

A NEW NOTION OF FUZZY PS-COMPACTNESS

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ABSTRACT. In this paper, using pre-semi-open L -sets and their inequality, a new notion of PS-compactness is introduced in L -topological spaces, where L is a complete De Morgan algebra. This notion does not depend on the structure of the basis lattice L and L does not need any distributivity.

1. Introduction

It is known that compactness and its stronger and weaker forms play very important roles in topology. Based on fuzzy topological spaces introduced by Chang [4], various kinds of fuzzy compactness [2-4,7,11] have been established. However, these concepts of fuzzy compactness rely on the structure of L and L is required to be completely distributive. In [10], for a complete De Morgan algebra L , Shi introduced a new definition of fuzzy compactness in L -topological spaces using open L -sets and their inequality. This new definition doesn't depend on the structure of L .

In this paper, following the lines of [10], we introduce a new notion of PS-compactness in L -topological spaces by means of pre-semi-open L -sets and their inequality. This notion can also be characterized by pre-semi-closed L -sets and their inequality and is a strong form of semi-compactness[9]. This form of PS-compactness is a good generalization and has many characterizations when L is completely distributive De Morgan algebra.

2. Preliminaries

Throughout this paper, $(L, \vee, \wedge, ')$ is a complete De Morgan algebra, X a nonempty set and L^X the set of all L -fuzzy sets (or L -sets for short) on X . The smallest element and the largest element in L^X are denoted by $\underline{0}$ and $\underline{1}$. An element a in L is called a prime element if $b \wedge c \leq a$ implies that $b \leq a$ or $c \leq a$. a in L is called a co-prime element if a' is a prime element [6] The set of nonunit prime elements in L is denoted by $P(L)$ and the set of nonzero co-prime elements in L by $M(L)$.

The binary relation \prec in L is defined as follows: for $a, b \in L$, $a \prec b$ iff for every subset $D \subseteq L$, the relation $b \leq \sup D$ always implies the existence of $d \in D$ with $a \leq d$ [5]. In a completely distributive De Morgan algebra L , each element b is a sup of $\{a \in L | a \prec b\}$. The set $\beta(b) = \{a \in L | a \prec b\}$ is called the greatest minimal family of b in the sense of [7,11]. Now, for $b \in L$, we define $\beta^*(b) = \beta(b) \cap M(L)$,

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$\alpha(b) = \{a \in L \mid a' \prec b'\}$ and $\alpha^*(b) = \alpha(b) \cap P(L)$. For $a \in L$ and $A \in L^X$, we write $A^{(a)} = \{x \in X \mid A(x) \not\leq a\}$ and $A_{(a)} = \{x \in X \mid a \in \beta(A(x))\}$ and for a subfamily $\psi \subseteq L^X$, $2^{(\psi)}$ will denote the set of all finite subfamilies of ψ .

An L -topological space (or L -ts for short) is a pair (X, δ) , where δ is a subfamily of L^X which contains $\underline{0}, \underline{1}$ and is closed for any suprema and finite infima. δ is called an L -topology on X . Each member of δ is called an open L -set and its quasi-complement is called a closed L -set.

Definition 2.1. [7,11] For a topological space (X, τ) , let $\omega_L(\tau)$ denote the family of all the lower semi-continuous maps from (X, τ) to L ; i.e. $\omega_L(\tau) = \{A \in L^X \mid A^{(a)} \in \tau, a \in L\}$. Then $\omega_L(\tau)$ is an L -topology on X and $(X, \omega_L(\tau))$ is topologically generated by (X, τ) .

Definition 2.2. [7,11] An L -ts (X, δ) is weak induced if, for all $a \in L$ and for all $A \in \delta$, it follows that $A^{(a)} \in [\delta]$, where $[\delta]$ denotes the topology formed by all crisp sets in δ . It is obvious that $(X, \omega_L(\tau))$ is weak induced.

Definition 2.3. [9] Let (X, δ) be an L -ts, $a \in L \setminus \{1\}$, and $A \in L^X$. A family $\mu \subseteq L^X$ is called

- (1) an a -shading of A if for any $x \in X$, $(A'(x) \vee \bigvee_{B \in \mu} B(x)) \not\leq a$.
- (2) a strong a -shading (briefly S- a -shading) of A if $\bigwedge_{x \in X} (A'(x) \vee \bigvee_{B \in \mu} B(x)) \not\leq a$.
- (3) an a -R-neighborhood family (briefly a -R-NF) of A if for any $x \in X$, $(A(x) \wedge \bigwedge_{B \in \mu} B(x)) \not\leq a$.
- (4) a strong a -R-neighborhood family (briefly S- a -R-NF) of A if $\bigvee_{x \in X} (A(x) \wedge \bigwedge_{B \in \mu} B(x)) \not\leq a$.

It is obvious that an S- a -shading of A is an a -shading of A , an S- a -R-NF of A is an a -R-NF of A , and μ is an S- a -R-NF of A iff μ' is an S- a -shading of A .

Definition 2.4. [9] Let (X, δ) be an L -ts, $a \in L \setminus \{0\}$ and $A \in L^X$. A family $\mu \subseteq L^X$ is called

- (1) a β_a -cover of A if for any $x \in X$, it follows that $a \in \beta(A'(x) \vee \bigvee_{B \in \mu} B(x))$.
- (2) a strong β_a -cover (briefly S- β_a -cover) of A if $a \in \beta(\bigwedge_{x \in X} (A'(x) \vee \bigvee_{B \in \mu} B(x)))$.
- (3) a Q_a -cover of A if for any $x \in X$, it follows that $A'(x) \vee \bigvee_{B \in \mu} B(x) \geq a$.

It is obvious that an S- β_a -cover of A must be a β_a -cover of A , and a β_a -cover of A must be a Q_a -cover of A .

Definition 2.5. [9] Let $a \in L \setminus \{0\}$ and $A \in L^X$. A family $\mu \subseteq L^X$ is said to have weak a -nonempty intersection in A if $\bigvee_{x \in X} (A(x) \wedge \bigwedge_{B \in \mu} B(x)) \geq a$. μ is said to have the finite weak a -intersection property in A if every finite subfamily ν of μ has weak a -nonempty intersection in A .

Lemma 2.6. [8] Let L be a complete Heyting algebra, $f : X \rightarrow Y$ a map and $f_L^\rightarrow : L^X \rightarrow L^Y$ the extension of f . Then for any family $\psi \subseteq L^Y$,

$$\bigvee_{y \in Y} (f_L^\rightarrow(A)(y) \wedge \bigwedge_{B \in \psi} B(y)) = \bigvee_{x \in X} (A(x) \wedge \bigwedge_{B \in \psi} f_L^\rightarrow(B)(x)).$$

Definition 2.7. [1] Let (X, δ) be an L -ts, $A \in L^X$. Then A is called a pre-semi-open set if $A \leq (A^-)_o$, and A is called a pre-semi-closed set if $A \geq (A^o)_-$, where A^o , A^- , A_o and A_- are the interior, closure, semi-interior and semi-closure of A , respectively.

Definition 2.8. [1,2] Let (X, δ) and (Y, τ) be two L -ts's. A map $f : (X, \delta) \rightarrow (Y, \tau)$ is called

- (1) pre-semi-continuous if $f_L^-(B)$ is pre-semi-open in (X, δ) for every $B \in \tau$.
- (2) pre-semi-irresolute if $f_L^-(B)$ is pre-semi-open in (X, δ) for every pre-semi-open L -set B in (Y, τ) .

3. Definition and Properties of PS-compactness

Definition 3.1. Let (X, δ) be an L -ts. $A \in L^X$ is called PS-compact if for every family μ of pre-semi-open L -sets, it follows that

$$\bigwedge_{x \in X} (A'(x) \vee \bigvee_{B \in \mu} B(x)) \leq \bigvee_{\nu \in 2^{(\mu)}} \bigwedge_{x \in X} (A'(x) \vee \bigvee_{B \in \nu} B(x)).$$

(X, δ) is called PS-compact if $\mathbb{1}$ is PS-compact.

Example 3.2. Let $X = \{x\}$ and $L = \{0, 1/3, 2/3, 1\}$. For each $a \in L$ define $a' = 1 - a$. Let $\delta = \{\emptyset, A, X\}$, where $A(x) = 2/3$, then δ is a topology on X . Clearly, any L -set in (X, δ) is PS-compact.

Example 3.3. Let X be an infinite set or a singleton, A and C be two $[0, 1]$ -sets on X defined as $A(x) = 0.5$, for all $x \in X$; $C(x) = 0.6$, for all $x \in X$. Take $\delta = \{\emptyset, A, X\}$, then δ is a topology on X . Obviously, any $[0, 1]$ -set in (X, δ) is pre-semi-open, and the set of all semi-open $[0, 1]$ -sets in (X, δ) is δ . In this case, we easily obtain that C is not PS-compact, and any $[0, 1]$ -set in (X, δ) is semi-compact.

Remark 3.4. Since every semi-open L -set is pre-semi-open[1], every PS-compact L -set is semi-compact. Example 3.3 shows that a semi-compact L -set needn't be PS-compact.

Theorem 3.5. Let (X, δ) be an L -ts. $A \in L^X$ is PS-compact iff for every family μ of pre-semi-closed L -sets, it follows that

$$\bigvee_{x \in X} (A(x) \wedge \bigwedge_{B \in \mu} B(x)) \geq \bigwedge_{\nu \in 2^{(\mu)}} \bigvee_{x \in X} (A(x) \wedge \bigwedge_{B \in \nu} B(x)).$$

Proof. This is immediate from Definition 3.1 and quasi-complements. \square

Theorem 3.6. Let (X, δ) be an L -ts and $A \in L^X$. Then the following conditions are equivalent.

- (1) A is PS-compact.
- (2) For any $a \in L \setminus \{1\}$, each pre-semi-open S - a -shading μ of A has a finite subfamily which is an S - a -shading of A .
- (3) For any $a \in L \setminus \{0\}$, each pre-semi-closed S - a -R-NF ψ of A has a finite subfamily which is an S - a -R-NF of A .
- (4) For any $a \in L \setminus \{0\}$, each family of pre-semi-closed L -sets which has the finite weak a -intersection property in A has weak a -nonempty intersection in A .

Proof. The theorem follows immediately from Definition 3.1 and Theorem 3.5. \square

Theorem 3.7. *Let L be a complete Heyting algebra. If both C and D are PS-compact, then $C \vee D$ is PS-compact.*

Proof. By Theorem 3.5 for any family μ of pre-semi-closed L -sets, we have

$$\begin{aligned} & \bigvee_{x \in X} ((C \vee D)(x) \wedge \bigwedge_{B \in \mu} B(x)) \\ &= \{ \bigvee_{x \in X} (C(x) \wedge \bigwedge_{B \in \mu} B(x)) \} \vee \{ \bigvee_{x \in X} (D(x) \wedge \bigwedge_{B \in \mu} B(x)) \} \\ &\geq \{ \bigwedge_{\nu \in 2^{(\mu)}} \bigvee_{x \in X} (C(x) \wedge \bigwedge_{B \in \nu} B(x)) \} \vee \{ \bigwedge_{\nu \in 2^{(\mu)}} \bigvee_{x \in X} (D(x) \wedge \bigwedge_{B \in \nu} B(x)) \} \\ &= \bigwedge_{\nu \in 2^{(\mu)}} \bigvee_{x \in X} ((C \vee D)(x) \wedge \bigwedge_{B \in \nu} B(x)). \end{aligned}$$

This shows that $C \vee D$ is PS-compact. \square

Theorem 3.8. *Let (X, δ) be an L -ts and $C, D \in L^X$. If C is PS-compact and D is pre-semi-closed, then $C \wedge D$ is PS-compact.*

Proof. By Theorem 3.5, for any family μ of pre-semi-closed L -sets, we have

$$\begin{aligned} & \bigvee_{x \in X} ((C \wedge D)(x) \wedge \bigwedge_{B \in \mu} B(x)) \\ &= \bigvee_{x \in X} (C(x) \wedge \bigwedge_{B \in \mu \cup \{D\}} B(x)) \\ &\geq \bigwedge_{\nu \in 2^{(\mu \cup \{D\})}} \bigvee_{x \in X} (C(x) \wedge \bigwedge_{B \in \nu} B(x)) \\ &= \{ \bigwedge_{\nu \in 2^{(\mu)}} \bigvee_{x \in X} (C(x) \wedge \bigwedge_{B \in \nu} B(x)) \} \wedge \{ \bigwedge_{\nu \in 2^{(\mu)}} \bigvee_{x \in X} (C(x) \wedge D(x) \wedge \bigwedge_{B \in \nu} B(x)) \} \\ &= \{ \bigwedge_{\nu \in 2^{(\mu)}} \bigvee_{x \in X} (C(x) \wedge D(x) \wedge \bigwedge_{B \in \nu} B(x)) \} \\ &= \bigwedge_{\nu \in 2^{(\mu)}} \bigvee_{x \in X} ((C \wedge D)(x) \wedge \bigwedge_{B \in \nu} B(x)). \end{aligned}$$

This shows that $C \wedge D$ is PS-compact. \square

Corollary 3.9. *Let (X, δ) be PS-compact and $D \in L^X$ be pre-semi-closed. Then D is PS-compact.*

Definition 3.10. Let (X, δ) and (Y, τ) be two L -ts's. A map $f : (X, \delta) \rightarrow (Y, \tau)$ is called

(1) strongly pre-semi-continuous if $f_L^-(B)$ is pre-semi-open in (X, δ) for every semi-open L -set B in (Y, τ) .

(2) strongly pre-semi-irresolute if $f_L^-(B)$ is semi-open in (X, δ) for every pre-semi-open L -set B in (Y, τ) .

It is obvious that a strongly pre-semi-continuous map is pre-semi-continuous, and a strongly pre-semi-irresolute map is pre-semi-irresolute.

From Lemma 2.6 and Definitions 2.7, 2.8, 3.10, we can obtain the following theorems.

Theorem 3.11. *Let L be a complete Heyting algebra and $f : (X, \delta) \rightarrow (Y, \tau)$ be a pre-semi-irresolute map. If A is a PS-compact L -set in (X, δ) , then $f_L^-(A)$ is a PS-compact L -set in (Y, τ) .*

Theorem 3.12. *Let L be a complete Heyting algebra and $f : (X, \delta) \rightarrow (Y, \tau)$ be a pre-semi-continuous map. If A is a PS-compact L -set in (X, δ) , then $f_L^{\rightarrow}(A)$ is a compact L -set in (Y, τ) .*

Theorem 3.13. *Let L be a complete Heyting algebra and $f : (X, \delta) \rightarrow (Y, \tau)$ be a strongly pre-semi-continuous map. If A is a PS-compact L -set in (X, δ) , then $f_L^{\rightarrow}(A)$ is a semi-compact L -set in (Y, τ) .*

Theorem 3.14. *Let L be a complete Heyting algebra and $f : (X, \delta) \rightarrow (Y, \tau)$ be a strongly pre-semi-irresolute map. If A is a semi-compact L -set in (X, δ) , then $f_L^{\rightarrow}(A)$ is a PS-compact L -set in (Y, τ) .*

4. Further Properties of PS-compactness and Goodness

In this section, we assume that L is a completely distributive de Morgan algebra.

Theorem 4.1. *Let (X, δ) be an L -ts and $A \in L^X$. Then the following conditions are equivalent.*

- (1) A is PS-compact.
- (2) For any $a \in L \setminus \{0\}$, each pre-semi-closed S - a - R -NF ψ of A has a finite subfamily which is an S - a - R -NF of A .
- (3) For any $a \in L \setminus \{0\}$, each pre-semi-closed S - a - R -NF ψ of A has a finite subfamily which is an a - R -NF of A .
- (4) For any $a \in L \setminus \{0\}$ and any pre-semi-closed S - a - R -NF ψ of A , there exist a finite subfamily φ of ψ and $b \in \beta(a)$ such that φ is an S - b - R -NF of A .
- (5) For any $a \in L \setminus \{0\}$ and any pre-semi-closed S - a - R -NF ψ of A , there exist a finite subfamily φ of ψ and $b \in \beta(a)$ such that φ is a b - R -NF of A .
- (6) For any $a \in M(L)$, each pre-semi-closed S - a - R -NF ψ of A has a finite subfamily which is an S - a - R -NF of A .
- (7) For any $a \in M(L)$, each pre-semi-closed S - a - R -NF ψ of A has a finite subfamily which is an a - R -NF of A .
- (8) For any $a \in M(L)$ and any pre-semi-closed S - a - R -NF ψ of A , there exist a finite subfamily φ of ψ and $b \in \beta^*(a)$ such that φ is an S - b - R -NF of A .
- (9) For any $a \in M(L)$ and any pre-semi-closed S - a - R -NF ψ of A , there exist a finite subfamily φ of ψ and $b \in \beta^*(a)$ such that φ is a b - R -NF of A .
- (10) For any $a \in L \setminus \{1\}$, each pre-semi-open S - a -shading μ of A has a finite subfamily which is an S - a -shading of A .
- (11) For any $a \in L \setminus \{1\}$, each pre-semi-open S - a -shading μ of A has a finite subfamily which is an a -shading of A .
- (12) For any $a \in L \setminus \{1\}$ and any pre-semi-open S - a -shading μ of A , there exist a finite subfamily ν of μ and $b \in \alpha(a)$ such that ν is an S - b -shading of A .
- (13) For any $a \in L \setminus \{1\}$ and any pre-semi-open S - a -shading μ of A , there exist a finite subfamily ν of μ and $b \in \alpha(a)$ such that ν is a b -shading of A .
- (14) For any $a \in P(L)$, each pre-semi-open S - a -shading μ of A has a finite subfamily which is an S - a -shading of A .
- (15) For any $a \in P(L)$, each pre-semi-open S - a -shading μ of A has a finite subfamily which is an a -shading of A .

(16) For any $a \in P(L)$ and any pre-semi-open S - a -shading μ of A , there exist a finite subfamily ν of μ and $b \in \alpha^*(a)$ such that ν is an S - b -shading of A .

(17) For any $a \in P(L)$ and any pre-semi-open S - a -shading μ of A , there exist a finite subfamily ν of μ and $b \in \alpha^*(a)$ such that ν is a b -shading of A .

(18) For any $a \in L \setminus \{0\}$, each pre-semi-open S - β_a -cover μ of A has a finite subfamily which is an S - β_a -cover of A .

(19) For any $a \in L \setminus \{0\}$, each pre-semi-open S - β_a -cover μ of A has a finite subfamily which is a β_a -cover of A .

(20) For any $a \in L \setminus \{0\}$ and any pre-semi-open S - β_a -cover μ of A , there exist a finite subfamily ν of μ and $b \in L$ with $a \in \beta(b)$ such that ν is an S - β_b -cover of A .

(21) For any $a \in L \setminus \{0\}$ and any pre-semi-open S - β_a -cover μ of A , there exist a finite subfamily ν of μ and $b \in L$ with $a \in \beta(b)$ such that ν is a β_b -cover of A .

(22) For any $a \in M(L)$, each pre-semi-open S - β_a -cover μ of A has a finite subfamily which is an S - β_a -cover of A .

(23) For any $a \in M(L)$, each pre-semi-open S - β_a -cover μ of A has a finite subfamily which is a β_a -cover of A .

(24) For any $a \in M(L)$ and any pre-semi-open S - β_a -cover μ of A , there exist a finite subfamily ν of μ and $b \in M(L)$ with $a \in \beta^*(b)$ such that ν is an S - β_b -cover of A .

(25) For any $a \in M(L)$ and any pre-semi-open S - β_a -cover μ of A , there exist a finite subfamily ν of μ and $b \in M(L)$ with $a \in \beta^*(b)$ such that ν is a β_b -cover of A .

(26) For any $a \in L \setminus \{0\}$ and any $b \in \beta(a) \setminus \{0\}$, each pre-semi-open Q_a -cover μ of A has a finite subfamily which is a Q_b -cover of A .

(27) For any $a \in L \setminus \{0\}$ and any $b \in \beta(a) \setminus \{0\}$, each pre-semi-open Q_a -cover μ of A has a finite subfamily which is a β_b -cover of A .

(28) For any $a \in L \setminus \{0\}$ and any $b \in \beta(a) \setminus \{0\}$, each pre-semi-open Q_a -cover μ of A has a finite subfamily which is an S - β_b -cover of A .

(29) For any $a \in M(L)$ and any $b \in \beta^*(a)$, each pre-semi-open Q_a -cover μ of A has a finite subfamily which is a Q_b -cover of A .

(30) For any $a \in M(L)$ and any $b \in \beta^*(a)$, each pre-semi-open Q_a -cover μ of A has a finite subfamily which is a β_b -cover of A .

(31) For any $a \in M(L)$ and any $b \in \beta^*(a)$, each pre-semi-open Q_a -cover μ of A has a finite subfamily which is an S - β_b -cover of A .

Proof. (1) \Leftrightarrow (2) : This follows directly from Theorem 3.6.

(2) \Rightarrow (3) : This is easy to prove if one notices that every S - a -R-NF of A is an a -R-NF of A .

(3) \Rightarrow (4) : Let $a \in L \setminus \{0\}$ and ψ be a pre-semi-closed S - a -R-NF of A . Then $\bigvee_{x \in X} (A(x) \wedge \bigwedge_{B \in \psi} B(x)) \not\geq a$. Take $c \in \beta(a)$ such that $\bigvee_{x \in X} (A(x) \wedge \bigwedge_{B \in \psi} B(x)) \not\geq c$. Obviously ψ is a pre-semi-closed S - c -R-NF of A . By (3) we know that ψ has a finite subfamily φ which is a c -R-NF of A . Take $b \in \beta(a)$ such that $c \in \beta(b)$, then φ is an S - b -R-NF of A .

(4) \Rightarrow (5) \Rightarrow (2) : Obvious.

(2) \Rightarrow (6) \Rightarrow (7) \Rightarrow (8) \Rightarrow (9) \Rightarrow (1) : the proof is similar.

(1) \Leftrightarrow (10) : This follows directly from Theorem 3.6.

(10) \Rightarrow (11) : This is easy to prove if one notices that every S- a -shading of A is an a -shading of A .

(11) \Rightarrow (12) : Let $a \in L \setminus \{1\}$ and μ be a pre-semi-open S- a -shading of A . Then $\bigwedge_{x \in X} (A'(x) \vee \bigvee_{B \in \mu} B(x)) \not\leq a$. Take $c \in \alpha(a)$ such that $\bigwedge_{x \in X} (A'(x) \vee \bigvee_{B \in \mu} B(x)) \not\leq c$. Obviously μ is a pre-semi-open S- c -shading of A . By (11) we know that μ has a finite subfamily ν which is a c -shading of A . Take $b \in \alpha(a)$ such that $c \in \alpha(b)$, then ν is an S- b -shading of A .

(12) \Rightarrow (13) \Rightarrow (10) : Obvious.

(10) \Rightarrow (14) \Rightarrow (15) \Rightarrow (16) \Rightarrow (17) \Rightarrow (10) : We can prove these in the similar way.

Similarly we can also prove the other results. \square

Lemma 4.2. *Let $(X, \omega_L(\tau))$ be generated topologically by (X, τ) . If A is a pre-semi-open L -set in (X, τ) , then χ_A is a pre-semi-open set in $(X, \omega_L(\tau))$. If B is a pre-semi-open L -set in $(X, \omega_L(\tau))$, then, $B_{(a)}$ is a pre-semi-open set in (X, τ) for every $a \in L$.*

Proof. This is easy to prove if one notices that $\chi_{D^-} = (\chi_D)^-$ and $\chi_{D_o} = (\chi_D)_o$ and applies Lemma 5.4 in [9]. \square

Theorem 4.3. *Let (X, τ) be a topological space and $(X, \omega_L(\tau))$ be generated topologically by (X, τ) . Then $(X, \omega_L(\tau))$ is PS-compact iff (X, τ) is PS-compact.*

Proof. Necessity: Let μ be a pre-semi-open cover of (X, τ) . Then $\{\chi_A | A \in \mu\}$ is a family of pre-semi-open L -sets in $(X, \omega_L(\tau))$ with $\bigwedge_{x \in X} (\bigvee_{A \in \mu} \chi_A(x)) = 1$. From PS-compactness of $(X, \omega_L(\tau))$, we have that

$$1 = \bigwedge_{x \in X} (\bigvee_{A \in \mu} \chi_A(x)) \leq \bigvee_{\nu \in 2^{(\mu)}} \bigwedge_{x \in X} (\bigvee_{A \in \nu} \chi_A(x)).$$

This implies that there exists $\nu \in 2^{(\mu)}$ such that $\bigwedge_{x \in X} (\bigvee_{A \in \nu} \chi_A(x)) = 1$. Hence, ν is a cover of (X, τ) . Therefore (X, τ) is PS-compact.

Sufficiency: Let μ be a family of pre-semi-open L -sets in $(X, \omega_L(\tau))$ and $\bigwedge_{x \in X} (\bigvee_{B \in \mu} B(x)) = a$. If $a = 0$, obviously we have that

$$\bigwedge_{x \in X} (\bigvee_{B \in \mu} B(x)) \leq \bigvee_{\nu \in 2^{(\mu)}} \bigwedge_{x \in X} (\bigvee_{B \in \nu} B(x)).$$

Now we suppose that $a \neq 0$. In this case, for any $b \in \beta(a) \setminus \{0\}$, we have that

$$b \in \beta(\bigwedge_{x \in X} (\bigvee_{B \in \mu} B(x))) \subseteq \bigcap_{x \in X} \beta(\bigvee_{B \in \mu} B(x)) = \bigcap_{x \in X} \bigcup_{B \in \mu} \beta(B(x)).$$

By Lemma 4.2, this implies that $\{B_{(b)} | B \in \mu\}$ is a pre-semi-open cover of (X, τ) and from the PS-compactness of (X, τ) , we know that there exists $\nu \in 2^{(\mu)}$ such that $\{B_{(b)} | B \in \nu\}$ is a cover of (X, τ) . Hence $b \leq \bigwedge_{x \in X} (\bigvee_{B \in \nu} B(x))$. Moreover, we have that

$$b \leq \bigwedge_{x \in X} (\bigvee_{B \in \nu} B(x)) \leq \bigvee_{\nu \in 2^{(\mu)}} \bigwedge_{x \in X} (\bigvee_{B \in \nu} B(x)).$$

This implies that

$$\bigwedge_{x \in X} (\bigvee_{B \in \mu} B(x)) = a = \bigvee \{b | b \in \beta(a)\} \leq \bigvee_{\nu \in 2^{(\mu)}} \bigwedge_{x \in X} (\bigvee_{B \in \nu} B(x)).$$

Therefore, $(X, \omega_L(\tau))$ is PS-compact. \square

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