GROUP GENERALIZED INTERVAL-VALUED INTUITIONISTIC FUZZY SOFT SETS AND THEIR APPLICATIONS IN DECISION MAKING

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Abstract. Interval-valued intuitionistic fuzzy sets (IVIFSs) are widely used to handle uncertainty and imprecision in decision making. However, in more complicated environment, it is difficult to express the uncertain information by an IVIFS with considering the decision-making preference. Hence, this paper proposes a group generalized interval-valued intuitionistic fuzzy soft set (G-GIVIFSS) which contains the basic description by interval-valued intuitionistic fuzzy soft set (IVIFSS) on the alternatives and a group of experts' evaluation of it. It contributes the following threefold: 1) A generalized interval-valued intuitionistic fuzzy soft set (GIVIFSS) is proposed by introducing an interval-valued intuitionistic fuzzy parameter, which reflects a new and senior expert's opinion on the basic description. The operations, properties and aggregation operators of GIVIFSS are discussed. 2) Based on GIVIFSS, a G-GIVIFSS is then proposed to reduce the impact of decision-making preference by introducing more parameters by a group of experts. Its important operations, properties and the weighted averaging operator are also defined. 3) A multi-attribute group decision making model based on G-GIVIFSS weighted averaging operator is built to solve the group decision making problems in the more universal IVIF environment, and two practical examples are taken to validate the efficiency and effectiveness of the proposed model.

1. Introduction

In decision making, it is very important to express the uncertain, imprecise and incomplete information in a suitable and reasonable way. Zadeh [36] proposed the fuzzy set (FS) to present the fuzzy information with single membership function. However, in real life, a person usually gives the preference degree to an object with some hesitation, i.e., neither supporting nor opposing. In the following decades, many researchers have carried on various works [1, 4, 5, 8, 12, 17, 20, 22, 29] to overcome this shortcoming. These efforts can be categorized as four main stages in FS's evolution, which are denoted as the intuitionistics, soft set, generalization and group concept, respectively. The main works in these stages are reviewed in the following.

Atanassov [4] for the first time proposed the intuitionistic fuzzy set (IFS) by improving the FS with a membership function, a non-membership function and a

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hesitation function. Later, Atanassov and Gargov [5] further introduced interval-valued intuitionistic fuzzy set (IVIFS) by extending the crisp numbers into intervals, and generated more precise and suitable description for uncertain information. With the advantage of IVIFS, the IVIFS has generated many excellent results [11, 13, 14, 28, 30, 31, 32]. For instance, Liu [14] presented some interval-valued intuitionistic fuzzy Hamacher aggregation operators to solve the multi-attribute group decision making (MAGDM) problems. Although the intuitionistics have their merits for fuzzy description, Molodtsov [22] pointed out that FS and IFS also had some difficulties, such as how to set the membership function in each particular case.

To overcome the shortcomings of FS and IFS, Molodtsov [22] took the soft set (SS) as a mathematical tool to deal with uncertainties, and many works [2, 3, 7, 10, 12, 15, 16, 17, 18, 19, 26] were derived from SS in following decades. Among them, Maji *et al.* [16, 17] originally extended the soft sets to fuzzy soft sets (FSS) and intuitionistic fuzzy soft set (IFSS). Jiang *et al.* [12] proposed the interval-valued intuitionistic fuzzy soft set (IVIFSS) and discussed its basic properties. Both of IFSS and IVIFSS reflect only an expert's hesitation in decision making. However, the hesitation corresponds to his/her own opinion, which may lead unfair decision. To solve this problem, a new kind of generalization is introduced in [1, 8, 20] by embedding another independent expert's evaluation.

With respect to the generalization of FS, Majumdar *et al.* [20] proposed the generalized fuzzy soft set (GFSS), in which a degree related to the parametrization of fuzzy sets. Dinda *et al.* [8] presented the generalized intuitionistic fuzzy soft set (GIFSS) and discussed the relations on GIFSS. Agarwal *et al.* [1] extended the GIFSS by introducing a generalization parameter which is an intuitionistic fuzzy number. Different from IVIFS, GIFSS provides the description of alternative by a junior expert and a senior expert, respectively. However, GIFSS still has some limitations on describing the imprecise and uncertain information. It is hard for a senior expert to give a suitable generalization parameter with more fuzzy information according to his/her own knowledge. To address this problem, We proposed a group generalized intuitionistic fuzzy soft set (G-GIFSS) in our previous work [29], by introducing more experts to provide their generalization parameters. However, G-GIFSS is not suitable for the more universal IVIF environment.

To sum up the previous works on FS's evolution, there are still some limitations which can be summarized as follows.

- In more complex IVIF environment, it may be imprecise for IVIFS and IVIFSS to describe the information by considering the decision-making preference.
- In addition, although both of GFSS and GIFSS have generalization parameters, given by another independent expert, they may still bring the personal preference into the whole description.

In view of these limitations, this paper proposes a group generalized interval-valued intuitionistic fuzzy soft set (G-GIVIFSS) and applies it to solve the MAGDM problems. The main contributions are described as follows.

1) A generalized interval-valued intuitionistic fuzzy soft set (GIVIFSS) is firstly proposed by extending the GIFSS with IVIFSs. Its basic operations, properties and aggregation operators are discussed.

2) Based on GIVIFSS, a G-GIVIFSS is then presented by introducing more experts into GIVIFSS, which reduces the impact of the single expert's preference.

3) A G-GIVIFSS weighted averaging (GGWA) operator-based model is built to solve the MAGDM problems in the IVIF environment.

The remainder of the paper is organized as follows: In Section 2, some basic concepts and notations are reviewed briefly. Section 3 proposes the GIVIFSS and discusses its important operations, properties and aggregation operators. In Section 4, the G-GIVIFSS is introduced, and its operations, properties and GGWA operator are defined. Section 5 presents the MAGDM model based on the GGWA operator in the more universal fuzzy environment. In Section 6, two practical examples are provided to validate the superiority of the proposed approach. The conclusion is summarized in Section 7.

2. **Preliminaries**

In this section, some definitions and properties corresponding to the IVIFS, IVIFSS and GIFSS are briefly reviewed as the basic of this paper.

Atanassov and Gargov [5] gave the definition of IVIFS as follows:

Definition 2.1. Let $X = \{x_1, x_2, ..., x_n\}$ be a universe of discourse, then an interval-valued intuitionistic fuzzy set α in X is given by

$$\alpha = \langle \mu_\alpha(x), \upsilon_\alpha(x) \rangle, x \in X, \tag{1}$$

where $\mu_\alpha(x) \subseteq [0,1]$,$\upsilon_\alpha(x) \subseteq [0,1]$ and $0 \le (\sup \mu_\alpha(x) + \sup \upsilon_\alpha(x)) \le 1, \forall x \in X$. The interval numbers, $\mu_\alpha(x)$ and $\upsilon_\alpha(x)$, denote the membership function and the non-membership function of element x to the set α, respectively.

Let α and β be any two IVIFSs. Some basic operations and operators were defined in [31], including $\alpha \cap \beta$, $\alpha \cup \beta$, $\alpha \oplus \beta$, $\alpha \otimes \beta$, $\lambda\alpha$ and α^λ. In addition, the interval-valued intuitionistic fuzzy weighted averaging (IVIFWA) operator and the interval-valued intuitionistic fuzzy weighted geometric (IVIFWG) operator were also introduced in [31].

Definition 2.2. [31] Let $\beta_i\,(i = 1, 2, \ldots, n)$ be a set of IVIFSs, the interval-valued intuitionistic fuzzy weighted averaging (IVIFWA) operator is given by

$$\begin{aligned} &IVIFWA(\beta_1, \beta_2, ..., \beta_n) = \sum_{i=1}^{n} w_i\beta_i \\ &= \left(\left[1 - \prod_{i=1}^{n} \left(1 - \mu_\beta^L\right)^{w_i}, 1 - \prod_{i=1}^{n} \left(1 - \mu_\beta^H\right)^{w_i} \right], \left[\prod_{i=1}^{n} \left(\upsilon_\beta^L\right)^{w_i}, \prod_{i=1}^{n} \left(\upsilon_\beta^H\right)^{w_i} \right] \right), \end{aligned} \tag{2}$$

where $w = (w_1, w_2, ..., w_n)^T$ is the weight vector of $\beta_i\,(i = 1, 2, ..., n)$, with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$.

Definition 2.3. [6] Let $\beta = \left(\left[\mu_\beta^L, \mu_\beta^H\right], \left[\upsilon_\beta^L, \upsilon_\beta^H\right]\right)$ be an IVIFS. A composite score function s is defined as follows:

$$s(\beta) = \delta \cdot [\mu_\beta^L \cdot (2 - \mu_\beta^L - 2\upsilon_\beta^H)] + (1-\delta) \cdot [1 - \upsilon_\beta^L \cdot (2 - \upsilon_\beta^L - 2\mu_\beta^H)], \tag{3}$$

where $\delta \in [0,1]$ is a parameter that reflects the attitudes of decision makers.

Definition 2.4. [6] Let $\beta = \left(\left[\mu_\beta^L, \mu_\beta^H\right], \left[\upsilon_\beta^L, \upsilon_\beta^H\right]\right)$ be an IVIFS. A composite accuracy function h, based on the measures of accuracy and not-hesitancy of IVIFS, is defined as follows:

$$h(\beta) = \phi \cdot 2\mu_\beta^L \cdot \upsilon_\beta^L + (1-\phi) \cdot [1 - (1 - \mu_\beta^H - \upsilon_\beta^H)^2], \tag{4}$$

where $\phi \in [0,1]$ is a parameter that reflects the attitudes of decision makers.

In the MADM model, an expert can express his/her hesitancy with IFS. However, this hesitancy presents an expert's own perception, which also needs to be supported by another independent expert. For this reason, Dinda *et al.* [8] proposed the GIFSS and Agarwal *et al.* [1] improved it with a generalization parameter, which is an intuitionistic fuzzy number.

Definition 2.5. [1] Let $\bar{U} = \{\bar{a}_1, \bar{a}_2, ..., \bar{a}_n\}$ denote a universal set of elements and the collection of all intuitionistic fuzzy subsets of $\bar{U}$ is denoted by $IF(\bar{U})$. Let $\bar{E} = \{\bar{e}_1, \bar{e}_2, ..., \bar{e}_m\}$ be a set of parameters. A pair $(\bar{U}, \bar{E})$ is called as a soft universe and $\bar{\alpha}$ be an intuitionistic fuzzy subset of $\bar{E}$. Let $\bar{A} \subseteq \bar{E}$ and $\bar{F}_{\bar{\alpha}}$ is a mapping given by $\bar{F}_{\bar{\alpha}} : \bar{A} \to IF(\bar{U}) \times IF$.

A generalized intuitionistic fuzzy soft set (GIFSS) over the soft universe $(\bar{U}, \bar{E})$ is defined as

$$\bar{F}_{\bar{\alpha}} = (\bar{F}(\bar{e}), \bar{\alpha}(\bar{e})), \tag{5}$$

where $\bar{F}(\bar{e}) \in IF(\bar{U})$ and $\bar{\alpha}(\bar{e}) \in IF$. $\bar{\alpha}(\bar{e})$ indicates a senior expert's assessment on the elements of $\bar{U}$ in $\bar{F}(\bar{e})$.

However, in the complicated environment, it is hard for each expert to give his/her evaluation with crisp numbers, and even intuitionistic fuzzy numbers, whose membership function and non-membership function are also crisp numbers. Therefore, a generalized interval-valued intuitionistic fuzzy soft set is proposed to overcome this disadvantage.

3. Generalized Interval-valued Intuitionistic Fuzzy Soft Set (GIVIFSS)

Definition 3.1. Let $U = \{a_1, a_2, ..., a_n\}$ denote a universe set of elements, and the collection of all interval-valued intuitionistic fuzzy subsets of U is denoted by $IVIF(U)$. Let $E = \{e_1, e_2, ..., e_m\}$ be a set of parameters and β be an interval-valued intuitionistic fuzzy subset of E. The pair (U, E) is called as a soft universe. Let $A \subseteq E$, and F be a mapping given by $F : A \to IVIF(U)$ and F_β by $F_\beta : A \to IVIF(U) \times IVIF$.

A generalized interval-valued intuitionistic fuzzy soft set (GIVIFSS) F_β over the soft universe (U, E) is defined as

$$F_\beta(e) = (F(e), \beta(e)), \tag{6}$$

where $F(e) \in IVIF(U)$ and $\beta(e) \in IVIF$. $F(e)$ represents the elements of U in the interval-valued intuitionistic fuzzy soft set (IVIFSS), and $\beta(e)$ indicates a senior expert's assessment on the elements of U in $F(e)$.

The Definition 3.1 is clarified by Example 3.2.

Example 3.2. A school library plans to purchase some books for students. There are a collection of four books $U = \{a_1, a_2, a_3, a_4\}$ and a collection of their attributes $E = \{e_1(paperback), e_2(helpful), e_3(best-selling), e_4(academic)\}$. A generalized interval-valued intuitionistic fuzzy soft set F_β over the soft universe (U, E) is described as follows:

$$F_\beta(e_1) = \{a_1|([0.1, 0.3], [0.4, 0.6]), a_2|([0.3, 0.5],\ [0.1, 0.4]), a_3|([0.6, 0.8], [0.1,\ 0.2]), a_4|([0.4, 0.9],\ [0, 0.1]), \beta|([0.7, 0.8]\ , [0.1, 0.2])\},$$

$$F_\beta(e_4) = \{a_1|([0.6, 0.7], [0.1, 0.2]), a_2|([0.5, 0.8],\ [0, 0.1]), a_3|([0.2, 0.3], [0.4, 0.6]), a_4|([0.4, 0.6],\ [0.2, 0.3]), \beta|([0.6, 0.7],\ [0.2, 0.3])\},$$

where $\beta(e_1) = ([0.7, 0.8], [0.1, 0.2])$ and $\beta(e_4) = ([0.6, 0.7], [0.2, 0.3])$ represent the generalized parameters, which indicate a senior expert's assessment for the basic description with IVIFSS. The $F_\beta(e_1)$ and $F_\beta(e_4)$ obtained above are used to make the final decision.

Definition 3.3. Let $F_\beta = (F(e), \beta(e))$ and $G_\gamma = (G(e), \gamma(e))$ be two GIVIFSSs over the universe (U, E). F_β is the subset of G_γ, i.e. $F_\beta \subseteq G_\gamma$, if
i) $\beta \leq \gamma$, and
ii) $F(e)$ is an interval-valued intuitionistic fuzzy subset of $G(e)$, $\forall e \in E$.

Definition 3.4. Let $F_\beta = (F(e), \beta(e))$ and $G_\gamma = (G(e), \gamma(e))$ be two GIVIFSSs over the universe (U, E). F_β and G_γ are equal, i.e. $F_\beta = G_\gamma$, if
i) $G(e)$ is an IVIF subset of $F(e)$, $\forall e \in E$, and
ii) $F(e)$ is an IVIF subset of $G(e)$, $\forall e \in E$.

Definition 3.5. The complement of F_β is presented by F_β^c and given as

$$F_\beta^c = (F^c(e), \beta^c(e)), \forall e \in E. \tag{7}$$

Example 3.6. One of the complements of $F_\beta(e_1)$ in Example 3.2 is

$$F_\beta^c(e_1) = \{a_1|([0.6, 0.7], [0.1, 0.3]), a_2|([0.2, 0.4],\ [0.5, 0.6]), a_3|([0.1, 0.2], [0.7, 0.8]), a_4|([0, 0.1], [0.4, 0.9]), \beta|([0, 0.2]\ ,\ [0.7, 0.8])\}.$$

Definition 3.7. The union of F_β and G_γ, denoted by $F_\beta \cup G_\gamma$, is given as follows:

$$F_\beta \cup G_\gamma = (F(e) \circ G(e), \beta(e) \circ \gamma(e)), \forall e \in E, \tag{8}$$

where $F(e) \circ G(e) \in IVIF(U)$, $\beta(e) \circ \gamma(e) \in IVIF$; $\circ$ is any t-conorm.

Example 3.8. Obtain the union of the two sets of GIVIFSS, $F_\beta(e)$ and $G_\gamma(e)$ by the standard union operation in [31].

$F_\beta(e) = \{a_1|\left([0.4, 0.5], [0.3, 0.4]\right), a_2|\left([0.1, 0.3],\ [0.4, 0.6]\right), \beta|\left([0.6, 0.7], [0.2, 0.3]\right)\}$,

$G_\gamma(e) = \{a_1|\left([0.5, 0.8], [0.1, 0.2]\right), a_2|\left([0.2, 0.5],\ [0.4, 0.5]\right), \beta|\left([0.4, 0.7], [0.1, 0.2]\right)\}$.

Let $R_\theta = F_\beta \cup G_\gamma, \forall e \in E$. According to Definition 3.7 , there is

$R_\theta(e) = \{a_1|\left([0.5, 0.8], [0.1, 0.2]\right), a_2|\left([0.2, 0.5],\ [0.4, 0.5]\right), \beta|\left([0.6, 0.7], [0.1, 0.2]\right)\}$.

Definition 3.9. The intersection of F_β and G_γ, denoted by $F_\beta \cap G_\gamma$, is given as

$$F_\beta \cap G_\gamma = (F(e) * G(e), \beta(e) * \gamma(e)), \forall e \in E, \tag{9}$$

where $F(e) * G(e) \in IVIF(U)$, $\beta(e) * \gamma(e) \in IVIF$; $*$ is any t-norm.

Example 3.10. Obtain the intersection of the two sets of GIVIFSS, $F_\beta(e)$ and $G_\gamma(e)$ of Example 3.8 by the standard intersection operation in [31].

Let $P_\delta = F_\beta \cap G_\gamma, \forall e \in E$. According to Definition 3.9, there is

$P_\delta(e) = \{a_1|\left([0.4, 0.5], [0.3, 0.4]\right), a_2|\left([0.1, 0.3],\ [0.4, 0.6]\right), \beta|\left([0.4, 0.7], [0.2, 0.3]\right)\}$.

Definition 3.11. A GIVIFSS $\Phi_\delta(e) = (F(e), \delta(e))$ is regarded as a null generalized interval-valued intuitionistic fuzzy soft set, if $F(e) = ([0, 0], [1, 1])$, $\delta(e) = ([0, 0], [1, 1])$, $\forall e \in E$.

Definition 3.12. A GIVIFSS $\Omega_\varphi(e) = (F(e), \varphi(e))$ is regarded as an absolute generalized interval-valued intuitionistic fuzzy soft set, if $F(e) = ([1, 1], [0, 0])$, $\varphi(e) = ([1, 1], [0, 0])$, $\forall e \in E$.

Definition 3.13. Let $F_{\beta_i} = (F_i(e), \beta_i(e)), (i = 1, ..., n)$ be any GIVIFSS over (U, E), where $F_i(e) = \{a_{i1}, a_{i2}, ..., a_{im}\}$ whose weight vector is $w = \{w_1, w_2, ..., w_m\}^T$, and $\beta_i(e)$ is a generalization parameter for the ith alternative a_i. The GIVIFSS weighted averaging (GWA) operator is defined as

$$\begin{aligned} & GWA(a_{i1}, a_{i2}, ..., a_{im}) \\ & = \beta_i(e) \otimes IVIFWA(a_{i1}, a_{i2}, ..., a_{im}) \\ & = \left(\left[\mu_{\beta_i}^L \cdot (1 - \prod_{j=1}^{m} (1 - \mu_{a_{ij}}^L)^{w_j}), \mu_{\beta_i}^H \cdot (1 - \prod_{j=1}^{m} (1 - \mu_{a_{ij}}^H)^{w_j})\right],\ \left[(\upsilon_{\beta_i}^L + \right.\right. \\ & \left.\left. \prod_{j=1}^{m} (\upsilon_{a_{ij}}^L)^{w_j} - \upsilon_{\beta_i}^L \cdot \prod_{j=1}^{m} (\upsilon_{a_{ij}}^L)^{w_j}), (\upsilon_{\beta_i}^H + \prod_{j=1}^{m} (\upsilon_{a_{ij}}^H)^{w_j} -\ \upsilon_{\beta_i}^H \cdot \prod_{j=1}^{m} (\upsilon_{a_{ij}}^H)^{w_j})\right]\right). \end{aligned} \tag{10}$$

where the operation $\otimes$ is the logical multiplicative operation.

Example 3.14. A company chooses a best plan from three alternatives $U = \{a_1, a_2, a_3\}$ with considering four attributes $E = \{e_1(innovative), e_2(practical), e_3(sustainable), e_4(green)\}$ for each one. The weight vector of the attributes is $w = \{0.2, 0.3, 0.3, 0.2\}^T$. A senior expert provides his evaluation on this basic description as the generalized parameters for each alternative.

$$F_\beta(a_1) = \{e_1|([0.6, 0.7], [0.1, 0.2]), e_2|([0.5, 0.8], [0, 0.1]), e_3|([0.2, 0.3], [0.4, 0.6]), e_4|([0.4, 0.6], [0.2, 0.3]), \beta|([0.6, 0.7], [0.2, 0.3])\},$$

$$F_\beta(a_2) = \{e_1|([0.4, 0.6], [0.2, 0.3]), e_2|([0.5, 0.7], [0.1, 0.2]), e_3|([0.5, 0.6], [0.2, 0.3]), e_4|([0.4, 0.6], [0.2, 0.3]), \beta|([0.6, 0.8], [0.1, 0.2])\},$$

$$F_\beta(a_3) = \{e_1|([0.5, 0.7], [0.1, 0.2]), e_2|([0.6, 0.7], [0.2, 0.3]), e_3|([0.5, 0.7], [0.1, 0.2]), e_4|([0.5, 0.6], [0.2, 0.4]), \beta|([0.7, 0.8], [0, 0.1])\}.$$

According to (10), each aggregated evaluation $GWA(a_{i1}, a_{i2}, ..., a_{im})$ of alternative a_i is shown as follows.

$$GWA(a_{11}, a_{12}, ..., a_{14}) = ([0.429, 0.637], [0, 0.245]),$$

$$GWA(a_{21}, a_{22}, ..., a_{24}) = ([0.462, 0.633], [0.162, 0.266]),$$

$$GWA(a_{31}, a_{32}, ..., a_{34}) = ([0.532, 0.682], [0.141, 0.260]).$$

Theorem 3.15. *Let the generalization parameter be $\beta_i(e) = ([1, 1], [0, 0])$. Then there is*

$$GWA(a_{i1}, a_{i2}, ..., a_{im}) = IVIFWA(a_{i1}, a_{i2}, ..., a_{im}). \tag{11}$$

Proof. According to Definition 3.13, (10) with $\beta_i(e) = ([1, 1], [0, 0])$ can be obtained as follows.

$$\begin{aligned}
&GWA(a_{i1}, a_{i2}, ..., a_{im})\\
&= ([1, 1], [0, 0]) \otimes IVIFWA(a_{i1}, a_{i2}, ..., a_{im})\\
&= \left(\left[1 \cdot (1 - \prod_{j=1}^{m} (1 - \mu_{a_{ij}}^L)^{w_j}), 1 \cdot (1 - \prod_{j=1}^{m} (1 - \mu_{a_{ij}}^H)^{w_j})\right],\right.\\
&\quad \left.\left[(0 + \prod_{j=1}^{m} (\upsilon_{a_{ij}}^L)^{w_j} - 0 \cdot \prod_{j=1}^{m} (\upsilon_{a_{ij}}^L)^{w_j}), (0 + \prod_{j=1}^{m} (\upsilon_{a_{ij}}^H)^{w_j} - 0 \cdot \prod_{j=1}^{m} (\upsilon_{a_{ij}}^H)^{w_j})\right]\right)\\
&= \left(\left[(1 - \prod_{j=1}^{m} (1 - \mu_{a_{ij}}^L)^{w_j}), (1 - \prod_{j=1}^{m} (1 - \mu_{a_{ij}}^H)^{w_j})\right], \left[\prod_{j=1}^{m} (\upsilon_{a_{ij}}^L)^{w_j}, \prod_{j=1}^{m} (\upsilon_{a_{ij}}^H)^{w_j}\right]\right)\\
&= IVIFWA(a_{i1}, a_{i2}, ..., a_{im}),
\end{aligned}$$

where $IVIFWA(a_{i1}, a_{i2}, ..., a_{im})$ is defined in [31]. □

Theorem 3.16. *Let the generalization parameter be $\beta_i(e) = ([0, 0], [1, 1])$. Then, there is*

$$GWA(a_{i1}, a_{i2}, ..., a_{im}) = ([0, 0], [1, 1]). \tag{12}$$

Proof. According to Definition 3.13, (10) with $\beta_i(e) = ([0,0],[1,1])$ can be obtained as follows.

$$\begin{aligned}
&GWA(a_{i1}, a_{i2}, ..., a_{im})\\
&= ([0,0],[1,1]) \otimes IVIFWA(a_{i1}, a_{i2}, ..., a_{im})\\
&= \left(\left[0 \cdot (1 - \prod_{j=1}^{m} (1-\mu_{a_{ij}}^L)^{w_j}), 0 \cdot (1 - \prod_{j=1}^{m} (1-\mu_{a_{ij}}^H)^{w_j})\right],\right.\\
&\quad \left.\left[(1 + \prod_{j=1}^{m} (\upsilon_{a_{ij}}^L)^{w_j} - 1 \cdot \prod_{j=1}^{m} (\upsilon_{a_{ij}}^L)^{w_j}), (1 + \prod_{j=1}^{m} (\upsilon_{a_{ij}}^H)^{w_j} - 1 \cdot \prod_{j=1}^{m} (\upsilon_{a_{ij}}^H)^{w_j})\right]\right)\\
&= ([0,0],[1,1]).
\end{aligned}$$

□

Definition 3.17. Let $F_{\beta_i} = (F_i(e), \beta_i(e)), (i = 1, ..., n)$ be any GIVIFSS over (U, E), where $F_i(e) = \{a_{i1}, a_{i2}, ..., a_{im}\}$ with the weight vector $w = \{w_1, w_2, ..., w_m\}^T$, and $\beta_i(e)$ is a generalization parameter for the ith alternative a_i. The GIVIFSS weighted geometric (GWG) operator is defined as

$$\begin{aligned}
&GWG(a_{i1}, a_{i2}, ..., a_{im})\\
&= \beta_i(e) \otimes IVIFWG(a_{i1}, a_{i2}, ..., a_{im})\\
&= \left(\left[\mu_{\beta_i}^L \cdot \prod_{j=1}^{m} (\mu_{a_{ij}}^L)^{w_j},\ \mu_{\beta_i}^H \cdot \prod_{j=1}^{m} (\mu_{a_{ij}}^H)^{w_j}\right],\right.\\
&\quad \left[\upsilon_{\beta_i}^L + (1 - \prod_{j=1}^{m} (1-\upsilon_{a_{ij}}^L)^{w_j}) - \upsilon_{\beta_i}^L \cdot (1 - \prod_{j=1}^{m} (1-\upsilon_{a_{ij}}^L)^{w_j}),\right.\\
&\quad \left.\left.\upsilon_{\beta_i}^H + (1 - \prod_{j=1}^{m} (1-\upsilon_{a_{ij}}^H)^{w_j}) - \upsilon_{\beta_i}^H \cdot (1 - \prod_{j=1}^{m} (1-\upsilon_{a_{ij}}^H)^{w_j})\right]\right).
\end{aligned} \tag{13}$$

Theorem 3.18. *Let the generalization parameter be $\beta_i(e) = ([1,1],[0,0])$. Then, there is*

$$GWG(a_{i1}, a_{i2}, ..., a_{im}) = IVIFWG(a_{i1}, a_{i2}, ..., a_{im}). \tag{14}$$

Proof. According to Definition 3.17, (13) with $\beta_i(e) = ([1,1],[0,0])$ can be obtained as follows.

$$\begin{aligned}
&GWG(a_{i1}, a_{i2}, ..., a_{im})\\
&= ([1,1],[0,0]) \otimes IVIFWG(a_{i1}, a_{i2}, ..., a_{im})\\
&= \left(\left[1 \cdot \prod_{j=1}^{m} (\mu_{a_{ij}}^L)^{w_j},\ 1 \cdot \prod_{j=1}^{m} (\mu_{a_{ij}}^H)^{w_j}\right], \left[0 + (1 - \prod_{j=1}^{m} (1-\upsilon_{a_{ij}}^L)^{w_j}) -\right.\right.\\
&\quad \left.\left. 0 \cdot (1 - \prod_{j=1}^{m} (1-\upsilon_{a_{ij}}^L)^{w_j}), 0 + (1 - \prod_{j=1}^{m} (1-\upsilon_{a_{ij}}^H)^{w_j}) - 0 \cdot (1 - \prod_{j=1}^{m} (1-\upsilon_{a_{ij}}^H)^{w_j})\right]\right)\\
&= \left(\left[\prod_{j=1}^{m} (\mu_{a_{ij}}^L)^{w_j},\ \prod_{j=1}^{m} (\mu_{a_{ij}}^H)^{w_j}\right], \left[1 - \prod_{j=1}^{m} (1-\upsilon_{a_{ij}}^L)^{w_j},\ 1 - \prod_{j=1}^{m} (1-\upsilon_{a_{ij}}^H)^{w_j}\right]\right).
\end{aligned}$$

where $IVIFWG(a_{i1}, a_{i2}, ..., a_{im})$ is defined in [31]. □

Theorem 3.19. *Let the generalization parameter be* $\beta_i(e) = ([0,0],[1,1])$. *Then, there is*

$$GWG(a_{i1}, a_{i2}, ..., a_{im}) = ([0,0],[1,1]). \tag{15}$$

Proof. According to Definition 3.17, (13) with $\beta_i(e) = ([0,0],[1,1])$ can be obtained as follows.

$$\begin{aligned}
&GWG(a_{i1}, a_{i2}, ..., a_{im}) \\
&= ([0,0],[1,1]) \otimes IVIFWG(a_{i1}, a_{i2}, ..., a_{im}) \\
&= \Bigg(\Bigg[0 \cdot \prod_{j=1}^{m} (\mu_{a_{ij}}^{L})^{w_j},\ 0 \cdot \prod_{j=1}^{m} (\mu_{a_{ij}}^{H})^{w_j}\Bigg], \Bigg[1 + (1 - \prod_{j=1}^{m} (1 - \upsilon_{a_{ij}}^{L})^{w_j}) - \\
&\quad 1 \cdot (1 - \prod_{j=1}^{m} (1 - \upsilon_{a_{ij}}^{L})^{w_j}), 1 + (1 - \prod_{j=1}^{m} (1 - \upsilon_{a_{ij}}^{H})^{w_j}) -\ 1 \cdot (1 - \prod_{j=1}^{m} (1 - \upsilon_{a_{ij}}^{H})^{w_j})\Bigg]\Bigg) \\
&= ([0,0],[1,1]).
\end{aligned}$$

□

In the more complicated environment, the GIVIFSS with one senior expert still has some disadvantages, i.e., some decision-making preference is introduced by his/her own knowledge and comprehension. Hence, we further propose a group generalized interval-valued intuitionistic fuzzy soft set (G-GIVIFSS) by extending GIVIFSS with a group of superior experts to reduce the impact of individual and make an appropriate decision.

4. Group Generalized Interval-valued Intuitionistic Fuzzy Soft Set (G-GIVIFSS)

Definition 4.1. Let $U = \{a_1, a_2, ..., a_n\}$ denote a universal set of elements, $E = \{e_1, e_2, ..., e_m\}$ be a set of parameters and $C = \{\beta_1, \beta_2, ..., \beta_l\}$ be an intuitionistic fuzzy subset of E. The pair (U, E) is called a soft universe. Let $A \subseteq E$, F be a mapping given by $F : A \to IVIF(U)$ and F_C be another mapping given by $F_C : A \to IVIF(U) \times IVIF$.

A group generalized interval-valued intuitionistic fuzzy soft set (G-GIVIFSS) F_C over the soft universe (U, E) is defined as

$$F_C(e) = (F(e), C_\beta(e)), \tag{16}$$

where $F(e) \in IVIF(U)$ and $C_\beta(e) \in IVIF$. $F(e)$ represents the elements of U in the IVIFSS, and $C_\beta(e)$ indicates the senior experts' assessments on the elements of U in $F_C(e)$.

Definition 4.2. Let $F_C = (F(e), C(e))$ and $G_D = (G(e), D(e))$ be two G-GIVIFSSs over the universe (U, E). F_C is called as the subset of G_D, i.e. $F_C \subseteq G_D$, if

i) $C(e)$ is an IVIF subset of $D(e)$, $\forall e \in E$, and

ii) $F(e)$ is an IVIF subset of $G(e)$, $\forall e \in E$.

Definition 4.3. Let $F_C = (F(e), C(e))$ and $G_D = (G(e), D(e))$ be two G-GIVIFSSs over the universe (U, E). F_C and G_D is called equal, i.e. $F_C = G_D$, if

i) $G_D(e)$ is an IVIF subset of $F_C(e)$, $\forall e \in E$, and

ii) $F_C(e)$ is an IVIF subset of $G_D(e)$, $\forall e \in E$.

Definition 4.4. The complement of F_C is presented by F_C^c and given as

$$F_C^c = (F^c(e), C^c(e)), \forall e \in E. \tag{17}$$

Definition 4.5. The union of F_C and G_D denoted by $F_C \cup G_D$ is given by

$$F_C \cup G_D = (F(e) \circ G(e), C(e) \circ D(e)), \tag{18}$$

where $F(e) \circ G(e) \in IVIF(U)$, $C(e) \circ D(e) \in IVIF$, $\forall e \in E$; $\circ$ is any t-conorm.

Definition 4.6. The intersection of F_C and G_D, denoted by $F_C \cap G_D$, is given by

$$F_C \cap G_D = (F(e) * G(e), C(e) * D(e)), \tag{19}$$

where $F(e) * G(e) \in IVIF(U)$, $C(e) * D(e) \in IVIF$, $\forall e \in E$; $*$ is any t-norm.

Definition 4.7. A G-GIVIFSS $\Phi_O(e) = (F(e), C(e))$ is regarded as a null group generalized intuitionistic fuzzy soft set, if $F(e) = ([0,0], [1,1])$, $C(e) = ([0,0], [1,1])$, $\forall e \in E$.

Definition 4.8. A G-GIVIFSS $\Omega_I(e) = (F(e), C(e))$ is regarded as an absolute group generalized intuitionistic fuzzy soft set, if $F(e) = ([1,1], [0,0])$, $C(e) = ([1,1], [0,0])$, $\forall e \in E$.

Definition 4.9. Let $F_{C_i} = (F_i(e), C_i(e))$ be any G-GIVIFSS over (U, E), where $F_i(e) = \{a_{i1}, a_{i2}, ..., a_{im}\}$, $C_i(e) = \{\beta_{i1}, \beta_{i2}, ..., \beta_{il}\}$, and their weight vectors are $w = \{w_1, w_2, ..., w_m\}^T$ and $\nu = \{\nu_1, \nu_2, ..., \nu_l\}^T$, respectively. The G-GIVIFSS weighted averaging (GGWA) operator is defined as

$$\begin{aligned}
&GGWA(a_{i1}, a_{i2}, ..., a_{im}) \\
&= IVIFWA(\beta_{i1}, \beta_{i2}, ..., \beta_{il}) \otimes IVIFWA(a_{i1}, a_{i2}, ..., a_{im}) \\
&= \Bigg(\Bigg[(1 - \prod_{k=1}^{l} (1 - \mu_{\beta_{ik}}^L)^{\nu_k}) \cdot (1 - \prod_{j=1}^{m} (1 - \mu_{a_{ij}}^L)^{w_j}),\ (1 - \prod_{k=1}^{l} (1 - \mu_{\beta_{ik}}^H)^{\nu_k}) \cdot (1 - \\
&\quad \prod_{j=1}^{m} (1 - \mu_{a_{ij}}^H)^{w_j})\Bigg], \Bigg[\prod_{k=1}^{l} (\upsilon_{\beta_{ik}}^L)^{\nu_k} + \prod_{j=1}^{m} (\upsilon_{a_{ij}}^L)^{w_j} - \prod_{k=1}^{l} (\upsilon_{\beta_{ik}}^L)^{\nu_k} \cdot \prod_{j=1}^{m} (\upsilon_{a_{ij}}^L)^{w_j}, \\
&\quad \prod_{k=1}^{l} (\upsilon_{\beta_{ik}}^H)^{\nu_k} + \prod_{j=1}^{m} (\upsilon_{a_{ij}}^H)^{w_j} - \prod_{k=1}^{l} (\upsilon_{\beta_{ik}}^H)^{\nu_k} \cdot \prod_{j=1}^{m} (\upsilon_{a_{ij}}^H)^{w_j}\Bigg]\Bigg).
\end{aligned} \tag{20}$$

Theorem 4.10. *Let the group generalization parameter be* $C_i(e) = (\beta_{ik} | ([1,1], [0,0]),$ $k = 1, 2, ..., l)$. *Then, there is*

$$GGWA(a_{i1}, a_{i2}, ..., a_{im}) = IVIFWA(a_{i1}, a_{i2}, ..., a_{im}) \tag{21}$$

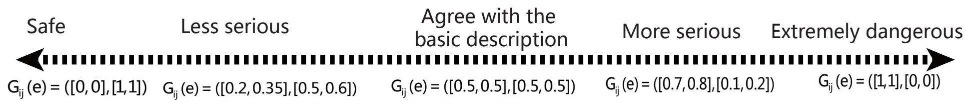

FIGURE 1. The Generalization Parameter with the Consideration of Current Situation

Theorem 4.11. *Let the group generalization parameter be* $C_i(e) = (\beta_{ik}|([0,0],[1,1]), k = 1,2,...,l)$. *Then, there is*

$$GGWA(a_{i1}, a_{i2}, ..., a_{im}) = ([0,0],[1,1]). \tag{22}$$

Proof. According to Definition 4.9, we can get Theorem 4.10 and Theorem 4.11. □

The multi-attribute group decision making is playing an important role in modern decision theory. The researchers achieve outstanding results in a variety of fields[9, 13, 14, 21, 23, 24, 25, 27, 33, 34, 35, 37]. The threat assessment of aerial targets also is an important application of MAGDM theory and methods.

5. A MAGDM Model Based on GGWA Operator

In this section, a MAGDM model based on GGWA operator is built for threat assessment in IVIF environment. In this application, we extend the concept of original G-GIVIFSS. It contains two parts, the basic description of targets calculated directly with original data and the generalization parameters provided by three experts which reflect their attitudes on the description in the current situation. In the threat assessment, there is a collection of targets $\{a_1, a_2, ..., a_n\}$ and a collection of attributes $\{e_1, e_2, ..., e_m\}$. The evaluation of each target against the attribute is given as $a_{ij}(e)$. A group of experts $\{\beta_1, \beta_2, ..., \beta_l\}$ provide generalization parameters of description, i.e. $\beta_{ij}(e)$. The steps of this method are described as follows.

Step 1: Build a basic description matrix $[F]_{n\times m}$ of the alternatives with IVIFSs.

Step 2: Obtain the evaluation matrix $[C_\beta]_{n\times l}$ of the alternatives according to l experts. The values are given by following the rule, which is shown in Figure 1.

- When $\beta_{ij}(e) = ([0.5, 0.5], [0.5, 0.5])$, it means that the expert absolutely agrees with the basic description of the target.
- When $\beta_{ij}(e) = ([1, 1], [0, 0])$, it means that the situation is extremely serious in the expert's opinion.
- When $\beta_{ij}(e) = ([0, 0], [1, 1])$, it means that it is safe, and the threat can be ignored in the expert's opinion.

The experts give generalization parameters for the basic description with considering the current situation. The generalization parameter is an interval-valued intuitionistic fuzzy set. Its membership function $[\mu_\beta^L, \mu_\beta^H]$ represents the expert with a positive attitude to the basic description of targets. The larger membership function given by the expert, means that the target in the situation is more aggressive than its basic description. Meanwhile the non-membership function $[\upsilon_\beta^L, \upsilon_\beta^H]$ represents the expert with a negative attitude to the basic description of targets. The larger non-membership function given by the expert, means that the target in the situation

	e_1	e_2	e_3	e_4	e_5
a_1	([0.92,0.96], [0,0.02])	([0.928,1], [0,0])	([1,1], [0,0])	([0.734,0.746], [0.21,0.25])	([0.98,1], [0,0])
a_2	([0.52,0.56], [0.29,0.32])	([0.563,0.662], [0.197,0.261])	([0.922,0.946], [0.03,0.039])	([0.676,0.702], [0.199,0.253])	([0.543,0.588], [0.314,0.336])
a_3	([0.78,0.82], [0.06,0.10])	([0.245,0.296], [0.470,0.585])	([0.754,0.789], [0.162,0.196])	([0.333,0.343], [0.482,0.519])	([0.677,0.712], [0.245,0.251])
a_4	([0.52,0.56], [0.395,0.42]	([0.035,0.04], [0.692,0.783])	([0.882,0.933], [0.03,0.06])	([0.553,0.612], [0.267,0.312])	([0.962,0.969], [0,0.02])
a_5	([0.38,0.42], [0.331,0.36])	([0.721,0.847], [0.104,0.142])	([0.963,0.970], [0,0.02])	([0.131,0.208], [0.491,0.526])	([0.961,1], [0,0])
a_6	([0.02,0.06], [0.82,0.86])	([0.288,0.292], [0.472,0.586])	([0.812,0.843], [0.122,0.131])	([0.299,0.325], [0.487,0.558])	([0.676,0.685], [0.225,0.249])

TABLE 1. The Basic Description of Aerial Targets

is less aggressive than its basic description. When $[\mu_\beta^L, \mu_\beta^H] = [\upsilon_\beta^L, \upsilon_\beta^H] = [0.5, 0.5]$, it represents that the expert agrees with the basic description. In other words, there is no extra suggestion for the basic description.

Step 3: Build a G-GIVIFSS matrix $[F_C]_{n\times(m+l)}$, which contains the basic information of alternatives and the views of experts.

Step 4: Compute the aggregated evaluation $GGWA(a_{i1}, a_{i2}, ..., a_{im})$ of each alternative a_i by applying (20).

Step 5: Calculate the score of $GGWA(a_{i1}, a_{i2}, ..., a_{im})$ by (3). Then, the final results are ranked by the values.

6. Practical Example Validation

6.1. **An Example for Threat Assessment.** Assume there are six different aerial targets $F = \{a_1, a_2, a_3, a_4, a_5, a_6\}$ and each of them has five attributes $E = \{e_1, e_2, e_3, e_4, e_5\}$ in this threat assessment, such as Target type, Flight time, Flight velocity, Short-cut route and Flight height in Table 1, and their weight vector is $w = \{0.19, 0.17, 0.19, 0.23, 0.22\}^T$. Three experts $C = \{\beta_1, \beta_2, \beta_3\}$ make their assessments on the basic description of each alternative, by considering the current situation. The weight vector of experts is $\nu = \{0.4, 0.3, 0.3\}^T$.

The MAGDM method based on the proposed GGWA operator is given to solve this threat assessment of aerial targets as follows:

Step 1: Acquire the basic description matrix $[F]_{n\times m}$ of the alternatives with IVIFSs, as shown in Table 1.

Step 2: Obtain the evaluation matrix $[C_\beta]_{n\times l}$ of the alternatives by three experts, shown in Table 2.

Step 3: Build a new G-GIVIFSS matrix $[F_C]_{n\times(m+l)}$.

Step 4: Compute the aggregated evaluation $GGWA(a_{i1}, a_{i2}, ..., a_{im})$ of alternative a_i by applying (20).

$$GGWA(a_{11}, a_{12}, ..., a_{15}) = ([0.9305, 1.0000], [0, 0]),$$

$$GGWA(a_{21}, a_{22}, ..., a_{25}) = ([0.5280, 0.5941], [0.2785, 0.3303]),$$

$$GGWA(a_{31}, a_{32}, ..., a_{35}) = ([0.3899, 0.4503], [0.3956, 0.4690]),$$

	β_1	β_2	β_3
a_1	([0.92,0.95],[0,0.05])	([0.96,1],[0,0])	([0.90,0.93],[0,0.05])
a_2	([0.71,0.76],[0.16,0.18])	([0.82,0.85],[0.1,0.12])	([0.74,0.78],[0.15,0.20])
a_3	([0.62,0.68],[0.20,0.26])	([0.65,0.7],[0.20,0.25])	([0.65,0.70],[0.27,0.30])
a_4	([0.78,0.82],[0,0.09])	([0.84,0.88],[0.07,0.10])	([0.62,0.66],[0.30,0.34])
a_5	([0.75,0.78],[0.20,0.22])	([0.80,0.85],[0,0.15])	([0.68,0.73],[0.21,0.24])
a_6	([0.82,0.86],[0.08,0.15])	([0.96,1],[0,0])	([0.9,0.92],[0,0.06])

Table 2. The Generalization Parameter Matrix Given by Three Experts

$$GGWA(a_{41}, a_{42}, ..., a_{45}) = ([0.5057, 0.5790], [0.2241, 0.3616]),$$

$$GGWA(a_{51}, a_{52}, ..., a_{55}) = ([0.6088, 0.7915], [0, 0.2013]),$$

$$GGWA(a_{61}, a_{62}, ..., a_{65}) = ([0.3030, 0.3424], [0.4721, 0.5331]).$$

Step 5: Calculate the score of $GGWA(a_{i1}, a_{i2}, ..., a_{im})$ by (3).

$$S_{GG}(a_1) = 0.9976, S_{GG}(a_2) = 0.6400, S_{GG}(a_3) = 0.4918,$$

$$S_{GG}(a_4) = 0.6257, S_{GG}(a_5) = 0.8009, S_{GG}(a_6) = 0.3966.$$

Then, the complete ranking is $a_1 > a_5 > a_2 > a_4 > a_3 > a_6$.

In order to validate the superiority of the proposed method, some comparisons are provided, including the multi-attribute decision making (MADM) method based on the IVIFWA operator [31], the MADM method based on GWA operator, the MAGDM method based on the interval-valued intuitionistic fuzzy Hamacher weighted averaging (IVIFHWA) operator [14] and the MAGDM method based on the interval-valued intuitionistic fuzzy continuous weighted entropy (IVIFCWE) [13]. The same data in Table 1 are used in all these comparative methods to compare the results on a common basis.

IVIFWA:[31] Without any additional judgments of experts, the example is a typical multi-attribute decision making problem. According to [31], we obtain the aggregated values by applying (2), and compute their scores by (3).

$$S(a_1) = 1.0000, S(a_2) = 0.7875, S(a_3) = 0.7032,$$

$$S(a_4) = 0.7041, S(a_5) = 0.9826, S(a_6) = 0.5807.$$

Then, the complete ranking is $a_1 > a_5 > a_2 > a_3 \approx a_4 > a_6$.

GWA: If only one expert gives his/her evaluation as a generalization parameter, we can solve the problem with the GWA of GIVIFSS in Section 3. To make a comparison, each expert's evaluation is obtained in Table 2.

- **GWA-1 with the first expert's evaluation**

  We obtain the aggregated values by applying (10), and compute their scores by (3).

$$S_{G1}(a_1) = 0.9508, S_{G1}(a_2) = 0.6181, S_{G1}(a_3) = 0.4918,$$

$$S_{G1}(a_4) = 0.6235, S_{G1}(a_5) = 0.7658, S_{G1}(a_6) = 0.4024.$$

  Then, the final ranking is $a_1 > a_5 > a_4 > a_2 > a_3 > a_6$.

	e_1	e_2	e_3	e_4	e_5
a_1	([0.8464,0.9120], [0,0.0690])	([0.8538,0.9500], [0,0.0500])	([0.9200,0.9500], [0,0.0500])	([0.6753,0.7087], [0.2100,0.2875])	([0.9016,0.9500], [0,0.0500])
a_2	([0.3692,0.4256], [0.4036,0.4424])	([0.3997,0.5031], [0.3255,0.3940])	([0.6546,0.7190], [0.1852,0.2120])	([0.4800,0.5335], [0.3272,0.3875])	([0.3855,0.4469], [0.4238,0.4555])
a_3	([0.4836,0.5576], [0.2480,0.3340])	([0.1519,0.2013], [0.5760,0.6929])	([0.4675,0.5365], [0.3296,0.4050])	([0.2065,0.2332], [0.5856,0.6441])	([0.4197,0.4842], [0.3960,0.4457])
a_4	([0.4056,0.4592], [0.3950,0.4722])	([0.0273,0.0328], [0.6920,0.8025])	([0.6880,0.7651], [0.0300,0.1446])	([0.4313,0.5018], [0.2670,0.3739])	([0.5164,0.5486], [0.2720,0.3912])
a_5	([0.2850,0.3276], [0.4640,0.5008])	([0.5408,0.6607], [0.2832,0.3308])	([0.7223,0.7566], [0.2000,0.2356])	([0.0983,0.1622], [0.5928,0.6303])	([0.7208,0.7800], [0.2000,0.2200])
a_6	([0.0124,0.0396], [0.8524,0.8950])	([0.1786,0.1927], [0.5670,0.6895])	([0.5034,0.5564], [0.2800,0.3483])	([0.1854,0.2145], [0.5793,0.6685])	([0.4191,0.4521], [0.3645,0.4367])

TABLE 3. IVIF Decision Matrix $\tilde{A}_1$ Provided by β_1

- **GWA-2 with the second expert's evaluation**

  We obtain the aggregated values by applying (10), and compute their scores by (3).

$$S_{G2}(a_1) = 0.9992, S_{G2}(a_2) = 0.6798, S_{G2}(a_3) = 0.5023,$$

$$S_{G2}(a_4) = 0.6262, S_{G2}(a_5) = 0.8414, S_{G2}(a_6) = 0.3963.$$

  Then, the final ranking is $a_1 > a_5 > a_2 > a_4 > a_3 > a_6$.

- **GWA-3 with the third expert's evaluation**

  We obtain the aggregated values by applying (10), and compute their scores by (3).

$$S_{G3}(a_1) = 0.9500, S_{G3}(a_2) = 0.6185, S_{G3}(a_3) = 0.4806,$$

$$S_{G3}(a_4) = 0.6033, S_{G3}(a_5) = 0.7207, S_{G3}(a_6) = 0.3865.$$

  Then, the final ranking is $a_1 > a_5 > a_2 > a_4 > a_3 > a_6$.

Both comparative methods, IVIFHWA and IVIFCWE, are based on the evaluations by each expert directly. Hence, each value in Table 1 has to be processed by GIVIFSS for fair comparison, such as $\tilde{a}_{ij} = a_{ij} \otimes \beta_{ik} (i = 1, 2, ..., 6; j = 1, 2, ..., 5; k = 1, 2, 3)$ in Tables. 3-5. However, in practice, these evaluations are not completely similar to the values in Tables. 3-5.

IVIFHWA:[14] First, because all the measured values are in the same type, each IVIF decision matrix is aggregated into the collective IVIF decision matrix $\tilde{A} = [\tilde{a}_{ij}]_{n\times m}$ by applying (23). Furthermore, the collective overall preference values $\tilde{a}_i (i = 1, 2, ..., n)$ are computed by (24).

$$\tilde{a}_{ij} = IVIFHWA(\tilde{a}_{ij}^1, \tilde{a}_{ij}^2, \tilde{a}_{ij}^3), \tag{23}$$

$$\tilde{a}_i = IVIFHWA(\tilde{a}_{i1}, \tilde{a}_{i2}, ..., \tilde{a}_{im}). \tag{24}$$

1) When $\gamma = 1$, the collective overall preference values $\tilde{a}_i (i = 1, 2, ..., n)$ by (24) are denoted as

$$\tilde{a}_1 = ([0.8648, 1.0000], [0, 0]), \tilde{a}_2 = ([0.5005, 0.5640], [0.3038, 0.3569]),$$

	e_1	e_2	e_3	e_4	e_5
a_1	([0.8832,0.9600], [0,0.0200])	([0.8909,1.0000], [0,0])	([0.9600,1.0000], [0,0])	([0.7046,0.7460], [0.2100,0.2500])	([0.9408,1.0000], [0,0])
a_2	([0.4264,0.4760], [0.3610,0.4016])	([0.4617,0.5627], [0.2773,0.3497])	([0.7560,0.8041], [0.1270,0.1543])	([0.5543,0.5967], [0.2791,0.3426])	([0.4453,0.4998], [0.3826,0.4157])
a_3	([0.5070,0.5740], [0.2480,0.3250])	([0.1593,0.2072], [0.5760,0.6888])	([0.4901,0.5523], [0.3296,0.3970])	([0.2165,0.2401], [0.5856,0.6393])	([0.4401,0.4984], [0.3960,0.4383])
a_4	([0.4368,0.4928], [0.4374,0.4780])	([0.0294,0.0352], [0.7136,0.8047])	([0.7409,0.8210], [0.0979,0.1540])	([0.4645,0.5386], [0.3183,0.3808])	([0.5561,0.5887], [0.3230,0.3979])
a_5	([0.3040,0.3570], [0.3300,0.4560])	([0.5768,0.7200], [0.1040,0.2707])	([0.7704,0.8245], [0,0.1670])	([0.1048,0.1768], [0.4910,0.5971])	([0.7688,0.8500], [0,0.1500])
a_6	([0.0112,0.0378], [0.8560,0.8880])	([0.1613,0.1840], [0.5776,0.6688])	([0.4547,0.5311], [0.2976,0.3048])	([0.1674,0.2048], [0.5896,0.6464])	([0.3786,0.4316], [0.3800,0.3992])

TABLE 4. IVIF Decision Matrix $\tilde{A}_2$ Provided by β_2

	e_1	e_2	e_3	e_4	e_5
a_1	([0.8280,0.8928], [0,0.0690])	([0.8352,0.9300], [0,0.0500])	([0.9000,0.9300], [0,0.0500])	([0.6606,0.6938], [0.2100,0.2875])	([0.8820,0.9300], [0,0.0500])
a_2	([0.3848,0.4368], [0.3965,0.4560])	([0.4166,0.5164], [0.3174,0.4088])	([0.6823,0.7379], [0.1755,0.2312])	([0.5002,0.5476], [0.3192,0.4024])	([0.4018,0.4586], [0.4169,0.4688])
a_3	([0.5070,0.5740], [0.3138,0.3700])	([0.1593,0.2072], [0.6131,0.7095])	([0.4901,0.5523], [0.3883,0.4372])	([0.2165,0.2401], [0.6219,0.6633])	([0.4401,0.4984], [0.4489,0.4757])
a_4	([0.4264,0.4816], [0.4555,0.5012])	([0.0287,0.0344], [0.7228,0.8134])	([0.7232,0.8024], [0.1270,0.1916])	([0.4535,0.5263], [0.3403,0.4083])	([0.5428,0.5753], [0.3448,0.4247])
a_5	([0.2584,0.3066], [0.4841,0.5264])	([0.4903,0.6183], [0.3101,0.3651])	([0.6548,0.7081], [0.2300,0.2748])	([0.0891,0.1518], [0.6081,0.6492])	([0.6535,0.7300], [0.2300,0.2600])
a_6	([0.0120,0.0372], [0.8560,0.8964])	([0.1728,0.1810], [0.5776,0.6936])	([0.4872,0.5227], [0.2976,0.3569])	([0.1794,0.2015], [0.5896,0.6729])	([0.4056,0.4247], [0.3800,0.4443])

TABLE 5. IVIF Decision Matric $\tilde{A}_3$ Provided by β_3

$$\tilde{a}_3 = ([0.3711, 0.4288], [0.4240, 0.4925]), \tilde{a}_4 = ([0.4798, 0.5432], [0.2802, 0.3900]),$$
$$\tilde{a}_5 = ([0.5222, 0.6040], [0, 0.3437]), \tilde{a}_6 = ([0.2730, 0.3092], [0.4971, 0.5632]).$$

Then, calculate the scores of $\tilde{a}_i (i = 1, 2, ..., n)$ by (3), and obtain

$$S_1(\tilde{a}_1) = 0.9909, S_1(\tilde{a}_2) = 0.6103, S_1(\tilde{a}_3) = 0.4672,$$
$$S_1(\tilde{a}_4) = 0.5888, S_1(\tilde{a}_5) = 0.7064, S_1(\tilde{a}_6) = 0.3621.$$

The final ranking is $a_1 > a_5 > a_2 > a_4 > a_3 > a_6$.

2) When $\gamma = 2$, the collective overall preference values $\tilde{a}'_i (i = 1, 2, ..., n)$ by (24) are specified as

$$\tilde{a}'_1 = ([0.8630, 1.0000], [0, 0]), \tilde{a}'_2 = ([0.4955, 0.5594], [0.3065, 0.3600]),$$
$$\tilde{a}'_3 = ([0.3641, 0.4206], [0.4296, 0.4985]), \tilde{a}'_4 = ([0.4648, 0.5261], [0.2929, 0.4021]),$$
$$\tilde{a}'_5 = ([0.5011, 0.5841], [0, 0.3522]), \tilde{a}'_6 = ([0.2614, 0.2965], [0.5094, 0.5770]).$$

Then, calculate the scores of $\tilde{a}'_i (i = 1, 2, ..., n)$ by (3), and obtain

$$S_2(\tilde{a}'_1) = 0.9906, S_2(\tilde{a}'_2) = 0.6063, S_2(\tilde{a}'_3) = 0.4597,$$
$$S_2(\tilde{a}'_4) = 0.5740, S_2(\tilde{a}'_5) = 0.6991, S_2(\tilde{a}'_6) = 0.3478.$$

	Ranking result
GGWA	$a_1 > a_5 > a_2 > a_4 > a_3 > a_6$
GWA-1	$a_1 > a_5 > a_4 > a_2 > a_3 > a_6$
GWA-2	$a_1 > a_5 > a_2 > a_4 > a_3 > a_6$
GWA-3	$a_1 > a_5 > a_2 > a_4 > a_3 > a_6$
IVIFWA [31]	$a_1 > a_5 > a_2 > a_3 \approx a_4 > a_6$
IVIFHWA ($\gamma = 1$)[14]	$a_1 > a_5 > a_2 > a_4 > a_3 > a_6$
IVIFHWA ($\gamma = 2$)[14]	$a_1 > a_5 > a_2 > a_4 > a_3 > a_6$
IVIFCWE[13]	$a_1 > a_5 > a_2 > a_4 > a_3 > a_6$

Table 6. The Comparison Results Between the Proposed Method and Comparative Examples

The final ranking is $a_1 > a_5 > a_2 > a_4 > a_3 > a_6$.

IVIFCWE: Jin *et al.* [13] proposed a method based on the weighted relative closeness and the IVIF attitudinal expected score function. The main steps of this approach are shown as follows:

Step 1: Take the BUM function $Q(y) = y$, and the attitudinal character $\lambda = 0.5$. The weights of attributes are $w_1 = 0.1692$, $w_2 = 0.2413$, $w_3 = 0.2504$, $w_4 = 0.1902$ and $w_5 = 0.1490$.

Step 2: Acquire the collective interval-valued intuitionistic fuzzy decision matrix $D = [a_{ij}^d]_{n\times m}$ by (7) in [33].

Step 3: Calculate the attitudinal expected scores of each a_{ij}^d by (32) in [13].

Step 4: Get the weighted relative closeness $T_i (i = 1, 2, ..., 6)$ as follows:

$$T_1 = 0.9188, T_2 = 0.5955, T_3 = 0.4491, T_4 = 0.5224, T_5 = 0.6050, T_6 = 0.3536.$$

The final ranking is $a_1 > a_5 > a_2 > a_4 > a_3 > a_6$.

To make a clearer comparison, the assessment results of different methods are demonstrated in Table 6.

From the assessment comparison results, it can be observed that the proposed method is similar to the last two group decision making methods (i.e., IVIFHWA ($\gamma = 1$), IVIFHWA ($\gamma = 2$) and IVIFCWE), different from GWA with each expert, and significantly different from the IVIFWA. The reasons for these results can be summarized as follows. 1) The IVIFWA is based on the value of each target against each attribute, without considering the current situation. Therefore, its description is incomplete and inaccurate. Then the ranking result of IVIFWA cannot reflect the real threat assessment. 2) GWA is similar to GGWA, including the basic description provided a junior expert and the evaluation given by a senior expert. However, different experts may give different generalization parameters, because of their different personal preferences, knowledge and comprehension. Then the ranking based on GWA-1 is obviously different from the rankings based on GWA-2 and GWA-3, such as the ranking of $a_4 > a_2$ and $a_2 > a_4$. 3) IVIFHWA ($\gamma = 1$), IVIFHWA ($\gamma = 2$) and IVIFCWE are multi-attribute group decision making methods with three experts' evaluation. But it is inconvenient for the expert to present the opinion of each target against each attribute. Furthermore,

	e_1	e_2	e_3	e_4	e_5
a_1	([0.81,0.86], [0.08,0.12])	([0.75,0.80], [0.11,0.15])	([0.78,0.82], [0.07,0.13])	([0.80,0.85], [0.05,0.12])	([0.72,0.76], [0.16,0.21])
a_2	([0.65,0.69], [0.21,0.25])	([0.64,0.69], [0.19,0.26])	([0.90,0.94], [0,0.05])	([0.66,0.72], [0.19,0.24])	([0.55,0.60], [0.28,0.33])
a_3	([0.52,0.56], [0.37,0.42]	([0.68,0.71], [0.19,0.23])	([0.88,0.93], [0,0.05])	([0.65,0.69], [0.26,0.31])	([0.92,0.96], [0,0.02])
a_4	([0.78,0.82], [0.05,0.10])	([0.54,0.59], [0.27,0.35])	([0.74,0.79], [0.14,0.18])	([0.82,0.86], [0.06,0.11])	([0.67,0.71], [0.20,0.25])

Table 7. The Basic Description of Technology Companies

it takes much time for experts to give reasonable evaluation directly, and it is not suitable in aerial target threat assessment, which is very urgent.

According to the practical example, it can be seen that the description of imprecise, incomplete information with G-GIVIFSS is more reasonable and accurate. Moreover, the multi-attribute group decision making method based on G-GIVIFSS weighted averaging (GGWA) operator is easier to operate, which can obviously save experts' time and their energy. Furthermore, the proposed G-GIVIFSS reduces the impact of unfair arguments on the final results by single expert.

6.2. **Extending Application.** In this example, the GGWA operator-based MAGDM method is used for assessing the innovation ability of technology companies. There are four companies $F = \{a_1, a_2, a_3, a_4\}$ as alternatives. The local government plans to choose the best one from these companies and support it by funding. In this survey, five attributes are considered, such as resource commitment, research and development, business management, production capacity and capability in shielding against risks, specified as $E = \{e_1, e_2, e_3, e_4, e_5\}$. The weight vector of these attributes is $w = \{0.17, 0.22, 0.21, 0.19, 0.21\}^T$. Each company against each attribute is shown in Table 7. Three experts $C = \{\beta_1, \beta_2, \beta_3\}$ make their assessments on the basic description of each alternative, by considering environmental protection and energy conservation, in Table 8. The weight vector of experts is $\nu = \{0.35, 0.35, 0.3\}^T$.

Step 1: Acquire the basic description matrix $[F]_{n\times m}$ of the alternatives with IVIFSs, as shown in Table 7.

Step 2: Obtain the evaluation matrix $[C_\beta]_{n\times l}$ of the alternatives by the three experts, shown in Table 8. In addition, the membership function $[\mu_\beta^L, \mu_\beta^H]$ represents the expert with a positive attitude to the basic description of the company against its innovation ability. If the membership function given by the expert is larger, its innovation ability is more than the basic description. Meanwhile, $[\upsilon_\beta^L, \upsilon_\beta^H]$ represents the expert with a negative attitude to the basic description of the company against its innovation ability. If the non-membership function given by the expert is larger, its innovation ability is less than the basic description. When $[\mu_\beta^L, \mu_\beta^H] = [\upsilon_\beta^L, \upsilon_\beta^H] = [0.5, 0.5]$, it represents that the expert agrees with the basic description.

Step 3: Build a new G-GIVIFSS matrix $[F_C]_{n\times(m+l)}$.

	β_1	β_2	β_3
a_1	([0.90,0.95],[0,0.05])	([0.94,0.98],[0,0])	([0.87,0.93],[0,0.05])
a_2	([0.81,0.86],[0.06,0.12])	([0.88,0.93],[0.10,0.15])	([0.72,0.78],[0.15,0.20])
a_3	([0.46,0.52],[0.32,0.36])	([0.53,0.57],[0.36,0.41])	([0.62,0.67],[0.27,0.30])
a_4	([0.78,0.82],[0,0.05])	([0.64,0.68],[0.24,0.30])	([0.72,0.76],[0.20,0.24])

Table 8. The Generalization Parameter Matrix Given by Three Experts

Step 4: Compute the aggregated evaluation $GGWA(a_{i1}, a_{i2}, ..., a_{i5})$ of alternative a_i by applying (20).

$$GGWA(a_{11}, a_{12}, ..., a_{15}) = ([0.7021, 0.7860], [0.0883, 0.1442]),$$
$$GGWA(a_{21}, a_{22}, ..., a_{25}) = ([0.5860, 0.6756], [0.0944, 0.3118]),$$
$$GGWA(a_{31}, a_{32}, ..., a_{35}) = ([0.4232, 0.4966], [0.3169, 0.4321]),$$
$$GGWA(a_{41}, a_{42}, ..., a_{45}) = ([0.5170, 0.5815], [0.1246, 0.3063]).$$

Step 5: Calculate the score of $GGWA(a_{i1}, a_{i2}, ..., a_{i5})$ by (3).

$$S_{GG}(a_1) = 0.8394, S_{GG}(a_2) = 0.7054, S_{GG}(a_3) = 0.5415, S_{GG}(a_4) = 0.6807.$$

Then the ranking order of four potential alternatives is $a_1 > a_2 > a_4 > a_3$. Hence, the best alternative is a_1.

In general, group decision-making methods can reduce the impact of individual preference in decision making. Hence, in this example, IVIFHWA and IVIFCWE are chosen as comparative methods, which are based on the evaluations by each expert directly. Hence, each value in Table 7 has to be processed by GIVIFSS for fair comparison, such as $\tilde{a}_{ij} = a_{ij} \otimes \beta_{ik} (i = 1, 2, ..., 4; j = 1, 2, ..., 5; k = 1, 2, 3)$. For concise and clear the computational process and tables of results are omitted here.

IVIFHWA:[14] the collective overall preference values are computed by (24).

1) When $\gamma = 1$, there are

$$\tilde{a}_1 = ([0.6988, 0.7816], [0.0883, 0.1712]), \tilde{a}_2 = ([0.5658, 0.6495], [0.2316, 0.3317]),$$
$$\tilde{a}_3 = ([0.4011, 0.4637], [0.4113, 0.4736]), \tilde{a}_4 = ([0.5083, 0.5713], [0.2356, 0.3374]).$$

Then, calculate the scores of $\tilde{a}_i (i = 1, 2, 3, 4)$ by (3), and obtain

$$S_2(\tilde{a}_1) = 0.8197, S_2(\tilde{a}_2) = 0.6637, S_2(\tilde{a}_3) = 0.4947, S_2(\tilde{a}_4) = 0.6344.$$

The final ranking is $a_1 > a_2 > a_4 > a_3$.

2) When $\gamma = 2$, there are

$$\tilde{a}'_1 = ([0.6984, 0.7812], [0.0886, 0.1716]), \tilde{a}'_2 = ([0.5616, 0.6450], [0.2347, 0.3342]),$$
$$\tilde{a}'_3 = ([0.3980, 0.4602], [0.4146, 0.4771]), \tilde{a}'_4 = ([0.5060, 0.5690], [0.2406, 0.3419]).$$

Then, calculate the scores of $\tilde{a}'_i (i = 1, 2, ..., n)$ by (3), and obtain

$$S_2(\tilde{a}'_1) = 0.8193, S_2(\tilde{a}'_2) = 0.6604, S_2(\tilde{a}'_3) = 0.4910, S_2(\tilde{a}'_4) = 0.6302.$$

The final ranking is $a_1 > a_2 > a_4 > a_3$.

IVIFCWE: [13] The weighted relative closeness $T_i (i = 1, 2, 3, 4)$ is obtained as follows:

$$T_1 = 0.8067, T_2 = 0.6517, T_3 = 0.4772, T_4 = 0.6299.$$

	Ranking result
GGWA	$a_1 > a_2 > a_4 > a_3$
IVIFHWA ($\gamma = 1$)[14]	$a_1 > a_2 > a_4 > a_3$
IVIFHWA ($\gamma = 2$)[14]	$a_1 > a_2 > a_4 > a_3$
IVIFCWE[13]	$a_1 > a_2 > a_4 > a_3$

Table 9. The Comparison Results Between the Proposed Method and Comparative Examples

The final ranking is $a_1 > a_2 > a_4 > a_3$.

To make a clearer comparison, the assessment results of different methods are demonstrated in Table 9.

From these comparison results, it can be also observed that the proposed method has the same ranking result as the other two group decision making methods, IVIFHWA and IVIFCWE. However, the proposed method is provided generalization parameters for each company as a whole. Whereas, by the comparative methods, the experts take much time and energy to give reasonable evaluation of each company against each attribute one by one. Hence, the multi-attribute group decision making method based on GGWA operator is easier to operate, which saves experts' time and their energy, and reduces the impact of their decision making preference.

7. **Conclusion**

In this paper, we propose a group generalized interval-valued intuitionistic fuzzy soft set (G-GIVIFSS) and discuss its application in decision making. For achieving G-GIVIFSS, a generalized interval-valued intuitionistic fuzzy soft set (GIVIFSS) is firstly proposed, and the important operations, properties and aggregation operators of GIVIFSS are discussed. Then, based on GIVIFSS, G-GIVIFSS is derived by embedding more senior experts to reduce the impact of single expert's preference. The important operations, properties and the weighted averaging operator of G-GIVIFSS are also discussed. Then, we build the multi-attribute group decision making model based on the G-GIVIFSS weighted averaging operator, and two examples about multi-attribute group decision making are provided to illustrate the validity and practicality of the proposed MAGDM model.

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