Iranian Journal of Fuzzy Systems **Vol. 14, No. 1,** (2017) pp. 23-41

SOFT COMPUTING BASED ON A MODIFIED MCDM APPROACH UNDER INTUITIONISTIC FUZZY SETS

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ABSTRACT. The current study set to extend a new VIKOR method as a compromise ranking approach to solve multiple criteria decision-making (MCDM) problems through intuitionistic fuzzy analysis. Using compromise method in MCDM problems contributes to the selection of an alternative as close as possible to the positive ideal solution and far away from the negative ideal solution, concurrently. Using Atanassov intuitionistic fuzzy sets (A-IFSs) may simultaneously express the degree of membership and non-membership to decision makers (DMs) to describe uncertain situations in decision-making problems. The proposed intuitionistic fuzzy VIKOR indicates the degree of satisfaction and dissatisfaction of each alternative with respect to each criterion and the relative importance of each criterion, respectively, by degrees of membership and non-membership. Thus, the ratings for the importance of criteria, DMs, and alternatives are in linguistic variables and expressed in intuitionistic fuzzy numbers. Using IFS aggregation operators and with respect to subjective judgment and objective information, the most suitable alternative is indicated among potential alternatives. Moreover, practical examples illustrate the procedure of the proposed method.

1. Introduction

Multiple criteria decision-making (MCDM) method is a widely used decision-making method [7, 23, 31]. The typical MCDM problem is concerned with the ranking order of decision alternatives according to different conflicting criteria. In practice, it is not possible for alternatives to concurrently satisfy all criteria [13, 46, 3, 22] . In fact, if one alternative obtains a good score with respect to one criterion, it is least likely that in other criteria the same good score is achieved. Alternatives possess their own strengths regarding different criteria that may not be consistent [2, 38, 39, 20, 48].

Different MCDM methods work based on Pareto optimality concept. The VIKOR (Vlse Kriterijumska Optimizacija I Kompromisno Resenje in Serbian) method by Opricovic [27] is one and was developed for multi-attribute optimizations of complex systems; it is a compromise ranking approach for MCDM problems. This method concentrates on the ranking and selecting from a set of alternatives in the presence of conflicting criteria, introducing the multi-criteria ranking index according to the particular measure of closeness to the ideal solution [6]. Compared to

Received: February 2016; Revised: June 2016; Accepted: August 2016

Keywords and phrases: Multiple criteria decision making (MCDM), Decision makers (DMs), Atanassov intuitionistic fuzzy sets (A-IFSs), Intuitionistic fuzzy numbers.

other MCDM methods (e.g., TOPSIS, PROMETHEE, and ELECTRE), VIKOR has its advantages [29, 30]. VIKOR determines a compromise solution accepted by DMs since it provides a maximum group utility for the majority and a minimum of individual regret for the opponent.

MCDM problems have an uncertainty because all MCDM methods operate according to DMs subjective preferences and judgments. They utilize linguistic variables by their knowledge and experience which contain uncertainty in real-life situations since it is difficult to apply precise numbers to describe DMs' assessments. On the other hand, conflicting criteria in MCDM problems cause further uncertainty. Hence, using decision approaches that can handle these uncertainties may be helpful in having realistic and reasonable judgments [25, 21, 42, 14, 40]. Turning their subjective perceptions of a fuzzy concept, in most real-life cases, such as importance or excellence, into crisp numbers may get the DMs into trouble [43, 37, 35, 34]. Thus, to solve MCDM problems in fuzzy environments, a number of MCDM methods using the concept of fuzzy sets have been presented [9, 11, 12, 19, 53, 47, 41, 32, 26].

The notion of intuitionistic fuzzy sets (IFSs) was initiated by Atanassov [4] characterized by a membership function and non-membership function and is the generalization of the concept of fuzzy sets [6], characterized only by a membership function and more suitable for dealing with vagueness compared with fuzzy sets. The ability of describing agreement, disagreement, and hesitancy values in the Atanassov intuitionistic fuzzy set (A-IFS) is a powerful tool to deal with vagueness [24, 36, 16]. It has received high attention in various research fields, especially many complex decision-making problems in which decision information is provided by a DM and is often imprecise or uncertain due to time pressure, lack of data, or DMs limited attention and information processing capabilities.

Since VIKOR is a highly powerful MCDM method, more attention has been attracted by extensions on this method and its applications under fuzzy environments. Chen and Wang [10] provided a rational and systematic process to develop the best alternative and compromise solution under each selection criterion. Chang [8] introduced a modified VIKOR method for multiple criteria analysis to avoid numerical difficulties in solving problems via the traditional VIKOR method. To improve service quality among domestic airlines in Taiwan, James [18] used a modified VIKOR method. Jahan et al. [16] developed a version of VIKOR method, covering all types of criteria with emphasis on compromise solution. Sanayei et al. [2] assessed the ratings and weights expressed by trapezoidal or triangular fuzzy numbers using linguistic values. Aghajani et al. [1] presented an evaluation model based on deterministic data, fuzzy numbers, interval numbers, and linguistic terms, and then they applied a combination of analytic hierarchy process (AHP) and entropy method for attributes weighting in their proposed MCDM method. Cristbal [33] applied VIKOR to select a renewable energy project corresponding to the renewable energy plan launched by Spanish government and combined it with AHP method for weighting the importance of different criteria. Vahdani et al. [45] applied a VIKOR method to solve fuzzy group decision-making problems. Developing two new indices was simultaneously taken into account to consider the relative distance of alternatives from positive and negative ideal solutions. A new index was presented to distinguish alternatives in the assessment process by combining the effect of majority of criteria weights and individual regrets weights; hence, they extended the method for an intuitionistic fuzzy environment. Wu and Yu [49] extended VIKOR for MCDM problems with ratings of alternatives expressed using intuitionistic fuzzy sets, and the weights of the criteria are completely unknown. They established an entropy weight model to determine the weights with respect to a set of criteria represented by IFSs.

According to the existing literature, most studies on VIKOR are under the environment of deterministic data or traditional fuzzy numbers; however, because of its extent, it seems to be almost unknown research field in solving MCDM problems. The purpose of present study is to extend a new VIKOR method for an intuitionistic fuzzy environment. The current paper is organized as follows: In the following, the VIKOR method is briefly introduced. Section 3 contains preliminaries related to IFSs. The proposed VIKOR method is represented in section 4. In section 5, an illustrative example of the proposed method is presented. The paper is concluded in section 6.

2. VIKOR Method

Since Opricovic and Tzeng [29, 28] developed VIKOR method, different researches have been focused on it. VIKOR was extended in different ways in traditional fuzzy or crisp environment. One of the most applicable extensions used in this study is based on Vahdani et al. [45] approach for the fuzzy environment. The procedure by a group of DMs is as follows [45]:

The MCDM problem containing crisp and fuzzy numbers can be expressed in the matrix format for each DM as:

$$\begin{bmatrix} x_{11} & \cdots & x_{1(k-1)} & \tilde{x}_{1k} & \cdots & \tilde{x}_{1n} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{m1} & \cdots & x_{m(k-1)} & \tilde{x}_{mk} & \cdots & \tilde{x}_{mn} \end{bmatrix}$$

Step 1: Establish a group of L DMs. A group of L DMs ($l = 1, 2, \ldots, L$) is assumed, responsible for evaluating m alternatives with respect to each n criterion.

Step 2: Define and describe a set of relevant criteria.

Step 3: Obtain the rating of a potential alternative versus each selected criterion for each DM.

Step 4: Aggregate the ratings of alternatives versus each subjective criterion ($\tilde{x}_{ij}$) and fuzzy weights of selected criteria ($\tilde{w}_j$).

Step 5: Compute the normalized decision matrix. Vector normalization is applied to calculate r_{ij} and $\tilde{r}_{ij}$.

Step 6: Construct the fuzzy weighted normalized decision matrix $\tilde{V} = [\tilde{v}_{ij}]_{m\times n}$. The fuzzy weighted normalized decision matrix is computed through multiplying each matrix column by the fuzzy weight ($\tilde{w}_j$), assigned by pairwise comparisons of elements. Thus,

$$\tilde{v}_{ij} = \tilde{w}_j r_{ij}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, k-1 \tag{1}$$

$$\tilde{v}_{ij} = \tilde{w}_j \tilde{r}_{ij}, \quad i = 1, 2, \ldots, m; \quad j = k, k+1, \ldots, n \tag{2}$$

If the supports of triangular fuzzy numbers do not belong to the interval $[0,1]$, scaling has to transform them back into the interval.

Step 7: Determine the positive ideal and negative ideal solutions. A^* and A^- values are defined as:

$$\begin{aligned} A^* &= n(v_1^*, v_2^*, \ldots, v_{k-1}^*, \tilde{v}_k^*, \tilde{v}_{k+1}^*, \ldots, \tilde{v}_n^*) \\ &= \left\{ \left(\max_i \tilde{v}_{ij} \ \ j \in J \right), \left(\min_i \tilde{v}_{ij} \ \ j \in J' \right) \mid i = 1, 2, \ldots, m \right\} \end{aligned} \tag{3}$$

$$\begin{aligned} A^- &= (v_1^-, v_2^-, \ldots, v_{k-1}^-, \tilde{v}_k^-, \tilde{v}_{k+1}^-, \ldots, \tilde{v}_n^-) \\ &= \left\{ \left(\min_i \tilde{v}_{ij} \ \ j \in J \right), \left(\max_i \tilde{v}_{ij} \ \ j \in J' \right) \mid i = 1, 2, \ldots, m \right\} \end{aligned} \tag{4}$$

Where

$$\max_i \tilde{v}_{ij} = \left(\max_i v_{ij1}, \max_i v_{ij2}, \max_i v_{ij3} \right)$$
$$\min_i \tilde{v}_{ij} = \left(\min_i v_{ij1}, \min_i v_{ij2}, \min_i v_{ij3} \right)$$
$$J = j = 1, 2, \ldots, n \mid j \text{ is associated with benefit criterion}$$
$$J' = j = 1, 2, \ldots, n \mid j \text{ is associated with cost criterion}$$

Step 8: Construct ideal separation matrix (D^*) and anti-ideal separation matrix (D^-).

Step 9: Compute $\mathfrak{h}_i$, $\mathfrak{I}_i$, ζ_i and ξ_i values for $j = 1, 2, \ldots, n$ as follows:

$$\mathfrak{h}_i = \sum_{j=1}^{n} w_j d_{ij}^* \tag{5}$$
$$\mathfrak{I}_i = \max_j \bar{w}_j d_{ij}^* \tag{6}$$
$$\zeta_i = \sum_{j=1}^{n} w_j d_{ij}^- \tag{7}$$
$$\xi_i = \max_j \bar{w}_j d_{ij}^- \tag{8}$$

Where $\bar{w}_j$ is obtained using the defuzzification method.

Step 10: Compute the values of τ_i and η_i indices.

$$\tau_i = \begin{cases} \dfrac{\mathfrak{I}_i - \mathfrak{I}^+}{\mathfrak{I}^- - \mathfrak{I}^+} & \text{if } \mathfrak{h}^- = \mathfrak{h}^+ \\ \dfrac{\mathfrak{h}_i - \mathfrak{h}^+}{\mathfrak{h}^- - \mathfrak{h}^+} & \text{if } \mathfrak{I}^- = \mathfrak{I}^+ \\ \left(\dfrac{\mathfrak{h}_i - \mathfrak{h}^+}{\mathfrak{h}^- - \mathfrak{h}^+} \right) v + \left(\dfrac{\mathfrak{I}_i - \mathfrak{I}^+}{\mathfrak{I}^- - \mathfrak{I}^+} \right) (1 - v) & \text{otherwise} \end{cases} \tag{9}$$

and

$$\eta_i = \begin{cases} \dfrac{\xi_i - \xi^+}{\xi^- - \xi^+} & \text{if } \zeta^- = \zeta^+ \\ \dfrac{\zeta_i - \zeta^+}{\zeta^- - \zeta^+} & \text{if } \xi^- = \xi^+ \\ \left(\dfrac{\xi_i - \xi^+}{\xi^- - \xi^+} \right) \gamma + \left(\dfrac{\zeta_i - \zeta^+}{\zeta^- - \zeta^+} \right) (1 - \gamma) & \text{otherwise} \end{cases} \tag{10}$$

where

$$\begin{cases} \mathfrak{h}^+ = \min_i \mathfrak{h}_i \\ \mathfrak{h}^- = \max_i \mathfrak{h}_i \end{cases}, \quad \begin{cases} \mathfrak{I}^+ = \min_i \mathfrak{I}_i \\ \mathfrak{I}^- = \max_i \mathfrak{I}_i \end{cases}$$

$$\begin{cases} \xi^+ = \max_i \xi_i \\ \xi^- = \min_i \xi_i \end{cases}, \quad \begin{cases} \zeta^+ = \min_i \zeta_i \\ \zeta^- = \max_i \zeta_i \end{cases}$$

v and γ are regarded as a weight for strategy of the majority of the criteria, whereas $(1-v)$ and $(1-\gamma)$ are individual regret weights. v and γ values of all within the range of 0 to 1 and these strategies can be compromised by $v = 0.5$ and $\gamma = 0.5$.

Step 11: Calculate collective index (CI). CI is calculated by:

$$CI_i = \tau_i + \frac{1}{\eta_{i(B)}} + \Phi_{i'} \tag{11}$$

Where the second term refers to all i for which $\eta_i > 0$ while $\Phi_{i'}$ refers to all i' for which $\eta_i = 0$ and $\Phi_{i'} = (\min_i \eta_{i(B)})^{\min_j w_j}$.

Step 12: Rank the preference order. The best satisfactory alternative can be determined according to preference rank order of τ_i and η_i. The minimum value of CI indicates the better performance for alternative i.

3. Preliminaries on Intuitionistic Fuzzy Sets

As a generalization of fuzzy set, intuitionistic fuzzy sets (IFSs) assign to each element a membership degree and a non-membership degree; therefore, it is powerful in dealing with uncertainty, imprecision, and vagueness. Atanassov [4] defined IFSs as follows:

Definition 3.1. Let a set X be fixed, the concept of IFS A on X as follows:

$$A = \{< x, \mu_A(x), _A(x) > \mid x \in X\} \tag{12}$$

Where functions $\mu_A(x)$ and $v_A(x)$ denote the degrees of membership and non-membership of the element to the set A, respectively, where $\mu_A : X \to [0,1]$ and $v_A : X \to [0,1]$ denote, respectively, the membership function and non-membership function of A with the condition $0 \le \mu_A(x) + v_A(x) \le 1$ for any $x \in X$. For each IFS A in X,

$$\pi_A(x) = 1 - \mu_A(x) - _A(x) \tag{13}$$

is called the degree of indeterminacy of x to A, or the degree of hesitancy of x to A.

For the convenience of computation, Xu and Yager [51] called $\alpha = (\mu_\alpha, _\alpha, \pi_\alpha)$ an intuitionistic fuzzy number (IFN), where

$$\mu_\alpha \in [0,1], _\alpha \in [0,1], \quad \mu_\alpha(x) + _\alpha(x) \le 1 \tag{14}$$

$$\pi_\alpha = 1 - \mu_\alpha - v_\alpha \tag{15}$$

For an IFN $\alpha = (\mu_\alpha, _\alpha, \pi_\alpha)$, if the value μ_α gets larger and the value v_α gets smaller, IFN increases, and thus from (4), $\alpha^+ = (1,0,0)$ and $\alpha^- = (0,1,0)$ are the largest and smallest IFNs, respectively.

Definition 3.2. Xu and Yager [51] and Xu [52] introduced some basic operations for two IFNs α_1 and α_2:

1) $\alpha_1 \oplus \alpha_2 = (\mu_{\alpha_1} + \mu_{\alpha_2} - \mu_{\alpha_1}\mu_{\alpha_2}, v_{\alpha_1}v_{\alpha_2})$;
2) $\alpha_1 \oplus \alpha_2 = (\mu_{\alpha_1}\mu_{\alpha_2}, v_{\alpha_1} + v_{\alpha_2} - v_{\alpha_1}v_{\alpha_2})$;
3) $\lambda\alpha_1 = (1 - (1 - \mu_{\alpha_1})^{\lambda}, {}_{\alpha_1})^{\lambda})$, $\lambda > 0$;
4) $\alpha_1^{\lambda} = (\mu_{\alpha_1})^{\lambda}, 1 - (1 - v_{\alpha_1})^{\lambda})$, $\lambda > 0$;
5) $\alpha_1 \wedge \alpha_2 = (\min\{\mu_{\alpha_1}, \mu_{\alpha_2}\}, \max\{{}_{\alpha_1}, {}_{\alpha_2}\})$;
6) $\alpha_1 \vee \alpha_2 = (\max\{\mu_{\alpha_1}, \mu_{\alpha_2}\}, \min\{{}_{\alpha_1}, {}_{\alpha_2}\})$;

Where $\wedge = \max$ and $\vee = \min$.

Xu and Yager [51] introduced a deviation factor $s(\alpha) = \mu_\alpha - {}_\alpha$, which is called a score of $\alpha.h(\alpha) = \mu_\alpha + {}_\alpha$ is called the accuracy degree of IFN α. Xu and Yager [51] gave an order relation between two IFNs α_1 and α_2 based on the score function s and the accuracy function h.

Definition 3.3.

$$\begin{aligned}
&\text{If } S(\alpha_1) < S(\alpha_2), \text{ then } \alpha_1 < \alpha_2;\\
&\text{If } S(\alpha_1) = S(\alpha_2), \text{ then}\\
&\qquad\qquad \text{If } h(\alpha_1) < h(\alpha_2), \text{ then } \alpha_1 = \alpha_2;\\
&\qquad\qquad \text{If } h(\alpha_1) < h(\alpha_2), \text{ then } \alpha_1 < \alpha_2.
\end{aligned}$$

4. Proposed Decision Method with the Intuitionistic Fuzzy Information

Assume a committee of LDMs $(D_1, D_2, \ldots, D_L)$, and for each DM, the MCDM problem is represented by an intuitionistic fuzzy decision matrix R as follows:

$$R = \begin{bmatrix} x_{11} & \cdots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \cdots & x_{mn} \end{bmatrix} \tag{16}$$

Let A be a set of alternatives and C a set of criteria, where $A = \{A_1, A_2, \ldots, A_m\}$ and $C = \{C_1, C_2, \ldots, C_n\}$. Assume that the characteristics of the alternative A_i are presented by IFS as $A_i = \{(C_1, \mu_{i1}, {}_{i1}), (C_2, \mu_{i2}, {}_{i2}), \ldots, (C_n, \mu_{in}, {}_{in})\}$, $i = 1, 2, \ldots, m$, where μ_{ij} indicates the degree to which alternative A_i satisfies criterion C_j; ${}_{ij}$ indicates the degree to which alternative A_i does not satisfy criterion $C_j({}_{ij}, {}_{ij}), i = 1, 2, \ldots, m;\ j = 1, 2, \ldots, n$.

The steps of the proposed VIKOR method for solving the MCDM problems are described as follows:

Step 1: A group of L DMs $(l = 1, 2, \ldots, L)$ is assumed responsible for evaluating alternatives with respect to each n criterion. The relative importance of DMs is evaluated in linguistic terms represented by IFNs. The weight of an IFND$_k$ = $[\mu_k, {}_k, \pi_k]$ which rates the kth DM is computed by Boran et al. [6]:

$$\lambda_k = \frac{\left(\mu_k + \pi_k\left(\dfrac{\mu_k}{\mu_k + v_k}\right)\right)}{\sum_{k=1}^{L}\left(\mu_k + \pi_k\left(\dfrac{\mu_k}{\mu_k + v_k}\right)\right)} \tag{17}$$

and

$$\sum_{k=1}^{L} \lambda_k = 1$$

Step 2: Define all objective and subjective criteria with benefit or cost types and based on these criteria, identify the alternatives.

Step 3: Determine the alternatives rating matrix $R^{(k)} = \left(r_{ij}^{(k)}\right)$ for each DM versus each criterion via linguistic terms.

Step 4: Construct an aggregated intuitionistic fuzzy decision matrix $R = (r_{ij})_{m\times n}$ and aggregate the weight of each criterion w_j based on DMs opinions applying IFWA operator proposed by Xu [50].

$$\begin{aligned} r_{ij} =& IFWA_\lambda\left(r_{ij}^{(1)}, r_{ij}^{(2)}, \ldots, r_{ij}^{(L)}\right) \\ =& \lambda_1 r_{ij}^{(1)} \oplus \lambda_2 r_{ij}^{(2)} \oplus \ldots \oplus \lambda_L r_{ij}^{(L)} \\ =& \left[1 - \Pi_{k=1}^{L}(1-\mu_{ij}^{(k)})^{\lambda_k}, \Pi_{k=1}^{L}(v_{ij}^{(k)})^{\lambda_k}, \Pi_{k=1}^{L}(1-\mu_{ij}^{(k)})^{\lambda_k} - \Pi_{k=1}^{L}(v_{ij}^{(k)})^{\lambda_k}\right] \end{aligned} \tag{18}$$

Where $\lambda = \{\lambda_1, \lambda_2 \ldots, \lambda_L\}$ is the weight of each DM and $\sum_{k=1}^{L} \lambda_k = 1, \lambda_k \in [0,1]$. Here $r_{ij} = (\mu_{ri}(x_j), {}_{ri}(x_j), \pi_{ri}(x_j))$, $i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n$.

Let $w_j^{(k)} = [\mu_j^{(k)}, {}_j^{(k)}, \pi_j^{(k)}]$ be an IFN assigned to criterion C_j by the kth DM. Then, the weight of each criterion is calculated as follows:

$$\begin{aligned} w_j &= IFWA_\lambda\left(w_j^{(1)}, w_j^{(2)}, \ldots, w_j^{(L)}\right) = \lambda_1 w_j^{(1)} \oplus \lambda_2 w_j^{(2)} \oplus \cdots \oplus \lambda_L w_j^{(L)} \\ &= \left[1 - \Pi_{k=1}^{L}(1-\mu_j^{(k)})^{\lambda_k}, \Pi_{k=1}^{L}({}_j^{(k)})^{\lambda_k}, \Pi_{k=1}^{L}(1-\mu_j^{(k)})^{\lambda_k} - \Pi_{k=1}^{L}({}_j^{(k)})^{\lambda_k}\right] \\ w &= [w_1, w_2, \ldots, w_j] \end{aligned} \tag{19}$$

Where $w_j = (\mu_{wj}, {}_{wj}, \pi_{wj})$, $j = 1, 2, \ldots, n$.

Step 5: Construct the fuzzy weighted decision matrix $V = WR$ using relation (2) of Definition 2.

Step 6: Determine the positive ideal and negative ideal solutions represented by A^* and A^-, respectively. The largest and smallest IFNs are $\alpha^+ = (1, 0, 0)$ and $\alpha^- = (0, 1, 0)$, respectively. For benefit criterion, DM desires to have a maximum value, and for cost criterion, desires minimum value among the alternatives; therefore, A^* indicates the most preferable alternatives, and A^- indicates least preferable alternatives. According to equations (3) and (4) and relations (5) and (6) of Definition 2, the positive ideal solution and negative ideal solution are described as follows:

$$\begin{aligned} A^* = \Big\{ &\Big((\max_i \mu_{ij} \ \ j \in J), (\min_i {}_{ij} \ \ j \in J)\Big) \& (\min_i \mu_{ij} \ \ j \in{}'), \\ &(\max_i {}_{ij} \ \ j \in J') \ \ i = 1, 2, \ldots, m\Big\} \end{aligned} \tag{20}$$

$$\begin{aligned} A^- = \Big\{ &\Big((\min_i \mu_{ij} \ \ j \in J), (\max_i {}_{ij} \ \ j \in J)\Big) \& (\max_i \mu_{ij} \ \ j \in{}'), \\ &(\min_i {}_{ij} \ \ j \in J') \ \ i = 1, 2, \ldots, m\Big\} \end{aligned} \tag{21}$$

Where:
$J = j = 1, 2, \ldots, n \mid j$ is associated with benefit coriterion
$J^{'} = j = 1, 2, \ldots, n \mid j$ is associated with cost coriterion

Step 7: Construct D^* and D^- are ideal and non-ideal separation matrixes, respectively.

$$D^* = \begin{bmatrix} d(v_{11}, v_1^*) & \cdots & d(v_{1n}, v_n^*) \\ \vdots & \ldots & \vdots \\ d(v_{m1}, v_1^*) & \cdots & d(v_{mn}, v_n^*) \end{bmatrix} \tag{22}$$

$$D^- = \begin{bmatrix} d(v_{11}, v_1^-) & \cdots & d(v_{1n}, v_n^-) \\ \vdots & \ldots & \vdots \\ d(v_{m1}, v_1^-) & \cdots & d(v_{mn}, v_n^-) \end{bmatrix} \tag{23}$$

Where $d(A, B)$ is the definition of distance according to Grzegorewski [15] and defined as follows:

$$d(A, B) = \max \left\{ |\mu_A(x) - \mu_B(x)|, |_A(x) - _B(x)| \right\}. \tag{24}$$

Step 8: Defuzzify w using the score function according to Bandyopadhyay et al. [5] and compute $\mathfrak{h}_i, \mathfrak{I}_i, \zeta_i$ and ξ_i values using equations (5) to (8).

Step 9: Compute τ_i and η_i using equations (9) and (10).

Step 10: Calculate collective index (CI) using equation (11).

Step 11: Rank the preference order. The best satisfactory alternative can be determined according to τ_i and η_i preference rank order. The minimum value of CI indicates the better performance for alternative i.

5. Application of the Proposed Method in Solving Problems

5.1. **Illustrative example one.** In this sub-section, a numerical example illustrates how the proposed VIKOR method can be used under uncertainty. Suppose that a company manufacturing tractor components desires to renew the manufacturing system. This company should evaluate and select the most appropriate one among the available alternative flexible manufacturing systems (FMSs) to produce a group of products. A_1, A_2, A_3, A_4, and A_5 are five alternative FMSs, determined and evaluated by a committee of three DMs against five selected criteria. The functional requirements (FRs) that must be satisfied by an FMS are provided as follows regarding the characteristic of the product group manufactured by a company [44]:

- C_1: Quality of results
- C_2: Ease of use
- C_3: Competitive
- C_4: Adaptability
- C_5: Expandability

The importance weights of three DMs and five criteria are defined in Table 1 using the following linguistic terms. The performance ratings of the alternatives are characterized by the following linguistic terms with respect to the criteria: Extreme Good (EG)/Extreme High (EH), Very Good (VG)/Very High (VH), Good (G)/High (H), Medium Good (MG)/Medium High (MH), Fair (F)/Medium (M), Medium poor (MP)/Medium Low (ML), Poor (P)/Low (L), Very Poor (VP)/Very Low (VL), Extreme Poor (EP)/Extreme Low (EL) (Table 2). In this example,

Linguistic variables	Interval valued fuzzy numbers
Very Poor (VP)	(0.90, 0.10, 0.00)
Poor (P)	(0.75, 0.20, 0.05)
Moderately Poor (MP)	(0.50, 0.45, 0.05)
Fair (F)	(0.35, 0.60, 0.05)
Moderately Good (MG)	(0.25, 0.60, 0.15)
Good (G)	(0.10, 0.78, 0.12)
Very Good (VG)	(0.10, 0.90, 0.00)

Table 1. Linguistic Variables for Rating the Importance of Criteria and the DMs

Linguistic variables	Intuitionistic fuzzy numbers
Extreme good (EG)/Extreme high (EH)	(1.00, 0.00, 0.00)
Very verygood (VVG)/ Very very high (VVH)	(0.90, 0.10, 0.00)
Very good (VG)/Very high (VH)	(0.80, 0.10, 0.10)
Good(G)/High(H)	(0.70, 0.20, 0.10)
Medium good (MG)/medium high (MH)	(0.60, 0.30, 0.10)
Fair (F)/medium (M)	(0.50, 0.40, 0.10)
Medium bad(MB)/Medium low (ML)	(0.40, 0.50, 0.10)
Bad (B)/Low (L)	(0.25, 0.60, 0.15)
Very bad (VB)/Very low (VL)	(0.10, 0.75, 0.15)
Very very bad (VVb)/ Very very low (VVL)	(0.10, 0.90,0.00)

Table 2. Linguistic Variables for the Rating of Alternatives

the DMs importance is not equal; hence, their relative importance is given in Table 3, containing the DMs weights calculated by equation (17).

The alternatives' ratings with respect to each criterion described by the DMs are illustrated in Table 4 using linguistic rating variables (Steps 1 to 3). In step

	DM_1	DM_3	DM_2
Linguistic terms	Important	medium	Very important
Intuitionistic fuzzy number	(0.75, 0.2, 0.05)	(0.50, 0.45, 0.05)	(0.90, 0.10, 0.00)
Weight	0.356	0.238	0.406

Table 3. The Relative Importance of the DMs

Criteria	Alternatives	Decision makers		
		DM1	DM2	DM3
C_1	A_1	H	H	MH
	A_2	VG	G	VG
	A_3	VG	VG	VG
	A_4	VH	VH	H
	A_5	F	F	MG
C_2	A_1	MG	MG	G
	A_2	MB	MB	MB
	A_3	VVG	VG	VG
	A_4	VVG	VG	VG
	A_5	MB	F F	
C_3	A_1	G	G	VG
	A_2	VG	G	VG
	A_3	VG	G	G
	A_4	VG	G	G
	A_5	G	MG	MG
C_4	A_1	H	H	H
	A_2	MB	F	MB
	A_3	VH	H	H
	A_4	H	MH	MH
	A_5	M	MH	M
C_5	A_1	MG	MG	MG
	A_2	MH	MH	M
	A_3	VG	G	G
	A_4	G	G	F
	A_5	MB	F	MB

Table 4. Ratings of the Alternatives

4, the intuitionistic fuzzy ratings of each alternative are described by the DMs with respect to each criterion and aggregated using equation (18) to construct decision matrix (Tables 5 and 6). The importance weight of each criterion is calculated by equation (19) (Table 7 and 8) applying the selected criteria' weights, evaluated by the DMs via linguistic variables and their respective IFNs (Table 1).

	C_1	C_2	C_3
A_1	(0.663, 0.236, 0.101)	(0.644, 0.254, 0.101)	(0.746, 0.151, 0.104)
A_2	(0.708, 0.118, 0.102)	(0.400, 0.500, 0.100)	(0.780, 0.118, 0.102)
A_3	(0.800, 0.100, 0.100)	(0.844, 0.100, 0.056)	(0.740, 0.156, 0.103)
A_4	(0.764, 0.133, 0.103)	(0.844, 0.100, 0.056)	(0.740, 0.156, 0.103)
A_5	(0.543, 0.356, 0.101)	(0.466, 0.433, 0.100)	(0.639, 0.260, 0.101)

Table 5. Aggregated Intuitionistic Fuzzy Decision Matrix

After constructing the intuitionistic fuzzy weighted decision matrix (Step 5), the positive ideal solution and negative ideal solution are determined for each criterion (Tables 9 and 10) (Step 6). Afterwards, the ideal separation matrix D^* and non-ideal separation matrix D^- are constructed (Step 7). Bandyopadhyay et al. [5]

	C_4	C_5
A_1	(0.700, 0.200, 0.100)	(0.600, 0.300, 0.100)
A_2	(0.425, 0.474, 0.100)	(0.562, 0.337, 0.101)
A_3	(0.740, 0.156, 0.103)	(0.780, 0.118, 0.102)
A_4	(0.639, 0.260, 0.101)	(0.631, 0.265, 0.104)
A_5	(0.526, 0.374, 0.101)	(0.425, 0.474, 0.100)

Table 6. Aggregated Intuitionistic Fuzzy Decision Matrix

	C_1	C_2	C_3
DM_1	M	MH	VH
DM_2	M	H	VH
DM_3	VL	MH	H
Aggregated intuitionistic weight	(0.258, 0.707, 0.034)	(0.576, 0.371, 0.053)	(0.855, 0.133, 0.013)

Table 7. Weights and Aggregated Intuitionistic Weights of Criteria

	C_4	C_5
DM_1	VL	MH
DM_2	VL	MH
DM_3	M	M
Aggregated intuitionistic weight	(0.211, 0.763, 0.025)	(0.444, 0.506, 0.050)

Table 8. Weights and Aggregated Intuitionistic Weights of Criteria

showed that, according to the score function, μ_j is usable as a defuzzified IFN, like $w_j(\mu_j, {}_j\pi_j)$; according to above,$\mathfrak{h}_i, \Im_i, \zeta_i$, and ξ_i values are computed using equations (5) to (8) (Step 8). Then, τ_i and η_i values are calculated using equations (9) and (10), where the weight values of v and γ are assumed 0.5 (Step 9). Finally, the collective index (CI) of all alternatives is calculated using equation (11), and the FMSs are ranked according to the results (Steps 10 and 11). All calculated indices values are provided in Table 11. According to the results, the ranking orders of five alternatives are A_3, A_4, A_5, A_1, and A_2.

5.2. **Illustrative Example Two.** In order to further clarify the proposed method, another example is presented for the contractor selection in a construction industry by Vahdani et al. [45]. After pre-evaluation, three contractors remained as alternatives for further evaluation. In order to evaluate alternative contractors, a committee composed of three DMs ($DM_1, DM_2,$ and DM_3) having equal degrees of importance was formed because of the same backgrounds and experience. Fifteen criteria ($C_i, i = 1, 2, \ldots, 15$) are selected (Steps 1 and 2).

	C_1	C_2	C_3
A_1	(0.171, 0.776, 0.052)	(0.371, 0.531, 0.098)	(0.637, 0.263, 0.099)
A_2	(0.201, 0.742, 0.057)	(0.230, 0.686, 0.084)	(0.667, 0.235, 0.099)
A_3	(0.207, 0.737, 0.057)	(0.486, 0.434, 0.080)	(0.633, 0.268, 0.099)
A_4	(0.197, 0.746, 0.057)	(0.486, 0.434, 0.080)	(0.633, 0.268, 0.099)
A_5	(0.140, 0.812, 0.048)	(0.269, 0.643, 0.088)	(0.546, 0.358, 0.096)
Positive ideal solution $(A)^*$	(0.207, 0.737, 0.057)	(0.486, 0.434, 0.080)	(0.667, 0.235, 0.099)
Negative ideal solution$(A)^-$	(0.140, 0.812, 0.048)	(0.230, 0.686, 0.084)	(0.546, 0.358, 0.096)

TABLE 9. Intuitionistic Fuzzy Weighted Decision Matrix

	C_4	C_5
A_1	(0.148, 0.811, 0.041)	(0.266, 0.654, 0.080)
A_2	(0.090, 0.876, 0.034)	(0.249, 0.672, 0.078)
A_3	(0.156, 0.800, 0.043)	(0.346, 0.564, 0.090)
A_4	(0.135, 0.825, 0.040)	(0.280, 0.637, 0.083)
A_5	(0.111, 0.852, 0.037)	(0.189, 0.740, 0.071)
Positive ideal solution $(A)^*$	(0.156, 0.800, 0.043)	(0.346, 0.564, 0.090)
Negative ideal solution $(A)^-$	(0.090, 0.876, 0.034)	(0.189, 0.740, 0.071)

TABLE 10. Intuitionistic Fuzzy Weighted Decision Matrix

	Indices values							
Alternative	$\mathfrak{h}_i$	$\Im_i$	ζ_i	ξ_i	τ_i	η_i	CI	Final ranking
A_1	0.109	0.066	0.231	0.089	0.231	0.590	1.927	4
A_2	0.167	0.147	0.153	0.105	0.694	0.530	2.581	5
A_3	0.078	0.029	0.337	0.147	0.009	1.000	1.009	1
A_4	0.073	0.040	0.297	0.147	0.047	0.935	1.116	2
A_5	0.314	0.125	0.029	0.024	0.907	0.000	1.781	3

TABLE 11. Indices Values and CIby the Proposed Method for the Illustrative Exampleone

The weights of these criteria obtained by the DMs according to linguistic terms are defined in Table 1, and their aggregated intuitionistic weights obtained by equation (19) are shown in Table 12. The alternatives ratings with respect to each criterion represented by the DMs in linguistic terms are given in Table 2 and Table 13 (Step3).

The intuitionistic fuzzy decision matrix is calculated in Step 4 using equation (18) and is illustrated in Table 13. The weighted decision matrix is constructed (Step 5). Subsequently, the positive ideal solution and negative ideal solution are determined, and the ideal and non-ideal separation matrixes are constructed (Steps 6 and 7) (Table 14).

Criteria	Description	Decision makers		
		DM_1	DM_2	DM_3
C_1	Tender price	H	MH	VH
C_2	Financial statement	VH	H	H
C_3	Financial references	H	MH	MH
C_4	Failure to have contract completed	H	VH	VH
C_5	Cost overruns	MH	H	MH
C_6	Delay	VH	MH	H
C_7	Quality	MH	M	H
C_8	Scale	M	MH	MH
C_9	Type	H	MH	H
C_{10}	Experience	VH	VH	H
C_{11}	Physical resources	H	H	MH
C_{12}	Human resources	H	VH	MH
C_{13}	Current workload	M	ML	ML
C_{14}	Past client/contractor relationship	MH	MH	M
C_{15}	Safety performance	M	MH	ML

Table 12. Weights and Aggregated Intuitionistic Weights of Criteria

Criteria	Alternatives	Decision makers		
		DM1	DM2	DM3
	A_1	VVH	G	VVH
C_1	A_2	G	MG	MG
	A_3	MG	G	MG
	A_1	G	G	G
C_2	A_2	MG	MG	L
	A_3	LG	MG	MG
	A_1	VVH	G	VVH
C_3	A_2	VGHH	MG	MG
	A_3	MG	G	MG
	A_1	VVH	VVH	G
C_4	A_2	G	MG	G
	A_3	MG	MG	MG
	A_1	G	G	MG
C_5	A_2	L	L	VL
	A_3	VL	VL	VL
	A_1	L	L	VL
C_6	A_2	VL	L	L
	A_3	VL	VL	VL
	A_1	G	MG	G
C_7	A_2	MG	G	G
	A_3	L	MG	MG
	A_1	EG	EG	VVH
C_8	A_2	VVH	VVH	MG
	A_3	MG	MG	VVH
	A_1	MG	L	L
C_9	A_2	L	VL	L
	A_3	MG	MG	VL
	A_1	L	L	VL
C_{10}	A_2	L	VL	L
	A_3	VVL	L	VVL
	A_1	VVH	VVH	G
C_{11}	A_2	MG	MG	G
	A_3	G	MG	MG
	A_1	G	G	G
C_{12}	A_2	L	MG	MG
	A_3	MG	L	MG
	A_1	G	MG	VG
C_{13}	A_2	MG	G	MG
	A_3	L	G	G
	A_1	EG	VVH	EG
C_{14}	A_2	VVH	G	VVH
	A_3	G	G	VVH
	A_1	G	MG	G
C_{15}	A_2	MG	G	MG
	A_3	L	L	G

Table 13. Ratings of the Alternatives

	C_1	C_2	C_3
A_1	(0.856, 0.126, 0.018)	(0.700, 0.200, 0.100)	(0.856, 0.126, 0.018)
A_2	(0.637, 0.262, 0.101)	(0.507, 0.378, 0.115)	(0.748, 0.208, 0.044)
A_3	(0.637, 0.262, 0.101)	(0.507, 0.378, 0.115)	(0.637, 0.262, 0.101)
	C_6	C_7	C_8
A_1	(0.203, 0.646, 0.151)	(0.670, 0.229, 0.101)	(1.000, 0.000, 0.000)
A_2	(0.203, 0.646, 0.151)	(0.670, 0.229, 0.101)	(0.841, 0.144, 0.015)
A_3	(0.100, 0.750, 0.150)	(0.507, 0.378, 0.115)	(0.748, 0.208, 0.044)
	C_{11}	C_{12}	C_{13}
A_1	(0.856, 0.126, 0.018)	(0.700, 0.200, 0.100)	(0.670, 0.229, 0.101)
A_2	(0.637, 0.262, 0.101)	(0.507, 0.378, 0.115)	(0.637, 0.262, 0.101)
A_3	(0.637, 0. 262, 0.101)	(0.507, 0.378, 0.115)	(0.593, 0.288, 0.119)

TABLE 14. Intuitionistic Fuzzy Weighted Decision Matrix

	C_4	C_5
A_1	(0.856, 0.126, 0.018)	(0.670, 0.229, 0.101)
A_2	(0.670, 0.229, 0.101)	(0.203, 0.646, 0.151)
A_3	(0.600, 0.300, 0.100)	(0.100, 0.750, 0.150)
	C_9	C_{10}
A_1	(0.392, 0.476, 0.132)	(0.203, 0.646, 0.151)
A_2	(0.203, 0.646, 0.151)	(0.153, 0.696, 0.151)
A_3	(0.476, 0.407, 0.117)	(0.153, 0.786, 0.061)
	C_{14}	C_{15}
A_1	(1.000, 0.000, 0.000)	(0.670, 0.229, 0.101)
A_2	(0.856, 0.126, 0.018)	(0.637, 0.262, 0.101)
A_3	(0.792, 0.159, 0.049)	(0.447, 0.416, 0.137)

TABLE 15. Intuitionistic Fuzzy Weighted Decision Matrix

	C_1	C_2	C_3
A_1	(0.435, 0.308, 0.257)	(0.499, 0.327, 0.174)	(0.435, 0.426, 0.139)
A_2	(0.324, 0.416, 0.261)	(0.361, 0.477 0.162)	(0.380, 0.480, 0.140)
A_3	(0.324, 0.416, 0.261)	(0.361, 0.477 0.162)	(0.324, 0.515, 0.161)
Positive ideal solution $(A)^*$	(0.435, 0.308, 0.257)	(0.499, 0.327, 0.174)	(0.435, 0.426, 0.139)
Negative ideal solution $(A)^-$	(0.324, 0.416, 0.261)	(0.361, 0.477 0.162)	(0.324, 0.515, 0.161)
	C_6	C_7	C_8
A_1	(0.129, 0.720, 0.151)	(0.217, 0.520, 0.263)	(0.324, 0.495, 0.181)
A_2	(0.129, 0.720, 0.151)	(0.217, 0.520, 0.263)	(0.273, 0.568, 0.159)
A_3	(0.064, 0.802, 0.134)	(0.164, 0.613, 0.223)	(0.242, 0.600, 0.157)
Positive ideal solution $(A)^*$	(0.129, 0.720, 0.151)	(0.217, 0.520, 0.263)	(0.324, 0.495, 0.181)
Negative ideal solution $(A)^-$	(0.064, 0.802, 0.134)	(0.164, 0.613, 0.223)	(0.242, 0.600, 0.157)
	C_{11}	C_{12}	C_{13}
A_1	(0.522, 0.355, 0.123)	(0.495, 0.366, 0.138)	(0.170, 0.692, 0.138)
A_2	(0.388, 0.455, 0.156)	(0.359, 0.507, 0.134)	(0.162, 0.705, 0.133)
A_3	(0.388, 0.455, 0.156)	(0.359, 0.507, 0.134)	(0.151, 0.715, 0.134)
Positive ideal solution $(A)^*$	(0.522, 0.355, 0.123)	(0.495, 0.366, 0.138)	(0.170, 0.692, 0.138)
Negative ideal solution $(A)^-$	(0.388, 0.455, 0.156)	(0.359, 0.507, 0.134)	(0.151, 0.715, 0.134)

TABLE 16. Intuitionistic Fuzzy Weighted Decision Matrix

5.3. **Discussion.** Sensitivity is recognized on different values of the majority criteria, i.e., v and γ; hence, the mentioned weights' values are changed simultaneously to perform sensitivity analysis. The maximal group utility is 1; the maximal regret is 0, and the combination of both is 0.5. To conduct sensitivity analysis, v and γ are set by six different values between 0 and 1. Then, seven different v and γ values are used in the proposed method to check the changes in the alternatives' ranking in the given example (Tables 19 and 20). In fact, the results of these changes could help the DMs by making the assessment process easier and through determining the priorities.

	C_4	C_5
A_1	(0.606, 0.236, 0.158)	(0.341, 0.494, 0.166)
A_2	(0.474, 0.326, 0.200)	(0.103, 0.768, 0.129)
A_3	(0.425, 0.388, 0.187)	(0.051, 0.836, 0.113)
Positive ideal solution $(A)^*$	(0.606, 0.236, 0.158)	(0.341, 0.494, 0.166)
Negative ideal solution $(A)^-$	(0.425, 0.388, 0.187)	(0.051, 0.836, 0.113)
	C_9	C_{10}
A_1	(0.199, 0.613, 0.187)	(0.159, 0.691, 0.150)
A_2	(0.103, 0.739, 0.158)	(0.120, 0.735, 0.145)
A_3	(0.242, 0.563, 0.196)	(0.120, 0.813, 0.067)
Positive ideal solution $(A)^*$	(0.242, 0.563, 0.196)	(0.159, 0.691, 0.150)
Negative ideal solution $(A)^-$	(0.103, 0.739, 0.158)	(0.120, 0.813, 0.067)
	C_{14}	C_{15}
A_1	(0.381, 0.495, 0.124)	(0.217, 0.649, 0.134)
A_2	(0.326, 0.559, 0.115)	(0.206, 0.664, 0.129)
A_3	(0.302, 0.575, 0.123)	(0.145, 0.734 , 0.121)
Positive ideal solution $(A)^*$	(0.381, 0.495, 0.124)	(0.217, 0.649, 0.134)
Negative ideal solution $(A)^-$	(0.302, 0.575, 0.123)	(0.145, 0.734 , 0.121)

TABLE 17. Intuitionistic Fuzzy Weighted Decision Matrix

	Indices values							
Alternative	$\mathfrak{h}_i$	$\Im_i$	ζ_i	ξ_i	τ_i	η_i	CI	Final ranking
A_1	0.026	0.026	1.044	0.174	0.000	1.000	1.000	1
A_2	0.785	0.139	0.297	0.062	0.781	0.108	10.000	3
A_3	0.980	0.174	0.090	0.090	1.000	0.125	9.008	2

TABLE 18. Indices Values and CI by the Proposed Method for the Illustrative Example Two

v and γ values	Alternatives	CI	Preference order ranking
$v = 0$ and $\gamma = 0$	A_1	2.217	4
	A_2	2.521	5
	A_3	1.000	1
	A_4	1.094	2
	A_5	1.687	3
$v = 0.2$ and $\gamma = 0.2$	A_1	2.095	4
	A_2	2.526	5
	A_3	1.004	1
	A_4	1.102	2
	A_5	1.733	3
$v = 0.4$ and $\gamma = 0.4$	A_1	1.981	4
	A_2	2.556	5
	A_3	1.007	1
	A_4	1.111	2
	A_5	1.771	3
$v = 0.4$ and $\gamma = 0.5$	A_1	1.927	4
	A_2	2.581	5
	A_3	1.009	1
	A_4	1.116	2
	A_5	1.781	3
$v = 0.6$ and $\gamma = 0.6$	A_1	1.874	4
	A_2	2.616	5
	A_3	1.011	1
	A_4	1.122	2
	A_5	1.791	3
$v = 0.8$ and $\gamma = 0.8$	A_1	1.773	3
	A_2	2.717	5
	A_3	1.014	1
	A_4	1.134	2
	A_5	1.809	4
$v = 1$ and $\gamma = 1$	A_1	1.677	3
	A_2	2.875	5
	A_3	1.018	1
	A_4	1.149	2
	A_5	1.825	4

TABLE 19. Different Values of v and γ and Preference Order Ranking by the Proposed Method for Illustrative Example One

v and γ values	Alternatives	CI	Preference order ranking
$v = 0$ and $\gamma = 0$	A_1	1.000	1
	A_2	5.376	3
	A_3	1.640	2
$v = 0.2$ and $\gamma = 0.2$	A_1	1.000	1
	A_2	6.534	2
	A_3	21.019	3
$v = 0.4$ and $\gamma = 0.4$	A_1	1.000	1
	A_2	8.460	2
	A_3	11.010	3
$v = 0.4$ and $\gamma = 0.5$	A_1	1.000	1
	A_2	10.000	3
	A_3	9.008	2
$v = 0.6$ and $\gamma = 0.6$	A_1	1.000	1
	A_2	12.307	3
	A_3	7.673	2
$v = 0.8$ and $\gamma = 0.8$	A_1	1.000	1
	A_2	23.836	3
	A_3	6.005	2
$v = 1$ and $\gamma = 1$	A_1	1.000	1
	A_2	1.498	2
	A_3	5.004	3

Table 20. Different Values of and γ and Preference Order Ranking by the Proposed Method for Illustrative Example One

6. **Concluding**

The intuitionistic fuzzy VIKOR method focuses on assessing a set of alternatives under an uncertain environment and ranking them with respect to multiple conflicting criteria. The multi-criteria decision making problems vagueness has been expressed through intuitionistic fuzzy sets by a group of experts due to its power in revealing DMs hesitancy, assuming their agreement and disagreement. The proposed method gives the chance of utilizing more flexible way to deal with real-life decision-making problems and help the DMs to reach an acceptable compromise of the maximum group utility of the majority and the minimum of the individual regret of the opponent. In the presented method, the relative importance of criteria, called their weights and the alternatives ratings, can be described by linguistic terms and then converted to IFNs. The intuitionistic fuzzy weighted averaging (IFWA) operator is used to aggregate DMs judgments and weights of criteria. The ideal separation matrix and anti-ideal separation matrix were constructed using two IF-distance measures. To consider the relative distance of alternatives from the positive ideal solution and negative ideal solution, two indices were used for ranking the alternatives based on the strategy of the majority criteria and the individual regret. Finally, to highlight the applicability of the proposed method, two application examples were used, and also different weights of the majority criteria for the sensitivity analysis purpose were changed in both flexible manufacturing systems (FMSs) selection and contractor selection examples. The proposed method can provide a simple and effective mechanism for solving decision-making problems by a group of experts under an intuitionistic fuzzy environment. Extending the proposed VIKOR method is recommended by a group of experts to other domains of engineering and management for further research studies.

References

[1] B. A. Aghajani, M. Osanloo and B. Karimi, *Deriving preference order of open pit minesequipment through MCDM methods: Application ofmodified VIKOR metho*, Expert Systems with Applications, **38(3)** (2011), 2550-2556.

[2] M. Amiri, S. A. Ayazi, L. Olfat, and J. Siahkali Moradi, *Group decision making process for supplier selection with VIKOR under fuzzy circumstance case study: an Iranian car parts supplier*, International Bulletin of Business Administration, **10(6)** (2011), 66-75.

[3] M. Amiri, M. Zandieh, B. Vahdani, R. Soltani and V. Roshanaei, *An integrated eigenvector DEATOPSIS methodology for portfolio risk evaluation in the FOREX spot market*, Expert Systems with Applications, **37(1)** (2010), 509-516.

[4] K. Atanassov, *Intuitionistic fuzzy sets*, Fuzzy Sets and Systems,**20(1)** (1986), 87-96.

[5] S. Bandyopadhyay, P. Kumar Nayak and M. Pal, *Solution of matrix game in intuitionistic fuzzy environment*, JCER, **3** (2013), 84-89.

[6] F. E. Boran, S. Genc, M. Kurt and D. Akay, *A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method*, Expert Systems with Applications, **36(8)** (2009), 11363-11368.

[7] D. Bouyssou, *Using DEA as a tool for MCDM: some remarks*, Journal of the operational Research Society, **50(9)** (1999), 974-978.

[8] C. L. Chang, *A modified VIKOR method for multiple criteria analysis*, Environmental monitoring and assessment, **168(1-4)** (2010), 339-344.

[9] C. T. Chen, *A fuzzy approach to select location of the distribution center*, Fuzzy sets and systems,**118(1)** (2001), 65-73.

[10] S. M. Chen and C. H. Wang, *A generalized model for prioritized multi-criteria decision making systems*, Expert Systems with Applications, **36(3)** (2009), 4773-4783.

[11] T. C. Chu, *Facility location selection using fuzzy TOPSIS under group decision*, International journal of uncertainty, fuzziness and knowledge-based systems, **10(6)** (2002), 687-701.

[12] T. C. Chu, *Selecting plant location via a fuzzy TOPSIS approach*, The International Journal of Advanced Manufacturing Technology, **20(11)** (2002), 859-864.

[13] S. Ebrahimnejad, S. M. Mousavi, R. Tavakkoli-Moghaddam, H. Hashemi and B. Vahdani, *A novel two-phase group decision-making approach for construction project selection in a fuzzy environment*, Applied Mathematical Modelling, **36(9)** (2012), 4197-4217.

[14] S. Ebrahimnejad, H. Hashemi, S. M. Mousavi and B. Vahdani, *A new interval-valued intuitionistic fuzzy model to group decision making for the selection of outsourcing providers*, Journal of Economic Computation and Economic Cybernetics Studies and Research, **49(2)** (2015), 269-290.

[15] P. Grzegorzewski, *Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric*, Fuzzy sets and systems, **148(2)** (2004), 319-328.

[16] H. Hashemi, J. Bazargan, S. M. Mousavi and B. Vahdani *An extended compromise ratio model with an application to reservoir flood control operation under an interval-valued intuitionistic fuzzy environment*, Applied Mathematical Modelling, **38(14)** (2014), 3495-3511.

[17] A. Jahan, M. Faizal, I. Md Yusof, S. M. Sapuan, and M. Bahraminasab, *A comprehensive VIKOR method for material selection*, Materials and Design, **32(3)** (2011), 1215-1221.

[18] L. James JH, C. Y. Tsai, R. H. Lin and and G. H. Tzeng, *A modifed VIKOR multiple-criteria decision method for improving domestic airlines service quality*, Journal of Air Transport Management, **17(2)** (2011), 57-61.

[19] G. S. Liang and M. J. J. Wang, *A fuzzy multi-criteria decision-making method for facility site selection*, The International Journal of Production Research, **29(11)** (1991), 2313-2330.

[20] S. M. Mousavi, F. Jolai, R. Tavakkoli-Moghaddam and B. Vahdani, *A fuzzy grey model based on the compromise ranking for multi-criteria group decision making problems in manufacturing systems*, Journal of Intelligent and Fuzzy Systems, **24(4)** (2013), 819-827.

[21] V. Mohagheghi, S. M. Mousavi and B. Vahdani, *A new optimization model for project portfolio selection under interval-valued fuzzy environment*, Arabian Journal for Science and Engineering, **40(11)** (2015), 3351-3361.

[22] S. M. Mousavi, H. Gitinavard and B. Vahdani, *Evaluating construction projects by a new group decision-making model based on intuitionistic fuzzy logic concepts*, International Journal of Engineering-Transactions C: Aspects, **28(9)** (2015), 1312-1319.

[23] S. M. Mousavi, B. Vahdani, M. Amiri, R. Tavakkoli-Moghaddam, S. Ebrahimnejad and M. Amiri *A multi-stage decision making process for multiple attributes analysis under an interval-valued fuzzy environment*, International Journal of Advanced Manufacturing Technology, **64(9-12)** (2013), 1263-1273.

[24] S. M. Mousavi B. Vahdani and S. Sadigh Behzadi, *Designing a model of intuitionistic fuzzy VIKOR in multi-attribute group decision-making problems*, Iranian Journal of Fuzzy Systems, **13(1)** (2016), 45-65.

[25] S. M. Mousavi and B. Vahdani *Cross-docking location selection in distribution systems: a new intuitionistic fuzzy hierarchical decision model*, International Journal of Computational Intelligence Systems, **9(1)** (2016), 91-109.

[26] S. M. Mousavi, B. Vahdani, R. Tavakkoli-Moghaddam and N. Tajik, *Soft computing based on a fuzzy grey compromise solution approach with an application to the selection problem of material handling equipment*, International Journal of Computer Integrated Manufacturing, **27(6)** (2014), 547-569.

[27] S. Opcorvic, *Multicriteria optimization of Civil engineering systems*, Faculty of Civil Engineering, **2(1)** (1998), 5-21.

[28] S. Opricovic and G. H. Tzeng, *Multi-criteria planning of post-earthquake sustainable reconstruction*, omputer-Aided Civil and Infrastructure Engineering, **17(3)** (2002), 211-220.

[29] S. Opricovic and G. H. Tzeng, *The compromise solution by MCDM methods: A comparative analysis of VIKOR and TOPSIS*, European Journal of Operational Research, **156(2)** (2004), 445-455.

[30] S. Opricovic and G. H. Tzeng, *Extended VIKOR method in comparison with outranking methods*, European Journal of Operational Research, **178(2)** (2007), 514-529.

[31] V. Roshanaei, B. Vahdani, S. M. Mousavi, M. Mousakhani and G. Zhang, *CAD/CAM system selection: A multi-component hybrid fuzzy MCDM model*, Arabian Journal for Science and Engineering, **38(9)** (2013), 2579-2594 .

[32] M. Salimi, B. Vahdani, S. M . Mousavi and R. Tavakkoli-Moghaddam, *A new method based on multi-segment decision matrix for solving decision making problems*, Scientia Iranica, **20(6)** (2013), 2259-2274.

[33] J. R. San Cristóbal, *Multi-criteria decision-making in the selection of a renewable energy project in Spain: The VIKOR method*, Renewable Energy, **36(2)** (2011), 498-502.

[34] B. Vahdani, H. Hadipour, J. S. Sadaghiani and M. Amiri, *Extension of VIKOR method based on interval-valued fuzzy sets*, The International Journal of Advanced Manufacturing Technology, **47(9-12)** (2010), 1231-1239.

[35] B. Vahdani and H. Hadipour, *Extension of the ELECTRE method based on interval-valued fuzzy sets*, Soft Computing, **15(3)** (2011), 569-579.

[36] B. Vahdani, M. Salimi, and M. Charkhchian, *A new FMEA method by integrating fuzzy belief structure and TOPSIS to improve risk evaluation process*, The International Journal of Advanced Manufacturing Technology, **77(1-4)** (2015), 357-368.

[37] B. Vahdani, M. Zandieh and A. Alem-Tabriz, *Supplier selection by balancing and ranking method*, Journal of Applied Sciences, **8(19)** (2008), 3467-3472.

[38] B. Vahdani, M. Zandieh, and R. Tavakkoli-Moghaddam, *Two novel FMCDM methods for alternative-fuel buses selection*, Applied Mathematical Modelling, **35(3)** (2011), 1396-1412.

[39] B. Vahdani, S. M. Mousavi and S. Ebrahimnejad, *Soft computing-based preference selection index method for human resource management*, Journal of Intelligent and Fuzzy Systems, **26(1)** (2014), 393-403.

[40] B. Vahdani, H. Hadipour and R. Tavakkoli-Moghaddam, *Soft computing based on interval valued fuzzy ANP-A novel methodology*, Journal of Intelligent Manufacturing, **23(5)** (2012), 1529-1544.

[41] B. Vahdani, M. Salimi and S. M. Mousavi, *A new compromise decision-making model based on TOPSIS and VIKOR for solving multi-objective large-scale programming problems with a block angular structure under uncertainty*, International Journal of Engineering Transactions B: Applications,**27(11)** (2014), 1673-1680.

[42] B. Vahdani, M. Salimi and S. M. Mousavi, *A compromise decision-making model based on VIKOR for multi-objective large-scale nonlinear programming problems with a block angular structure under uncertainty*, Scientia Iranica. Transaction E, Industrial Engineering,**22(6)** (2015), 2571-2584.

[43] B. Vahdani, S. M. Mousavi and R. Tavakkoli-Moghaddam, *Group decision making based on novel fuzzy modified TOPSIS method*, Applied Mathematical Modelling, **35(9)** (2011), 4257-4269.

[44] B. Vahdani, S. M. Mousavi, R. Tavakkoli-Moghaddam and H. Hashemi, *A new design of the elimination and choice translating reality method for multi-criteria group decision making in an intuitionistic fuzzy environment*, Applied Mathematical Modeling ,**37(4)** (2013), 1781-1799.

[45] B, Vahdani, S. M. Mousavi, H. Hashemi, M. Mousakhani and R. Tavakkoli-Moghaddam, *A new compromise solution method for fuzzy group decision-making problems with an application to the contractor selection*, Engineering Applications of Artificial Intelligence, **26(2)** (2012), 779-788.

[46] B. Vahdani and M. Zandieh *Selecting suppliers using a new fuzzy multiple criteria decision model: the fuzzy balancing and ranking method*, International Journal of Production Research, **48(18)** (2010) 5307-5326.

[47] B. Vahdani, S. M. Mousavi, R. Tavakkoli-Moghaddam, A. Ghodratnama and M. Mohammadi, *Robot selection by a multiple criteria complex proportional assessment method under an interval-valued fuzzy environment*, International Journal of Advanced Manufacturing Technology, **73(5-8)** (2014), 687-697.

[48] B. Vahdani, R. Tavakkoli-Moghaddam, S. M. Mousavi and A. Ghodratnama, *Soft computing based on new interval-valued fuzzy modified multi-criteria decision-making method*, Applied Soft Computing, **13(1)** (2013), 165-172.

[49] Y. Y. Wu and D. J. Yu, *Extended VIKOR for Multi-criteria decision making problems under intuitionistic environment*, 18th International Conference on Management Science & Engineering, 2011.

[50] Z. Xu, *An overview of methods for determining OWA weights*, International Journal of Intelligent Systems,**20(8)** (2005), 843-865.

[51] Z. Xu and R. R. Yager, *Some geometric aggregation operators based on intuitionistic fuzzy sets*, International Journal of General Systems, **35(4)** (2006), 417-433.

[52] Z. Xu, *Intuitionistic fuzzy aggregation operators*, IEEE Transactions on Fuzzy Systems, **15(6)** (2007), 1179-1187.

[53] D. Yong, *Plant location selection based on fuzzy TOPSIS*, The International Journal of Advanced Manufacturing Technology ,**28(7-8)** (2006), 839-844.

[54] L. A. Zadeh, *Fuzzy sets*, Information and control , **8(3)** (1965), 338-353.

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