OPTIMAL LOT-SIZING DECISIONS WITH INTEGRATED PURCHASING, MANUFACTURING AND ASSEMBLING FOR REMANUFACTURING SYSTEMS

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ABSTRACT. This work applies fuzzy sets to the integration of purchasing, manufacturing and assembling of production planning decisions with multiple suppliers, multiple components and multiple machines in remanufacturing systems. The developed fuzzy multi-objective linear programming model (FMOLP) simultaneously minimizes total costs, total CO_2 emissions and total lead time with reference to customer demand, due date, supplier/manufacturer capacity, lot-size release and machine yield. The proposed FMOLP model provides a recoverable remanufacturing framework that facilitates fuzzy decision-making, enabling the decision maker (DM) to adjust interactively the membership function or parameters during the solution procedure to obtain a preferred and satisfactory solution. To test the model, it was implemented in various scenarios with a remanufacturing production system. The analytical results in this work can help planner by enabling systematic analysis of the cost-effectiveness of remanufacturing systems and their potential for improving CO_2 emissions and lead time in terms of remanufacturing planning. Future investigations may apply the related patterns of non-linear membership functions to develop an actual remanufacturing planning decision.

1. Introduction

In recent years, increasing problems of resource depletion and waste production have caused countries around the world to begin to focus on environmental issues, including reverse logistics and the reuse and remanufacturing of recoverable products. Therefore, as well as adhering to strict environmental regulations that govern corporations, corporations should take the initiative to reduce the environmental impact of their products; the most efficient means of so doing is to recycle products for reuse and remanufacture. Reverse logistics refer to the recycling of end-of-life products after they are dismantled, cleaned and refurbished, as part of a cycle of repeated reuse and remanufacture. For example, Hewlett-Packard recycles carbon cartridges from clients for reuse [13]. The recycling of components reduces net CO_2 emissions and environmental effects, and enables a waste product with low value to be converted into a product with added value. The supply chain system is commonly associated with purchasing, manufacturing and assembly as well as distribution to markets. For environmental reasons, upstream suppliers

of material to downstream manufacturers should use recoverable components and reduce waste. Production planning of recycling and remanufacturing system is complex and requires that various scenarios must be considered; these include many components, many materials (both new and recycled), many vendors and various machines. Several researchers in this field [6, 7, 15, 17, 20, 27] have considered a single goal, (such as minimizing the total costs or maximizing the total profits). In practice, a DM must consider many objectives simultaneously (minimizing total costs, minimizing total CO_2 emissions, minimizing total lead time) to ensure smooth operations. Since recycle and manufacture programs involve uncertainties, associated with cost, power consumption, and lead time for example, the objectives frequently lack precision. Therefore, the objectives may be mutually conflicting, raising issues of fuzzy multi-objective linear programming (FMOLP). Even if the objectives of minimizing the total costs, total CO_2 emissions and total lead time are satisfied. This work addresses procurement, manufacturing and assembly plan problems with respect to recoverable remanufacturing systems by considering various vendors (of both new and recycled materials), various materials (both new and recycled), and various machines (for manufacturing and assembling), as well as yield parameter of the manufacturing machine while minimizing total cost, CO_2 emissions and lead time. The FMOLP is utilized to construct the optimal lot size for recycling and remanufacturing to satisfy the objectives of low cost, low carbon emissions and low lead time. The remainder of this paper is organized as follows. Section 2 presents a literature review; Section 3 formulates the fuzzy multi-objective recoverable remanufacturing planning model; Section 4 utilizes a remanufacturing curtain and shutter components case, which is used to assess the feasibility of the proposed model. Section 5 draws conclusions and recommends future research.

2. **Literature Review**

This section reviews the literature on recoverable remanufacturing planning. Van der Laan and Salomon [29] developed a general manufacturing/remanufacturing inventory system that included production and disposal operations. Van der Laan et al. [30] extended push and pull control strategies to evaluate numerically the effects of lead-time and the variability thereof on total expected costs in manufacturing/remanufacturing systems. Guide Jr et al. [9] schematically represented a remanufacturing system that comprised three highly dependent subsystems that conducted disassembly, processing, and reassembly operations. Guide Jr [10] identified and discussed seven complicating characteristics of production planning and control activities for remanufacturing firms. Marx-Gómez et al. [18] developed a forecasting method that combined a simulation model with fuzzy reasoning and a neuro-fuzzy method to forecast the returns of scrapped products with a view to forecasting return values over time. Dobos [5] designed a reverse logistics model with continuous disposal to optimize inventory policies in a reverse logistics system. This model minimized the sum of the holding cost in the stores and the costs of manufacturing, remanufacturing and disposal. Guide Jr et al. [8] studied the development of a contingency planning problem for closed-loop supply chains with

product recovery. They utilized a foundation of remanufacturing systems to study three cases of remanufacturing systems that represented remanufacturing-to-stock, reassemble-to-order, and remanufacturing-to-order. Lebreton and Tuma [16] formulated a linear programming model for solving the product mix problem that involve car and truck tire remanufacturing operations. The goal was to find a profit-maximizing product mix. Tang et al. [26] developed a newsboy model for solving the planned lead time and for determining the component purchasing strategy with yield probability and stochastic lead times of disassembly and purchasing. Vlachos et al. [31] developed a system-dynamic model for strategic remanufacturing and collection capacity planning for a single product reverse supply chain for product recovery. This model can be utilized to evaluate long-term capacity planning policies for implementation in closed-loop supply chains. Kernbaum et al. [14] developed a mixed integer program model for optimizing a remanufacturing process, which comprises three steps - data analysis, process design and remanufacturing. This model is based on a three-step process, which remanufacturing process integrated into a single user-friendly software interface and used for flat screen monitors [14] Subramoniam et al. [23] developed a remanufacturing decision-making framework that was based on comprehensively researched strategic factors for use in the automotive aftermarket remanufacturing industry. This framework provides valuable guidance for the suppliers of original equipment in making strategic decisions concerning the remanufacturing of products. Sutherland et al. [25] developed a remanufacturing facility cost model to solve the optimal lot size problem for a diesel engine remanufacturing facility by minimizing the total annual cost of operation. Vahdani et al. [28] explored the completion time of supply chain network operations; its determination is often imprecise or fuzzy. They developed a multistage hybrid model by using a fuzzy program evaluation and review technique to evaluate the supply chain performance in an uncertain environment. Alamri et al. [1] formulated a unified general reverse logistics inventory model for use in the integrated production of new items and the remanufacturing of returned items. This model involves three shops, which are the shop for remanufacturing returned items, the shop for manufacturing new items, the shop for collecting returned items. Wei et al. [34] developed a robust linear programming model to solve the inventory control and production planning problem with uncertain returns and demand, whose objective is to minimize the total production cost. The product return process is integrated into the manufacturing process over a finite planning horizon. Georgiadis and Athanasiou [7] presented a simulation-based system-dynamic optimization method for flexible long-term capacity planning with remanufacturing, to generate flexible policies for implementation in closed-loop supply chains. Subramoniam et al. [24] developed a remanufacturing decision-making framework that was based on an extensive literature review of strategic factors for use by the automotive industry and they utilized the analytic hierarchy process method to refine and prioritize the factors that are involved in strategic decision-making. Zangiabadi and Maleki [35] presented a fuzzy multi-objective programming technique to solve multi-objective transportation problem with some non-linear (exponential and hyperbolic) membership functions. Su [21] developed a fuzzy multi-objective linear

programming model to solve recoverable remanufacturing planning decisions problem with multiple components and multiple machines. This model simultaneously minimizes total production cost and total CO_2 emissions as functions of production cost, machine yield, capacity and energy consumption. Su and Lin [22] developed a fuzzy multi-objective linear programming model to solve the lot-sizing production planning problem for recoverable manufacturing systems. This model simultaneously minimizes total cost and total lead time as functions of supplier capacity, lead time, input lot size, machine yield and customer demand. Moghaddam [19] developed a fuzzy multi-objective mathematical model to solve the supplier selection and order allocation problem for reverse logistics systems under supply and demand uncertainty. This model simultaneously maximizes total net profit, minimizes total number of defective parts, total number of late delivered parts and total risk factors of economic environment associated with each supplier. Cárdenas-Barrón et al. [3] developed a mixed integer linear programming model to solve the multi-product multi-period inventory lot sizing with supplier selection problem. The objective was to minimize the total cost, including the total purchase cost of the products, the total ordering cost, and the total holding cost for carrying inventory in each period. Carvalho and Nascimento [4] developed a Lagrangian heuristics to solve the multi-plant capacity lot sizing problem with multiple periods and items; this heuristic can determine the best production planning in different industrial sectors. Hwang and Kang [12] applied a two-phase approach for solving multi-period production and transportation planning problems with backlogging in a distribution center. Vörös and Rappai [32] investigated the quality problems of the production process with random yields in Toyota's production system. They designed a mathematical model to solve an inventory control problem in order to minimize the expected values of the inventory costs; they used this model to determine the optimal lot size. Based on the above literature review, although several scholars have investigated recycling and remanufacturing, few have comprehensively examined the integration of purchasing, manufacturing and assembling, as follows.

(1) Work on recycling and remanufacturing has not produced a systematic plan. Such a plan should consider various suppliers (of new and recycled materials), various materials (new and recycled), and various machines (for manufacturing and assembling) so as to be effective in a real environment in which decisions concerning recycling and remanufacturing are made.

(2) Most research on recycling and remanufacturing issues has a single objective, but in practice, cost, CO_2 emissions and lead time affect each other. Few multi-objective models for use in recycle and remanufacture plans have been developed.

(3) In practical decision-making, decision variables related to recycle and remanufacture planning include uncertainties; such variables include costs, power consumption, and lead time. Related objectives are not obvious and they may be in conflict with each other. Further development of the FMOLP model is necessary to increase applicability to DM but relevant research is lacking.

3. **Problem Formulation**

Definition 3.1. This work considers procurement, manufacturing and assembly plan problems with respect to recoverable remanufacturing systems. The following issues arise in each stage. During purchasing, the DM must procure various materials - both new and recycled - from various vendors. These vendors may be recycling companies. Each vendor and recycling company has its own procurement costs, CO_2 emissions and lead time. The DM must decide the optimal lot size that minimizes the total cost, CO_2 emissions and lead time. In the manufacturing stage, the identical components are processed on particular machinery. As every batch of recycled materials suffers different rate of imperfections and every machine has its own defective rate, component manufacturing costs, production lead time, power consumption and production output may all be uncertain. During assembly, the DM must consider the defective rate of each machine, the assembly costs and the power consumption for work-in-process to optimize the lot size. During the remanufacturing process, costs, CO_2 emissions and lead times of new and recycled materials actually exhibit a trade-off relationship. For example, new components have a high procurement cost and short procurement lead time, but their manufacture consumes less power, whereas recycled materials have a lower procurement cost, but a greater procurement lead time because the recyclable materials must be disassembled, repaired and refurbished, so their manufacture consumes more power. The DMs must consider the trade-offs among multiple fuzzy objectives. For example, the objective function for total production costs may be \$0.8 million, annual CO_2 emissions are limited to 1 ton, and the lead time is approximately 10 days. This imprecision requires a set of fuzzy objectives and a set of compromise solutions. When companies consider differences in costs, CO_2 emissions and lead times, they tend to compromise by simultaneously considering new and recycled materials. Therefore, the DMs must simultaneously achieve the targets of low cost, low CO_2 emissions and short lead times. The proposed FMOLP model is based on the following assumptions.

(1) The required order volume is known.
(2) Only one product is remanufactured.
(3) The recycle and remanufacture plan includes purchasing, manufacturing and assembling.
(4) The production capacities of various vendors, recycling companies and machines are known.
(5) The defective rates of the various manufacturing and assembling machines are known.

Remark 3.2. Indices

i: number of components, $i = 1, 2, \ldots, I$
k: number of vendors of new materials, $k = 1, 2, \ldots, K$
l: number of vendors of recycled materials, $l = 1, 2, \ldots, L$
m: number of manufacturing machines, $m = 1, 2, \ldots, M$
n: number of assembling machines, $n = 1, 2, \ldots, N$

Remark 3.3. Parameters

α_m: cost of setting up m^{th} manufacturing machine (\$/time)

α'_n: cost of setting up n^{th} assembling machine (\$/time)

β_m: variable operating cost per unit for m^{th} manufacturing machine (\$/unit)

β'_n: variable operation cost per unit for n^{th} assembling machine (\$/unit)

Ck_{ik}: CO_2 emissions per unit of ith component purchased from k^{th} vendor of new materials (kg/unit)

Cl_{il}: CO_2 emissions per unit of ith component purchased from l^{th} vendor of recycled materials (kg/unit)

Cm_{im}: CO_2 emissions per unit of ith component released to m^{th} manufacturing machine (kg/unit)

Cs_n: CO_2 emissions per unit assembled on n^{th} machine (kg/unit)

CO_2^{max}: maximal CO_2 emissions (kg)

Dd: due date for customer order (day)

e_{ik}^{new}: defect rate of ith component purchased from k^{th} vendor of new materials (%)

e_{il}^{old}: defect rate of ith component purchased from l^{th} vendor of recycled materials (%)

g_{ik}^{new}: delivery capacity per truck when i^{th} component is purchased from k^{th} vendor of new materials (unit/truck)

g_{il}^{old}: delivery capacity per truck when i^{th} components is purchased from l^{th} vendor of recycled materials (unit/truck)

nq_m: unit cost of disposal of defective products that are produced on m^{th} manufacturing machine (\$/unit)

nq'_m: unit cost of disposal of defective products that are produced on n^{th} assembling machine (\$/unit)

P_{ik}^{new}: cost of ordering i^{th} component from k^{th} vendor of new materials (\$/time)

pc_{ik}^{new}: unit cost of ith component purchased from k^{th} vendor of new materials (\$/unit)

P_{il}^{old}: costs of ordering i^{th} component from l^{th} vendor of recycled materials (\$/time)

pc_{il}^{old}: unit cost of ith component purchased from l^{th} vendor of recycled materials (\$/unit)

θ_m: yield parameter of mth manufacturing machine, $0 \le \theta_m \le 1$

θ'_n: yield parameter of nth assembling machine, $0 \le \theta'_n \le 1$

S_n^{max}: maximal capacity of nth assembling machine

t_{ik}^{new}: delivery time per truck when i^{th} component is purchased from k^{th} vendor of new materials (day/truck)

t_{il}^{old}: delivery time per truck when i^{th} component is purchased from l^{th} vendor of recycled materials (day/truck)

O_k^{new}: binary variable, specifying costs are incurred upon ordering i^{th} component from k^{th} vendor of new materials $O_k^{new} = \begin{cases} 1 & \text{if } x_{ik}^{new} > 0 \\ 0 & \text{if } x_{ik}^{new} = 0 \end{cases}$

O_l^{old}: binary variable, specifying whether costs are incurred upon ordering ith component from l^{th} vendor of recycled materials $O_l^{old} = \begin{cases} 1 & \text{if } x_{il}^{old} > 0 \\ 0 & \text{if } x_{il}^{old} = 0 \end{cases}$

W_m: binary variable, specifying need for setting up mth manufacturing machine for release of ith component to m^{th} manufacturing machine

$$W_m = \begin{cases} 1 & \text{if } y_{im}^{new} > 0 \text{ or } y_{im}^{old} > 0 \\ 0 & \text{if } y_{im}^{new} = 0 \text{ or } y_{im}^{old} = 0 \end{cases}$$

W_n': binary variable, specifying need for setting up nth assembling machine for release of ith component to n^{th} assembling machine $W_n' = \begin{cases} 1 & \text{if } s_n > 0 \\ 0 & \text{if } s_n = 0 \end{cases}$

M: a very large value

U_{ik}^{max}: maximum number purchasable units of ith component from k^{th} vendor of new materials (unit)

U_{ik}^{min}: minimum number of purchasable units of ith component from k^{th} vendor of new materials (unit)

R_{il}^{max}: maximum number of purchasable units of ith component from l^{th} vendor of recycled materials (unit)

R_{il}^{min}: minimum number of purchasing units of ith component from l^{th} vendor of recycled materials (unit)

M_m^{max}: maximal capacity of the mth manufacturing machine (unit)

Remark 3.4. Decision variables

x_{ik}^{new}: size of lot of ith component purchased from l^{th} vendor of recycled materials (unit)

x_{il}^{old}: size of lot of ith component of new materials released to m^{th} manufacturing machine (unit)

y_{im}^{new}: size of lot of ith component of recycled materials released to m^{th} manufacturing machine (unit)

y_{im}^{old}: size of lot of ith component for work-in-process released to m^{th} assembling machine (unit)

s_n: size of lot of ith component for work-in-process released to n^{th} assembling machine (unit)

Remark 3.5. Minimize total production cost

Total production cost includes purchasing cost, manufacturing cost and assembly cost, as specified by (1):

$$\text{Min } Z_1 \simeq \sum_{i=1}^{I} \sum_{k=1}^{K} (P_{ik}^{new} O_k^{new} + pc_{ik}^{new} x_{ik}^{new}) + \sum_{i=1}^{I} \sum_{l=1}^{L} (P_{il}^{old} O_l^{old} + pc_{il}^{old} x_{il}^{old})$$

$$+ \sum_{i=1}^{I} \sum_{m=1}^{M} [\alpha_m W_m + \beta_m (y_{im}^{new} + y_{im}^{old}) + nq_m (y_{im}^{new} + y_{im}^{old})(1 - \theta_m)]$$

$$+ \sum_{n=1}^{N} [\alpha_n' W_n' + \beta_n' s_n + nq_n' s_n (1 - \theta_n')] \tag{1}$$

Remark 3.6. Minimize total CO_2 emissions

Total CO_2 emissions include CO_2 emissions procured materials, manufacturing and assembling stages, as specified by (2):

$$\text{Min } Z_2 \simeq \sum_{i=1}^{I}\sum_{k=1}^{K}(Ck_{ik}x_{ik}^{new}) + \sum_{i=1}^{I}\sum_{l=1}^{L}(Cl_{il}x_{il}^{old})$$
$$+ \sum_{i=1}^{I}\sum_{m=1}^{M}[Cm_{im}(y_{im}^{new} + y_{im}^{old})] + \sum_{n=1}^{N}(Cs_n s_n) \tag{2}$$

Remark 3.7. Minimize total lead time

Total lead time includes lead time for procurement of new materials and that of recycled materials, as specified by (3):

$$\text{Min } Z_3 \simeq LT = \max\left\{\frac{(t_{ik}^{new}x_{ik}^{new})}{g_{ik}^{new}}, \frac{(t_{il}^{old}x_{il}^{old})}{g_{il}^{old}}\right\} \tag{3}$$

In (1)-(3), objective function is the fuzzified version of "=" and indicates the fuzzification of aspiration levels of DM. For each objective function in the proposed FMOLP model, the DM is assumed to have a fuzzy objective. (1)-(3) are fuzzy with imprecise aspiration levels, and variations in the judgment of DM over time are incorporated into the solutions to the fuzzy optimization problem. These fuzzy objectives require simultaneous optimization by a DM in the framework of a fuzzy aspiration levels.

Remark 3.8. Constraints

$$\sum_{n=1}^{N} s_n \theta_n' \geq D \tag{4}$$

$$s_n \leq (y_{im}^{new} + y_{im}^{old})\theta_m \quad \forall i, m, n \tag{5}$$

$$s_n \leq S_n^{max} \quad \forall n \tag{6}$$

$$y_{im}^{new} + y_{im}^{old} \leq M_m^{max} \quad \forall m \tag{7}$$

$$y_{im}^{new} + y_{im}^{old} \leq M \cdot W_m' \tag{8}$$

$$s_n \leq M \cdot W_m' \tag{9}$$

$$\sum_{k=1}^{K} x_{ik}^{new}(1 - e_{ik}^{new}) \geq y_{im}^{new} \quad \forall i, m \tag{10}$$

$$\sum_{l=1}^{L} x_{il}^{old}(1 - e_{il}^{old}) \geq y_{im}^{old} \quad \forall i, m \tag{11}$$

$$x_{ik}^{new} \leq U_{ik}^{max} \quad \forall i, k \tag{12}$$

$$x_{ik}^{new} \geq U_{ik}^{min} \quad \forall i, k \tag{13}$$

$$x_{il}^{old} \leq R_{il}^{max} \quad \forall i, l \tag{14}$$

$$x_{il}^{old} \geq R_{il}^{min} \quad \forall i, l \tag{15}$$

$$\sum_{i=1}^{I}\sum_{k=1}^{K}(Ck_{ik}x_{ik}^{new}) + \sum_{i=1}^{I}\sum_{l=1}^{L}(Cl_{il}x_{il}^{old}) + \sum_{i=1}^{I}\sum_{m=1}^{M}[Cm_{im}(y_{im}^{new} + y_{il}^{old})] +$$

$$\sum_{n=1}^{N}(Cs_n s_n) \leq CO_2^{max} \tag{16}$$

$$\frac{(t_{ik}^{new}x_{ik}^{new})}{g_{ik}^{new}} \leq Dd \quad \forall i, k \tag{17}$$

$$\frac{(t_{il}^{old}x_{il}^{old})}{g_{il}^{old}} \leq Dd \quad \forall i, l \tag{18}$$

$$x_{ik}^{new}, \ x_{il}^{old}, \ y_{im}^{new}, \ y_{im}^{old}, \ s_n \geq 0 \text{ and integer,} \quad \forall i, k, l, m, n \tag{19}$$

Note: (4) specifies that the total amount of non-defective products must satisfy demand. (5) specifies the number of non-defective products that are produced on a manufacturing machine in terms of the size of the lotsreleased by the assembling machine. (6) and (7) set the limits on the size of the lots released if the manufacturing and assembling machines do not exceed the maximum available manufacturing and assembling machines capacity. (8) and (9) present the constraints on the sizes of the lots released for manufacturing and assembling, respectively. (10) and (11) ensure that the total amount of new and recycledmaterials purchased equals or exceeds the sizes of released lots in all instances, respectively. (12) and (13) specify the quantity ordered from the vendor of new materials, which should be between the materials purchased upper and lower limits. (14) and (15) specify the quantity ordered from the vendor of recycled materials, which should be between the materials purchased upper and lower limits. (16) specifies the limit on annul total CO_2 emission. (17) and (18) ensure that the delivery time of purchased new materials and recycled materialsis not after the due date of the customer's order. (19) specifies constraints on decision variables.

Remark 3.9. Solving the FMOLP model

Wang and Liang [33] compared various membership functions; they found that the piecewise linear membership function has the most efficiency and flexibility. The original FMOLP model for solving previous problems can apply the piecewise linear membership function developed by Hannan[11]. Hannan proposed that piecewise linear membership functions should be specified to represent the fuzzy objectives, together with the fuzzy decision-making of Bellman and Zadeh[2].

z_1	$> X_{10}$	X_{10}	X_{11}	$\dots$	X_{1N}	$X_{1,N+1}$	$< X_{1,N+1}$
$f_1(z_1)$	0	0	q_{11}	$\dots$	q_{1N}	1.0	1.0
z_2	$> X_{20}$	X_{20}	X_{21}	$\dots$	X_{2N}	$X_{2,N+1}$	$< X_{2,N+1}$
$f_2(z_2)$	0	0	q_{21}	$\dots$	q_{2N}	1.0	1.0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
z_k	$> X_{k0}$	X_{k0}	X_{k1}	$\dots$	X_{kN}	$X_{k,N+1}$	$< X_{k,N+1}$
$f_k(z_k)$	0	0	q_{k1}	$\dots$	q_{kN}	1.0	1.0

where $0 \leq a_{kn} \leq 1.0$ and $q_{kn} \leq q_{k,n+1}$, $k = 1, 2, 3, \dots, K$; $n = 1, 2, 3, \dots, N$.

TABLE 1. Membership Function $f_k(z_k)$

Here, a minimum operator is used to integrate the fuzzy set and to transform the original FMOLP model into a single objective linear programming model. By adding an auxiliary variable L ($0 \leq L \leq 1$), all the objective functions in fuzzy multi-objective recyclable remanufacturing lot-sizing decision problem can be solved easily using the simple method of linear programming.

Remark 3.10. Solution Procedure for the FMOLP Model

The proposed interactive FMOLP method uses the following procedure to solve the fuzzy multi-objective recyclable remanufacturing lot-sizing decision problem; it is derived [11] as follows.

Assume that $X_{10}, X_{11}, \dots, X_{1,N+1}$ and $q_{11}, \dots, q_{1N}$ represent objective values and $f_k(z_k)$ scales, respectively.

Step1: Derive a membership function $f_k(z_k)$ for each objective function z_k ($k = 1, 2, 3$). Table 1 presents the piecewise linear membership functions, $f_1(z_1)$, $f_2(z_2)$ and $f_3(z_3)$.

Step2: Connect the points in the discrete membership function using line segments: $(z_k, f_k(_zk))$, for $k = 1, 2, 3$ (see Figure 1).

**Step3**: Formulate each piecewise linear membership function $f_k(z_k)$ as follows.

$$f_k(z_k) = \sum_{n=1}^{N} \alpha_{kn} |z_k - X_{kn}| + \beta_k z_k + \gamma_k, \quad k = 1, 2, 3, \dots, K \tag{20}$$

where

$$\alpha_{kn} = \frac{t_{k,n+1} - t_{kn}}{2}, \quad \beta_k = \frac{t_{k,N+1} + t_{k1}}{2}, \quad \gamma_k = \frac{S_{k,N+1} + S_{k1}}{2}$$

For each segment $X_{k,r-1} \leq L \leq X_{kr}$, assume that $f_k(z_k) = t_{kr} z_k + S_{kr}$ where t_{kr} represents the slope, and let S_{kr} represent the y-intercept of the line segment at $[X_{k,r-1}, X_{kr}]$ in the piecewise linear membership function. Hence,

$$f_1(z_1) = \left(\frac{t_{12} - t_{11}}{2}\right) |z_1 - X_{11}| + \left(\frac{t_{13} - t_{12}}{2}\right) |z_1 - X_{12}| + \dots$$

$$+ \left(\frac{t_{1,N+1} - t_{1N}}{2}\right) |z_1 - X_{1N}| + \left(\frac{t_{1,N+1} + t_{11}}{2}\right) z_1 + \frac{S_{1,N+1} + S_{11}}{2} \tag{21}$$
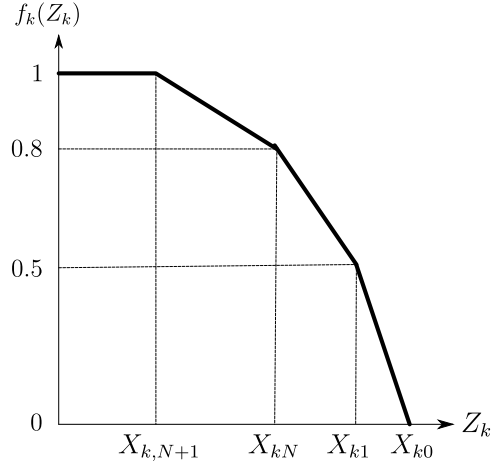
FIGURE 1. Piecewise Membership Function $f_k(z_k)$

where $t_{11} = \left(\dfrac{q_{11} - 0}{X_{11} - X_{10}}\right), t_{12} = \left(\dfrac{q_{12} - q_{11}}{X_{12} - X_{11}}\right), \dots, t_{1,N+1} = \left(\dfrac{1.0 - q_{1N}}{X_{1,N+1} - X_{1N}}\right)$
and $S_{1,N+1}$ denotes between the line segment from X_{1N} to $X_{1,N+1}$ and the vertical line, which is given by $f_1(z_1) = t_{1r}z_1 + S_{1r}$.

$$f_2(z_2) = \left(\frac{t_{22} - t_{21}}{2}\right)|z_2 - X_{21}| + \left(\frac{t_{23} - t_{22}}{2}\right)|z_2 - X_{22}| + \dots$$

$$+ \left(\frac{t_{2,N+1} - t_{2N}}{2}\right)|z_2 - X_{2N}| + \left(\frac{t_{2,N+1} + t_{21}}{2}\right)z_2 + \frac{S_{2,N+1} + S_{21}}{2} \qquad (22)$$

where $t_{21} = \left(\dfrac{q_{21} - 0}{X_{21} - X_{20}}\right), t_{22} = \left(\dfrac{q_{22} - q_{21}}{X_{22} - X_{21}}\right), \dots, t_{2,N+1} = \left(\dfrac{1.0 - q_{2N}}{X_{2,N+1} - X_{2N}}\right)$
and $S_{2,N+1}$ denotes between the line segment from X_{2N} to $X_{2,N+1}$ and the vertical line, which is given by $f_2(z_2) = t_{2r}z_2 + S_{2r}$.

$$f_3(z_3) = \left(\frac{t_{32} - t_{31}}{2}\right)|z_3 - X_{31}| + \left(\frac{t_{33} - t_{32}}{2}\right)|z_3 - X_{32}| + \dots$$

$$+ \left(\frac{t_{3,N+1} - t_{3N}}{2}\right)|z_3 - X_{3N}| + \left(\frac{t_{3,N+1} + t_{31}}{2}\right)z_3 + \frac{S_{3,N+1} + S_{31}}{2} \qquad (23)$$

where $t_{31} = \left(\dfrac{q_{31} - 0}{X_{31} - X_{30}}\right), t_{32} = \left(\dfrac{q_{32} - q_{31}}{X_{32} - X_{31}}\right), \dots, t_{3,N+1} = \left(\dfrac{1.0 - q_{3N}}{X_{3,N+1} - X_{3N}}\right)$
and $S_{3,N+1}$ denotes between the line segment from X_{3N} to $X_{3,N+1}$ and the vertical line, which is given by $f_3(z_3) = t_{3r}z_3 + S_{3r}$.

Step4: Introduce the nonnegative deviational variables d_{kn}^+ and d_{kn}^-

$$\left\{\sum_{i=1}^{I}\sum_{k=1}^{K}(P_{ik}^{new}\,O_k^{new}) + (pc_{ik}^{new}x_{ik}^{new}) + \sum_{i=1}^{I}\sum_{l=1}^{L}(P_{il}^{old}O_l^{old} + pc_{il}^{old}x_{il}^{old})\right.$$

$$+ \sum_{i=1}^{I} \sum_{m=1}^{M} [\alpha_m W_m + \beta_m(y_{im}^{new} + y_{im}^{old}) + nq_m(y_{im}^{new} + y_{im}^{old})(1 - \theta_m)]$$

$$+ \sum_{n=1}^{N} [\alpha'_m W' + \beta'_n s_n + nq'_n s_n(1 - \theta'_n)] \Bigg\} + d_{1n}^{-} - d_{1n}^{+} = X_{1n},$$

$$n = 1, 2, \ldots, N \tag{24}$$

$$\Bigg\{ \sum_{i=1}^{I} \sum_{k=1}^{K} (Ck_{ik} x_{ik}^{new}) + \sum_{i=1}^{I} \sum_{l=1}^{L} (Cl_{il} x_{il}^{old}) + \sum_{i=1}^{I} \sum_{m=1}^{M} [Cm_{im}(y_{im}^{new} + y_{im}^{old})]$$

$$+ \sum_{n=1}^{N} (Cs_n s_n) \Bigg\} + d_{2n}^{-} - d_{2n}^{+} = X_{2n}, n = 1, 2, \ldots, N \tag{25}$$

$$\{LT\} + d_{3n}^{-} - d_{3n}^{+} = X_{3n}, \qquad n = 1, 2, \ldots, N \tag{26}$$

where d_{kn}^{+} and d_{kn}^{-} denote the negative and positive deviational variables, respectively, at the jth point; X_{kn} denotes the value of the kth objective function at the nth point.

Step5: Substitute equations (24)-(26) into equations (21)-(23), respectively. Here the substitution yielding: d_{kn}^{-}

$$f_1(z_1) = \left(\frac{t_{12} - t_{11}}{2} \right) (d_{11}^{-} - d_{11}^{+}) + \left(\frac{t_{13} - t_{12}}{2} \right) (d_{12}^{-} - d_{12}^{+}) + \ldots$$

$$+ \left(\frac{t_{1,N+1} - t_{1N}}{2} \right) (d_{1N}^{-} - d_{1N}^{+}) + \left(\frac{t_{1,N+1} - t_{11}}{2} \right)$$

$$\Bigg\{ \sum_{i=1}^{I} \sum_{k=1}^{K} (P_{ik}^{new} \, O_k^{new} + pc_{ik}^{new} x_{ik}^{new}) + \sum_{i=1}^{I} \sum_{l=1}^{L} (P_{il}^{old} O_l^{old} + pc_{il}^{old} x_{il}^{old})$$

$$+ \sum_{i=1}^{I} \sum_{m=1}^{M} [\alpha_m W_m + \beta_m(y_{im}^{new} + y_{im}^{old}) + nq_m(y_{im}^{new} + y_{im}^{old})(1 - \theta_m)]$$

$$+ \sum_{n=1}^{N} [\alpha'_m W' + \beta'_n s_n + nq'_n s_n(1 - \theta'_n)] + \Bigg\} \frac{S_{1,N+1} + S_{11}}{2} \tag{27}$$

$$f_2(z_2) = \left(\frac{t_{22} - t_{21}}{2} \right) (d_{21}^{-} - d_{21}^{+}) + \left(\frac{t_{23} - t_{22}}{2} \right) (d_{22}^{-} - d_{22}^{+}) + \ldots$$

$$+ \left(\frac{t_{2,N+1} - t_{2N}}{2} \right) (d_{2N}^{-} - d_{2N}^{+}) + \left(\frac{t_{2,N+1} - t_{21}}{2} \right) \Bigg\{ \sum_{i=1}^{I} \sum_{k=1}^{K} (Ck_{ik} x_{ik}^{new})$$

$$+ \sum_{i=1}^{I} \sum_{l=1}^{L} (Cl_{il} x_{il}^{old}) + \sum_{i=1}^{I} \sum_{m=1}^{M} [Cm_{im}(y_{im}^{new} + y_{im}^{old})] + \sum_{n=1}^{N} (Cs_n s_n) \Bigg\}$$

$$+ \frac{S_{2,N+1} + S_{21}}{2} \tag{28}$$

$$f_3(z_3) = \left(\frac{t_{32} - t_{31}}{2}\right)(d_{31}^- - d_{31}^+) + \left(\frac{t_{33} - t_{32}}{2}\right)(d_{32}^- - d_{32}^+) + \dots$$

$$+ \left(\frac{t_{3,N+1} - t_{3N}}{2}\right)(d_{3N}^- - d_{3N}^+) + \left(\frac{t_{3,N+1} - t_{31}}{2}\right)\{LT\} + \frac{S_{3,N+1} + S_{31}}{2} \quad (29)$$

Step6: Introduce the auxiliary variable $L(0 \leq\leq L \leq 1)$, and transform the original FMOLP model for the recyclable remanufacturing planning problem into an equivalent crisp LP form by applying the minimum operator to aggregate fuzzy sets. The resulting equivalent crisp LP form that is used to solve the fuzzy multi-objective recyclable remanufacturing lot-sizing decision problem is formulated as follows.

max L

s.t.

$$L \leq \left(\frac{t_{12} - t_{11}}{2}\right)(d_{11}^- - d_{11}^+) + \left(\frac{t_{13} - t_{12}}{2}\right)(d_{12}^- - d_{12}^+) + \dots$$

$$+ \left(\frac{t_{1,N+1} - t_{1N}}{2}\right)(d_{1N}^- - d_{1N}^+) + \left(\frac{t_{1,N+1} - t_{11}}{2}\right)$$

$$\left\{\sum_{i=1}^{I}\sum_{k=1}^{K}(P_{ik}^{new} O_k^{new} + pc_{ik}^{new} x_{ik}^{new}) + \sum_{i=1}^{I}\sum_{l=1}^{L}(P_{il}^{old}O_l^{old} + pc_{il}^{old}x_{il}^{old})\right.$$

$$+ \sum_{i=1}^{I}\sum_{m=1}^{M}[\alpha_m W_m + \beta_m(y_{im}^{new} + y_{im}^{old}) + nq_m(y_{im}^{new} + y_{im}^{old})(1 - \theta_m)]$$

$$\left. + \sum_{n=1}^{N}[\alpha_m' W' + \beta_n' s_n + nq_n' s_n(1 - \theta_n')]\right\} + \frac{S_{1,N+1} + S_{11}}{2}$$

$$L \leq \left(\frac{t_{22} - t_{21}}{2}\right)(d_{21}^- - d_{21}^+) + \left(\frac{t_{23} - t_{22}}{2}\right)(d_{22}^- - d_{22}^+) + \dots$$

$$+ \left(\frac{t_{2,N+1} - t_{2N}}{2}\right)(d_{2N}^- - d_{2N}^+) + \left(\frac{t_{2,N+1} - t_{21}}{2}\right)$$

$$\left\{\sum_{i=1}^{I}\sum_{k=1}^{K}(Ck_{ik}x_{ik}^{new}) + \sum_{i=1}^{I}\sum_{l=1}^{L}(Cl_{il}x_{il}^{old}) + \sum_{i=1}^{I}\sum_{m=1}^{M}[Cm_{im}(y_im^{new} + y_{im}^{old})]\right.$$

$$\left. + \sum_{n=1}^{N}(Cs_n s_n)\right\} + \frac{S_{2,N+1} + S_{21}}{2}$$

$$L \leq \left(\frac{t_{32} - t_{31}}{2}\right)(d_{31}^- - d_{31}^+) + \left(\frac{t_{33} - t_{32}}{2}\right)(d_{32}^- - d_{32}^+) + \dots$$

$$+ \left(\frac{t_{3,N+1} - t_{3N}}{2}\right)(d_{3N}^- - d_{3N}^+) + \frac{t_{3,N+1} - t_{31}}{2}\{LT\} + \frac{S_{3,N+1} + S_{31}}{2}$$

$$\left\{ \sum_{i=1}^{I} \sum_{k=1}^{K} (P_{ik}^{new} O_k^{new} + pc_{ik}^{new} x_{ik}^{new}) + \sum_{i=1}^{I} \sum_{l=1}^{L} (P_{il}^{old} O_l^{old} + pc_{il}^{old} x_{il}^{old}) \right.$$

$$+ \sum_{i=1}^{I} \sum_{m=1}^{M} [\alpha_m W_m + \beta_m (y_{im}^{new} + y_{im}^{old}) + nq_m (y_{im}^{new} + y_{im}^{old})(1 - \theta_m)]$$

$$\left. + \sum_{n=1}^{N} [\alpha'_m W' + \beta'_n s_n + nq'_n s_n (1 - \theta'_n)] \right\} + d_{1n}^- - d_{1n}^+ = X_{1n}$$

$$\left\{ \sum_{i=1}^{I} \sum_{k=1}^{K} (Ck_{ik} x_{ik}^{new}) + \sum_{i=1}^{I} \sum_{l=1}^{L} (Cl_{il} x_{il}^{old}) \right.$$

$$+ \sum_{i=1}^{I} \sum_{m=1}^{M} [Cm_{im}(y_{im}^{new} + y_{im}^{old})] + \sum_{n=1}^{N} (Cs_n s_n) \left. \right\} + d_{2n}^- - d_{2n}^+ = X_{2n}$$

$$\{LT\} + d_{3n}^- + d_{3n}^+ = X_{3n}$$

$$0 \le L \le 1$$

Equations (4)-(18)

$$d_{1n}^-, d_{1n}^+, d_{2n}^-, d_{2n}^+, d_{3n}^-, d_{3n}^+, x_{ik}^{new}, x_{il}^{old}, y_{im}^{new}, y_{im}^{old}, s_n \ge 0 \text{ and integer}, \forall i, k, l, m, n$$

4. Model Implementation

Definition 4.1. Case Descriptions. A test model is applied to a curtain and shutter components company in southern Taiwan. The firm specializes in producing vertical blinds and shutter components, and exports all over the world, including to the U.S., Mexico, Australia, China, India, and Malaysia. In procurement and the release of lots production planning, considered factors include the defective rates of manufacturing and assembling, material processing times and power consumption; the objectives are the minimization of total production costs, CO_2 emissions, and lead time. The FMOLP model herein enables the DM to determine the optimal lot size that is released in a fuzzy environment. Case-related variables include the size of the quantity ordered (6500 sets), the projected CO_2 emissions, which are limited to 3,000,000kg, the due date (15 days), the three vendors of new materials and recycling materials, the three manufacturing machines and the two assembling machines that these numbers of machines and vendors are the variables. Table 2 presents the data of manufacturing and assembling machines. Table 3 and Table 4 present data concerning the suppliers of new material and the vendors of recycled material.

Example 4.2. Solution the FMOLP Problem Procedure. This section evaluates the accuracy and performance of the proposed model in efficient production planning in recyclable remanufacturing systems, by considering a real-world test case in southern Taiwan. The problem in this case is solved by using the procedure that is described in Remark 3.10.

Manufacturing machines	m_1	m_2	m_3	Assembling machines	n_1	n_2
α_m	1000	800	12000	α'_n	1500	1800
β_m	30	40	25	β'_n	50	45
Cm_{im}	20	25	15	Cs_n	60	80
n_{qm}	15	18	13	nq'_n	50	40
θ_m	0.95	0.99	0.90	θ'_n	0.8	0.8
M	40000	50000	45000	s_n^{max}	5000	6500

TABLE 2. Parameters Related to Manufacturing and Assembling Machines

Suppliers(k)	1			2		
components(i)	1	2	3	1	2	3
Ck_{ik}	20	25	28	30	40	45
e_{ik}^{new}	0.01	0.015	0.008	0.008	0.012	0.005
g_{ik}^{new}	1000	2000	1500	1500	3000	2000
P_{ik}^{new}	700	700	700	900	900	900
pc_{ik}^{new}	100	150	180	110	145	160
t_{ik}^{new}	1.5	2	3	1.5	2	3
U_{ik}^{max}	2000	4000	3000	2800	5000	4000
U_{ik}^{min}	500	500	500	800	800	800

TABLE 3. Costs, CO_2 Emissions, Lead Time and Capacity Constraints Associated with Procurement of New Materials From Suppliers

Recyclers(l)	1			2		
components(i)	1	2	3	1	2	3
Cl_{il}	40	50	45	50	65	60
e_{il}^{old}	5	7	8	3	4	6
g_{il}^{old}	800	1600	1300	1200	1800	1500
P_{il}^{old}	500	500	500	700	700	700
pc_{il}^{old}	80	120	150	75	110	130
t_{il}^{old}	4	6	8	3	6	9
R_{il}^{max}	2500	4500	4000	3000	6000	5000
R_{il}^{min}	1000	1000	1000	800	800	800

TABLE 4. Costs, CO_2 Emissions, Lead Time and Capacity Constraints Associated with Procurement of Recycled Materials From Recyclers

Imprecise production planning decisions for recoverable remanufacturing can be formulated as a fuzzy multi-objective problem using piecewise linear functions [11], as follows.

Step1: Use the conventional LP model to obtain initial solutions for each objective function.

z_1	>4300000	4300000	4100000	3800000	3600000	<3600000
$f_1(z_1)$	0	0	0.5	0.8	1	1
z_2	>2300000	2300000	2200000	2100000	1900000	<1900000
$f_2(z_2)$	0	0	0.5	0.8	1	1
z_3	>15	15	13	11	6	<6
$f_3(z_3)$	0	0	0.5	0.8	1	1

TABLE 5. Piecewise Membership Functions

The results obtained are z_1=\$3,670,504, z_2=1,972,345 kg, and z_3=6.15 days. Then, formulate the FMOLP model using these initial solutions. Table 5 present the piecewise linear membership functions of the proposed model.

Step2: Use Table 2 to plot the piecewise linear membership functions $(z_1, f_1(z_1))$, $(z_2, f_2(z_2))$ and $(z_3, f_3(z_3))$ (Figs.2-4). The curves of the fuzzy objective should be almost linearly related to the Hannan membership function.
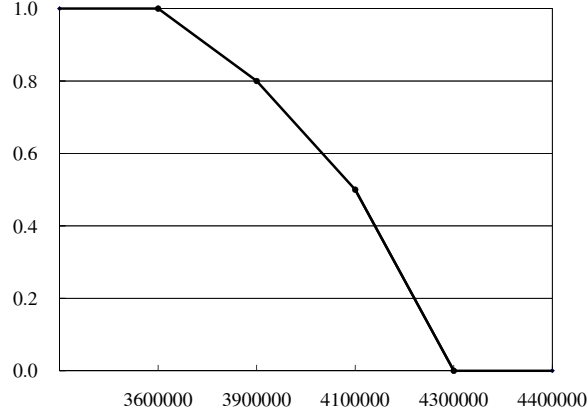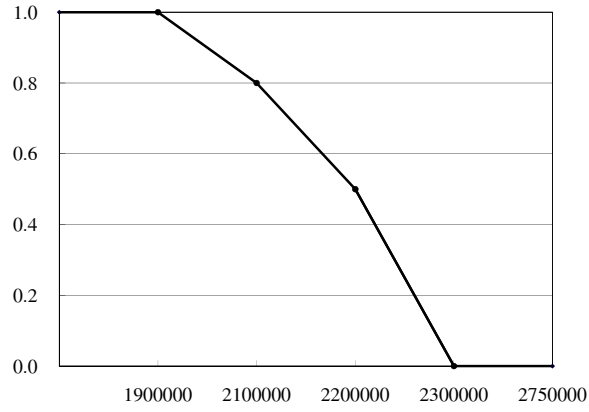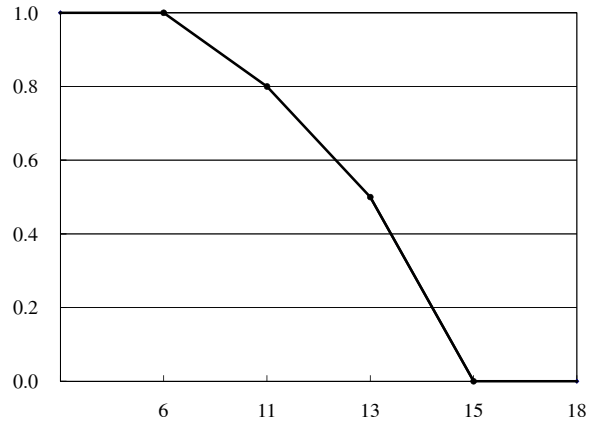

FIGURE 2. Piecewise Linear Membership Function $(z_1, f_1(z_1))$

Step3: Express the piecewise linear membership functions in the following form.

$$f_1(z_1) = -0.0000005\,|z_1 - 3900000| - 0.00000042\,|z_1 - 4100000| \\ -0.00000158 \cdot z_1 + 7.075$$

$$f_2(z_2) = -0.000001\,|z_2 - 2100000| - 0.000001\,|z_2 - 2200000| \\ -0.000003 \cdot z_2 + 7.2$$

$$f_3(z_3) = -0.055|z_3 - 11| - 0.055|z_3 - 13| - 0.145 \cdot z_3 + 2.495$$

FIGURE 3. Piecewise Linear Membership Function $(z_2, f_2(z_2))$

FIGURE 4. Piecewise Linear Membership Function $(z_3, f_3(z_3))$

Step4: Introduce the nonnegative deviational variables.

$$\sum_{i=1}^{I}\sum_{k=1}^{K}(P_{ik}^{new}O_{ik}^{new} + pc_{ik}^{new}x_{ik}^{new}) + \sum_{i=1}^{I}\sum_{l=1}^{L}(P_{il}^{old}O_{il}^{old} + pc_{il}^{old}x_{il}^{old})$$

$$+\sum_{i=1}^{I}\sum_{m=1}^{M}[\alpha_m W_m + \beta\left(y_{im}^{new} + y_{im}^{old}\right) + nq_m\left(y_{im}^{new} + y_{im}^{old}\right)(1-\theta)]$$

$$+\sum_{n=1}^{N}[\alpha_n' W_n' + \beta_n' s_n + nq_n' s_n\left(1-\theta_n'\right)] + d_{11}^- - d_{11}^+ = 3900000$$

$$\sum_{i=1}^{I}\sum_{k=1}^{K}(P_{ik}^{new}O_{ik}^{new} + pc_{ik}^{new}x_{ik}^{new}) + \sum_{i=1}^{I}\sum_{l=1}^{L}(P_{il}^{old}O_{il}^{old} + pc_{il}^{old}x_{il}^{old})$$

$$+\sum_{i=1}^{I}\sum_{m=1}^{M}\left[\alpha_m W_m + \beta\left(y_{im}^{new}+y_{im}^{old}\right)+nq_m\left(y_{im}^{new}+y_{im}^{old}\right)(1-\theta)\right]$$

$$+\sum_{n=1}^{N}\left[\alpha_n' W_n' + \beta_n' s_n + nq_n' s_n\left(1-\theta_n'\right)\right]+d_{12}^{-}-d_{12}^{+}=4100000$$

$$\sum_{i=1}^{I}\sum_{k=1}^{K}Ck_{ik}x_{ik}^{new}+\sum_{i=1}^{I}\sum_{l=1}^{L}Cl_{il}x_{il}^{old}+\sum_{i=1}^{I}\sum_{m=1}^{M}\left[Cm_{im}(y_{im}^{new}+y_{im}^{old})\right]+\sum_{n=1}^{N}\left(Cs_n s_n\right)$$

$$+d_{21}^{-}-d_{21}^{+}=2100000$$

$$\sum_{i=1}^{I}\sum_{k=1}^{K}Ck_{ik}x_{ik}^{new}+\sum_{i=1}^{I}\sum_{l=1}^{L}Cl_{il}x_{il}^{old}+\sum_{i=1}^{I}\sum_{m=1}^{M}\left[Cm_{im}(y_{im}^{new}+y_{im}^{old})\right]+\sum_{n=1}^{N}\left(Cs_n s_n\right)$$

$$+d_{22}^{-}-d_{22}^{+}=2200000$$

$$\left\{\frac{(t_{ik}^{new}x_{ik}^{new})}{g_{ik}^{new}},\frac{(t_{il}^{old}x_{il}^{old})}{g_{il}^{old}}\right\}+d_{31}^{-}-d_{31}^{+}=11$$

$$\left\{\frac{(t_{ik}^{new}x_{ik}^{new})}{g_{ik}^{new}},\frac{(t_{il}^{old}x_{il}^{old})}{g_{il}^{old}}\right\}+d_{32}^{-}-d_{32}^{+}=13$$

Step5: Formulate the piecewise linear equationseach membership function,where $f_k(z_k), k=1,2,3$.

$$f_1(z_1)=-0.0000005(d_{11}^{-}-d_{11}^{+})-0.00000042(d_{12}^{-}-d_{12}^{+})-0.00000158$$

$$\left\{\sum_{i=1}^{I}\sum_{k=1}^{K}P_{ik}^{new}O_{ik}^{new}+pc_{ik}^{new}x_{ik}^{new}+\sum_{i=1}^{I}\sum_{l=1}^{L}P_{il}^{old}O_{il}^{old}+pc_{il}^{old}x_{il}^{old}\right.$$

$$\sum_{i=1}^{I}\sum_{mk=1}^{M}\alpha_m W_m+\beta\left(y_{im}^{new}+y_{im}^{old}\right)+nq_m\left(y_{im}^{new}+y_{im}^{old}\right)(1-\theta)$$

$$\left.+\sum_{n=1}^{N}\alpha_n' W_n'+\beta_n' s_n+nq_n' s_n\left(1-\theta_n'\right)\right\}+7.075$$

$$f_2(z_2)=0.000001(d_{21}^{-}-d_{21}^{+})-0.000001(d_{22}^{-}-d_{22}^{+})-0.000003$$

$$\left\{\sum_{i=1}^{I}\sum_{k=1}^{K}Ck_{ik}x_{ik}^{new}+\sum_{i=1}^{I}\sum_{l=1}^{L}Cl_{il}x_{il}^{old}+\sum_{i=1}^{I}\sum_{m=1}^{M}\left[Cm_{im}(y_{im}^{new}+y_{im}^{old})\right]\right.$$

$$\left.+\sum_{n=1}^{N}Cs_n s_n\right\}+7.2$$

$$f_3(z_3)=-0.055(d_{31}^{-}-d_{31}^{+})-0.055(d_{32}^{-}-d_{32}^{+})$$

$$-0.145\left\{\frac{(t_{ik}^{new}x_{ik}^{new})}{g_{ik}^{new}},\frac{(t_{il}^{old}x_{il}^{old})}{g_{il}^{old}}\right\}+2.495$$

Step6: Introduce the auxiliary variable $L(0 \leq L \leq 1)$, and transform the original FMOLP model for the recyclable remanufacturing planning problem into an equivalent crisp LP form by applying the minimum operator to aggregate fuzzy sets. The resulting equivalent crisp LP form that is used to solve the fuzzy multi-objective recyclable remanufacturing lot-sizing problem is formulated as follows.

Max L

s.t.

$$L \leq -0.0000005(d_{11}^- - d_{11}^+) - 0.00000042(d_{12}^- - d_{12}^+) - 0.00000158$$

$$\left\{ \sum_{i=1}^{I} \sum_{k=1}^{K} (P_{ik}^{new} O_{ik}^{new} + pc_{ik}^{new} x_{ik}^{new}) + \sum_{i=1}^{I} \sum_{l=1}^{L} (P_{il}^{old} O_{il}^{old} + pc_{il}^{old} x_{il}^{old}) \right.$$

$$+ \sum_{i=1}^{I} \sum_{mk=1}^{M} [\alpha_m W_m + \beta \left(y_{im}^{new} + y_{im}^{old} \right) + nq_m \left(y_{im}^{new} + y_{im}^{old} \right) (1 - \theta)]$$

$$\left. + \sum_{n=1}^{N} [\alpha_n' W_n' + \beta_n' s_n + nq_n' s_n (1 - \theta_n')] \right\} + 7.075$$

$$L \leq 0.000001(d_{21}^- - d_{21}^+) - 0.000001(d_{22}^- - d_{22}^+) - 0.000003$$

$$\left\{ \sum_{i=1}^{I} \sum_{k=1}^{K} (Ck_{ik} x_{ik}^{new}) + \sum_{i=1}^{I} \sum_{l=1}^{L} (Cl_{il} x_{il}^{old}) + \sum_{i=1}^{I} \sum_{m=1}^{M} [Cm_{im}(y_{im}^{new} + y_{im}^{old})] \right.$$

$$\left. + \sum_{n=1}^{N} (Cs_n s_n) \right\} + 7.2$$

$$L \leq -0.055(d_{31}^- - d_{31}^+) - 0.055(d_{32}^- - d_{32}^+) - 0.145 \left\{ \frac{(t_{ik}^{new} x_{ik}^{new})}{g_{ik}^{new}}, \frac{(t_{il}^{old} x_{il}^{old})}{g_{il}^{old}} \right\} + 2.495$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} (P_{ik}^{new} O_{ik}^{new} + pc_{ik}^{new} x_{ik}^{new}) + \sum_{i=1}^{I} \sum_{l=1}^{L} (P_{il}^{old} O_{il}^{old} + pc_{il}^{old} x_{il}^{old})$$

$$+ \sum_{i=1}^{I} \sum_{mk=1}^{M} [\alpha_m W_m + \beta \left(y_{im}^{new} + y_{im}^{old} \right) + nq_m \left(y_{im}^{new} + y_{im}^{old} \right) (1 - \theta)]$$

$$+ \sum_{n=1}^{N} [\alpha_n' W_n' + \beta_n' s_n + nq_n' s_n (1 - \theta_n')] + d_{11}^- - d_{11}^+ = 3900000$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} (P_{ik}^{new} O_{ik}^{new} + pc_{ik}^{new} x_{ik}^{new}) + \sum_{i=1}^{I} \sum_{l=1}^{L} (P_{il}^{old} O_{il}^{old} + pc_{il}^{old} x_{il}^{old})$$

$$+ \sum_{i=1}^{I} \sum_{mk=1}^{M} [\alpha_m W_m + \beta \left(y_{im}^{new} + y_{im}^{old} \right) + nq_m \left(y_{im}^{new} + y_{im}^{old} \right) (1 - \theta)]$$

k, l, m		1			2		
i, m		1	2	3	1	2	3
purchasing	x_{ik}^{new}	2000	3818	1426	1180	800	3878
quantity	x_{il}^{old}	2021	2493	1641	3000	800	1685
Lot-sizing	Manufacturing machines	$y_{11}^{new} = 3150, y_{22}^{new} = 4551, y_{33}^{new} = 5273,$ $y_{11}^{old} = 4976, y_{22}^{old} = 3246, y_{33}^{old} = 3304$					
	Assembling machines	$S_1 = 1219, S_2 = 6500$					
Objective value		$z_1 = 3,887,702, z_2 = 2,078,883, z_3 = 10.11$					
L (%)		82.11%					

TABLE 6. Recoverable Remanufacturing Lot-Sizing Plangenerated Using Proposed FMOLP Method

$$+ \sum_{n=1}^{N} [\alpha_n' W_n' + \beta_n' s_n + n q_n' s_n (1 - \theta_n')] + d_{12}^- - d_{12}^+ = 4100000$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} (Ck_{ik} x_{ik}^{new}) + \sum_{i=1}^{I} \sum_{l=1}^{L} (Cl_{il} x_{il}^{old}) + \sum_{i=1}^{I} \sum_{m=1}^{M} [Cm_{im}(y_{im}^{new} + y_{im}^{old})]$$

$$+ \sum_{n=1}^{N} Cs_n s_n + d_{21}^- - d_{21}^+ = 2100000$$

$$\sum_{i=1}^{I} \sum_{k=1}^{K} (Ck_{ik} x_{ik}^{new}) + \sum_{i=1}^{I} \sum_{l=1}^{L} (Cl_{il} x_{il}^{old}) + \sum_{i=1}^{I} \sum_{m=1}^{M} [Cm_{im}(y_{im}^{new} + y_{im}^{old})]$$

$$+ \sum_{n=1}^{N} Cs_n s_n + d_{22}^- - d_{22}^+ = 2200000$$

$$\left\{ \frac{(t_{ik}^{new} x_{ik}^{new})}{g_{ik}^{new}}, \frac{(t_{il}^{old} x_{il}^{old})}{g_{il}^{old}} \right\} + d_{31}^- - d_{31}^+ = 11$$

$$\left\{ \frac{(t_{ik}^{new} x_{ik}^{new})}{g_{ik}^{new}}, \frac{(t_{il}^{old} x_{il}^{old})}{g_{il}^{old}} \right\} + d_{32}^- - d_{32}^+ = 13$$

$$0 \le L \le 1$$

Equations (4)-(18)

$d_{1n}^-, d_{1n}^+, d_{2n}^-, d_{2n}^+, d_{3n}^-, d_{3n}^+, x_{ik}^{new}, x_{il}^{old}, y_{im}^{new}, y_{im}^{old}, s_n \ge 0$ and integer, $\forall i, k, l, m, n$

The linear programming software program LINGO version 11.0 is utilized to apply the ordinary single-objective LP model for fuzzy multi-objectives in recyclable remanufacturing lot-sizing problems, yielding $z_1 = \$3,887,702$, $z_2 = 2,078,883$ kg, and $z_3 = 10.11$ days. The overall satisfaction with the determined goal values is 82.11%. Table 6 provides the recoverable remanufacturing lot-sizing decisions.

Example 4.3. Sensitivity Analysis. The sensitivity of the decision parameters in the FMOLP modelis analyzed for numerical examples that involve two scenarios, which are as follows.

Item		Run 1	Run 2	Run 3	Run 4	Run 5
		-6%	-3%	0%	3%	6%
Demand quantity		6110	6305	6500	6695	6890
x_{ik}^{new}	x_{11}^{new}	2000	2000	2000	2000	2000
	x_{12}^{new}	1879	1304	1180	1094	1218
	x_{21}^{new}	3200	3652	3818	3874	3941
	x_{22}^{new}	800	800	800	800	800
	x_{31}^{new}	2000	1666	1426	1653	1889
	x_{32}^{new}	4000	3999	3878	3355	2977
x_{il}^{old}	x_{11}^{old}	1268	1637	2021	2368	2500
	x_{12}^{old}	2535	3000	3000	2998	3000
	x_{21}^{old}	1689	2164	2493	2688	2871
	x_{22}^{old}	1721	1045	800	800	800
	x_{31}^{old}	1030	1329	1641	1924	2128
	x_{32}^{old}	1056	1364	1685	1972	2183
y_{im}^{new}	y_{11}^{new}	3843	3273	3150	3065	3188
	y_{22}^{new}	3942	4387	4551	4606	4672
	y_{33}^{new}	5964	5631	5273	4978	4836
y_{im}^{old}	y_{11}^{old}	3770	4569	4976	3438	5451
	y_{22}^{old}	3364	3164	3246	5251	3618
	y_{33}^{old}	2072	2675	3304	3870	4283
s_n	s_1	744	975	1223	1471	1719
	s_2	6488	6500	6496	6492	6488
z_1		3,687,871	3,785,634	3,887,702	3,996,565	4,115,615
z_2		1,946,995	2,011,518	2,078,883	2,143,949	2,205,207
z_3		6.34	8.16	10.11	11.84	13.10
$L(\%)$		95.30%	88.85%	82.11%	66.82%	47.40%

TABLE 7. Analysis of Sensitivity of Decision-Making Variables to Demand

Remark 4.4. Scenario 1: effect of demand on satisfaction level L. In this scenario, quantity demanded is adjusted from small to large across five runs, in which the variable is adjusted the base value by -6%, -3%, 0%, +3% and +6%.Run 3 provides the basis for the scenario, so the other runs represent a two-step variation in both increasing (Run 4 and Run 5) and decreasing (Run 1 and Run 2) directions, where each step is 3%.Table 7 presents the effect of demand on the decision-making variables, objectives and satisfaction level L.

The results of the sensitivity analysis demonstrate that quantity demanded and the total production costs, the total CO_2 emissions and the total lead time influence the objective functions (z_1, z_2, z_3) and the degree of satisfaction (L). The total production cost (z_1) is increased from \$3,687,871 to \$4,115,615;the total CO_2 emissions (z_2) are increased from 1,946,995 kg to 2,205,207kg; the total lead time (z_3) is increased from 6.34 days to 13.10 days, and the satisfaction level L falls from 95.30% to 47.40% from Run 1 to Run 5. Therefore, as demand increases, production costs, CO_2 emissions, and lead time also increase, resulting in a corresponding increase in shortage risks and a decrease in the overall satisfaction level. Accordingly, firms should optimize order quantity to reduce shortage risks.

Remark 4.5. Scenario 2: effect of due date on satisfaction level L. In this scenario, the demanded quantity is increased from small to large in 5 runs by -8%, -6%, -3%, 0%, +3% from a base value. Run 4 sets the base case for the scenario with the originaldue date. Table 8 presents the effect of due date on decision variables, objectives and satisfaction level L.

The results of the sensitivity analysis reveal that when due date is brought forward, to ensure that order deadlines are met, the company must use emergency procurement, require overtime work or procure new materials, increasing total production cost from \$3,887,702 to \$3915611, and satisfaction level L to decline from 82.11% to 78.96%.

Item		Run 1	Run 2	Run 3	Run 4	Run 5
		-8%	-6%	3%	0%	3%
Due data value		7	9	12	15	18
x_{ik}^{new}	x_{11}^{new}	2000	2000	2000	2000	2000
	x_{12}^{new}	1999	1400	1179	1180	1181
	x_{21}^{new}	1686	3493	3818	3818	3818
	x_{22}^{new}	2252	800	800	800	800
	x_{31}^{new}	2328	1696	1427	1426	1422
	x_{32}^{new}	4000	3973	3876	3878	3884
x_{il}^{old}	x_{11}^{old}	1400	1800	022	2021	2021
	x_{12}^{old}	2800	3000	3000	3000	2999
	x_{21}^{old}	1866	2400	2493	2493	2493
	x_{22}^{old}	2100	1217	800	800	800
	x_{31}^{old}	1137	1462	1642	1641	1642
	x_{32}^{old}	1166	1500	1685	1685	1682
y_{im}^{new}	y_{11}^{new}	3963	3368	3149	3150	3151
	y_{22}^{new}	3885	4231	4551	4551	4551
	y_{33}^{new}	6289	5635	5272	5273	5275
y_{im}^{old}	y_{11}^{old}	4163	4758	4977	4976	4975
	y_{22}^{old}	3912	3566	4551	3246	4551
	y_{33}^{old}	2288	2942	3305	3305	5275
s_n	s_1	1223	1223	1223	1223	1223
	s_2	6496	6496	6496	6496	6496
z_1		3,915,611	3,894,931	3,887,681	3,887,702	3,887,766
z_2		2,103,469	2,083,653	2,078,876	2,078,883	2,078,886
z_3		7.00	9.00	10.11	10.11	10.12
$L(\%)$		78.96%	81.63%	82.11%	82.11%	82.11%

TABLE 8. Analysis of Sensitivity of Decision Variables to Due Date

These results indicate that the due date and costs are inversely related, so in receiving orders, attention must be paid to the deadline for delivery to the customer. The due date should be set with account taken of manufacturing capacity and material supply, to prevent a shortage of materials, overtime, and instances of overdue delivery.

Remark 4.6. Computational analysis. The interactive solution procedure with the proposed FMOLP approach to solve the fuzzy multi-objective recyclable remanufacturing lot-sizing decision problem forthe curtain and shutter company is as follows. The proposed approach is solved using the ordinary single objective LP model to obtain initial solutions for each objective function. Table 9 compares the solutions from the original LP model and the proposed FMOLP approach.

The figures shown in Table 9 demonstrate that the solutions obtained using the proposed FMOLP approach reflects an efficient compromise solution. The proposed FMOLP approach is a practical method since it simultaneously minimizes total costs, total CO_2 emissions and total lead time with reference to customer demand, due date, supplier and manufacturer capacity, lot-size release and machine yield.We assume the DM specified the corresponding possible value interval for each fuzzy objective, as the precise value can be determined based on the experience of DMs, and the equal membership group of the DM is normally in the interval [0, 1].

Model	LP-1	LP-2	LP-3	The proposed FMOLP approach
Objective function	Min z_1	Min z_2	Min z_3	Max L
L (Satisfactory degree)	100%	100%	100%	0.8211%
z_1 ($)	3,670504†	4,270,164	3,942,282	3,887,702
z_2 (kg)	2,272,530	1,972,345†	2,221,500	2,078,883
z_3 (days)	15	15	6.15†	10.11

TABLE 9. Solutions Comparisons $f_k(z_k)$

†Denotes the optimal value with the ordinary single-objective LP model.

The proposed FMOLP approach developed in this work provides overall DM satisfaction with the determined goal values in uncertain environments. For example, for the overall DM satisfaction with the given goal values in the curtain and shutter company, z_1=$3,887,702, z_2=2,078,883 kg, z_3=10.11 days, the overall satisfaction with the determined goal values is 82.11%.The proposed approach can greatly facilitate DMs during production planning in uncertain environments and satisfy practical managerial requirements.

5. Conclusions

Production planning of recycling and remanufacturing systems is complex and requires consideration of various production stages, vendors, components,materials and machines. During the remanufacturing process, costs, CO_2 emissions and lead times of new and recycled materials actually exhibit a trade-off relationship. The objectives may be mutually conflicting, raising issues of fuzzy multi-objective recyclable remanufacturing planning decisions. The aim of recyclable remanufacturing planning problem decision-making is to simultaneously achieve the targets of low cost, low CO_2 emissions and short lead times. This work addresses a new fuzzy multi-objective remanufacturing planning decisions problem distinguished by the integrated procurement, manufacturing and assembling planning in the recoverable remanufacturing systems. A new integrated recycling and remanufacture FMOLP model is developed. This work also develops an interactive approach that uses a piecewise linear membership function to capture DMs' preferences for various objectives and enables a multi-objective purchasing and production lot-sizing problem with recycling and remanufacturing to be solved. The major contribution of this work to the literature is its fuzzy mathematical programming methodology for solving recyclable remanufacturing planning problems in a fuzzy environment, and providing a systematic decision-making procedure that allows a decision maker to interactively adjust search direction until a satisfactory solution is obtained.The proposed model simultaneously considers such objectives as minimizing total production costs, CO_2 emissions and lead-time, and is therefore more useful than other models for remanufacturers. Establishing the accuracy of production planning by analyses of actual cases in which the model is applied, the work analyzes the sensitivity of decision-making variable to demand and due date.Based on the results

of this analysis, management is advised of management policies concerning purchasing and remanufacturing while ensuring low costs, low carbon emission and short lead times, significantly improving remanufacturing systems.Computational methodology can easily be extended to other industries and can handle the practical remanufacturing planning problems in fuzzy situations. Therefore, the proposed approach is a practical method; it can satisfy practical managerial requirements in uncertain environments. The major limitations of the proposed approach concern the assumptions that the piecewise linear membership function is the applicable representation imprecise/fuzzygoals of the human DM for the practical remanufacturing planning problems. Hence, future investigations may apply the related patterns of non-linear membership functions to develop an actual remanufacturing planningdecision.Furthermore, the assumption of deterministic demand can be relaxed by incorporating fuzzy or stochastic demand. In addition, the proposed FMOLP model is based on Hannan's approach [11], which assumes that the minimum operator is an appropriate representation of a DM's judgment in combining fuzzy sets by logical 'and' operations. Therefore, further work may also apply the averaging or other operators to solve the recyclable remanufacturing planning problem in an uncertain environment.

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