DISTURBANCE REJECTION IN NONLINEAR SYSTEMS USING NEURO-FUZZY MODEL

A. AOUICHE, K. CHAFAA AND F. BOUTTOUT

Abstract. The problem of disturbance rejection in the control of nonlinear systems with additive disturbance generated by some unforced nonlinear systems, was formulated and solved by *Mukhopadhyay* and *Narendra*, they applied the idea of increasing the order of the system, using neural networks the model of multilayer perceptron on several systems of varying complexity, so the objective of this work is using the same idea with two other recent methods fast and reliable ; fuzzy set systems and hybrid neuro-fuzzy systems respectively to compute the control law which minimizes the effect of the disturbance at the output of nonlinear systems. The application of the methods previously cited in form of results is presented to determine the identification model and to provide theoretical justification to existence a solution of disturbance rejection. Our better results with fuzzy systems and neuro-fuzzy systems are presented and discussed in detail in this paper with several systems of increasing complexity.

1. Introduction

The problem of rejection of input disturbances when the disturbances can be modeled as the outputs of unforced linear or nonlinear dynamic systems is considered in this paper. The principal objective is to determine the identification model of order increased, and the control law to minimize the effect of the disturbance at the output, this work has been realized by Mukhopadhyay and Narendra using the artificial neural networks (ANNs) [9]. In this paper two techniques are presented based on fuzzy set systems (FS) and adaptive neuro-fuzzy systems (ANFIS) to resolve the problem of disturbance rejection. The FS has been successfully used as building blocks in the design of practically feasible identifiers and controllers for nonlinear dynamical systems [16]. Extensive simulation studies have shown that in robust control designs, a fixed control law based on some prior information on the uncertainties is designed to compensate for their effects, such identifiers and controllers result in satisfactory performance [3]. Robust control has some advantages such as its ability to deal with disturbances, quickly varying parameters, and unmodeled dynamics [13]. As well as the modern techniques of artificial intelligence based in the ANFIS are frequently applied together in almost all the fields,

Received: March 2017; Revised: August 2017; Accepted: December 2017

Key words and phrases: Disturbance rejection, Fuzzy set systems, Neuro-fuzzy systems, Identification of dynamical systems, Nonlinear systems.

the reasons to combine these two techniques come out of the difficulties and inherent limitations of each isolated paradigm [14]. ANNs with Back propagation (BP) learning algorithm is widely used in solving various problems even though BP convergence is slow but it is guaranteed [9]. However, ANN is black box learning approach, cannot intepretе relationship between input and output and cannot deal with uncertainties. To overcome this several approches have been combined with ANN such as feature selection [6]. Meanwhile FS is good in handling uncertainties and can intepretе relationship between input/output by producing rules. Therefore, to increase the capability of FS and ANN, hybridization of ANN and FS (ANFIS) is usually implemented, that what we are going to show in this paper, we attempt to demonstrate that those models apt to obtain better results, on the other hand we judge in this work that it is a matter of interest to investigate a maximum number of modeling methods. That will make confidence in the developed of the method purposed in [9], also those models provide a very strong performance as being the most accurate algorithms in such high dimensional input problems even when robustness of the controller is desired in the presence of bounded external disturbances. The training data for fuzzy systems and neuro-fuzzy systems are derived from numerical simulations using the analytic formulas. Once, the training dataset is prepared, it is ready for FS and ANFIS training. The Takagi-Sugeno-Kang (TSK) method is selected for back-propagation training of the FS and ANFIS too. The TSK method is chosen due to its guaranteed convergence and numerical robustness. The trained FS and ANFIS are used to predict the identification model for eliminate the effect of the disturbance at the output of the linear or nonlinear dynamic systems that it was not exposed during the training phase. Using the models previously cited with suitable training algorithms has the following advantages: 1) The number of the input variables can be increased especially for FS, for our studied models we use until nine inputs vector. 2) The trained FS and ANFIS are able to predict the identification models without using Jacobian special functions which are time consuming. This work is organized as follows: In section I, the mathematical formulation is presented for illustrate how calculate the identification models after increasing their orders for eliminate the disturbance at the output. Section II and III are devoted to theoretical formulation of the FS and ANFIS modeling, respectively. In section IV, we present numerical results and discussions, using non-trivial examples of increasing levels of complexity to demonstrate the effectiveness of the techniques suggested.

2. Declaration of the Problem

A plant in the absence of external disturbances is represented by

$$y(k+1) = f\left[y(k), ..., y(k-n+1)\right] + \sum_{j=0}^{n-1} \left[u(k-j) + v(k-j)\right] \tag{1}$$

When an external disturbance is present, the objective is to follow the output of a reference model. The latter is represented by the input-output pair $\{r(k), y_m(k)\}$ where $y_m(k)$ represents the desired output.

The theoretical objective is to determine a control input $u(k)$ so that the error $e_1(k) \overset{\Delta}{=} y(k) - y_m(k)$ is bounded and tends to zero asymptotically in many practical cases. However, as we discuss in next sections, in a certain way we can only attempt to minimize it.

3. The Problem of Disturbance Rejection

When an external disturbance is present, the overall system is described by the equations

$$\mathbf{y}(k+1) = \sum_{i=0}^{n} \alpha_i f_i[Y(k)] + D(z)[u(k) + v(k)]$$
$$u(k+1) = R(z)u(k) \tag{2}$$

Where

$$R(z) = \lambda_0 + \lambda_1 z^{-1} + ... + \lambda_{p-1} z^{-(p-1)}$$

This implies that the disturbance is generated as the output of an unforced linear difference equation. It is assumed that the disturbance is bounded, implying that the polynomial $R_1(z) = 1 - z^{-1}R(z)$ is stable [12].

Noting from (2) that $\forall\ k$:

$$D(z)v(k) = y(k+1) - \sum_{i=0}^{n} \alpha_i f_i[Y(k)] - D(z)u(k)$$

The following can be deduced

$$D(z)v(k) = D(z)R(z)v(k-1)$$

$$D(z)v(k) = \lambda_0 \left\{ y(k) - \sum_{i=1}^{N} \alpha_1 f_i \left[Y(k-1)\right] - D(z)u(k-1) \right\} +$$
$$\lambda_{p-1} \left\{ y(k-p+1) - \sum_{i=1}^{N} \alpha_i f_i \left[Y(k-p)\right] - D(z)u(k-p) \right\}$$

Thus the output of the system given by (2) can also be written as

$$y(k+1) = \sum_{i}^{\bar{N}} \bar{\alpha}_i \bar{f}_i \left[\bar{Y}(k)\right] + R(z)y(k) + \bar{D}(z)u(k)$$

$$y(k+1) = \sum_{i}^{\bar{\bar{N}}} \bar{\bar{\alpha}}_i \bar{\bar{f}}_i \left[\bar{Y}(k)\right] + \bar{D}(z)u(k) \tag{3}$$

Where

$$\bar{Y}(k) = [y(k), y(k-1), ..., y(k-p-n+1)]^T$$

And

$$\bar{N} = (p+1)N$$

$$\bar{\bar{N}} = (p+1)N + p$$

$$\bar{D}(z) = \left[\bar{\beta}_0 + \bar{\beta}_1 z^{-1} + ... + \bar{\beta}_{n+p-1} z^{-(n+p-1)}\right].D(z) \tag{4}$$

$\bar{D}(z)$: Stable since it is a product of a stable and strictly stable polynomials. $\bar{B}_0$ is equal to B_0 which was assumed to be known. Hence, the same procedure described earlier for the disturbance-free case, can be employed to adaptively control the plant in the presence of disturbances to ensure that the tracking error tends to zero [9].

The function (model) $\bar{\bar{f}}_i\left[\bar{Y}(k)\right]$ is unknown and has to be estimated. Both the FS and ANFIS can be used for this purpose in the identification model. $\bar{\bar{f}}_i\left[\bar{Y}(k)\right]$ is approximated as

$$\hat{f}[y(k),....,y(k-q),u(k-1),...,u(k-q)] \tag{5}$$

Where $q \geq n+p$ we use the series-parallel identification model in which the output of the nonlinear identification plant (rather than the model) is fed back into the identification model. Because we only use the series-parallel identification model, the static BP algorithms developed in the next sections are sufficient to train the identifiers [10].

The input-output representations of the plant and the disturbance considered in this paper are given below.

- **Stage 1**

A plant is described by the input-output equation

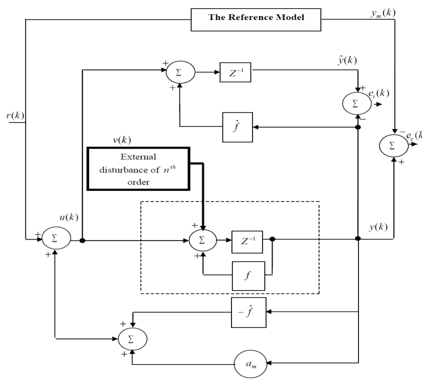
$$\begin{aligned} y(k+1) &= f\left[y(k),...,y(k-n+1)\right] + \sum_{j=0}^{n-1} \mathrm{B}_j\left[u(k-j)+v(k-j)\right] \\ v(k+1) &= \sum_{i=0}^{p-1} \lambda_i v(k-i) \end{aligned} \tag{6}$$

- **Stage 2**

A plant is described by the input-output equation

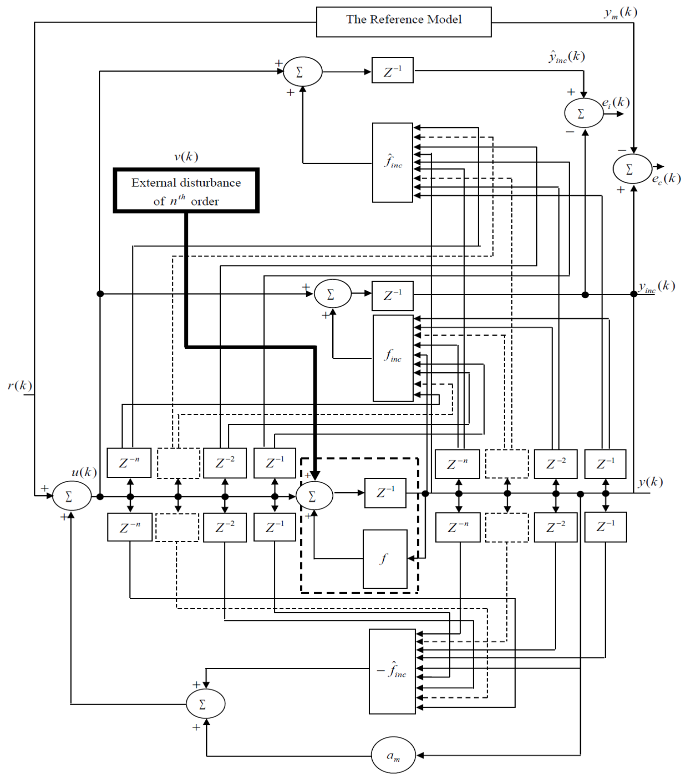
$$\begin{aligned} y(k+1) &= f\left[y(k),...,y(k-n+1)\right] + \mathrm{B}_0\left[u(k)+v(k)\right] \\ v(k+1) &= g\left[v(k),...,v(k-p+1)\right] \end{aligned} \tag{7}$$

In the following section, we present the general structure of system of n^{th} order before and after full disturbance rejection ($q=0$ and $q=n$) , noting that in all this work we consider the perturbation as a linear or nonlinear system, this structure shows the role of the control input to follow the reference model .



3.1. **Structure of System with No Disturbance Rejection $q=0$.**

3.2. **Structure of System with Full Disturbance Rejection** $q = n$.



4. **Fuzzy Set Systems**

The main part of the Fuzzy Systems is a knowledge-base consisting of fuzzy IF-THEN rules, for example:

IF the speed of a car is slow; THEN apply more force to the accelerator

At the beginning of designing fuzzy systems, we should collect fuzzy IF-THEN rules from human experts or domain knowledge [17]. The typical steps of a fuzzy reasoning consist of:

- Fuzzification: comparison of the input variables with the membership functions of the premise part in order to obtain the membership values;
- Weighing: combination, by means of a specific operator, normally multiplication or minimum, of the premise part membership values to get the firing strength of each rule;
- Generation: creation of the consequents relative to each rule;

- Defuzzification: aggregation of the consequents to produce the output.

Fuzzy models are categorized into three models according to the expressions of consequent parts:

1- Mamdani model: $y = A$ (A is a fuzzy number);
2- Takagi-Sugeno-Kang (TSK) model: $y^i_j = f^i_j(x)$ (x is an input variable);
3- Simplified fuzzy model: $y = c$ (c is a constant).

The most fundamental difference between Mamdani model and Sugeno (TSK) is the way the crisp output is generated from the fuzzy inputs [4]. The operation of the Mamdani rule base can be broken down into four parts:

1) Mapping each of the crisp inputs into a fuzzy variable (fuzzification);
2) Determining the output of each rule given its fuzzy antecedents ;
3) Determining the aggregate outputs of all of the fuzzy rules;
4) Mapping the fuzzy output(s) to crisp output(s) (defuzzification) [7].

The second type Takagi-Sugeno-Kang proposed real-valued variables inputs and outputs, an example fuzzy IF-THEN rule of TSK as follows:

IF the speed x of a car is slow; THEN the force to the accelerator is

$$y^i_j = y^i_{j0} + y^i_{j1}x_1 + y^i_{j2}x_2 + ... + y^i_{jp}x_p$$

Noted

$$y^i_j = f^i_j(x) \tag{8}$$

This third model is the special case of both the Mamdani type and the TSK models, the biggest advantage of the simplified fuzzy models is that the models are easy to design [8].

For all simulations in this work we will use the TSK fuzzy model of order 0 (singleton pattern) for modeling our nonlinear systems.

5. Back-Propagation Training for the Fuzzy System

Our task in this part is to update the parameters, Mean of antecedent part (c^l_i), Sigma of antecedent part (σ^l_i)and the consequent part (b^l), the numerical value of the output is:

$$y = \hat{f}(\underline{x}) = \frac{\sum\limits_{l=1}^{M} \omega_l \, b_l}{\sum\limits_{l=1}^{M} \omega_l} = \frac{\sum\limits_{l=1}^{M} b_l \left[\prod_{i=1}^{n} \exp\left(- \left(\frac{x_i - c^l_i}{\sigma^l_i} \right)^2 \right) \right]}{\sum\limits_{l=1}^{M} \left[\prod_{i=1}^{n} \exp\left(- \left(\frac{x_i - c^l_i}{\sigma^l_i} \right)^2 \right) \right]} \tag{9}$$

Such the mean-squared error $J = \frac{1}{2}\Big(d - \hat{f}\,(\underline{x}) \Big)^2$ is minimized to determine a fuzzy system $\hat{f}(x)$, w_l is the degree of truth of l^{th} rules IF-THEN which can be obtained by : $\mu_{A^i_1}(x_1) \wedge ... \wedge \mu_{A^i_p}(x_p)$ who $\mu_{A^i_j}(x_j)$ is the degree of membership value x_j to the fuzzy set A^i_j we have chosen a Gaussian membership function of the form shown in (9).

For adjusting the parameters b^l according to the parametric adaptation algorithm:

$$b^l(k+1) = b^l(k) - \alpha_b \frac{\partial J}{\partial b^l} \mid_k \tag{10}$$

Where $l = 1, 2, \ ... \ , M, \ k = 0, 1, 2, \ ... \ ,(\hat{f}(\underline{x})$ will be denoted by $\hat{f})$, α_b:learning rate of permise part, and therefore J depends only on b^l through A

Where also $\hat{f} = A/B$, $A = \sum_{l=1}^{M} b^l \omega_l$ and $B = \sum_{l=1}^{M} \omega_l$

Then, by deriving J compared with b^l we have:

$$\frac{\partial J}{\partial b^l} = -\left(d - \hat{f}\right) \frac{\partial \hat{f}}{\partial A} \frac{\partial A}{\partial b^l} = -\left(d - \hat{f}\right) \frac{1}{B} \omega_l \tag{11}$$

Where

$$\frac{\partial J}{\partial b^l} = -\frac{e}{B} \omega_l \tag{12}$$

Substituent (12) in (10) we obtain the algorithm of adjustment of b^l as follows:

$$b^l(k+1) = b^l(k) + \alpha_b \frac{e}{B} \omega_l \tag{13}$$

For adjusting c_i^l we have

$$\overset{l}{\underset{i}{c}}(k+1) = \overset{l}{\underset{i}{c}}(k) - \alpha_c \frac{\partial J}{\partial c_i^l} \mid_k \tag{14}$$

$l = 1, 2, \ ... \ , M, \ k = 0, 1, 2, \ ... \ ,, \ i = 1, 2, \ ... \ , n$ and α_c:learning rate of antecedent part , and we see $\hat{f}$(and therefore J) depends only on c_i^l through ω_l

Therefore:

$$\frac{\partial J}{\partial c_i^l} = -\left(d - \hat{f}\right) \frac{\partial \hat{f}}{\partial \omega_l} \frac{\partial \omega_l}{\partial c_i^l} \tag{15}$$

We have

$$\begin{aligned} \frac{\partial \hat{f}}{\partial \omega_l} &= \frac{B\,(\partial A/\partial \omega_l) - A\,(\partial B/\partial \omega_l)}{B^2} \\ &= \frac{(\partial A/\partial \omega_l) - (A/B\,)\,(\partial B/\partial \omega_l)}{B} \\ &= \frac{b^l - \hat{f}}{B} \end{aligned} \tag{16}$$

And we still have

$$\frac{\partial \omega_l}{\partial c_i^l} = \frac{2\,\left(x_i - c_i^l\right)}{\sigma_i^{l2}} \omega_l \tag{17}$$

Substituting (16) and (17) in (15) we have:

$$\frac{\partial J}{\partial c_i^l} = \frac{-\left(d - \hat{f}\right)}{B} (b^l - \hat{f}) \frac{2\,\left(x_i - c_i^l\right)}{\sigma_i^{l2}} \omega_l \tag{18}$$

So we obtain the algorithm of adjustment of parameters c_i^l :

$$\underset{i}{\overset{l}{c}}(k+1) = \underset{i}{\overset{l}{c}}(k) + \alpha_c \frac{\left(d-\hat{f}\right)}{B}(b^l - \hat{f})\frac{2\left(x_i - c_i^l\right)}{\sigma_i^{l2}}\omega_l \tag{19}$$

Using the same method for σ_i^l , we obtain the algorithm of adjustment as follows:

$$\underset{i}{\overset{l}{\sigma}}(k+1) = \underset{i}{\overset{l}{\sigma}}(k) + \alpha_\sigma \frac{e}{B}(b^l - \hat{f})\frac{2\left(x_i - c_i^l\right)^2}{\sigma_i^{l3}}\omega_l \tag{20}$$

The learning algorithm (13), (19) and (20) perform a procedure error back-propagation. For adjusting the parameters the normalized error $\frac{e}{B}$ is back propagated to the layer then its updated using (10) when is the input of b^l.

For adjusting the parameters c_i^l and σ_i^l, the normalized error $\frac{e}{B}$ adjusts $(b^l - \hat{f})$ then is back propagated to the processing unit of layer l its output is w_l; then c_i^l and σ_i^l are updated by (19) and (20) [18, 11].

For the identification of all our systems in this paper, we used the fuzzy model described by (9) with:

- The singleton fuzzification;
- Number of fuzzy rules are 20, for each input is adjusted two parameters of membership functions corresponding input and output (Gaussian, singleton respectively) that are the premise part and the consequent part;
- Defuzzification by the method of FM (fuzzy mean) [1].

6. Neuro-fuzzy Systems

One example of ANFIS would be the use of neural networks for the membership function elicitation and mapping between fuzzy sets that are utilized as fuzzy rules as shown in (Figure 1). In the training process a neural network adjusts its weights in order to minimize the mean square error between the output of the network and the desired output [2].

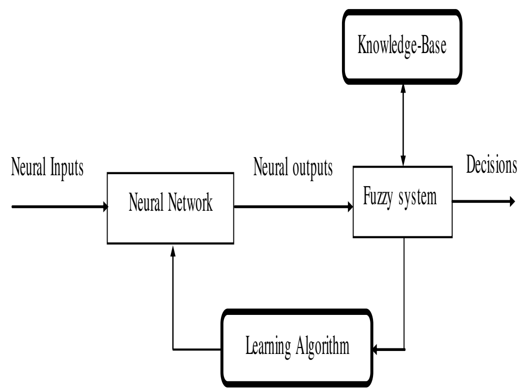


FIGURE 1. Neural Fuzzy System

The architecture of this approach is shown below. The circular nodes represent nodes that are fixed whereas the square nodes are nodes that have parameters to be learnt [5]. A Two Rule Sugeno ANFIS has rules of the form:

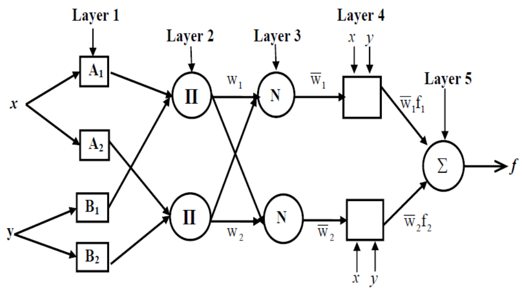


FIGURE 2. An ANFIS Architecture for a Two Rule Sugeno System

$$If\ x\ is\ A_1\ and\ y\ is\ B_1 \quad THEN \quad f_1 = p_1x + q_1y + r_1$$
$$If\ x\ is\ A_2\ and\ y\ is\ B_2 \quad THEN \quad f_2 = p_2x + q_2y + r_2$$

For the training of the network, there is a forward pass and a backward pass. We now look at each layer in turn for the forward pass. The forward pass propagates the input vector through the network layer by layer. In the backward pass, the error is sent back through the network in a similar manner to back-propagation [18].

Layer 1: The output of each node is:

$$O_{1,i} = \mu_{A_i}(x) \qquad for \quad i = 1,2$$
$$O_{1,i} = \mu_{B_{(i-2)}}(y) \quad for\ i = 3,4$$

So, the $O_{1,i}(x)$ is essentially the membership grade for x and y. The membership functions could be anything but for illustration purposes we will use the bell shaped function given by:

$$\mu_A(x) = \frac{1}{1 + \left|\frac{x-c_i}{a_i}\right|^{2b_i}} \tag{21}$$

Where $a_i, b_i, c_i$are parameters to be learnt. These are the premise parameters.

Layer 2: Every node in this layer is fixed. This is where the t-norm is used to AND the membership grades , for example the product:

$$O_{2,i} = w_i = \mu_{A_i}(x)\mu_{B_i}(y), \qquad i = 1,2$$

Layer 3: Layer 3 contains fixed nodes which calculate the ratio of the firing strengths of the rules:

$$O_{3,i} = \overline{w}_i = \frac{w_i}{w_1 + w_2} \tag{22}$$

Layer 4: The nodes in this layer are adaptive and perform the consequent of the rules:

$$O_{4,i} = \overline{w_i} f_i = \overline{w_i}(p_i x + q_i y + r_i) \tag{23}$$

The parameters in this layer (p_i, q_i, r_i) are to be determined and are referred to as the consequent parameters.

Layer 5: There is a single node here that computes the overall output [15]:

$$O_{5,i} = \sum_i \overline{w_i} f_i = \frac{\sum_i w_i f_i}{\sum_i w_i} \tag{24}$$

7. Neuro-Fuzzy Synthesis Model

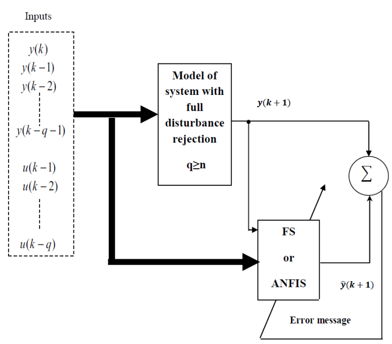


Figure 3. FS and ANFIS Synthesis Model

We verify that the FS and ANFIS models after learning are actually able to predict the desired output for values given at the entry which are not used in the learning.

8. Simulation Results

We discuss the results of a simulation experiment where the disturbance is generated by a linear or nonlinear dynamical system.

8.1. Stage 1.

- System 1

A first order plant in the presence of an external disturbance $v(k)$

$$y(k+1) = \frac{y(k)}{1 + y(k)^2} + u(k) + v(k) \tag{25}$$

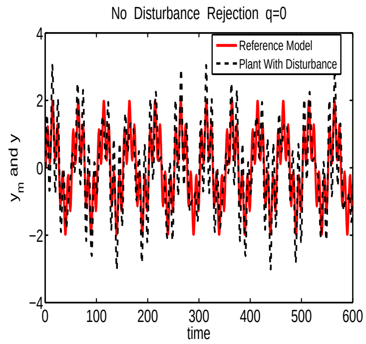


FIGURE 4. System 1 with No Disturbance Rejection

The disturbance is sinusoidal in nature and can be modelled as the output of the following linear unforced dynamical system :

$$\begin{aligned} v_1(k+1) &= \cos(\Psi)v_1(k) + \sin(\Psi)v_2(k) \\ v_2(k+1) &= \cos(\Psi)v_2(k) - \sin(\Psi)v_1(k) \\ v(k) &= v_1(k) \end{aligned} \tag{26}$$

Where $(2\Pi)/\Psi$ is the period of the sinusoidal disturbance .In the simulation Ψ was chosen to be ${}^{\Pi}/_{6}$ and the initial conditions of the disturbance generating system was chosen to be $[v_{10}, v_{20}] = [1, 0]$ so that the disturbance was a cosine function of time with amplitude 1 and period 12, the disturbance of second order can also be expressed in an autoregressive form given by

$$v(k+1) = \lambda_1 v(k) + \lambda_2 v(k-1)$$

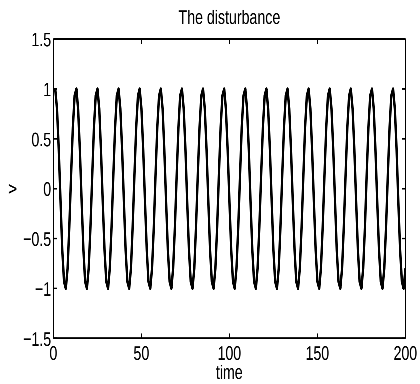


FIGURE 5. Sinusoidal Disturbance

Eliminating $v(k)$ from (25), the plant with the disturbance can be described by

$$y_{inc}(k+1) = f_{inc}[\underbrace{y(k), y(k-1), y(k-2),\ u(k-1),\ u(k-2)}_{5\ inputs}] + u(k) \tag{27}$$

To identify the plant with the disturbance, an identification model is set up as described below:

$$\hat{y}(k+1) = F[y(k), y(k-1), ..., y(k-q),\ \ u(k-1), u(k-2), ..., u(k-q)] + u(k)$$

Here $F(..)$ is realised in our paper by a fuzzy set system and neuro-fuzzy system model of Takagi-Sugeno as an approximation to f_{inc}, for exact identification of the system described by (27) q should be greater than or equal to 2 (the order of the disturbance and the original system).

The identification was carried out off-line in the presence of the disturbance with uniformly random input in the range [-1, 1].

The results of the tracking problem for $q = 0$ (no disturbance rejection) is shown in (Figure 4) and for $q = 2$ (full disturbance rejection) is shown in (Figure 6(a), Figure 7(a)) using FS and ANFIS respectively.

It is evident that $q = 2$ results in much better response than that obtained with $q = 0$.

The desired output is assumed to be the output of the reference model

$$y_m(k+1) = 0.6y_m(k) + r(k)$$

With the reference input

$$r(k) = 0.5\sin((2\Pi k)/50)) + 0.5\sin((2\Pi k)/10))$$

The control input is computed as

$$u_r(k) = (y_m(k+1) - F[y(k), y(k-1), ..., y(k-q),\ u(k-1), u(k-2), ..., u(k-q)])$$

f_{inc}: Increased function (system) with disturbance.

u_r : Control law to reject the disturbance at the output.

$q = 2$:Full disturbance rejection

$q = 0$:No disturbance rejection.

a) **Using Fuzzy System**

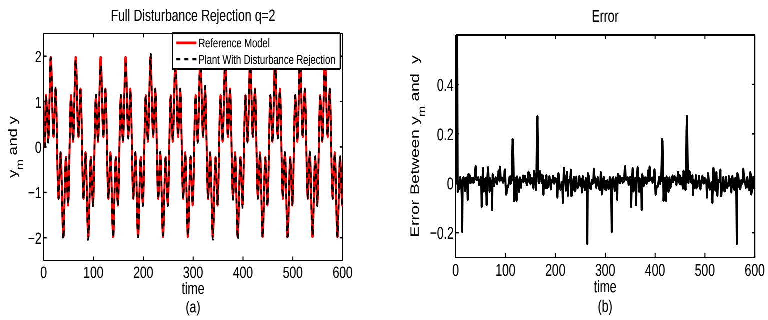


FIGURE 6. Simulation Result of System 1 Using Fuzzy System:(a) Full Disturbance Rejection; Solid Line (Desired Output y_m),Dotted Line (Plant Output y),(b) Error Between the Model and the Plant

b) **Using Neuro-Fuzzy System**

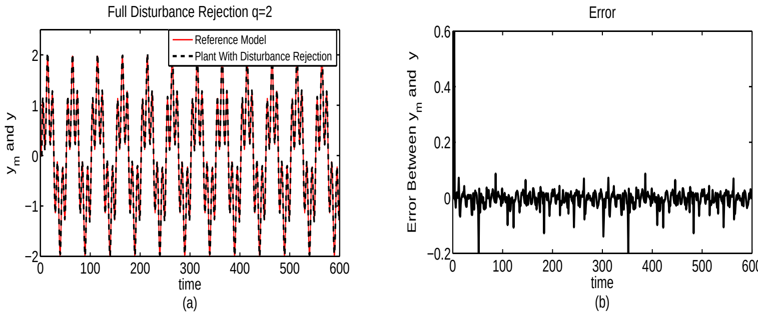


FIGURE 7. Simulation Result of System 1 Using Neuro-fuzzy System:(a) Full Disturbance Rejection; Solid Line (Desired Output),Dotted Line (Plant Output),(b) Error Between the Model and the Plant

- System 2

A first order system in the presence of an external disturbance $v(k)$ given by

$$y(k+1) = \frac{\cos(y(k))}{1+\sin{(y(k))}^2} + \sin(y(k))\exp(-y(k)^2) + \frac{y(k)}{1+y(k)^2} + u(k) + v(k) \tag{28}$$

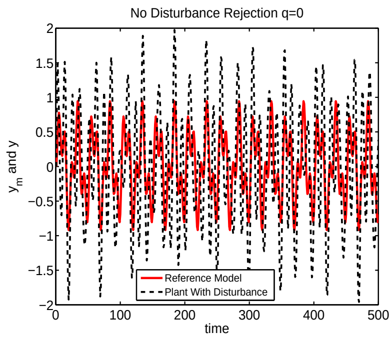


FIGURE 8. System 2 with No Disturbance Rejection

The disturbance of the fourth order can be modelled as

$$x_v(k+1) = \begin{bmatrix} \cos\psi_1 & \sin\psi_1 & 0 & 0 \\ -\sin\psi_1 & \cos\psi_1 & 0 & 0 \\ 0 & 0 & \cos\psi_2 & \sin\psi_2 \\ 0 & 0 & -\sin\psi_2 & \cos\psi_2 \end{bmatrix} x_v(k)$$
$$v(k) = x_{v1}(k) + x_{v3}(k) \tag{29}$$

Where $x_v(k) \in R^4$, Ψ_1 and Ψ_2 determine the periods of two sine functions constituting the disturbance and are chosen to be $\Psi_1 = \frac{\Pi}{6}$, $\Psi_2 = \frac{\Pi}{12}$ and $x_{vi}(k)$ is the i^{th} element of $x_v(k)$.

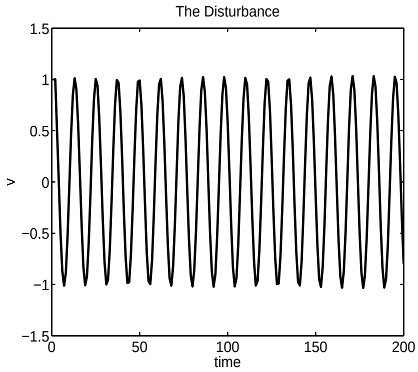


FIGURE 9. The Disturbance of the Fourth Order

Eliminating $v(k)$ from (28), the plant with the disturbance can be described by

$$y_{inc}(k+1) = f_{inc}\underbrace{[y(k), y(k-1), ..., y(k-4),\ u(k-1), u(k-2), ..., u(k-4)]}_{9\ inputs} + u(k) \tag{30}$$

To identify the plant with the disturbance, an identification model is set up as described below

$$\hat{y}(k+1) = F[y(k), y(k-1), ..., y(k-q-1),\ u(k-1), u(k-2), ..., u(k-q)] + u(k)$$

$F(..)$ is realised in this paper by a fuzzy set system and neuro-fuzzy system model of Takagi-Sugeno as an approximation to f_{inc}, for exact identification of the system described by (30) q should be greater than or equal to 4 (Figure 10(a), Figure 11(a)).

The identification was carried out off-line in the presence of the disturbance with uniformly random input in the range [-1, 1].

The desired output is assumed to be the output of the reference model

$$y_m(k+1) = 0.6y_m(k) + r(k)$$

With the reference input

$$r(k) = 0.25\sin(2\Pi k/25) + 0.25\sin(2\Pi k/10)$$

The control input is computed as

$$u_r(k) = (y_m(k+1) - F(..))$$

$q = 4$:Full disturbance rejection

$q = 0$:No disturbance rejection.

a) **Using Fuzzy System**

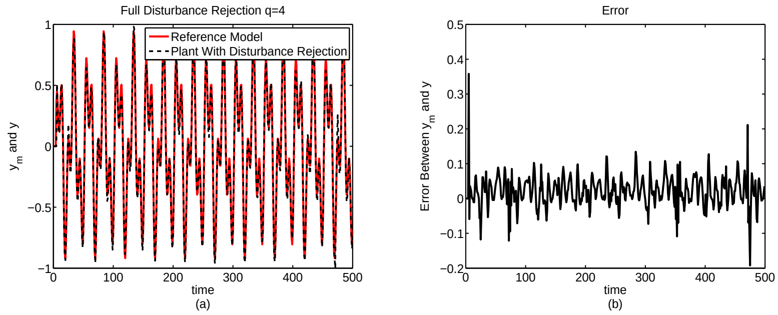


FIGURE 10. Simulation Result of System 2 Using Fuzzy System:(a) Full Disturbance Rejection; Solid Line (Desired Output),Dotted Line (Plant Output),(b) Error Between the Model and the Plant

b) **Using Neuro-Fuzzy System**

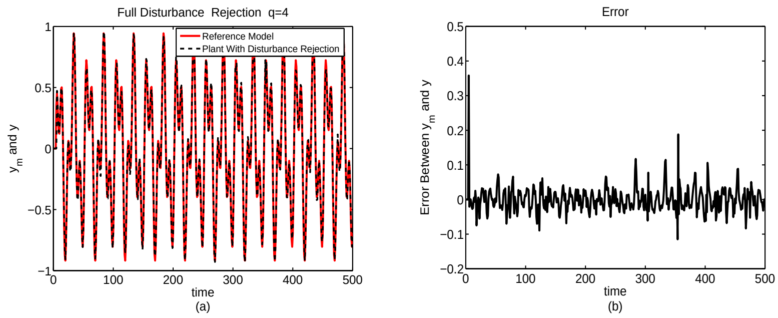


FIGURE 11. Simulation Result of System 2 Using Neuro-fuzzy System:(a) Full Disturbance Rejection; Solid Line (Desired Output),Dotted Line (Plant Output),(b) Error Between the Model and the Plant

8.2. **Stage 2.**

A second order plant in the presence of an external disturbance $v(k)$ given by

$$y(k+1) = \frac{\cos(y(k))}{1+\sin\left(y(k-1)\right)^2} + \sin(y(k-1))\exp(-y(k)^2) + \frac{y(k)}{1+y(k)^2} + u(k) + v(k) \tag{31}$$

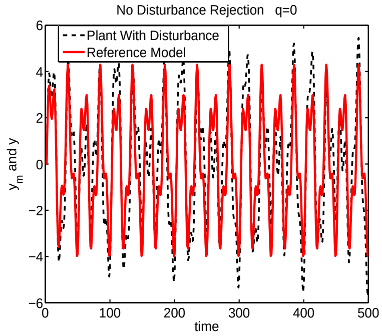


FIGURE 12. The Plant with No Disturbance Rejection

The disturbance of van der pol is assumed to be generated as the output of a second-order nonlinear system:

$$
\begin{aligned}
x_{v1}(k+1) &= x_{v1}(k) + 0.2x_{v2}(k) \\
x_{v2}(k+1) &= -0.2x_{v1}(k) + x_{v2}(k) - 0.1(x_{v1}(k)^2 - 1)x_{v2}(k) \\
v(k) &= x_{v1}(k)
\end{aligned}
\tag{32}
$$

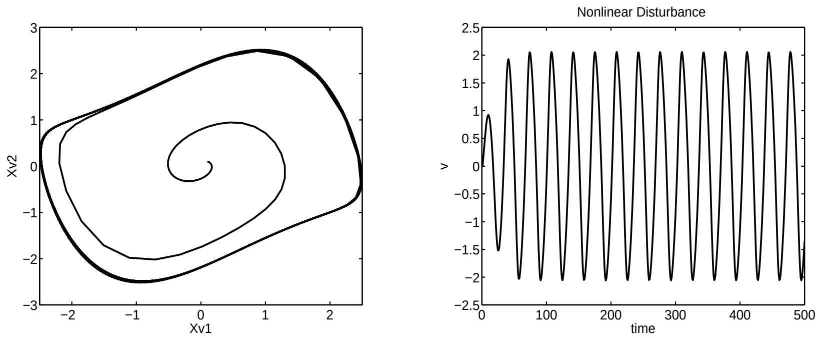


FIGURE 13. Disturbance of Van Der Pol

Eliminating $v(k)$ from (31), the plant in the presence of the disturbance can be described by

$$y_{inc}(k+1) = f_{inc}\underbrace{[y(k), y(k-1), y(k-2), y(k-3),\ u(k-1), u(k-2)]}_{6\ inputs} + u(k)$$

To identify the plant with the disturbance, an identification model is set up as described below

$$\hat{y}(k+1) = F[y(k), y(k-1), ..., y(k-q-1),\ u(k-1), u(k-2), ..., u(k-q)] + u(k)$$

The identification was carried out off-line in the presence of the disturbance with uniformly random input in the range [-2, 2], where $q \geq 2$ (Figure 14(a) , Figure 15(a)).

The desired output is assumed to be the output of the reference model

$$y_m(k+1) = 0.6y_m(k) + r(k)$$

With the reference input

$$r(k) = \sin((2\Pi k)/10)) + \sin((2\Pi k)/25))$$

The control input is computed as

$$u_r(k) = (y_m(k+1) - Fs(..))$$

$q = 2$ (Full disturbance rejection), $q = 0$ (No disturbance rejection).

a) **Using Fuzzy System**

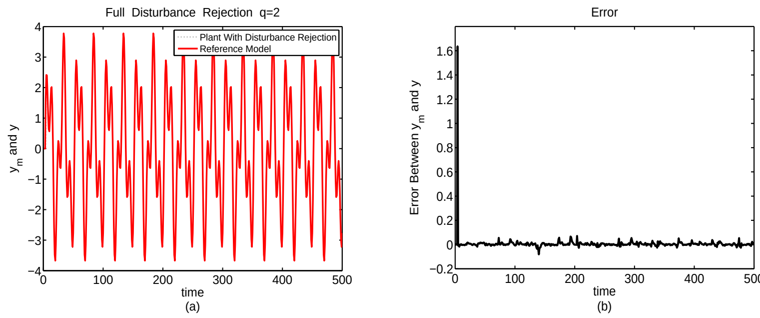


FIGURE 14. Simulation Result of the Plant Using Fuzzy System:(a) Full Disturbance Rejection; Solid Line (Desired Output),Dotted Line (Plant Output),(b) Error Between the Model and the Plant

b) **Using Neuro-Fuzzy System**

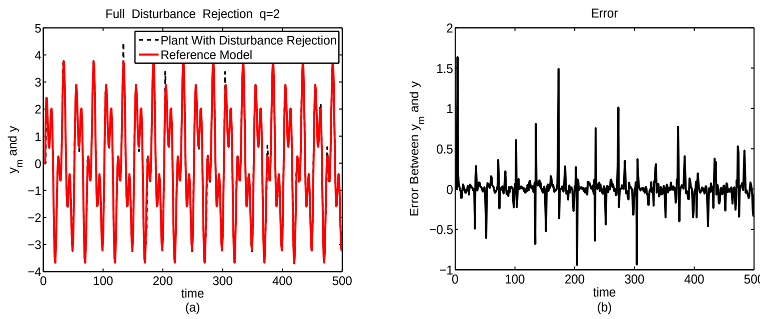


FIGURE 15. Simulation Result of the Plant Using Neuro-fuzzy System:(a) Full Disturbance Rejection; Solid Line (Desired Output),Dotted Line (Plant Output),(b) Error Between the Model and the Plant

8.3. Architectures of Our Models Used To Predict The Outputs Of Systems With Full Disturbance Rejection.

a) **Fuzzy Logic Model**

Structure	Stage 1		Stage 2
	System 1	System 2	
Number of rules	20	20	20
Input variables	5	9	6
Learning rate α_c	0.1	0.09	0.05
Learning rate α_b	0.03	0.09	0.4
Sigma of antecedent part σ	0.9	0.9	1
Number of iterations	844	8083	3000
Training time (second)	2901	1.8904e+003	2.2636e+003
MSE between (y) and (y_m)	0.0023	0.0024	0.0055

Table 1. Structure of Fuzzy Logic Model Used to Identify the Increased Order Model

b) **Neuro-Fuzzy Model**

Structure	Stage 1		Stage 2
	System 1	System 2	
Number of rules	32	512	64
Input variables	5	9	6
Number of nodes	92	1072	161
Number of linear parameters	192	5120	448
Number of nonlinear parameters	30	54	36
Total number of parameters	222	5174	484
Number of training data pairs	600	100	500
Training time (second)	7.5972	841.4070	2.7227e+003
MSE between (y) and (y_m)	0.0015	0.0013	0.0367

Table 2. Structure of Neuro-Fuzzy Model Used to Identify the Increased Order Model

9. Numerical Results

The FS and ANFIS are used in this work to identify the model of order q increased, after that the control law to minimize the effect of the disturbance at the output is obtained. FS and ANFIS are trained to have very effective results with less complicated architectures. To confirm the effectiveness of the techniques suggested, two stages have been studied and their results are obtained.

9.1. **Stage 1.**

- System 1

A first order system in the presence of an external sinusoidal disturbance of second order ($q = 2$), the order of the model to identify by FS and ANFIS must be ($q \geq 2$) it means greater or equal five inputs vector for full disturbance rejection. The mean square error MSE between the desired outputs and the outputs of models is 0.0023 for FS (Figure (6b)) after 2901 seconds (48.35 minutes) and 0.0015 for ANFIS (Figure (7b)) after 7.9 seconds. In this system the hybrid ANFIS demonstrates its effectiveness compared to FS.

- System 2

A first order system in the presence of an external sinusoidal disturbance of fourth order ($q = 4$), the order of the model to identify by FS and ANFIS must be ($q \geq 4$) it means greater or equal nine inputs vector. The mean square error MSE between the desired outputs and the outputs of models is 0.0024 for FS (Figure (10b)) after 31.5067 minutes and 0.0013 for ANFIS (Figure (11b)) after 14.0235 minutes. Also in this system the hybrid ANFIS demonstrates its effectiveness compared to FS.

9.2. **Stage 2.**

A second order plant in the presence of an external nonlinear disturbance of second order ($q = 2$), the order of the model to identify by FS and ANFIS must be ($q \geq 2$) it means greater or equal six inputs vector. The mean square error MSE between the desired outputs and the outputs of the models is 0.0055 for FS (Figure (14b)) after 37.6667 minutes and 0.0367 for ANFIS (Figure (15b)) after 45.3333 minutes. Because of the nonlinearity of the disturbance, we see in this system the hybrid ANFIS gave a less efficient result than FS.

10. **Conclusion**

In this paper, the problem of disturbance rejection has been studied in order to calculate the control law from the nonlinear dynamical model of order greater than or equal the order of the disturbance and system, this model have been identified by FS and ANFIS techniques. This study based on the consideration of the external disturbance is modeled as the output of linear or nonlinear unforced dynamic system. First, the obtained simulation data are calculated from the theoretical justification previously mentioned. Second, FS and ANFIS architectures are implemented using the back-propagation method in conjunction with TSK training algorithms coding under Matlab @ R13, two stages in the form of simulation studies have been shown for problems of increasing complexity; we demonstrated the efficiency of FS and ANFIS , the very good agreement between the aforementioned methods is obtained. Finally, the presented technique of disturbance rejection can easily be applied to other physical systems. And for the techniques of artificial intelligence we can use the method of fuzzy logic type 2, fast and reliable who can be developed in this subject.

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