FINITE-TIME PASSIVITY OF DISCRETE-TIME T-S FUZZY NEURAL NETWORKS WITH TIME-VARYING DELAYS

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ABSTRACT. This paper focuses on the problem of finite-time boundedness and finite-time passivity of discrete-time T-S fuzzy neural networks with time-varying delays. A suitable Lyapunov–Krasovskii functional(LKF) is established to derive sufficient condition for finite-time passivity of discrete-time T-S fuzzy neural networks. The dynamical system is transformed into a T-S fuzzy model with uncertain parameters. Furthermore, the obtained passivity criteria is established in terms of Linear matrix inequality (LMI), which can be easily checked by using the efficient MATLAB LMI toolbox. Finally, some numerical cases are given to illustrate the effectiveness of the proposed approach.

1. Introduction

In the midst of the earlier decades, a speedy development has been made in the examination of artificial neural networks (NNs) which have wide applications in various locales, for instance, reproducing moving pictures, signal processing pattern recognition and optimization issues. For the late progress, since time delays can't be kept up a key separation from and they frequently provoke insecurity of neural networks, it essentially focuses on progressing different sorts of stability conditions of different sorts of delayed neural frameworks. Thus, the artificial delayed neural frameworks have been widely studied by many authors and a assortment of results have been derived (see [14, 10, 22, 42, 37, 38, 39, 40, 41, 1] and references therein).

In addition, in actualizing the continuous-time neural system for computer simulation, experimental or computational purposes, it is fundamental to detail a discrete-time framework which is a simple of the continuous-time neural system. In any case, as a rule, the discretization can't preserve the dynamics of the continuous-time counterpart. In this manner, it is critical to study the dynamics of the discrete-time frameworks and numerous outcomes have been accomplished in the literature [24, 2, 29, 12, 19, 6, 15, 16]. Further, it is notable that the connection weights of the neurons are inherently dependent on certain resistance and capacitance values that unavoidably acquire uncertainties amid the parameter identification process. The deviations and perturbations in parameters affect the execution of neural systems. In this way, it is vital to study the dynamical behaviors of neural frameworks

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by considering the uncertainties. Many researchers have talked about the flow of delayed systems with uncertainties, (see [24, 2, 12] and references therein.)

It is worth pointing out that the theory of "Fuzzy Sets" plays a vital role in the modeling and controlling of nonlinear systems [27]. The T-S fuzzy dynamic model is described by a family of fuzzy IF-THEN rules that represent local linear input-output relations of a nonlinear system. The T-S fuzzy model introduced in [27] is essentially a multi-model approach in which some linear models are blended into an overall single model through nonlinear membership functions. Specifically, the T-S fuzzy neural network has the advantages of both the fuzzy logic and neural networks which has many important applications in moving image processing and pattern classification [27, 11, 31, 43, 30, 25, 3, 32, 33, 21].

In practice, the passivity theory, which was proposed first in circuit investigation, has likewise pulled in impressive consideration since it is a valuable instrument to the stability analysis of linear and nonlinear systems, especially for high-order systems. Passive properties of frameworks can keep the frameworks inside stable. Because of its significance, the issue of passivity analysis for delayed dynamic systems has been investigated [7, 34, 4, 26, 13, 18, 20]. For example, the passivity condition for discrete-time switched neural systems with different functions and mixed time delays was derived [34]. The problem of robust passivity for discrete-time delayed standard neural system model with norm-bounded parameter uncertainties was investigated [4, 26]. In [13], the delay-dependent robust passivity criterion for uncertain discrete-time neural networks with interval time-varying delay was proposed.

However, it should be mentioned that all these existing studies about the passivity analysis are performed using the conventional Lyapunov asymptotic stability theory, which is defined over the infinite-time interval. But in many practical applications, the transient behavior of system is concerned over a fixed finite-time interval, in which the system states need to grip below a prescribed upper bound and larger values are not permitted during this time-interval [8, 23, 35]. Recently, many biologists are focusing on the transient values of the actual network states. Many interesting results for finite-time stability of various types of systems can be found (see [23, 35, 36, 17, 28] and references therein). However, there is no work is studied in finite-time passivity of discrete-time T-S fuzzy neural networks with time-varying delays.

Motivated by the above discussion, in this paper the problem of finite-time stability for discrete-time T-S fuzzy neural networks with time-varying delays based on passive theory is investigated. Delay-dependent results for finite-time boundedness and finite-time passivity are derived by using finite-time stability method and Lyapunov-Krasovskii functional approach. Finite-time bounded condition for the considered system with norm bounded uncertainties is given separately. Furthermore, the obtained passivity criteria is established in terms of Linear matrix inequality (LMI), which can be easily checked by using the efficient MATLAB LMI toolbox. Finally, numerical examples are provided to demonstrate the effectiveness of the proposed method.

Notations: Throughout the paper, $\mathcal{R}^n$ denotes the n dimensional Euclidean space, and $\mathcal{R}^{m\times n}$ is the set of all $m \times n$ real matrices. For symmetric matrices $\mathcal{X}$ and $\mathcal{Y}$, the notation $\mathcal{X} \geq \mathcal{Y}$ means that $\mathcal{X} - \mathcal{Y}$ is positive-semi definite; $\mathcal{M}^T$ is transpose of the matrix $\mathcal{M}$; I is the identity matrix with appropriate dimension; In symmetric block matrices, we use $'*'$ to represent a term that is induced by symmetry; λ_{max} and λ_{min} denote the maximum and minimum eigenvalue of a matrix.

2. Problem Statement and Preliminaries

Consider the following uncertain discrete-time delayed neural networks described as

$$\begin{cases} y(k+1) = & -(\mathscr{E} + \Delta\mathscr{E}(k))y(k) + (\mathscr{A} + \Delta\mathscr{A}(k))f(y(k)) \\ & +(\mathscr{B} + \Delta\mathscr{B}(k))f(y(k-\tau(k))) + u(k), \\ z(k) = & \mathscr{D}f(y(k)), \\ y(k) = & \phi(k), \ k \in [-\tau_M, 0], \end{cases} \tag{1}$$

where $y(k) = [y_1(k), y_2(k), ..., y_n(k)]^T \in R^n$ is the neuron state vector, $u(k)$ is the exogenous disturbance input vector belongs to $\mathcal{L}_2[0,\infty)$, and $z(k)$ is the output vector of the neural network. The nonlinear function $f(y(k)) = [f_1(y_1(k)), f_2(y_2(k)), ..., f_n(y_n(k))]^T \in \mathcal{R}^n$ denotes the neuron activation function, $\phi(k) \in R^n$ is a vector-valued initial condition function. $\mathscr{E} = diag\{e_1, ..., e_n\}$ represents the state feedback coefficient matrix with $|e_i| < 1$, $\mathscr{A}$, $\mathscr{B}$, $\mathscr{D}$ are interconnection weight matrices with appropriate dimensions. $\Delta\mathscr{E}(k)$, $\Delta\mathscr{A}(k)$, and $\Delta\mathscr{B}(k)$ are parametric uncertainties. $\tau(k)$ are the time-varying delays assumed to satisfy

$$0 \leq \tau(k) \leq \tau_M, \ for\ all\ k \geq 0, \tag{2}$$

where τ_M is constant.

Assumption (H1). [9]. The uncertain parameters $\Delta\mathscr{E}(k)$, $\Delta\mathscr{A}(k)$, and $\Delta\mathscr{B}(k)$ in DNNs (1) are time-varying and norm bounded, which admit

$$\begin{bmatrix}\Delta\mathscr{E}(k) & \Delta\mathscr{A}(k) & \Delta\mathscr{B}(k)\end{bmatrix} = \mathscr{H}\mathscr{F}(k)[\mathscr{G}_1 \quad \mathscr{G}_2 \quad \mathscr{G}_3], \tag{3}$$

where $\mathscr{H}$ and $\mathscr{G}_j, (j = 1, 2, 3)$ are known constant matrices of appropriate dimensions and $\mathscr{F}(k)$ is an unknown time-varying matrix with Lebesgue measurable elements bounded by

$$\mathscr{F}^T(k)\mathscr{F}(k) \leq I.$$

As mentioned before, the uncertainties and vagueness are always unavoidable in practical implementation. To reflect such a reality, we use the fuzzy system approach to model the vagueness, in which the system parameters are with uncertainties. The i^{th} rule of the T-S fuzzy model is of the following form:

Model i: $IF\{w_1$ is $\zeta_{i1}\}$ and, ..., and $\{w_p$ is $\zeta_{ip}\}$, THEN

$$\begin{cases} y(k+1) = & -(\mathscr{E}_i + \Delta\mathscr{E}_i(k))y(k) + (\mathscr{A}_i + \Delta\mathscr{A}_i(k))f(y(k)) \\ & +(\mathscr{B}_i + \Delta\mathscr{B}_i(k))f(y(k-\tau(k))) + u(k), \\ z(k) = & \mathscr{D}_i f(y(k)), \\ y(k) = & \phi(k), \ k \in [-\tau_M, 0], \end{cases} \tag{4}$$

where $w_1(k)$,...,$w_p(k)$ are premise variables. ζ_{ij} $(i = 1, 2, ..., r,\ j = 1, 2, ..., p)$ are fuzzy sets and r is the number of IF-THEN rules.

Using a standard fuzzy inference method, the system (4) is inferred as follows

$$\begin{cases} y(k+1) = & \sum_{i=1}^{r} h_i(w(k))\{-(\mathcal{E}_i + \Delta\mathcal{E}_i(k))y(k) + (\mathcal{A}_i + \Delta\mathcal{A}_i(k))f(y(k)) \\ & \quad +(\mathcal{B}_i + \Delta\mathcal{B}_i(k))f(y(k-\tau(k))) + u(k)\}, \\ z(k) = & \sum_{i=1}^{r} h_i(w(k))\mathcal{D}_i f(y(k)), \quad i = 1, ..., r, \end{cases} \tag{5}$$

where $h_i(w(k))$ is the normalised membership function of the inferred fuzzy set $\beta_i(w(k))$, that is,

$$h_i(w(k)) = \frac{\beta_i(w(k))}{\sum_{i=1}^{r} \beta_i(w(k))}, \quad \beta_i(w(k)) = \prod_{j=1}^{p} \zeta_{ij}(w_j(k)),$$

and $\zeta_{ij}(\cdot)$ is the grade membership function of $w_p(k)$ in ζ_{ij}. We assume $\beta_i(w(k) \geq 0.$ i=1,...,r, $\sum_{i=1}^{r} \beta_i(w(k)) > 0$ for any $w(k)$.

Hence $h_i(w(k))$ satisfies

$$h_i(w(k) \geq 0. \quad i = 1, ..., r, \quad \sum_{i=1}^{r} h_i(w(k)) = 1,$$

for any $w(k)$.

Assumption (H2). [14] In this brief, we assume that activation functions $f_i(\cdot)$, $(i \in [1, m])$ are bounded and satisfy

$$\mathcal{F}_i^- \leq \frac{f_i(a) - f_i(b)}{a - b} \leq \mathcal{F}_i^+, \ a \neq b, \ a, b \in \mathbb{R}, \ i = 1, 2, ..., m, \tag{6}$$

where $\mathcal{F}_i^-$ *and* $\mathcal{F}_i^+$ are known real scalars. For notation convenience, we define the following matrices

$$\mathcal{F}_1 = diag\Big\{\mathcal{F}_1^- \mathcal{F}_1^+, \mathcal{F}_2^- \mathcal{F}_2^+, ..., \mathcal{F}_m^- \mathcal{F}_m^+\Big\},$$
$$\mathcal{F}_2 = diag\Big\{\frac{\mathcal{F}_1^- + \mathcal{F}_1^+}{2}, \frac{\mathcal{F}_2^- + \mathcal{F}_2^+}{2}, ..., \frac{\mathcal{F}_m^- + \mathcal{F}_m^+}{2}\Big\}$$

Assumption (H3). [18] The disturbance input vector $u(k)$ is time-varying and for a given $\vartheta > 0$, satisfies $u^T(k)u(k) \leqslant \vartheta$.

The following definitions and Lemmas will be used in the proof of main results.

Definition 2.1. [18] The DNN (1) is said to be robustly finite-time bounded with respect to $(c_1, c_2, L, N, \vartheta)$, where $0 < c_1 < c_2$ and $L > 0$, if

$$y^T(k_1)Ly(k_1) \leqslant c_1 \Rightarrow y^T(k_2)Ly(k_2) \leqslant c_2,$$
$$\forall \ k_1 \in \{-\tau_M, \tau_M + 1, ..., 0\}, \ k_2 \in \{1, 2, ..., N\},$$

holds for any nonzero $u(k)$ satisfies Assumption (H3).

Definition 2.2. [18] The DNN (1) is said to be robustly finite-time passive with respect to $(c_1, c_2, L, N, \gamma, \vartheta)$, where $0 < c_1 < c_2$, γ is a prescribed positive scalar and $L > 0$, if DNN (1) with output (2) is robustly finite-time bounded with respect to $(c_1, c_2, L, N, \vartheta)$, and under the zero initial condition the output $z(k)$ satisfies

$$2\sum_{k=0}^{N} z^T(k)u(k) \geqslant -\gamma \sum_{k=0}^{N} u^T(k)u(k)$$

holds for any nonzero $u(k)$ satisfies Assumption (H3).

Lemma 2.3. [5] Let $\mathcal{M},\ \mathcal{P},\ \mathcal{Q}$ be given matrices such that $\mathcal{Q} > 0$, then

$$\begin{bmatrix} \mathcal{P} & \mathcal{M}^T \\ \mathcal{M} & -\mathcal{Q} \end{bmatrix} < 0 \ \Leftrightarrow \mathcal{P} + \mathcal{M}^T\mathcal{Q}^{-1}\mathcal{M} < 0.$$

Lemma 2.4. [9] Given matrices $\mathcal{Q} = \mathcal{Q}^T, \mathcal{H}, \mathcal{E}$ and $\mathcal{R} = \mathcal{R}^T > 0$ with appropriate dimensions

$$\mathcal{Q} + \mathcal{H}\mathcal{F}(k)\mathcal{E} + \mathcal{E}^T\mathcal{F}^T(k)\mathcal{H}^T < 0$$

for all $\mathcal{F}(k)$ satisfying $\mathcal{F}^T(k)\mathcal{F}(k) \leq \mathcal{R}$ if and only if there exists a scalar $\varepsilon > 0$ such that

$$\mathcal{Q} + \varepsilon^{-1}\mathcal{H}\mathcal{H}^T + \varepsilon\mathcal{E}^T\mathcal{R}\mathcal{E} < 0.$$

3. **Finite-time Boundedness**

In this section, we provide the condition for the following nominal system of (5), (without uncertainties)

$$\begin{aligned} y(k+1) =& \sum_{i=1}^{r} h_i(w(k))\{-\mathcal{E}_i y(k) + \mathcal{A}_i f(y(k) + \mathcal{B}_i f(y(k-\tau(k))) + u(k)\}, \\ y(k) =& \phi(k),\ k \in [-\tau_M, 0], \quad i = 1, ..., r, \end{aligned} \tag{7}$$

to be finite-time bounded with respect to $(c_1, c_2, R, N, \vartheta)$.

Theorem 3.1. Under assumptions (H2) & (H3), for given scalars $\mu \geq 1,\ \tau_M$, the system (7) is finite-time bounded with respect to $(c_1, c_2, R, N, \vartheta)$, if there exist matrices $\mathcal{P} > 0,\ \mathcal{Q} > 0$, diagonal matrices $\mathcal{W}_i > 0,\ (i = 1, 2, 3)$, and matrix $\mathcal{W}$ of appropriate dimension, such that the following inequalities hold:

$$\Omega_i = \begin{bmatrix} (1,1) & \mathcal{F}_2\mathcal{W}_3 & 0 & \mathcal{E}_i^T\mathcal{P}\mathcal{A}_i + \mathcal{F}_2\mathcal{W}_1 + \mathcal{F}_1\mathcal{W}_3 & \mathcal{E}_i^T\mathcal{P}\mathcal{B}_i - \mathcal{F}_2\mathcal{W}_3 & \mathcal{E}_i^T\mathcal{P} \\ * & -\mathcal{F}_1\mathcal{W}_2 - \mathcal{W}_3 & 0 & -\mathcal{F}_2\mathcal{W}_3 & \mathcal{W}_3 & 0 \\ * & * & -\mu^{\tau_M}\mathcal{Q} & 0 & 0 & 0 \\ * & * & * & \mathcal{A}_i^T\mathcal{P}\mathcal{A}_i - \mathcal{F}_1\mathcal{W}_3 - \mathcal{W}_1 & \mathcal{A}_i^T\mathcal{P}\mathcal{B}_i + \mathcal{F}_2\mathcal{W}_3 & \mathcal{A}_i^T\mathcal{P} \\ * & * & * & * & \mathcal{B}_i^T\mathcal{P}\mathcal{B}_i - \mathcal{W}_2 - \mathcal{W}_3 & 0 \\ * & * & * & * & * & -\mathcal{P} - \mathcal{W} \end{bmatrix} < 0, \tag{8}$$

$$0 \leq \overline{\mathcal{Q}} \leq \lambda_{max}(\overline{\mathcal{Q}})I,\ i = 1, 2, ..., r, \tag{9}$$

$$\frac{\mu^N\big((\lambda_{max}(\overline{\mathcal{P}}) + \tau_M\mu^{\tau_M-1}\lambda_{max}(\overline{\mathcal{Q}}))c_1 + \lambda_{max}(\mathcal{W})\vartheta\big)}{\lambda_{min}(\overline{\mathcal{P}})} < c_2, \tag{10}$$

where

$$(1,1) = \mathcal{E}_i^T\mathcal{P}\mathcal{E}_i - \mathcal{F}_1\mathcal{W}_1 - \mathcal{F}_1\mathcal{W}_3 + \mathcal{Q} - \mu\mathcal{P},\ \overline{\mathcal{P}} = R^{-1/2}\mathcal{P}R^{-1/2},\ \overline{\mathcal{Q}} = R^{-1/2}\mathcal{Q}R^{-1/2}.$$

Proof. We consider the LKF candidate as

$$V(y(k),k) = V_1(y(k),k) + V_2(y(k),k) \tag{11}$$

where

$$\begin{aligned} V_1(y(k),k) =& y(k)^T \mathcal{P} y(k), \\ V_2(y(k),k) =& \sum_{i=k-\tau_M}^{k-1} \mu^{k-i-1} y(i)^T \mathcal{Q} y(i) \end{aligned}$$

Calculating the forward difference of $V(y(k),k)$ by defining $\Delta V(y(k),k) = V(y(k+1),k) - V(y(k),k)$ along the solutions of (7), we obtain

$$\begin{aligned} \Delta V(y(k),k) - (\mu-1)V(y(k),k) =& \big(\Delta V_1(y(k),k) - (\mu-1)V_1(y(k),k)\big) \\ &+ \big(\Delta V_2(y(k),k) - (\mu-1)V_2(y(k),k)\big), \end{aligned} \tag{12}$$

then

$$\begin{aligned} &\Delta V_1(y(k),k) - (\mu-1)V_1(y(k),k) = y^T(k+1)\mathcal{P}y(k+1) - \mu y^T(k)\mathcal{P}y(k) \\ &= \sum_{i=1}^{r} h_i(w(k)) \otimes \begin{bmatrix} y(k) \\ f(y(k)) \\ f(y(k-\tau(k))) \\ u(k) \end{bmatrix}^T \begin{bmatrix} \mathcal{E}_i^T\mathcal{P}\mathcal{E}_i & \mathcal{E}_i^T\mathcal{P}\mathcal{A}_i & \mathcal{E}_i^T\mathcal{P}\mathcal{B}_i & \mathcal{E}_i^T\mathcal{P} \\ \mathcal{A}_i^T\mathcal{P}\mathcal{E}_i & \mathcal{A}_i^T\mathcal{P}\mathcal{A}_i & \mathcal{A}_i^T\mathcal{P}\mathcal{B}_i & \mathcal{A}_i^T\mathcal{P} \\ \mathcal{B}_i^T\mathcal{P}\mathcal{E}_i & \mathcal{B}_i^T\mathcal{P}\mathcal{A}_i & \mathcal{B}_i^T\mathcal{P}\mathcal{B}_i & \mathcal{B}_i^T\mathcal{P} \\ \mathcal{P}\mathcal{E}_i & \mathcal{P}\mathcal{A}_i & \mathcal{P}\mathcal{B}_i & \mathcal{P} \end{bmatrix} \\ &\begin{bmatrix} y(k) \\ f(y(k)) \\ f(y(k-\tau(k))) \\ u(k) \end{bmatrix} - \mu y^T(k)\mathcal{P}y(k), \end{aligned} \tag{13}$$

$$\Delta V_2(y(k),k) - (\mu-1)V_2(y(k),k) \leq y^T(k)\mathcal{Q}y(k) - \mu^{\tau_M} y^T(k-\tau_M)\mathcal{Q}y(k-\tau_M). \tag{14}$$

Moreover, it follows from Assumption (H2), that

$$\Big[f_i(y_i(k)) - \mathcal{F}_i^- y_i(k)\Big]^T \Big[f_i(y_i(k)) - \mathcal{F}_i^+ y_i(k)\Big] \leq 0, \ i = 1,2,...,n,$$

which is equivalent to

$$\begin{bmatrix} y(k) \\ f(y(k)) \end{bmatrix}^T \begin{bmatrix} \mathcal{F}_i^-\mathcal{F}_i^+ y_i y_i^T & -\frac{\mathcal{F}_i^- + \mathcal{F}_i^+}{2} y_i y_i^T \\ -\frac{\mathcal{F}_i^- + \mathcal{F}_i^+}{2} y_i y_i^T & y_i y_i^T \end{bmatrix} \begin{bmatrix} y(k) \\ f(y(k)) \end{bmatrix} \leq 0,$$

where y_i denotes the units column vector having element 1 on its i^{th} row and zeros elsewhere. Let $\mathcal{W}_1 = diag\{w_{11}, w_{12}, ..., w_{1n}\} > 0$, it is easy to see that

$$\sum_{i=1}^{n} w_{1i} \begin{bmatrix} y(k) \\ f(y(k)) \end{bmatrix}^T \begin{bmatrix} \mathcal{F}_i^-\mathcal{F}_i^+ y_i y_i^T & -\frac{\mathcal{F}_i^- + \mathcal{F}_i^+}{2} y_i y_i^T \\ -\frac{\mathcal{F}_i^- + \mathcal{F}_i^+}{2} y_i y_i^T & y_i y_i^T \end{bmatrix} \begin{bmatrix} y(k) \\ f(y(k)) \end{bmatrix} \leq 0.$$

That is,

$$\begin{bmatrix} y(k) \\ f(y(k)) \end{bmatrix}^T \begin{bmatrix} -\mathcal{F}_1\mathcal{W}_1 & \mathcal{F}_2\mathcal{W}_1 \\ * & -\mathcal{W}_1 \end{bmatrix} \begin{bmatrix} y(k) \\ f(y(k)) \end{bmatrix} \geq 0. \tag{15}$$

Similar to above, for any positive diagonal matrices $\mathcal{W}_2$ and $\mathcal{W}_3$, we can obtain the following inequalities:

$$\begin{bmatrix} y(t-\tau(k)) \\ f(y(t-\tau(k))) \end{bmatrix}^T \begin{bmatrix} -\mathcal{F}_1\mathcal{W}_2 & \mathcal{F}_2\mathcal{W}_2 \\ * & -\mathcal{W}_2 \end{bmatrix} \begin{bmatrix} y(t-\tau(k)) \\ f(y(t-\tau(k))) \end{bmatrix} \geq 0, \tag{16}$$

and

$$\begin{bmatrix} y(k) \\ f(y(k)) \\ y(k-\tau(k)) \\ f(y(t-\tau(k))) \end{bmatrix}^T \begin{bmatrix} -\mathcal{F}_1\mathcal{W}_3 & \mathcal{F}_2\mathcal{W}_3 & \mathcal{F}_1\mathcal{W}_3 & -\mathcal{F}_2\mathcal{W}_3 \\ * & -\mathcal{W}_3 & -\mathcal{F}_2\mathcal{W}_3 & \mathcal{W}_3 \\ * & * & -\mathcal{F}_1\mathcal{W}_3 & \mathcal{F}_2\mathcal{W}_3 \\ * & * & * & -\mathcal{W}_3 \end{bmatrix} \begin{bmatrix} y(k) \\ f(y(k)) \\ y(k-\tau(k)) \\ f(y(t-\tau(k))) \end{bmatrix} \geq 0. \tag{17}$$

Then from (12) and adding (13)–(17), gives

$$\Delta V(y(k),k) - (\mu-1)V(y(k),k) - u^T(k)\mathcal{W}u(k) \leq \sum_{i=1}^{r} h_i(w(k))[\xi^T(k)\Omega_i\xi(k)], \tag{18}$$

where

$\xi(k) = \left[y^T(k)\ y^T(k-\tau(k))\ y^T(k-\tau_M)\ f^T(y(k))\ f^T(y(k-\tau(k)))\ u^T(k)\right]^T.$

Hence if the LMIs (8) hold, it is easy to get

$$\begin{aligned} \Delta V(y(k),k) - (\mu-1)V(y(k),k) - u^T(k)\mathcal{W}u(k) &\leq 0 \\ V(k+1) - V(k) &\leq (\mu-1)V(y(k),k) + u^T(k)\mathcal{W}u(k) \\ &\leq (\mu-1)V(y(k),k) + \lambda_{max}(\mathcal{W})u^T(k)u(k). \end{aligned} \tag{19}$$

Simple computation gives $V(k+1) \leq \mu V(y(k),k) + \lambda_{max}(\mathcal{W})u^T(k)u(k)$. Noticing $\mu \geq 1$, it follows that

$$V(k) \leq \mu^k V(0) + \lambda_{max}(\mathcal{W}) \sum_{i=0}^{k-1} \mu^{k-i-1} u^T(i)u(i),$$

from the Assumption (H3), we have

$$V(k) \leq \mu^k V(0) + \mu^k \lambda_{max}(\mathcal{W})\vartheta. \tag{20}$$

Further, from (11), we get

$$V(0) = y^T(0)\mathcal{P}y(0) + \sum_{i=-\tau_M}^{-1} \mu^{-i-1} y^T(i)\mathcal{Q}y(i).$$

Letting $\overline{\mathcal{P}} = R^{-1/2}\mathcal{P}R^{-1/2}$, $\overline{\mathcal{Q}} = R^{-1/2}\mathcal{Q}R^{-1/2}$, we obtain

$$\begin{aligned} V(0) &= y^T(0)R^{1/2}\overline{\mathcal{P}}R^{1/2}y(0) + \sum_{i=-\tau_M}^{-1} \mu^{-i-1} y^T(i)R^{1/2}\overline{\mathcal{Q}}R^{1/2}y(i) \\ &\leq \lambda_{max}(\overline{\mathcal{P}})y^T(0)Ry(0) + \lambda_{max}(\overline{\mathcal{Q}}) \sum_{i=-\tau_M}^{-1} \mu^{-i-1} y^T(i)Ry(i) \\ &\leq \left[\lambda_{max}(\overline{\mathcal{P}}) + \tau_M \mu^{\tau_M - 1} \lambda_{max}(\overline{\mathcal{Q}})\right] c_1. \end{aligned}$$

On the other hand, from (11), we can obtain that

$$V(k) \geq y^T(k)\mathcal{P}y(k) \geq y^T(k)R^{1/2}\overline{\mathcal{P}}R^{1/2}y(k) \geq \lambda_{min}(\overline{\mathcal{P}})y^T(k)Ry(k) \tag{21}$$

From (20) and (21), we get

$$y^T(k)Ry(k) < \frac{\mu^k\big((\lambda_{max}(\overline{\mathcal{P}}) + \tau_M\mu^{\tau_M-1}\lambda_{max}(\overline{\mathcal{Q}})\big)c_1 + \lambda_{max}(\mathcal{W})\vartheta\big)}{\lambda_{min}(\overline{\mathcal{P}})} < c_2.$$

This implies $y^T(k)Ry(k) < c_2,\ \forall\ k \in \{1,2,...,N\}$. Thus by Definition 2.1 the DNNs (7) is finite-time bounded with respect to $(c_1, c_2, R, N, \vartheta)$. This completes the proof. □

4. **Finite-time Passivity**

In this section, we focus on the finite-time passivity of DNN (7) with output $z(t)$.

Theorem 4.1. Under assumptions (H2) & (H3) hold, for given scalars $\mu \geq 1,\ \tau_M$, the system (7) is finite-time bounded with respect to $(c_1, c_2, R, N, \gamma, \vartheta)$, if there exist matrices $\mathcal{P} > 0$, and $\mathcal{Q} > 0$, diagonal matrices $\mathcal{W}_i > 0,\ i = 1,2,3$, matrix $\mathcal{W}$ of appropriate dimension, and positive scalar γ, such that the following inequalities hold:

$$\Omega_i^1 = \begin{bmatrix} (1,1) & \mathcal{F}_2\mathcal{W}_3 & 0 & \mathcal{E}_i^T\mathcal{P}\mathcal{A}_i + \mathcal{F}_2\mathcal{W}_1 + \mathcal{F}_1\mathcal{W}_3 & \mathcal{E}_i^T\mathcal{P}\mathcal{B}_i - \mathcal{F}_2\mathcal{W}_3 & 0 \\ * & -\mathcal{F}_1\mathcal{W}_2 - \mathcal{W}_3 & 0 & -\mathcal{F}_2\mathcal{W}_3 & \mathcal{W}_3 & 0 \\ * & * & -\mu^{\tau_M}\mathcal{Q} & 0 & 0 & 0 \\ * & * & * & \mathcal{A}_i^T\mathcal{P}\mathcal{A}_i - \mathcal{F}_1\mathcal{W}_3 - \mathcal{W}_1 & \mathcal{A}_i^T\mathcal{P}\mathcal{B}_i + \mathcal{F}_2\mathcal{W}_3 & -\mathcal{D}_i^T \\ * & * & * & * & \mathcal{B}_i^T\mathcal{P}\mathcal{B}_i - \mathcal{W}_2 - \mathcal{W}_3 & 0 \\ * & * & * & * & * & -\gamma I \end{bmatrix} < 0, \tag{22}$$

$$0 \leq \overline{\mathcal{Q}} \leq \lambda_{max}(\overline{\mathcal{Q}})I,\ i = 1,2,...,r, \tag{23}$$

$$\frac{\mu^N\left((\lambda_{max}(\overline{\mathcal{P}}) + \tau_M\mu^{\tau_M-1}\lambda_{max}(\overline{\mathcal{Q}}))c_1 + \mu^{-N}\gamma\vartheta\right)}{\lambda_{min}(\overline{\mathcal{P}})} < c_2, \tag{24}$$

where

$$(1,1) = \mathcal{E}_i^T\mathcal{P}\mathcal{E}_i - \mathcal{F}_1\mathcal{W}_1 - \mathcal{F}_1\mathcal{W}_3 + \mathcal{Q} - \mu\mathcal{P},\ \overline{\mathcal{P}} = R^{-1/2}\mathcal{P}R^{-1/2},\ \overline{\mathcal{Q}} = R^{-1/2}\mathcal{Q}R^{-1/2},$$

Proof. The proof is similar to that in Theorem 3.1. In Theorem 3.1 by choosing $\mathcal{W} = \gamma\mu^{-N}$ in $\mathcal{J}$ and using similar lines of (19), it follows that

$$\Delta V(k) - (\mu-1)V(k) - 2z^T(k)u(k) - \gamma\mu^{-N}u^T(k)u(k) \leq 0$$
$$V(k+1) - V(k) \leq (\mu-1)V(k) + 2z^T(k)u(k) + \gamma\mu^{-N}u^T(k)u(k)$$

By simple computation

$$V(k) \leq \mu^kV(0) + 2\sum_{i=0}^{k-1}\mu^{k-i-1}z^T(i)u(i) + \gamma\mu^{-N}\sum_{i=0}^{k-1}\mu^{k-i-1}u^T(i)u(i). \tag{25}$$

Under the zero initial condition and noticing $V(k) \geq 0,\ \forall\ k \in \{1, 2, ..., N\}$, we have

$$2\sum_{i=0}^{k-1} \mu^{k-i-1} z^T(i)u(i) \geq -\gamma\mu^{-N} \sum_{i=0}^{k-1} \mu^{k-i-1} u^T(i)u(i).$$

Noticing that $\mu \geq 1$, we have

$$2\sum_{k=0}^{N} \mu^{N-k} z^T(k)u(k) \geq -\gamma\mu^{-N} \sum_{k=0}^{N} \mu^{N-k} u^T(k)u(k). \tag{26}$$

By Definition 2.2, can be concluded that the nominal system (7) is finite-time passive. This completes the proof. □

Based on Theorem 4.1 and Lemma 2.4, we derive the finite-time passivity for discrete-time T-S fuzzy neural networks with uncertainties in the following Theorem(4.2).

Theorem 4.2. Under assumptions (H2) & (H3), for given scalars $\mu \geq 1,\ \tau_M$, the system (1) is finite-time bounded with respect to $(c_1, c_2, R, N, \gamma, \vartheta)$, if there exist matrices $\mathcal{P} > 0,\ \mathcal{Q} > 0$, diagonal matrices $\mathcal{W}_i > 0,\ (i = 1, 2, 3)$, and matrix $\mathcal{W}$ of appropriate dimension, and positive scalar γ, such that the following inequalities hold:

$$\Omega_i^2 = \begin{bmatrix} \Omega_i^1 + \varepsilon_i \mathcal{H}_i \mathcal{H}_i^T & \mathcal{G}_i^T \\ * & -\varepsilon_i I \end{bmatrix} < 0, \quad i = 1, 2, ..., r \tag{27}$$

$$0 \leq \overline{\mathcal{Q}} \leq \lambda_{max}(\overline{\mathcal{Q}})I,\ i = 1, 2, ..., r, \tag{28}$$

$$\frac{\mu^N \Big((\lambda_{max}(\overline{\mathcal{P}}) + \tau_M \mu^{\tau_M - 1} \lambda_{max}(\overline{\mathcal{Q}}))c_1 + \mu^{-N}\gamma\vartheta \Big)}{\lambda_{min}(\overline{\mathcal{P}})} < c_2, \tag{29}$$

where Ω_i^1 is given in Theorem 4.1 and

$$\mathcal{H}_i = \begin{bmatrix} \mathcal{H}_i & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T,$$
$$\mathcal{G}_i = \begin{bmatrix} \mathcal{G}_{i1} & 0 & 0 & \mathcal{G}_{i2} & \mathcal{G}_{i3} & 0 \end{bmatrix}^T,$$

Proof. Substituting $\mathcal{E}_i = \mathcal{E}_i + \Delta\mathcal{E}_i,\ \mathcal{A}_i = \mathcal{A}_i + \Delta\mathcal{A}_i,\ \mathcal{B}_i = \mathcal{B}_i + \Delta\mathcal{B}_i$, into (22) and considering (3), we have

$$\Omega_i^1 + \mathcal{H}_i \mathcal{F}_i(k)\mathcal{G}_i + \mathcal{G}_i^T \mathcal{F}_i^T(k)\mathcal{H}_i < 0, \quad i = 1, 2, ..., r.$$

It follows from Lemma 2.4,

$$\Omega_i^1 + \varepsilon_i \mathcal{H}_i \mathcal{H}_i^T + \varepsilon_i^{-1} \mathcal{G}_i \mathcal{G}_i^T < 0,\ i = 1, 2, ..., r. \tag{30}$$

Applying Schur compliment Lemma 2.3 in (30), the T-S fuzzy DNN model (5) is robustly finite-time passive. This completes the proof. □

Remark 4.3. The equation (1) is described by a discrete-time T-S fuzzy neural networks with time-varying delays modelled in (4). In this model the system dynamics are captured by a set of fuzzy IF-THEN rules that represent local linear input-output relations of a nonlinear system.

Remark 4.4. Theorem 4.1 develops a finite-time passivity criterion of discrete-time T-S fuzzy neural networks with time-varying delays. Theorem 4.1 makes full use of the information of the subsystems upper bounds of the time-varying delays, which also brings us the less conservativeness.

5. Numerical Examples

In this section, simulation examples are given to demonstrate the feasibility and efficiency of theoretic results.

Example 5.1. Consider the following discrete-time T-S fuzzy neural networks:

$$\begin{aligned} y(k+1) =& \sum_{i=1}^{r} h_i(w(k))\{-\mathcal{E}_i y(k) + \mathcal{A}_i f(y(k) + \mathcal{B}_i f(y(k-\tau(k))) + u(k)\}, \\ y(k) =& \phi(k),\ k \in [-\tau_M, 0], \quad i = 1, ..., r, \end{aligned} \tag{31}$$

Mode Rule 1: IF $w(k) = y_1(k)$ is "about 0", THEN

$$y(k+1) = -\mathcal{E}_1 y(k) + \mathcal{A}_1 f(y(k) + \mathcal{B}_1 f(y(k-\tau(k))) + u(k)$$

where

$$\mathcal{E}_1 = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.4 \end{bmatrix},\ \mathcal{A}_1 = \begin{bmatrix} -0.1 & 0.8 \\ 0.1 & 0.2 \end{bmatrix},\ \mathcal{B}_1 = \begin{bmatrix} -0.2 & -0.9 \\ 0.1 & 0.2 \end{bmatrix},$$

Mode Rule 2: IF $w(k) = y_2(k)$ is "about 1", THEN

$$y(k+1) = -\mathcal{E}_2 y(k) + \mathcal{A}_2 f(y(k) + \mathcal{B}_2 f(y(k-\tau(k))) + u(k)$$

where

$$\mathcal{E}_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.4 \end{bmatrix},\ \mathcal{A}_2 = \begin{bmatrix} -0.1 & 0.12 \\ 0.23 & 0.3 \end{bmatrix},\ \mathcal{B}_2 = \begin{bmatrix} 0.4 & 0.5 \\ 0.15 & 0.25 \end{bmatrix},$$

the activation functions are $F_1 = 0$, $F_2 = 0.3I$, the membership functions are given as $h_1 = (1 - sin(y_1(k)))/2$, and $h_2 = (1 + sin(y_1(k)))/2$. Take $R = I$, $c_1 = 1$, $\vartheta = 5$, $N = 30$, $\tau_M = 0.9$, and $\mu = 2$, by solving the LMI based finite-time bounded conditions in Theorem 3.1 using Matlab LMI toolbox, we obtain the feasible solutions as follows:

$$\begin{aligned} &\mathcal{P} = 10^7 \times \begin{bmatrix} 1.0616 & -0.1867 \\ -0.1867 & 0.9422 \end{bmatrix},\ \mathcal{Q} = 10^6 \times \begin{bmatrix} 6.7889 & -3.0254 \\ -3.0254 & 6.1954 \end{bmatrix}, \\ &\mathcal{W}_1 = 10^7 \times \begin{bmatrix} 3.0026 & 0 \\ 0 & 3.0026 \end{bmatrix},\ \mathcal{W}_2 = 10^7 \times \begin{bmatrix} 3.0466 & 0 \\ 0 & 3.0466 \end{bmatrix}, \\ &\mathcal{W}_3 = 10^6 \times \begin{bmatrix} 8.5088 & 0 \\ 0 & 8.5088 \end{bmatrix},\ \mathcal{W} = 10^7 \times \begin{bmatrix} 4.1306 & 0.1567 \\ 0.1567 & 3.9937 \end{bmatrix}, \\ &c_2 = 1.9005e+010. \end{aligned}$$

Example 5.2. Consider the following discrete-time T-S fuzzy neural networks:

$$\begin{aligned} y(k+1) =& \sum_{i=1}^{r} h_i(w(k))\{-\mathcal{E}_i y(k) + \mathcal{A}_i f(y(k) + \mathcal{B}_i f(y(k-\tau(k))) + u(k)\}, \\ z(k) =& \sum_{i=1}^{r} h_i(w(k))\mathcal{D}_i f(y(k)), \\ y(k) =& \phi(k),\ k \in [-\tau_M, 0], \quad i = 1, ..., r, \end{aligned} \tag{32}$$

Mode Rule 1: IF $w(k) = y_1(k)$ is "about 0", THEN

$$\begin{aligned} y(k+1) =& -\mathcal{E}_1 y(k) + \mathcal{A}_1 f(y(k) + \mathcal{B}_1 f(y(k-\tau(k))) + u(k) \\ z(k) =& \mathcal{D}_1 f(y(k)) \end{aligned}$$

where

$$\mathcal{E}_1 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.2 \end{bmatrix},\ \mathcal{A}_1 = \begin{bmatrix} 0.2 & 0.4 \\ 0.3 & 0.1 \end{bmatrix},\ \mathcal{B}_1 = \begin{bmatrix} -0.5 & -0.3 \\ 0.4 & 0.2 \end{bmatrix},\ \mathcal{D}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

Mode Rule 2: IF $w(k) = y_2(k)$ is "about 1", THEN

$$\begin{aligned} y(k+1) =& -\mathcal{E}_2 y(k) + \mathcal{A}_2 f(y(k) + \mathcal{B}_2 f(y(k-\tau(k))) + u(k) \\ z(k) =& \mathcal{D}_2 f(y(k)) \end{aligned}$$

where

$$\mathcal{E}_2 = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix},\ \mathcal{A}_2 = \begin{bmatrix} -0.6 & 0.32 \\ 0.43 & 0.15 \end{bmatrix},\ \mathcal{B}_2 = \begin{bmatrix} 0.24 & 0.25 \\ 0.45 & 0.15 \end{bmatrix},\ \mathcal{D}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

the activation functions are $F_1 = 0,\ F_2 = 0.3I$, the membership functions are given as $h_1 = (1 - sin(y_1(k)))/2$, and $h_2 = (1 + sin(y_1(k)))/2$.

Let $R = I,\ c_1 = 1,\ \vartheta = 5,\ N = 30,\ \tau_M = 0.5$, and $\mu = 2$, by solving the LMI based finite-time passivity conditions in Theorem 4.1 using Matlab LMI toolbox, we obtain the feasible solutions as follows:

$$\mathcal{P} = \begin{bmatrix} 30.7703 & 0.1543 \\ 0.1543 & 30.9746 \end{bmatrix},\ \mathcal{Q} = \begin{bmatrix} 26.6650 & 0.0752 \\ 0.0752 & 27.3428 \end{bmatrix},\ \mathcal{W}_1 = \begin{bmatrix} 35.5137 & 0 \\ 0 & 35.5137 \end{bmatrix},$$
$$\mathcal{W}_2 = \begin{bmatrix} 31.9156 & 0 \\ 0 & 31.9156 \end{bmatrix},\ \mathcal{W}_3 = \begin{bmatrix} 11.9073 & 0 \\ 0 & 11.9073 \end{bmatrix},$$
$\gamma = 34.7872,\ c_2 = 1.8330e + 009.$

Example 5.3. Consider the following T-S fuzzy neural networks with norm bounded uncertainties.

$$\begin{cases} y(k+1) = & \sum_{i=1}^{r} h_i(w(k))\{-(\mathcal{E}_i + \Delta\mathcal{E}_i(k))y(k) + (\mathcal{A}_i + \Delta\mathcal{A}_i(k))f(y(k)) \\ & \qquad +(\mathcal{B}_i + \Delta\mathcal{B}_i(k))f(y(k-\tau(k))) + u(k)\}, \\ z(k) = & \sum_{i=1}^{r} h_i(w(k))\mathcal{D}_i f(y(k)), \quad i = 1, ..., r, \end{cases} \tag{33}$$

Mode Rule 1: IF $w(k) = y_1(k)$ is "about 0", THEN

$$\begin{aligned} y(k+1) =& -(\mathcal{E}_1 + \mathcal{H}_1\mathcal{F}(k)\mathcal{G}_{11})y(k) + (\mathcal{A}_1 + \mathcal{H}_1\mathcal{F}(k)\mathcal{G}_{12}) \\ &+ (\mathcal{B}_1 + \mathcal{H}_1\mathcal{F}(k)\mathcal{G}_{13})f(y(k-\tau(k))) + u(k) \\ z(k) =& \mathcal{D}_1 f(y(k)) \end{aligned}$$

where

$$\mathcal{E}_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},\ \mathcal{A}_1 = \begin{bmatrix} 0.2 & 0.4 \\ 0.4 & 0.1 \end{bmatrix},\ \mathcal{B}_1 = \begin{bmatrix} -0.6 & -0.5 \\ 0.5 & 0.4 \end{bmatrix},\ \mathcal{D}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathcal{H}_1 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \mathcal{G}_{11} = \begin{bmatrix} 0.51 & 0.32 \\ 0.15 & 0.25 \end{bmatrix},\ \mathcal{G}_{12} = \begin{bmatrix} 0.2 & 0.6 \\ 0.5 & 0.4 \end{bmatrix},\ \mathcal{G}_{13} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

Mode Rule 2: IF $w(k) = y_2(k)$ is " 1", THEN

$$\begin{aligned} y(k+1) =& -(\mathcal{E}_2 + \mathcal{H}_2\mathcal{F}(k)\mathcal{G}_{21})y(k) + (\mathcal{A}_2 + \mathcal{H}_2\mathcal{F}(k)\mathcal{G}_{22}) \\ &+ (\mathcal{B}_2 + \mathcal{H}_2\mathcal{F}(k)\mathcal{G}_{23})f(y(k-\tau(k))) + u(k) \\ z(k) =& \mathcal{D}_2 f(y(k)) \end{aligned}$$

where

$$\mathcal{E}_2 = \begin{bmatrix} 0.6 & 0 \\ 0 & 0.6 \end{bmatrix},\ \mathcal{A}_2 = \begin{bmatrix} -0.4 & 0.22 \\ 0.6 & 0.25 \end{bmatrix},\ \mathcal{B}_2 = \begin{bmatrix} 0.34 & 0.33 \\ 0.4 & 0.5 \end{bmatrix},\ \mathcal{D}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\mathcal{H}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},\ \mathcal{G}_{21} = \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.1 \end{bmatrix},\ \mathcal{G}_{22} = \begin{bmatrix} 0.5 & 0.6 \\ 0.4 & 0.2 \end{bmatrix},\ \mathcal{G}_{23} = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},$$

and $\mathcal{F}(k) = diag\{sin(k),\ cos(k)\}$ the activation functions are $F_1 = 0,\ F_2 = 0.3I$, the membership functions are given as $h_1 = (1 - sin(y_1(k)))/2$, and $h_2 = (1 + sin(y_1(k)))/2$.

Let $R = I,\ c_1 = 1,\ \vartheta = 5,\ N = 30,\ \tau_M = 0.5$, and $\mu = 2$, by solving the LMI based robustly finite-time passivity conditions in Theorem 4.2 using Matlab LMI toolbox, we can obtain the feasible solutions as follows:

$$\mathcal{P} = \begin{bmatrix} 15.3083 & 0.6592 \\ 0.6592 & 12.0839 \end{bmatrix},\ \mathcal{Q} = \begin{bmatrix} 8.1040 & -0.3226 \\ -0.3226 & 4.8648 \end{bmatrix},\ \mathcal{W}_1 = \begin{bmatrix} 17.8159 & 0 \\ 0 & 17.8159 \end{bmatrix},$$

$$\mathcal{W}_2 = \begin{bmatrix} 22.2845 & 0 \\ 0 & 22.2845 \end{bmatrix},\ \mathcal{W}_3 = \begin{bmatrix} 6.9637 & 0 \\ 0 & 6.9637 \end{bmatrix},$$

$\gamma = 12.6626, \varepsilon_1 = 6.4903,\ \varepsilon_2 = 4.7010,\ c_2 = 1.8330e + 009.$

6. **Conclusion**

This paper investigated the problem of discrete-time T-S fuzzy neural networks with time-varying delays. By constructing suitable Lyapunov-Krasovskii functional and using passivity theory sufficient conditions are derived to guarantee stability of concerned neural networks. Thus conditions for finite-time boundedness and passivity of T-S fuzzy neural networks are given in terms of LMIs which can be easily verified via the LMI toolbox. Finally, the effectiveness and superiority has been shown through numerical examples.

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References

[1] C. K. Ahn, P. Shi, R. K. Agarwal and J. Xu, *L_∞ performance of single and interconnected neural networks with time-varying delay*, Information Sciences, **346** (2016), 412–423.

[2] A. Arunkumar, R. Sakthivel, K. Mathiyalagan and S. Marshal Anthoni, *Robust stability criteria for discrete-time switched neural networks with various activation functions*, Applied Mathematics and Computation , **218** (2012), 10803–10816.

[3] A. Arunkumar, R. Sakthivel, K. Mathiyalagan and J. H. Park, *Robust stochastic stability of discrete-time fuzzy Markovian jump neural networks*, ISA transactions, **53** (2014), 1006–1014.

[4] J. Bai, R. Lu, A. Xue, Q. She and Z. Shi, *Finite-time stability analysis of discrete-time fuzzy Hopfield neural network*, Neurocomputing, **159** (2015), 263–267.

[5] B. Boyd, L. Ghoui, E. Feron and V. Balakrishnan, *Linear matrix inequalities in system and control theory,* Philadephia, PA: SIAM. (1994).

[6] J. Cao, R. Rakkiyappan, K. Maheswari and A. Chandrasekar, *Exponential H_∞ filtering analysis for discrete-time switched neural networks with random delays using sojourn probabilities*, Science China Technological Sciences, **59** (2016), 387–402.

[7] Y. L. Chien, W. L. Chin and H. T. Hsun, *Delay-range-dependent global robust passivity analysis of discrete-time uncertain recurrent neural networks with interval time-varying delay*, Discrete Dynamics in Nature and Society, (2009), 1–14.

[8] P. Dorato, *Short time stability in linear time-varying systems*, Proc IRE Int Convention Record Part 4, (1961), 83–87.

[9] K. Gu, *An integral inequality in the stability problem of time-delay systems*, in: Proc. 39th IEEE Conf. Decision and Control, Sydney, Australia, (2000), 2805-2810.

[10] M. Gupta, L. Jin and N. Homma, *Static and dynamic neural networks: from fundamentals to advanced theory*, Wiley-IEEE Press, 2013.

[11] M. Han, Y. N. Sun and Y. N. Fan, *An improved fuzzy neural network based on T-S model*, Expert Systems with Applications, **34** (2008), 2905–2920.

[12] L. Jarina Banu and P. Balasubramaniam, *Robust stability analysis for discrete-time neural networks with time-varying leakage delays and random parameter uncertainties*, Neurocomputing, **179** (2016), 126–134.

[13] R. Li, J. Cao and Z. Tu, *Passivity analysis of memristive neural networks with probabilistic time-varying delays*, Neurocomputing, **191** (2016), 249–262.

[14] Y. Liu, Z. Wang and X. Liu, *Global exponential stability of generalized recurrent neural networks with discrete and distributed delays*, Neural Networks, **19** (2006), 667–675.

[15] H. Liu, Z. Wang, B. Shen and F. E. Alsaadi, *state estimation for discrete-time memristive recurrent neural networks with stochastic time-delays*, International Journal of General Systems, **45** (2016), 633–647.

[16] X. G. Liu, F. X. Wang and Y. J. Shu, *A novel summation inequality for stability analysis of discrete-time neural networks*, Journal of Computational and Applied Mathematics, **304** (2016), 160–171.

[17] A. Liu, L. Yun, D. Zhang and W. Zhang, *Finite-time H_∞ control for discrete-time genetic regulatory networks with random delays and partly unknown transition probabilities*, Journal of the Franklin Institute, **350** (2013), 1944–1961.

[18] K. Mathiyalagan, J. H. Park and R. Sakthivel, *Novel results on robust finite-time passivity for discrete-time delayed neural networks*, Neurocomputing, **177** (2016), 585–593.

[19] K. Mathiyalagan, H. Su, P. Shi and R. Sakthivel, *Exponential $\mathcal{H}_\infty$ filtering for discrete-time switched neural networks with random delays*, IEEE transactions on cybernetics, **45** (2015), 676–687.

[20] G. Nagamani and S. Ramasamy, *Dissipativity and passivity analysis for discrete-time T-S fuzzy stochastic neural networks with leakage time-varying delays based on Abel lemma approach*, Journal of the Franklin Institute, **353** (2016), 3313–3342.

[21] S. Ramasamy, G. Nagamani and Q. Zhu, *Robust dissipativity and passivity analysis for discrete-time stochastic T–S fuzzy Cohen–Grossberg Markovian jump neural networks with mixed time delays*, Nonlinear Dynamics, **85(4)** (2016), 2777–2799.

[22] R. Saravanakumar, M. Syed Ali, C. K. Ahn, H. R. Karimi and P. Shi, *Stability of Markovian jump generalized neural networks with interval time-varying delays*, IEEE transactions on neural networks and learning systems **28(8)** (2017), 1840–1850.

[23] P. Shi, Y. Zhang and R. K. Agarwal, *Stochastic finite-time state estimation for discrete time-delay neural networks with Markovian jumps*, Neurocomputing, **151** (2015), 168–174.

[24] M. Syed Ali and M. Marudai, *Stochastic stability of discrete-time uncertain recurrent neural networks with Markovian jumping and time-varying delays*, Mathematical and Computer Modelling, **54** (2011), 1979-1988.

[25] M. Syed Ali, N. Gunasekaran and Q. Zhu, *State estimation of T-S fuzzy delayed neural networks with Markovian jumping parameters using sampled-data control*, Fuzzy Sets and Systems, **306** (2017), 87-104.

[26] M. Syed Ali, R. Saravanakumar and J. Cao, *New passivity criteria for memristor-based neutral-type stochastic BAM neural networks with mixed time-varying delays*, Neurocomputing, **171** (2016), 1533–1547.

[27] T. Takagi and M. Sugeno, *Fuzzy identification of systems and its applications to modeling and control*, *IEEE transactions on systems, man, and cybernetics*, **15** (1985), 116–132.

[28] H. Wang and Q. Zhu, *Finite-time stabilization of high-order stochastic nonlinear systems in strict-feedback form*, Automatica, **54** (2015), 284–291.

[29] L. Wu, Z. Feng and J. Lam, *Stability and synchronization of discrete-time neural networks with switching parameters and time-varying delays*, IEEE transactions on neural networks and learning systems, **24** (2013), 1957–1972.

[30] W. Xie and Q. Zhu, *Mean square exponential stability of stochastic fuzzy delayed Cohen-Grossberg neural networks with expectations in the coefficients*, Neurocomputing, **166** (2015), 133–139.

[31] E. Yucel, M. Syed Ali, N. Gunasekaran and S. Arik, *Sampled-data filtering of Takagi-Sugeno fuzzy neural networks with interval time-varying delays*, Fuzzy Sets and Systems, **316** (2017), 69-81.

[32] H. Zhang and J. Wang *State estimation of discrete-time Takagi–Sugeno fuzzy systems in a network environment*, IEEE Transactions on Cybernetics, **45** (2015), 1525–1536.

[33] Y. Zhang, P. Shi, R. K. Agarwal and Y. Shi, *Dissipativity analysis for discrete time-delay fuzzy neural networks with Markovian jumps*, IEEE Transactions on Fuzzy Systems, **24** (2016), 432–443.

[34] D. Zhang and L. Yu, *Passivity analysis for discrete-time switched neural networks with various activation functions and mixed time delays*, Nonlinear Dynamics, **67** (2012), 403–411.

[35] Y. Zhang, P. Shi, S. K. Nguang, J. Zhang and H. R. Karimi, *Finite-time boundedness for uncertain discrete neural networks with time-delays and Markovian jumps*, Neurocomputing, **140** (2014), 1–7.

[36] C. Zheng, J. Cao, M. Hu and X. Fan, *Finite-time stabilisation for discrete-time T-S fuzzy model system with channel fading and two types of parametric uncertainty*, International Journal of Systems Science, **48** (2017), 34–42.

[37] Q. Zhu, R. Rakkiyappan and A. Chandrasekar, *Stochastic stability of Markovian jump BAM neural networks with leakage delays and impulse control*, Neurocomputing, **136** (2014), 136–151.

[38] Q. Zhu and J. Cao, *Mean-square exponential input-to-state stability of stochastic delayed neural networks*, Neurocomputing, **131** (2014), 157–163.

[39] Q. Zhu and J. Cao, *Stability analysis of Markovian jump stochastic BAM neural networks with impulse control and mixed time delays*, IEEE Transactions on Neural Networks and Learning Systems, **23** (2012), 467–479.

[40] Q. Zhu and J. Cao, *Stability of Markovian jump neural networks with impulse control and time varying delays*, Nonlinear Analysis: Real World Applications, **13** (2012), 2259–2270.

[41] Q. Zhu and J. Cao, *Exponential stability of stochastic neural networks with both Markovian jump parameters and mixed time delays*, IEEE Transactions on Systems, Man, and Cybernetics, Part B (Cybernetics), **41** (2011), 341–353.

[42] Q. Zhu, J. Cao and R. Rakkiyappan, *Exponential input-to-state stability of stochastic Cohen-Grossberg neural networks with mixed delays*, Nonlinear Dynamics, **79** (2015), 1085–1098.

[43] Q. Zhu and X. Li, *Exponential and almost sure exponential stability of stochastic fuzzy delayed Cohen-Grossberg neural networks*, Fuzzy Sets and Systems, **203** (2012), 74–94.

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