^{m-1} + \cdots + b_{im}$ with $m = n - 1$.

The state-space realization of (31) can be written as

$$\begin{cases} \dot{\underline{\tilde{e}}}_{is} = A_{ioo}\underline{\tilde{e}}_{is} - B_{is}f_{if}'(\underline{x}) - b_{is}\{e_{iu}f_{iu_{\lambda f}} + d_{if}(t) \\ \quad + m_{if}(\underline{x}_1,\underline{x}_2,...,\underline{x}_N) + v_{if}' + k_{inof}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)\left|C_i^T\underline{\tilde{e}}_{is}\right|\} \\ \tilde{e}_{iy} = C_{is}^T\tilde{e}_{is} \end{cases} \tag{32}$$

The ideal controller can be represented as:

$$u_i^* = f_i(\underline{z}_i) + \varepsilon_{iu} \tag{33}$$

where $z_i = [x_i, v_i]^T$ and $f_i(\underline{z}) = \theta_{i1}^{*} w_{i1}(\underline{z})$, and θ_{i1}^{*} and $w_{i1}(\underline{z}_i)$ are consequent parameters and a set of normalized basis functions, respectively. ε_{iu} is an approximation error that satisfies $|\varepsilon_{iu}| \le \varepsilon_{\max}$ and $\varepsilon_{\max} > 0$. The parameters θ_{i1}^{*} are determined through the following optimization.

$$\theta_{i1}^{*} = \underset{\theta_{i1}}{\arg\min}\left[\sup\left|\theta_{i1}^{T} w_{i1}(\underline{z}) - f_i(\underline{z})\right|\right] \tag{34}$$

Denoting the estimate of θ_{i1}^{*} as θ_{i1} and u_{irob} as a robust controller to compensate for the approximation error, uncertainties, disturbance and the interconnection term,and defining $\xi_{ij}\left(\left|C_i^T \underline{\tilde{e}}_i\right|\right)$ by $\eta_{ji}\left\|C_i^T \underline{\tilde{e}}_i\right\|$, we rewrite the controller given in (33) as:

$$u_i = \theta_{i1}^{T} w_{i1}(\underline{z}_i) + u_{irob} + \underline{\hat{e}}_i^{T} P_{i2} \mathrm{K}_{i0} \tag{35}$$

where u_{irob} is defined by

$$u_{irob} = sign\left(C_{is}^T \underline{\tilde{e}}_{is}\right)\begin{pmatrix} \dfrac{N}{2 f_{\min}}\left|C_{is}^T \underline{\tilde{e}}_{is}\right| + \dfrac{\hat{\xi}_{i0}}{f_{\min}} \\ + \dfrac{1}{2 f_{\min}}\sum_{j=1}^{N}\hat{\eta}_{ij}\left\|\underline{\hat{x}}_i\right\|^2\left|C_{is}^T \underline{\tilde{e}}_{is}\right| + u_{icom} + \dfrac{u_{ir}}{f_{\min}} \\ + \dfrac{\hat{v}_i'}{f_{\min}} + \hat{k}_{inof}(\underline{\tilde{e}}_i, \underline{\hat{e}}_i)\left(\dfrac{\left|C_{is}^T \underline{\tilde{e}}_{is}\right|}{f_{\min}} + \left|\underline{\hat{e}}_i^T P_{i1} b_i\right|\right)\end{pmatrix} \tag{36}$$

In the above, $\theta_{i1}^{T} w_{i1}(\underline{z})$ approximates the ideal controller, $\hat{\xi}_{i0} + \frac{1}{2}\sum_{j=1}^{N}\hat{\eta}_{ij}\left\|\underline{\hat{x}}_j\right\|^2\left|C_{is}^T \underline{\tilde{e}}_{is}\right|$ represents the estimate of the interconnection term, u_{icom} compensates for the approximation error and uncertainties, u_{ir} is designed to compensate for bounded external disturbances, the term $\hat{k}_{inof}(\underline{\tilde{e}}_i, \underline{\hat{e}}_i)(\left|C_{is}^T \underline{\tilde{e}}_{is}\right| / f_{\min} + \left|\underline{\hat{e}}_i^T P_{i1} b_i\right|))$ estimates the nonlinear gain of the observer, and $\hat{v}_i'$ is the estimate of v_i'.

Define the error vector $\tilde{\theta}_{i1} = \theta_{i1} - \theta_{i1}^{*}$ and use (35) and (36) to rewrite the error equation (21) as:

$$\begin{cases} \dot{\underline{e}}_i = A_{i0}\underline{e}_i - b_i k_{ic}^T \hat{\underline{e}}_i - f_i'(\underline{x}) - b_i\{(\tilde{\theta}_{i1}^T w_{i1}(\underline{z}_i) + u_{irob} \\ \quad -\varepsilon_{iu}) f_{iu_\lambda} + m_i(\underline{x}_1, \underline{x}_2, ..., \underline{x}_N) + d_i(t) + v_i'\} \\ e_{iy} = C_i^T \underline{e}_i \end{cases} \tag{37}$$

Based on equation (35) and (36), the state-space realization of equation (32) can be written as

$$\begin{cases} \dot{\tilde{\underline{e}}}_{is} = A_{ioo}\tilde{\underline{e}}_{is} - B_{is} f_{if}'(\underline{x}) - b_{is}\{(\tilde{\theta}_{i1}^T w_{i1}(\underline{z}_i) + u_{irob} \\ \quad -\varepsilon_{iu}) f_{iu_{\lambda f}} + d_{if}(t) + v_{if}' + k_{inof}(\tilde{\underline{e}}_i, \hat{\underline{e}}_i)\left|C_i^T \tilde{\underline{e}}_i\right|\} \\ \tilde{e}_{iy} = C_{is}^T \tilde{\underline{e}}_{is} \end{cases} \tag{38}$$

Assume that P_{i1} and P_{i2} are positive definite solutions of the following matrix Riccati equations, respectively.

$$\begin{aligned} A_{ioo}^T P_{i1} + P_{i1} A_{ioo} &= -Q_{i1} \\ A_{ioc}^T P_{i2} + P_{i2} A_{ioc} &= -Q_{i2} \\ b_{is}^T P_{i1} &= C_{is}^T \end{aligned} \tag{39}$$

In equations (39), Q_{i1} and Q_{i2} are the given positive definite matrices.

Consider the following update laws.

$$\begin{aligned} \dot{\hat{k}}_{ino} &= \gamma_{iko}\left(\frac{\left|C_{is}^T \tilde{\underline{e}}_{is}\right|^2}{f_{\min}} + \left|C_{is}^T \tilde{\underline{e}}_{is}\right|\left|b_i^T P_{i2} \hat{\underline{e}}_i\right|\right) \\ \dot{\theta}_{i1} &= \Gamma_1 C_{is}^T \tilde{\underline{e}}_{is} w_{i1}(\underline{z}_i) \\ \dot{\hat{\xi}}_{i0} &= \frac{\gamma_{\xi_{i0}}}{f_{\min}}\left|C_{is}^T \tilde{\underline{e}}_{is}\right| \\ \dot{\hat{\eta}}_{ji} &= \frac{\gamma_{\eta_{ji}}}{2 f_{\min}}\left|C_{is}^T \tilde{\underline{e}}_{is}\right|\left\|\hat{\underline{x}}_i\right\|^2 \\ \dot{u}_{ir} &= \gamma_{u_{ir}}\left|C_{is}^T \tilde{\underline{e}}_{is}\right| \\ \dot{u}_{icom} &= \gamma_{u_{icom}}\left|C_{is}^T \tilde{\underline{e}}_{is}\right| \\ \dot{\hat{v}}_i' &= \gamma_{\hat{v}_i'}\left|C_{is}^T \tilde{\underline{e}}_{is}\right| \end{aligned} \tag{40}$$

where $\Gamma_1 = \Gamma_1^T > 0, \gamma_{u_{ir}}, \gamma_{\eta_{ji}}, \gamma_{u_{icom}}, \gamma_{\hat{v}_i'}, \gamma_{iko} > 0$ are constant parameters properly defined.

Let's $\lambda_{\max(.)}$ and $\sigma_{\max}(.)$ be the maximum eigenvalue and maximum singular value, respectively.

Lemma 4.1. The following inequality holds if $\lambda_{\max}(Q_{i1}) \geq -\,{f_{d\,\mathrm{m}}}/{f_{\min}}\,\lambda_{\max}(P_{i1})$.

$$\frac{1}{f_{iu_{\lambda f}}}\,\underline{\tilde{e}}_{is}^T Q_{i1}\underline{\tilde{e}}_{is} + \frac{\dot{f}_{iu_{\lambda f}}}{f_{iu_{\lambda f}}^2}\,\underline{\tilde{e}}_{is}^T P_{i1}\underline{\tilde{e}}_{is} \geq 0 \tag{41}$$

Proof:

From assumption (1) and using the fact that $\lambda_{\max}(Q_{i1}) \geq -\,{f_{d\,\mathrm{m}}}/{f_{\min}}\,\lambda_{\max}(P_{i1})$, obviously we can have

$$\left(\lambda_{\max}(Q_{i1})\, f_{\min} + \lambda_{\max}(P_{i1})\, f_{d\,\mathrm{m}}\right) \geq 0 \tag{42}$$

This in turn leads to the following inequality.

$$\frac{1}{f_{iu_{\lambda}}^2}\left(\lambda_{\max}(Q_{i1})\, f_{\min} + \lambda_{\max}(P_{i1})\, f_{d\,\mathrm{m}}\right)\left\|\underline{\tilde{e}}_{is}\right\|^2 \geq 0 \tag{43}$$

After some algebraic manipulations, the following inequality is obtained.

$$\frac{1}{f_{iu_{\lambda f}}}\,\underline{\tilde{e}}_{is}^T Q_{i1}\underline{\tilde{e}}_{is} + \frac{\dot{f}_{iu_{\lambda f}}}{f_{iu_{\lambda f}}^2}\,\underline{\tilde{e}}_{is}^T P_{i1}\underline{\tilde{e}}_{is} \geq \frac{1}{f_{iu_{\lambda f}}^2}\begin{pmatrix} \lambda_{\max}(Q_{i1})\, f_{\min}\left\|\underline{\tilde{e}}_{is}\right\|^2 \\ +\lambda_{\max}(P_{i1})\, f_{d\,\mathrm{m}}\left\|\underline{\tilde{e}}_{is}\right\|^2 \end{pmatrix} \tag{44}$$

And finally use (42) to have the following, which completes the proof.

$$\frac{1}{f_{iu_{\lambda f}}}\,\underline{\tilde{e}}_{is}^T Q_{i1}\underline{\tilde{e}}_{is} + \frac{\dot{f}_{iu_{\lambda f}}}{f_{iu_{\lambda f}}^2}\,\underline{\tilde{e}}_{is}^T P_{i1}\underline{\tilde{e}}_{is} \geq 0 \tag{45}$$

Q.E.D.

Lemma 4. 2. Based on lemma (4.1) and equation (42), the following inequality holds.

$$\sigma_{\max}(A_{ioo}) \leq -\,{f_{dm}}/{2 f_{\min}} \tag{46}$$

Proof. using equation (42) and after some algebraic manipulations, the following inequality is obtained.

$$\|Q_{i1}\| \le \|A_{ioo}^T P_{i1}\| + \|P_{i1}A_{ioo}\| = 2\|P_{ioo}A_{ioo}\| \tag{47}$$

Using the above equation, we get

$$\|Q_{i1}\| \le 2\|P_{i1}\|\|A_{ioo}\| = 2\lambda_{\max}(P_{i1})\sigma_{\max}(A_{ioo}) \tag{48}$$

Use (42) and (48) to have the following; this completes the proof.

$$\sigma_{\max}(A_{ioo}) \le -f_{dm}\big/2f_{\min} \tag{49}$$

Q.E.D.

Theorem 4.3. Consider the error dynamical system given in (27) and (38) for the large scale system (1) satisfying assumption (1), the interconnection term satisfying assumption (3), the external disturbances satisfying assumption (4), and a desired trajectory satisfying assumption (2), then the controller structure given in (35), (36) with adaptation laws (40) makes the tracking error and the observer error ultimately bounded and all signals in the closed loop system bounded as well.

Proof. consider the following Lyapunov function.

$$V = \sum_{i=1}^{N} \frac{1}{2} \left(\begin{array}{l} \dfrac{1}{f_{iu_{\lambda f}}} \underline{\tilde{e}}_{is}^T P_{i1} \underline{\tilde{e}}_{is} + \underline{\hat{e}}_i^T P_{i2} \underline{\hat{e}}_i + \tilde{\theta}_{i1}^T \Gamma_1^{-1} \tilde{\theta}_{i1} \\ + \dfrac{\tilde{\xi}_{i0}^2}{\gamma_{\xi_{i0}}} + \dfrac{\sum_{j=1}^{N} \tilde{\eta}_{ji}^2}{\gamma_{\eta_{ji}}} + \dfrac{\tilde{u}_{ir}^2}{\gamma_{u_{ir}}} + \dfrac{\tilde{u}_{icom}^2}{\gamma_{u_{icom}}} + \dfrac{\tilde{k}_{ino}^2}{\gamma_{iko}} + \dfrac{\tilde{v}_i'^2}{\gamma_{\hat{v}_i'}} \end{array} \right) \tag{50}$$

where $\tilde{\theta}_{i1} = \theta_{i1} - \theta_{i1}^*$, $\tilde{u}_{ir} = u_{ir} - d_{\max}$, $\tilde{u}_{icom} = u_{icom} - \varepsilon_{\max}$, $\tilde{k}_{ino} = \hat{k}_{ino} - k_{ino}$, $\tilde{\eta}_{ji} = \hat{\eta}_{ji} - \eta_{ji}$, $\tilde{\xi}_{i0} = \hat{\xi}_{i0} - \xi_{i0}$ and $\tilde{v}_i' = \hat{v}_i' - |v_i'|$. The time derivative of the Lyapunov function becomes.

$$\dot{V} = \sum_{i=1}^{N} \frac{1}{2} \left(\frac{1}{f_{iu_{\lambda f}}} \dot{\underline{\tilde{e}}}_{is}^T P_{i1} \underline{\tilde{e}}_{is} + \frac{1}{f_{iu_{\lambda f}}} \underline{\tilde{e}}_{is}^T P_{i1} \dot{\underline{\tilde{e}}}_{is} + \frac{\dot{f}_{iu_{\lambda f}}}{f_{iu_{\lambda f}}^2} \underline{\tilde{e}}_{is}^T P_{i1} \underline{\tilde{e}}_{is} \right)$$

$$+ \frac{1}{2}\left(\dot{\underline{\hat{e}}}_i^T P_{i2} \underline{\hat{e}}_i + \underline{\hat{e}}_i^T P_{i2} \dot{\underline{\hat{e}}}_i \right) + \tilde{\theta}_{i1}^T \Gamma_1^{-1} \dot{\theta}_{i1} + \frac{\tilde{\xi}_{i0} \dot{\hat{\xi}}_{i0}}{\gamma_{\xi_{i0}}} + \frac{\sum_{j=1}^{N} \tilde{\eta}_{ji} \dot{\hat{\eta}}_{ji}}{\gamma_{\eta_{ji}}}$$

$$+\frac{\tilde{u}_{ir}\dot{u}_{ir}}{\gamma_{u_{ir}}}+\frac{\tilde{u}_{icom}\dot{u}_{icom}}{\gamma_{u_{icom}}}+\frac{\tilde{v}_i'\dot{\hat{v}}_i'}{\gamma_{\hat{v}_i'}}+\frac{\tilde{k}_{no}\dot{\hat{k}}_{no}}{\gamma_{ko}} \tag{51}$$

Use equations (27) and (38) to rewrite the above equation as:

$$\begin{aligned}\dot{V}=\sum_{i=1}^{N}\ &\frac{1}{2}\frac{1}{f_{iu_{\lambda f}}}(A_{ioo}\underline{\tilde{e}}_{is}-B_{is}f_{if}'(\underline{x})\text{-b}_{\text{is}}\{(\tilde{\theta}_{i1}^{T}w_{i1}(\underline{z}_i)\\ &+u_{irob}-\varepsilon_{iu})f_{iu_{\lambda f}}+d_{if}(t)+m_i(\underline{x}_1,\underline{x}_2,...,\underline{x}_N)+v_{if}'\\ &+k_{inof}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|\})^{T}P_{i1}\underline{\tilde{e}}_{is}+\frac{1}{2f_{iu_{\lambda f}}}\underline{\tilde{e}}_{is}^{T}P_{i1}(A_{ioo}\underline{\tilde{e}}_{is}\\ &-B_{is}f_{if}'(\underline{x})\text{-b}_{\text{is}}\{(\tilde{\theta}_{i1}^{T}w_{i1}(\underline{z}_i)+u_{irob}-\varepsilon_{iu})f_{iu_{\lambda f}}+d_{if}(t)\\ &+m_{if}(\underline{x}_1,\underline{x}_2,...,\underline{x}_N)+v_{if}'+k_{inof}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|\})\\ &+\frac{\dot{f}_{iu_{\lambda f}}}{2f_{iu_{\lambda f}}^{2}}\underline{\tilde{e}}_{is}^{T}P_{i1}\underline{\tilde{e}}_{is}+\frac{1}{2}\left(\text{A}_{\text{ioc}}\underline{\hat{\text{e}}}_i+\text{K}_{\text{i0}}C_{is}^{T}\underline{\tilde{e}}_{is}+b_ik_{ino}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|\right)^{T}P_{i2}\underline{\hat{e}}_i\\ &+\frac{1}{2}\underline{\hat{e}}_i^{T}P_{i2}\left(\text{A}_{\text{ioc}}\underline{\hat{\text{e}}}_i+\text{K}_{\text{i0}}C_{is}^{T}\underline{\tilde{e}}_{is}+b_ik_{ino}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|\right)\\ &+\tilde{\theta}_{i1}^{T}\Gamma_1^{-1}\dot{\theta}_{i1}+\frac{\tilde{\xi}_{i0}\dot{\hat{\xi}}_{i0}}{\gamma_{\xi_{i0}}}+\frac{\sum_{j=1}^{N}\tilde{\eta}_{ji}\dot{\hat{\eta}}_{ji}}{\gamma_{\eta_{ji}}}+\frac{\tilde{u}_{ir}\dot{u}_{ir}}{\gamma_{u_{ir}}}+\frac{\tilde{u}_{icom}\dot{u}_{icom}}{\gamma_{u_{icom}}}+\frac{\tilde{v}_i'\dot{\hat{v}}_i'}{\gamma_{\hat{v}_i'}}+\frac{\tilde{k}_{no}\dot{\hat{k}}_{no}}{\gamma_{ko}}\end{aligned} \tag{52}$$

Using assumption (1) yields $1/f_{iu_\lambda}\le 1/f_{\min}$ and by assumptions (3), (4) and equation (39), we can rewrite (52) as:

$$\begin{aligned}\dot{V}\le\sum_{i=1}^{N}\ &\frac{1}{2f_{iu_{\lambda f}}}\underline{\tilde{e}}_{is}^{T}\underbrace{\left(A_{ioo}^{T}P_{i1}+P_{i1}A_{ioo}\right)}_{-Q_{i1}}\underline{\tilde{e}}_{is}-\frac{\dot{f}_{iu_{\lambda f}}}{2f_{iu_{\lambda f}}^{2}}\underline{\tilde{e}}_{is}^{T}P_{i1}\underline{\tilde{e}}_{is}\\ &+\frac{1}{f_{\min}}\left\|f_i'^{T}(\underline{x})\right\|\left\|B_{is}^{T}P_{i1}\underline{\tilde{e}}_{is}\right\|\\ &+\frac{1}{2}\underline{\hat{e}}_i^{T}\underbrace{\left(\text{A}_{\text{ioc}}^{\text{T}}P_{i2}+P_{i2}\text{A}_{\text{ioc}}\right)}_{-Q_{i2}}\underline{\hat{e}}_i\text{-}(\tilde{\theta}_{i1}^{T}w_{i1}(\underline{z}_i)+u_{irob}-\varepsilon_{iu})C_{is}^{T}\underline{\tilde{e}}_{is}\end{aligned}$$

$$+\frac{d_{\max}}{f_{\min}}\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|+\frac{\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|}{f_{\min}}\left(\xi_{i0}+\sum\nolimits_{j=1}^{N}\xi_{ij}\left\|\underline{\hat{x}}_{j}\right\|\right)+\frac{\left|v_i'\right|}{f_{\min}}\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|$$

$$+k_{ino}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)\frac{\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|^2}{f_{\min}}+\underline{\hat{e}}_i^{T}P_{i2}\mathrm{K}_{i0}C_{is}^{T}\underline{\tilde{e}}_{is}+k_{ino}(\underline{\tilde{e}}_i,\underline{\hat{e}}_i)\underline{\hat{e}}_i^{T}P_{i2}b_i\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right| \tag{53}$$

$$+\tilde{\theta}_{i1}^{T}\Gamma_1^{-1}\dot{\theta}_{i1}+\frac{\tilde{\xi}_{i0}\dot{\hat{\xi}}_{i0}}{\gamma_{\xi_{i0}}}+\frac{\sum_{j=1}^{N}\tilde{\eta}_{ji}\dot{\hat{\eta}}_{ji}}{\gamma_{\eta_{ji}}}+\frac{\tilde{u}_{ir}\dot{u}_{ir}}{\gamma_{u_{ir}}}+\frac{\tilde{u}_{icom}\dot{u}_{icom}}{\gamma_{u_{icom}}}+\frac{\tilde{v}_i'\dot{\hat{v}}_i'}{\gamma_{\hat{v}_i'}}+\frac{\tilde{k}_{no}\dot{\hat{k}}_{no}}{\gamma_{ko}}$$

Using the sigmoid properties, equations (35)-(36), and the fact $\alpha^2+\beta^2\geq 2\alpha\beta$, equation (53) can be rewritten as given below.

$$\dot{V}\leq\sum_{i=1}^{N}\ \frac{-1}{2f_{iu_{\lambda f}}}\underline{\tilde{e}}_{is}^{T}Q_{i1}\underline{\tilde{e}}_{is}-\frac{\dot{f}_{iu_{\lambda f}}}{2f_{iu_{\lambda f}}^{2}}\underline{\tilde{e}}_{is}^{T}P_{i1}\underline{\tilde{e}}_{is}+\frac{1}{2}\underline{\hat{e}}_i^{T}Q_{i2}\underline{\hat{e}}_i$$

$$-\frac{\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|}{f_{\min}}\underbrace{\left(\hat{v}_i'-\left|v_i'\right|\right)}_{\tilde{v}_i'}-\tilde{\theta}_{i1}^{T}w_{i1}(\underline{z}_i)C_{is}^{T}\underline{\tilde{e}}_{is}-\frac{\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|^2\left\|\underline{\hat{x}}_i\right\|^2}{2f_{\min}}\sum\nolimits_{j=1}^{N}\underbrace{\left(\hat{\eta}_{ji}-\eta_{ji}\right)}_{\tilde{\eta}_{ji}}$$

$$-\frac{\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|}{f_{\min}}\underbrace{\left(\hat{\xi}_{i0}-\xi_{i0}\right)}_{\tilde{\xi}_{i0}}-\underbrace{(\hat{k}_{ino}-k_{ino})}_{\tilde{k}_{ino}}\left(\frac{\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|^2}{f_{\min}}+\left|\underline{\hat{e}}_i^{T}P_{i2}b_i\right|\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|\right)$$

$$-\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|\underbrace{\left(u_{icom}-\varepsilon_{\max}\right)}_{\tilde{u}_{icom}}-\frac{\left|C_{is}^{T}\underline{\tilde{e}}_{is}\right|}{f_{\min}}\underbrace{\left(u_{ir}-d_{\max}\right)}_{\tilde{u}_{ir}}+\frac{1}{f_{\min}}\left\|f_i'^{T}(\underline{x})\right\|\left\|B_{is}^{T}P_{i1}\underline{\tilde{e}}_{is}\right\|$$

$$+\underline{\hat{e}}_i^{T}P_{i2}\mathrm{K}_{i0}C_{is}^{T}\underline{\tilde{e}}_{is}+\tilde{\theta}_{i1}^{T}\Gamma_1^{-1}\dot{\theta}_{i1}+\frac{\tilde{\xi}_{i0}\dot{\hat{\xi}}_{i0}}{\gamma_{\xi_{i0}}}+\frac{\sum_{j=1}^{N}\tilde{\eta}_{ji}\dot{\hat{\eta}}_{ji}}{\gamma_{\eta_{ji}}}$$

$$+\frac{\tilde{u}_{ir}\dot{u}_{ir}}{\gamma_{u_{ir}}}+\frac{\tilde{u}_{icom}\dot{u}_{icom}}{\gamma_{u_{icom}}}+\frac{\tilde{v}_i'\dot{\hat{v}}_i'}{\gamma_{\hat{v}_i'}}+\frac{\tilde{k}_{no}\dot{\hat{k}}_{no}}{\gamma_{ko}} \tag{54}$$

This after some more algebraic manipulations becomes

$$\begin{aligned}
\dot{V} \le \sum_{i=1}^{N} & \frac{-1}{2 f_{iu_{\lambda f}}} \underline{\tilde{e}}_{is}^{T} \left(Q_{i1} + \frac{\dot{f}_{iu_{\lambda}}}{f_{iu_{\lambda}}} P_{i1} \right) \underline{\tilde{e}}_{is} + \frac{1}{2} \underline{\hat{e}}_{i}^{T} Q_{i2} \underline{\hat{e}}_{i} \\
& + \frac{1}{f_{\min}} \left\| f_i'^{T}(\underline{x}) \right\| \left\| B_{is}^{T} P_{i1} \underline{\tilde{e}}_{is} \right\| - \tilde{v}_i' \left(\frac{\left| C_{is}^{T} \underline{\tilde{e}}_{is} \right|}{f_{\min}} - \frac{\dot{\hat{v}}_i'}{\gamma_{\hat{v}_i'}} \right) \\
& -\tilde{\theta}_{i1}^{T} \left(w_{i1}(\underline{z}_i) C_{is}^{T} \underline{\tilde{e}}_{is} - \Gamma_1^{-1} \dot{\theta}_{i1} \right) - \sum_{j=1}^{N} \tilde{\eta}_{ji} \left(\frac{\left| C_{is}^{T} \underline{\tilde{e}}_{is} \right|^2 \left\| \underline{\hat{x}}_i \right\|^2}{2 f_{\min}} - \frac{\dot{\hat{\eta}}_{ji}}{\gamma_{\eta_{ji}}} \right) \\
& -\tilde{\xi}_{i0} \left(\frac{\left| C_{is}^{T} \underline{\tilde{e}}_{is} \right|}{f_{\min}} - \frac{\dot{\hat{\xi}}_{i0}}{\gamma_{\xi_{i0}}} \right) - \tilde{k}_{ino} \left(\frac{\left| C_{is}^{T} \underline{\tilde{e}}_{is} \right|^2}{f_{\min}} + \left| \underline{\hat{e}}_{i}^{T} P_{i2} b_i \right| \left| C_{is}^{T} \underline{\tilde{e}}_{is} \right| - \frac{\dot{\hat{k}}_{no}}{\gamma_{ko}} \right) \\
& -\tilde{u}_{icom} \left(\left| C_{is}^{T} \underline{\tilde{e}}_{is} \right| - \frac{\dot{u}_{ir}}{\gamma_{u_{ir}}} \right) - \tilde{u}_{ir} \left(\frac{\left| C_{is}^{T} \underline{\tilde{e}}_{is} \right|}{f_{\min}} - \frac{\dot{u}_{icom}}{\gamma_{u_{icom}}} \right)
\end{aligned} \tag{55}$$

Use (40) in (55) and write

$$\begin{aligned}
\dot{V} \le \sum_{i=1}^{N} & -\frac{1}{2 f_{iu_{\lambda f}}} \underline{\tilde{e}}_{is}^{T} \underbrace{\left(Q_{i1} + \frac{\dot{f}_{iu}}{f_{iu}} P_{i1} \right)}_{M_i} \underline{\tilde{e}}_{is} \\
& -\frac{1}{2} \underline{\hat{e}}_{i}^{T} Q_{i2} \underline{\hat{e}}_{i} + \frac{1}{f_{\min}} \left\| f_i'^{T}(\underline{x}) \right\| \left\| B_{is}^{T} P_{i1} \right\| \left\| \underline{\tilde{e}}_{is} \right\|
\end{aligned} \tag{56}$$

Based on the boundedness of the reference signals and $\left\| f_i'^{T}(\underline{x}) \right\| \le c_1 \left\| x_i \right\| + c_2$, the following inequality is easily obtained.

$$\begin{aligned}
\left\| f_i'^{T}(\underline{x}) \right\| & \le c_1 \left\| \underline{x}_i \right\| + c_2 \le c_1 \left\| \underbrace{\underline{x}_i - \underline{\hat{x}}_i}_{\underline{\tilde{e}}_{is}} + \underbrace{\underline{\hat{x}}_i - \underline{x}_{im}}_{\underline{\hat{e}}_i} + \underline{x}_{im} \right\| + c_2 \\
& \le c_1 \left\| \underline{\tilde{e}}_{is} \right\| + c_1 \left\| \underline{\hat{e}}_i \right\| + \underbrace{c_1 \left\| \underline{x}_{im} \right\| + c_2}_{\le c_2'}
\end{aligned} \tag{57}$$

Using (57), equation (56) becomes

$$\begin{aligned}\dot{V} \le -\frac{1}{f_{\min}}\lambda_{\min}(M_i)\left\|\underline{\tilde{e}}_{is}\right\|^2 - \frac{1}{2}\lambda_{\min}(Q_{i2})\left\|\underline{\hat{e}}_i\right\|^2 + \frac{1}{f_{\min}}c_1\left\|\underline{\tilde{e}}_{is}\right\|^2\left\|B_{is}^T P_{i1}\right\| \\ + \frac{1}{f_{\min}}c_1\left\|\underline{\hat{e}}_i\right\|\left\|\underline{\tilde{e}}_{is}\right\|\left\|B_{is}^T P_{i1}\right\| + \frac{1}{f_{\min}}c_2'\left\|\underline{\tilde{e}}_{is}\right\|\left\|B_{is}^T P_{i1}\right\|\end{aligned} \tag{58}$$

The following inequality is then easily obtained:

$$\begin{aligned}\dot{V} \le -\frac{\left\|\underline{\tilde{e}}_{is}\right\|}{f_{\min}}\Big(\lambda_{\min}(M_i)\left\|\underline{\tilde{e}}_{is}\right\| - c_1\left\|\underline{\tilde{e}}_{is}\right\|\left\|B_{is}^T P_{i1}\right\| - c_2'\left\|B_{is}^T P_{i1}\right\|\Big) \\ -\left(\frac{1}{2}\lambda_{\min}(Q_{i2})\left\|\underline{\hat{e}}_i\right\| - \frac{1}{f_{\min}}c_1\left\|\underline{\tilde{e}}_{is}\right\|\left\|B_{is}^T P_{i1}\right\|\right)\left\|\underline{\hat{e}}_i\right\|\end{aligned} \tag{59}$$

By choosing matrices M_i, Q_{i2} appropriately we can guarantee that $\dot{V}$ is negative definite as long as $\underline{\tilde{e}}_{is}, \underline{\hat{e}}_i$ lie outside the compact set Ω_e defined as

$$\Omega_e = \left\{ \underline{\tilde{e}}_{is}, \underline{\hat{e}}_i \;\middle|\; \begin{array}{l} \left\|\underline{\tilde{e}}_{is}\right\| \le \dfrac{c_2'\left\|B_{is}^T P_{i1}\right\|}{\lambda_{\min}(M_i) - c_1\left\|B_{is}^T P_{i1}\right\|} \\ \left\|\underline{\hat{e}}_i\right\| \le \dfrac{2c_1\left\|B_{is}^T P_{i1}\right\| c_2'\left\|B_{is}^T P_{i1}\right\|}{\lambda_{\min}(Q_{i2}) f_{\min}\left(\lambda_{\min}(M_i) - c_1\left\|B_{is}^T P_{i1}\right\|\right)} \end{array} \right\} \tag{60}$$

According to the standard Lyapunov theorem, we conclude that the observation error and accordingly the observer error and the tracking error are all ultimately bounded and $\underline{\tilde{e}}_{is}, \underline{\hat{e}}_i$ will converge to Ω_e. In addition, the boundedness of the coefficient parameters is guaranteed. This completes the proof.

Q.E.D.

To guarantee the boundedness of the parameters in the presence of the unavoidable approximation error, the proposed adaptive laws (40) is modified by introducing a $\sigma -$ modification term as follows:

$$\begin{aligned}\dot{\hat{k}}_{ino} &= \gamma_{iko}\Big(\frac{\left|C_{is}^T \underline{\tilde{e}}_{is}\right|^2}{f_{\min}} + \left|C_{is}^T \underline{\tilde{e}}_{is}\right|\left|b_i^T P_{i2}\underline{\hat{e}}_i\right|\Big) - \sigma\gamma_{iko}\hat{k}_{ino} \\ \dot{\theta}_{i1} &= \Gamma_1 C_{is}^T \underline{\tilde{e}}_{is} w_{i1}(\underline{z}_i) - \sigma\Gamma_1\theta_{i1}\end{aligned}$$

$$\begin{aligned}
&\dot{\hat{\xi}}_{i0} = \frac{\gamma_{\xi_{i0}}}{f_{\min}}\left|C_{is}^T \underline{\tilde{e}}_{is}\right| - \sigma\gamma_{\xi_{i0}}\hat{\xi}_{i0} \\
&\dot{\hat{\eta}}_{ji} = \frac{\gamma_{\eta_{ji}}}{2f_{\min}}\left|C_{is}^T \underline{\tilde{e}}_{is}\right|\left\|\underline{\hat{x}}_i\right\|^2 - \sigma\gamma_{\xi_{ij}}\hat{\eta}_{ij} \\
&\dot{u}_{ir} = \gamma_{u_{ir}}\left|C_{is}^T \underline{\tilde{e}}_{is}\right| - \sigma\gamma_{u_{ir}}u_{ir} \\
&\dot{u}_{icom} = \gamma_{u_{icom}}\left|C_{is}^T \underline{\tilde{e}}_{is}\right| - \sigma\gamma_{u_{icom}}u_{icom} \\
&\dot{\hat{v}}'_i = \gamma_{\hat{v}'_i}\left|C_{is}^T \underline{\tilde{e}}_{is}\right| - \sigma\gamma_{\hat{v}'_i}\hat{v}'_i
\end{aligned} \tag{61}$$

5.Simulation Results

In this section, we apply the proposed observer based decentralized fuzzy model reference adaptive controller to the large scale system consists of two subsystems, where each subsystem's dynamics are described by the following nonlinear non-affine form.

$$\begin{cases}
\begin{cases}
\dot{x}_{11} = \sin(x_{11}) + x_{12} - x_{11} \\
\dot{x}_{12} = \sin(x_{11}) + 1 - x_{12} + 50\tanh(u_1) + 4\sin(x_{21}) + d_1(t) \\
y_1 = x_{11}
\end{cases} \\
\begin{cases}
\dot{x}_{21} = \sin(x_{21}) + x_{22} - x_{21} \\
\dot{x}_{22} = \sin(x_{21}) + 1 - x_{22} + 40\tanh(u_2) + 4\sin(x_{11}) + d_2(t) \\
y_2 = x_{21}
\end{cases}
\end{cases} \tag{62}$$

It has been considered that the first leader's dynamics are described as:

$$r_1 = 2\sin(\pi t) + 2\sin(3\pi t)$$

and the second one can be described as:

$$r_2 = 3\sin(2\pi t) + \sin(3\pi t)\,.$$

Furthermore, it is assumed that $d_1(t) = \sin(200\pi t)$ and $d_2(t) = \sin(120\pi t)$.

Now we apply the proposed controller defined in (35), (36) to the system defined in equation (1). Based on the experts' knowledge, let to define $x_i = [x_{i1}, x_{i2}]^T$, $z_i = [x_{i1}, x_{i2}, v_i]^T$ and the states of the subsystems are in the range of $[5,-5]$, furthermore v_i are defined over $[-45,45]$. For each fuzzy system input, we define 6 membership functions over the defined sets. Consider that all of the membership functions are defined by the Gaussian function $\mu_j(\chi) = \exp\left((\chi - c)^2 \big/ 2\delta^2 \right)$, where c is center of the membership function and δ is its variance. We assume that all the initial values of controller parameters are set to zero. Furthermore, it has been assumed that $f_{\min} = 1$, $\Gamma_1 = 10$, $\gamma_{\xi_{i0}} = 2$, $\gamma_{\xi_{ij}} = 2$, $\gamma_{u_{icom}} = 2$, $\gamma_{u_{ir}} = 5$, $\gamma_{\hat{v}_i'} = 2$ and $\gamma_{iko} = 10$. In addition, we assume that $\sigma = 0.01, \varepsilon = 0.01$. The parameters f_{dm}, $f_{\min}$ and the vector k_{ic} are chosen so that lemma 2 holds.

As shown in Figures 2 and 3, it is obvious that the performance of the proposed controller is promising. Furthermore, in the presence of $d_i(t)$ and ε_{iu}, it is evident the controller has satisfactory performance and is robust against uncertainties and disturbances as easily observed in these figures. Figures 4, and 5 show the total input of each subsystem as given in equation (35) and (36).

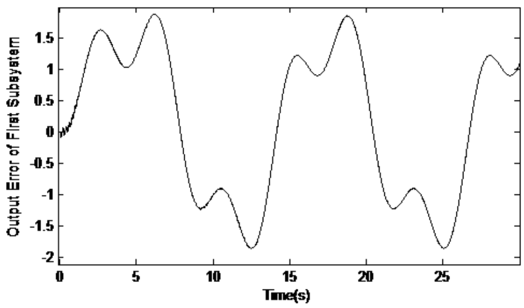

FIGURE 2. Performance of the Proposed Controller in First Subsystem of the First Subsystem

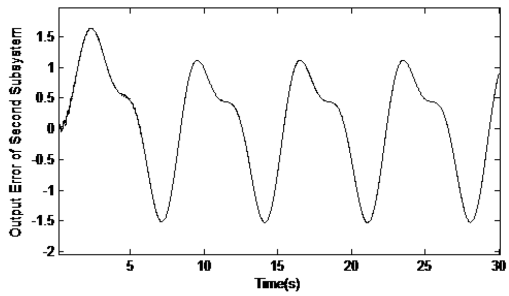

FIGURE 3. Performance of the Proposed Controller in Second Subsystem of the First Subsystem

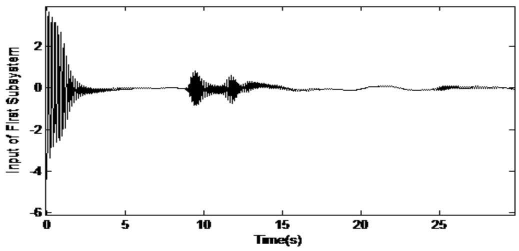

FIGURE4. Control Input of the First Subsystem of the First Subsystem

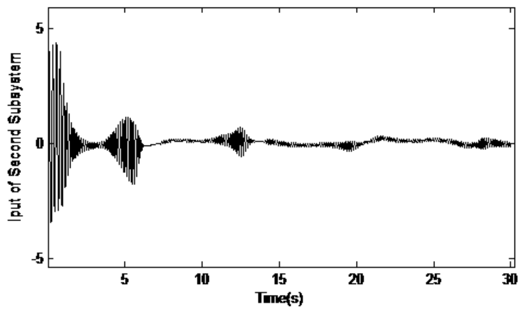

FIGURE 5. Control Input of the Second Subsystem of the First Subsystem

Figures 6 and 7 present the estimation of the first and second states of the first subsystem with their desired values.

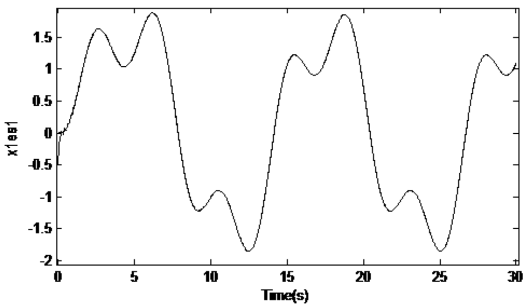

FIGURE 6. The Estimation of the First State of the First Subsystem and the Desired Value of the First Subsystem

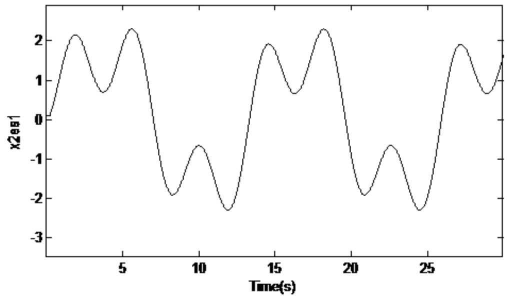

FIGURE 7. The Estimation of the Second State of the first Subsystem of

the First Subsystem

The performance of the proposed observer on the second subsystem and their desired trajectories are shown in figure 8 and 9.

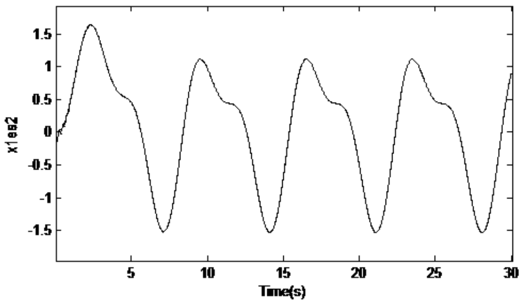

FIGURE 8. The Estimation of the First State of the Second Subsystem and the Desired Value of the First Subsystem

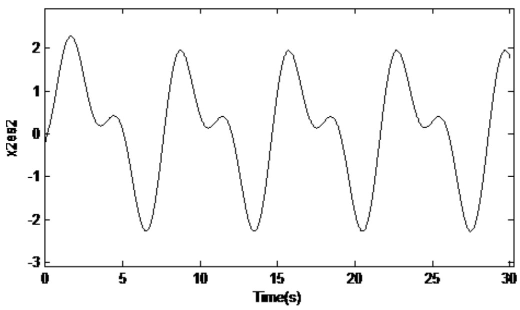

FIGURE9. The Estimation of the Second State of the Second Subsystem of

the First Subsystem

As shown in Figures 6 through 9, it is obvious that the nonlinear state observer can generate the estimated states and perform perfectly. Moreover, it is also clear that the output of the system converges to the desired value. The stability of the closed loop system, the convergence of the tracking error and the observer error to zero, and robustness against external disturbances as well as approximation error are the merits of the proposed controller and observer.

To verify the boundedness of the controller and observer parameters in the first subsystem, the trajectories of some of them are depicted in Figures 10 and 11.

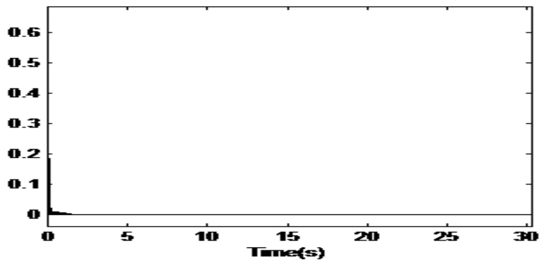

FIGURE10. Time Trajectory of the Nonlinear Gain of the Observer in the First Subsystem ($\hat{k}_{1no}$) of the First Subsystem

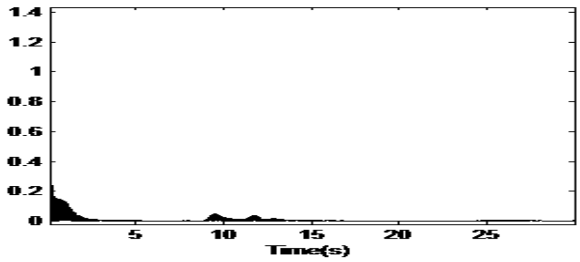

FIGURE 11. Time Trajectory of the Compensation Term in First Subsystem(u_{1com})of the First Subsystem

To validate the boundedness of the controller and observer parameters the in first subsystem, the trajectories of some of them are given in Figures 12 –13.

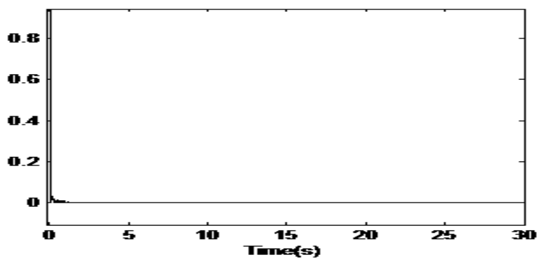

FIGURE12. Time Trajectory of the Nonlinear Gain of the Observer in the Second Subsystem ($\hat{k}_{2no}$)of the First Subsystem

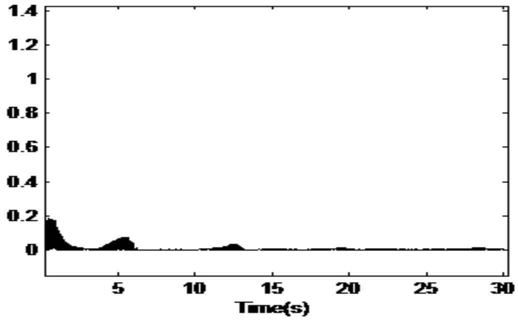

FIGURE13. Time Trajectory of the Compensation Term in Second Subsystem (u_{2com}) of the First Subsystem

The performance of the second subsystem is also demonstrated in the following figure.

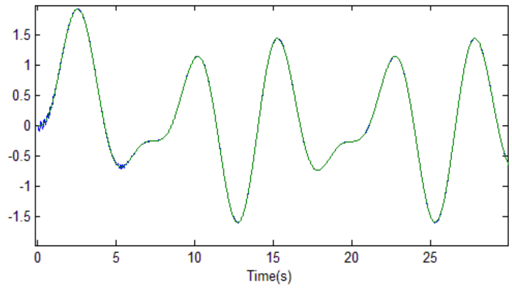

FIGURE 14. Performance of the Proposed Controller in First Subsystem of the First Subsystem

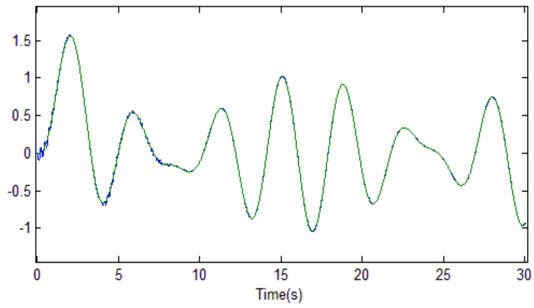

FIGURE 15. Performance of the Proposed Controller in Second Subsystem of the First Subsystem

As shown in Figures 14 and 15, it is obvious that the performance of the proposed controller is indeed satisfactory. Furthermore, in the presence of $d_i(t)$ and ε_{iu}, it is evident the controller has a promising performance and is robust against uncertainties and disturbances as shown in figures 14 and 15. Figures 16, 17 represent the total input history of each subsystem defined in equation (35) and (36).

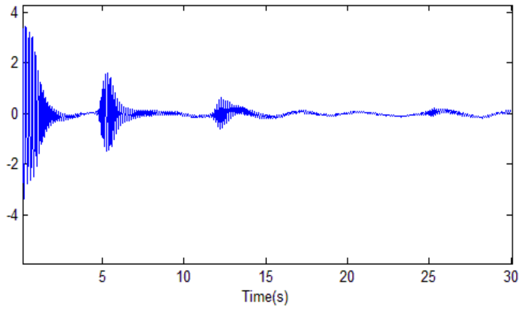

FIGURE16. Control Input of the First Subsystem of the First Subsystem

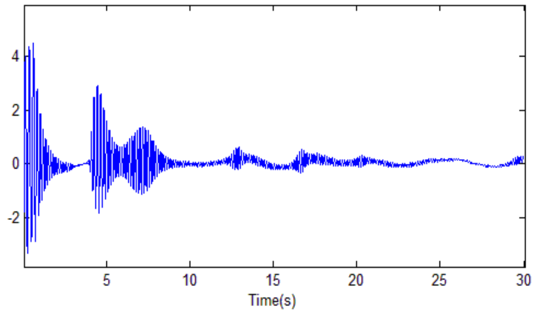

FIGURE17. Control Input of the Second Subsystem of the First Subsystem

Figure 18 and 19 presents the estimation of the first and second states of the first subsystem with their desired value.

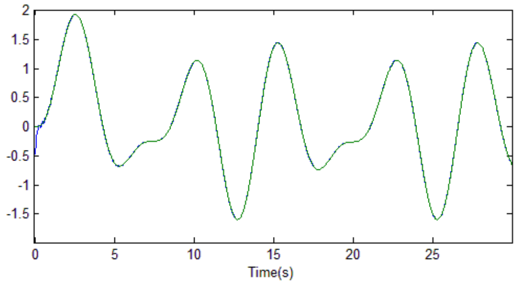

FIGURE18. The Estimation of the First State of the First Subsystem and the Desired Value of the First Subsystem

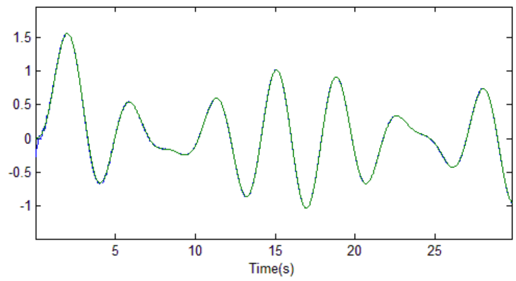

FIGURE19. The Estimation of the Second State of the First Subsystem of the First Subsystem

The performance of the proposed observer on the second subsystem and their desired trajectories are shown in figure 20 and 21.

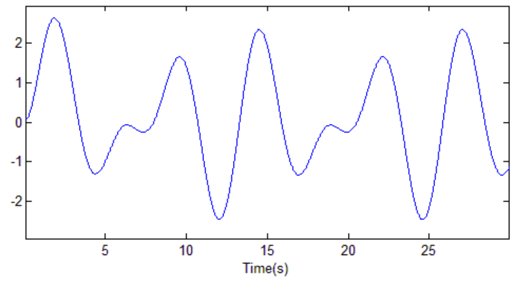

FIGURE 20.the Estimation of the First State of the Second Subsystem and the Desired Value of the First Subsystem

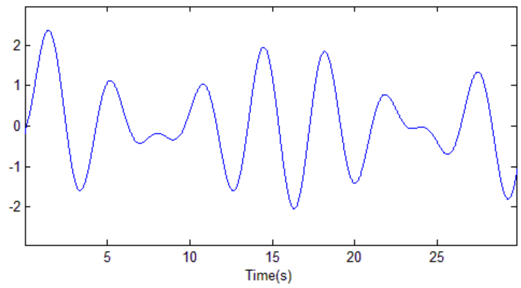

FIGURE 21. The Estimation of the Second State of the Second Subsystem of the First Subsystem

As shown in Figures 18 through 21, it is obvious that the nonlinear state observer can generate properly the estimated states and it indeed performs really well. Moreover, it is also clear that the output of the system converges to the desired value. The stability of the closed loop system, the convergence of both the tracking error and the observer error to zero, and robustness against both external disturbances and approximation errors are the merits of the proposed controller and observer as well.

6.Conclusion

This paper proposed a decentralized fuzzy adaptive consensus controller method for a class of non-affine nonlinear multi-subsystem systems with non-canonical uncertain nonlinear dynamics. In order to resolve the obstacle deriving from the immeasurable states, the adaptive nonlinear state observers are constructed while the interconnections and the functions of the subsystems are unknown. Both the observer-based controller structure and the derived adaptation rules guarantee the boundedness of the tracking error and observer error. Robustness against external disturbances and approximation errors and using knowledge of experts are the merits of the proposed method. The effectiveness of the proposed control method is further verified through simulation studies. The future work of the authors is to apply the proposed method on the cognitive science. We are currently working on formulating a cognitive based structure by extending the main concepts of the proposed intelligent method to furnish a better and yet solid framework for the so-called cognitive control. Furthermore, we are working on its applications in modeling driver's steering behavior in turns.

Acknowledgments. We are deeply grateful to QIAU's Incubator Center of Technology Units (Center of Cognitive Systems) for providing us the necessary support.

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