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DIRECT ADAPTIVE FUZZY PI SLIDING MODE CONTROL OF SYSTEMS WITH UNKNOWN BUT BOUNDED DISTURBANCES

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ABSTRACT. An asymptotically stable direct adaptive fuzzy PI sliding mode controller is proposed for a class of nonlinear uncertain systems. In contrast to other existing approaches of handling disturbances, the proposed approach does not require this bound to be known, only requiring that it exists. Moreover, a PI control structure is used to attenuate chattering. The approach is applied to stabilize an open-loop unstable nonlinear system as well as the Duffing forced-oscillation chaotic nonlinear system amid significant disturbances. Analysis of simulations reveals the effectiveness of the proposed method in terms of a significant reduction in chattering while maintaining asymptotic convergence.

1. Introduction

The control of nonlinear uncertain systems has always been a challenging and yet rewarding problem and, since conventional methods such as feedback linearization do not adequately address the inherent complexities and uncertainties in real systems and may fail [13], many new approaches for solving the problem have been proposed [8, 13]. Among methods that address uncertainty in plant dynamics is adaptive control, which aims to model and track changing system/control parameters in real time. Adaptive control methods, however, generally guarantee parameter convergence only if parameter changes are slow enough [2,11]. Furthermore, the existing adaptive control approaches require the general structure of the plant, such as its order, to be known.

In comparison, variable structure control (VSC) and in particular sliding mode control (SMC) aim to provide good robustness against system uncertainties and external disturbances [13,16]. Assuming certain characteristics of the mathematical model of the plant under control and that system uncertainties remain within known intervals, SMC generally provides guaranteed asymptotic stability. However, due to its discontinuous control design, this approach suffers from another drawback, commonly referred to as chattering. The chattering problem is usually resolved by introducing a boundary layer around the sliding surface and applying continuous

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control inside this boundary layer. However, the state trajectory of the resulting system may no longer converge to the sliding surface and asymptotic convergence is usually lost [9].

Fuzzy logic, an alternative to conventional control methodologies, and particularly in control of complex uncertain systems, has been the focus of numerous studies in the past two decades [19]. This is mainly due to the possibility of making use of fuzzy knowledge-based control as a complement to mathematical knowledge in dealing with systems whose dynamics are not so well understood and whose models can not be so conveniently established [10,18]. However, pure fuzzy logic based systems generally suffer from the increasing number of rules, i.e. the curse of dimensionality, while also being unable to generally guarantee closed-loop stability. A hybrid combination of the VSC methodology, fuzzy logic, and adaptive control may therefore provide an attractive ground for designing robust control systems with high degrees of nonlinearities and uncertainties [6, 14].

In following the above analysis, recent research has proposed various adaptive fuzzy sliding mode control (AFSMC) strategies such as in [3,4,5,7,15,17]. Specifically, [17] proposed an adaptive fuzzy sliding mode for a class of non-affine nonlinear systems with relative degree equal to the system's degree. A new scheme of adaptive fuzzy sliding mode control, which incorporates adaptive model tracking with sliding mode and fuzzy systems, has been investigated in [4]. In [15] a new design of an adaptive fuzzy sliding mode controller for linear systems with mismatched time varying uncertainties has been presented. In [3], fuzzy systems as universal approximators and SMC have been used to control a class of nonlinear systems. An adaptive law has been introduced that is robust to approximation error to improve the approximation accuracy. An indirect adaptive control strategy by using a PI control term, which approximated the switching control to minimize the amount of chattering, has been devised in [7]. In [5] an adaptive fuzzy sliding mode control for robotic manipulators with asymptotic stability has been established. Finally, the authors in [12] have proposed an indirect adaptive fuzzy controller for a class of nonlinear uncertain systems with bounded disturbances. However, while adequately addressing several of the issues in control of complex systems, the above approaches are common in their requiring of a *known* bound for system uncertainties. In other words, their results may no longer be held if the bound assumptions on system uncertainties are violated.

In contrast, we here propose a novel approach to *direct* adaptive fuzzy PI sliding mode control for a class of uncertain nonlinear systems with disturbances that are bounded but where these bounds are *not* known. The proposed direct adaptive PI structure attenuates chattering as well as guaranteeing asymptotic stability. The method has a general algorithm as previously reported in [1]; here we provide a more detailed description and extend the simulation by adding more comparisons/analysis. Although our earlier algorithm in [12] was able to control a more general class of nonlinear uncertain systems than the one in this paper, it required that the bound of disturbances be known in advance. Therefore, compared with the other existing approaches of handling external disturbances, the proposed approach has the advantage that it only requires a bound to *exist,* while the magnitude of this bound does not need to be *known*. One may argue that an alternative solution to this problem is to assume a *larger* bound that *would* be sufficiently large. However, such a large bound would also result in larger chattering.

This paper is organized as follows. Section 2 formulates the class of nonlinear systems under consideration here, and describes the assumptions based on the theory of SMC, fuzzy logic systems and universal approximation theorem. In Section 3, the proposed direct adaptive fuzzy PI sliding mode control approach is presented. To show the effectiveness of the proposed method, in section 4, it is applied to two nonlinear systems — an open-loop unstable nonlinear system and a chaotic Duffing forced-oscillation stabilization amid significant uncertainties. Simulation results indicate the superiority of the approach in presence of disturbances.

2. **Problem Formulation, Sliding Mode Control and Fuzzy Systems**

2.1. **Problem Formulation.** Consider a class of SISO *n*-th order nonlinear systems in the following form,

$$\begin{aligned} x^{(n)} &= f(X,t)+bu(t)+d(X,t) \\ y &= x \end{aligned} \tag{1}$$

Where f is an unknown bounded nonlinear function, $X^T=[x,\dot{x},\cdots,x^{(n-1)}]$ $=[x_1,x_2,\cdots,x_n]\in R^n$ is the state vector of the system which is assumed to be available for measurement, $u\in R$ and $y\in R$ are respectively the input and the output of the system and $d(X,t)$ is an unknown, bounded external disturbance, but the bound is unknown.. Without loss of generality, b is assumed to be an unknown positive constant; since if b is negative, the control law can be derived similarly. In other words we have following assumptions.

Assumption 2.1. The external disturbance $d(X,t)$ is bounded by a constant D , i.e.,

$$\left|d(X,t)\right|\le D. \tag{2}$$

Remark 2.1. In comparison with other approaches to handling external disturbances, the proposed approach only requires that a bound *D* exist, but its magnitude does not need to be known.

The control objective is to design u such that the state of the system X follows the desired state X_d in presence of uncertainties and disturbances, that is the tracking error

$$E = X - X_d = [e, \dot{e}, \cdots, e^{(n-1)}]^T \tag{3}$$

should converge to zero.

Assumption 2.2. The desired trajectory vector X_d is a known continuous and bounded function as below,

$$\|X_d\| < c\ . \tag{4}$$

2.2. **Sliding Mode Control.** The SMC control problem can be consequently stated as follows. For system (1) define a sliding surface by the following scalar function

$$s = e^{(n-1)} + c_{n-1}e^{(n-2)} + \cdots + c_2\dot{e} + c_1 e\ . \tag{5}$$

This is a time-varying surface in the state space R^n. If the coefficients $c_i, i = 1, \cdots, n-1$ are chosen such that the polynomial $\lambda^{n-1} + c_{n-1}\lambda^{n-2} + \cdots + c_2\lambda + c_1$ is Hurwitz, the differential equation $s(E) = 0$ with the initial condition $E(0) = 0.$ has the unique solution $E(t) = 0$. Therefore the objective becomes to design a control law that forces the state trajectory to a sliding surface $s(E) = s(X,t) = 0$ in finite time and to remain on this surface. For achieving this objective the control law must be chosen such that

$$\frac{1}{2}\frac{d}{dt}s^2 \le -\eta_m |s|$$

or,

$$s\dot{s} \le -\eta_m |s| \tag{6}$$

where η_m is a positive constant and,

$$\dot{s} = \sum_{i=1}^{n-1} c_i e^{(i)} + f(X,t) + bu(t) - x_d^{(n)} + d(X,t)\ . \tag{7}$$

Therefore,

$$s\Big(\sum_{i=1}^{n-1} c_i e^{(i)} + f(X,t) + bu(t) - x_d^{(n)} + d(X,t)\Big) \le -\eta_m |s|\ . \tag{8}$$

Solving the above inequality for $u(t)$ and using Assumption 2, we have the input $u^*(t)$ as follows

$$\begin{aligned} u^*(t) &= \frac{1}{b}\left[-\sum_{i=1}^{n-1} c_i e^{(i)} - f(X,t) + x_d^{(n)} - d(X,t) - \eta_b \operatorname{sgn}(s)\right] \\ &= u_{eq}^* - u_{sw}^* \end{aligned} \tag{9}$$

where, $\eta_b \geq \eta_m$, and guarantees the sliding condition (6), where u_{eq}^*, u_{sw}^* are defined as follows:

$$u_{eq}^*(t) = \frac{1}{b}\left[-\sum_{i=0}^{n-1} c_i e^{(i)} - f(X,t) + x_d^{(n)} - d(X,t)\right] \tag{10}$$

$$u_{sw}^*(t) = \frac{\eta_b}{b} \operatorname{sgn}(s) \tag{11}$$

and $\operatorname{sgn}(.)$ is the sign function.

However, $f(X,t)$, $d(X,t)$ and b are generally unknown and cannot be used for constructing the control law (9). Furthermore, the chattering due to the switching function can invoke undesirable dynamics.

Here, we attend to the above challenges by proposing a hybrid direct adaptive fuzzy sliding mode and an adaptive PI structure. The following section outlines the basic characteristics of fuzzy systems that are used in this approach.

2.3. **Fuzzy Logic Systems and Universal Approximation Theorem.** The fuzzy logic systems (FLS) detailed in [20] are briefly reviewed below for continuity of discussion. FLS perform a mapping from $U_1 \times U_2 \times \cdots \times U_n = U \subset R^n$ to $V \subset R$. A fuzzy system consists of a fuzzifier, fuzzy rule base, fuzzy inference engine and defuzzifier. The fuzzy rule base consists of a collection of canonical fuzzy If-Then rules such as,

$$R^{(l)}: \text{if } x_1 \text{ is } F_1^l \text{ and} \cdots \text{ and } x_n \textit{ is } F_n^l \text{ then } y \text{ is } G^l \qquad l = 1, \cdots, M \tag{12}$$

where $X = [x_1, x_2, \cdots, x_n]^T \in U$ and $y \in V$ are respectively the input and output of the fuzzy system, M is the total number of rules; and F_i^l and G^l are fuzzy sets in U_i and V, respectively. The fuzzy inference engine performs a mapping from fuzzy sets in U to fuzzy sets in V, based on the fuzzy rule base. Furthermore, the fuzzifier maps a crisp point $X = [x_1, x_2, \cdots, x_n]^T \in U$ to a fuzzy set in U and the defuzzifier maps fuzzy sets in V to a crisp point in V.

Using the singleton fuzzifier, a product inference engine and a center average defuzzifier, the output of fuzzy system can be expressed as,

$$y = \frac{\sum_{l=1}^{M} \bar{y}^{l} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)} = \theta^T \xi(X) \tag{13}$$

where, $\theta = [\bar{y}^1, \bar{y}^2, \cdots, \bar{y}^M]^T$ is the center of the output fuzzy membership functions and is also the adjustable parameter vector, and $\xi(X) = [\xi_1(X), \xi_2(X), \cdots, \xi_M(X)]^T$ is the vector of fuzzy basis functions defined as below,

$$\xi_j(X) = \frac{\prod_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{l=1}^{M} \left(\prod_{i=1}^{n} \mu_{F_i^l}(x_i)\right)}, \qquad j = 1, \cdots, M \tag{14}$$

We have the following theorem:

Theorem 2.1. [20] *For any given real continuous function g on the compact set $U \subset R^n$ and arbitrary $\varepsilon > 0$, there exists a fuzzy system $f^*(X) = \theta^{*T} \xi(X)$ of the form (13) such that*

$$\sup_{X \in U} \left| f^*(X) - g(X) \right| < \varepsilon . \tag{15}$$

The above theorem states that the fuzzy systems of the form (13) can approximate any real continuous function to any degree of accuracy. This means the fuzzy systems of the form (13) have the universal approximation property reported earlier in [20].

3. **The Proposed Control Law**

The SMC-based controller in Section 2.2 has two terms as mentioned in (10) and (11). Since the nonlinear functions $f(X,t)$ and $d(X,t)$ are not known in (10), we use the fuzzy system,

$$\hat{u}_{eq}(X \mid \theta_1) = \theta_1^T \xi(X) \tag{16}$$

with free parameter θ_1, to approximate (10). Furthermore, to attenuate chattering, when the state is within the boundary layer $|s| < \varphi$ (φ is the thickness of the boundary layer), a PI-type controller,

$$\hat{u}_{sw}(s \mid \theta_2) = \theta_2^T \psi(s) = K_p s + K_I \int s \, dt \tag{17}$$

with $\psi^T(s) = [s, \int s \, dt]$, and $\theta_2^T = [K_P, K_I]$ is used instead of switching control (11). The control action is kept at the saturated value η when the state is outside the boundary layer, i.e. $|\hat{u}_{sw}(s \mid \theta_2)| \geq \eta$ (η is a control parameter) when $|s| > \varphi$. We propose a controller in the following form:

$$u(X \mid \theta) = \hat{u}_{eq}(X \mid \theta_1) - \hat{u}_{sw}(s \mid \theta_2) \,. \tag{18}$$

Consequently, u_{eq}^* and u_{sw}^* as mentioned in (10) and (11) are the ideal for $\hat{u}_{eq}$ and $\hat{u}_{sw}$, respectively. They can be expressed by the following equations,

$$u_{eq}^*(t) = \frac{1}{b}\left[-\sum_{i=1}^{n-1} c_i e^{(i)} - f(X,t) + x_d^{(n)} - d(X,t)) \right]. \tag{19}$$

$$u_{sw}^*(t) = \frac{\eta_b}{b} \operatorname{sgn}(s) \tag{20}$$

Thus, the desired control can be written as,

$$u^*(t) = u_{eq}^*(t) - u_{sw}^*(t) \,. \tag{21}$$

The optimal θ_1 and θ_2 are defined as follows,

$$\theta_1^* = \arg \min_{\theta_1 \in R^M} \left[\sup_{X \in R^n} \left| \hat{u}_{eq}(X \mid \theta_1) - u_{eq}^* \right| \right], \tag{22}$$

$$\theta_2^* = \arg \min_{\theta_2 \in R^2} \left[\sup_{s \in R} \left| \hat{u}_{sw}(s \mid \theta_2) - u_{sw}^* \right| \right]. \tag{23}$$

and the minimum approximation error of the fuzzy system can be obtained as

$$\omega = \hat{u}_{eq}(X \mid \theta_1^*) - u_{eq}^*(t) \tag{24}$$

Therefore we have the following theorem :

Theorem 3.1. *Consider the nonlinear system (1) and control law (18). The closed-loop system signals are bounded and the tracking error converges to zero asymptotically if the following adaptation laws hold:*

$$\dot{\theta}_1 = -\gamma_1 s \xi(X) \tag{25}$$

$$\dot{\theta}_2 = \gamma_2 s \psi(s). \tag{26}$$

Proof. Consider the Lyapunov function candidate:

$$V = \frac{1}{2}s^2 + \frac{b}{2\gamma_1}\tilde{\theta}_1^T\tilde{\theta}_1 + \frac{b}{2\gamma_2}\tilde{\theta}_2^T\tilde{\theta}_2 \tag{27}$$

where, $\tilde{\theta}_1 = \theta_1 - \theta_1^*$, $\tilde{\theta}_2 = \theta_2 - \theta_2^*$ and γ_1, γ_2 are positive constant. The time derivative of (27) along (1) and using (7) leads to

$$\begin{aligned}
\dot{V} &= s\dot{s} + \frac{b}{\gamma_1}\tilde{\theta}_1^T\dot{\theta}_1 + \frac{b}{\gamma_2}\tilde{\theta}_2^T\dot{\theta}_2 \\
&= s\Big(\sum_{i=1}^{n-1} c_i e^{(i)} + f(X,t) + bu(t) - x_d^{(n)} + d(X,t)\Big) + \frac{b}{\gamma_1}\tilde{\theta}_1^T\dot{\theta}_1 + \frac{b}{\gamma_2}\tilde{\theta}_2^T\dot{\theta}_2 \\
&= s\Big(\sum_{i=1}^{n-1} c_i e^{(i)} + f(X,t) + bu(t) - b(u_{eq}^*(t) + \hat{u}_{eq}(X\mid\theta_1^*)) + b(u_{eq}^*(t) + \hat{u}_{eq}(X\mid\theta_1^*)) \\
&\quad - x_d^{(n)} + d(X,t)\Big) + \frac{b}{\gamma_1}\tilde{\theta}_1^T\dot{\theta}_1 + \frac{b}{\gamma_2}\tilde{\theta}_2^T\dot{\theta}_2.
\end{aligned} \tag{28}$$

Using (18), (19) and (24) we have

$$\begin{aligned}
\dot{V} &= sb(\hat{u}_{eq}(X\mid\theta_1) - \hat{u}_{eq}(X\mid\theta_1^*)) + sb(\hat{u}_{eq}(X\mid\theta_1^*) - u_{eq}^*(t)) \\
&\quad + sb(\hat{u}_{sw}(s\mid\theta_2^*) - \hat{u}_{sw}(s\mid\theta_2)) - sb\hat{u}_{sw}(s\mid\theta_2^*) + \frac{b}{\gamma_1}\tilde{\theta}_1^T\dot{\theta}_1 + \frac{b}{\gamma_2}\tilde{\theta}_2^T\dot{\theta}_2 \\
&= sb\tilde{\theta}_1^T\xi(X) - sb\tilde{\theta}_2^T\psi(s) + sb\omega - sb\hat{u}_{sw}(s\mid\theta_2^*) + \frac{b}{\gamma_1}\tilde{\theta}_1^T\dot{\theta}_1 + \frac{b}{\gamma_2}\tilde{\theta}_2^T\dot{\theta}_2.
\end{aligned}$$

It should be noted that since $u_{sw}(s\mid\theta_2^*)$ lies in the first and third quadrant, $u_{sw}(s\mid\theta_2^*) = 0$, for $s = 0$, and $s\,u_{sw}(s\mid\theta_2^*) \ge 0$ for all s. Therefore, $s\,u_{sw}(s\mid\theta_2^*) = |s|\,|u_{sw}(s\mid\theta_2^*)|$, so

$$\begin{aligned}
\dot{V} &= b\tilde{\theta}_1^T\Big(s\xi(X) + \frac{1}{\gamma_1}\dot{\theta}_1\Big) + b\tilde{\theta}_2^T\Big(-s\psi(s) + \frac{1}{\gamma_2}\dot{\theta}_2\Big) + b\big(s\omega - |s|\,|u_{sw}(s\mid\theta_2^*)|\big) \\
&\le b\tilde{\theta}_1^T\Big(s\xi(X) + \frac{1}{\gamma_1}\dot{\theta}_1\Big) + b\tilde{\theta}_2^T\Big(-s\psi(s) + \frac{1}{\gamma_2}\dot{\theta}_2\Big) + b(s\omega - \eta|s|).
\end{aligned} \tag{29}$$

Using the adaptation laws (25) and (26) in (29) and recalling that b is positive we have

$$\dot{V} \le b(s\omega - \eta|s|). \tag{30}$$

From the universal approximation theorem, it can be expected that the term $s\omega$ will be very small so

$$\dot{V} \le 0 . \tag{31}$$

From (31) it can be seen that s, θ_1, θ_2 are bounded. Furthermore, by Assumption 2.2 the signal X_d is bounded, so the system states X will be bounded. In order to prove that $\lim_{t\to\infty}|e(t)| = 0$, it is necessary to show that $\lim_{t\to\infty}|s(t)| = 0$. Assuming $|s| \le \eta_s$, then (30) can be written as

$$\dot{V} \le b|s||\omega| - b\,\eta|s| \le b\,\eta_s|\omega| - b\,\eta|s| . \tag{32}$$

Integrating both sides of (32), we have

$$\int_0^t |s(\tau)| d\tau \le \frac{1}{b\eta}(V(0) - V(t)) + \frac{\eta_s}{\eta}\int_0^t |\omega| d\tau . \tag{33}$$

If $\omega \in L_1$, then from (33) we have $s \in L_1$. It can be seen that all the terms in the right-hand side of (7) are bounded, so $\dot{s} \in L_\infty$. Using the Barbalat lemma [13], we have $\lim_{t\to\infty} |s(t)| = 0$, thus $\lim_{t\to\infty} |e(t)| = 0$. □

The algorithm of the proposed method is depicted in Figure 1.

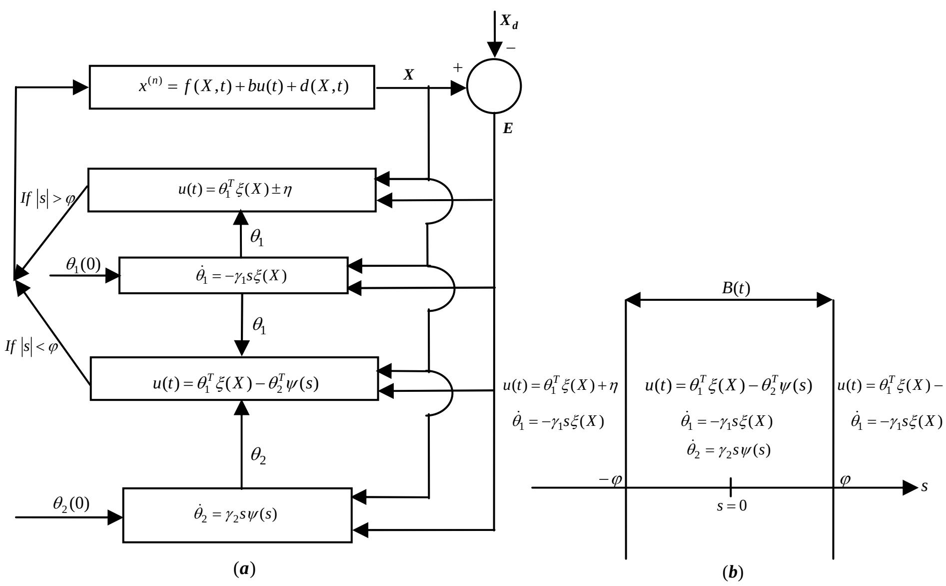

FIGURE 1. The overall scheme of the proposed method (a) Block diagram (b) Boundary layer representation

4. **Simulation Examples**

In this section the proposed controller is applied to two nonlinear systems. In the first example a first order uncertain nonlinear system is considered. In the second example the Duffing forced-oscillation system is used.

Example 4.1. Consider a first order nonlinear system as follows

$$\dot{x} = \frac{1-e^{-x}}{1+e^{x}} + u(t) + d(t) \tag{34}$$

where $u(t)$ is the control signal and $d(t)$ is a bounded disturbance, in which the bound is also unknown. Without loss of generality we use, $d(t) = 5 + \sin(3t)$. The desired trajectory is $x_d = \sin(t)$, $x(0) = 1.5$ and $s = e$. Two cases are considered: the first uses the conventional SMC proposed in [13] and the second uses the method proposed in this paper.

Case 1 (Conventional SMC): By [13], if we write $f(x) = \hat{f}(x) + \Delta f(x)$ the conventional SMC method needs to know $\hat{f}(x)$ and the upper bound of $\Delta f(x)$. For system (34), if $\hat{f}(x) = 1$, $\Delta f(x) = -2\frac{e^{-x}}{1+e^{-x}}$, and so, $|\Delta f(x)| \le F = 2$. Therefore, from [13], the conventional SMC can be written as:

$$u(t) = -\hat{f}(x) - (\eta + F)\,\text{sgn}(s) \tag{35}$$

where η is a positive constant as a control parameter. Here, we choose $\eta = 0.1$. First, it is assumed that there is no external disturbance, *i.e.* $d(t) = 0$. Figure 2 represents the results.

From Figure 2, the desirable tracking is achieved at the expense of high frequency oscillation (chattering). Now assume an external disturbance ($d(t) = 5 + \sin(3t)$) at $t = 10^s$ that had not been earlier accounted for in the control law. The results are depicted in Figure 3. It can be seen from Figure 3 that the conventional SMC is not only fragile with respect to such unknown external disturbances, but also suffers from significant chattering which is a harmful phenomenon in practical applications. One may suggest that this problem can be overcome by assuming a larger bound on the uncertainty $\Delta f(x)$ in the design of control law, however one should consider the following two points. First, external disturbances are not always predictable in real systems, and second, the assumption of a larger bound for uncertainties, even though it may help guarantee stability, is always at a significant cost of deteriorating performance by increasing chattering. The proposed method, as shown in the following set of simulations, is significant since it does not require a bound on these uncertainties to be known, and therefore builds robustness against such unknown disturbances.

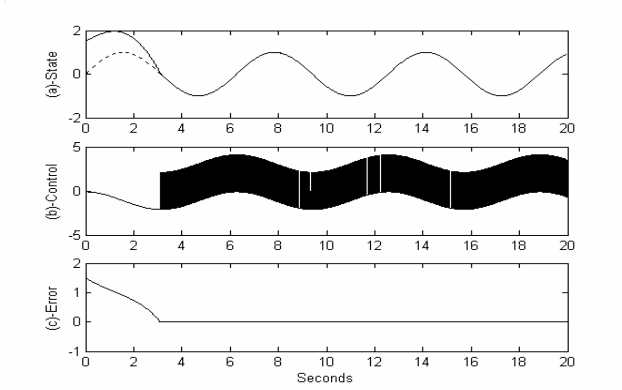

FIGURE 2. Conventional SMC without any external disturbance (a) Desired output (dashed), actual state (Solid) (b) Control input (c) Error signal

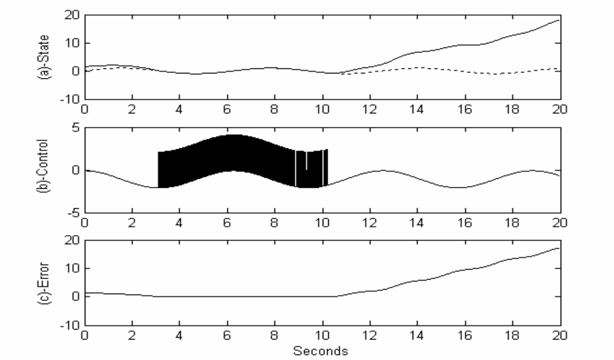

FIGURE 3. Conventional SMC with external disturbance (a) Desired output (dashed), actual state (solid) (b) Control input (c) Error signal

Case 2 (Proposed method): In this case, we utilize the proposed method for controlling the uncertain nonlinear system (34). The parameters $\eta = 0.01$, $\gamma_1 = 100$, $\gamma_2 = 800$ and $\varphi = 0.5$ are chosen. $K_P(0) = 800$, $K_I(0) = 900$ are selected. All the parameters are chosen to achieve the best transient control performance considering the requirement of stability and possible operating conditions. The universe [-3,3] is partitioned into the following six fuzzy input membership functions:

$$\begin{aligned}
\mu_{N3} &= \frac{1}{1+\exp(5(x+2))} \\
\mu_{N2} &= \exp(-(x+1.5)^2) \\
\mu_{N1} &= \exp(-(x+0.5)^2) \\
\mu_{P1} &= \exp(-(x-0.5)^2) \\
\mu_{P2} &= \exp(-(x-1.5)^2) \\
\mu_{P3} &= \frac{1}{1+\exp(-5(x-2))}
\end{aligned} \tag{36}$$

as shown in Figure 4.

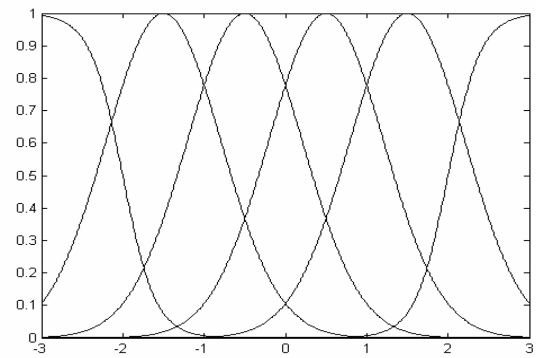

FIGURE 4. Membership functions defined in universe [-3,3], from left to right $\mu_{N3}, \mu_{N2}, \mu_{N1}, \mu_{P1}, \mu_{P2}, \mu_{P3}$.

The initial fuzzy parameters are chosen randomly in the interval [0,1] and at first it is assumed that initially there is no external disturbance, *i.e.* $d(t) = 0$. Figures 5 and 6 represent the results.

It can be seen from Figures 5 and 6 that perfect tracking is achieved while chattering is not observed.
Now assume a sudden external disturbance ($d(t) = 5 + \sin(3t)$) occurs at $t = 10^s$. The results are depicted in Figures 7 and 8. Simulation results show the effectiveness of the proposed method to cope with uncertainty, disturbances and chattering .

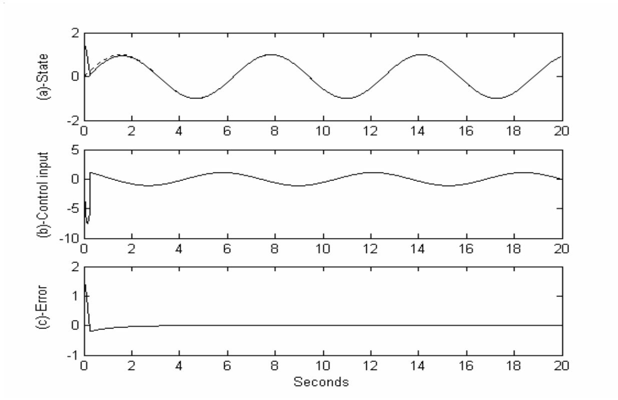

FIGURE 5. Proposed method without external disturbance (a) Desired output (dashed), actual state (solid) (b) Control input (c) Error signal

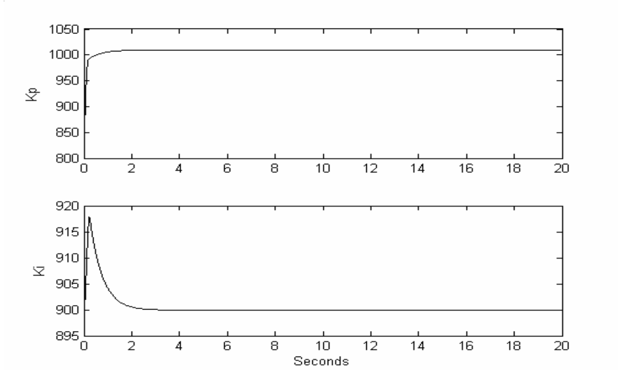

FIGURE 6. Variation of K_P and K_I with respect to time

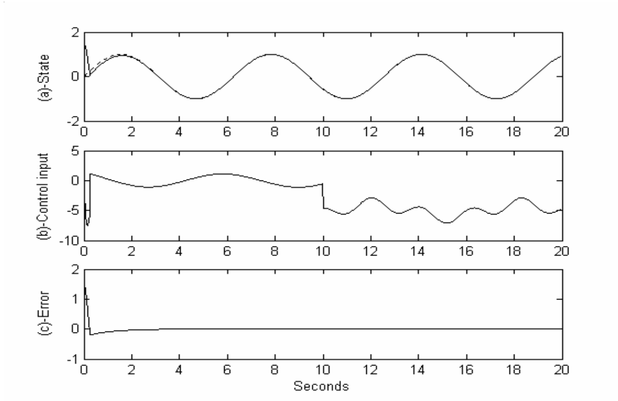

FIGURE 7. Proposed method with external disturbance (a) Desired output (dashed), actual state (solid) (b) Control input (c) Error signal

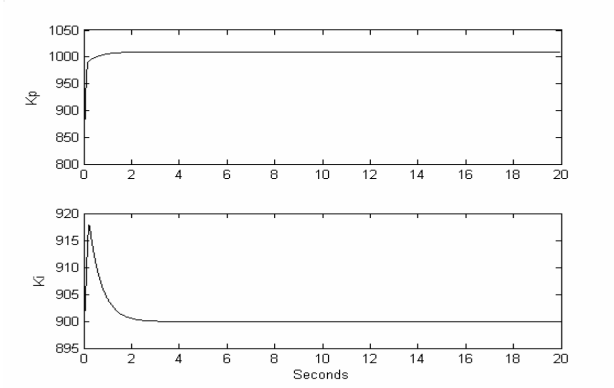

FIGURE 8. Variation of K_P and K_I with respect to time

Example 4.2. Consider the Duffing forced-oscillation system in the form of

$$\dot{x}_1 = x_2$$
$$\dot{x}_2 = -0.1x_2 - x_1^3 + 12\cos(t) + u(t) + d(t) \quad (37)$$

where $u(t)$ is the control signal and $d(t) = 4 + \sin(4t)$ is an unknown bounded disturbance, and the bound is also unknown. The desired trajectory is $x_d = \sin(t)$, the initial condition is $[2,2]^T$ and $s = \dot{e} + 2e$. Again, as in example 4.1, two cases are considered :

Case 1 (Conventional SMC method): From [13], if we write $f(x) = \hat{f}(x) + \Delta f(x)$ the conventional SMC method needs to know $\hat{f}(x)$ and the upper bound of $\Delta f(x)$. For system (37), it can be determined if $\hat{f}(x) = -0.1x_2 - x_1^3$ and $\Delta f(x) = 12\cos(t)$, then, $|\Delta f(x)| \le F = 12$. Therefore, from [13], the conventional SMC can be written as (35). We choose $\eta = 0.1$, As in the previous example, at first we assume that there are no external disturbances, *i.e.,* $d(t) = 0$. Figure 9 represents the results. From Figure 9, the perfect tracking is achieved at the expense of high frequency oscillation (chattering). Now, assume a sudden external disturbance ($d(t) = 4 + \sin(4t)$) occurs at $t = 20^s$. The results are depicted in Figure 10. It can be seen from Figure 10 that the conventional SMC is not only fragile when external disturbances exceeds its assumed bounds, but also suffers from chattering which is a harmful phenomenon in practical applications.

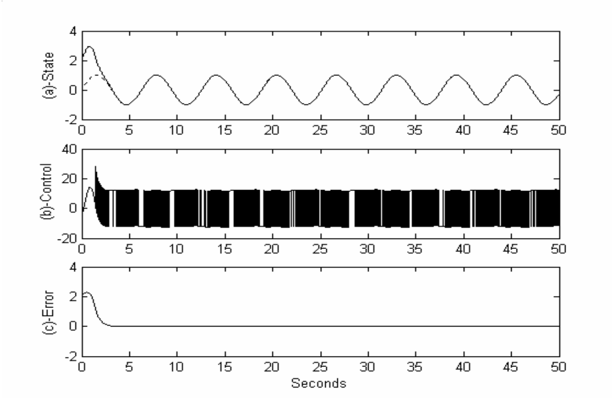

FIGURE 9. Conventional SMC without any axternal disturbance (a) Desired output (dashed), actual state (solid) (b) Control input (c) Error signal

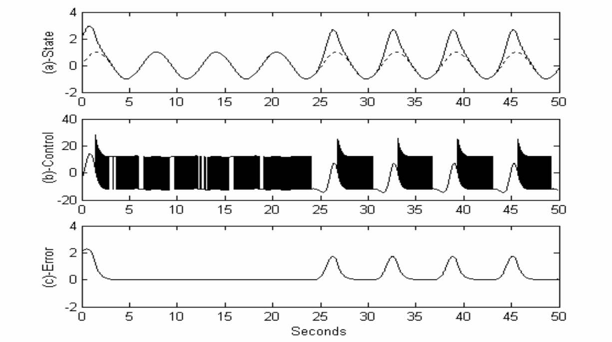

FIGURE 10. Conventional SMC with external disturbance (a) Desired output (dashed), actual state (solid) (b) Control input (c) Error signal

Case 2 (Our method): In this case, we utilize the proposed method for controlling the uncertain nonlinear Duffing-forced system (37). The parameters $\eta = 0.01,\ \gamma_1 = 10,\ \gamma_2 = 100,\ \varphi = 1,\ K_P(0) = 10,\ K_I(0) = 80$ are selected to achieve the best transient control performance considering the requirement of stability and possible operating conditions. The universe [-3,3] is partitioned into six fuzzy input memberships as in (36). The initial condition of fuzzy parameters are chosen randomly in the interval [0,1]. At first it is assumed there is no external disturbance, *i.e.,* $d(t) = 0$. Figures 11 and 12 represent the results. It can be seen from Figures 11 and 12 that tracking is achieved and chattering is not observed. Now, assume a sudden external disturbance ($d(t) = 4 + \sin(4t)$) occurs at $t = 20^s$. The results are depicted in Figures 13 and 14. From the simulation results it can be seen that the proposed controller not only controls the system amid uncertainty and external disturbances, but it also copes well with chattering.

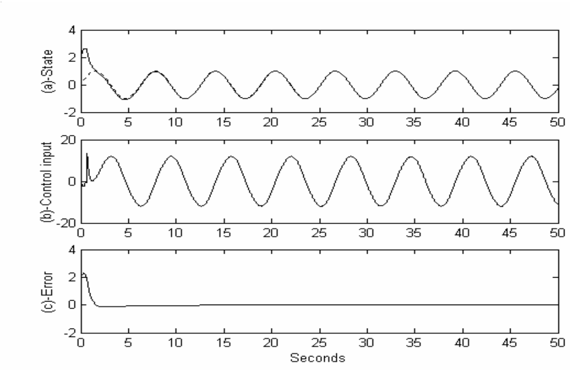

FIGURE 11. Proposed method without external disturbance (a) Desired output (dashed), actual state (solid) (b) Control input (c) Error signal

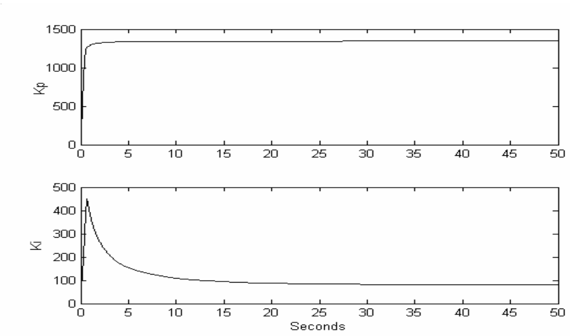

FIGURE 12. Variation of K_P and K_I with respect to time

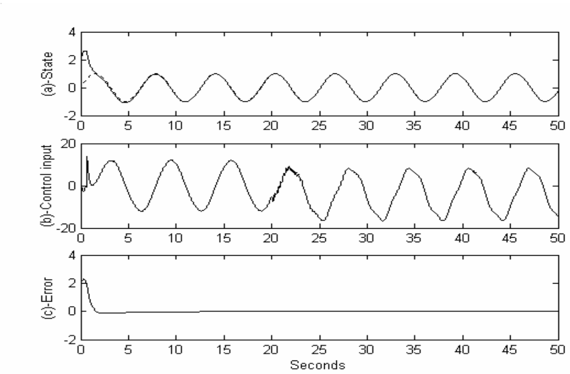

FIGURE 13. Proposed method with external disturbance (a) Desired output (dashed), actual state (solid) (b) Control input (c) Error signal

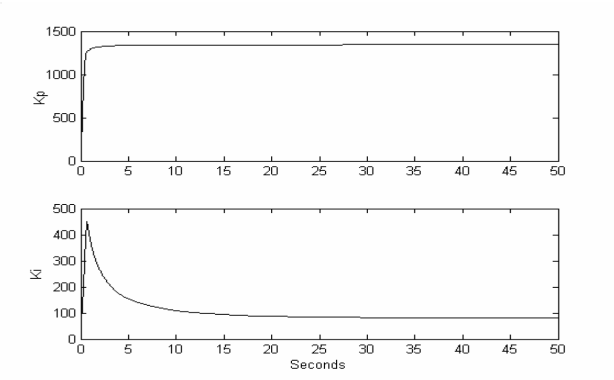

FIGURE 14. Variation of K_P and K_I with respect to time

Finally Figure 15 shows the states in phase plane for four different initial conditions.

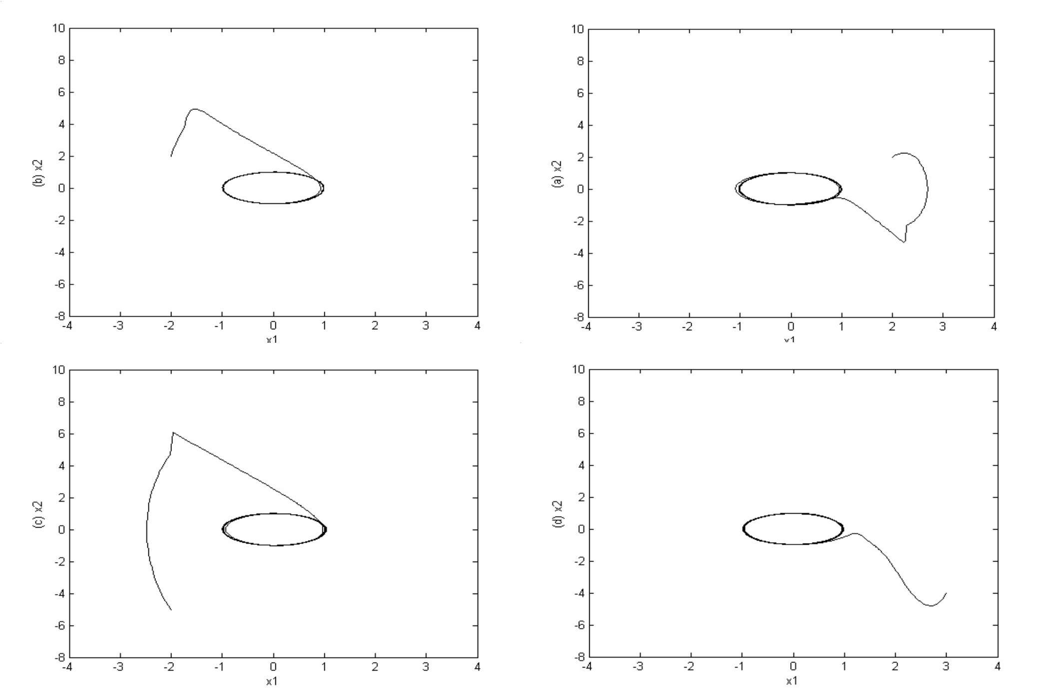

FIGURE 15. Phase-plane representation for different initial conditions:

(a) $x(0) = [2,2]^T$ (b) $x(0) = [-2,2]^T$ (c) $x(0) = [-2,-5]^T$ (d) $x(0) = [3,-4]^T$.

5. **Discussion and Conclusion**

In this paper, a direct adaptive fuzzy sliding mode controller for a class of nonlinear uncertain systems is investigated. The proposed method is robust in the presence of uncertainties and bounded external disturbances. Comparing the proposed method with the other existing approaches, there is an important difference when external disturbances are assumed. Here, the value of a bound does not need to be known, i.e. knowing that it exists is sufficient to prove asymptotic stability of the closed loop system. For attenuating the chattering problem, the control law was furthermore designed with an adaptive PI term. In future work, we aim to extend this methodology to more general forms of nonlinear systems. We also aim to propose a methodology for auto tuning the adaptive gains.

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References

[1] M.-R. Akbarzadeh-T. and R. Shahnazi, *Direct adaptive fuzzy PI sliding mode control for a class of uncertain nonlinear systems,* In Proceeding of IEEE International Conference on Systems, Man and Cybernetics, (2005), 2566-2571.

[2] K. J. Astrom and B. Wittenmark, *Adaptive control,* second Edition, Addison-Wesley Pub Co, Newyork, December (1994).

[3] Y. Byungkook and H. Woonchul, *Adaptive fuzzy sliding mode control of nonlinear systems,* IEEE Transaction Fuzzy systems, 6 (**2**) (1998).

[4] Z. M. Chen, J. G. Zhang, Z. C. Zhang and J. C. Zeng, *Adaptive fuzzy sliding mode control for uncertain nonlinear systems,* In Proceeding of the Second International Conference on Machine Learning and Cybernetics, Xian, 2-5 November (2003).

[5] Y. Guo and P. Y. Woo, *Adaptive fuzzy sliding mode control for robotic manipulators,* In Proceeding of 42nd Conference on Decision and Control, Maui, Hawaii USA, December (2003).

[6] H.G. Han and C. Y. Su, *Further results on adaptive control of a class of nonlinear systems with fuzzy logic,* In Proceedings of IEEE Conference on Fuzzy Systems, Seoul, Korea, (1999), 1309-1314.

[7] H. F. Ho, Y. K. Wong and A. B. Rad, *Adaptive fuzzy sliding mode control design: Lyapunov approach,* In Proceeding of 5th Asian Control Conference, **3** (2004), 1502- 1507.

[8] H. K. Khalil. *Nonlinear systems,* Prentice-Hall Inc., second edition, (1996).

[9] Y. K. Kim and G. J. Jeon, *Error reduction of sliding mode control using sigmoid-type nonlinear interpolation in the boundary layer,* International Journal of Control, and Systems, 2 (**4**) (2004), 523-529.

[10] C. C. Lee, *Fuzzy logic in control systems: fuzzy logic controller,* part I and part II, IEEE Transactions on Systems, Man, and Cybernetics, **20** (1990), 404-435.

[11] K. S. Narenda and A. M. Annaswamy, *Stable adaptive systems,* Prentice-Hall Inc., (1989).

[12] R. Shahnazi and M.-R. Akbarzadeh-T., *Robust PI adaptive fuzzy control for a class of uncertain nonlinear systems,* In Proceeding of IEEE International Conference on Systems, Man and Cybernetics, (2005), 2548-2553.

[13] J. J. Slotine and W. Li, *Applied nonlinear control,* Prentice Hall, Inc.: Englewood Cliffs, New Jersy, (1991).

[14] C. Y. Su and Y. Stepanenko, *Adaptive control of a class of nonlinear systems with fuzzy logic,* IEEE Transactions on Fuzzy Systems, 2 (**2**) (1994), 285-294.

[15] C. W. Tao, M.L. Chan and T. T. Lee, *Adaptive fuzzy sliding mode controller for linear systems with mismatched time-varying uncertainties,* IEEE Transaction on Systems Man and Cybernetics, Part B: Cybernetics, 33 (**2**)(2003).

[16] V. I. Utkin, *Sliding modes and their application in variable structure systems,* Moscow, Russia: Mir, (1978).

[17] J. Wang, S. S. Get and T. H. Lee, *Adaptive fuzzy sliding mode control of a class of nonlinear systems,* In Proceedings of the 3rd Asian Control Conference, Shanghai, July 4-7, (2000).

[18] L. X. Wang, *A course in fuzzy systems and control*, Prentice Hall., New Jersy, August (1996).

[19] L. X. Wang, *Adaptive fuzzy systems and control: design and stability analysis,* Prentice Hall, Englewood Cliffs, NJ., (1994).

[20] L. X. Wang, *Stable adaptive fuzzy control of nonlinear systems,* IEEE Transaction on Fuzzy systems, 1 (**2**) (1993).

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