INVENTORY MODEL WITH DEMAND AS TYPE-2 FUZZY NUMBER: A FUZZY DIFFERENTIAL EQUATION APPROACH

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Abstract. An inventory model is formulated with type-2 fuzzy parameters under trade credit policy and solved by using Generalized Hukuhara derivative approach. Representing demand parameter of each expert's opinion is a membership function of type-1 and thus, this membership function again becomes fuzzy. The final opinion of all experts is expressed by a type-2 fuzzy variable. For this present problem, to get corresponding defuzzified values of the triangular type-2 fuzzy demand parameters, first critical value (CV)-based reduction methods are applied to reduce corresponding type-1 fuzzy variables which becomes pentagonal in form. After that α- cut of a pentagonal fuzzy number is used to construct the upper α- cut and lower α- cut of the fuzzy differential equation. Different cases are considered for fuzzy differential equation: gH-(i) differentiable and gH-(ii) differentiable systems. The objective of this paper is to find out the optimal time so as to minimize the total inventory cost. The considered problem ultimately reduces to a multi-objective problem which is solved by weighted sum method and global criteria method. Finally the model is solved by generalised reduced gradient method using LINGO (13.0) software. The proposed model and technique are lastly illustrated by providing numerical examples. Results from two methods are compared and some sensitivity analyses both in tabular and graphical forms are presented and discussed. The effects of total cost with respect to the change of demand related parameter (β), holding cost parameter (r), unit purchasing cost parameter (p), interest earned (i_e) and interest payable (i_p) are discussed. We also find the solutions for type-1 and crisp demand as particular cases of type-2 fuzzy variable. This present study can be applicable in many aspects in many real life situations where type-1 fuzzy set is not sufficient to formulate the mathematical model. From the numerical studies, it is observed that under both gH-(i) and gH-(ii) cases, total cost of the system gradually reduces for the sub-cases - 1.1, 1.2 and 1.3 depending upon the positions of N(trade credit for customer) and M (trade credit for retailer) with respect to T (time period).

1. Introduction

Type-2 fuzzy sets (T2 FSs) are extensions of type-1 fuzzy sets was first introduced by Zadeh [41, 42]. Normally the membership grade of a type-1 fuzzy set is a real number in [0, 1], when the membership function (MF) of a type-1 fuzzy set is imprice, then type-2 fuzzy set can formulated from the type-1 MFS. So, the membership grade of a type-2 fuzzy set (T2 FS) is a fuzzy number with a support

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bounded by the interval [0, 1]. The logical operations of type-2 fuzzy sets were explored by Mizumoto and Tanaka [20] and Dubois and Prade [10]. Later on, a significant number of theoretical research works on the property of type-2 fuzzy sets and its many applications have been developed e.g., [21, 33]. In case of a T2 FS, complete defuzzification process consists of two parts- type reduction and defuzzification. Type reduction is the procedure by which a T2 FS is transformed to the corresponding T1 FS, known as type reduced set (TRS). The TRS is then easily defuzzified to a crisp value. Karnik and Mendel [16] proposed a centroid type reduction method to reduce IT2 FS into T1 FS. Qin et al. [32] introduced three kinds of reduction methods called optimistic CV, pessimistic CV and CV reduction methods for type-2 fuzzy variables (T2 FVs) based on CVs (critical values) of regular fuzzy variables α-cut and the extension principle forms a methodology for extending mathematical concepts from crisp sets to fuzzy sets. These have been applied to many operations, and have also been extended to interval valued fuzzy sets. In the present situation, uses of T2 FSs in the real life inventory problems is a burning topic.

Now-a-days, due to the advent of multinationals for globalization, competition among the national merchants is quite stiff and business people adopt different means to allure/attract the customers. Goyal [13] established a single item inventory model under permissible delay in payment. Huang [15] determined optimal retailer's replenishment decisions in the EPQ model under two levels of trade credit policy. Recently, Barrón et al. [37, 38, 36, 34, 8, 7, 6, 39, 5] formulated EOQ/EPQ models of under trade credit policy. Inventory problems had been studied extensively during second worldwide by Mousavi [24, 25, 26, 30, 27].

Demand has been always one of the most effective factors in the decisions relating to economic ordered quantity (EOQ) model as well as economic production quantity (EPQ) model. Due to this reason, various formations of consumption tendency have been studied by inventory control practitioners, such as stock dependent demand [19], selling price dependent demand [28]. Recently, You et al. [40] developed an inventory model incorporating trial period dependent demand. All of them developed their models in crisp environment, i.e. demand coefficients are considered as crisp number. But as discussed earlier, it is better to estimate demand coefficients with fuzzy numbers. The presence of fuzzy demand as well as fuzzy production rate leads to fuzzy differential equation of instantaneous state of inventory level. Till now fuzzy differential equation is little used to solve fuzzy inventory models though the topics on fuzzy differential equations have been rapidly growing in the recent years. The first impetus on solving fuzzy differential equation was made by Kandel and Byatt [17]. After that different approaches have been made by several authors to solve fuzzy differential equations [12, 2, 1].

The concept of fuzzy numbers is the generalization of the concept of real numbers. Dubois and Prade [11] has defined fuzzy number as a fuzzy subset of the real line. So far fuzzy numbers like triangular fuzzy numbers, trapezoidal fuzzy numbers, Pentagonal fuzzy numbers [31] have been introduced with its membership functions. These numbers have got many applications in practical field and many operations were performed using fuzzy numbers. In fuzzy differential equation, all derivatives are deliberated as either Hukuhara or generalized derivatives. The Hukuhara differentiability [3] has a deficiency that the solution turns fuzzfier as time goes by. Recently, Stefanini and Bede [35] introduced the concept of generalization of the Hukuhara difference for compact convex set, introduced generalized Hukuhara differentiability for fuzzy valued function and they displayed that. There are many approaches for solving FDE. There are very few development found in inventory and fuzzy differential equation. In last two years, some authors considered the fuzzy differential equation in inventory control problem and used the approaches (different approaches) of fuzzy differential equation such as [9, 14].

Though there are some articles on fuzzy inventory models which were solved by fuzzy differential equation approach, till now no one has solved inventory problem with fuzzy differential equation approach by generalized Hukuhara derivative concepts and where fuzzy numbers are taken as type-2 fuzzy number. Considering the inventory parameter as a type-2 fuzzy set is one of the important factor in real life situation. It is due to the fact that, in our real life problem there are some parameters like as demand which cannot be explained as a type-1 fuzzy number because the membership functions obtained by another experts again build another membership function. In this case taking demand parameter as type-2 fuzzy number is more realistic. Due to this situation we are motivated to formulate one model where demand parameter is taken as type-2 fuzzy number. But in our work, we are considering EPQ model with trade credit financing with fuzzy demand where Hukuhara gH derivative approach is used for solving the fuzzy differential equation. Some papers of the above literature survey in the area of inventory control and our proposed model are summarized and presented in Table 1.

Authors	Fuzzy environment	Stock dependent demand	Selling price dependent demand	Trade credit	T2FS & gH-derivative
Maiti and Maiti [23]	✓				
Huang [15]				✓	
Das et al. [9]	✓				
Ouyang et al. [28]			✓	✓	
Pal et al. [29]	✓		✓		
Mousavi et al. [26]	✓				
Shilpi et al. [19]		✓	✓	✓	
Guchhait et al. [14]	✓				
Shah and Barrón [34]				✓	
Our work	✓	✓	✓	✓	✓

TABLE 1. Contribution of Different Authors

The rest of the paper is organized as follows: In Section 2, preliminary definitions are introduced. Section 3 is a bout the defuzzification methods of Type-2 fuzzy variables. In section 4, we have discussed solution techniques. In section 5, different assumptions and notations are listed and also mathematical description are given. Numerical examples and their representations in graphs and tables are given in section 6 and also discussion have been given in section 7. Real-world problems and managerial decision are drawn in sections 8. Lastly, conclusions and future research works are discussed in section 9.

2. Preliminaries

2.1. **Fuzzy Set:** A fuzzy set $\tilde{A}$ is defined by $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in A, \mu_{\tilde{A}}(x) \in [0,1]\}$. In the pair (x,$\mu_{\tilde{A}}(x)$) the first element x belongs to the classical set A, the second element $\mu_{\tilde{A}}(x)$, belongs to the interval [0,1], called Membership function.

2.2. **Fuzzy Numder:** A fuzzy number is fuzzy set like $u : R \longrightarrow I = [0,1]$ which satisfies

(1) u is upper semi-continuous.
(2) $u(x) = 0$ outside the interval $[c, d]$.
(3) There are real numbers a,b such $c \leq a \leq b \leq d$ and
 (i) $u(x)$ is monotonic increasing on $[c, a]$,
 (ii)$u(x)$ is monotonic decreasing on $[b, d]$,
 (iii)$u(x) = 1$ when $a \leq x \leq b$.

2.3. α**-cut of a Fuzzy Set:** The α-level set (or interval of confidence at level α or α-cut) of the fuzzy set $\tilde{A}$ of X is a crisp set A_α that contains all the elements of X that have membership values in $\tilde{A}$ greater than or equal to α, i.e $\tilde{A} = (x : \mu_{\tilde{A}}(x) \geq \alpha, x \in X, \alpha \in [0,1])$

2.4. **Pentagonal Fuzzy Numder** [31]**:** A pentagonal fuzzy number $\tilde{A_p} = (a_1, a_2, a_3, a_4, a_5)$ where a_1, a_2, a_3, a_4, a_5 are real numbers and its membership is given below

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & \text{if } x < a_1 \\ \frac{1}{2}[\frac{x-a_1}{a_2-a_1}], & \text{if } a_1 \leq x \leq a_2 \\ \frac{1}{2} + \frac{1}{2}[\frac{x-a_2}{a_3-a_2}], & \text{if } a_2 \leq x \leq a_3 \\ 1 - \frac{1}{2}[\frac{a_4-x}{a_4-a_3}], & \text{if } a_3 \leq x \leq a_4 \\ \frac{1}{2}[\frac{a_5-x}{a_5-a_4}], & \text{if } a_4 \leq x \leq a_5 \\ 0, & \text{if } x > a_5 \end{cases}$$

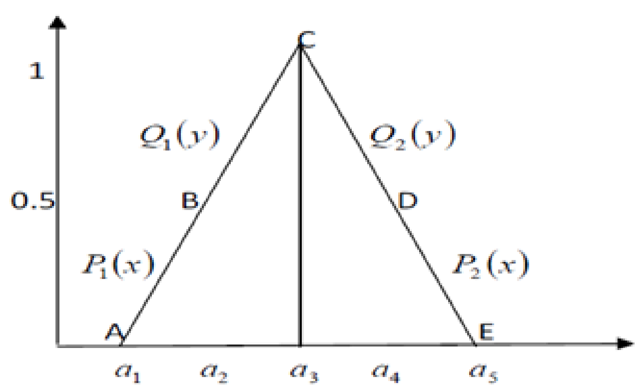

FIGURE 1. Graphical Representation of a Normal Pentagonal Fuzzy Number

Remark 2.1. Pentagonal fuzzy number $\tilde{A_p}$ is the ordered quadruple $(P_1(x), Q_1(x), Q_2(x), P_2(x))$, for $x \in [0, 0.5)$ and $x \in [0.5, 1)$ where, $P_1(x) = \frac{1}{2}[\frac{x-a_1}{a_2-a_1}]$, $P_2(x) = \frac{1}{2}[\frac{a_5-x}{a_5-a_4}]$, $Q_1(x) = \frac{1}{2} + \frac{1}{2}[\frac{x-a_2}{a_3-a_2}]$, $Q_2(x) = 1 - \frac{1}{2}[\frac{a_4-x}{a_4-a_3}]$

2.5. **Alpha Cut:** The classical alpha-cut is the set of elements whose degree of membership in $\tilde{A_p} = (a_1, a_2, a_3, a_4, a_5)$ is no less than α, it is defined as $A_\alpha = (x \in X/\mu_{\tilde{A_p}}(x) \geq x)$

$$= \begin{cases} [P_1(\alpha), P_2(\alpha)] & \text{for } \alpha \in [0, 0.5] \\ [Q_1(\alpha), Q_2(\alpha)] & \text{for } \alpha \in [0.5, 1] \end{cases}$$

2.6. α**-cut Operations**[31]**:** If we get crisp interval by α-cut operations interval A_α shall be obtained as follows, for all $\alpha \in [0, 1]$
Consider, $Q_1(x) = \alpha$
$\frac{1}{2} + \frac{1}{2}[\frac{x-a_2}{a_3-a_2}] = \alpha$
Hence, $Q_1(\alpha) = 2\alpha(a_3 - a_2) + 2a_2 - a_3$
Similarly, $Q_2(x) = \alpha$, $Q_2(\alpha) = 2\alpha(a_4 - a_3) + 2a_3 - a_4$, $P_1(\alpha) = 2\alpha(a_2 - a_1) + a_1$
$P_2(\alpha) = -2\alpha(a_5 - a_4) + a_5$
Hence,

$$A_\alpha = \begin{cases} [2\alpha(a_2 - a_1) + a_1, -2\alpha(a_5 - a_4) + a_5] & \text{for } \alpha \in [0, 0.5] \\ [2\alpha(a_3 - a_2) + 2a_2 - a_3, 2\alpha(a_4 - a_3) + 2a_3 - a_4] & \text{for } \alpha \in [0.5, 1] \end{cases}$$

By CV reduction method membership function of type two variable reduces to membership function of type one variable $\tilde{\xi_3} = (r_1, r_2, r_3, \theta_r, \theta_l)$
Now, $\tilde{\xi_3}$ is just like a pentagonal fuzzy number (c.f. equation (3))
Therefore α-cut of $\tilde{\xi_3}$ is

$$\left[\tilde{\xi_3}\right]_\alpha = \begin{cases} [P_1(\alpha), P_2(\alpha)] & \text{for } \alpha \in [0, 0.5] \\ [Q_1(\alpha), Q_2(\alpha)] & \text{for } \alpha \in [0.5, 1] \end{cases}$$

where,

$$P_1(\alpha) = \frac{r_1(1+\theta_r) + (r_2 - r_1)\alpha - 2\theta_r r_1 \alpha}{1 + \theta_r - 2\theta_r \alpha}, \quad P_2(\alpha) = \frac{(r_3 - r_2)\alpha - r_3 + (2\alpha - 1)\theta_r r_3}{(2\alpha - 1)\theta_r - 1}$$
$$Q_1(\alpha) = \frac{(r_2 - r_1)\alpha + r_1 + (2\alpha - 1)\theta_l r_2}{(2\alpha - 1)\theta_l + 1}, \quad Q_2(\alpha) = \frac{\alpha(r_3 - r_2) + (1 - 2\alpha)\theta_l r_2 - r_3}{-1 + (1 - 2\alpha)\theta_l}$$

2.7. **Generalised Hukuhara Derivative**[4]**:** The generalised Hukuhara derivative of a fuzzy valued function $f : (a, b) \longrightarrow \Re_F$ at t_0 is defined as
$f^{'}(t_0) = \lim_{h \longrightarrow 0} \frac{f(t_0+h) \ominus_g f(t_0)}{h}$
In parametric form we say that
f(t) is gH-(i) differentiable at t_0 if $[f^{'}(t_0)]_\alpha = [f^{'}_L(t_0, \alpha), f^{'}_R(t_0, \alpha)]$
and
f(t) is gH-(ii) differentiable at t_0 if $[f^{'}(t_0)]_\alpha = [f^{'}_R(t_0, \alpha), f^{'}_L(t_0, \alpha)]$

2.8. **Type-2 Fuzzy Set (T2FS):.** Type-2 fuzzy set $\tilde{A}$ defined on a universe of discourse X, which is denoted as $\tilde{A} \subseteq X$, is a set of pairs $\{x, \mu_{\tilde{A}}(x)\}$, where x an element of $\tilde{A}$ and its grade of membership $\{\mu_{\tilde{A}}(x)\}$ in the fuzzy set $\tilde{A}$ is a type-1 fuzzy set defined in the interval $J_x \subset [0,1]$, i.e. A T2 FS $\tilde{A}$ defined by Mendel and John [22] is

$$\tilde{A} = \{((x,u), \mu_{\tilde{A}}(x,u)) : \forall x \in X, J_x \subset [0,1]\},$$

where $0 \le \mu_{\tilde{A}}(x,u) \le 1$ is the type-2 membership function.

2.9. **Regular Fuzzy Variable (RFV):.** For a possibility space (φ, p, Pos), a regular fuzzy variable $\tilde{\xi}$ is defined as a measurable map from φ to $[0,1]$ in the sense that for every $t \in [0,1]$, one has $\{\gamma \in \varphi \,|\, \tilde{\xi}(\gamma) \le t\} \in p$. A discrete RFV is represented as $\tilde{\xi} \sim \begin{pmatrix} r_1\, r_2 \cdots r_n \\ \mu_1\, \mu_2 \cdots \mu_n \end{pmatrix}$ where $r_i \in [0,1]$ and $\mu_i > 0, \forall i$ and $max_i(\mu_i) = 1$.

If $\tilde{\xi} = (r_1, r_2, r_3)$ with $0 \le r_1 < r_2 < r_3 \le 1$, then $\tilde{\xi}$ is called a triangular RFV.

2.10. **Critical Values (CVs) for RFVs:** Qin et al. [32] introduced three kinds of critical values (CVs). Let $\tilde{\xi}$ be a RFV. Then,

I. The optimistic CV of $\tilde{\xi}$, denoted by $CV^*[\tilde{\xi}]$ is given by,

$$CV^*[\tilde{\xi}] = \sup_{\alpha \in [0,1]} [\alpha \wedge Pos(\tilde{\xi} \ge \alpha)]$$

II. The pessimistic CV of $\tilde{\xi}$, denoted by $CV_*[\tilde{\xi}]$ is given by,

$$CV_*[\tilde{\xi}] = \sup_{\alpha \in [0,1]} [\alpha \wedge Nec(\tilde{\xi} \ge \alpha)]$$

III. The CV of $\tilde{\xi}$, denoted by $CV[\tilde{\xi}]$ is given by,

$$CV[\tilde{\xi}] = \sup_{\alpha \in [0,1]} [\alpha \wedge Cr(\tilde{\xi} \ge \alpha)]$$

2.11. **Critical Values (CVs) of Triangular RFVs.:** The following theorems introduced the critical values (CVs) of triangular RFVs.

Theorem 2.2. *(Qin et al. [32]) Let $\tilde{\xi} = (r_1, r_2, r_3, r_4)$ be a trapezoidal RFV. Then we have,*

I. *The optimistic CV of $\tilde{\xi}$ is $CV^*[\tilde{\xi}] = \frac{r_4}{(1+r_4-r_3)}$.*

II. *The pessimistic CV of $\tilde{\xi}$ is $CV_*[\tilde{\xi}] = \frac{r_2}{(1+r_2-r_1)}$.*

III. *The CV of $\tilde{\xi}$ is* $CV[\tilde{\xi}] = \begin{cases} \frac{2r_2-r_1}{1+2(r_2-r_1)}, & if\ r_2 \ge \frac{1}{2} \\ \frac{1}{2}, & if\ r_2 \le \frac{1}{2} < r_3 \\ \frac{r_4}{1+2(r_4-r_3)}, & if\ r_3 \le \frac{1}{2} \end{cases}$

Theorem 2.3. *(Qin et al. [32]) Let $\tilde{\xi} = (r_1, r_2, r_3)$ be a triangular RFV. Then we have,*

I. *The optimistic CV of $\tilde{\xi}$ is $CV^*[\tilde{\xi}] = \frac{r_3}{(1+r_3-r_2)}$.*

II. *The pessimistic CV of $\tilde{\xi}$ is $CV_*[\tilde{\xi}] = \frac{r_2}{(1+r_2-r_1)}$.*

III. *The CV of $\tilde{\xi}$ is* $CV[\tilde{\xi}] = \begin{cases} \frac{2r_2-r_1}{1+2(r_2-r_1)}, & if\ r_2 \ge \frac{1}{2} \\ \frac{r_3}{1+2(r_3-r_2)}, & if\ r_2 \le \frac{1}{2} \end{cases}$

3. Defuzzification Methods for Type-2 Fuzzy Variables

For application purpose, some detailed defuzzication methods for type-2 fuzzy variables will be introduced in this section, which can be conceived as a simplification process for two-fold uncertain information. The defuzzification process of a type-2 fuzzy variable have two stage. In the first stage the type-2 fuzzy variable is reduced to its corresponding type-1 fuzzy variable and in the second stage the crisp value is obtained by applying different methods like as centroid method [16], expected value method ([18]) to the reduced fuzzy variables. In this paper we first apply the CV-Based reduction method to the type-2 fuzzy variables, so that we get the reduced form, i.e. a type-1 fuzzy variable and then we apply the centroid method to the type-1 fuzzy variables, resulting a crisp value. We now discuss defuzzification of a type-2 variable using CV-Based reduction method and Centroid method.

3.1. **Defuzzification Using CV-Based Reduction Method:** In type-2 fuzzy set, the membership function itself is a fuzzy set. So computation related to type-2 fuzzy is a very difficult job. To avoid this difficulty some defuzzification method and methodology have been used for defuzzification of type-2 fuzzy variable. Since we cannot apply the methodologies that are related to type-1 fuzzy sets directly to the type-2 fuzzy sets, we reduce the type-2 fuzzy sets into a type-1 fuzzy sets at first and then apply the methodologies. There are so many researchers who have developed different methods to defuzzify a type-2 fuzzy sets. Recently Qin et al. [32] introduced a new method viz. CV-Based reduction method that reduces a type-2 fuzzy variables into a type-1 fuzzy variables which may or may not be normal. This method basically based on to find out three critical values viz. optimistic CV denoted as $CV^*[\tilde{\xi}]$, pessimistic CV denoted as $CV_*[\tilde{\xi}]$ and CV reduction denoted as $CV[\tilde{\xi}]$. Using this critical values we easily reduce a type-2 fuzzy variables into a type-1 fuzzy variables. Here we discuss the method with an example.

Theorem 3.1. *(Qin et al.* [32]*) Let $\tilde{\xi} = (r_1, r_2, r_3; \theta_l, \theta_r)$ be a type -2 triangular fuzzy variable. Then we have,*

I. *Using the optimistic CV reduction method, the reduction ξ_1 of $\tilde{\xi}$ has the following possibility distribution,*

$$\mu_{\xi_1}(x) = \begin{cases} \frac{(1+\theta_r)(x-r_1)}{r_2-r_1+\theta_r(x-r_1)}, & \text{if } x \in \left[r_1, \frac{r_1+r_2}{2}\right] \\ \frac{(1-\theta_r)x-r_1+\theta_r r_2}{r_2-r_1+\theta_r(r_2-x)}, & \text{if } x \in \left(\frac{r_1+r_2}{2}, r_2\right] \\ \frac{(-1+\theta_r)x-\theta_r r_2+r_3}{r_3-r_2+\theta_r(x-r_2)}, & \text{if } x \in \left(r_2, \frac{r_2+r_3}{2}\right] \\ \frac{(1+\theta_r)(r_3-x)}{r_3-r_2+\theta_r(r_3-x)}, & \text{if } x \in \left(\frac{r_2+r_3}{2}, r_3\right]. \end{cases} \tag{1}$$

II. *Using the pessimistic CV reduction method, the reduction ξ_2 of $\tilde{\xi}$ has the following possibility distribution,*

$$\mu_{\xi_2}(x) = \begin{cases} \frac{(x-r_1)}{r_2-r_1+\theta_l(x-r_1)}, & \text{if } x \in \left[r_1, \frac{r_1+r_2}{2}\right] \\ \frac{(x-r_1)}{r_2-r_1+\theta_l(r_1-x)}, & \text{if } x \in \left(\frac{r_1+r_2}{2}, r_2\right] \\ \frac{(r_3-x)}{r_3-r_2+\theta_l(x-r_2)}, & \text{if } x \in \left(r_2, \frac{r_2+r_3}{2}\right] \\ \frac{(r_3-x)}{r_3-r_2+\theta_l(r_3-x)}, & \text{if } x \in \left(\frac{r_2+r_3}{2}, r_3\right]. \end{cases} \tag{2}$$

III. *Using the CV reduction method, the reduction ξ_3 of $\tilde{\xi}$ has the following possibility distribution,*

$$\mu_{\xi_3}(x) = \begin{cases} \frac{(1+\theta_r)(x-r_1)}{r_2-r_1+2\theta_r(x-r_1)}, & \text{if } x \in \left[r_1, \frac{r_1+r_2}{2}\right] \\ \frac{(1-\theta_l)x+\theta_l r_2-r_1}{r_2-r_1+2\theta_l(r_2-x)}, & \text{if } x \in \left(\frac{r_1+r_2}{2}, r_2\right] \\ \frac{(-1+\theta_l)x-\theta_l r_2+r_3}{r_3-r_2+2\theta_l(x-r_2)}, & \text{if } x \in \left(r_2, \frac{r_2+r_3}{2}\right] \\ \frac{(1+\theta_r)(r_3-x)}{r_3-r_2+2\theta_r(r_3-x)}, & \text{if } x \in \left(\frac{r_2+r_3}{2}, r_3\right]. \end{cases} \tag{3}$$

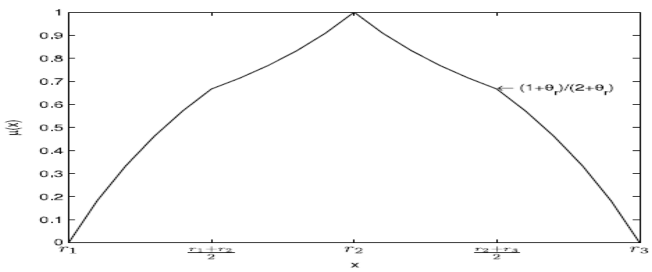

FIGURE 2. The Possibility Distribution $\mu_{\xi_1}(x)$ of ξ_1

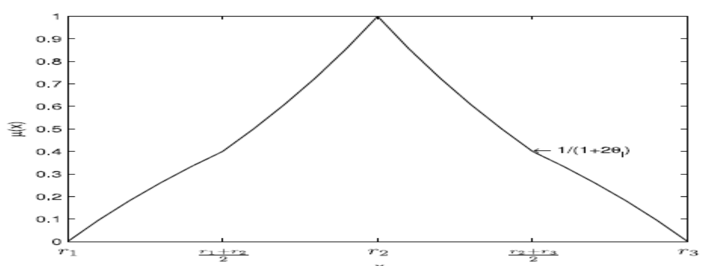

FIGURE 3. The Possibility Distribution $\mu_{\xi_2}(x)$ of ξ_2

3.2. **Centroid Defuzzification Technique:** The centroid method is also known as center of gravity or center of area defuzzification. It is the most commonly used method and is more accurate compare to other existing method. The method can be expressed as,

$$x^* = \begin{cases} \frac{\sum_{i=1}^n x_i \, \mu_\xi(x_i)}{\sum_{i=1}^n \mu_\xi(x_i)}, & \text{for discrete case,} \\ \frac{\int x_i \, \mu_\xi(x_i)}{\int \mu_\xi(x_i)}, & \text{for continuous case,} \end{cases}$$

where $\xi = (\xi_1, \xi_2, \cdots, \xi_n)$ is a fuzzy variable, x^* is the corresponding crisp value to be obtained, $\mu_\xi(x_i)$ is the aggregated membership function and x is the output variable.

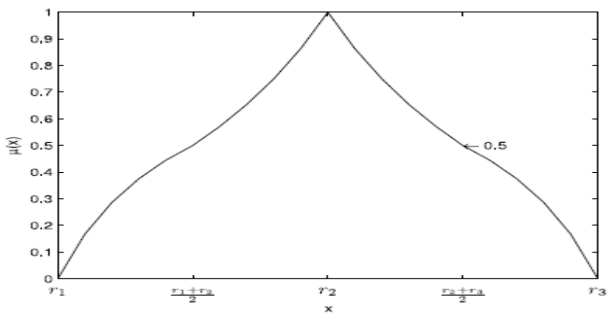

FIGURE 4. The Possibility Distribution $\mu_{\xi_3}(x)$ of ξ_3

On observing the above result, it is suggested to a decision maker to use the CV based reduction method instead of optimistic CV^* and pessimistic CV_* as it gives the more centroid compromised crisp value, compare to other CV values. The reason behind of it, the optimistic CV^* and pessimistic CV_* methods based on possibility and necessity measure respectively, whereas the CV reduction method based on the average of these two measures.

4. **Solution Techniques for Multi-Objective Non-Linear Problem**

4.1. **Weighted Sum Method:** The weighted sum method salaries a set of objectives into a single objective by multiplying each objective with user's supplied weights. The weights of an objective are usually chosen in proportion to the objective's relative importance in the problem. However, setting up an appropriate weight vector depends on the scaling of each objective function. It is likely that different objectives take different orders of magnitude. When such objectives are weighted to form a composite objective function, it would be better to scale them appropriately so that each objective possesses more or less the same order of magnitude. This process is called normalization of objectives. After the objectives are normalized, a composite objective function F can be formed by summing the weighted normalized objectives and the relative problem is then converted to a single-objective optimization problem as follows:
Minimize F=$\sum_{l=1}^{L} w_l f_l$, $w_l \in [0,1]$.
Here, w_l is the weight of the $l-$th objective function. Since the minimum of the above problem does not change if all the weights are multiplied by a constant, it is the usual practice to choose weights such that their sum is one, i.e. $\sum_{l=1}^{L} w_l = 1$.

4.2. **Global Criteria Method:** The Multi-Objective Non-linear Programming (MONLP) problems may be solved by Global Criteria Method converting it to a single objective optimization problem. The solution procedure is as follows:
Step-1: Solve the multi-objective programming problem as a single objective problem using one objective at a time ignoring the others.
Step-2: From the results of Step-1, determine the ideal objective vector, say (f_1^{min},

$f_2^{min}, f_3^{min}, ..., f_k^{min}$) and the corresponding values of ($f_1^{max}, f_2^{max}, f_3^{max}, ...f_k^{max}$). Here the ideal objective vector is used as a reference point. The problem is then to solve the following auxiliary problem:
Find $x = (x_1, x_2, x_3, ..., x_n)^T$
which minimizes GC
subject to
$x \in X$
where $GC = Minimize\{\sum_{i=1}^{k}(\frac{f_i(x)-f_i^{min}}{f_i^{max}-f_i^{min}})^p\}^{\frac{1}{p}}$
or, $GC = Minimize\{\sum_{i=1}^{k}(\frac{f_i(x)-f_i^{min}}{f_i^{max}-f_i^{min}})^p\}^{\frac{1}{p}}$
where $1 \leq p < \infty$. The usual value of p is 2.
Step-3: Now, solve the above single objective problem described in step-2 by GRG method to obtain the compromise solution.

5. Model Formulation

5.1. **Notations and Assumptions:** To formulate the mathematical model for the proposed inventory system, the following notations and assumptions are made.

Notations:

(i) $q(t)$=inventory level at time t (variable).
(ii) Q = inventory level at time $t = 0$(parameter) .
(iii) c_3 = set up cost for each cycle(parameter).
(iv) T = length of each cycle(decision variable).
(v) M = permissible delay period by wholesaler to the retailer (decision variable).
(vi) N= permissible delay period by retailer to the customer(decision variable).
(vii) s = selling price per unit quantity (variable) .
(viii) p = purchasing cost per unit quantity .
(ix) $D(s, q(t))$= rate of demand (variable).
(x) i_e= rate of interest earned by the retailer from the revenue(parameter).
(xi) i_p= rate of interest payable by the retailer(parameter).
(xii) h = inventory holding cost per unit quantity per unit time(parameter).
(xiii) TVC is the cost function for the whole period(variable).

Assumptions:

(i) Lead time is zero.
(ii) Shortages are not allowed .
(iii) Demand is stock and selling price dependent i.e $D(s, q(t)) = \tilde{d_0}s^{-\epsilon}q(t)^{1-\beta}$; where $\tilde{d_0} = (r_1, r_2, r_3, \theta_l, \theta_r)$ is a type two fuzzy number,$\epsilon > 0; 0 < \beta < 1$, $r_1, r_2, r_3 > 0$ and $0 < \theta_l, \theta_r < 1$.
(iv) Holding cost per unit per unit time $h = rp$ where r is a real number which lies between 0 and 1.
(v) $i_e < i_p$.
(vi) Time horizon is infinite.

5.2. **Mathematical Description:** The inventory equation with boundary conditions, $q(0) = Q$ and $q(T) = 0$ is given by

$$\frac{d\tilde{q}(t)}{dt} = -\tilde{d}_0 s^{-\epsilon} \tilde{q}(t)^{1-\beta} \quad 0 < t \leq T$$

or

$$\frac{1}{\tilde{q}(t)^{1-\beta}} \frac{d\tilde{q}(t)}{dt} = -\tilde{d}_0 s^{-\epsilon} \quad 0 < t \leq T \tag{4}$$

Depending upon the value of α two possibility aries

(i) $\alpha \in [0, 0.5]$
(ii) $\alpha \in [0.5, 1]$

If $\alpha \in [0, 0.5]$, then equation (4) reduces to

$$[\frac{1}{q_L(t)^{1-\beta}} \frac{dq_L(t)}{dt}, \frac{1}{q_R(t)^{1-\beta}} \frac{dq_R(t)}{dt}] = [-d_{0R}, -d_{0L}] s^{-\epsilon} \quad 0 < t \leq T \tag{5}$$

Therefore two cases arise

Case 1: gH-(i) differentiable
Case 2: gH-(ii) differentiable

We will now discuss case 1: gH-(i) differentiable
Then from equation (5), two ordinary differential equation can be written as

$$\frac{1}{q_L(t)^{1-\beta}} \frac{dq_L(t)}{dt} = -d_{0R} s^{-\epsilon} \quad ,0 < t \leq T$$
$$\frac{1}{q_R(t)^{1-\beta}} \frac{dq_R(t)}{dt} = -d_{0L} s^{-\epsilon} \quad ,0 < t \leq T$$

Solving the above equations by using the boundary conditions we get,

$$q_L(t) = [\frac{d_{0R}\beta(T-t)}{s^\epsilon}]^{\frac{1}{\beta}} \quad ,0 < t \leq T$$
$$q_R(t) = [\frac{d_{0L}\beta(T-t)}{s^\epsilon}]^{\frac{1}{\beta}} \quad ,0 < t \leq T$$

where, $d_{0L} = \frac{r_1(1+\theta_r)+(r_2-r_1)\alpha-2\theta_r r_1 \alpha}{(1+\theta_r-2\theta_r\alpha)}$; $d_{0R} = \frac{-r_3+(r_3-r_2)\alpha+(2\alpha-1)\theta_r r_3}{(2\alpha-1)\theta_r-1}$
and $q_L(t)$ and $q_R(t)$ denotes the upper and lower α-cut of inventory level respectively.
Various inventory costs are given by
Lower α-cut and Upper α-cut of ordering cost (OC_L, OC_R), stock holding cost (THC_L, THC_R) are given by

$$OC_L = c_2$$
$$OC_R = c_2$$
$$THC_L = h \int_0^T q_L(t)dt = h[\frac{d_{0R}\beta}{s^\epsilon}]^{\frac{1}{\beta}} \frac{\beta}{1+\beta} T^{\frac{1+\beta}{\beta}}$$
$$THC_R = h \int_0^T q_R(t)dt = h[\frac{d_{0L}\beta}{s^\epsilon}]^{\frac{1}{\beta}} \frac{\beta}{1+\beta} T^{\frac{1+\beta}{\beta}}$$

Three sub cases aries according to the values of M, N, T.

Sub case 1.1: $0 < N \leq M \leq T$
Sub case 1.2: $0 < N \leq T \leq M$
Sub case 1.3: $0 < T \leq N \leq M$

Sub case 1.1: $0 < N \leq M \leq T$

For sub case 1.1, Lower α-cut and Upper α-cut of interest payable (TIP_L, TIP_R), interest earned (TIE_L, TIE_R) are given by

$$TIP_L = i_p[\int_M^T q_L(t)dt] = i_p \frac{\beta}{1+\beta}[\frac{d_{0R}\beta}{s^\epsilon}]^{\frac{1}{\beta}}(T-M)^{\frac{1+\beta}{\beta}}$$

$$TIP_R = i_p[\int_M^T q_R(t)dt] = i_p \frac{\beta}{1+\beta}[\frac{d_{0L}\beta}{s^\epsilon}]^{\frac{1}{\beta}}(T-M)^{\frac{1+\beta}{\beta}}$$

$$TIE_L = si_e[(T-N)\int_0^N d_{0L}s^{-\epsilon}{q_L}^{1-\beta}(t)dt + (1+T-M)\int_N^M d_{0L}s^{-\epsilon}{q_L}^{1-\beta}(t)(M-t)dt$$
$$+\int_M^T d_{0L}s^{-\epsilon}{q_L}^{1-\beta}(t)(T-t)dt]$$
$$= si_e d_{0L}(d_{0R})^{\frac{1-\beta}{\beta}}(\frac{\beta}{s^\epsilon})^{\frac{1}{\beta}}[(T-N)(T^{\frac{1}{\beta}} - (T-N)^{\frac{1}{\beta}}) + (1+T-M)\{\frac{1}{1+\beta}((T-N)^{\frac{1+\beta}{\beta}} -$$
$$(T-M)^{\frac{1+\beta}{\beta}}) + (T-M)((T-M)^{\frac{1}{\beta}} - (T-N)^{\frac{1}{\beta}})\} + \frac{1}{1+\beta}(T-M)^{\frac{\beta+1}{\beta}}]$$

$$TIE_R = si_e[(T-N)\int_0^N d_{0R}s^{-\epsilon}{q_R}^{1-\beta}(t)dt + (1+T-M)\int_N^M d_{0R}s^{-\epsilon}{q_R}^{1-\beta}(t)(M-t)dt$$
$$+\int_M^T d_{0R}s^{-\epsilon}{q_R}^{1-\beta}(t)(T-t)dt]$$
$$= si_e d_{0R}(d_{0L})^{\frac{1-\beta}{\beta}}(\frac{\beta}{s^\epsilon})^{\frac{1}{\beta}}[(T-N)(T^{\frac{1}{\beta}} - (T-N)^{\frac{1}{\beta}}) + (1+T-M)\{\frac{1}{1+\beta}((T-N)^{\frac{1+\beta}{\beta}} -$$
$$(T-M)^{\frac{1+\beta}{\beta}}) + (T-M)((T-M)^{\frac{1}{\beta}} - (T-N)^{\frac{1}{\beta}})\} + \frac{1}{1+\beta}(T-M)^{\frac{\beta+1}{\beta}}]$$

Subcase 1.2: $0 < N \leq T \leq M$

In this case, lower and upper α-cut of annual interest payable is zero
i.e., $TIP_L = 0\ and\ TIP_R = 0$.
For sub case 1.2, Lower α-cut and Upper α-cut of interest earned (TIE_L, TIE_R) are given by

$$TIE_L = si_e[(M-N)\int_0^N d_{0L}s^{-\epsilon}{q_L}^{1-\beta}(t)dt + (1+M-T)\int_N^T d_{0L}s^{-\epsilon}{q_L}^{1-\beta}(t)(T-t)dt$$
$$= si_e d_{0R}(d_{0L})^{\frac{1-\beta}{\beta}}(\frac{\beta}{s^\epsilon})^{\frac{1}{\beta}}[(M-N)(T^{\frac{1}{\beta}} - (T-N)^{\frac{1}{\beta}}) + (1+M-T)\{\frac{1}{1+\beta}(T-N)^{\frac{1+\beta}{\beta}}\}]$$
$$TIE_R = si_e[(M-N)\int_0^N d_{0R}s^{-\epsilon}{q_R}^{1-\beta}(t)dt + (1+M-T)\int_N^T d_{0R}s^{-\epsilon}{q_R}^{1-\beta}(t)(T-t)dt$$
$$= si_e d_{0R}(d_{0L})^{\frac{1-\beta}{\beta}}(\frac{\beta}{s^\epsilon})^{\frac{1}{\beta}}[(M-N)(T^{\frac{1}{\beta}} - (T-N)^{\frac{1}{\beta}}) + (1+M-T)\{\frac{1}{1+\beta}(T-N)^{\frac{1+\beta}{\beta}}\}]$$

Subcase 1.3: $0 < T \leq N \leq M$

In this case, lower and upper α-cut of annual interest payable is zero
i.e., $TIP_L = 0\ and\ TIP_R = 0$.
For sub case 1.3, Lower α-cut and Upper α-cut of interest earned (TIE_L, TIE_R) are given by

$TIE_L = si_e[(M-T)\int_0^T d_{0L}s^{-\epsilon}q_L{}^{1-\beta}(t)dt] = si_e d_{0L}(d_{0R})^{\frac{1-\beta}{\beta}}(\frac{\beta}{s^\epsilon})^{\frac{1}{\beta}}[(M-T)(T)^{\frac{1}{\beta}}]$

$TIE_R = si_e[(M-T)\int_0^T d_{0R}s^{-\epsilon}q_R{}^{1-\beta}(t)dt] = si_e d_{0R}(d_{0L})^{\frac{1-\beta}{\beta}}(\frac{\beta}{s^\epsilon})^{\frac{1}{\beta}}[(M-T)(T)^{\frac{1}{\beta}}]$

The unit cost incurred by the retailer for all sub cases of all cases are

$$TVC = \frac{1}{T}(THC + OC + TIP - TIE) \tag{6}$$

Therefore total variable cost per unit time is a fuzzy quantity and is defined by

$$TVC = [TVC_L, TVC_R] \tag{7}$$

where,

$$TVC_R = \frac{1}{T}(THC_R + OC_R + TIP_R - TIE_L) \tag{8}$$

and

$$TVC_L = \frac{1}{T}(THC_L + OC_L + TIP_L - TIE_R) \tag{9}$$

Therefore this model mathematically can be written as
Minimize $\{TVC_L, TVC_R\}$
subject to

$$0 \le \alpha \le 0.5 \tag{10}$$

Therefore the problem is a multi-objective optimization problem. Weighted sum method is used to solve this multi-objective optimization problem.

By using weighted sum method the above problem reduces to
Minimize $Z = \lambda_1 Z_1 + \lambda_2 Z_2$
subject to $0 \le \alpha \le 0.5$
where,

$$Z_1 = TVC_L \text{ and } Z_2 = TVC_R \quad (\lambda_1 + \lambda_2) = 1 \tag{11}$$

By using global criteria method the above problem reduces to
Minimize GC
where

$$GC = \{\sum_{i=1}^{2}(\frac{Z_i(x) - Z_i^{min}}{Z_i^{max} - Z_i^{min}})^2\}^{\frac{1}{2}} \ , Z_1 = TVC_L \text{ and } Z_2 = TVC_R \tag{12}$$

We will now discuss case 2: gH-(ii) differentiable
Then from equation (5), two ordinary differential equation can be written as

$$\frac{1}{q_L(t)^{1-\beta}}\frac{dq_L(t)}{dt} = -d_{0L}s^{-\epsilon} \quad ,0 < t \le T$$
$$\frac{1}{q_R(t)^{1-\beta}}\frac{dq_R(t)}{dt} = -d_{0R}s^{-\epsilon} \quad ,0 < t \le T$$

Solving the above equations by using the boundary conditions we get,

$$q_L(t) = [\frac{d_{0L}\beta(T-t)}{s^\epsilon}]^{\frac{1}{\beta}} \quad 0 < t \le T$$
$$q_R(t) = [\frac{d_{0R}\beta(T-t)}{s^\epsilon}]^{\frac{1}{\beta}} \quad 0 < t \le T$$

where, $d_{0L} = \frac{r_1(1+\theta_r) + (r_2 - r_1)\alpha - 2\theta_r r_1 \alpha}{(1+\theta_r - 2\theta_r\alpha)}$; $d_{0R} = \frac{-r_3 + (r_3 - r_2)\alpha + (2\alpha - 1)\theta_r r_3}{(2\alpha - 1)\theta_r - 1}$
and $q_L(t)$ and $q_R(t)$ denotes the upper and lower α-cut of inventory level respectively.

Various inventory costs are given by
Lower α-cut and Upper α-cut of ordering cost(OC_L, OC_R), stock holding cost (THC_L, THC_R) are given by

$$OC_L = c_2$$
$$OC_R = c_2$$
$$THC_L = h\int_0^T q_L(t)dt = h[\frac{d_{0L}\beta}{s^\epsilon}]^{\frac{1}{\beta}}\frac{\beta}{1+\beta}T^{\frac{1+\beta}{\beta}}$$
$$THC_R = h\int_0^T q_R(t)dt = h[\frac{d_{0R}\beta}{s^\epsilon}]^{\frac{1}{\beta}}\frac{\beta}{1+\beta}T^{\frac{1+\beta}{\beta}}$$

Three sub cases aries according to the values of M, N, T.
Subcase 2.1: $0 < N \leq M \leq T$
Subcase 2.2: $0 < N \leq T \leq M$
Subcase 2.3: $0 < T \leq N \leq M$

Subcase 2.1: $0 < N \leq M \leq T$

For sub case 2.1, Lower α-cut and Upper α-cut of interest payable (TIP_L, TIP_R), interest earned (TIE_L, TIE_R) are given by

$$TIP_L = i_p[\int_M^T q_L(t)dt] = i_p\frac{\beta}{1+\beta}[\frac{d_{0L}\beta}{s^\epsilon}]^{\frac{1}{\beta}}(T-M)^{\frac{1+\beta}{\beta}}$$
$$TIP_R = i_p[\int_M^T q_R(t)dt] = i_p\frac{\beta}{1+\beta}[\frac{d_{0R}\beta}{s^\epsilon}]^{\frac{1}{\beta}}(T-M)^{\frac{1+\beta}{\beta}}$$
$$TIE_L = si_e[(T-N)\int_0^N d_{0L}s^{-\epsilon}{q_L}^{1-\beta}(t)dt + (1+T-M)\int_N^M d_{0L}s^{-\epsilon}{q_L}^{1-\beta}(t)(M-t)dt]$$
$$= si_e d_{0L}(d_{0L})^{\frac{1-\beta}{\beta}}(\frac{\beta}{s^\epsilon})^{\frac{1}{\beta}}[(T-N)(T^{\frac{1}{\beta}} - (T-N)^{\frac{1}{\beta}}) + (1+T-M)\{\frac{1}{1+\beta}((T-N)^{\frac{1+\beta}{\beta}} - (T-M)^{\frac{1+\beta}{\beta}}) + (T-M)((T-M)^{\frac{1}{\beta}} - (T-N)^{\frac{1}{\beta}})\} + \frac{1}{1+\beta}(T-M)^{\frac{\beta+1}{\beta}}]$$

$$TIE_R = si_e[(T-N)\int_0^N d_{0R}s^{-\epsilon}{q_R}^{1-\beta}(t)dt + (1+T-M)\int_N^M d_{0R}s^{-\epsilon}{q_R}^{1-\beta}(t)(M-t)dt]$$
$$= si_e d_{0R}(d_{0R})^{\frac{1-\beta}{\beta}}(\frac{\beta}{s^\epsilon})^{\frac{1}{\beta}}[(T-N)(T^{\frac{1}{\beta}} - (T-N)^{\frac{1}{\beta}}) + (1+T-M)\{\frac{1}{1+\beta}((T-N)^{\frac{1+\beta}{\beta}} - (T-M)^{\frac{1+\beta}{\beta}}) + (T-M)((T-M)^{\frac{1}{\beta}} - (T-N)^{\frac{1}{\beta}})\} + \frac{1}{1+\beta}(T-M)^{\frac{\beta+1}{\beta}}]$$

Subcase 2.2: $0 < N \leq T \leq M$

In this case, Lower α-cut and Upper α-cut of annual interest payable is zero
i.e., $TIP_L = 0$ *and* $TIP_R = 0$.
For sub case 2.2, Lower α-cut and Upper α-cut of interest earned (TIE_L, TIE_R) are given by

$$TIE_L = si_e[(M-N)\int_0^N d_{0L}s^{-\epsilon}{q_L}^{1-\beta}(t)dt + (1+M-T)\int_N^T d_{0L}s^{-\epsilon}{q_L}^{1-\beta}(t)(T-t)dt]$$
$$= si_e d_{0L}(d_{0L})^{\frac{1-\beta}{\beta}}(\frac{\beta}{s^\epsilon})^{\frac{1}{\beta}}[(M-N)(T^{\frac{1}{\beta}} - (T-N)^{\frac{1}{\beta}}) + (1+M-T)\{\frac{1}{1+\beta}(T-N)^{\frac{1+\beta}{\beta}}\}]$$
$$TIE_R = si_e[(M-N)\int_0^N d_{0R}s^{-\epsilon}{q_R}^{1-\beta}(t)dt + (1+M-T)\int_N^T d_{0R}s^{-\epsilon}{q_R}^{1-\beta}(t)(T-t)dt]$$

$= si_e d_{0R}(d_{0R})^{\frac{1-\beta}{\beta}}(\frac{\beta}{s^{\epsilon}})^{\frac{1}{\beta}}[(M-N)(T^{\frac{1}{\beta}}-(T-N)^{\frac{1}{\beta}})+(1+M-T)\{\frac{1}{1+\beta}(T-N)^{\frac{1+\beta}{\beta}}\}]$

Sub case 2.3: $0 < T \leq N \leq M$

In this case, Lower α-cut and Upper α-cut of annual interest payable is zero i.e., $TIP_L = 0 \ and \ TIP_R = 0$.

For sub case 2.3, Lower α-cut and Upper α-cut of interest earned (TIE_L, TIE_R) are given by

$TIE_L = si_e[(M-T)\int_0^T d_{0L}s^{-\epsilon}q_L{}^{1-\beta}(t)dt] = si_e d_{0L}(d_{0L})^{\frac{1-\beta}{\beta}}(\frac{\beta}{s^{\epsilon}})^{\frac{1}{\beta}}[(M-T)(T)^{\frac{1}{\beta}}]$

$TIE_R = si_e[(M-T)\int_0^T d_{0R}s^{-\epsilon}q_R{}^{1-\beta}(t)dt] = si_e d_{0R}(d_{0R})^{\frac{1-\beta}{\beta}}(\frac{\beta}{s^{\epsilon}})^{\frac{1}{\beta}}[(M-T)(T)^{\frac{1}{\beta}}]$

The unit cost incurred by the retailer for all sub cases of all cases are

$$TVC = \frac{1}{T}(THC + OC + TIP - TIE) \tag{13}$$

Therefore total variable cost per unit time is a fuzzy quantity and is defined by

$$TVC = [TVC_L, TVC_R] \tag{14}$$

Where,

$$TVC_R = \frac{1}{T}(THC_R + OC_R + TIP_R - TIE_L) \tag{15}$$

$$\& \ TVC_L = \frac{1}{T}(THC_L + OC_L + TIP_L - TIE_R) \tag{16}$$

Therefore this model mathematically can be written as
Minimize $\{TVC_L, TVC_R\}$
subject to

$$0 \leq \alpha \leq 0.5 \tag{17}$$

Therefore the problem is a multi objective optimization problem. Weighted sum method and global criteria method are used to solve this multi-objective optimization problem.

By using weighted sum method, the above problem reduces to
Minimize $Z = \lambda_1 Z_1 + \lambda_2 Z_2$
subject to $0 \leq \alpha \leq 0.5$ Where,

$$Z_1 = TVC_L \text{ and } Z_2 = TVC_R$$
$$(\lambda_1 + \lambda_2) = 1 \tag{18}$$

By using global criteria method the above problem reduces to
Minimize GC
where

$$GC = \{\sum_{i=1}^{2}(\frac{Z_i(x) - Z_i^{min}}{Z_i^{max} - Z_i^{min}})^2\}^{\frac{1}{2}} \ , Z_1 = TVC_L \text{ and } Z_2 = TVC_R \tag{19}$$

If $\alpha \in [\mathbf{0.5}, \mathbf{1}]$, then equation (4) reduces to

$$[\frac{1}{q_L(t)^{1-\beta}}\frac{dq_L(t)}{dt}, \frac{1}{q_R(t)^{1-\beta}}\frac{dq_R(t)}{dt}] = [-d_{0R}, -d_{0L}]s^{-\epsilon} \ \ 0 < t \leq T \tag{20}$$

Therefore two cases arise

Case 1: gH-(i) differentiable
Case 2: gH-(ii) differentiable

We will now discuss case 1: gH-(i) differentiable
Then from equation (20), two ordinary differential equation can be written as

$$\frac{1}{q_L(t)^{1-\beta}}\frac{dq_L(t)}{dt} = -d_{0R}s^{-\epsilon} \quad ,0 < t \le T$$
$$\frac{1}{q_R(t)^{1-\beta}}\frac{dq_R(t)}{dt} = -d_{0L}s^{-\epsilon} \quad ,0 < t \le T$$

Solving the above equations by using the boundary conditions we get,

$$q_L(t)=[\frac{d_{0R}\beta(T-t)}{s^{\epsilon}}]^{\frac{1}{\beta}} \quad ,0 < t \le T$$
$$q_R(t)=[\frac{d_{0L}\beta(T-t)}{s^{\epsilon}}]^{\frac{1}{\beta}} \quad ,0 < t \le T$$

where, $d_{0L} = \frac{r_1(1+\theta_r)+(r_2-r_1)\alpha-2\theta_r r_1\alpha}{(1+\theta_r-2\theta_r\alpha)}$; $d_{0R} = \frac{-r_3+(r_3-r_2)\alpha+(2\alpha-1)\theta_r r_3}{(2\alpha-1)\theta_r-1}$
and $q_L(t)$ and $q_R(t)$ denotes the upper and lower α-cut of inventory level respectively.
In this case, all the relevant costs (holding cost, ordering cost), total interest earned, total interest payable are same as the the previous case 1 where $\alpha \in [0, 0.5]$, but the values of d_{0L} *and* d_{0R} are different from the previous case 1. In this case

$$d_{0L} = \frac{r_1(1+\theta_r)+(r_2-r_1)\alpha-2\theta_r r_1\alpha}{(1+\theta_r-2\theta_r\alpha)}; \ d_{0R} = \frac{-r_3+(r_3-r_2)\alpha+(2\alpha-1)\theta_r r_3}{(2\alpha-1)\theta_r-1}$$

Here the objective function have the same expression as previous Case 1 where $\alpha \in [0, 0.5]$ with different values of d_{0L} and d_{0R} and also for $\alpha \in (0.5, 1]$
We will now discuss case 2: gH-(ii) differentiable
Then from equation (20), two ordinary differential equation can be written as

$$\frac{1}{q_L(t)^{1-\beta}}\frac{dq_L(t)}{dt} = -d_{0L}s^{-\epsilon} \quad ,0 < t \le T$$
$$\frac{1}{q_R(t)^{1-\beta}}\frac{dq_R(t)}{dt} = -d_{0R}s^{-\epsilon} \quad ,0 < t \le T$$

Solving the above equations by using the boundary conditions we get,

$$q_L(t)=[\frac{d_{0L}\beta(T-t)}{s^{\epsilon}}]^{\frac{1}{\beta}} \quad ,0 < t \le T$$
$$q_R(t)=[\frac{d_{0R}\beta(T-t)}{s^{\epsilon}}]^{\frac{1}{\beta}} \quad ,0 < t \le T$$

where, $d_{0L} = \frac{r_1+(r_2-r_1)\alpha+(2\alpha-1)\theta_l r_2}{(1+(2\alpha-1)\theta_l)}$; $d_{0R} = \frac{-r_3+(r_3-r_2)\alpha+(1-2\alpha)\theta_l r_2}{(1-2\alpha)\theta_l-1}$
and $q_L(t)$ and $q_R(t)$ denote the upper and lower α-cuts of inventory level respectively.
In this case also, all the relevant costs (holding cost, ordering cost), total interest earned, total interest payable are the same as the previous Case 2 where $\alpha \in [0, 0.5]$, but the values of d_{0L} *and* d_{0R} are different from the previous Case 2. In this case

$$d_{0L} = \frac{r_1(1+\theta_r)+(r_2-r_1)\alpha-2\theta_r r_1\alpha}{(1+\theta_r-2\theta_r\alpha)}; \ d_{0R} = \frac{-r_3+(r_3-r_2)\alpha+(2\alpha-1)\theta_r r_3}{(2\alpha-1)\theta_r-1}$$

Here the objective function has the same expression as previous Case 1 where $\alpha \in [0, 0.5]$, with different values of d_{0L} and d_{0R} and also for $\alpha \in (0.5, 1]$
. The flowchart of the solution procedure of our proposed model is given as follows:

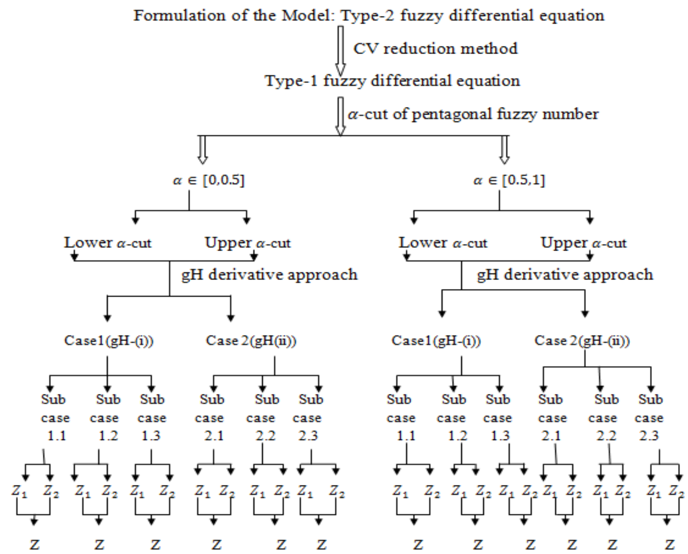

FIGURE 5. Flowchart of the Solution Procedure

6. Numerical Experiment

A medium grocery shop (Aparupa Varieties) at Agartala, Tripura, India sells different types of products. We visited this store and collected the data for the item noddles for every month of 2015. The corresponding input data are as follows:

6.1. **Input Data:** Let $i_p = 0.31$, $i_e = 0.11$, $\theta_r = 0.3$, $\theta_l = 0.5$, $\beta = 0.8$, $r_1 = 1600$, $r_2 = 1700$, $r_3 = 1800$, $\alpha = 0.3$ $(when\ \alpha \in [0, 0.5])$, $\alpha = 0.7$ $(when\ \alpha \in [0.5, 1])$, $p = Rs10/unit$, $\epsilon = 0.2$, $r = 0.4$, $h = 17$, $c_3 = 200$, $\lambda_1 = 0.5$, $\lambda_2 = 0.5$.
Then the optimal results for different cases are given in Table 2 and Table 3.
In case of Type-1, there exists only one possibility where $\alpha \in [0, 1]$. Let α=0.3 and $\theta_l = \theta_r$=0. Then Type-2 variable reduces to Type-1 variable. The corresponding optimum values are given in Table 6 by using weighted sum method.
In case of crisp demand, there is no need of α cut. Let θ_l=θ_r=0 and $r_1 = r_2 = r_3$=1700. Then Type-2 variable reduces to crisp number. The corresponding optimum values are given in Table 7.

7. Discussion

Table 2 and 3 describe the optimal results of fuzzy inventory model by weighted sum and global criteria methods. From these tables, it is observed that in all cases, weighted sum method gives better results than the global criteria method.

Different cases		Different sub cases	T	M	N	(Z_1, Z_2)	Z
$\alpha \in [0, 0.5]$	Case 1	Sub case 1.1	0.9061	0.5323	0.5011	(286.66, 509.00)	397.83
		Sub case 1.2	1.2064	1.2127	0.6673	(139.49, 461.13)	300.31
		Sub case 1.3	1.2896	1.7835	1.4288	(103.17, 455.13)	279.15
	Case 2	Sub case 2.1	0.9097	0.5326	0.5031	(364.93, 427.44)	396.19
		Sub case 2.2	1.3221	1.3380	0.7312	(227.03, 328.53)	277.78
		Sub case 2.3	1.4762	2.0523	1.9309	(185.05, 302.66)	243.85
$\alpha \in (0.5, 1]$	Case 1	Sub cases 1.1	0.9122	0.5334	0.5045	(260.85, 529.29)	395.07
		Sub case 1.2	1.3257	1.3417	0.7332	(59.23, 494.91)	277.07
		Sub case 1.3	1.2996	1.8008	1.6998	(53.32, 479.65)	266.22
	Case 2	Sub cases 2.1	1.1372	0.5993	0.6290	(264.49, 369.89)	317.19
		Sub case 2.2	1.3222	1.3381	0.7313	(215.73, 339.80)	277.76
		Sub case 2.3	1.5042	2.9248	1.9676	(165.64, 312.98)	239.31

Table 2. Optimization Results for Different Sub Cases for Type-2 Fuzzy Demand by Using Weighted Sum Method

Different cases		Different sub cases	T	M	N	(Z_1, Z_2)	GC
$\alpha \in [0, 0.5]$	Case 1	Sub cases 1.1	0.8195	0.4823	0.4706	(289.57, 514.12)	0.2778
		Sub case 1.2	0.8309	0.8352	0.4828	(143.32, 467.21)	0.8600
		Sub case 1.3	1.3888	1.9214	1.5402	(110.21, 461.24)	0.5893
	Case 2	Sub cases 2.1	0.9871	0.5779	0.5459	(371.74, 429.42)	0.0200
		Sub case 2.2	1.4078	1.4248	0.7786	(231.14, 334.24)	0.5996
		Sub case 2.3	1.4334	1.9927	1.8749	(191.12, 312.45)	0.5362
$\alpha \in (0.5, 1]$	Case 1	Sub cases 1.1	1.3511	0.7900	0.7473	(264.75, 534.34)	0.1959
		Sub case 1.2	1.2857	1.2929	0.7005	(61.41, 495.17)	0.6261
		Sub case 1.3	1.2896	1.7870	1.6868	(57.54, 484.42)	0.7034
	Case 2	Sub cases 2.1	1.1033	0.5801	0.6102	(269.21, 371.18)	0.0016
		Sub case 2.2	1.4231	1.4403	0.7871	(225.13, 340.71)	0.5609
		Sub case 2.3	1.6030	2.2298	2.0968	(171.20, 318.14)	0.6425

Table 3. Optimization Results for Different Sub Cases for Type-2 Fuzzy Demand by Using Global Criteria Method

r	Subcase 1.1		Sub case 1.1		Sub case 1.1	
	(Z_1, Z_2)	Z	(Z_1, Z_2)	Z	(Z_1, Z_2)	Z
0.2	(262.847,318.708)	290.777	(211.356,246.037)	228.697	(146.909,241.817)	194.3634
0.4	(264.496,369.896)	317.196	(215.731,339.800)	277.766	(165.649,312.984)	239.3168
0.6	(743.059,784.894)	763.977	(700.037,745.272)	722.654	(689.074,735.470)	712.2727
0.8	(971.315,1007.300)	989.307	(941.159,978.776)	959.967	(933.462,971.700)	952.5818

Table 4. Effects of r on Z_1 and Z_2 of Different Sub Cases of Case 2 for Type-2 Fuzzy Demand When $\alpha \in (0.5, 1]$

p	Subcase 1.1		Sub case 1.1		Sub case 1.1	
	(Z_1, Z_2)	Z	(Z_1, Z_2)	Z	(Z_1, Z_2)	Z
10	(286.66,509.00)	397.8329	(139.493,461.139)	300.3167	(103.17,455.13)	279.1541
11	(414.73,588.74)	501.7386	(332.530,538.348)	435.4394	(314.92,529.71)	422.3173
13	(585.51,721.69)	653.6050	(529.659,673.571)	601.6155	(519.46,666.76)	593.1182
15	(710.57,830.54)	770.5625	(656.958,777.965)	717.4622	(649.30,772.38)	710.8450
17	(813.10,924.23)	868.6717	(754.852,863.801)	809.3267	(748.56,859.01)	803.7895
13	(901.68,1007.44)	954.5664	(835.834,937.423)	886.6288	(830.41,933.20)	881.8111
20	(942.16,1046.01)	994.0897	(871.894,970.816)	921.3554	(866.81,966.82)	916.8172
22	(1017.22,1118.26)	1067.7464	(937.373,1032.256)	984.8145	(932.82,1028.63)	980.7297

Table 5. Effects of Unit Purchasing Cost p on Z_1 and Z_2 of Different Sub Cases of Case 1 for Type-2 Fuzzy Demand When $\alpha \in (0, 0.5]$

Different cases		Different sub cases	T	M	N	(Z_1, Z_2)	Z
$\alpha \in [0, 1]$	Case 1	Sub case 1.1	0.9060	0.5323	0.5011	$(291.57, 504.14)$	397.8587
		Sub case 1.2	1.2663	1.2126	0.6672	$(146.61, 454.09)$	300.3565
		Sub case 1.3	1.2894	1.7832	1.4286	$(110.97, 447.42)$	279.1980
	Case 2	Sub case 2.1	0.9098	0.5326	0.5032	$(350.32, 442.05)$	396.4856
		Sub case 2.2	1.3223	1.3383	0.7313	$(203.288, 352.19)$	277.7345
		Sub case 2.3	1.4766	2.0527	1.9314	$(157.52, 330.07)$	243.8018

TABLE 6. Optimization Results for Different Sub Cases of All Cases for Type-1 Fuzzy Demand

	Different sub cases	T	M	N	Z
Case 1	Sub cases 1.1	0.90543	0.53198	0.5007909	398.1329
	Sub case 1.2	1.204584	1.210857	0.6662524	300.7800
	Sub case 1.3	1.287259	1.780305	1.426283	279.6640

TABLE 7. Optimization Results for All Sub Cases of All Cases for Crisp Demand

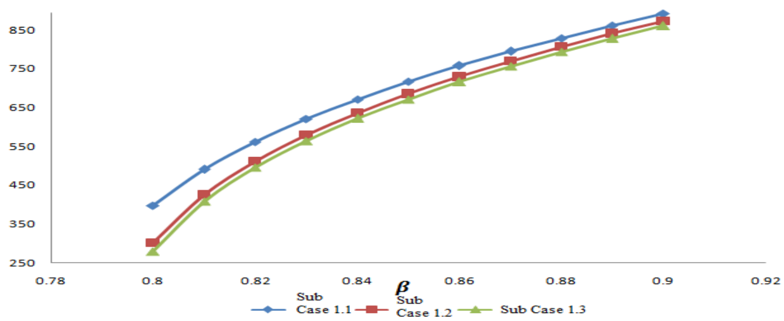

FIGURE 6. Effect of β on Z of Different Sub Cases of Case 1 when $\alpha \in [0, 0.5]$

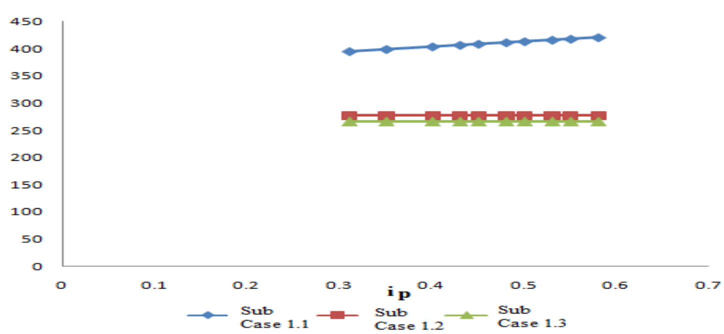

FIGURE 7. Effect of Interest Earned i_p on Z of Different Sub Cases of Case 1 when $\alpha \in [0.5, 1]$

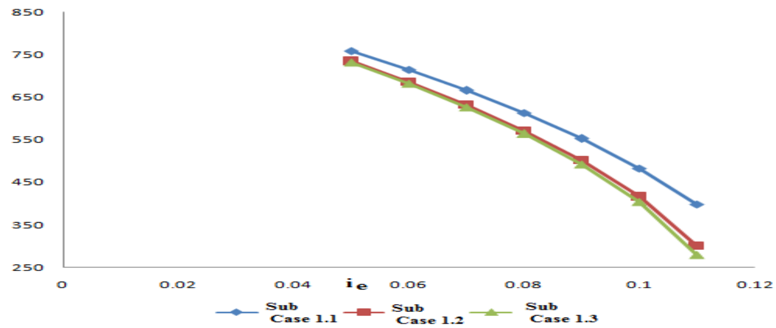

FIGURE 8. Effect of i_p on Z of Different Sub Cases of Case 1 when $\alpha \in [0, 0.5]$

In this proposed model, we observe that for $\alpha \in [0, 0.5]$ in sub case 1.3 of case 1, the annual inventory cost is minimum because in sub case 1.1 retailers trade credit period is much greater than total cycle time T. The same scenario is depicted for case 2. We also observe that for $\alpha \in [0.5, 1]$ in sub case 1.3 of case 1, the annual inventory cost is minimum and for case 2 in sub-case 2.3, we get the minimum cost. These natures/ behaviors of cost are the same in case of both methods. From Table 4, we observe that if the value of parameter r increases, the total holding cost increases and as a result annual inventory cost increases for all sub-cases of $\alpha \in (0.5, 1]$. Table 5 represents the effect of purchasing cost p on Z_1 and Z_2 of case 1 when $\alpha \in [0, 0.5]$. From this table we observe that if we increase unit purchasing cost p, the annual inventory cost also increases. Table 6 shows the optimal results for all sub cases of all cases where demand is a type-1 fuzzy variable. We also notice that the results of all sub cases of case 1 and case 2 of type-2 fuzzy demand is nearly similar to the the results of the type-1 fuzzy variable. So, we can say that type-1 is a particular case of type-2 where $\theta_l = 0$ and $\theta_r = 0$ and the value of α is 0.3. Table 7 shows the optimal results for crisp demand. In this case also retailer's minimum cost is obtained for sub case 1.3 as for type-2 and type-1 fuzzy demand. Figure 6 shows that if we increase the value of the parameter β, the value of annual inventory cost increases. It is due to the fact that with the increase of β, the inventory level increases and for the increase of inventory, the annual inventory cost increases. Figure 7 shows that with the increase of i_p interest payable by the retailer increases and as a result total annual cost increases for sub case 1.1. But for the other sub cases, i.e 1.2 and 1.3, the total relevant cost remain unchanged. It is due to the fact that in those cases, total interest payable by the retailer is zero. Figure 8 shows that with the increase of i_e, total annual inventory cost decreases. It is due to the reason that if we increase the value of i_e, then total interest earned by the retailer increases and as a result, total relevant cost of the retailer decreases.

8. **Real-world Problems and Managerial Decision**

When a new product is launched in the market by a company, demand of the product is not known. Normally some experts are asked about the possible demand.

Let there are N experts, each one is asked to give three estimates. From these three estimates of each expert, a membership function can be formed, which are different for different experts. Now the values of these membership functions are different and called type-1 membership function. From this membership functions, another membership functions can be formed and this membership function is called type-2 membership function. Thus, the proposed model can be applied in case of introducing a new product in the market through a supply chain. Normally, demand of a new product is type-2 fuzzy in nature. So, this present analysis is helpful for the retailer of reputed departmental stores specially in multinational developing countries.

Again in real life problems, customers are more attracted by the stock of the items in any departmental stores. This phenomenon is supported by the present market research. For this reason, demands normally depends on the stock level.

Moreover, demand always depends on price. It is the fact that more customers buck away as the selling price of an item increases. Thus, demand is selling price dependent. For more economic decisions, decision maker (DM) may decide the appropriate minimum cost depending upon the practical situation following the present analysis. Thus DM is able to take more appropriate precise decisions with the help of present analysis.

9. **Conclusions and Future Research Work**

In this paper, some useful ideas were presented to deal with inventory control problem with type-2 fuzzy parameters. The main contributions can be summarized as the following three aspects:

1. CV based reduction method proposed by Qin et al. [32] is discussed and successfully applied to the proposed model to find the total variable cost.
2. According to literature survey for the first time, we introduced an economic order quantity model with stock and selling price dependent demand in fuzzy environment where demand is taken as type-2 fuzzy number. With the use of CV based reduction method and α-cut of pentagonal fuzzy number the proposed model is solved for the minimum cost of the retailer.
3. Some new real life based important facts are provided as managerial decisions in this paper, which will help the retailers in developing the business management.
4. We have also analyzed the result of the current inventory model for type-1 and crisp demand which are particular cases of type-2 demand. The optimal results are obtained by generalized reduced gradient method using LINGO 13.0. The methodologies proposed in this paper are quite general and these can be applied to the decision making problems in different areas with type-2 fuzzy parameters.
5. Limitations of our proposed model is that we have considered the pentagonal type-2 fuzzy number only, which is the easiest form and from which the triangular form is easily reduced.

As a future work, the presented models can be extended to different types of inventory problems taking selling price, ordering cost, base demand as trapezoidal type-2 fuzzy number, gamma type-2 fuzzy number, Gaussian type-2 fuzzy number etc. We may also extend our model by considering price discounts, shortages,

quantity discounts etc.
6. As it is assumed that the unit selling price is greater than the unit purchasing price, the retailer must have sufficient amounts to pay the dues to the wholesaler at a time before the end of the total cycle and in this situation, he/she will have to pay less interest to the wholesaler. Moreover, the retailer can earn more interest after that payment time up to the end of the business period. This new approach to calculate the interest earned by the retailer may also be applied in this model and the result can be compared with the conventional approach also [19].
7. The concept of immediate part payment and the delay-payment for the rest can also be allowed over a finite planning horizon or random planning horizon. We can also extend the current model for partial trade credit i.e. supplier offers partial trade credit to retailer and retailer offers full trade credit to customers. In this paper, weighted sum method is used for solving multi-objective optimization. In future we may also extend the model by using another type of multi-objective optimization method like multi-objective genetic algorithm,fuzzy programming technique, etc.

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