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SOME COUPLED FIXED POINT RESULTS ON MODIFIED INTUITIONISTIC FUZZY METRIC SPACES AND APPLICATION TO INTEGRAL TYPE CONTRACTION

V. GUPTA, R. K. SAINI AND A. KANWAR

Abstract. In this paper, we introduce fruitful concepts of common limit range and joint common limit range for coupled mappings on modified intuitionistic fuzzy metric spaces. An illustrations are also given to justify the notion of common limit range and joint common limit range property for coupled maps. The purpose of this paper is to prove fixed point results for coupled mappings on modified intuitionistic fuzzy metric spaces. Moreover, we extend the notion of common limit range property and E.A property for coupled maps on modified intuitionistic fuzzy metric spaces. As an application, we extend our main result to integral type contraction condition and also for finite number of mappings on modified intuitionistic fuzzy metric spaces.

1. **Introduction**

According to fuzzy principal, everything is a matter of degree. Zadeh [24] coined the idea of a fuzzy set in his seminal paper. Later on, Kramosil and Michalek [15] first introduced the concept of a fuzzy metric space, which can be regarded as a generalization of statistical metric space. It serves as a beacon for the construction of fixed point theory in fuzzy metric space. After slightly modification in the concept of fuzzy metric space given by Kramosil and Michalek [15], the notion of fuzzy metric space is reintroduced by George and Veeramani [9]. Some of the impotrant results of Aamri and Moutawakil in [1] are common fixed point theorems under strict contractive conditions using a property (E.A) which generalized the concept of non-compatible mappings.

Atanassov [4] presented and studied the concept of intuitionistic fuzzy set as a noted generalization of fuzzy set which has inspired intense research activity around this introduced notion. Using the idea of intuitionistic fuzzy sets, Park [17] defined intuitionistic fuzzy metric spaces as a generalization of fuzzy metric spaces due to George and Veeramani [9]. Saadati et al. [19] introduced the modified intuitionistic fuzzy metric space and proved some fixed point theorems for compatible and weakly compatible maps. After that Jain et al. [13] discussed the notion of the compatibility of type (P). The paper [19] is the inspiration of a large number of

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eminent mathematician that employ the use of modified intuitionistic fuzzy metric space and its applications.

Branciari [6] gave integral contractive condition which was the generalization of Banach contraction map. The concepts of coupled fixed points and mixed monotone property on fuzzy metric space are propounded by Bhaskar and Lakshmikantham [5]. Lakshmikantham and Ćirić [16] discussed the mixed monotone mappings and gave some coupled fixed point theorems, which can be used to execute the existence and uniqueness of solution for a periodic boundary value problem. For more significant results in the field of fixed point theory, we refer to [2], [3], [10], [11], [12], [14],[18], [20], [21], [22] and [23].

2. Preliminaries

Lemma 2.1. [8] *Consider the set L^* and operation $\leq_{L^*}$ defined by*

$$L^* = \left\{(u,v) : (u,v) \in [0,1]^2 \text{ and } u+v \leq 1\right\},$$

$(u,v) \leq_{L^*} (w,z) \Leftrightarrow u \leq w$ *and* $v \geq z$ *for every* $(u,v),(w,z) \in L^*$. *Then* $(L^*, \leq_{L^*})$ *is a complete lattice.*

Definition 2.2. [7] A triangular norm (t-norm) on L^* is a mapping $\mathcal{T} : (L^*)^2 \to L^*$ satisfying the following conditions:
i. $\mathcal{T}(x, 1_{L^*}) = x$,
ii. $\mathcal{T}(x,y) = \mathcal{T}(y,x)$,
iii. $\mathcal{T}(x, \mathcal{T}(y,z)) = \mathcal{T}(\mathcal{T}(x,y),z)$,
iv. $x \leq_{L^*} x'$ and $y \leq_{L^*} y' \Rightarrow \mathcal{T}(x,y) \leq_{L^*} \mathcal{T}(x', y')$,
for all $x,y,z,x',y' \in L^*$.

Definition 2.3. [7] A continuous t-norm $\mathcal{T}$ on L^* is called continuous t- representable if and only if there exists a continuous t- norm $*$ and a continuous t-conorm $\diamond$ on $[0,1]$ such that, for all $x=(u,v) \in L^*$ and $y=(z,w) \in L^*, \mathcal{T}(u*z, v\diamond w)$. Define a sequence $\{\mathcal{T}^n\}$ recursively by $\left\{\mathcal{T}^1 = \mathcal{T}\right\}$

$$\mathcal{T}^n(x^1, \cdots, x^{n+1}) = \mathcal{T}(\mathcal{T}^{n-1}(x^1, \cdots, x^n), x^{n+1})$$

for $n \geq 2$ and $x^{(i)} \in L^*$.

Definition 2.4. [7] A negator on L^* is a decreasing mapping $\mathcal{N} : L^* \to L^*$ satisfying $\mathcal{N}(0_{L^*}) = 1_{L^*}$ and $\mathcal{N}(1_{L^*}) = 0_{L^*}$. A negator on $[0,1]$ is a decreasing mapping $N : [0,1] \to [0,1]$ satisfying $N(0) = 1$ and $N(1) = 0$. In what follows, N_s denotes the standard negator on $[0,1]$ defined as $N_s(x) = 1-x$, for all $x \in [0,1]$.

Definition 2.5. [19] The 3-tuple $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ is said to be an $\mathcal{MIFM}$-space if Y is a non-empty set, $\mathcal{T}$ is a continuous t-representable. Let U and V be fuzzy sets such that $U(x,y,t) + V(x,y,t) \leq 1$, for all $x,y,z \in Y$, $t,s > 0$ and $\mathcal{F}_{U,V}$ is a mapping $Y \times Y \times (0,\infty) \to L^*$ satisfying the following conditions:
i. $\mathcal{F}_{U,V}(x,y,t) >_{L^*} 0_{L^*}$,
ii. $\mathcal{F}_{U,V}(x,y,t) = 1_{L^*} \Leftrightarrow x = y$,
iii. $\mathcal{F}_{U,V}(x,y,t) = \mathcal{F}_{U,V}(y,x,t)$,
iv. $\mathcal{F}_{U,V}(x,y,t+s) \geq_{L^*} \mathcal{T}(\mathcal{F}_{U,V}(x,z,t), F_{U,V}(z,y,s))$,

v. $\mathcal{F}_{U,V}(x, y, \cdot) : (0, \infty) \to L^*$ is continuous.

In this case, $\mathcal{F}_{U,V}$ is called a modified intuitionistic fuzzy metric.
Here, $\mathcal{F}_{U,V}(x, y, t) = (U(x, y, t), V(x, y, t))$.

Lemma 2.6. [19] *Let $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ be a modified intuitionistic fuzzy metric space ($\mathcal{MIFM}$-space). Then for all $x, y \in Y$, $t > 0$, $\mathcal{F}_{U,V}(x, y, t)$ is non-decreasing with respect to t in $(L^*, \leq_{L^*})$.*

Definition 2.7. [19] The sequence $\{x_n\}$ is said to be convergent to $x \in Y$ in the $\mathcal{MIFM}$-space $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ and is generally denoted by $x_n \to x$ if

$$\mathcal{F}_{U,V}(x_n, x, t) \to 1_{L^*}$$

whenever $n \to \infty$ for every $t > 0$.

Definition 2.8. [19] A sequence $\{x_n\}$ in a $\mathcal{MIFM}$-space $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ is called a Cauchy sequence if for any $0 < \epsilon < 1$ and $t > 0$, there exists $n_0 \in N$ such that

$$\mathcal{F}_{U,V}(x_n, x_m, t) >_{L^*} (N_s(\epsilon), \epsilon)$$

and for any $n, m \geq n_0$, where N_s is the standard negator.

Definition 2.9. [19] An $\mathcal{MIFM}$-space $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ is said to be complete if and only if every Cauchy sequence is convergent.

Lemma 2.10. [19] *Let $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ be a $\mathcal{MIFM}$-space. Then $\mathcal{F}_{U,V}$ is said to be continuous on $Y^2 \times (0, \infty)$ if $\lim_{n\to\infty} \mathcal{F}_{U,V}(x_n, y_n, t_n) = \mathcal{F}_{U,V}(x, y, t)$ whenever a sequence $\{x_n, y_n, t_n\}$ in $Y^2 \times (0, \infty)$ converges to a point $(x, y, t) \in Y^2 \times (0, \infty)$ i.e*

$$\lim_{n\to\infty} \mathcal{F}_{U,V}(x_n, x, t_n) = \lim_{n\to\infty} \mathcal{F}_{U,V}(y_n, y, t_n) = 1_{L^*}$$

and

$$\lim_{n\to\infty} \mathcal{F}_{U,V}(x, y, t_n) = \mathcal{F}_{U,V}(x, y, t).$$

Lemma 2.11. [19] *Let $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ be a $\mathcal{MIFM}$-space. Then $\mathcal{F}_{U,V}$ is continuous on $Y^2 \times (0, \infty)$.*

Lemma 2.12. [19] *Let $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ be a $\mathcal{MIFM}$-space. If*

$$\mathcal{F}_{U,V}(x_n, x_{n+1}, t) \geq_{L^*} \mathcal{F}_{U,V}(x_0, x_1, k^n t)$$

for some $k > 1$ and $n \in N$ (set of natural numbers). Then $\{x_n\}$ is a Cauchy sequence.

3. Main Results

Motivated by the previous research work, we introduce the concept of the common limit range property and joint common limit range property, E.A property for coupled maps on modified intuitionistic fuzzy metric spaces as follows:

Definition 3.1. The functions $g : Y \times Y \to Y$ and $H : Y \to Y$ satisfy property (E.A) on $\mathcal{MIFM}$-space $(Y, \mathcal{F}_{U,V}, \mathcal{T})$, if there exist sequences $\{\alpha_n\}$ and $\{\beta_n\}$ in Y such that

$$\lim_{n\to\infty} \mathcal{F}_{U,V}\left(g\left(\alpha_n, \beta_n\right), l, t\right) = \lim_{n\to\infty} \mathcal{F}_{U,V}\left(H\alpha_n, l, t\right) = 1_{L^*},$$

$$\lim_{n\to\infty} \mathcal{F}_{U,V}\left(g\left(\beta_n,\alpha_n\right),m,t\right) = \lim_{n\to\infty} \mathcal{F}_{U,V}\left(H\beta_n,m,t\right) = 1_{L^*}$$

for some $l, m \in Y$.

Definition 3.2. The pair (g, H) satisfies CLR_H property on $\mathcal{MIFM}$-space $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ if there exist sequences $\{\alpha_n\}$ and $\{\beta_n\}$ in Y such that

$$\lim_{n\to\infty} \mathcal{F}_{U,V}\left(g\left(\alpha_n,\beta_n\right),H(l),t\right) = 1_{L^*} = \lim_{n\to\infty} \mathcal{F}_{U,V}\left(H\alpha_n,H(l),t\right),$$

$$\lim_{n\to\infty} \mathcal{F}_{U,V}\left(g\left(\beta_n,\alpha_n\right),H(m),t\right) = 1_{L^*} = \lim_{n\to\infty} \mathcal{F}_{U,V}\left(H\beta_n,H(m),t\right)$$

for some $l, m \in Y$.

Example 3.3. Let $g : Y \times Y \to Y$ and $H : Y \to Y$ where $Y = [0, 1]$ are mappings defined on $\mathcal{MIFM}$-space $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ as $g(x, y) = x+y$, $H(x) = x$ for all $x, y \in Y$. Now consider the sequence $\{x_n\} = -\frac{1}{n}$, $t > 0$.

$$\lim_{n\to\infty} g\left(\alpha_n,\beta_n\right) = \lim_{n\to\infty} H\left(\alpha_n\right) = 0 = H(0),$$

$$\lim_{n\to\infty} g\left(\beta_n,\alpha_n\right) = \lim_{n\to\infty} H\left(\beta_n\right) = 0 = H(0).$$

This implies

$$\lim_{n\to\infty} \mathcal{F}_{U,V}\left(g\left(\alpha_n,\beta_n\right),H(l),t\right) = 1_{L^*} = \lim_{n\to\infty} \mathcal{F}_{U,V}\left(H\left(\alpha_n\right),H(l),t\right),$$

$$\lim_{n\to\infty} \mathcal{F}_{U,V}\left(g\left(\beta_n,\alpha_n\right),H(m),t\right) = 1_{L^*} = \lim_{n\to\infty} \mathcal{F}_{U,V}\left(H\left(\beta_n\right),H(m),t\right).$$

This implies that the pair (g, H) satisfies CLR_H property.

Definition 3.4. Let $g, S : Y \times Y \to Y$ and $H, R : Y \to Y$ be mappings on $\mathcal{MIFM}$-space $(Y, \mathcal{F}_{U,V}, \mathcal{T})$. The pairs (g, H) and (S, R) satisfy the joint common limit property in the range of H and R (say$JCLR_{HR}$) if there exist sequences $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ and $\{\delta_n\}$ such that

$$\begin{aligned} &\lim_{n\to\infty} \mathcal{F}_{U,V}\left(g\left(\alpha_n,\beta_n,H(l),t\right)\right) = \lim_{n\to\infty} \mathcal{F}_{U,V}\left(H\left(\alpha_n,H(l),t\right)\right) \\ &= \lim_{n\to\infty} \mathcal{F}_{U,V}\left(S\left(\gamma_n,\delta_n,R(l),t\right)\right) = \lim_{n\to\infty} \mathcal{F}_{U,V}\left(R\left(\gamma_n,R(l),t\right)\right) \\ &= \mathcal{F}_{U,V}\left(H(l),R(l),t\right) = 1_{L^*}, \end{aligned}$$

$$\begin{aligned} &\lim_{n\to\infty} \mathcal{F}_{U,V}\left(g\left(\beta_n,\alpha_n,H(m),t\right)\right) = \lim_{n\to\infty} \mathcal{F}_{U,V}\left(H\left(\beta_n,H(m),t\right)\right) \\ &= \lim_{n\to\infty} \mathcal{F}_{U,V}\left(S\left(\delta_n,\gamma_n,R(m),t\right)\right) = \lim_{n\to\infty} \mathcal{F}_{U,V}\left(R\left(\delta_n,R(m),t\right)\right) \\ &= \mathcal{F}_{U,V}\left(H(m),R(m),t\right) = 1_{L^*} \end{aligned}$$

for some $l, m \in Y$.

An illustration is discussed below which satisfies the above property.

Example 3.5. Let $Y = (0, 2)$ and $g, S : Y \times Y \to Y$ and $H, R : Y \to Y$ be mappings defined as

$$g(x,y) = \begin{cases} \frac{x+y}{2}, & x,y \in (0,1], \\ 0, & e.w, \end{cases}, \quad H(x) = \{x, x \in Y\},$$

$$S(x,y) = \begin{cases} x-y+1, x,y \in (0,1], \\ 0, \qquad e.w, \end{cases}, \quad R(x) = \begin{cases} 1, & x = 1, \\ 0, & x \in (0,1) \cup (1,2) \end{cases}.$$

Consider the sequences $\{\alpha_n\} = 1 + \frac{1}{n}$, $\{\beta_n\} = 1 - \frac{1}{n}$,$\{\delta_n\} = \frac{1}{n}$ and $\{\gamma_n\} = -\frac{1}{n}$ such that

$$\lim_{n\to\infty} g(\alpha_n, \beta_n) = \lim_{n\to\infty} H(\alpha_n) = \lim_{n\to\infty} S(\gamma_n, \delta_n) = \lim_{n\to\infty} R(\gamma_n) = 1 = H(1) = R(1),$$

$$\lim_{n\to\infty} g(\beta_n, \alpha_n) = \lim_{n\to\infty} H(\beta_n) = \lim_{n\to\infty} S(\delta_n, \gamma_n) = \lim_{n\to\infty} R(\delta_n) = 1 = H(1) = R(1).$$

This implies that the pairs (g, H) and (S, R) satisfy $JCLR_{HR}$ property.

Note: Throughout the main results, we are using following notion:
Let Φ be the set of all continuous $\phi : L^* \to L^*$ and $\phi(t) >_{L^*} t$ for all $t \in L^*/\{0_{L^*}, 1_{L^*}\}$.
Using the concept of the joint common limit for coupled maps, we are ready to prove our first main result.

Theorem 3.6. *Let $g, S : Y \times Y \to Y$ and $H, R : Y \to Y$ be mappings defined on $\mathcal{MIFM}$-space $(Y, \mathcal{F}_{U,V}, \mathcal{T})$, such that the pairs (g, H) and (S, R) satisfy the joint common limit in range of H and R. For all $x, y, u, v \in Y$ and the following condition holds:*
a_1. for all $t > 0$, $\phi \in \Phi$

$$\mathcal{F}_{U,V}(g(x,y), S(u,v), t) \geq_{L^*} \phi\left\{\min\begin{pmatrix} \mathcal{F}_{U,V}(g(x,y), R(u), t), \\ \mathcal{F}_{U,V}(S(u,v), H(x), t), \\ \mathcal{F}_{U,V}(g(x,y), H(x), t), \\ \mathcal{F}_{U,V}(S(u,v), R(u), t), \\ \mathcal{F}_{U,V}(H(x), R(u), t) \end{pmatrix}\right\}.$$

Then the pairs (g, H) and (S, R) have common coupled coincident point. Moreover, if (g, H) and (S, R) are weakly compatible, then (g, H) and (S, R) have a unique common fixed point in Y.

Proof. The pairs (g, H) and (S, R) satisfy the joint common limit property in the range of H and R property (say $JCLR_{HR}$) if there exist sequences $\{\alpha_n\}, \{\beta_n\}, \{\gamma_n\}$ and $\{\delta_n\}$ such that

$$\begin{aligned} &\lim_{n\to\infty} \mathcal{F}_{U,V}(g(\alpha_n, \beta_n), H(l), t)) = \lim_{n\to\infty} \mathcal{F}_{U,V}(H(\alpha_n), H(l), t)) \\ &= \lim_{n\to\infty} \mathcal{F}_{U,V}(S(\gamma_n, \delta_n), R(l), t)) = \lim_{n\to\infty} \mathcal{F}_{U,V}(R(\gamma_n), R(l), t)) \\ &= \lim_{n\to\infty} \mathcal{F}_{U,V}(H(l), R(l), t) = 1_{L^*} \end{aligned}$$

and

$$\begin{aligned} &\lim_{n\to\infty} \mathcal{F}_{U,V}(g(\beta_n, \alpha_n), H(m), t)) = \lim_{n\to\infty} \mathcal{F}_{U,V}(H(\beta_n), H(m), t)) \\ &= \lim_{n\to\infty} \mathcal{F}_{U,V}(S(\delta_n, \gamma_n), R(m), t)) = \lim_{n\to\infty} \mathcal{F}_{U,V}(R(\delta_n), R(m), t)) \\ &= \lim_{n\to\infty} \mathcal{F}_{U,V}(H(m), R(m), t) = 1_{L^*} \end{aligned} \quad (1)$$

for some $l, m \in Y$.
From condition (a_1), we have

$$\mathcal{F}_{U,V}(g(\alpha_n, \beta_n), S(l,m), t) \geq_{L^*} \phi\left\{\min\begin{pmatrix} \mathcal{F}_{U,V}(g(\alpha_n, \beta_n), R(l), t), \\ \mathcal{F}_{U,V}(S(l,m), H(\alpha_n), t), \\ \mathcal{F}_{U,V}(g(\alpha_n, \beta_n), H(\alpha_n), t), \\ \mathcal{F}_{U,V}(S(l,m), R(l), t), \\ \mathcal{F}_{U,V}(H(\alpha_n), R(l), t) \end{pmatrix}\right\}.$$

Letting $n \to \infty$ and using (1)

$$\mathcal{F}_{U,V}(R(l), S(l,m), t) \geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(R(l), R(l), t), \\ \mathcal{F}_{U,V}(S(l,m), R(l), t), \\ \mathcal{F}_{U,V}(R(l), R(l), t), \\ \mathcal{F}_{U,V}(S(l,m), R(l), t), \\ \mathcal{F}_{U,V}(R(l), R(l), t) \end{pmatrix} \right\}.$$

This implies

$$\mathcal{F}_{U,V}(R(l), S(l,m), t) \geq_{L^*} \phi\left(\mathcal{F}_{U,V}(R(l), S(l,m), t)\right) >_{L^*} \mathcal{F}_{U,V}(R(l), S(l,m), t).$$

We have

$$S(l,m) = R(l),$$

and in the same way, we get

$$S(m,l) = R(m). \tag{2}$$

By using condition (a_1), we get

$$\mathcal{F}_{U,V}(g(l,m), S(\gamma_n, \delta_n), t) \geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(l,m), R(\gamma_n), t), \\ \mathcal{F}_{U,V}(S(\gamma_n, \delta_n), H(l), t), \\ \mathcal{F}_{U,V}(g(l,m), H(l), t), \\ \mathcal{F}_{U,V}(S(\gamma_n, \delta_n), R(\gamma_n), t), \\ \mathcal{F}_{U,V}(H(l), R(\gamma_n), t) \end{pmatrix} \right\}.$$

Assuming $n \to \infty$ and using (1), we can get $g(l,m) = H(l)$.
In the same way, we can show that

$$g(m,l) = H(m). \tag{3}$$

From eqs.(1), (2) and (3), we obtain $g(l,m) = H(l) = S(l,m) = R(l)$ and

$$g(m,l) = H(m) = S(m,l) = R(m). \tag{4}$$

Hence, we conclude that the pairs (g, H) and (S, R) have common coupled coincident point in Y.

Now, we assume that

$$\begin{aligned} g(l,m) = H(l) = S(l,m) = R(l) = r_1, \\ g(m,l) = H(m) = S(m,l) = R(m) = r_2 \end{aligned} \tag{5}$$

where $r_1, r_2 \in Y$
Since the pair (g, H) is weakly compatible, we get that

$$H(g(l,m)) = g(H(l), H(m)),\ H(g(m,l)) = g(H(m), H(l)).$$

From (5), we have

$$H(r_1) = g(r_1, r_2), \quad H(r_2) = g(r_2, r_1). \tag{6}$$

Also the pair (S, R) is weakly compatible, therefore

$$R(S(l,m)) = S(R(l), R(m)),\ R(S(m,l)) = S(R(m), R(l)).$$

From (5), we have

$$R(r_1) = S(r_1, r_2), \quad R(r_2) = S(r_2, r_1). \tag{7}$$

Using (a_1) and (4), (5), (6), we get

$$\mathcal{F}_{U,V}(g(r_1,r_2),S(l,m),t) \geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(r_1,r_2),R(l),t), \\ \mathcal{F}_{U,V}(S(l,m),H(r_1),t), \\ \mathcal{F}_{U,V}(g(r_1,r_2),H(r_1),t), \\ \mathcal{F}_{U,V}(S(l,m),R(l),t), \\ \mathcal{F}_{U,V}(H(r_1),R(l),t) \end{pmatrix} \right\}.$$

This implies

$$\mathcal{F}_{U,V}(g(r_1,r_2),r_1,t) \geq_{L^*} \phi\left(\mathcal{F}_{U,V}(g(r_1,r_2),r_1,t)\right) >_{L^*} \mathcal{F}_{U,V}(g(r_1,r_2),r_1,t).$$

We have $r_1 = g(r_1,r_2)$.
In the same way, we can prove that $r_2 = g(r_2,r_1)$.
From eq. (6), we obtain

$$r_1 = g(r_1,r_2) = H(r_1), \quad r_2 = g(r_2,r_1) = H(r_2). \tag{8}$$

Again using (a_1) and $(4-6)$, we get

$$\mathcal{F}_{U,V}(g(l,m),S(r_1,r_2),t) \geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(l,m),R(r_1),t), \\ \mathcal{F}_{U,V}(S(r_1,r_2),H(l),t), \\ \mathcal{F}_{U,V}(g(l,m),H(l),t), \\ \mathcal{F}_{U,V}(S(r_1,r_2),R(r_1),t), \\ \mathcal{F}_{U,V}(H(l),R(r_1),t) \end{pmatrix} \right\}.$$

Thus we get

$$r_1 = S(r_1,r_2) = R(r_1), \quad r_2 = S(r_2,r_1) = R(r_2). \tag{9}$$

From equations $(6-9)$, we see that

$$r_1 = g(r_1,r_2) = H(r_1) = S(r_1,r_2) = R(r_1),$$
$$r_2 = g(r_2,r_1) = H(r_2) = S(r_2,r_1) = R(r_2).$$

We shall assert that (g,H) and (S,R) have common fixed point in Y.
For this, we prove that $r_1 = r_2$.
Let $r_1 \neq r_2$.
By using condition (a_1), we have

$$\begin{aligned} \mathcal{F}_{U,V}(r_1,r_2,t) &= \mathcal{F}_{U,V}(g(r_1,r_2),S(r_2,r_1),t) \\ &\geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(r_1,r_2),R(r_2),t), \\ \mathcal{F}_{U,V}(S(r_2,r_1),H(r_1),t), \\ \mathcal{F}_{U,V}(g(r_1,r_2),H(r_1),t), \\ \mathcal{F}_{U,V}(S(r_2,r_1),R(r_2),t), \\ \mathcal{F}_{U,V}(H(r_1),R(r_2),t) \end{pmatrix} \right\} \\ &\geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(r_1,r_2,t), \\ \mathcal{F}_{U,V}(r_2,r_1,t), \\ \mathcal{F}_{U,V}(r_1,r_1,t), \\ \mathcal{F}_{U,V}(r_2,r_2,t), \\ \mathcal{F}_{U,V}(r_1,r_2,t) \end{pmatrix} \right\}. \end{aligned}$$

This is contradiction to our assumption. This implies that $r_1 = r_2$.
Thus, we proved that $r_1 = g(r_1,r_2) = H(r_1) = S(r_1,r_2) = R(r_1)$.
So, we conclude that the pairs (g,H) and (S,R) have common fixed point in Y.
The uniqueness of the fixed point can be easily proved in the same way as above, by using condition (a_1). This completes the proof of Theorem 3.6. □

Now, we extend Theorem 3.6 be finite numbers of self-mappings which stated below:

Theorem 3.7. *Let $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ be a $\mathcal{MIFM}$-space. Let $g, S : Y \times Y \to Y$ and $H_1, H_2, \cdots, H_s : Y \to Y$, $R_1, R_2, \cdots, R_w : Y \to Y$ are finite number of mappings such $H = H_1 H_2 \cdots H_s$, $R = R_1 R_2 \cdots R_w$ which satisfy the following conditions:*
b_1. the pairs (g, H) and (S, R) satisfy $JCLR_{HR}$ property,
b_2. for all $x, y \in Y$, $t > 0$

$$\mathcal{F}_{U,V}(g(x,y), u, v, t) \geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(x,y), R(u), t), \\ \mathcal{F}_{U,V}(S(u,v), H(x), t) \\ \mathcal{F}_{U,V}(g(x,y), H(x), t), \\ \mathcal{F}_{U,V}(S(u,v), R(u), t), \\ \mathcal{F}_{U,V}(H(x), R(u), t) \end{pmatrix} \right\},$$

where $\phi \in \Phi$, then pairs (g, H) and (S, R) have common coupled coincident point. Moreover, if the pairs (g, H) and (S, R) are weakly compatible, then (g, H) and (S, R) have a unique common fixed point in Y.

Proof. The result can be justified by considering $H = H_1 H_2 \cdots H_s$, $R = R_1 R_2 \cdots R_w$ and following the same steps in Theorem 3.6. □

Now, we state next theorem in which a pair of mappings satisfy the common limit range property (or CLR property).

Theorem 3.8. *Let $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ be a $\mathcal{MIFM}$-space. Let $g : Y \times Y \to Y$ and $H : Y \to Y$ be mappings such that the pair (g, H) satisfies CLR_H property and the following condition:*

$$\mathcal{F}_{U,V}(g(x,y), g(u,v), t) \geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(x,y), H(u), t), \\ \mathcal{F}_{U,V}(g(u,v), H(x), t), \\ \mathcal{F}_{U,V}(g(x,y), H(x), t), \\ \mathcal{F}_{U,V}(g(u,v), H(u), t), \\ \mathcal{F}_{U,V}(H(x), H(u), t) \end{pmatrix} \right\}$$

for all $x, y \in Y$, $t > 0$ and $\phi \in \Phi$. Then the pair (g, H) has common coupled coincident point. Moreover, if the pair (g, H) is weakly compatible then the pair (g, H) has a unique common fixed point in Y.

Proof. By considering $g = S$ and $H = R$ in Theorem 3.6 and the pair (g, H) satisfies CLR_H property then there exist sequences $\{\alpha_n\}$, $\{\beta_n\}$ in Y such that

$$\lim_{n\to\infty} \mathcal{F}_{U,V}(g(\alpha_n, \beta_n), H(l), t) = 1_{L^*} = \lim_{n\to\infty} \mathcal{F}_{U,V}(H(\alpha_n), H(l), t),$$

$$\lim_{n\to\infty} \mathcal{F}_{U,V}(g(\beta_n, \alpha_n), H(l), t) = 1_{L^*} = \lim_{n\to\infty} \mathcal{F}_{U,V}(H(\beta_n), H(m), t)$$

for some $l, m \in Y$.
Using the same argument as in Theorem 3.6, we get the above result. □

In the following, we state a theorem in which a pair of mappings satisfy E.A property and have a unique fixed point on modified intuitionistic fuzzy metric space.

Theorem 3.9. *Let $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ be a $\mathcal{MIFM}$-space. Let $g : Y \times Y \to Y$ and $H : Y \to Y$ satisfy the E.A property and the range of $H(Y)$ is a closed subspace of Y satisfying the following condition:*

$$\mathcal{F}_{U,V}(g(x,y), g(u,v), t) \geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(x,y), H(u), t), \\ \mathcal{F}_{U,V}(g(u,v), H(x), t), \\ \mathcal{F}_{U,V}(g(x,y), H(x), t), \\ \mathcal{F}_{U,V}(g(u,v), H(u), t), \\ \mathcal{F}_{U,V}(H(x), H(u), t) \end{pmatrix} \right\}$$

for all $x, y \in Y$, $t > 0$ and $\phi \in \Phi$. Then the pair (g, H) has common coupled coincident point. Moreover, if the pair (g, H) is weakly compatible then the pair (g, H) has a unique common fixed point in Y.

Proof. Since the pair (g, H) satisfies the $E.A$ property, there exist sequences $\{\alpha_n\}$, $\{\beta_n\}$ in Y such that

$$\lim_{n\to\infty} \mathcal{F}_{U,V}(g(\alpha_n, \beta_n), l, t) = 1_{L^*} = \lim_{n\to\infty} \mathcal{F}_{U,V}(H(\alpha_n), l, t),$$

$$\lim_{n\to\infty} \mathcal{F}_{U,V}(g(\beta_n, \alpha_n), m, t) = 1_{L^*} = \lim_{n\to\infty} \mathcal{F}_{U,V}(H(\beta_n), m, t).$$

Here, if the range of $H(Y)$ is a closed subspace of Y, then there exist $a, b \in Y$ such that $l = H(a)$, $m = H(b)$. So, we have

$$\lim_{n\to\infty} \mathcal{F}_{U,V}(g(\alpha_n, \beta_n), H(a), t) = 1_{L^*} = \lim_{n\to\infty} \mathcal{F}_{U,V}(H(\alpha_n), H(a), t),$$

$$\lim_{n\to\infty} \mathcal{F}_{U,V}(g(\beta_n, \alpha_n), H(b), t) = 1_{L^*} = \lim_{n\to\infty} \mathcal{F}_{U,V}(H(\beta_n), H(b), t).$$

Therefore, the pair (g, H) satisfies (CLR_H) property. So, from Theorem 3.8, we get result immediately. □

Next, we define joint common limit property in range of one mapping with using subset condition and then we prove a fixed point theorem on modified intuitionistic fuzzy metric space in which we use joint common limit property in range of either H or R mapping.

Theorem 3.10. *Let $g, S : Y \times Y \to Y$ and $H, R : Y \to Y$ be mappings on $\mathcal{MIFM}$-space satisfying the following conditions:*
c_1. $g(Y \times Y) \subset R(Y)$ or $(S(Y \times Y) \subset H(Y))$,
c_2. the pairs (g, H) and (S, R) share $JCLR_H$ property (or $JCLR_R$ property),
c_3. for all $x, y \in Y$

$$\mathcal{F}_{U,V}(g(x,y), S(u,v), t) \geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(x,y), R(u), t), \\ \mathcal{F}_{U,V}(S(u,v), H(x), t) \\ \mathcal{F}_{U,V}(g(x,y), H(x), t), \\ \mathcal{F}_{U,V}(S(u,v), R(u), t), \\ \mathcal{F}_{U,V}(H(x), R(u), t) \end{pmatrix} \right\},$$

where $\phi \in \Phi$, $t > 0$. Then the pairs (g, H) and (S, R) have common coupled coincident point. Moreover, if the pairs (g, H) and (S, R) are weakly compatible then (g, H) and (S, R) have a unique common fixed point in Y.

Proof. Let the pairs (g, H) and (S, R) share the joint common limit in the range of H property, then there exist sequences $\{\alpha_n\}$, $\{\beta_n\}$,$\{\gamma_n\}$ and $\{\delta_n\}$ such that

$$\lim_{n\to\infty} \mathcal{F}_{U,V}(g(\alpha_n,\beta_n), H(l_1), t) = \lim_{n\to\infty} \mathcal{F}_{U,V}(H(\alpha_n), H(l_1), t)$$
$$= \lim_{n\to\infty} \mathcal{F}_{U,V}(S(\gamma_n,\delta_n), H(l_1), t) = \lim_{n\to\infty} \mathcal{F}_{U,V}(R(\gamma_n), H(l_1), t) = 1_{L^*},$$

$$\lim_{n\to\infty} \mathcal{F}_{U,V}(g(\beta_n,\alpha_n), H(l_2), t) = \lim_{n\to\infty} \mathcal{F}_{U,V}(H(\beta_n), H(l_2), t)$$
$$= \lim_{n\to\infty} \mathcal{F}_{U,V}(S(\delta_n,\gamma_n), H(l_2), t) = \lim_{n\to\infty} \mathcal{F}_{U,V}(R(\delta_n), H(l_2), t) = 1_{L^*} \tag{10}$$

for some $l_1, l_2 \in Y$.
By using condition (c_3), we have

$$\mathcal{F}_{U,V}(g(l_1,l_2), S(\gamma_n,\delta_n), t) \geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(l_1,l_2), R(\gamma_n), t), \\ \mathcal{F}_{U,V}(S(\gamma_n,\delta_n), H(l_1), t) \\ \mathcal{F}_{U,V}(g(l_1,l_2), H(l_1), t), \\ \mathcal{F}_{U,V}(S(\gamma_n,\delta_n), R(\gamma_n), t), \\ \mathcal{F}_{U,V}(H(l_1), R(\gamma_n), t) \end{pmatrix} \right\}.$$

Taking the limit as $n \to \infty$, we get

$$g(l_1,l_2) = H(l_1), g(l_2,l_2) = H(l_2). \tag{11}$$

Hence (g, H) has coupled coincident point. Since $g(Y \times Y) \subset R(Y)$, there exist $m_1, m_2 \in Y$ such that

$$g(l_1,l_2) = R(m_1), \quad g(l_2,l_1) = R(m_2). \tag{12}$$

By using condition (c_3), we obtain

$$\mathcal{F}_{U,V}(g(l_1,l_2), S(m_1,m_2), t) \geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(l_1,l_2), R(m_1), t), \\ \mathcal{F}_{U,V}(S(m_1,m_2), H(l_1), t), \\ \mathcal{F}_{U,V}(g(l_1,l_2), H(l_1), t), \\ \mathcal{F}_{U,V}(S(m_1,m_2), R(m_1), t), \\ \mathcal{F}_{U,V}(H(l_1), R(m_1), t) \end{pmatrix} \right\}.$$

By using eqs.(11) and (12), we get

$$\mathcal{F}_{U,V}(R(m_1), S(m_1,m_2), t) \geq_{L^*} \phi\left(\mathcal{F}_{U,V}(R(m_1), S(m_1,m_2), t)\right).$$

This implies

$$S(m_1,m_2) = R(m_1), S(m_2,m_1) = R(m_2). \tag{13}$$

Thus (S, R) has common coupled coincident point.
We have $H(l_1) = g(l_1,l_2) = R(m_1) = S(m_1,m_2) = s_1$ (say) and

$$H(l_2) = g(l_2,l_1) = R(m_2) = S(m_2,m_1) = s_2 \tag{14}$$

(say).

Since the pair (S, R) is weakly compatible, this gives

$$H(g(l_1, l_2)) = g(H(l_1), H(l_2)), H(l_2) = g(s_1, s_2),$$

$$H(s_2) = H(g(l_2, l_1)) = g(H(l_2), H(l_1)) = g(s_2, s_1).$$

From condition (c_3), we have

$$\mathcal{F}_{U,V}(g(s_1, s_2), S(m_1, m_2), t) \geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(s_1, s_2), R(m_1), t), \\ \mathcal{F}_{U,V}(S(m_1, m_2), H(s_1), t), \\ \mathcal{F}_{U,V}(g(s_1, s_2), H(s_1), t), \\ \mathcal{F}_{U,V}(S(m_1, m_2), R(m_1), t), \\ \mathcal{F}_{U,V}(H(s_1), R(m_1), t) \end{pmatrix} \right\}.$$

By using (14), we get

$$g(s_1, s_2) = H(s_1) = s_1, \quad g(s_2, s_1) = H(s_2) = s_2.$$

Hence (g, H) has coupled fixed point.
Again, weak compatibility of the pair (S, R) gives that

$$R(S(m_1, m_2)) = S(R(m_1), R(m_2)), R(S(m_2, m_1)) = S(R(m_2), R(m_1)).$$

From eq. (14), we get

$$R(s_1) = R(S(m_1, m_2)) = S(R(m_1), R(m_2)) = S(s_1, s_2),$$

$$R(s_2) = R(S(m_2, m_1)) = S(R(m_2), R(m_1)) = S(s_2, s_1).$$

From (c_3), one can obtain

$$\mathcal{F}_{U,V}(g(l_1, l_2), S(s_1, s_2), t) \geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(l_1, l_2), R(s_1), t), \\ \mathcal{F}_{U,V}(S(s_1, s_2), H(l_1), t), \\ \mathcal{F}_{U,V}(g(l_1, l_2), H(l_1), t), \\ \mathcal{F}_{U,V}(S(s_1, s_2), R(s_1), t), \\ \mathcal{F}_{U,V}(H(l_1), R(s_1), t) \end{pmatrix} \right\}.$$

This gives

$$S(s_1, s_2) = R(s_1) = s_1, S(s_2, s_1) = R(s_2) = s_2.$$

Here, (S, R) has coupled fixed point.
This implies $g(s_1, s_2) = H(s_1) = S(s_1, s_2) = R(s_1)$.
Similarly, one can easily prove that

$$g(s_2, s_1) = H(s_2) = S(s_2, s_1) = R(s_2). \tag{15}$$

We assert that (g, H) and (S, R) have common coupled fixed point in Y.
For this, we shall prove that $s_1 = s_2$.
Let suppose that $s_1 \neq s_2$.Thus we have

$$\mathcal{F}_{U,V}(s_1, s_2, t) = \mathcal{F}_{U,V}(g(s_1, s_2), S(s_2, s_1), t)$$

$$\geq_{L^*} \phi \left\{ \min \begin{pmatrix} \mathcal{F}_{U,V}(g(s_1, s_2), R(s_2), t), \\ \mathcal{F}_{U,V}(S(s_2, s_1), H(s_1), t), \\ \mathcal{F}_{U,V}(g(s_1, s_2), H(s_1), t), \\ \mathcal{F}_{U,V}(S(s_2, s_1), R(s_2), t), \\ \mathcal{F}_{U,V}(H(s_1), R(s_2), t) \end{pmatrix} \right\}.$$

By using (15), we have $\mathcal{F}_{U,V}(s_1, s_2, t) > \mathcal{F}_{U,V}(s_1, s_2, t)$.
This implies $s_1 = s_2$.
Thus we proved that $s_1 = g(s_1, s_2) = H(s_1) = S(s_1, s_2) = R(s_1)$.
So, we conclude that the pairs (g, H) and (S, R) have common coupled fixed point in Y. The uniqueness of the fixed point can be easily proved in the same way as above by using condition (c_3) of this theorem. □

4. **Application**

In the next result, we give an application related to our result on modified intuitionistic fuzzy metric space.

Theorem 4.1. *Let $(Y, \mathcal{F}_{U,V}, \mathcal{T})$ be a $\mathcal{MIFM}$-space. Let $g, S : Y \times Y \to Y$ and $H, R : Y \to Y$ be mappings satisfying the following conditions:*
d_1. the pairs (g, H) and (S, R) are satisfy $JCLR_{HR}$ property,
d_2. for $x, y \in Y$, $t > 0$

$$\int_0^{s_1} \chi(t)dt \geq_{L^*} \phi\left(\int_0^{s_2} \chi(t)dt\right)$$

where $\phi : L^* \to L^*$ and $\phi(t) >_{L^*} t$, for all $t \in L^*/\{0_{L^*}, 1_{L^*}\}$, $\chi(t)$ is Lebesgue-integrable function and

$$s_1 = \mathcal{F}_{U,V}(g(x, y), S(u, v), kt),$$

$$s_2 = \min \left\{ \begin{array}{c} \mathcal{F}_{U,V}(g(x, y), R(u), t), \mathcal{F}_{U,V}(S(u, v), H(x), t), \\ \mathcal{F}_{U,V}(g(x, y), H(x), t), \mathcal{F}_{U,V}(S(u, v), R(u), t), \\ \mathcal{F}_{U,V}(H(x), R(u), t) \end{array} \right\}.$$

Then the pairs (g, H) and (S, R) have common coupled coincident point. Moreover, if the pairs (g, H) and (S, R) are weakly compatible then (g, H) and (S, R) have a unique common fixed point in Y.

Proof. Since the pairs (g, H) and (S, R) satisfy the joint common limit in the range of H and R property, so it satisfies equation (1) of Theorem 3.6.
From condition (d_2), we have

$$\int_0^{\mathcal{F}_{U,V}(g(\alpha_n, \beta_n), S(l,m), t)} \chi(t)dt \geq_{L^*} \phi\left(\int_0^{s_2} \chi(t)dt\right)$$

where

$$s_2 = \min \left(\begin{array}{c} \mathcal{F}_{U,V}(g(\alpha_n, \beta_n), R(l), t), \mathcal{F}_{U,V}(S(l, m), H(\alpha_n), t), \\ \mathcal{F}_{U,V}(g(\alpha_n, \beta_n), H(\alpha_n), t), \mathcal{F}_{U,V}(S(l, m), R(l), t), \\ \mathcal{F}_{U,V}(H(\alpha_n), R(l), t) \end{array} \right).$$

Letting $n \to \infty$, we get

$$S(l, m) = R(l) \quad , \quad S(m, l) = R(m). \tag{16}$$

By using condition (d_2) and taking $n \to \infty$, we get

$$\int_0^{\mathcal{F}_{U,V}(g(l,m), S(\gamma_n, \delta_n), t)} \chi(t)dt \geq_{L^*} \phi\left(\int_0^{s_2} \chi(t)dt\right)$$

where

$$s_2 = \min \begin{pmatrix} \mathcal{F}_{U,V}(g(l,m), R(\gamma_n), t), \mathcal{F}_{U,V}(S(\gamma_n, \delta_n), H(l), t), \\ \mathcal{F}_{U,V}(g(l,m), H(l), t), \mathcal{F}_{U,V}(S(\gamma_n, \delta_n), R(\gamma_n), t), \\ \mathcal{F}_{U,V}(H(l), R(\gamma_n), t) \end{pmatrix}.$$

This implies

$$g(l,m) = H(l), \quad g(m,l) = H(m). \tag{17}$$

From (16) and (17), we obtain

$$g(l,m) = H(l) = S(l,m) = R(l), g(m,l) = H(m) = S(m,l) = R(m). \tag{18}$$

Hence we conclude that the pairs (g, H) and (S, R) have common coupled coincident point in Y.
Now we assume that

$$\begin{aligned} g(l,m) = H(l) = S(l,m) = R(l) = r_1, \\ g(m,l) = H(m) = S(m,l) = R(m) = r_2 \end{aligned} \tag{19}$$

where $r_1, r_2 \in Y$.
The definition of weakly compatible mappings implies that

$$\begin{aligned} H(r_1) = g(r_1, r_2), H(r_2) = g(r_2, r_1), \\ R(r_1) = S(r_1, r_2), R(r_2) = S(r_2, r_1). \end{aligned} \tag{20}$$

With the help of (d_2) and $(18-20)$, we get

$$\int_0^{\mathcal{F}_{U,V}(g(r_1,r_2),S(l,m),t)} \chi(t)dt \geq_{L^*} \phi\left(\int_0^{s_2} \chi(t)dt\right)$$

where

$$s_2 = \min \begin{pmatrix} \mathcal{F}_{U,V}(g(r_1,r_2), R(l), t), \mathcal{F}_{U,V}(S(l,m), H(r_1), t), \\ \mathcal{F}_{U,V}(g(r_1,r_2), H(r_1), t), \mathcal{F}_{U,V}(S(l,m), R(l), t), \\ \mathcal{F}_{U,V}(H(r_1), R(l), t) \end{pmatrix}.$$

This implies that

$$\mathcal{F}_{U,V}(g(r_1,r_2), r_1, t) \geq_{L^*} \phi\left\{\mathcal{F}_{U,V}(g(r_1,r_2), r_1, t))\right\} >_{L^*} \mathcal{F}_{U,V}(g(r_1,r_2), r_1, t).$$

We obtain

$$r_1 = g(r_1, r_2) = H(r_1) \quad and \quad r_2 = g(r_2, r_1) = H(r_2). \tag{21}$$

Using (d_2), we get

$$\int_0^{\mathcal{F}_{U,V}(g(l,m),S(r_1,r_2),t)} \chi(t)dt \geq_{L^*} \phi\left(\int_0^{s_2} \chi(t)dt\right)$$

where

$$s_2 = \min \left\{ \begin{matrix} \mathcal{F}_{U,V}(g(l,m), R(r_1), t), \mathcal{F}_{U,V}(S(r_1,r_2), H(l), t), \\ \mathcal{F}_{U,V}(g(l,m), H(l), t), \mathcal{F}_{U,V}(S(r_1,r_2), R(r_1), t), \\ \mathcal{F}_{U,V}(H(l), R(r_1), t) \end{matrix} \right\}.$$

Thus we get

$$r_1 = g(r_1, r_2) = H(r_1) = S(r_1, r_2) = R(r_1),$$

$$r_2 = g(r_2, r_1) = H(r_2) = S(r_2, r_1) = R(r_2). \tag{22}$$

We shall prove that (g, H) and (S, R) have common fixed point in Y.
For this, we shall prove that $r_1 = r_2$.
Let us suppose that $r_1 \neq r_2$.
By using condition (d_2), we have

$$\int_0^{\mathcal{F}_{U,V}(r_1,r_2,t)} \chi(t)dt = \int_0^{\mathcal{F}_{U,V}(g(r_1,r_2),S(r_1,r_2),t)} \chi(t)dt \geq_{L^*} \phi\left(\int_0^{s_2} \chi(t)dt\right)$$

where

$$s_2 = \min\left\{\begin{array}{c} \mathcal{F}_{U,V}(g(r_1,r_2), R(r_2), t), \mathcal{F}_{U,V}(S(r_2,r_1), H(r_1), t), \\ \mathcal{F}_{U,V}(g(r_1,r_2), H(r_1), t), \mathcal{F}_{U,V}(S(r_2,r_1), R(r_2), t), \\ \mathcal{F}_{U,V}(g(r_1), R(r_2), t) \end{array}\right\}.$$

This is a contradiction to our assumption. This implies that $r_1 = r_2$. We conclude that the pairs (g, H) and (S, R) have common fixed point in Y. The uniqueness of the fixed point can be easily proved by using condition (d_2) of this theorem. □

5. Conclusion

Fuzzy set theory offers a strict mathematical basis in which vague conceptual phenomenon can be exactly and thoroughly studied. The information of intuitionistic fuzzy set become more meaningful and applicable since it comprises the degree of belongingness, degree of non-belongingness and the hesitation margin. Distance measure between intuitionistic fuzzy sets is an important perception in fuzzy mathematics because of its wide applications in real world, such as pattern recognition, machine learning, decision making and market prediction. In this paper we make a contribution to the intuitionistic fuzzy fixed point theory by proving fixed point results on modified intuitionistic fuzzy metric spaces with the help of common limit range and joint common limit range property for coupled functions.

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Vishal Gupta*, Department of Mathematics, Maharishi Markandeshwar University, Mullana-133207, Ambala, Haryana, India
E-mail address: vishal.gmn@gmail.com

Rajesh Kumar Saini, Department of Mathematics, Statistics and Computer Applications, Bundelkhand University, Jhansi, U.P., India
E-mail address: rksaini.bu@gmail.com

Ashima Kanwar, Department of Mathematics, Maharishi Markandeshwar University, Mullana-133207, Ambala, Haryana, India
E-mail address: kanwar.ashima87@gmail.com

*corresponding author