AN OPTIMAL FUZZY SLIDING MODE CONTROLLER DESIGN BASED ON PARTICLE SWARM OPTIMIZATION AND USING SCALAR SIGN FUNCTION

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Abstract. This paper addresses the problems caused by an inappropriate selection of sliding surface parameters in fuzzy sliding mode controllers via an optimization approach. In particular, the proposed method employs the parallel distributed compensator scheme to design the state feedback based control law. The controller gains are determined in offline mode via a linear quadratic regular. The particle swarm optimization is incorporated into the linear quadratic regular technique for determining the optimal weight matrices. Consequently, an optimal sliding surface is obtained using the scalar *sign* function. This latter is used to design the proposed control law. Finally, the effectiveness of the proposed fuzzy sliding mode controller based on parallel distributed compensator and using particle swarm optimization is evaluated by comparing the obtained results with other reported in literature.

1. Introduction

The concept of sliding mode was firstly proposed by Utkin [37] as a variable structure system controller and showed that a sliding mode could be achieved by changing the controller structure [32] . Sliding mode control theory, especially in the case of linear system, has been widely used to solve the nonlinear dynamic control problems such as uncertainty parameter, time varying delay and external disturbances [37, 47, 14].

In recent years, the sliding mode control approaches based on Takagi-Sugeno (TS) fuzzy model [35] were often used to deal with a variety of challenging control applications. It was widely applied in many research fields such as chemical processes [29, 24], robots [22, 18, 19], servo motors [39] and satellites [11]. It was proved as a robust control method for uncertain nonlinear systems, insensitivity to parameter variations and external disturbances [3, 28, 46]. However, its implementation suffers from a chattering problem which may degrade the performance and may even lead to instability of the closed-loop control systems. This has led to the development of various methods in order to reduce the effects of the chattering phenomenon. In [4], a robust fuzzy model-based controller has developed for Single-Input-Single-Output (SISO) nonlinear systems with or without uncertainties. However, there still exist some weaknesses in their approach, i.e., it depends

Received: March 2016; Revised: July 2016; Accepted: January 2017

Key words and phrases: Sliding mode control, Takagi-Sugeno fuzzy model, Particle swarm optimization, Parallel distributed compensator, Linear quadratic regulator.

highly on a good choice of boundary constant. Iglesias et *al.* [16] proposed a new controller based on a combination of sliding mode control and fuzzy logic, where the conventional sliding surface was modified using a set of fuzzy rules. A thin boundary layer neighboring the sliding plane is introduced to alleviate the chattering problem. However, this method can eliminate chattering effect, but a finite steady state error will exist. Later, Chen et *al.* [7] proposed an adaptive Fuzzy Sliding Mode Controller (FSMC) based on Genetic Algorithm (GA) in order to achieve good control performance. In their method, the initial values of the consequent parameters are decided via a genetic algorithm. Furthermore, in order to ensure the stability of the nonlinear system, the stability criterion is derived from the Lyapunov's direct method. However, this method has the disadvantages such as time-consuming and complex tuning parameters.

Several studies have shown that the Particle Swarm Optimization (PSO) algorithm can produce high-quality solutions within shorter calculation time, easy to implement, and more stable convergence characteristic methods compared with GA [20, 23]. This nice property has attracted most attention of FSMC works. Indeed, the PSO algorithm was exploited to optimize the sliding surface coefficients of the controller for a Buck Converter [38]. Additionally, in [1], the PSO was used to optimal tune the sliding surface coefficient in the design of decentralized intelligent FSMC. The approaches reported by [38] and [1] can only tune the sliding surface coefficients, whereas the designing of the control gains are specified by the designer.

Many decoupling control algorithms have been proposed to deal with coupling in nonlinear system based artificial neural works [15, 25, 21]. These decoupling control methods are based on complicated control algorithms, which are difficult not only to achieve accurate requirement and stability but also to be implemented in industrial applications.

Moreover, Parallel Distributed Compensation (PDC) method is applied to many applications successfully [34, 12, 2]. However, its procedure design by the means of the Linear Quadratic Regulator (LQR) technique is always difficult. Indeed, the choice of Q and R weight matrices of LQR method is done by trial.

The aim of this study is to overcome the mentioned problems and reserve favorable control performance. The main new contributions of this paper with respect to the current treated in the literature are:

- The PDC concept is applied to design the state-feedback based control law whereby the control gains are offline determined via LQR technique. This latter uses the PSO to ensure the optimal tuning the control gains.
- An optimal sliding surface is obtained.
- The scalar and matrix *sign* function is used instead to the one of the standard FSMC.
- The designed controller system has faster system response.

This study presents the development of a new method based on particle swarm optimization and FSMC (PSOFSMC). PSO algorithm is incorporated into the LQR technique in order to determine the optimal weight matrices by minimizing a defined objective function. Then, the control gains are used to design the sliding surface.

As a result, the obtained optimal sliding surface using the scalar *sign* function gives a good behavior during the sliding mode. Once the sliding mode surface has been determined, we determine the control law of the proposed method such that the global fuzzy model presents the desired dynamic characteristics. Consequently, the proposed design procedure is conceptually simple and easy to implement.
This paper is organized as follows. In Section 2, mathematical preliminaries are given. The proposed fuzzy sliding mode control approach based on PSO is detailed in Section 3. A numerical example is treated to validate the theoretical concept in Section 4. Finally, Section 5 summarizes the important features of the proposed approach.

2. Mathematical Preliminaries

2.1. **T-S Fuzzy Model and PDC Design.** A Takagi-Sugeno fuzzy model consists of a set of fuzzy rules, each describing a local input-output relation as follows:

$$rule_i : IF\ z_1(t)\ is\ M_{1i}\ and\ \dots\ and\ z_p(t)\ is\ M_{pi}$$
$$THEN \left\{ \begin{array}{l} \dot{x}(t) = A_i x(t) + B_i u(t), \quad i = 1, \dots, r \\ y(t) = C_i x(t) \end{array} \right. \tag{1}$$

where $rule_i$ denotes the i-th IF-THEN rule, M_{ji} is the fuzzy subset, r is the number of rules, $x(t) \in \Re^n$ is the state vector , $u(t) \in \Re^m$ is the input vector, $A_i \in \Re^{n\times n}$, $B_i \in \Re^{n\times m}$ and $C_i \in \Re^{q\times n}$. Here $z(t) = [z_1(t), z_2(t), ..., z_p(t)]$ denotes the vector containing some nonlinear functions of the states variables obtained from the original nonlinear.

Given a pair of $[x(t), u(t), z(t)]$, the overall output of the fuzzy system using the center of gravity defuzzification method can be represented as

$$\left\{ \begin{array}{l} \dot{x}(t) = \sum\limits_{i=1}^{r} h_i(z(t)) \left(A_i x(t) + B_i u(t)\right) \\ y(t) = \sum\limits_{i=1}^{r} h_i(z(t)) \left(C_i x(t)\right) \end{array} \right. \tag{2}$$

$$h_i(z(t)) = \frac{w_i(z(t))}{\sum\limits_{i=1}^{r} w_i(z(t))}$$

and

$$w_i(z(t)) = \prod_{j=1}^{p} M_{ji}(z_j(t))$$

Here $h_i(z(t))$ is regarded as the normalized weight of each model rule and $M_{ji}(z_j(t))$ denotes the grade of membership of $z_j(t)$ in M_{ji}.
The membership values $h_i(z(t))$ have to satisfy the following conditions:

$$\left\{ \begin{array}{l} \sum\limits_{i=1}^{r} h_i(z(t)) = 1 \\ 0 \le h_i(z(t)) \le 1 \qquad i = 1, ..., r \\ \sum\limits_{i=1}^{r} \dot{h}_i(z(t)) = 0 \end{array} \right. \tag{3}$$

Using the weighted average fuzzy inferences approach, we obtain the following global fuzzy state space model:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) \\ y(t) = Cx(t) \end{cases} \tag{4}$$

where

$$A = \sum_{i=1}^{r} h_i(z(t))A_i, B = \sum_{i=1}^{r} h_i(z(t))B_i$$

and

$$C = \sum_{i=1}^{r} h_i(z(t))C_i.$$

Assumption: Each linear subsystem of the global fuzzy model is controllable, *i.e.* the matrices $W_i = \left[B_i,\ A_iB_i, A_i^2B_i, ...,\ A_i^{n-1}B_i\right] \quad for\ i = 1, ..., r$ have full ranks, *i.e.* $rank(W_i) = n$ [45].

Assumption: The global fuzzy model equation (1) is controllable in the state space, *i.e.* the matrix $W = \left[B,\ AB,\ A^2B,\ ...,\ A^{n-1}B\right]$ has full rank, *i.e.* $rank(W) = n$, in the state space [17].

The concept of the PDC technique is introduced in [40, 36]. It is utilized to design fuzzy controller and to stabilize fuzzy system equation (1) [41]. In this concept, the fuzzy controller rule shares the same premise part as the fuzzy system equation (1) and uses the same number of fuzzy rules. The idea is to design a compensator for each rule of the fuzzy model. For each rule, we can use linear control design techniques. The resulting overall fuzzy controller, which is nonlinear in general, is a fuzzy blending of each individual linear controller. The general structure of the *i-th* controller is then as

$$\begin{array}{l} rule_i : IF\ z_1(t)\ is\ M_{1i}\ and\ \dots\ and\ z_p(t)\ is\ M_{pi} \\ THEN\ \ u(t) = -K_i\, x(t) \end{array} \tag{5}$$

The fuzzy controller is inferred as follows:

$$u(t) = -\sum_{i=1}^{r} h_i(z)K_i\, x(t) \tag{6}$$

The fuzzy regulator design is to determine the local feedback gains K_i in the consequent part. The feedback gains K_i are determined using an LMI-based design technique.

2.2. **PSO Algorithm.** Many studies have proposed the evolutionary computation technique based on particle swarm optimization. They have been successfully applied to solve various optimization problems [48, 27, 26, 33]. It is initialized with a population of random solutions, called particles, to find the optimal result. Each particle has a position and a velocity, representing a possible solution to the optimization problem and a search direction in the search space. In each iterative process, the particle adjusts the velocity and position according to the best experiences that are called the *pbest*, found by itself, and *gbest*, found by all its neighbors [23].

For every generation, the velocity and position can be updated by the following equations [13]:

$$V_{gd}^{s+1} = \omega V_{gd}^{s} + c_1 r_1 (pbest^s - X_{gd}^{s}) + c_2 r_2 (gbest^s - X_{gd}^{s}) \tag{7}$$

$$X_{gd}^{s+1} = X_{gd}^{s} + V_{gd}^{s+1}, \tag{8}$$

where s is the number of iterations; V_{gd}^{s} is the velocity in the d-th dimension of the g-th particle; X_{gd}^{s} is the position in the d-th dimension of the g-th particle; *pbest* and *gbest* are the memory of the particle; c_1 and c_2 are the cognition and the social factor, respectively; r_1 and r_2 are random functions uniformly distributed in [0 1]. ω is the inertia weight and can be determined by:

$$\omega = \omega_{\max} - (\omega_{\max} - \omega_{\min})\, s/s_{\max} \tag{9}$$

3. Design of the Proposed Fuzzy Sliding Mode Controller

The proposed fuzzy sliding mode controller is composed of two design objectives, which are:

- An optimal sliding surface which makes the states converging quickly to the equilibrium point.
- The designed control law is robustly stable.

3.1. Design of the Sliding Surface.

The objective is to design an optimal sliding surface in order to achieve a good behaviour during the sliding mode. The sliding mode occurs when $S = \tilde{C}\,x = 0$. By differentiating S with respect the time and using equation (3)

$$\dot{S} = \tilde{C}\,\dot{x} = 0 \tag{10}$$

where $\tilde{C} \in \Re^{m\times n}$ and $\tilde{C} = \sum_{i=1}^{r} h_i(z)\,\tilde{C}_i$ is the sliding mode parameter matrix. From equations (1) and (10), we get:

$$\dot{S} = \tilde{C}\,A_i x(t) + \tilde{C}\,B_i u(t) = 0 \tag{11}$$

if $(\tilde{C}B_i)^{-1}$ exists, then

$$u_{eq} = -\,(\tilde{C}B_i)^{-1}\tilde{C}A_i x = -k_i x \tag{12}$$

with

$$k_i = (\tilde{C}B_i)^{-1}\tilde{C}A_i \tag{13}$$

As a result, the dynamics $\dot{x} = \left(I - B_i(\tilde{C}B_i)^{-1}\tilde{C}\right) A_i x$ describes the motion on the sliding surface which is independent of the actual value of the control and depends only on the choice of the matrix $\tilde{C}$. In order to do so this, we present a new design procedure that allows to determine the optimal sliding surface parameters.

The canonical form used in [8] for VSC design can be applied for all the local models in order to select the gain matrix $\tilde{C}_i$.

Assumption: There exists an $(n \times n)$ orthogonal transformation matrix $T_{1,i}$ such that $Y = T_{1,i}\,x$ and $T_{1,i}B_i = \begin{bmatrix} 0 \\ B_{2,i} \end{bmatrix}$, $i = 1, ..., r$, where B_i has full rank m and $B_{2,i}$ is non-singular.

The transformed state variable vector is defined as

$$\dot{Y} = \sum_{i=1}^{r} h_i(z)(T_{1,i}A_i{T_{1,i}}^T Y + T_{1,i}B_i u) \tag{14}$$

The switching function S will be identically equal to zero during the sliding motion, then $S = \sum_{i=1}^{r} h_i(z)\,\tilde{C}_i x = \sum_{i=1}^{r} h_i(z)\,\tilde{C}_i T_{1,i}^T Y = 0$ with

$$\tilde{C}_i {T_{1,i}}^T = \begin{bmatrix} \tilde{C}_{1,i} & \tilde{C}_{2,i} \end{bmatrix} \tag{15}$$

If the transformed state Y is partitioned as $Y^T = \begin{bmatrix} Y_1^T & Y_2^T \end{bmatrix}$, with $Y_1 \in \Re^{n-m}$ and $Y_2 \in \Re^m$ such as

$$T_{1,i}A_i T_{1,i}^T = \begin{bmatrix} A_{11,i} & A_{12,i} \\ A_{21,i} & A_{22,i} \end{bmatrix} \tag{16}$$

Using equations (15) and (16), we get

$$\begin{cases} \dot{Y}_1 = \sum\limits_{i=1}^{r} h_i(z)(A_{11,i}Y_1 + A_{12,i}Y_2) \\ \dot{Y}_2 = \sum\limits_{i=1}^{r} h_i(z)(A_{21,i}Y_1 + A_{22,i}Y_2 + B_{2,i}u) \end{cases} \tag{17}$$

Assumption: $\tilde{C}_i B_i$ non singular implies that $\tilde{C}_{2,i}$ must also be non-singular and the condition defining the sliding mode is:

$$Y_2 = -\sum_{i=1}^{r} h_i(z)\,\tilde{C}_{2,i}^{-1}\tilde{C}_{1,i}Y_1 = -\sum_{i=1}^{r} h_i(z)\,F_i Y_1 \tag{18}$$

with

$$F_i = \tilde{C}_{2,i}^{-1}\tilde{C}_{1,i} \tag{19}$$

where $F_i \in \Re^{m\times(n-m)}$ and the order of the reduced system is $(n-m)$. The sliding mode is governed by the following equations:

$$\dot{Y}_1 = \sum_{i=1}^{r} h_i(z)(A_{11,i}Y_1 + A_{12,i}Y_2) \tag{20}$$

$$Y_2 = -\sum_{i=1}^{r} h_i(z)F_i Y_1 \tag{21}$$

with Y_2 playing the role of a state feedback control. This indicates that the design of a stable sliding mode requires the selection of state feedback gains F_i. Indeed, there are a variety of methods to select F_i [30], one of them being the PDC-based LQR technique [9, 10]. This latter minimizes the following cost function

$$J_i^{(1)} = \int_0^\infty \left(Y_1^T Q_i^{(1)}\,Y_1 + Y_2^T R_i^{(1)}\,Y_2\right) dt \tag{22}$$

where $Q_i^{(1)}$ is a symmetric positive semi-definite matrix and $R_i^{(1)}$ is a symmetric positive-definite matrix with appropriate dimensions.

The state feedback control law in the i-th subspace which minimizes $J_i^{(1)}$, is expressed as

$$Y_2 = -F_i\,Y_1 = -\left(R_i^{(1)}\right)^{-1} A_{12,i}^T P_i\,Y_1 \tag{23}$$

where P_i is a positive definite and symmetric matrix solution of the following Riccati equation:

$$P_i A_{11,i} + A_{11,i}^T P_i + Q_i^{(1)} - P_i A_{12,i}^T \left(R_i^{(1)}\right)^{-1} P_i = 0 \tag{24}$$

The final model-based fuzzy controller is analytically represented by equation (23). The overall closed-loop fuzzy system obtained by combining equations (22) and (23) is given as follows:

$$\dot{Y}_1 = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t)) \left[(A_{11,i} - A_{12,i} F_j) \, Y_1 \right] \tag{25}$$

such that $\Psi_i = A_{11,i} - A_{12,i} F_j$ has $(n - m)$ left-half-plane eigen-values. A sufficient condition that guarantees the stability of the fuzzy system equation (25); is given by the following theorem.

Theorem 3.1. *The equilibrium of fuzzy control system equation (25) is asymptotically stable in the large if there exists a common positive definite matrix P such that the following two inequalities are satisfied* [42]*:*

$$(A_{11,i} - A_{12,i} F_i)^T P + P \left(A_{11,i} - A_{12,i} F_i\right) < 0 \;\; i = 1, ..., r \tag{26}$$

and

$$G_{i\,j}{}^T P + P \, G_{i\,j} < 0 \quad (1 \le i < j \le r) \tag{27}$$

where

$$G_{i\,j} = [(A_{11,i} - A_{12,i} \, F_j) + (A_{11,j} - A_{12,j} \, F_i)]/2 \tag{28}$$

Proof. Consider the Lyapunov function

$$V\left(Y_1\right) = Y_1^T \, P \, Y_1 \tag{29}$$

with $P > 0$. The time derivative Lyapunov of $V\left(Y_1\right)$ can be obtained as

$$\dot{V}\left(Y_1\right) = \dot{Y}_1^T \, P \, Y_1 + Y_1^T \, P \, \dot{Y}_1 \tag{30}$$

Equation (25) can be rewritten as

$$\dot{Y}_1 = \sum_{i=1}^{r} h_i(z(t)) h_i(z(t)) G_{ii} Y_1 + 2 \sum_{i=1}^{r} \sum_{i<j}^{r} h_i(z(t)) h_j(z(t)) G_{ij} Y_1 \tag{31}$$

with $G_{ii} = (A_{11,i} - A_{12,i} F_i)$ and $G_{i\,j} = \left[(A_{11,i} - A_{12,i} \, F_j) + (A_{11,j} - A_{12,j} \, F_i)\right]/2$. Substituting equation (31) into equation (30), we obtain

$$\begin{aligned} \dot{V}\left(Y_1\right) = & \left\{ \sum_{i=1}^{r} h_i(z(t)) h_i(z(t)) G_{ii} Y_1 + 2 \sum_{i=1}^{r} \sum_{i<j}^{r} h_i(z(t)) h_j(z(t)) G_{ij} Y_1 \right\}^T P \, Y_1 \\ & + Y_1^T \, P \left\{ \sum_{i=1}^{r} h_i(z(t)) h_i(z(t)) G_{ii} Y_1 + 2 \sum_{i=1}^{r} \sum_{i<j}^{r} h_i(z(t)) h_j(z(t)) G_{ij} Y_1 \right\} \end{aligned} \tag{32}$$

Then

$$\begin{aligned} \dot{V}\left(Y_1\right) = & \sum_{i=1}^{r} h_i^2(z(t)) Y_1^T \left[G_{ii}^T P + P \, G_{ii} \right] Y_1 \\ & + \sum_{i=1}^{r} \sum_{i<j}^{r} h_i(z(t)) h_j(z(t)) Y_1^T \left[G_{ij}{}^T P + P \, G_{ij} \right] Y_1 \end{aligned} \tag{33}$$

Assuming conditions equations (26) and (27) are satisfied, one gets $\dot{V}(Y_1) < 0$, for every $Y_1(t) \neq 0$, hence, the asymptotic stability can be deduced. $\square$

Once the stabilizing matrix F_i is determined, the matrix $\tilde{C}_i$ can be obtained by

$$\tilde{C}_i = [F_i \quad I_m] \, T_{1,i} \tag{34}$$

3.2. **Design of the Control Law.** The proposed control law u consists of the sum of a linear control law u_L and a nonlinear part u_N, which has the following form:

$$\begin{aligned} u &= \sum_{i=1}^{r} h_i(z) \left(\left(L_i^{(1)} + L_i^{(2)} \right) x + E_i sign_l\left(S(t)\right) \right) \\ &= \sum_{i=1}^{r} h_i(z) \; \left(\left(u_{L,i}^{(1)} + u_{L,i}^{(2)} \right) + u_{N,i} \right) \end{aligned} \tag{35}$$

with $u_{L,i} = u_{L,i}^{(1)} + u_{L,i}^{(2)}$, $L_i = L_i^{(1)} + L_i^{(2)}$ and E_i are the control gains and $S(t) \in \Re^m$ is defined as the sliding surface to be determined and "*sign*" is the smooth *sign* function.

To accomplish the control system design, we form a second transformation $T_{2,i} : \Re^n \to \Re^n$ such that:

$$Z = T_{2,i} Y = \begin{bmatrix} Z_1^T & Z_2^T \end{bmatrix}^T, \; i = 1, ..., r \tag{36}$$

with $Z_1 \in \Re^{n-m}$ and $Z_2 \in \Re^m$ where

$$T_{2,i} = \begin{bmatrix} I_{n-m} & 0 \\ F_i & I_m \end{bmatrix} \tag{37}$$

Then, the state variables $Z_1$and Z_2 are

$$\begin{cases} Z_1 = Y_1 \\ Z_2 = F_i Y_1 + Y_2 \end{cases} \tag{38}$$

The transformed system is given by

$$\begin{cases} \dot{Z}_1 = \sum_{i=1}^{r} h_i(z)(\Gamma_{1,i} Z_1 + \Gamma_{2,i} Z_2) \\ \dot{Z}_2 = \sum_{i=1}^{r} h_i(z)(\Gamma_{3,i} Z_1 + \Gamma_{4,i} Z_2 + B_{2,i} u) \end{cases} \tag{39}$$

with

$$\begin{cases} \Gamma_{1,i} = A_{11,i} - A_{12,i} F_i \\ \Gamma_{2,i} = A_{12,i} \\ \Gamma_{3,i} = F_i \Gamma_{1,i} - A_{22,i} F_i + A_{21,i} \\ \Gamma_{4,i} = A_{22,i} + A_{12,i} F_i \end{cases} \tag{40}$$

3.2.1. **Design of the Linear Control Law $u_{L,i}^{(1)}$.** The i-th linear control law $u_{L,i}^{(1)}$ is obtained by taking $Z_2 = \dot{Z}_2 = 0$, which is defined as

$$u_{L,i}^{(1)}(Z) = -B_{2,i}^{-1} \left\{ \Gamma_{3,i} Z_1 + \left(\Gamma_{4,i} - \Gamma_{4,i}^{*} \right) Z_2 \right\} \tag{41}$$

where $\Gamma_{4,i}^{*} \in \Re^{m \times m}$ is a design matrix such that its eigen-values are in the left half complex plane. In particular, we may set $\Gamma_{4,i}^{*} = diag\{\mu_{e,i}\}$ such that $Re\{\mu_{e,i}\} < 0$ for $e = 1, ..., m$. Transforming back into the original x-space yields

$$u_{L,i}^{(1)}(x) = -L_i^{(1)} x \tag{42}$$

with

$$L_i^{(1)} = B_{2,i}^{-1} \left[\Gamma_{3,i} \;\; \left(\Gamma_{4,i} - \Gamma_{4,i}^{*} \right) \right] T_{2,i} T_{1,i} \tag{43}$$

3.2.2. **Design of the Linear Control Law $u_{L,i}^{(2)}$.** Once the gains of the first linear control law are obtained, we design the second linear control law $u_{L,i}^{(2)}$ so that the closed loop of the designed system is able to quickly converge and stay in the desirable optimal fuzzy sliding surface. For simplicity, we consider a SISO system model as

$$\begin{cases} \dot{x}(t) = A_i^{(1)} x(t) + B_i u(t) \\ S(t) = \tilde{C}_i x(t) \end{cases} \quad i = 1, ..., r \tag{44}$$

with

$$A_i^{(1)} = A_i - B_i L_i^{(1)} \tag{45}$$

where $A_i^{(1)} \in \Re^{4\times 4}$, $u \in \Re$ and $\tilde{C}_i \in \Re^{1\times 4}$. The sliding surface equation (44) can be expressed as [32]

$$S(t) = \tilde{C}_i x(t) = \begin{bmatrix} \tilde{C}_{1,i} & \tilde{C}_{2,i} & \tilde{C}_{3,i} & \tilde{C}_{4,i} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = 0 \tag{46}$$

From equation (46), we define the following constrained equation

$$\begin{aligned} x_1(t) &= -\tilde{C}_{1,i}^{-1} \begin{bmatrix} \tilde{C}_{2,i} & \tilde{C}_{3,i} & \tilde{C}_{4,i} \end{bmatrix} \begin{bmatrix} x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} \\ &= \begin{bmatrix} q_{2,i} & q_{3,i} & q_{4,i} \end{bmatrix} = 0 \end{aligned} \tag{47}$$

and by taking the output of the system in equation (44) has the following form

$$y(t) = x_1(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \\ x_4(t) \end{bmatrix} = C_i x(t) \tag{48}$$

then, the performance index for the tracking problem can be defined as

$$J_i^{(2)} = \int_0^\infty \Big([y(t) - S(t)]^T Q_i^{(2)} [y(t) - S(t)] + u^T(t) R_i^{(2)} u(t) \Big) dt \tag{49}$$

The higher value of $Q_i^{(2)}$would result in the lower tracking energy loss in the output $y(t)$ in equation (60) [43]. Taking $Q_i^{(2)} = I$ and substituting equations (47) and (48) into equation (49), we obtain $S(t) = 0$ and

$$\begin{aligned} J_i^{(2)} &= \textstyle\int_0^\infty \Big(x_1^2(t) + u^T(t) R_i^{(2)} u(t) \Big) dt \\ &= \textstyle\int_0^\infty \begin{pmatrix} q_{2,i} x_2^2(t) + q_{3,i} x_3^2(t) + q_{4,i} x_4^2(t) + 2q_{2,i} q_{3,i} x_2(t) x_3(t) \\ +2q_{3,i} q_{4,i} x_3(t) x_4(t) + 2q_{2,i} q_{4,i} x_2(t) x_4(t) + u^T(t) R_i^{(2)} u(t) \end{pmatrix} dt \\ &= \textstyle\int_0^\infty \Big(x^T(t) \hat{Q}_i x(t) + u^T(t) R_i^{(2)} u(t) \Big) dt \end{aligned} \tag{50}$$

with

$$\hat{Q}_i = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \hat{q}_{22,i} & \hat{q}_{23,i} & \hat{q}_{24,i} \\ 0 & \hat{q}_{23,i} & \hat{q}_{33,i} & \hat{q}_{34,i} \\ 0 & \hat{q}_{24,i} & \hat{q}_{34,i} & \hat{q}_{44,i} \end{bmatrix} \tag{51}$$

where, $\hat{q}_{22,i} = q_{2,i}^2, \hat{q}_{33,i} = q_{3,i}^2, \hat{q}_{44,i} = q_{4,i}^2, \hat{q}_{23,i} = q_{2,i} q_{3,i}, \hat{q}_{24,i} = q_{2,i} q_{4,i}$ and $\hat{q}_{34,i} = q_{3,i} q_{4,i}$.

To solve the constrained problem for the fuzzy sliding surface tracking, the equations (44), (46) and (49) can be transformed into an equivalent PDC-based LQR problem. Thus, equations (44) and (50) are rewritten as follows

$$\dot{x}(t) = A_i^{(1)} x(t) + B_i u(t) \tag{52}$$

$$J_i^{(2)} = \int_0^{\infty} \left(x(t)^T \hat{Q}_i x(t) + u^T(t) R_i^{(2)} u(t) \right) dt \tag{53}$$

The global linear control law becomes

$$u(t) = -\sum_{i=1}^{r} h_i(z) L_i x(t) \tag{54}$$

where $P > 0$ is the solution of the following Riccati equation

$$A_i^{(1)T} P + P A_i^{(1)} + \hat{Q}_i - P B_i \left(R_i^{(2)} \right)^{-1} P = 0 \tag{55}$$

From equations (52) and (53), the second optimal controller for the system equation (52) is

$$u_{L,i}^{(2)}(t) = -L_i^{(2)} x(t) = -\left(R_i^{(2)} \right)^{-1} B_i^T P x(t) \tag{56}$$

where $L_i^{(2)}$ is determined from the solution of the equivalent LQR problem described in equations (52), (53) and (55). The designed system becomes

$$\dot{x}(t) = A_i^{(2)} x(t) \tag{57}$$

where $A_i^{(2)} = A_i^{(1)} - B_i L_i^{(2)}$. Subsequently, the total sliding mode controller in equation (35) for the system equation (1) can be determined from equation (42) and equation (56) as

$$u(t) = \sum_{i=1}^{r} h_i(z) \left(u_{L,i}^{(1)}(t) + u_{L,i}^{(2)}(t) + u_{N,i} \right) = \sum_{i=1}^{r} h_i(z) \left(-L_i x(t) + u_{N,i} \right) \tag{58}$$

with

$$L_i = L_i^{(1)} + L_i^{(2)}. \tag{59}$$

The optimally designed sliding mode controlled system becomes

$$\begin{cases} \dot{x}(t) = (A_i - B_i L_i) x(t) + B_i u_N(t) \\ y(t) = C_i x(t) \end{cases} \quad i = 1, ..., r \tag{60}$$

The optimally designed controller in equation (58) is able to minimize the tracking error between the output trajectory $y(t)$ and the sliding surface $S(t)$ in equation (49) and enables the controlled state trajectory to satisfy the constrained equation in equation (47) or to stay in the sliding surface $S(t) = 0$ in equation (46).

3.2.3. **Design of the Nonlinear Control Law $u_{N,i}$.** The scalar/ matrix *sign* function [31] is used to design the nonlinear control law $u_{N,i}$. It is characterized by the interesting property of replacing the non-smooth function by a smooth and differentiable continuous-time function [44]. In addition, it is able to reduce the chattering phenomenon. The matrix *sign* function is defined as [6]

$$sign_l(S) = \left[(I_m + S)^l - (I_m - S)^l \right] \left[(I_m + S)^l + (I_m - S)^l \right]^{-1} \tag{61}$$

where $S \in \Re^m$ and I_m is the identity matrix. Using equation (58), the nonlinear control law $u_{N,i}$ is

$$u_{N,i} = E_i \, sign_l(S(t)) \tag{62}$$

where $E_i = -\left[C_i\left(A_i - B_iL_i\right)^{-1}B_i\right]^{-1}$. The scalar function with different values of l is given in Figure 1.

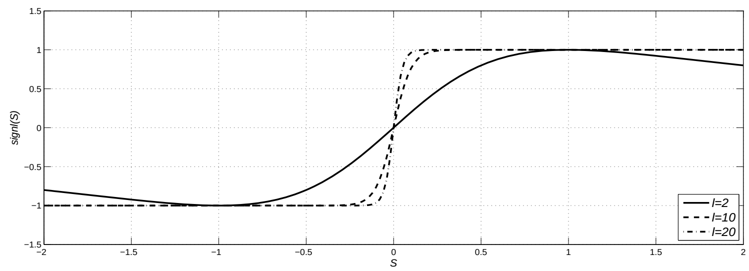

FIGURE 1. Scalar *sign* Function

3.3. **The Proposed PSOFSMC Design.** The proposed FSMC based on PSO combines both of the advantages of FSMC and PSO algorithm. PSO algorithm is used to search and to fine tune the weight matrices Q_i and R_i of PDC-based LQR controller. So, the P in Riccati matrix algebraic equation is calculated, and then the feedback gains F_i can be solved. Consequently, the optimal sliding mode surface is obtained. Once this latter having established, the next step, is to design the global control law such that the proposed PSOFSMC presents the desired dynamic characteristics.

The general design steps of the proposed controller are summarized as follows:

- **Step 1.** Computes the optimal weight matrices of $Q_i^{(1)} \geq 0$ and $R_i^{(1)} > 0$ that minimize the restricted function (equation (22)) using the procedure of Table 1.
- **Step 2.** Determine the sliding surface parameters $\tilde{C}_i$ for each local state model via equation (34)
- **Step 3.** Calculate the state feedback gains $L_i^{(1)}$for each local state model via equation (43)
- **Step 4.** Compute $R_i^{(2)}$that minimize the restricted function (equation (53)) using the procedure of Table 1 and by replacing S2 and S3 with:

$$Fitness = \int_0^\infty \left(x(t)^T\hat{Q}_ix\left(t\right) + u^T\left(t\right)R_i^{(2)}u\left(t\right)\right)dt \tag{63}$$

  and find the best particle labeled as $R^2 = \left[\begin{array}{cccc} R_1^{best2}, & R_2^{best2}, & ..., & R_r^{best2}\end{array}\right]$.
- **Step 5.** Calculate the state feedback gains L_i for each local state model via equation (59).
- **Step 6.** Design the control law u via equation (35).

S1. Choose the weighting matrices $Q_i^{(1)} \geq 0$ and $R_i^{(1)} > 0$ of PDC-based LQR controller and the number of particles N_P. Initialize the position and velocity of each particle; fix learning factors c_1 and c_2; ω_{max}; ω_{min} and the number of iterations s_{max}.
S2. For $s = 1$ to s_{max} do
for each particle do

(1) Calculate the fitness value of each particle by minimizing the following equation:
$$Fitness = \int_0^{\infty} \left(Y_1^T {Q_i}^{(1)} Y_1 + Y_2^T {R_i}^{(1)} Y_2 \right) dt \tag{64}$$
so as to obtain a compromise among response time and control effort.

(2) Find the individual best *pbest* for each particle and the global best *gbest*.

(3) Update the velocity and the position of each particle using equations (7) and (8), respectively.
end for
end for

S3. Find the best particle labeled as $Q^1 = \left[\ Q_1^{best1},\ \ Q_2^{best1},\ \ ...,\ \ Q_r^{best1}\ \right], R^1 = \left[\ R_1^{best1},\ \ R_2^{best1},\ \ ...,\ \ R_r^{best1}\ \right]$.

TABLE 1. Procedure of Weight Matrices Optimization

3.4. **Reaching mode.** The designed state trajectory would optimally track and stay in the fuzzy sliding surface $S(t)$ in order that the proposed control law is stable for a desired performance.
Consider the Lyapunov function
$$V = S^T S \tag{65}$$
$$V = x^T \tilde{C}^T \tilde{C} x = x^T \tilde{P}\, x \geq 0 \tag{66}$$
where $\dot{x}(t) = \sum\limits_{i=1}^{r}\sum\limits_{j=1}^{r} h_i(z(t)) h_j(z(t)) \left[(A_i - B_i L_j)\, x(t) + B_i\, E_j\, sign_l(S(t)) \right]$ and $\tilde{P} = \tilde{C}^T \tilde{C} \geq 0$.
The time derivative of V can be obtained as
$$\dot{V} = \dot{x}^T \tilde{P}\, x + x^T \tilde{P}\, \dot{x} \tag{67}$$
$$\begin{aligned} \dot{V} = &\sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))\, h_j(z(t))\, x^T \left[(A_i - B_i L_j)^T \tilde{P} + \tilde{P}(A_i - B_i L_j) \right] x \\ &+2x^T \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(z(t))\, h_j(z(t))\, \tilde{P} B_i E_j \tilde{S} \end{aligned} \tag{68}$$
where $\tilde{S}$ designated as $sign_l(S(t))$ and $(A_i - B_i L_j)$ is designed to be asymptotically stable, with $\tilde{P} \geq 0$. According to Lyapunov equation$(A_i - B_i L_j)^T \tilde{P} + \tilde{P}(A_i - B_i L_j) = -\hat{Q}_i$, where $\hat{Q}_i \geq 0$ and $\tilde{P} \geq 0$. The equation (68) can be

written as

$$\dot{V}=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i\left(z\left(t\right)\right)h_j\left(z\left(t\right)\right)\left[\begin{array}{cc} x^T & \tilde{S}^T \end{array}\right]\left[\begin{array}{cc} -\hat{Q}_i & \tilde{P}B_iE_j \\ \left(\tilde{P}B_iE_j\right)^T & 0 \end{array}\right]\left[\begin{array}{c} x \\ \tilde{S} \end{array}\right]\leq 0 \tag{69}$$

This result implies that the system trajectory could reach the sliding surface in finite time and maintain on the sliding surface.

4. **Simulation Results**

In this section, we are going to examine the performance and effectiveness of the proposed controller developed above. We consider the link flexible joint manipulator as shown in Figure 2. Its dynamics given as:

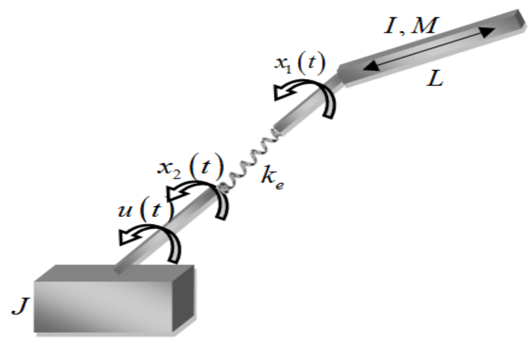

FIGURE 2. A Flexible Joint Manipulator

$$\left\{\begin{array}{l} \dot{x}_1=x_2 \\ \dot{x}_2=x_3 \\ \dot{x}_3=x_4 \\ \quad\dot{x}_4=\frac{MgL}{I}\sin x_1\left[x_2^2+\frac{MgL}{I}\cos x_1+\frac{k_e}{I}\right] \\ \quad+\left(x_3+\frac{MgL}{I}\sin x_1\right)\left[\frac{MgL}{I}\cos x_1+\frac{k_e}{I}+\frac{k_e}{J}\right]+\frac{k_e}{IJ}u \end{array}\right. \tag{70}$$

where I link inertia moment, J is the motor inertia moment, M is the link mass, k_e is the joint elastic constant, L is the distance from the axis of the rotation to the link center of mass, the gravitational acceleration $g=9.8\,ms^{-2}$ and u is the control input. For the simplicity of calculation, it is assumed that $I=1Kg\,m^2$, $J=1Kg\,m^2$, $M=1Kg$, $k_e=1N\,m^{-1}$ and $L=1m$.

We compare our results with those obtained with different controllers design. The T-S fuzzy rules of the system obtained around the equilibrium points $[x_1,\ x_2]=[-\pi,\ 0,\ \pi]$ are:

$$rule_1 : IF\ x_1\ is\ about\ 0\ and\ x_2\ is\ about\ 0\ \ THEN\ \dot{x}(t)=A_1x(t)+B_1u(t)$$
$$rule_2 : IF\ x_1\ is\ about\ 0\ and\ x_2\ is\ about\ \pi\ THEN\ \dot{x}(t)=A_2x(t)+B_2u(t)$$
$$rule_3 : IF\ x_1\ is\ about\ 0\ and\ x_2\ is\ about\ -\pi\ THEN\ \dot{x}(t)=A_3x(t)+B_3u(t)$$
$$rule_4 : IF\ x_1\ is\ about\ \pi\ and\ x_2\ is\ about\ 0\ \ THEN\ \dot{x}(t)=A_4x(t)+B_4u(t)$$

$$rule_5 : IF\ x_1\ is\ about\ \pi\ and\ x_2\ is\ about\ \pi\ THEN\ \dot{x}(t) = A_5x(t) + B_5u(t)$$
$$rule_6 : IF\ x_1\ is\ about\ \pi\ and\ x_2 is\ about\ -\pi\ THEN\ \dot{x}(t) = A_6x(t) + B_6u(t)$$
$$rule_7 : IF\ x_1\ is\ about\ -\pi\ and\ x_2 is\ about\ 0\ THEN\ \dot{x}(t) = A_7x(t) + B_7u(t)$$
$$rule_8 : IF\ x_1\ is\ about\ -\pi\ and\ x_2\ is\ about\ \pi\ THEN\ \dot{x}(t) = A_8x(t) + B_8u(t)$$

where

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -9.8 & 0 & -11.8 & 0 \end{bmatrix},\ A_2 = A_3 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 86.9 & 0 & -11.8 & 0 \end{bmatrix},$$

$$A_4 = A_7 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 9.8 & 0 & 7.8 & 0 \end{bmatrix}, A_5 = A_6 = A_8 = A_9 = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 86.9 & 0 & 7.8 & 0 \end{bmatrix},$$

,

$$B_i = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}^T,\ i = 1, 2,\ ..., 9.$$

The membership functions for the states are presented in Figure 3. The parameter

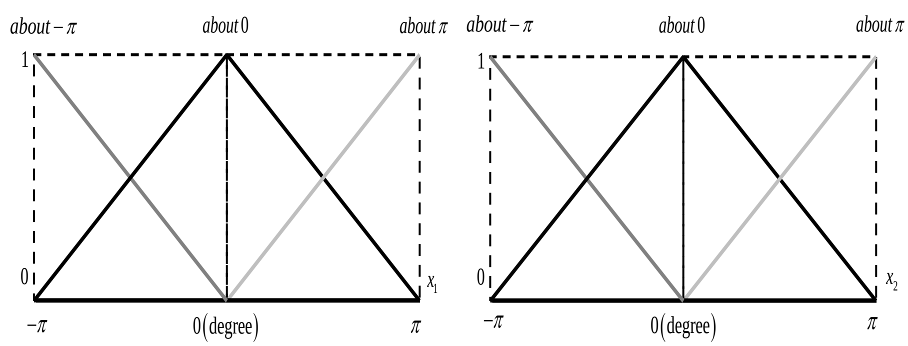

Figure 3. The Membership Functions

settings are $R_i = 1, Q_i = diag\{50 \quad 50 \quad 50 \quad 50\}, \quad (i = 1, \ldots, 9)$ for the PDC controller by the mean of the LQR. In addition, inner angle $\theta = \frac{\pi}{8}(rad)$, circle of centre $(0, 0)$, radius r_a is 10, vertical upper bound at $\sigma = -3.5$ for area LMI of the FSMC reported by [5]. The PSOPDC and PSOFSMC controllers perform best under the following settings: $\omega_{max} = 0.9$, $\omega_{min} = 0.4$, $c_1 = 0.2$, $c_2 = 1$, $N_P = 50$, $s_{max} = 500$ and $l = 2$. The optimized weight matrices $R_i^{(1)}$, $Q_i^{(1)}$, F_i, $R_i^{(2)}$, L_i and E_i of PSOFSMC are

$$R_1^{(1)} = 0.1590, R_2^{(1)} = 0.2171, R_3^{(1)} = 0.0816, R_4^{(1)} = 0.1710, R_5^{(1)} = 0.1955,$$

$$R_6^{(1)} = 0.0165, R_7^{(1)} = 0.1729, R_8^{(1)} = 0.0371, R_9^{(1)} = 0.0979,$$

$$Q_1^{(1)} = \begin{bmatrix} 37.8136 & 0 & 0 \\ 0 & 65.7290 & 0 \\ 0 & 0 & 92.2257 \end{bmatrix}, Q_2^{(1)} = \begin{bmatrix} 31.8804 & 0 & 0 \\ 0 & 31.8804 & 0 \\ 0 & 0 & 31.8804 \end{bmatrix},$$
$$Q_3^{(1)} = \begin{bmatrix} 60.5297 & 0 & 0 \\ 0 & 51.8357 & 0 \\ 0 & 0 & 46.9305 \end{bmatrix}, Q_4^{(1)} = \begin{bmatrix} 6.6095 & 0 & 0 \\ 0 & 6.6095 & 0 \\ 0 & 0 & 6.6095 \end{bmatrix},$$
$$Q_5^{(1)} = \begin{bmatrix} 47.7357 & 0 & 0 \\ 0 & 47.4725 & 0 \\ 0 & 0 & 46.9381 \end{bmatrix}, Q_6^{(1)} = \begin{bmatrix} 81.7115 & 0 & 0 \\ 0 & 92.9407 & 0 \\ 0 & 0 & 68.7792 \end{bmatrix},$$
$$Q_7^{(1)} = \begin{bmatrix} 28.0203 & 0 & 0 \\ 0 & 33.0662 & 0 \\ 0 & 0 & 33.6318 \end{bmatrix}, Q_8^{(1)} = \begin{bmatrix} 24.6126 & 0 & 0 \\ 0 & 31.1340 & 0 \\ 0 & 0 & 28.5967 \end{bmatrix},$$
$$Q_9^{(1)} = \begin{bmatrix} 39.5685 & 0 & 0 \\ 0 & 31.4591 & 0 \\ 0 & 0 & 49.8385 \end{bmatrix}, F_1 = \begin{bmatrix} 15.4198 & -22.0057 & -35.4761 \end{bmatrix},$$
$$F_2 = \begin{bmatrix} 12.1184 & -13.8111 & -21.9453 \end{bmatrix}, F_3 = \begin{bmatrix} 27.2293 & -26.9315 & -45.1830 \end{bmatrix},$$
$$F_4 = \begin{bmatrix} 6.2177 & -7.8763 & -11.6879 \end{bmatrix}, F_5 = \begin{bmatrix} 15.6277 & -17.2844 & -27.9352 \end{bmatrix},$$
$$F_6 = \begin{bmatrix} 70.3000 & -76.5874 & -122.1807 \end{bmatrix}, F_7 = \begin{bmatrix} 12.7320 & -15.4865 & -24.2676 \end{bmatrix},$$
$$F_8 = \begin{bmatrix} 25.7578 & -30.5966 & -48.4465 \end{bmatrix}, F_9 = \begin{bmatrix} 20.1068 & -19.8428 & -36.1547 \end{bmatrix}.$$

We get for $\Gamma_{4,i}^{*} = \{-4\}$, the following results:

$$R_1^{(2)} = 0.0852, R_2^{(2)} = 0.0524, R_3^{(2)} = 0.0412, R_4^{(2)} = 0.0272, R_5^{(2)} = 0.0424,$$
$$R_6^{(2)} = 0.0042, R_7^{(2)} = 0.0199, R_8^{(2)} = 0.0216, R_9^{(2)} = 0.0408,$$
$$L_1 = \begin{bmatrix} 51.8791 & 157.4737 & 111.7725 & 26.0091 \end{bmatrix},$$
$$L_2 = \begin{bmatrix} 135.3737 & 100.3091 & 65.5992 & 17.8242 \end{bmatrix},$$
$$L_3 = \begin{bmatrix} 195.8172 & 208.2129 & 141.2296 & 30.9364 \end{bmatrix},$$
$$L_4 = \begin{bmatrix} 34.6710 & 54.4608 & 51.8263 & 11.9501 \end{bmatrix},$$
$$L_5 = \begin{bmatrix} 52.7109 & 127.7641 & 85.4698 & 21.2950 \end{bmatrix},$$
$$L_6 = \begin{bmatrix} 368.1002 & 559.9017 & 417.1248 & 80.5959 \end{bmatrix},$$
$$L_7 = \begin{bmatrix} 137.8282 & 110.7639 & 74.8976 & 19.5145 \end{bmatrix},$$
$$L_8 = \begin{bmatrix} 112.8313 & 219.9815 & 178.8389 & 34.6042 \end{bmatrix},$$
$$L_9 = \begin{bmatrix} 167.3270 & 165.0446 & 123.4873 & 23.8508 \end{bmatrix},$$
$$E_1 = \begin{bmatrix} 61.6791 \end{bmatrix}, E_2 = \begin{bmatrix} 48.4737 \end{bmatrix}, E_3 = \begin{bmatrix} 108.9172 \end{bmatrix},$$
$$E_4 = \begin{bmatrix} 24.8710 \end{bmatrix}, E_5 = \begin{bmatrix} 62.5109 \end{bmatrix}, E_6 = \begin{bmatrix} 281.2002 \end{bmatrix},$$
$$E_7 = \begin{bmatrix} 50.9282 \end{bmatrix}, E_8 = \begin{bmatrix} 103.0313 \end{bmatrix}, E_9 = \begin{bmatrix} 80.4270 \end{bmatrix}.$$

The simulation results with $x(0) = \begin{bmatrix} 0 & 0 & 0.25 & 0 \end{bmatrix}^T$ are shown in Figures 4 and 5. It can be clearly shown that the proposed PSOPDC controller gives better performance compared with the standard PDC controller. The state feedback gains of the designed PSOPDC are determined using the optimal values of weighting matrices Q and R. Indeed, their choice is done via PSO algorithm. We also note that only the PSOFSMC method always retained better performance than that is proposed in [5]. Under the PSOFSMC, the states converge to zero with a shorter time response. As it is presented in Table 2, the proposed method has Steady-State Error (SSE) around 10^{-5}, while the FSMC proposed in [5] has a SSE of 3.6750e-4. To stabilize the closed-loop system, PSOFSMC needs the least control effort compared to the others (Figure 5). On the whole, we note that, our proposed PSOFSMC always keeps the best performance with a short response time and a much lower magnitude.

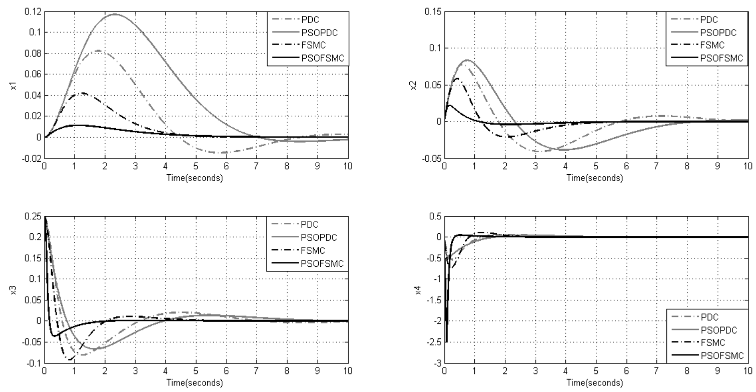

Figure 4. System State for Different Control Designs

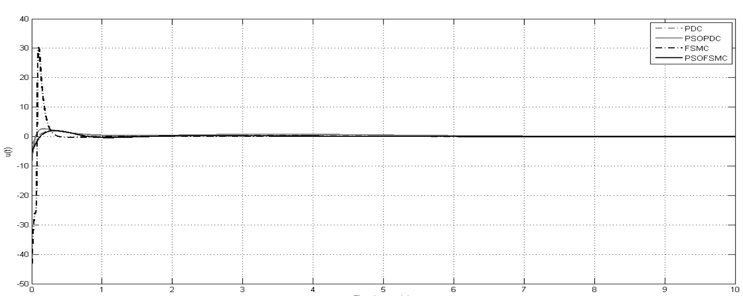

Figure 5. Control Signals of Different Control Designs

Table 2 shows that the proposed controller exhibits the best performance in terms of Steady-State Error.

	PDC	PSOPDC	FSMC	PSOFSMC
Steady-State Error	0.0013	0.0015	3.6750e-4	**3.0966e-5**

Table 2. SSEs of Regulation Results

5. Conclusion

In this paper, a new fuzzy sliding mode control based on particle swarm optimization is presented. The application of the particle swarm optimization improves the performance of the PDC based on LQR method. Indeed, all parameters concerning the LQR were determined using the PSO algorithm. As a result, the obtained controller gains are used to design the stable sliding surface using scalar *sign* function. Consequently, an optimal fuzzy sliding mode control is obtained considering the performance of the closed-loop system. In the proposed method, the required restriction such as the choice of weight matrices of the PDC controller and the sliding surface parameters are released compared to that of the existing approaches reported in literature. The applicability and the efficiency of these modifications are demonstrated by simulation results of selected highly nonlinear systems. Through these results, the proposed control scheme has been shown favorable results in terms of mathematical computation and response time compared with the other techniques.

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