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A new vector valued similarity measure for intuitionistic fuzzy sets based on OWA operators

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Abstract

Plenty of researches have been carried out, focusing on the measures of distance, similarity, and correlation between intuitionistic fuzzy sets (IFSs). However, most of them are single-valued measures and lack of potential for efficiency validation. In this paper, a new vector valued similarity measure for IFSs is proposed based on OWA operators. The vector is defined as a two-tuple consisting of the similarity measure and uncertainty measure, in which the latter is the uncertainty of the former. OWA operators have the ability to aggregate all values in the universe of discourse of IFSs, and to determine the weights according to specific applications. A framework is built to measure similarity between IFSs. A series of definitions and theorems are given and proved to satisfy the corresponding axioms defined for IFSs. In order to illustrate the effectiveness of the proposed vector valued similarity measure, a classification problem is used as an application.

Keywords: Similarity measure, Uncertainty measure, Intuitionistic fuzzy set, OWA operator, Classification.

1 Introduction

Zadeh firstly proposed the concept of fuzzy sets in 1965 [62]. With the maturity of fuzzy sets theory [13], Atanassov [1, 2] extended it and introduced intuitionistic fuzzy sets (IFSs) as the generalization of fuzzy sets. IFSs have been applied widely in many fields, such as decision making [68] and countermeasure analysis [33, 40]. It should be pointed out that, due to the complexity and incompleteness of information in these fields [20, 15, 27], there are many other theories and models to handle uncertainty information, such as evidence theory [28, 11, 29, 10, 56, 69, 70], D numbers method [73, 35, 41, 18, 16, 8], fuzzy dematel method [73, 52, 53, 31], fuzzy numbers method [23, 12, 30] and other decision making methodologies [67, 39, 39, 42]. One of the key issues in the application of IFSs is how to determine the similarity between two IFSs. With the development of IFSs theory, it attracts much researchers' attention, and more and more scholars have joint in the studies of these problems. A similarity measure method of IFSs based on Hausdorff distance was introduced in [22]. Ye proposed a kind of similarity measure using Cosine theory in [60], and other approaches have also been presented in [37, 51, 71, 66]. There are many other authors presented corresponding definitions of similarity measures for IFSs [54, 47, 6]. However, there exist limitations in most of the existing methods [63, 64] from the typical single-valued measures between vectors which denote the membership and non-membership values of the elements. These methods may result in counter-intuitive conclusions, for example, different IFSs can have the same similarity based on the approaches in [4, 49]. Szmidt and Kacprzyk put forward that the hesitancy membership and non-membership values were necessary (i.e., the gap between membership and non-membership) to improve the accuracy of similarity measure. In this paper, the uncertainty of similarity measure between IFSs is considered as the performance of the similarity measure itself.

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There is a controversy about whether counterintuitive results are inevitable or not for all the definitions of similarity measure between IFSs. It is obvious that the single-valued measure cannot reflect both similarity and uncertainty simultaneously since the uncertain information is often incomplete [45, 72, 65, 14]. So, both the two aspects are considered in this paper and represented by the vector of similarity measure, which can compensate for the shortcomings of conventional single-valued measure methods. For example, four IFSs are given as $A = \{(x, 0, 0)\}$, $B = \{(x, 0.01, 0.01)\}$, $C = \{(x, 1, 0)\}$ and $D = \{(x, 0.98, 0)\}$. Obviously, their hesitancy degree are $\pi_A = 1$, $\pi_B = 0.98$, $\pi_C = 0$ and $\pi_D = 0.02$. They can also be denoted as $A = \{(0, 0, 1)\}$, $B = \{(0.01, 0.01, 0.98)\}$, $C = \{(1, 0, 0)\}$ and $D = \{(0.98, 0, 0.02)\}$. According to intuition, the similarity between A and B is close to 1, and so is C and D. However, the uncertainty of similarity can be calculated as $U(A, B) = 1$ and $U(C, D) \approx 0.0002$ using the newly proposed vector valued similarity measure method. The results indicate that the high similarity between IFSs A and B is suspicious, but the similarity between C and D is not.

There may exist multiple elements in the universe of discourse of IFSs, and each element will obtain a calculation result. But the existing methods [17, 7, 44] to calculate the similarity and uncertainty act on single element. Therefore, how to aggregate multi-values into a final result is an important issue. Another key point in this paper is to introduce the ordered weight averaging (OWA) operator, which was proposed by Yager [59] to assign the ith weight to the ith largest input value. OWA operator has a lot of applications in many fields, such as pattern recognition [21], classification [50, 61, 36], decision making [24, 46], as well as other related fields [25, 57, 26]. Three main steps for OWA operator are as follows: (1) Descend order input elements. (2) Assign appropriate weights for operators. (3) Determine weighted values of reordered elements. It is an effective methodology for many applications under uncertain environment. In this paper, OWA operator is combined with the proposed vector valued similarity measure for aggregating the results of measure from multiple elements.

Classification is an important problem in many integrated learning applications, and one of the most effective strategies is fuzzy method. A margin-based model for rule weighting and reduction in fuzzy rule-based classification systems was proposed in [50]. A cognitive style and aggregation operator model was presented in [61] for classification. In this paper, the proposed measure is used to build a classifier, which is illustrated to be more effective in dealing with classification problems.

The remainder of this paper is constituted as follows. Section 2 introduces some necessary related concepts of IFSs and OWA operators. Distance and similarity measures of IFSs are presented in Section 3. The vector valued similarity measure is proposed in Section 4. Section 5 conducts some correlative applications in classification. Conclusion is given in Section 6.

2 Preliminaries

Definition 2.1. [1] *An intuitionistic fuzzy set A in universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$ can be denoted as follows:*

$$A = \{(x, \mu_A(x), \nu_A(x))|x \in X\}. \tag{1}$$

which is characterized by $\mu_A(x)$, $\nu_A(x) : X \to [0, 1]$ with the condition $0 \le \mu_A(x) + \nu_A(x) \le 1$. Where $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x in set A.

Some common operations of IFSs are shown as follows.

Definition 2.2. *Suppose two IFSs $A = \{(x, \mu_A(x), \nu_A(x))|x \in X\}$ and $B = \{(x, \mu_B(x), \nu_B(x))|x \in X\}$ have the same universe of discourse X. And then:*

(1) $A \bigcup B = \{(x, max(\mu_A(x), \mu_B(x)), min(\nu_A(x), \nu_B(x)))|x \in X\}$
(2) $A \bigcap B = \{(x, min(\mu_A(x), \mu_B(x)), max(\nu_A(x), \nu_B(x)))|x \in X\}$
(3) $A \subseteq B$ if and only if $\mu_A(x) \le \mu_B(x) and \nu_A(x) \ge \nu_B(x)$ for all $x \in X$
(4) $A = B$ if and only if $A \subseteq B$ and $B \subseteq A$
(5) The complement of A: $A_c = \{(x, \nu_A(x), \mu_A(x))|x \in X\}$

Definition 2.3. *A mapping, $F : \Re^n \to \Re$, represents an Ordered Weighted Averaging(OWA) operator [59] of dimension* n*, which has a corresponding weighting vector $W = (w_1, w_2, \ldots, w_n)^T$, such that $\sum_{i=1}^{n} w_i = 1$, $w_i \in [0,\ 1]$ and $F(a_1,\ \ldots,\ a_n) = w_1b_1 + w_2b_2 + \cdots + w_nb_n$, where b_j is the jth largest element of the aggregated objects $\{a_1,\ a_2,\ \ldots,\ a_n\}$.*

How to determine the corresponding weights is an open and significant issue for OWA operators. Yager presented two methods based on *orness* and *dispersion* in [59]. O'Hagan [43] applied these two methods to determine OWA operators, and which have been further improved based on Lagrange multipliers method introduced by Fullér and Majlender in [19].

3 Distance and similarity measures for IFSs

3.1 The basic metric spaces

The metric distance and similarity between IFSs can be defined as follows according to [6, 48].

Definition 3.1. *A mapping D: $IFS(X)^2 \to [0,1]$ is determined to be a distance measure if the following requirements hold:*
(1)$0 \leq D(A,B) \leq 1$;
(2)$D(A,B) = 0 \Leftrightarrow A = B$;
(3)$D(A,B) = D(B,A)$;
(4) *If* $A \subseteq B \subseteq C$, *then* $D(A,B) \leq D(A,C)$ *and* $D(B,C) \leq D(A,C)$;
(5)$D(A,B) + D(B,C) \geq D(A,C)$.

Similarity measure can be defined as the complementary of distance measure as follows.

Definition 3.2. *A mapping S: $IFS(X)^2 \to [0,1]$ is called a similarity measure if it satisfies the following axioms:*
(1)$0 \leq S(A,B) \leq 1$;
(2)$S(A,B) = 1 \Leftrightarrow A = B$;
(3)$S(A,B) = S(B,A)$;
(4)$S(A,C) \leq S(A,B)$ *and* $S(A,C) \leq S(B,C)$ *if* $A \subseteq B \subseteq C$.

Theorem 3.3. *If D represents the distance measure between two IFSs. Then, $S = 1 - D$ is the similarity measure between them.*

Proof. (I). As D is the distance measure, so it should meet the requirements in Definition 3.1 as:

$$0 \leq D(A,B) \leq 1,\ D(A,B) = 0 \Leftrightarrow A = B \text{ and } D(A,B) = D(B,A)$$

Based on the condition $S = 1 - D$, the following formulas can be obtained naturally as:

$$0 \leq S(A,B) \leq 1,\ S(A,B) = 1 \Leftrightarrow A = B \text{ and } S(A,B) = S(B,A)$$

(II). Suppose A, B, C are three IFSs, and meet $A \subseteq B \subseteq C$, then we have $D(A,B) \leq D(A,C)$ and $D(B,C) \leq D(A,C)$. Then, $1 - D(A,B) \geq 1 - D(A,C)$ and $1 - D(B,C) \geq 1 - D(A,C)$. That is, $S(A,B) \geq S(A,C)$ and $S(B,C) \geq S(A,C)$.

□

3.2 The framework to measure similarity between IFSs

As introduced in [6], the similarity measure between two IFSs has fuzziness or uncertainty to some extent. So the hesitancy index $\pi_A(x)$ is applied to describe the uncertainty of similarity measure in some proposed methods. However, the accuracy of the measure cannot be guaranteed if only $\pi_A(x)$ is used. Therefore, in this paper, a two-dimensional vector is defined, which consists of similarity and uncertainty measures. The definition of the basic framework is as follows.

Definition 3.4. *Let S and U be two mappings S, U: $IFS(X)^2 \to [0,1]$, then $SM = (S,U)$ can be defined as a vector valued similarity measure if the following requirements hold:*
(1) $S(A,B), U(A,B) \in [0,1]$;
(2) $S(A,B) = 1 \Leftrightarrow A = B$;
(3) $U(A,B) = 0$ *if and only if* A, B *are crisp sets;*
(4) $S(A,B) = S(B,A)$;
(5) $S(A,B) \geq S(A,C)$ *and* $S(B,C) \geq S(A,C)$ *if* $A \subseteq B \subseteq C$.

4 The proposed vector valued similarity measure

4.1 The new similarity measure between two IFSs

Definition 4.1. *Let* $A = \{(x, \mu_A(x), \nu_A(x)) | x \in X\}$, $B = \{(x, \mu_B(x), \nu_B(x)) | x \in X\}$ *be two IFSs under universe of discourse* $X = \{x_1, x_2, ..., x_n\}$.

For the ith element in A, a vector is used to represent it as, $I_A(x_i) = [\mu_A(x_i), \nu_A(x_i), 1-\mu_A(x_i)-\nu_A(x_i)]$*, likewise, for B, it should be,* $I_B(x_i) = [\mu_B(x_i), \nu_B(x_i), 1-\mu_B(x_i)-\nu_B(x_i)]$*, then,*

$$I_A(x_i) - I_B(x_i) = [\mu_A(x_i)-\mu_B(x_i), \nu_A(x_i)-\nu_B(x_i), (1-\mu_A(x_i)-\nu_A(x_i)) - (1-\mu_B(x_i)-\nu_B(x_i))].$$

The mixed matrix M is defined as follows:

$$M = \begin{array}{c} \\ \mu \\ \nu \\ 1-\mu-\nu \end{array} \begin{array}{c} \begin{array}{ccc} \mu & \nu & 1-\mu-\nu \end{array} \\ \begin{pmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{pmatrix} \end{array}$$

The distance measure between A and B for x_i *can be defined as follows:*

$$d_i(A,B) = \sqrt{\frac{1}{2}(I_A(x_i) - I_B(x_i)) M (I_A(x_i) - I_B(x_i))^T}. \tag{2}$$

To obtain the final distance between A and B, OWA operators are used to aggregate the n distance measures of all the elements in X as

$$Dis_{(d_1,\ \ldots,\ d_n)}(A,B) = w_1 b_1 + w_2 b_2 + \cdots + w_n b_n. \tag{3}$$

where w_i *denotes the weight of the ith element in X, and* b_i *represents the ith largest distance measure of all the elements between A and B.*

In order to clearly illustrate how to calculate the distance in equation (2), an example is shown below.

Example 4.2. *Let* $A = \{\ldots, (x_i, 0.5, 0.5), \ldots\}$ *and* $B = \{\ldots, (x_i, 0.49, 0.51), \ldots\}$ *be two IFSs. The calculation process of distance for* x_i *using the proposed method is shown as*

$$I_A(x_i) = [0.50, 0.50, 0.00],\ I_B(x_i) = [0.49, 0.51, 0.00],\ I_A(x_i) - I_B(x_i) = [0.01, -0.01, 0.00],$$

$$d_i(A,B) = \sqrt{\frac{1}{2}[0.01, -0.01, 0.00] \begin{bmatrix} 1 & 0 & \frac{1}{2} \\ 0 & 1 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 0.01 \\ -0.01 \\ 0.00 \end{bmatrix}} = 0.01.$$

Theorem 4.3. *Let* $Dis_{(d_1,\ \ldots,\ d_n)}(A,B) = w_1 b_1 + w_2 b_2 + \cdots + w_n b_n$ *represent the distance between IFSs A and B in the universe of discourse* $X = \{x_1, x_2, ..., x_n\}$*. Then,* $0 \leq Dis_{(d_1,\ \ldots,\ d_n)}(A,B) \leq 1$*.*

Proof. At first, $0 \leq d_i(A,B) \leq 1$ will be proved. Based on equation (2), the distance measure for x_i between two IFSs A and B in universe of discourse X can be unfolded as the following expression. Let $d_{Temp}(A,B) = (I_A(x_i) - I_B(x_i)) M (I_A(x_i) - I_B(x_i))^T$, then

$$d_i(A,B) = \sqrt{\frac{1}{2} d_{Temp}(A,B)}. \tag{4}$$

That is,

$$d_{Temp}(A,B) = \begin{pmatrix} \mu_A(x_i) - \mu_B(x_i) \\ \nu_A(x_i) - \nu_B(x_i) \\ (1-\mu_A(x_i)-\nu_A(x_i)) - (1-\mu_B(x_i)-\nu_B(x_i)) \end{pmatrix}^T$$

$$\begin{bmatrix} 1 & 0 & 1/2 \\ 0 & 1 & 1/2 \\ 1/2 & 1/2 & 1 \end{bmatrix} \begin{pmatrix} \mu_A(x_i) - \mu_B(x_i) \\ \nu_A(x_i) - \nu_B(x_i) \\ (1-\mu_A(x_i)-\nu_A(x_i)) - (1-\mu_B(x_i)-\nu_B(x_i)) \end{pmatrix} = (\mu_A(x_i) - \mu_B(x_i))^2 + (\nu_A(x_i) - \nu_B(x_i))^2. \tag{5}$$

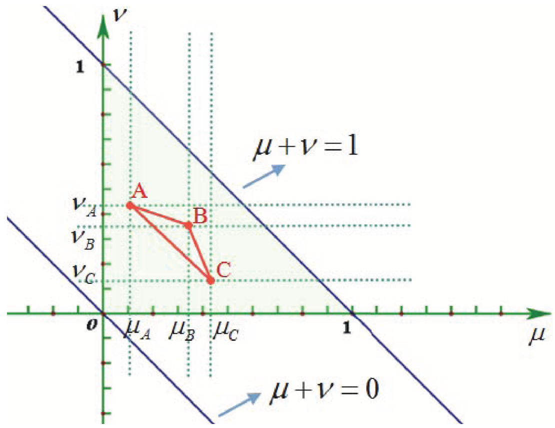

Figure 1: The coordinate system for μ and ν.

$$s.t. \begin{cases} 0 \le \mu_A(x_i) + \nu_A(x_i) \le 1 \\ 0 \le \mu_B(x_i) + \nu_B(x_i) \le 1 \\ 0 \le \mu_A(x_i), \nu_A(x_i), \mu_B(x_i), \nu_B(x_i) \le 1 \end{cases}$$

where $\mu_A(x_i)$, $\nu_A(x_i)$, $\mu_B(x_i)$ and $\nu_B(x_i)$ are from IFSs A and B. For equation (5), μ and ν denote x-coordinate and y-coordinate, respectively, in coordinate system shown in Figure 1. There are two lines satisfy the linear equation: $\mu + \nu = 0$ and $\mu + \nu = 1$. (μ_A, ν_A) and (μ_B, ν_B) are arbitrary two points of the shaded area in the coordinate system. The value of $d_{Temp}(A, B)$ in equation (5) is equal to the square of the distance between the two points. It is obvious that the minimum and maximum distance in this area are 0 and $\sqrt{2}$ respectivly. That is, $d_{Temp}(A, B) \in [0, 2]$. Based on equation (4), we have $0 \le d_i(A, B) \le 1$. For equation (3), as w and b are nonnegative, so $Dis_{(d_1,\ ...,\ d_n)}(A, B) \ge 0$. Similarly, when all values of b satisfy that $b_i = 1$, the distance reaches maximum value as $Dis_{(d_1,\ ...,\ d_n)}(A, B) = 1$, such that $\sum_{i=1}^{n} w_i = 1, \quad 0 \leqslant w_i \leqslant 1, \quad i = 1, \ \ldots, \ n$. Therefore, $0 \le Dis_{(d_1,\ ...,\ d_n)}(A, B) \le 1$. □

Theorem 4.4. *Let $Dis_{(d_1,\ ...,\ d_n)}(A, B) = w_1 b_1 + w_2 b_2 + \cdots + w_n b_n$ represent the distance measure between two IFSs A and B in the universe of discourse $X = \{x_1, x_2, ..., x_n\}$. Then, $Dis_{(d_1,\ ...,\ d_n)}(A, B) = 0$ if and only if $A = B$.*

Proof. As $A = B$, so we have $\mu_A = \mu_B$ and $\nu_A = \nu_B$. Based on equation (5), we know that $d_{Temp}(A, B) = 0$, so $d_i(A, B) = 0$. That is, $b_i = 0$. Therefore, $Dis_{(d_1,\ ...,\ d_n)}(A, B) = 0$. □

Theorem 4.5. *Let $Dis_{(d_1,\ ...,\ d_n)}(A, B) = w_1 b_1 + w_2 b_2 + \cdots + w_n b_n$ represent the distance measure between two IFSs A and B in the universe of discourse $X = \{x_1, x_2, ..., x_n\}$. Then, $Dis_{(d_1,\ ...,\ d_n)}(A, B) = Dis_{(d_1,\ ...,\ d_n)}(B, A)$.*

Proof. Obviously, $d_{Temp}(A, B) = d_{Temp}(B, A)$ based on equation (5), so $d_i(A, B) = d_i(B, A)$. That is, $Dis_{(d_1,\ ...,\ d_n)}(A, B)$ $= Dis_{(d_1,\ ...,\ d_n)}(B, A)$. □

Theorem 4.6. *Let $Dis_{(d_1,\ ...,\ d_n)}(A, B) = w_1 b_1 + w_2 b_2 + \cdots + w_n b_n$ represent the distance measure between two IFSs A and B in the universe of discourse $X = \{x_1, x_2, ..., x_n\}$. If $A \subseteq B \subseteq C$, then $Dis_{(d_1,\ ...,\ d_n)}(A, B) \le Dis_{(d_1,\ ...,\ d_n)}(A, C)$ and $Dis_{(d_1,\ ...,\ d_n)}(B, C) \le Dis_{(d_1,\ ...,\ d_n)}(A, C)$.*

Proof. As $A \subseteq B \subseteq C$, so $\mu_A \le \mu_B \le \mu_C$ and $\nu_A \ge \nu_B \ge \nu_C$ based on Definition 2.2. As shown in Figure 1, we take arbitrary values μ_A, μ_B, μ_C and ν_A, ν_B, ν_C on the coordinate axis. Their intersection points A, B and C form a triangle. Obviously, $\angle ABC$ is always an obtuse angle, so $AB < AC$ and $BC < AC$. So the distance between A and B is smaller than A and C. And the distance between B and C is smaller than A and C. That is, $d_{Temp}(A, B) \le d_{Temp}(A, C)$ and $d_{Temp}(B, C) \le d_{Temp}(A, C)$. So $Dis_{(d_1,\ ...,\ d_n)}(A, B) \le Dis_{(d_1,\ ...,\ d_n)}(A, C)$ and $Dis_{(d_1,\ ...,\ d_n)}(B, C) \le Dis_{(d_1,\ ...,\ d_n)}(A, C)$. □

Theorem 4.7. *Let $Dis_{(d_1,\ ...,\ d_n)}(A, B) = w_1 b_1 + w_2 b_2 + \cdots + w_n b_n$ represent the distance measure between two IFSs A and B in $X = \{x_1, x_2, ..., x_n\}$. Then, $Dis_{(d_1,\ ...,\ d_n)}(A, B) + Dis_{(d_1,\ ...,\ d_n)}(B, C) \ge Dis_{(d_1,\ ...,\ d_n)}(A, C)$.*

Proof. From the proof of Theorem 4.3, we know $d_i(A, B)$ is the distance between any two points of the shadows in the coordinate system shown in Figure 1. According to triangle inequality, we have $d_i(A, B) + d_i(B, C) > d_i(A, C)$, and $d_i(A, B)$
$+d_i(B, C) = d_i(A, C)$ when A, B, C on the same line. So, $Dis_{(d_1,\ ...,\ d_n)}(A, B) + Dis_{(d_1,\ ...,\ d_n)}(B, C) \ge Dis_{(d_1,\ ...,\ d_n)}(A, C)$. □

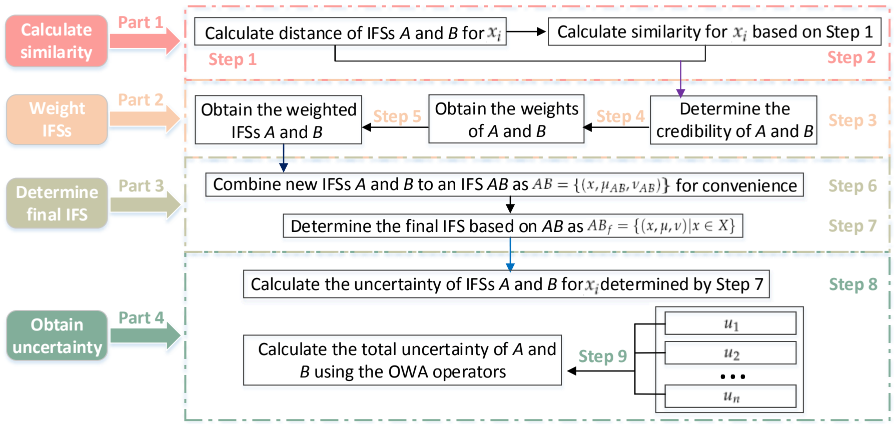

Figure 2: The uncertainty measure of similarity between two IFSs

Through the above theorems and proofs, the proposed distance measure satisfies all the axioms defined in [6, 48]. The similarity measure between two IFSs can be determined naturally based on equation (3) and Definition 3.3 as

$$S(A,B) = 1 - Dis_{(d_1,\ ...,\ d_n)}(A,B). \tag{6}$$

which satisfies the axioms for similarity measure introduced in Definition 3.2.

4.2 The uncertainty measure of similarity between two IFSs

To determine the uncertainty of the obtained similarity measure is quite important for ensuring the performance. In this part, a new method is proposed to calculate the uncertainty of similarity measure.

Definition 4.8. *Let $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle | x \in X\}$, $B = \{\langle x, \mu_B(x), \nu_B(x)\rangle | x \in X\}$ be two IFSs in $X = \{x_1, x_2, ..., x_n\}$. The proposed method is described as following steps. And the flow chart of this method is shown in Figure 2.*

Step 1: Calculate the distance between A and B for x_i as $d_i(A,B)$.

Step 2: Calculate the similarity between A and B for x_i as $S_i(A,B) = 1 - d_i(A,B)$.

Step 3: Determine the credibility between A and B for x_i as $Cre_A(x_i) = S_i(A,A) + S_i(A,B)$ and $Cre_B(x_i) = S_i(B,A) + S_i(B,B)$.

Step 4: Determine the weights of A and B for x_i as $w_A(x_i) = \frac{Cre_A(x_i)}{Cre_A(x_i)+Cre_B(x_i)}$ and $w_B(x_i) = \frac{Cre_B(x_i)}{Cre_A(x_i)+Cre_B(x_i)}$. Because $S_i(A,B) = S_i(B,A)$, so $w_A(x_i) = w_B(x_i) = 0.5$.

Step 5: Obtain the weighted IFSs for x_i as $A_w(x_i) = \{\langle x_i, w_A(x_i) \times \mu_A(x_i), w_A(x_i) \times \nu_A(x_i)\rangle\}$ and $B_w(x_i) = \{\langle x_i, w_B(x_i) \times \mu_B(x_i), w_B(x_i) \times \nu_B(x_i)\rangle\}$.

Step 6: Combine the new IFSs $A_w(x_i)$ and $B_w(x_i)$ as $AB(x_i) = \{\langle x_i, w_A(x_i) \times \mu_A(x_i) + w_B(x_i) \times \mu_B(x_i), w_A(x_i) \times \nu_A(x_i) + w_B(x_i) \times \nu_B(x_i)\rangle\}$, which is denoted as $AB(x_i) = \{\langle x_i, \mu_{AB}, \nu_{AB}\rangle\}$ for convenience.

Step 7: Determine the final IFS for x_i as $AB_f(x_i) = \{\langle x_i, \mu, \nu\rangle\}$ with

$$\mu = \frac{\mu_{AB}^2 + 2\mu_{AB}\pi}{1 - 2\mu_{AB}\nu_{AB}} = \frac{\mu_{AB}^2 + 2\mu_{AB}(1 - \mu_{AB} - \nu_{AB})}{1 - 2\mu_{AB}\nu_{AB}} \quad \text{and} \quad \nu = \frac{\nu_{AB}^2 + 2\nu_{AB}\pi}{1 - 2\mu_{AB}\nu_{AB}} = \frac{\nu_{AB}^2 + 2\nu_{AB}(1 - \mu_{AB} - \nu_{AB})}{1 - 2\mu_{AB}\nu_{AB}}$$

Step 8: Calculate the uncertainty between IFSs A and B for x_i as $u_i(A,B) = \frac{1-|\mu-\nu|+\pi}{1+\pi}$ [38].

Step 9: Calculate the total uncertainty of similarity measure between A and B using OWA operators as $U_{(u_1,\ ...,\ u_n)}(A,B) = w_1v_1 + w_2v_2 + \cdots + w_nv_n$, where v_i is the ith largest uncertainty measure of all the elements between A and B.

Theorem 4.9. *Let $U_{(u_1,\ ...,\ u_n)}(A,B) = w_1v_1 + w_2v_2 + \cdots + w_nv_n$ represent the uncertainty measure of similarity between two IFSs A and B in the universe of discourse $X = \{x_1, x_2, ..., x_n\}$. Then, $0 \le U_{(u_1,\ ...,\ u_n)}(A,B) \le 1$.*

Proof. As formula $u_i(A,B)=\frac{1-|\mu-\nu|+\pi}{1+\pi}$ in 4.2, and $0\le\mu,\nu\le 1,\pi=1-\mu-\nu$.

(I). $u_i(A,B)=1-\frac{|\mu-\nu|}{1+\pi}=1-\frac{|\mu-\nu|}{2-\mu-\nu}$. It is obvious that $|\mu-\nu|\ge 0$ and $2-\mu-\nu\ge 0$. So, $\frac{|\mu-\nu|}{2-\mu-\nu}\ge 0$, that is, $u_i(A,B)=1-\frac{|\mu-\nu|}{2-\mu-\nu}\le 1$;

(II). When $\mu\ge\nu$, $2\mu\le 2$ holds, and when $\mu\le\nu$, $2\nu\le 2$ holds. We have $|\mu-\nu|+\mu+\nu-2\le 0$, next, $|\mu-\nu|\le 2-\mu-\nu$, so, $\frac{|\mu-\nu|}{2-\mu-\nu}\le 1$, that is, $u_i(A,B)=1-\frac{|\mu-\nu|}{2-\mu-\nu}\ge 0$. Based on the proof process of Theorem 4.3, it is easy to draw the conclusion $0\le U_{(u_1,\ \dots,\ u_n)}(A,B)\le 1$. □

Theorem 4.10. *Let $U_{(u_1,\ \dots,\ u_n)}(A,B)=w_1v_1+w_2v_2+\cdots+w_nv_n$ represent the uncertainty measure of similarity between two IFSs A and B in the universe of discourse $X=\{x_1,x_2,...,x_n\}$. If A and B are crisp sets, then $U_{(u_1,\ \dots,\ u_n)}(A,B)=0$.*

Proof. As the crisp sets A and B, so $\pi=0$ and $|\mu-\nu|=1$. Then $u_i(A,B)=1-\frac{|\mu-\nu|}{1+\pi}=0$ holds, likes Theorem 4.3, $U_{(u_1,\ \dots,\ u_n)}(A,B)=0$. □

4.3 Comparison between the proposed similarity measure and existing methods

To show the effectiveness of the proposed measure, several comparative examples are conducted between the proposed similarity measure and some existing methods.

Let $A=\{(x,\mu_A(x),\nu_A(x))|x\in X\}$ and $B=\{(x,\mu_B(x),\nu_B(x))|x\in X\}$ be two IFSs in $X=\{x_1,x_2,...,x_n\}$. Several existing methods are introduced briefly as follows. The similarity measure between vague sets was proposed in [9]. The similarity between vague sets A and B can be evaluated by function T:

$$T(A,B)=\frac{1}{n}\sum_{i=1}^{n}(1-|\frac{\mu_A(x_i)-\nu_A(x_i)-(\mu_B(x_i)-\nu_B(x_i))}{2}|). \tag{7}$$

In [32], a new similarity measure between IFSs were proposed and corresponding proofs were given. The core definitions are as follows:

$$S_L(A,B)=1-\sqrt[p]{\frac{\sum_{i=1}^{n}|\Psi_A(x_i)-\Psi_B(x_i)|^p}{n}}. \tag{8}$$

where p is a parameter with $1\le p<+\infty$, and for each i, $\Psi_A(x_i)=\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}$, $\Psi_B(x_i)=\frac{\mu_B(x_i)+1-\nu_B(x_i)}{2}$.

In addition, the main forms of similarity measure in [48] and [58] are shown below:

$$Sim=1-\frac{1}{2n}\sum_{i=1}^{n}(|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|+|\pi_A(x_i)-\pi_B(x_i)|). \tag{9}$$

$$S_X(A,B)=\frac{1}{n}\sum_{i=1}^{n}\frac{d(\alpha_A(x_i),\alpha_B^c(x_i))}{d(\alpha_A,\alpha_B)+d(\alpha_A,\alpha_B^c)}. \tag{10}$$

where $\alpha_A(x_i)$ and $\alpha_B(x_i)$ are the ith IFSs of A and B, and $d(\alpha_A(x_i),\alpha_B(x_i))=\frac{1}{2}(|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|+||\mu_A(x_i)-\mu_B(x_i)|-|\nu_A(x_i)-\nu_B(x_i)||)$.

Similarity measure $S_{XX}^{\eta,\kappa}$ was defined by Xia and Xu in [55] as follows:

$$S_{XX}^{\eta,\kappa}(A,B)=1-(\frac{1}{n}\sum_{i=1}^{n}|(1-\kappa)(\mu_A(x_i)-\mu_B(x_i))-\kappa(\nu_A(x_i)-\nu_B(x_i))|^{\eta})^{\frac{1}{\eta}}. \tag{11}$$

Another similarity measure was introduced by Li and Xu in [34] as follows:

$$S_{LX}(A,B)=1-\frac{\sum_{i=1}^{n}|s_A(x_i)-s_B(x_i)|}{4n}-\frac{\sum_{i=1}^{n}(|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|)}{4n}. \tag{12}$$

where $S_A(x_i)=\mu_A(x_i)-\nu_A(x_i)$ and $S_B(x_i)=\mu_B(x_i)-\nu_B(x_i)$.

In order to highlight the superiority of our method, two cases are conducted to compare our proposed method with these existing methods. The experimental process is shown in Table 1, and the related analyses are as follows.

For the two cases in Table 1, it is obvious that the similarity between A and B is less than B and C. However, the magnitude relations of methods T, S_L, S_X, S_{XX} and S_{LX} are contrary to our common sense. Therefore, the conclusion is that the similarity measures Sim and S are more reasonable than T, S_L, S_X, $S_{XX}^{\eta,\kappa}$ and S_{LX}. In addition, the similarity of A and B in method Sim is 0 by Sim seems to be inconsistent with the facts. Therefore, the proposed similarity measure can play a good performance under actual situation, but the other methods cannot. What's more, although some methods (mentioned above and others [3]) perform well, most of them are single-valued measures. The proposed vector valued measure can give similarity and uncertainty, which compensates for the shortcomings of single-valued measures.

Case 1: $A = \{(x, 0, 0)\}$, $B = \{(x, 0.5, 0.5)\}$, $C = \{(x, 0.49, 0.51)\}$ be three IFSs			
Measure method	$S(A,B)$	Magnitude relation	$S(B,C)$
T[9]	1.0000	$>$	0.9900
S_L[32]	1.0000	$>$	0.9900
S_X[58]	0.5000	$=$	0.5000
$S_{XX}^{2,0.5}$[55]	1.0000	$>$	0.9900
Sim[48]	0.0000	$<$	0.9900
Our method	0.5000	$<$	0.9900
Case 2: $A = \{(x, 0.5, 0.3)\}$, $B = \{(x, 0.4, 0.2)\}$, $C = \{(x, 0.5, 0.2)\}$ be three IFSs			
S_{LX}[34]	0.9500	$=$	0.9500
Our method	0.9000	$<$	0.9293

Table 1: Comparison between existing similarity measures and the proposed method

Value size	Intuitionistic fuzzy set
Extremely large	(1.00, 0.00)
Very very large	(0.90, 0.10)
Very large	(0.80, 0.15)
Large	(0.75, 0.20)
Medium large	(0.60, 0.28)
Fair	(0.52, 0.40)
Medium small	(0.38, 0.50)
Small	(0.25, 0.55)
Very small	(0.15, 0.75)
Very very small	(0.10, 0.90)

Table 2: Value variables for the performance rating of iris plant

5 Application and experiment

To illustrate the effectiveness of our proposed similarity measure [5], a real application of classification is conducted using the data from UCI Machine Learning Repository. In the following part, a classifier is built based on the database of iris plant. There are 150 samples in this database, and they are divided into three categories, namely $Setosa$, $Versicolour$ and $Virginica$. Each sample has four attributes, namely $Sepal\ Length$ (SL), $Sepal\ Width$ (SW), $Petal\ Length$ (PL) and $Petal\ Width$ (PW). In this experiment, 30 samples are randomly selected from each class as the training set, and the remaining 20 samples will be used as the testing set. The primary task is to convert the data type in iris database into IFS representation that can be used in our proposed method. An approach is introduced as follows. Lorem ipsum dolor sit amet, consectetuer adipiscing elit. Ut purus elit, vestibulum ut, placerat ac, adipiscing vitae, felis. Curabitur dictum gravida mauris. Nam arcu libero, nonummy eget, consectetuer id, vulputate a, magna. Donec vehicula augue eu neque. Pellentesque habitant morbi tristique senectus et netus et malesuada fames ac turpis egestas. Mauris ut leo. Cras viverra metus rhoncus sem. Nulla et lectus vestibulum urna fringilla ultrices. Phasellus eu tellus sit amet tortor gravida placerat. Integer sapien est, iaculis in, pretium quis, viverra ac, nunc. Praesent eget sem vel leo ultrices bibendum. Aenean faucibus. Morbi dolor nulla, malesuada eu, pulvinar at, mollis ac, nulla. Curabitur auctor semper nulla. Donec varius orci eget risus. Duis nibh mi, congue eu, accumsan eleifend, sagittis quis, diam. Duis eget orci sit amet orci dignissim rutrum.

Step 1: The universe of discourse $X = \{x_1, x_2, ..., x_n\}$ is constructed by all attributes of sample in the tested database.

Step 2: Randomly select part of the samples as training set $T = \{t_1, t_2, ..., t_m\}$, and the remaining samples as the testing set.

Step 3: For each attribute x_i in X, calculate the maximum value x_{max} and minimum value x_{min} of the training set, where $x_{max} = max\{x_{ij}\}$ and $x_{min} = min\{x_{ij}\}$. The region between x_{max} and x_{min} will be divided into 10 levels ($R = \{r_1, r_2, ..., r_{10}\}$) by following method:
$r_k = [min\{x_{ij}\} + \frac{max\{x_{ij}\}-min\{x_{ij}\}}{10}(k-1), min\{x_{ij}\} + \frac{max\{x_{ij}\}-min\{x_{ij}\}}{10}k]$. Each level corresponds to an IFS in Table

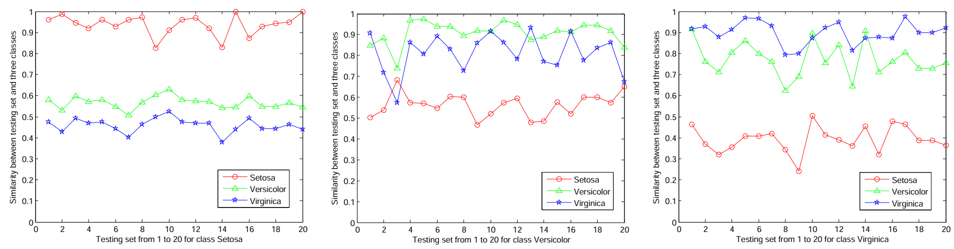

Figure 3: The similarity measures between testing sets and IFSs of three classes

	Sepal length	Sepal width	Petal length	Petal width
Setosa	(0.15, 0.75)	(0.75, 0.20)	(0.10, 0.90)	(0.10, 0.90)
Versicolour	(0.52, 0.40)	(0.38, 0.50)	(0.60, 0.28)	(0.60, 0.28)
Virginica	(0.75, 0.20)	(0.52, 0.40)	(0.80, 0.15)	(0.80, 0.15)

Table 3: IFS representations of iris database for classification

2.

Step 4: For each class C_i, calculate the average $ave_{ij} = ave\{x_{ij}\}$ of each attribute A_j in training set. The level r_k will be determined by the interval ave_{ij} belongs to. So the IFS_k can be considered as the IFS representation of this class.

Step 5: The membership degree and non-membership degree of attribute x_i under each class have been obtained above. The corresponding IFS representations of testing set can be obtained by repeating Step 1-4.

The method to convert iris database into IFSs has been introduced above. The IFS representations of three classes $Setosa$,$Versicolour$ and $Virginica$ in $X = \{x_1, x_2, \ldots, x_4\}$ are obtained and shown in Table 3. Similarly, IFSs of each testing set under each class can also be obtained and shown in Table 4. Next, similarity will be calculated between IFSs of the three classes and the 20 testing sets of each class. The purpose is to determine which category the target belongs to. Taking class $Virginica$ as an example, the similarity measures are conducted between the 20 testing sets of $Virginica$ and other three IFSs, and the results are shown in Table 5. In addition, the similarity measures between testing sets of each class and other three classes are shown in Figure 3. Some conclusions can be drawn from Figure 3, the accuracy of classification for the three classes are 100%, 80% and 90%, respectively, and the average accuracy is 90%. It is worth noting that the uncertainty measure also play a key role in the classification process. For example, for the first testing set of class $Virginica$, the similarity between class $Virginica$ and $Versicolour$ is the same, so this testing set cannot be accurately classified by similarity measure only. But the uncertainty of $Virginica$ is less than $Versicolour$ (see Table 5), so the testing set is classified into class $Virginica$ naturally. What's more, we know that this testing set really comes from class $Virginica$, so the uncertainty measure can ensure the accuracy of classification.

6 Conclusions

Intuitionistic fuzzy sets (IFSs) have been developed and applied widely since it was introduced by Atanassov. Similarity measures have always been the focus of researches. However, most of the existing methods are based on single-valued measures, which may result in an incorrect conclusion because the effectiveness cannot be guaranteed. In this paper, the vector valued similarity measure based on OWA operators is proposed, which makes up the shortcomings of single-valued measures by similarity and uncertainty measures. Some definitions and theorems are presented and proved to satisfy the axioms for IFSs. A practical application of classification problem is conducted to illustrate the effectiveness of the new vector valued similarity measure.

However, there are still some potential problems, which needs to be solved later to improve the current work. In the case of decision making, for example, such a problem inevitably occurs. When using our proposed method, the uncertainty measures between different IFSs are very large, then how do we make decisions? Is it possible to consider the sum of similarity and uncertainty as the final criterion for decision, or other more reasonable approaches? Therefore, in the future study, the framework of the presented similarity measure methodology based on OWA operators can be improved better. For example, the standard method for decision making based on the proposed two-tuple needs to be perfected. And in order to display the superiority of the proposed similarity measure, more applications, such as supply chain management and environmental impact assessment, should be conducted.

Setosa			
SL	SW	PL	PW
(0.25, 0.55)	(0.75, 0.2)	(0.1, 0.9)	(0.1, 0.9)
(0.15, 0.75)	(0.8, 0.15)	(0.1, 0.9)	(0.1, 0.9)
(0.15, 0.75)	(0.52, 0.4)	(0.1, 0.9)	(0.1, 0.9)
(0.15, 0.75)	(0.52, 0.4)	(0.1, 0.9)	(0.1, 0.9)
(0.25, 0.55)	(0.75, 0.2)	(0.1, 0.9)	(0.1, 0.9)
(0.25, 0.55)	(0.9, 0.1)	(0.1, 0.9)	(0.1, 0.9)
(0.1, 0.9)	(0.8, 0.15)	(0.1, 0.9)	(0.1, 0.9)
(0.15, 0.75)	(0.75, 0.2)	(0.1, 0.9)	(0.15, 0.75)
(0.52, 0.4)	(1, 0)	(0.1, 0.9)	(0.15, 0.75)
(0.38, 0.5)	(0.75, 0.2)	(0.1, 0.9)	(0.15, 0.75)
(0.25, 0.55)	(0.75, 0.2)	(0.1, 0.9)	(0.1, 0.9)
(0.15, 0.75)	(0.6, 0.28)	(0.1, 0.9)	(0.1, 0.9)
(0.1, 0.9)	(0.52, 0.4)	(0.1, 0.9)	(0.1, 0.9)
(0.1, 0.9)	(0.15, 0.75)	(0.1, 0.9)	(0.1, 0.9)
(0.15, 0.75)	(0.75, 0.2)	(0.1, 0.9)	(0.1, 0.9)
(0.25, 0.55)	(0.9, 0.1)	(0.15, 0.75)	(0.15, 0.75)
(0.25, 0.55)	(0.9, 0.1)	(0.1, 0.9)	(0.1, 0.9)
(0.1, 0.9)	(0.6, 0.28)	(0.1, 0.9)	(0.1, 0.9)
(0.25, 0.55)	(0.8, 0.15)	(0.1, 0.9)	(0.1, 0.9)
(0.15, 0.75)	(0.75, 0.2)	(0.1, 0.9)	(0.1, 0.9)

Table 4: IFS representations of each testing set under each class

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Testing set	Setosa		Versicolor		Virginica	
	S	U	S	U	S	U
1	0.4630	0.7269	**0.9157**	**0.5196**	**0.9157**	**0.4093**
2	0.3707	0.8097	0.7603	0.4020	0.9286	0.2440
3	0.3207	0.7155	0.7110	0.3604	0.8794	0.2246
4	0.3544	0.8971	0.8050	0.3597	0.9131	0.2859
5	0.4092	0.8203	0.8618	0.4314	0.9696	0.3469
6	0.4084	0.8089	0.7992	0.4473	0.9677	0.2702
7	0.4197	0.7298	0.7626	0.4020	0.9303	0.2303
8	0.3427	0.6289	0.6254	0.2150	0.7939	0.0945
9	0.2417	0.7542	0.6923	0.2549	0.8004	0.2318
10	0.5055	0.6615	0.8974	0.5881	0.8729	0.4655
11	0.4136	0.8157	0.7550	0.3828	0.9225	0.2170
12	0.3906	0.8567	0.8421	0.4111	0.9498	0.3350
13	0.3614	0.5925	0.6451	0.2352	0.8136	0.1065
14	0.4549	0.7390	0.9077	0.4978	0.8732	0.3780
15	0.3207	0.7155	0.7110	0.3604	0.8794	0.2246
16	0.4786	0.6465	0.7618	0.3979	0.8719	0.2162
17	0.4633	0.7429	0.8068	0.4400	0.9745	0.2508
18	0.3886	0.7662	0.7305	0.3650	0.8980	0.2085
19	0.3886	0.7662	0.7305	0.3650	0.8980	0.2085
20	0.3647	0.7958	0.7551	0.4093	0.9235	0.2498

Table 5: The similarity and uncertainty measures between testing sets and IFSs of class $Virginica$

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