SOME PROPERTIES FOR FUZZY CHANCE CONSTRAINED PROGRAMMING

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Abstract. Convexity theory and duality theory are important issues in mathematical programming. Within the framework of credibility theory, this paper first introduces the concept of convex fuzzy variables and some basic criteria. Furthermore, a convexity theorem for fuzzy chance constrained programming is proved by adding some convexity conditions on the objective and constraint functions. Finally, a duality theorem for fuzzy linear chance constrained programming is proved.

1. Introduction

Since Zadeh proposed the concept of fuzzy set in 1965, fuzziness has been widely studied as a basic uncertainty for describing subjective information. In order to quantify the chance of a fuzzy event occurring, Zadeh [21] defined a possibility measure as a nonadditive set function satisfying normality and maximality, and defined a necessity measure [22] as its dual part. However, neither possibility measure nor necessity measure is self-dual. Since the self-duality is intuitive and important, Liu and Liu [14] defined a credibility measure as the average of possibility measure and necessity measure. Based on the concept of credibility measure, a credibility theory [15, 16] is developed for dealing with fuzzy phenomena.

The fuzziness occurring in the optimization problems is categorized as the fuzzy programming problems, which not only avoids various unrealistic assumptions, but also retains the original realistic information. In 1970, Bellman and Zadeh [1] first proposed the basic concepts of fuzzy decision making. Since then, many papers have appeared to investigate the fuzzy programming theory and applications [2, 4, 5, 19]. Within the framework of credibility theory, Liu and Iwamura [10] and Liu [11] suggested a spectrum of fuzzy chance constrained programming, which provides a means of allowing the decision maker to maximize the optimistic value and pessimistic value of the fuzzy objective function under certain constraints in term of the credibility of their attainment. The model has been applied in many problems such as parallel machine scheduling problem [18], redundancy optimization problem [23], vehicle routing problem [24], newsboy problem [3], facility location-allocation problem [25], portfolio selection problem [8] and so on.

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The convexity theory and duality theory are important issues in mathematical programming. In stochastic programming, convexity theorem and duality theorem for chance constrained programming models have been well investigated. The purpose of this paper is to study the convexity theorem and duality theorem for fuzzy chance constrained programming model. For this purpose, the rest of this paper is organized as follows. After some preliminary summaries on fuzzy variables and credibility measure in Section 2, some properties of convex fuzzy variables are given in Section 3. In Section 4, a convexity theorem is proved for fuzzy chance constrained programming by adding some convexity conditions on the objective and constraint functions. In Section 5, the definition of dual programming for fuzzy linear chance constrained programming is proposed and a duality theorem is proved. At the end of this paper, a brief summary is given.

2. Preliminary

Let ξ be a fuzzy variable with membership function μ. For any set $B \subseteq \Re$, in order to measure the chance of the event that ξ takes values in set B, Liu and Liu [14] gave the concept of credibility measure as the average of possibility measure and necessity measure, that is,

$$\mathrm{Cr}\{\xi \in B\} = \frac{1}{2}\left(\sup_{x \in B} \mu(x) + 1 - \sup_{x \in B^c} \mu(x)\right). \tag{1}$$

Furthermore, Li and Liu [6] proved that a set function Cr is a credibility measure if and only if it satisfies

(a) (normality) $\mathrm{Cr}\{\xi \in \Re\} = 1$;
(b) (monotonicity) $\mathrm{Cr}\{\xi \in A\} \le \mathrm{Cr}\{\xi \in B\}$ whenever $A \subseteq B$;
(c) (self-duality) $\mathrm{Cr}\{\xi \in A\} + \mathrm{Cr}\{\xi \in A^c\} = 1$ for each set A;
(d) (partial maximality) $\mathrm{Cr}\{\xi \in \cup_i A_i\} = \sup_i \mathrm{Cr}\{\xi \in A_i\}$ for any sets $\{A_i\}$ with $\sup_i \mathrm{Cr}\{\xi \in A_i\} < 0.5$.

If ξ is a fuzzy variable, then its membership function may be derived from the credibility measure as

$$\mu(x) = \min\{2\mathrm{Cr}\{\xi = x\}, 1\}, \ \forall x \in \Re. \tag{2}$$

The concept of independent fuzzy variables characterized by credibility measure was given by Liu and Gao [17]. That is, fuzzy variables $\xi_1, \xi_2, \cdots, \xi_n$ are said to be independent if and only if

$$\mathrm{Cr}\left\{\cap_{i=1}^n \{\xi_i \in B_i\}\right\} = \min_{1 \le i \le n} \mathrm{Cr}\{\xi_i \in B_i\} \tag{3}$$

for any sets $B_1, B_2, \cdots, B_n \in \Re$. For any $\alpha \in (0, 1]$, the α-optimistic value and α-pessimistic value of fuzzy variable ξ were defined by Liu [13] as

$$\xi_{\sup}(\alpha) = \sup\{r | \mathrm{Cr}\{\xi \ge r\} \ge \alpha\}, \quad \xi_{\inf}(\alpha) = \inf\{r | \mathrm{Cr}\{\xi \le r\} \ge \alpha\}. \tag{4}$$

It has been proved that $\xi_{\inf}(\alpha)$ is an increasing and left-continuous function with respect to α, and $\xi_{\sup}(\alpha)$ is a decreasing and left-continuous function with respect to α. Furthermore, Li and Liu [7] proved the linearity of the optimistic value and pessimistic value of independent fuzzy variables, that is, if fuzzy variables ξ and η are independent, then we have

$$(\xi+\eta)_{\sup}(\alpha)=\xi_{\sup}(\alpha)+\eta_{\sup}(\alpha), \quad (\xi+\eta)_{\inf}(\alpha)=\xi_{\inf}(\alpha)+\eta_{\inf}(\alpha). \tag{5}$$

In 1998, Liu and Iwamura [10] and Liu [11] suggested a spectrum of fuzzy chance constrained programming model. If we want to maximize the optimistic value of the fuzzy objective function, we have the following fuzzy maximax chance constrained programming model

$$\begin{cases} \max & f(\boldsymbol{x},\boldsymbol{\xi})_{\sup}(\alpha) \\ \text{s.t.} & \mathrm{Cr}\{g_j(\boldsymbol{x},\boldsymbol{\xi})\ge 0, j=1,2,\cdots,p\}\ge\beta, \end{cases} \tag{6}$$

if we want to maximize the pessimistic value of the fuzzy objective function, we get the following fuzzy minimax chance constrained programming model

$$\begin{cases} \max & f(\boldsymbol{x},\boldsymbol{\xi})_{\inf}(\alpha) \\ \text{s.t.} & \mathrm{Cr}\{g_j(\boldsymbol{x},\boldsymbol{\xi})\ge 0, j=1,2,\cdots,p\}\ge\beta, \end{cases} \tag{7}$$

where $\boldsymbol{x}$ is an n-dimensional decision vector, $\boldsymbol{\xi}$ is a fuzzy parameter vector, and α,β are the predetermined confidence levels.

3. Convex Fuzzy Variable

A real-valued function f defined on a convex subset $X\in\Re^n$ is said to be quasiconcave if and only if for any $\boldsymbol{x},\boldsymbol{y}\in X$ and $0<\lambda<1$, we have

$$f(\lambda\boldsymbol{x}+(1-\lambda)\boldsymbol{y})\ge\min\{f(\boldsymbol{x}),f(\boldsymbol{y})\}.$$

Let f be a quasiconcave function defined on convex subset X. Then for any $r\in\Re$, the collection $\{\boldsymbol{x}\in X|f(\boldsymbol{x})\ge r\}$ is a convex set. Furthermore, it has been proved that any local maximum of f on X is also a global maximum. Let $\boldsymbol{\xi}$ be an n-dimensional fuzzy vector with membership function μ. Then $\boldsymbol{\xi}$ is said to be convex if and only if μ is a quasiconcave function.

Theorem 3.1. If convex fuzzy variables $\xi_1,\xi_2,\cdots,\xi_n$ are mutually independent, then $\boldsymbol{\xi}=(\xi_1,\xi_2,\cdots.\xi_n)$ is a convex fuzzy vector.

Proof. Suppose that μ and μ_1, μ_2, $\cdots$, μ_n are the membership functions of fuzzy vector $\boldsymbol{\xi}$ and fuzzy variables ξ_1, ξ_2, $\cdots$, ξ_n, respectively. For any $\boldsymbol{x},\boldsymbol{y}\in X$ and $0<\lambda<1$, since fuzzy variables $\xi_1,\xi_2,\cdots,\xi_n$ are independent, it follows from formulation (3) that

$$\begin{aligned} \mu(\lambda\boldsymbol{x}+(1-\lambda)\boldsymbol{y}) &= \min_{1\le i\le n}\mu_i(\lambda x_i+(1-\lambda)y_i) \\ &\ge \min_{1\le i\le n}\min\{\mu_i(x_i),\mu_i(y_i)\} \\ &= \min\{\mu(\boldsymbol{x}),\mu(\boldsymbol{y})\}, \end{aligned}$$

which implies that fuzzy vector $\boldsymbol{\xi}$ is convex. The proof is complete. □

Theorem 3.2. [9] Let $\boldsymbol{\xi}$ be an n-dimensional fuzzy vector. Then $\boldsymbol{\xi}$ is convex if and only if we have

$$\mathrm{Cr}\{\boldsymbol{\xi} \in \lambda A + (1-\lambda)B\} \geq \min\{\mathrm{Cr}\{\boldsymbol{\xi} \in A\}, \mathrm{Cr}\{\boldsymbol{\xi} \in B\}\} \tag{8}$$

for any $A, B \in \Re^n$ and $0 < \lambda < 1$.

4. **Convexity Theorem**

In this section, we prove a convexity theorem for fuzzy chance constrained programming within the framework of credibility theory.

Theorem 4.1. Suppose that $\boldsymbol{\xi}$ is an m-dimensional convex fuzzy vector. If functions $f_i(\boldsymbol{x}, \boldsymbol{y}) : \Re^{n+m} \to \Re$ are quasiconcave for all $1 \leq i \leq I$, then for any $r \in \Re$, the function $\mathrm{Cr}\{f_i(\boldsymbol{x}, \boldsymbol{\xi}) \geq r, i = 1, 2, \cdots, I\}$ is quasiconcave with respect to $\boldsymbol{x}$.

Proof. Consider the sets depending on the parameter $\boldsymbol{x}$:

$$H(\boldsymbol{x}) = \{\boldsymbol{y} \in \Re^m | f_i(\boldsymbol{x}, \boldsymbol{y}) \geq r, i = 1, 2, \cdots, I\}.$$

Let $D = \{\boldsymbol{x} \in \Re^n | H(\boldsymbol{x}) \neq \emptyset\}$, we show that $\{H(\boldsymbol{x}), \boldsymbol{x} \in D\}$ is a concave family, that is, for any $\boldsymbol{x}_1, \boldsymbol{x}_2 \in D$ and $0 < \lambda < 1$, we have

$$H(\lambda \boldsymbol{x}_1 + (1-\lambda)\boldsymbol{x}_2) \supset \lambda H(\boldsymbol{x}_1) + (1-\lambda)H(\boldsymbol{x}_2).$$

In fact, assume that $\boldsymbol{y}_1 \in H(\boldsymbol{x}_1)$ and $\boldsymbol{y}_2 \in H(\boldsymbol{x}_2)$. Then we have $f_i(\boldsymbol{x}_1, \boldsymbol{y}_1) \geq r$ and $f_i(\boldsymbol{x}_2, \boldsymbol{y}_2) \geq r$ for all $1 \leq i \leq I$. It follows from the quasiconcavity of f_i that

$$f_i(\lambda \boldsymbol{x}_1 + (1-\lambda)\boldsymbol{x}_2, \lambda \boldsymbol{y}_1 + (1-\lambda)\boldsymbol{y}_2) \geq \min\{f_i(\boldsymbol{x}_1, \boldsymbol{y}_1), f_i(\boldsymbol{x}_2, \boldsymbol{y}_2)\} \geq r$$

for all $1 \leq i \leq I$. Hence, we have $\lambda \boldsymbol{y}_1 + (1-\lambda)\boldsymbol{y}_2 \in H(\lambda \boldsymbol{x}_1 + (1-\lambda)\boldsymbol{x}_2)$. Thus, if $H(\boldsymbol{x}_1)$ and $H(\boldsymbol{x}_2)$ are nonempty, then according to Theorem 3.2, we have

$$\begin{aligned}\mathrm{Cr}\{\boldsymbol{\xi} \in H(\lambda \boldsymbol{x}_1 + (1-\lambda)\boldsymbol{x}_2)\} &\geq \mathrm{Cr}\{\boldsymbol{\xi} \in \lambda H(\boldsymbol{x}_1) + (1-\lambda)H(\boldsymbol{x}_2)\} \\ &\geq \min\{\mathrm{Cr}\{\boldsymbol{\xi} \in H(\boldsymbol{x}_1)\}, \mathrm{Cr}\{\boldsymbol{\xi} \in H(\boldsymbol{x}_2)\}\}.\end{aligned}$$

If at least one of the sets $H(\boldsymbol{x}_1)$ and $H(\boldsymbol{x}_2)$ is empty, the second row of the above inequality is zero, hence we have in general

$$\begin{aligned}&\mathrm{Cr}\{f_i(\lambda \boldsymbol{x}_1 + (1-\lambda)\boldsymbol{x}_2), \boldsymbol{\xi}) \geq r, i = 1, 2, \cdots, I\} \\ &= \mathrm{Cr}\{\boldsymbol{\xi} \in H(\lambda \boldsymbol{x}_1 + (1-\lambda)\boldsymbol{x}_2)\} \\ &\geq \min\{\mathrm{Cr}\{\boldsymbol{\xi} \in H(\boldsymbol{x}_1)\}, \mathrm{Cr}\{\boldsymbol{\xi} \in H(\boldsymbol{x}_2)\}\} \\ &= \min\{\mathrm{Cr}\{f_i(\boldsymbol{x}_1, \boldsymbol{\xi}) \geq r, i = 1, 2, \cdots, I\}, \mathrm{Cr}\{f_i(\boldsymbol{x}_2, \boldsymbol{\xi}) \geq r, i = 1, 2, \cdots, I\}\}.\end{aligned}$$

The proof is complete. □

Theorem 4.2. Suppose that $\boldsymbol{\xi}$ is an m-dimensional convex fuzzy vector. If function $f(\boldsymbol{x}, \boldsymbol{y}) : \Re^{n+m} \to \Re$ is quasiconcave, then for any $\alpha \in (0, 1]$, the optimistic value $f(\boldsymbol{x}, \boldsymbol{\xi})_{\sup}(\alpha)$ is a quasiconcave function with respect to $\boldsymbol{x}$.

Proof. Define $f_0 = \min\{f(\boldsymbol{x}_1,\boldsymbol{\xi})_{\sup}(\alpha), f(\boldsymbol{x}_2,\boldsymbol{\xi})_{\sup}(\alpha)\}$. For any $\varepsilon > 0$, it follows from Theorem 4.1 that

$$\begin{aligned}&\mathrm{Cr}\left\{f(\lambda\boldsymbol{x}_1 + (1-\lambda)\boldsymbol{x}_2,\boldsymbol{\xi}) \geq f_0 - \varepsilon\right\}\\ &\geq \min\{\mathrm{Cr}\{f(\boldsymbol{x}_1,\boldsymbol{\xi}) \geq f_0-\varepsilon\}, \mathrm{Cr}\{f(\boldsymbol{x}_2,\boldsymbol{\xi}) \geq f_0-\varepsilon\}\}\\ &\geq \alpha\end{aligned}$$

which implies that $f(\lambda\boldsymbol{x}_1 + (1-\lambda)\boldsymbol{x}_2,\boldsymbol{\xi})_{\sup}(\alpha) \geq f_0 - \varepsilon$. Letting $\varepsilon \to 0$, we get

$$f(\lambda\boldsymbol{x}_1 + (1-\lambda)\boldsymbol{x}_2,\boldsymbol{\xi})_{\sup}(\alpha) \geq f_0 = \min\{f(\boldsymbol{x}_1,\boldsymbol{\xi})_{\sup}(\alpha), f(\boldsymbol{x}_2,\boldsymbol{\xi})_{\sup}(\alpha)\}.$$

Hence, $f(\boldsymbol{x},\boldsymbol{\xi})_{\sup}(\alpha)$ is quasiconcave with respect to $\boldsymbol{x}$. The proof is complete. □

Theorem 4.3. Let $\boldsymbol{\xi}$ be an m-dimensional convex fuzzy vector. If functions $f(\boldsymbol{x},\boldsymbol{y}) : \Re^{n+m} \to \Re$ and $g_j(\boldsymbol{x},\boldsymbol{y}) : \Re^{n+m} \to \Re$, $j = 1,2,\cdots,p$ are all quasiconcave, then any strict local maximum of the fuzzy chance constrained programming model (6) is also the strict global maximum.

Proof. For any $\alpha \in (0,1]$, it follows from Theorem 4.2 that the objective function of (6) is quasiconcave. On the other hand, we have $\mathrm{Cr}\{g_j(\boldsymbol{x},\boldsymbol{\xi}) \geq 0, j = 1,2,\cdots,p\}$ is quasiconcave with respect to $\boldsymbol{x}$ by Theorem 4.1, which implies that $\mathrm{Cr}\{g_j(\boldsymbol{x},\boldsymbol{\xi}) \geq 0, j = 1,2,\cdots,p\} \geq \beta$ is a convex subset of $\Re^n$ for any $\beta \in (0,1]$. Thus, fuzzy chance constrained programming (6) is convex, and then any strict local maximum is also the strict global maximum. The proof is complete. □

5. **Duality Theorem**

In this section, we consider the duality theorem of the following fuzzy linear chance constrained programming model

$$\begin{cases}\max & (\xi_1x_1+\xi_2x_2+\cdots+\xi_nx_n)_{\inf}(\alpha)\\ \text{s.t.} & \mathrm{Cr}\{a_{i1}x_1+a_{i2}x_2+\cdots+a_{in}x_n \leq \eta_i, i=1,2,\cdots,m\} \geq \beta\\ & x_1,x_2,\cdots,x_n \geq 0,\end{cases} \tag{9}$$

where $\boldsymbol{x} = (x_1,x_2,\cdots,x_n)$ is the decision vector, $\boldsymbol{\xi} = (\xi_1,\xi_2,\cdots,\xi_n)$ and $\boldsymbol{\eta} = (\eta_1,\eta_2,\cdots,\eta_m)$ are fuzzy parameter vectors, $A = (a_{ij})_{m\times n}$ is a crisp matrix, and α,β are the predetermined confidence levels.

Definition 5.1. *The dual programming of fuzzy linear chance constrained programming* (9) *is defined as*

$$\begin{cases}\min & (\eta_1y_1+\eta_2y_2+\cdots+\eta_my_m)_{\sup}(\beta)\\ s.t. & \mathrm{Cr}\{a_{1j}y_1+a_{2j}y_2+\cdots+a_{mj}y_m \geq \xi_j, j=1,2,\cdots,n\} \geq \alpha\\ & y_1,y_2,\cdots,y_m \geq 0.\end{cases} \tag{10}$$

Theorem 5.2. Suppose that $\bar{\boldsymbol{x}}$ and $\bar{\boldsymbol{y}}$ are feasible solutions of programmings (9) and (10). Then, we have

$$(\xi_1\bar{x}_1+\xi_2\bar{x}_2+\cdots+\xi_n\bar{x}_n)_{\inf}(\alpha) \leq (\eta_1\bar{y}_1+\eta_2\bar{y}_2+\cdots+\eta_m\bar{y}_m)_{\sup}(\beta).$$

Proof. Since $\bar{\boldsymbol{x}}$ and $\bar{\boldsymbol{y}}$ are feasible solutions, we have $\bar{\boldsymbol{x}} \geq 0$ and $\mathrm{Cr}\left\{\bar{\boldsymbol{y}}A \geq \boldsymbol{\xi}\right\} \geq \alpha$. It follows from the monotonity of credibility measure that

$$\mathrm{Cr}\left\{\bar{\boldsymbol{y}}A\bar{\boldsymbol{x}}' \geq \boldsymbol{\xi}\bar{\boldsymbol{x}}'\right\} \geq \mathrm{Cr}\left\{\bar{\boldsymbol{y}}A \geq \boldsymbol{\xi}\right\} \geq \alpha.$$

That is, $(\boldsymbol{\xi}\bar{\boldsymbol{x}}')_{\inf}(\alpha) \leq \bar{\boldsymbol{y}}A\bar{\boldsymbol{x}}'$. Similarly, since $\bar{\boldsymbol{y}} \geq 0$ and $\mathrm{Cr}\{A\bar{\boldsymbol{x}}' \leq \boldsymbol{\eta}\} \geq \beta$, we have

$$\mathrm{Cr}\left\{\bar{\boldsymbol{y}}A\bar{\boldsymbol{x}}' \leq \boldsymbol{\eta}\bar{\boldsymbol{y}}'\right\} \geq \mathrm{Cr}\{A\bar{\boldsymbol{x}}' \leq \boldsymbol{\eta}\} \geq \beta,$$

which implies that $(\boldsymbol{\eta}\bar{\boldsymbol{y}}')_{\sup}(\beta) \geq \bar{\boldsymbol{y}}A\bar{\boldsymbol{x}}'$. Thus we have

$$(\boldsymbol{\xi}\bar{\boldsymbol{x}}')_{\inf}(\alpha) \leq \bar{\boldsymbol{y}}A\bar{\boldsymbol{x}}' \leq (\boldsymbol{\eta}\bar{\boldsymbol{y}}')_{\sup}(\beta).$$

The proof is complete. □

Corollary 5.3. *Suppose that $\bar{\boldsymbol{x}}$ and $\bar{\boldsymbol{y}}$ are the feasible solutions of programmings* (9) *and* (10)*. If we have $(\boldsymbol{\xi}\bar{\boldsymbol{x}}')_{\inf}(\alpha) = (\boldsymbol{\eta}\bar{\boldsymbol{y}}')_{\sup}(\beta)$, then $\bar{\boldsymbol{x}}$ and $\bar{\boldsymbol{y}}$ are the optimum solutions.*

Proof. It follows immediately from Theorem 5.2. □

Theorem 5.4. Assume that ξ is a continuous fuzzy variable. Then for any $\alpha \in (0,1]$, we have

$$\mathrm{Cr}\{\xi \geq \xi_{\sup}(\alpha)\} \geq \alpha, \quad \mathrm{Cr}\{\xi \leq \xi_{\inf}(\alpha)\} \geq \alpha.$$

Proof. Since ξ is a continuous fuzzy variable, functions $\mathrm{Cr}\{\xi \leq x\}$ and $\mathrm{Cr}\{\xi \geq x\}$ are continuous with respect to x. Then for any $\alpha \in (0,1]$, we have

$$\mathrm{Cr}\{\xi \leq \xi_{\inf}(\alpha)\} = \lim_{\varepsilon \downarrow 0} \mathrm{Cr}\{\xi \leq \xi_{\inf}(\alpha) + \varepsilon\} \geq \alpha,$$

$$\mathrm{Cr}\{\xi \geq \xi_{\sup}(\alpha)\} = \lim_{\varepsilon \downarrow 0} \mathrm{Cr}\{\xi \geq \xi_{\sup}(\alpha) - \varepsilon\} \geq \alpha.$$

The proof is complete. □

Theorem 5.5. Suppose that $\xi_1, \xi_2, \cdots, \xi_n$ and $\eta_1, \eta_2, \cdots, \eta_m$ are independent continuous fuzzy variables. If programming (9) has a finite maximum value, then programming (10) also has a finite minimum value, and the optimal values are equal.

Proof. Since $\xi_1, \xi_2, \cdots, \xi_n$ are independent, it follows from (5) that

$$(\xi_1 x_1 + \xi_2 x_2 + \cdots + \xi_n x_n)_{\inf}(\alpha) = (\xi_1)_{\inf}(\alpha)x_1 + (\xi_2)_{\inf}(\alpha)x_2 + \cdots + (\xi_n)_{\inf}(\alpha)x_n.$$

On the other hand, since fuzzy variables $\eta_1, \eta_2, \cdots, \eta_m$ are independent, it is easy to prove that

$$\min_{1 \leq i \leq m} \mathrm{Cr}\{a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \leq \eta_i\} \geq \beta.$$

According to Theorem 5.4, this condition is equivalent to $a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \leq (\eta_i)_{\sup}(\beta)$ for all $1 \leq i \leq m$. Hence, programming (9) degenerates to the following crisp linear programming model

$$\begin{cases} \max & (\xi_1)_{\inf}(\alpha)x_1 + (\xi_2)_{\inf}(\alpha)x_2 + \cdots + (\xi_n)_{\inf}(\alpha)x_n \\ \text{s.t.} & a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \leq (\eta_i)_{\sup}(\beta), i = 1, 2, \cdots, m \\ & x_1, x_2, \cdots, x_n \geq 0. \end{cases}$$

Similarly, it may prove that programming (10) degenerates to its dual programming model

$$\begin{cases} \min & (\eta_1)_{\sup}(\beta)y_1 + (\eta_2)_{\sup}(\beta)y_2 + \cdots + (\eta_m)_{\sup}(\beta)y_m \\ \text{s.t.} & a_{1j}y_1 + a_{2j}y_2 + \cdots + a_{mj}y_m \geq (\xi_j)_{\inf}(\alpha), j = 1, 2, \cdots, n \\ & y_1, y_2, \cdots, y_m \geq 0. \end{cases}$$

The proof follows immediately from the duality theorem of classical linear programming theory. □

6. Conclusions

This paper contributes to the research area of fuzzy programming in the following two aspects: (a) a convexity theorem for fuzzy chance constrained programming with convex fuzzy parameters was proved; and (b) a duality theorem for fuzzy linear chance constrained programming was proved.

References

[1] R. E. Bellman and L. A. Zadeh, *Decision making in a fuzzy environment*, Management Science, **17** (1970), 141-164.

[2] M. Delgado, J. Kacprzyk, J. L. Verdegay and M. A. Vila, *Fuzzy optimization: recent advances*, Physica-Verlag, Wurzburg, 1994.

[3] X. Ji X and Z. Shao, *Model and algorithm for bilevel newsboy problem with fuzzy demands and discounts*, Applied Mathematics and Computation, **172(1)** (2006), 163-174.

[4] Y. J. Lai and C. L. Hwang, *Fuzzy mathematical programming: methods and applications*, Lecture Notes in Economics and Mathematical Systems, Springer, Berlin, **394** (1992).

[5] Y. J. Lai and C. L. Hwang, *Fuzzy multiple objective decision making: methods and applications*, Lecture notes in Economics and Mathematical Systems, Springer, Berlin, **404** (1992).

[6] X. Li and B. Liu, *A sufficient and necessary condition for credibility measures*, International Journal of Uncertainty, Fuzziness & Knowledge-Based Systems, **14(5)** (2006), 527-535.

[7] X. Li and B. Liu, *The independence of fuzzy variables with applications*, International Journal of Natural Sciences and Technology, **1(1)** (2006), 95-100.

[8] X. Li, Z. Qin and L. X. Yang, *A chance-constrained portfolio selection model with risk constraints*, Applied Mathematics and Computation, **217** (2010), 949-951.

[9] X. Li, Z. Qin, L. X. Yang and K. P. Li, *Entropy maximization model for trip distribution problem with fuzzy and random parameters*, Journal of Computational and Applied Mathematics, doi:10.1016/j.cam.2010.09.004.

[10] B. Liu and K. Iwamura, *Chance constrained programming with fuzzy parameters*, Fuzzy Sets and Systems, **94(2)** (1998), 227-237.

[11] B. Liu, *Minimax chance constrained programming models for fuzzy decision systems*, Information Sciences, **112(1-4)** (1998), 25-38.

[12] B. Liu, *Dependent-chance programming in fuzzy environment*, Fuzzy Sets and Systems, **109(1)** (2000), 97-106.

[13] B. Liu, *Theory and practice of uncertain programming*, Physica-Verlag, Heidelberg, 2002.

[14] B. Liu and Y. K. Liu, *Expected value of fuzzy variable and fuzzy expected value models*, IEEE Transactions on Fuzzy Systems, **10(4)** (2002), 445-450.

[15] B. Liu, *Uncertainty theory: an introduction to its axiomatic foundations*, Springer-Verlag, Berlin, 2004.

[16] B. Liu, *Uncertainty theory*, 2nd ed., Springer-Verlag, Berlin, 2007.

[17] Y. K. Liu and J. Gao, *The independence of fuzzy variables with applications to fuzzy random optimization*, International Journal of Uncertainty, Fuzziness & Knowledge-Based Systems, **15(Suppl.2)** (2007), 1-20.
[18] J. Peng and B. Liu, *Parallel machine scheduling models with fuzzy processing times*, Information Sciences, **166(1-4)** (2004), 49-66.
[19] R. Slowinski, *Fuzzy sets in decision analysis, operations research and statistics*, Kluwer Academic Publishers, Dordrecht, 1998.
[20] S. Vajda, *Probabilistic programming*, Academic Press, New York, 1972.
[21] L. A. Zadeh, *Fuzzy sets as a basis for a theory of possibility*, Fuzzy Sets and Systems, **1** (1978), 3-28.
[22] L. A. Zadeh, *A theory of approximate reasoning*, In: Hayes J et al., Mathematical Frontiers of the Social and Policy Sciences, Westview Press, Boulder, Cororado, (1979), 69-129.
[23] R. Zhao and B. Liu, *Standby redundancy optimization problems with fuzzy lifetimes*, Computers & Industrial Engineering, **149(2)** (2005), 318-338.
[24] Y. Zheng and B. Liu, *Fuzzy vehicle routing model with credibility measure and its hybrid intelligent algorithm*, Applied Mathematics and Computation, **176(2)** (2006), 673-683.
[25] J. Zhou and B. Liu B, *Modeling capacitated location-allocation problem with fuzzy demands*, Computers & Industrial Engineering, **53(3)** (2007), 454-468.

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