Iranian Journal of Fuzzy Systems

Volume 16, Number 3, (2019), pp. 127-143

University of Sistan and Baluchestan

Decentralized prognosis of fuzzy discrete-event systems

B. Benmessahel[1], M. Touahria[2], F. Nouioua[3], J. Gaber[4] and P. Lorenz[5]

[1,2] *Computer Science Department, University of Ferhat Abbas Sétif 1, Pole 2 - El Bez, 19000 Sétif, Algeria*
[3] *Aix-Marseille University, CNRS, ENSAM, University of Toulon, LSIS UMR 7296, Marseille, France*
[4] *Computer Science Department, Universite de Technologie de Belfort Montbeliard, Rue Thierry Mieg, 90010 Belfort, France*
[5] *Computer Science Department, IUT, University of Haute Alsace, Colmar, France*

bilal.benmessahel@univ-setif.dz, mohamed.touahria@univ-setif.dz, farid.nouioua@lsis.org, gaber@utbm.fr, lorenz@ieee.org

Abstract

This paper gives a decentralized approach to the problem of failure prognosis in the framework of fuzzy discrete event systems (FDES). A notion of co-predictability is formalized for decentralized prognosis of FDESs, where several local agents with fuzzy observability rather than crisp observability are used in the prognosis task. An FDES is said to be co-predictable if each faulty event can be predicted prior to its occurrence by at least one local agent using the observability of fuzzy events. The verification of the decentralized predictability is performed by constructing a fuzzy co-verifier from a given FDES. The complexity of the fuzzy co-verifier is polynomial with respect to the FDES being predicted, and is exponential with respect to the number of the local prognosis agents. Then, a necessary and sufficient condition for the co-predictability of FDESs is given. In addition, we show that the proposed method may be used to deal with the decentralized prognosis for both FDESs and crisp DESs. Finally, to illustrate the effectiveness of the approach, some examples are provided.

Keywords: Co-predictability, Discrete-event systems, Decentralized prognosis, Failure detection, Fuzzy automata, Fuzzy systems.

1 Introduction

Actually there are many technological and engineering systems such as transportation systems, manufacturing systems and computer networks can be considered as discrete event systems (DES) [4] at some level of abstraction. A DES is a dynamical process with discrete states. Its dynamic evolves from a state to another state in response to certain occurrences of discrete events. A large amount of research has been done around the task of failure diagnosis which concerns the detection of a failure after its occurrence. However, For certain critical infrastructures such as high speed railway systems and large scale manufacturing systems, it is very expensive to recover the system after failure occurrence, which motivates the work on the task of prognosis of failure before its occurrence. The task of failure prognosis in crisp DES consists in predicting with certainty the future occurrences of failures based on the observations from the system whose current state is normal. If the failure is predicted, the system operator can be warned and may decide to halt the system or otherwise take preventive measures. The Failure prognosis in the framework of fuzzy discrete event systems (FDES) has not received enough attention since all recent research on the failure prognosis problem in the literature [16, 42, 12, 37, 39, 28, 3, 5, 13] deals with crisp discrete event systems. In this paper, we try to fill this gap and address the problem of decentralized prognosis in FDES.

1.1 Motivation

Studying the problem of prognosis in fuzzy discrete event systems is motivated by the fact that in real life, designers and engineers are often faced with difficulties related to the presence of different forms of uncertainties or imprecision in

Corresponding Author: B. Benmessahel
Received: January 2017; Revised: June 2017; Accepted: December 2018.

the available information [29]. In order to cope with such imperfections in the available information, two main solutions are proposed in discrete event systems literature. The first solution is based on stochastic discrete event systems [28, 5, 6, 17] used to model uncertain systems in which the uncertainty is modeled by a probability value associated with each transition of the system. The second solution is based on fuzzy discrete event systems [24, 23] used to model imprecise systems where imprecision is modeled by possibility degrees associated with states and transitions. In fact, stochastic DESs and FDESs are modeling approaches of DESs that give solutions for two completely different situations that may be encountered in real applications. The stochastic DESs are used in situations where one is uncertain about the transition to be fired from a given state and the probability values reflect frequencies of occurrence which can be extracted from the study of the history of the system. Also, in an effective running of the system, if a transition is fired then the system changes from a state to only one particular state. However fuzzy DESs are used in situations where one cannot classify exactly the states, the transitions and the events of the system in completely crisp categories. So the system can be in different states simultaneously with different membership degrees and can change its state to several states with different degrees.

As we said above, uncertainty and impression are typical characteristics that are present in most real systems. The idea of computing with words as suggested by Zadeh [44] as a methodology in which the objects of computation are words is a good tool for handling uncertainty, and imprecision. For example, young , small, old, good, and large, etc. can be regarded as a possibility distribution which may conform more to humans perception when describing the real world problems. There exist many systems which can be described better using computing with words methodology, especially those in biomedical applications [23] in which the states and transitions of a system (e.g., a patients condition) are always somewhat uncertain and vague. For example (this example is given in [24, 23]), we can describe the patients condition with "Good", "Fair" , and "Poor", where the patients condition state can simultaneously belong to "good", "fair" and "poor" with some possibilities. Thus the condition can be written with a word "Patients Condition" (PC): PC = x/G + y/F + z/P where $x, y, z \in [0, 1]$ denote the possibility distributions of "good", "fair" and "poor", respectively. Hence, the Patients Condition can be represented by a vector [xyz]. Recently, Lin and Ying [24, 23] have been inspired from this representation and proposed the fuzzy DESs. Lin and Ying [24, 23], initiated the study of FDESs by combining fuzzy logic [44] to the conventional or crisp discrete event systems (DESs) [4], with the aim to solve those problems that cannot be treated by crisp DESs. Since the first proposition of fuzzy discrete systems, increasing attention has been received by this domain, and many important and successful works have been proposed using FDESs. Among these works we can cite those in [25, 19, 1, 20, 41, 22, 35, 36, 8, 9, 10, 27]. Especially, FDESs have been successfully applied to biomedical systems, to handle treatment planning and control for HIV/AIDS patients [40].

To sum up, FDESs are quite suitable for modeling imprecision that cannot be handled neither by crisp DESs nor by probabilistic DESs. Indeed, FDESs have been successfully applied to several real applications. This motivates our interest to study diagnosis and prognosis in the framework of FDESs.

The problems of diagnosis and diagnosability have been studied in the framework of FDESs (see [19, 25] and the references there in). The problem of diagnosis is to detect a failure after its occurrence. However, the problem of failure prognosis is to predict the occurrence of failure prior to its occurrence. So the system operator can be warned and may decide to halt the system or otherwise take preventive measures. So considering the importance of the prognosis task there is obviously a real need to study this problem in general and especially in the FDES setting. The purpose of this paper is to give designers of systems valuable information about future occurrence of specials events like failures. To the best of our knowledge, the present paper is the first one which tackles the problem of defining and checking fault prognosis in the framework of FDES.

1.2 Related work

Due to its practical and theoretical importance, failure prognosis has received considerable attention in the DES literature. The first work that deals with the problem of failure prognosis was considered for partially observed discrete event systems in [7]. After that many works have been proposed (see, e.g., [39, 28, 11, 3, 5, 6, 17, 18, 38]).

The decentralized prognosis approach proposed in the present paper is different from those present in the literature [19, 16, 2, 34, 30, 43]. In this work, the prognosis framework is fuzzy and has a decentralized structure. Also observability of events is defined to be fuzzy, so that each event may be observable with some membership degree and failure occurs on each event with a certain possibility degree. This represents a generalization of conventional observability [16] where all events are either absolutely observable or absolutely unobservable, and failure occurs only on some events. In this approach, an FDES being co-predictable means that the occurrence of any failure event with a possibility of failure exceeds a specified upper bound can be prognosed by at least one local prognosis agent within a bounded delay.

This work is different from [39, 28], and [5, 6]. The approaches proposed in, [39, 28], and [5, 6] deal with the prognosis of probabilistic DESs, where the prognosis problem is modeled by probabilistic automata. Also the model of FDES

used in this paper is different from the model used in the approach proposed by Kilic et al for the problem of diagnosis in [15, 14]. The model of FDES in the approach of Kilic et al is considered as a fuzzy expert system with IFTHEN rules. In contrast, this paper studies the prognosis of fuzzy discrete event systems represented by maxmin systems with fuzzy events and states [31, 32].
In reality most real systems such as transportation systems, computer networks and communication protocols among others are large scale systems. They are constituted by interconnected and decentralized components in terms of information and geographical location. Therefore, the centralized prognosis methods are not well suited for this type of systems. A decentralized prognosis approach is therefore more appropriate and effective for the prognosis of real systems. In fact the decentralized predictability approaches are divided into two main categories: (1) the approaches considering a fault as the violation of specifications and (2) the approaches considering a fault as the execution of an event. The problem of decentralized prognosis for DESs was first considered by R. Kumar et al. [16] where the authors present a decentralized prognosis framework without involving any communication among the local prognosers, in which the notion of co-prognosability (Co-predictability) was introduced to capture the requirement for a decentralized prognosable DES. In the approach proposed by R. Kumar et al., the failure is considered as the violation of specifications but in our approach a fault is considered as the execution of an event.

1.3 Contributions of the paper

In this paper, we study the decentralized prognosis of fuzzy discrete event systems. We introduce the notion of co-predictability for decentralized prognosis of FDESs, in which the events observability is defined to be fuzzy rather than crisp. A FDES is said to be co-predictable if each failure can be prognosed by at least one local prognosis agent within a bounded delay. We construct a co-verifier from a given FDES to perform the decentralized prognosis. Especially, the co-verifier is used to deduce a necessary and sufficient condition for the co-predictability of FDESs. The contributions of this work are threefold:

- A decentralized approach for the prognosis of fuzzy discrete event systems is presented by introducing the notion of co-predictability of FDESs.
- The design of a fuzzy co-verifier from a given fuzzy discrete event system to perform the decentralized prognosis of FDESs.
- A necessary and sufficient condition for the co-predictability of FDESs is presented. And some examples are given to show that the proposed decentralized approach can be used for DESs as well as FDESs.

The remainder of this paper is organized as follows. Section II recalls some preliminaries and definitions concerning FDESs. In section III, we present the notion of co-predictability for FDESs. Section IV is devoted to the construction steps of the observability-based co-verifier of a FDES. In addition, a necessary and sufficient condition for co-predictability in FDESs is provided. Finally, to illustrate the condition of co-predictability for FDESs some examples are given in Section V. To conclude, in Section VI, a summary of results and some perspectives to future work are presented.

2 Preliminaries and Definitions

Fuzzy discrete event systems generalize traditional or crisp discrete event system to better cope with the problems of fuzziness and impreciseness arising in reality. As we said in the introduction, the advantage of FDESs is that their capability to represent fuzziness in states and events permits them to deal with the uncertainty and imprecision in the observability of the system, which is very interesting to real-world phenomena. In the following definition we give the model of the system used in this paper by defining the fuzzy finite automaton. In the FDESs framework, the FDES is composed of a set of fuzzy states and a set of fuzzy events. Each fuzzy state $\tilde{q}$ is represented by a vector $[a_0, a_1, ..., a_{n-1}]$, where $a_i \in [0, 1]$ represents the possibility that the system being in state q_i for $i = 0, 1, , n-1$ where n is the number of possibilities. For example let us take the patients condition $\tilde{q}$ which belongs to "Good" with the membership degree 0.2, to "Poor" with the membership degree 0.6, and to "Bad" with the membership degree 0.2. Hence the patients condition can be represented by the vector $\tilde{q} = [0.2, 0.6, 0.2]$. In addition, each event $\tilde{\sigma}$ is a represented by a state transition $matrix[a_{ij}]_{n\times n}$, where $a_{ij} \in [0, 1]$ represents the likelihood of the system changing from state q_i to state q_j.

Definition 2.1. [31] *A fuzzy automaton is a four-tuple*

$$\tilde{G} = (\tilde{Q}, \tilde{E}, \tilde{\delta}, \tilde{q}_0) \tag{1}$$

Here, $\tilde{Q}$ and $\tilde{E}$ are sets of fuzzy states and fuzzy events, respectively. $\tilde{q}_0$ is the initial state; $\tilde{\delta} : \tilde{Q} \times \tilde{E} \to \tilde{Q}$ describes the state transition function and is defined as $\tilde{\delta}(\tilde{q}, \tilde{\sigma}) = \tilde{q} \odot \tilde{\sigma}$. The symbol $\odot$ represents a max-min operation: Let $A[a_{ij}]_{n\times m}$ and $B[b_{ij}]_{m\times k}$ be two matrixes, define $A \odot B = [c_{ij}]_{n\times k}$, where

$$c_{ij} = max_{l=1}^{m} min\{a_{il} b_{lj}\} \tag{2}$$

In fact, in practice the transition function $\tilde{\delta}$ can be considered a partial function.

Example 2.2. *Let $\tilde{G} = (\tilde{Q}, \tilde{E}, \tilde{\delta}, \tilde{q}_0)$ be an FDES, if Fuzzy state $\tilde{q} = [0.8, 0.2] \in \tilde{Q}$ and fuzzy event $\tilde{\sigma}$ occur in $\tilde{q}$ where $\tilde{\sigma} = \begin{pmatrix} 0.4 & 0.8 \\ 0.2 & 0.3 \end{pmatrix} \in \tilde{E}$ then $\tilde{\delta}(\tilde{q}, \tilde{\sigma}) = \tilde{q} \odot \tilde{\sigma} = [0.8, 0.2] \odot \begin{pmatrix} 0.4 & 0.8 \\ 0.2 & 0.3 \end{pmatrix} = [0.4, 0.8] = \tilde{q}'$.*

In the framework of Fuzzy DESs, each fuzzy event has simultaneously membership in the observable event set, in the unobservable event set, and in the failure event set; with different degrees. Suppose that $\tilde{G}$ is an FDES and $\tilde{E}$ is the set of the fuzzy events. We use three fuzzy subsets: the unobservable events fuzzy subset: $\tilde{\Sigma}_{uo} : \tilde{E} \to [0, 1]$, the observable events fuzzy subset $\tilde{\Sigma}_o : \tilde{E} \to [0, 1]$ and the failure events fuzzy subset $\tilde{\Sigma}_f : \tilde{E} \to [0, 1]$. $\tilde{\Sigma}_f(\tilde{\sigma})$ denotes the likelihood of failure occurring on $\tilde{\sigma} \in \tilde{E}$, $\tilde{\Sigma}_o(\tilde{\sigma})$ and $\tilde{\Sigma}_{uo}(\tilde{\sigma})$ denote the observability and unobservability degree of $\tilde{\sigma}$. We have, $\tilde{\Sigma}_{uo}(\tilde{\sigma}) + \tilde{\Sigma}_o(\tilde{\sigma}) = 1$. The language generated by $\tilde{G}$, denoted as $L_{\tilde{G}}$ (or simply L when it is clear from the context), is defined as follows:

$$L_{\tilde{G}} = \{\tilde{s} \in \tilde{E}^* : (\exists \tilde{q} \in \tilde{Q})\ \tilde{\delta}(\tilde{q}_0, \tilde{s}) = \tilde{q}\} \tag{3}$$

$\tilde{\Sigma}_f(\tilde{s}) = max\{\tilde{\Sigma}_f(\tilde{\sigma}) : \tilde{\sigma} \in \tilde{s}\}$ and $\tilde{\Sigma}_o(\tilde{s}) = min\{\tilde{\Sigma}_o(\tilde{\sigma}) : \tilde{\sigma} \in \tilde{s}\}$ represent the failure degree and observability degree of the string $\tilde{s} \in L_{\tilde{G}}$, respectively.

The notation $L/\tilde{s}$ represents the post language of L after $\tilde{s}$ and defined as follows:

$$L/\tilde{s} = \{\tilde{t} \in \tilde{E}^*\ (\exists \tilde{q} \in \tilde{Q})\ \tilde{\delta}(\tilde{q}_0, \tilde{s}\tilde{t}) = \tilde{q}\} \tag{4}$$

Definition 2.3. *[32] Let $\tilde{\sigma} \in \tilde{E}$, the $\tilde{\sigma}$-projection operation or $\tilde{\sigma}$-Mask denoted $M_{\tilde{\sigma}} : \tilde{E}^* \to \tilde{E}^*$ is defined as: $M_{\tilde{\sigma}}(\epsilon) = \epsilon$, and $M_{\tilde{\sigma}}(\tilde{s}\tilde{a}) = M_{\tilde{\sigma}}(\tilde{s}) M_{\tilde{\sigma}}(\tilde{a})$ for $\tilde{a} \in \tilde{E}$ and $\tilde{s} \in \tilde{E}^*$, where*

$$M_{\tilde{\sigma}}(\tilde{a}) = \begin{cases} \tilde{a}, & if\ \ \tilde{\Sigma}_o(\tilde{a}) > \tilde{\Sigma}_o(\tilde{\sigma}) \\ \epsilon, & otherwise \end{cases} \tag{5}$$

The inverse projection operation is given by:

$$M_{\tilde{\sigma}}^{-1}(\tilde{t}) = \{\tilde{s} \in \tilde{E}^*\ (\exists \tilde{q} \in \tilde{Q})\ \ \tilde{\delta}(\tilde{q}_0, \tilde{s}) = \tilde{q} \wedge M_{\tilde{\sigma}}(\tilde{s}) = \tilde{t}\ \} \tag{6}$$

Remark 2.4. . *The aim of $\tilde{\sigma}$-projection is to erase the fuzzy events where the observability is not greater than $\tilde{\Sigma}_o(\tilde{\sigma})$.*

In order to make a correct prognosis decision, we specify an upper bound $\tilde{\Sigma}_f(\tilde{\sigma})$ for each $\tilde{\sigma} \in \tilde{E}$. If the failure degree of a string $\tilde{s}$ exceeds a specified bound (i.e., $\tilde{\Sigma}_f(\tilde{s}) > \tilde{\Sigma}_f(\tilde{\sigma})$) then we consider that $\tilde{s}$ is a failure string. In the next section, we give an approach of fuzzy predictability to predict the occurrence of those failure strings that exceed the specified upper bound.

We denote the set of faulty events by $\tilde{E}_f = \{\tilde{\sigma} \in \tilde{E} : \tilde{\Sigma}_f(\tilde{\sigma}) > 0\}$. For $\tilde{\sigma} \in \tilde{E}_f$, sequences that end with an event where the likelihood of failure occurring is greater than $\tilde{\Sigma}_f(\tilde{\sigma})$ is defined as follow:

$$\tilde{\Psi}_{\tilde{\sigma}} = \{\tilde{s}\tilde{e} \in L : \tilde{s} \in \tilde{E}^*, \tilde{e} \in \tilde{E}_f\} \tag{7}$$

A prefix of a string $s = a_0 a_1 \cdots a_n$, is a string $s' = a_0 a_1 \cdots a_m$, where $m \leq n$. A proper prefix of a string is not equal to the string itself ($0 \leq m < n$).
The set of prefixes of $s = a_0 a_1 \cdots a_n$, which is composed of all proper prefixes of s is denotes $Pref(s)$. After defining the prefix of a string, we give the definition of the maximal non faulty prefix of a faulty trace:

Definition 2.5. *Let $\tilde{s} \in \tilde{\Psi}_{\tilde{\sigma}}$ be a trace of $L_{\tilde{G}}$ ending by a faulty event. The maximal prefix of $\tilde{s}$ without a fault denoted $MaxPref(\tilde{s})$ is defined by, $\tilde{t} = MaxPref(\tilde{s})$ if and only if,*

- *$\tilde{t} \in Pref(\tilde{s})$ and $\tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{\sigma})$.*
- *for all $\tilde{t}' \in Pref(\tilde{s})$, if $\tilde{\Sigma}_f(\tilde{t}') < \tilde{\Sigma}_f(\tilde{\sigma})$ and $\tilde{t}' \neq \tilde{t}$ then $\tilde{t}' \in Pref(\tilde{t})$.*

Intuitively, $MaxPref(\tilde{s})$ is the prefix of $\tilde{s}$ followed by the first occurrence of an event $\tilde{e}$ in $\tilde{s}$ such that $\tilde{\Sigma}_f(\tilde{e}) > \tilde{\Sigma}_f(\tilde{\sigma})$.

3 Predictability in Fuzzy DESs

In this section, we define the notion of predictability and co-predictability for FDESs.

3.1 Fuzzy predictability notion

We define the notion of the fuzzy predictability to characterize the predictability of fuzzy discrete event systems based on the observability degree of events.

Definition 3.1. *Let $\tilde{G}$ be an FDES. $L_{\tilde{G}}$ is the generated language by $\tilde{G}$. The fuzzy event $\tilde{\sigma} \in \tilde{E}$ is said to be predictable in the FDES $\tilde{G}$ with respect to projections mask $M_{\tilde{\sigma}}$ if*

$$(\exists n_0 \in \mathbb{N})(\forall n > n_0)(\forall \tilde{s} \in \tilde{\Psi}_{\tilde{\sigma}})(\exists \tilde{t} \in Pref(\tilde{s}))(\tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{\sigma}))$$
$$(\tilde{\Sigma}_f(\tilde{\sigma}) \leq min\{(\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in L_{\tilde{G}} \wedge \tilde{w} = \tilde{u}\tilde{v} \wedge M_{\tilde{\sigma}}(\tilde{u}) = M_{\tilde{\sigma}}(\tilde{t}) \wedge \tilde{\Sigma}_f(\tilde{u}) < \tilde{\Sigma}_f(\tilde{\sigma}) \wedge \|\tilde{v}\| = n\} \tag{8}$$

The definition 3.1 states that for any trace $\tilde{s}$ that ends with a failure event, there exists a non-faulty prefix $\tilde{t}$ i.e. the possibility of failure on this prefix is less than $\tilde{\Sigma}_f(\tilde{\sigma})$ such that for any indistinguishable trace with the same observable prefix as $\tilde{t}$ (i.e., observability of the prefix of such traces is the same as the observability of $\tilde{t}$) has a possibility of failure which is equal or superior than the possibility of failure of the fuzzy event $\tilde{\sigma}$ to be predicted. Only in such case the prognoser can predict the failure after the execution of the non-faulty prefix $\tilde{t}$ of a failure trace $\tilde{s}$.

3.2 Fuzzy Co-predictability Notion

The notion of fuzzy co-predictability allows verifying whether a set of predefined faults can be predicted in decentralized manner using a set of local fuzzy prognosers. Each fault must be predicted by at least one local prognoser by using its proper local observation of the system. The co-predictability property is stronger than the predictability property. If a system is co-predictable, then it is predictable. However a predictable system is not necessarily co-predictable. The notion of co-predictability was first considered by R. Kumar et al. [16] where the authors present a decentralized prognosis framework for crisp DES without involving any communication among the local prognosers, in which the notion of co-predictability was introduced to capture the requirement for a decentralized prognosable crisp DES. In the following we give the definition of fuzzy co-predictability in the framework of fuzzy discrete-event systems.

Definition 3.2. *let $\tilde{t} \in L_{\tilde{G}}$ and $n \in \mathbb{N}$ such that $\tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{\sigma})$, We define $\Phi_{\tilde{\sigma},i}$ as follows:*

$$\Phi_{\tilde{\sigma},i}(\tilde{t}, n) = \{\tilde{w} \in L_{\tilde{G}} : \tilde{w} = \tilde{u}\tilde{v} \wedge M_{\tilde{\sigma},i}(\tilde{u}) = M_{\tilde{\sigma},i}(\tilde{t}) \wedge \tilde{\Sigma}_f(\tilde{u}) < \tilde{\Sigma}_f(\tilde{\sigma}) \wedge \|\tilde{v}\| = n\} \tag{9}$$

In definition 3.2, the set $\Phi_{\tilde{\sigma},i}$ that contains all traces in $L_{\tilde{G}}$ with a prefix $\tilde{u}$ is defined. The prefix $\tilde{u}$ has same observables as $\tilde{t}$ with respect to the projection mask $M_{\tilde{\sigma},i}$ and its faulty degree is less than the faulty degree of $\tilde{\sigma}$ and the continuation length of this prefix is equal or greater than n. In fact the set $\Phi_{\tilde{\sigma},i}$ is used to collect all traces in $L_{\tilde{G}}$ with the same observables as $\tilde{t}$.

Definition 3.3. *Let $\tilde{G}$ be an FDES with local projections masks $M_1, M_2, \cdots, M_m$. $L_{\tilde{G}}$ is the generated language. The fuzzy event $\tilde{\sigma} \in \tilde{E}$. $\tilde{G}$ is said to be copredictable with respect to observation masks $\{M_i : i = 1, 2, \cdots, m\}$ if*

$$(\exists n_0 \in \mathbb{N})(\forall n > n_0)(\forall \tilde{s} \in \tilde{\Psi}_{\tilde{\sigma}}(\tilde{\Sigma}_f)(\exists \tilde{t} \in Pref(\tilde{s}))(\tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{\sigma}))$$
$$(\exists i \in \{1, 2, \cdots, m\})(\tilde{\Sigma}_f(\tilde{\sigma}) \leq min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\sigma},i}(\tilde{t}, n)\} \tag{10}$$

The above definition 3.3 states that for any trace $\tilde{s}$ that ends with failure event, there exists a non-faulty prefix $\tilde{t}$ i.e. the possibility of failure on this prefix is less than $\tilde{\Sigma}_f(\tilde{\sigma})$ such that for some local prognoser, any locally indistinguishable trace with the same observable prefix as $\tilde{t}$ (i.e., observability of the prefix of such traces is the same as the observability of $\tilde{t}$) has a possibility of failure which is equal or superior than the possibility of failure of the fuzzy event $\tilde{\sigma}$ to be predicted. Then the prognoser can predict the inevitability of a failure following the execution of the non-faulty prefix $\tilde{t}$ of a failure trace $\tilde{s}$. Let us consider an example of an FDES $\tilde{G}$ with two prognosers P_1 and P_2.

Example 3.4. *The FDES $\tilde{G}$ presented in Figure 1 is not predictable with respect to P_1, and $\tilde{G}$ is not predictable with respect to P_2, but $\tilde{G}$ is co-predictable with respect to $\{P_1, P_2\}$.*
(1) *First, we prove that $\tilde{G}$ is not predictable with respect to P_1. Consider the fuzzy event $\tilde{\theta}$ in the FDES $\tilde{G}$. We have: $\tilde{\Psi}_{\tilde{\theta}} = \{\tilde{\beta}\tilde{\alpha}\tilde{\theta}\}$.*

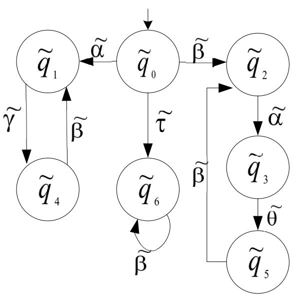


Figure 1: Fuzzy Discrete-event System $\tilde{G}$

- *For $\tilde{s} = \tilde{\beta}\tilde{\alpha}\tilde{\theta}$, $\tilde{t} = MaxPref(\tilde{s}) = \tilde{\beta}\tilde{\alpha}$. We have:*

$$\begin{cases} \tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{\theta}) \\ \Phi_{\tilde{\theta},1}(\tilde{t},\infty) = \{\tilde{\beta}(\tilde{\alpha}\tilde{\theta}\tilde{\beta})^*, \tilde{\alpha}(\tilde{\gamma}\tilde{\beta})^*\} \end{cases}$$

By Definition 3.3, the fuzzy event $\tilde{\theta}$ is predictable in the FDES $\tilde{G}$ with respect to the prognoser P_1 if the inequation (10) holds:

$$\tilde{\Sigma}_f(\tilde{\theta}) \leq min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\theta},1}(\tilde{t},n)\}$$

Note that : $\tilde{\Sigma}_f(\tilde{\theta}) = 0.4$ and $min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\theta},1}(\tilde{t},\infty)\} = min\{0.3, 0.4\}$. Then $(\forall n > 0)$ $\tilde{\Sigma}_f(\tilde{\theta}) > min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\theta},1}(\tilde{t},n)\}$. So, from the above result the inequation (10) does not hold. Thus the fuzzy event $\tilde{\theta}$ is not predictable in the FDES $\tilde{G}$ with respect to the prognoser P_1.

Event	$\tilde{\alpha}$	$\tilde{\beta}$	$\tilde{\tau}$	$\tilde{\gamma}$	$\tilde{\theta}$
Observability	0.6	0.2	0.7	0.5	0.3
Faulty	0.1	0.2	0.2	0.3	0.4

Table 1: prognoser P1

Event	$\tilde{\alpha}$	$\tilde{\beta}$	$\tilde{\tau}$	$\tilde{\gamma}$	$\tilde{\theta}$
Observability	0.2	0.8	0.4	0.5	0.3
Faulty	0.1	0.2	0.2	0.3	0.4

Table 2: prognoser P2

(2) *Second, we prove that $\tilde{G}$ is not predictable with respect to P2. Consider the fuzzy event $\tilde{\gamma}$ in the FDES $\tilde{G}$. The set of the faulty traces is $\tilde{\Psi}_{\tilde{\gamma}} = \{\tilde{\alpha}\tilde{\gamma}, \tilde{\beta}\tilde{\alpha}\tilde{\theta}\}$.*

- *For $\tilde{s} = \tilde{\alpha}\tilde{\gamma}$, $\tilde{t} = MaxPref(\tilde{s}) = \tilde{\alpha}$. We have:*

$$\begin{cases} \tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{\gamma}) \\ \Phi_{\tilde{\gamma},2}(\tilde{t},\infty) = \{\tilde{\alpha}(\tilde{\gamma}\tilde{\beta})^*, \tilde{\beta}(\tilde{\alpha}\tilde{\theta}\tilde{\beta})^*, \tilde{\tau}\tilde{\beta}^*\} \end{cases}$$

By Definition 3.3, the fuzzy event $\tilde{\gamma}$ is predictable in the FDES $\tilde{G}$ with respect to the prognoser P_1 if the inequation (10) holds, we find that: $\tilde{\Sigma}_f(\tilde{\gamma}) = 0.3$ and $min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\gamma},2}(\tilde{t},n)\} = min\{0.3, 0.2, 0.4\}$
Then $(\forall n > 0)$ $\tilde{\Sigma}_f(\tilde{\gamma}) > min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\gamma},i}(\tilde{t},n)\}$
Thus, the fuzzy string $\tilde{s} = \tilde{\alpha}\tilde{\gamma}$ is not predictable in the FDES $\tilde{G}$ with respect to the prognoser P_2. So, because there exists a string that is not predictable in FDES $\tilde{G}$ with respect to the prognoser P_2, the fuzzy event $\tilde{\gamma}$ is not predictable in the FDES $\tilde{G}$ with respect to the prognoser P_2.

(3) Third, we prove that $\tilde{G}$ is c-opredictable with respect to P_1 and P_2.

- *Consider the fuzzy event $\tilde{\alpha}$ in the FDES $\tilde{G}$. The set of the faulty traces with respect to the prognoser P_1 is $\tilde{\Psi}_{\tilde{\alpha}} = \{\tilde{\alpha}, \tilde{\beta}, \tilde{\tau}\}$.*

  - *For $\tilde{s} = \tilde{\alpha}$, $\tilde{t} = MaxPref(\tilde{s}) = \epsilon$. We have:*

$$\begin{cases} \tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{\alpha}) \\ \Phi_{\tilde{\alpha},1}(\tilde{t}, \infty) = \{\tilde{\alpha}(\tilde{\gamma}\tilde{\beta})^*, \tilde{\beta}(\tilde{\alpha}\tilde{\theta}\tilde{\beta})^*, \tilde{\tau}\tilde{\beta}^*\} \end{cases}$$

  By Definition 3.3, the fuzzy event $\tilde{\alpha}$ is predictable in the FDES $\tilde{G}$ with respect to the prognoser P_1 if the inequation (10) holds, we find that: $\tilde{\Sigma}_f(\tilde{\alpha}) = 0.1$ and $min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\alpha},1}(\tilde{t}, 1)\} = min\{0.1, 0.2, 0.2\}$ Then $(\exists n_0 = 1)(\forall n > n_0)$ $\tilde{\Sigma}_f(\tilde{\alpha}) \leq min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\alpha},1}(\tilde{t}, 1)\}$. Thus, the fuzzy string $\tilde{s} = \tilde{\alpha}$ is predictable in the FDES $\tilde{G}$ with respect to the prognoser P_1.

  For $\tilde{s} = \tilde{\beta}$ and $\tilde{s} = \tilde{\tau}$ a similar method is used as for the fuzzy event $\tilde{\alpha}$. We obtain that the faulty events $\tilde{\beta}$ and $\tilde{\tau}$ in the FDES $\tilde{G}$ are predictable with respect to the prognoser P_1.

- *Consider the fuzzy events $\tilde{\beta}$ and $\tilde{\tau}$ in the FDES $\tilde{G}$. We do same as for the fuzzy event $\tilde{\alpha}$, we obtain that the fuzzy event $\tilde{\beta}$ and $\tilde{\tau}$ in the FDES $\tilde{G}$ are predictable with respect to the prognoser P_1.*

- *Consider the fuzzy event $\tilde{\gamma}$ in the FDES $\tilde{G}$. The set of the faulty traces with respect to the prognoser P_1 is $\tilde{\Psi}_{\tilde{\gamma}} = \{\tilde{\alpha}\tilde{\gamma}, \tilde{\beta}\tilde{\alpha}\tilde{\theta}\}$.*

  - *For $\tilde{s} = \tilde{\alpha}\tilde{\gamma}$, $\tilde{t} = MaxPref(\tilde{s}) = \tilde{\alpha}$. We have:*

$$\begin{cases} \tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{\gamma}) \\ \Phi_{\tilde{\gamma},1}(\tilde{t}, \infty) = \{\tilde{\alpha}(\tilde{\gamma}\tilde{\beta})^*, \tilde{\beta}(\tilde{\alpha}\tilde{\theta}\tilde{\beta})^*\} \end{cases}$$

  By Definition 3.3, the fuzzy event $\tilde{\gamma}$ is predictable in the FDES $\tilde{G}$ with respect to the prognoser P_1 if the inequation (10) holds:

$$\tilde{\Sigma}_f(\tilde{\gamma}) \leq min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\gamma},1}(\tilde{t}, n)\}$$

  Note that: $\tilde{\Sigma}_f(\tilde{\gamma}) = 0.3$ and $min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\gamma},1}(\tilde{t}, 3)\} = min\{0.3, 0.4\}$ then $(\exists n_0 = 3)(\forall n > n_0)$ $\tilde{\Sigma}_f(\tilde{\gamma}) \leq min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\gamma},1}(\tilde{t}, 3)\}$. Thus, the fuzzy string $\tilde{s} = \tilde{\alpha}\tilde{\gamma}$ is predictable in the FDES $\tilde{G}$ with respect to the prognoser P_1.

  - *For $\tilde{s} = \tilde{\beta}\tilde{\alpha}\tilde{\theta}$ a similarly method is used as for the fuzzy string $\tilde{s} = \tilde{\alpha}\tilde{\gamma}$ and we obtain that the faulty string $\tilde{s} = \tilde{\beta}\tilde{\alpha}\tilde{\theta}$ is predictable with respect to fuzzy event $\tilde{\gamma}$ and the prognoser P_1.*

Because all faulty traces in $\tilde{\Psi}_{\tilde{\gamma}} = \{\tilde{\alpha}\tilde{\gamma}, \tilde{\beta}\tilde{\alpha}\tilde{\theta}\}$ are predictbale with respect to the prognoser P_1 then the fuzzy event $\tilde{\gamma}$ in the FDES $\tilde{G}$ is predictable with respect to the prognoser P_1.

- *Consider the fuzzy event $\tilde{\theta}$ in the FDES $\tilde{G}$. The set of the faulty traces with respect to the prognoser P_2 is $\tilde{\Psi}_{\tilde{\theta}} = \{\tilde{\beta}\tilde{\alpha}\tilde{\theta}\}$.*

  - *For $\tilde{s} = \tilde{\beta}\tilde{\alpha}\tilde{\theta}$, $\tilde{t} = MaxPref(\tilde{s}) = \tilde{\beta}\tilde{\alpha}$. We have:*

$$\begin{cases} \tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{\theta}) \\ \Phi_{\tilde{\theta},2}(\tilde{t}, \infty) = \{\tilde{\beta}(\tilde{\alpha}\tilde{\theta}\tilde{\beta})^*\} \end{cases}$$

  By Definition 3.3, the fuzzy event $\tilde{\theta}$ is predictable in the FDES $\tilde{G}$ with respect to the prognoser P_2 if the below inequation holds:

$$\tilde{\Sigma}_f(\tilde{\gamma}) \leq min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\theta},2}(\tilde{t}, n)\}$$

  Note that : $\tilde{\Sigma}_f(\tilde{\theta}) = 0.4$ and $min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\theta},2}(\tilde{t}, 2)\} = min\{0.4\}$ then $(\exists n_0 = 2)(\forall n > n_0)$ $\tilde{\Sigma}_f(\tilde{\gamma}) \leq min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\theta},2}(\tilde{t}, 2)\}$. Thus, the fuzzy string $\tilde{s} = \tilde{\beta}\tilde{\alpha}\tilde{\theta}$ is predictable in the FDES $\tilde{G}$ with respect to the prognoser P_2.
  Because all faulty traces in $\tilde{\Psi}_{\tilde{\theta}} = \{\tilde{\beta}\tilde{\alpha}\tilde{\theta}\}$ are predictbale with respect to the prognoser P_2 then the fuzzy event $\tilde{\theta}$ in the FDES $\tilde{G}$ is predictable with respect to the prognoser P_2.

Now, to conclude, we have found that:

1. *The fuzzy events $\tilde{\alpha}$, $\tilde{\beta}$ and $\tilde{\tau}$ are predictable with respect to the prognoser P_1.*
2. *$\tilde{\gamma}$ is predictable with respect to the prognoser P_1 but it's not predictable with respect to the prognoser P_2.*
3. *$\tilde{\theta}$ is predictable with respect to the prognoser P_2 but it's not predictable with respect to the prognoser P_1.*

Because for any local fuzzy discrete event in the FDES $\tilde{G}$ there is a prognoser P_1 or P_2 that can predict it, the the FDES $\tilde{G}$ is co-predictable with respect to P_1 and P_2.

4 Verification of Co-predictability of FDESs

In this section, the Fuzzy co-verifier will be designed to perform the decentralized prognosis of FDESs. Especially, we present a necessary and sufficient condition for co-predictability of FDESs.

4.1 Verification approach

The steps to follow in order to verify the co-predicta- bility of FDESs are summed up as follows.

Step 1: Construct the Non-Faulty Accessible Part (NFAP) $\tilde{G}_{\tilde{\sigma}}$.

We denote by $\tilde{G}_{\tilde{\sigma}}$ the accessible part of the FDES $\tilde{G} = (\tilde{Q}, \tilde{E}, \tilde{\delta}, \tilde{q}_0)$ after removing all fuzzy events where the possibility of the failure occurring is greater or equal to the possibility of the failure occurring on the fuzzy event $\tilde{\sigma}$ i.e. $(\tilde{\Sigma}_f(\tilde{a}) \geq \tilde{\Sigma}_f(\tilde{\sigma}) : \tilde{a} \in \tilde{E}_f)$. In fact $\tilde{G}_{\tilde{\sigma}}$ models the non-faulty behavior of the FDES $\tilde{G}$ with respect to the fuzzy event $\tilde{\sigma}$. The fuzzy FDES $\tilde{G}_{\tilde{\sigma}}$ is constructed as an automaton $\tilde{G}_{\tilde{\sigma}} = (\tilde{Q}_{\tilde{\sigma}}, \tilde{E}_{\tilde{\sigma}}, \tilde{\delta}_{\tilde{\sigma}}, \tilde{q}_{\tilde{\sigma},0})$ where:

- $\tilde{q}_{\tilde{\sigma},0} = \tilde{q}_0$
- $\tilde{E}_{\tilde{\sigma}} = \{\tilde{a} \in \tilde{E} : \tilde{\Sigma}_f(\tilde{a}) < \tilde{\Sigma}_f(\tilde{\sigma})\}$
- $\tilde{\delta}_{\tilde{\sigma}} = \tilde{\delta}$
- $\tilde{Q}_{\tilde{\sigma}} = \{\tilde{q}_0 \cup\ \tilde{\delta}(\tilde{q}_0, \tilde{s}\tilde{a})\ is\ defined,\ and\ \tilde{s} \in \tilde{E}_f^*\ and\ \tilde{a} \in \tilde{E}_{\tilde{\sigma}}\}$

Step 2: Construct of the fuzzy Co-Verifier "F-co-verifier"

We construct the F-co-verifier $\tilde{G}_V = (\tilde{Q}_V, \tilde{E}_V, \tilde{\delta}_V, \tilde{q}_{V,0})$ automaton with n local prognoser as follows:

- $\Delta = \{\mu, F\}$ is the set of failure labels: F means that the possibility of the failure occurring is greater than the specified degree $\tilde{\Sigma}_f(\tilde{\sigma})$ and the system is in a faulty state, while μ means that the likelihood of the failure occurring is less than the specified bound and the system is still in a normal state.
- $\tilde{Q}_V = \tilde{Q} \times \Delta \times \underbrace{\tilde{Q}_{\tilde{\sigma}} \times \Delta \times \tilde{Q}_{\tilde{\sigma}} \times \Delta \times \cdots \times \tilde{Q}_{\tilde{\sigma}} \times \Delta}_{\text{n times}}$
- $\tilde{q}_{V,0} = (\tilde{q}_0, 0, \tilde{q}_{\tilde{\sigma},0}, 0, \tilde{q}_{\tilde{\sigma},0}, 0, \cdots, \tilde{q}_{\tilde{\sigma},0}, 0)$
- $\tilde{E}_V = \underbrace{(\tilde{E} \cup \{\epsilon\}) \times (\tilde{E} \cup \{\epsilon\}) \times \cdots \times (\tilde{E} \cup \{\epsilon\})}_{\text{n+1 times}} - \{(\epsilon, \epsilon, \cdots, \epsilon)\}$
- $\tilde{\delta}_V : \tilde{Q}_V \times \tilde{E}_V \to \tilde{Q}_V$ is the transition function of the F-co-verifier and it is defined as follows: for each $\tilde{q}_V = (\tilde{q}, l, \tilde{q}_{\sigma,1}, l_1, \tilde{q}_{\sigma,2}, l_2, \cdots, \tilde{q}_{\sigma,n}, l_n) \in \tilde{Q}_V$ and $\tilde{a}_V = (\tilde{a}, \tilde{a}_1, \tilde{a}_2, \cdots, \tilde{a}_n) \in \tilde{E}_V, \tilde{\delta}_V(\tilde{q}_V, \tilde{a}_V)$ is defined if and only if :

$$\begin{cases} if \tilde{a} \neq \epsilon\ \ then\ \ \tilde{\delta}(\tilde{q}, \tilde{a})\ \ is\ \ defined; \\ For\ \ each\ \ i\ \ \in\ \ \{1, 2, ..., m\}\ \ M_{\tilde{\sigma},i}(\tilde{a}) = M_{\tilde{\sigma},i}(\tilde{a}_i); \\ For\ \ each\ \ i\ \ \in\ \ \{1, 2, ..., m\}\ \ if \tilde{a}_i \neq \epsilon \\ \qquad then\ \ \tilde{\delta}_{\tilde{\sigma}}(\tilde{q}_{\sigma,i}, \tilde{a}_i)\ \ is\ \ defined. \end{cases}$$

If $\tilde{\delta}_V(\tilde{q}_V, \tilde{a}_V)$ is defined, then $\tilde{\delta}_V(\tilde{q}_V, \tilde{a}_V) = (\tilde{q}', l', \tilde{q}'_{\sigma,1}, l'_1, \tilde{q}'_{\sigma,2}, l'_2, \cdots, \tilde{q}'_{\sigma,n}, l'_n)$ where

$$(\tilde{q}', l') = \begin{cases} (\tilde{\delta}(\tilde{q}, \tilde{a}), l) & if \;\; \tilde{a}_V \;\; \neq \;\; \epsilon \;\; and \;\; \tilde{\Sigma}_f(\tilde{a}) \leq l \\ (\tilde{\delta}(\tilde{q}, \tilde{a}), \tilde{\Sigma}_f(\tilde{a})) & if \;\; \tilde{a} \;\; \neq \;\; \epsilon \;\; and \;\; \tilde{\Sigma}_f(\tilde{a}) > l \\ (\tilde{\delta}(\tilde{q}, \tilde{a}), F) & if \;\; \tilde{a} \;\; \neq \;\; \epsilon \;\; and \;\; (l = F) \\ & or \;\; \tilde{\Sigma}_f(\tilde{a}) > \tilde{\Sigma}_f(\tilde{\sigma}) \\ (\tilde{q}, l) & if \;\; \tilde{a} \;\; = \;\; \epsilon \end{cases}$$

$$(\tilde{q}'_{\sigma,i}, l'_i) = \begin{cases} (\tilde{\delta}_{\tilde{\sigma}}(\tilde{q}_{\tilde{\sigma},i}, \tilde{a}_i), l) & if \;\; \tilde{a}_i \;\; \neq \;\; \epsilon \;\; and \\ & \tilde{\Sigma}_f(\tilde{a}_i) \leq l \\ (\tilde{\delta}_{\tilde{\sigma}}(\tilde{q}_{\tilde{\sigma},i}, \tilde{a}_i), \tilde{\Sigma}_f(\tilde{a}_i)) & if \;\; \tilde{a}_i \;\; \neq \;\; \epsilon \;\; and \\ & \tilde{\Sigma}_f(\tilde{a}_i) > l \\ (\tilde{\delta}_{\tilde{\sigma}}(\tilde{q}_{\tilde{\sigma},i}, \tilde{a}_i), F) & if \;\; \tilde{a}_i \;\; \neq \;\; \epsilon \;\; and \;\; (l = F) \\ & or \;\; \tilde{\Sigma}_f(\tilde{a}_i) > \tilde{\Sigma}_f(\tilde{\sigma}) \\ (\tilde{q}_{\tilde{\sigma},0}, l) & if \;\; \tilde{a}_i \;\; = \;\; \epsilon \end{cases}$$

Remark 4.1. *The construction of the fuzzy co-verifier is inspirited by the design of M machine introduced by Rudie and Willems* [33] *for the decentralized supervisory control and the automaton T introduced by R. Kumar et al.* [16] *for co-prognosability of crisp DESs..*

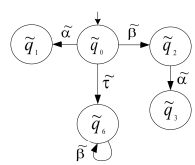
A part of the fuzzy co-verifier with respect to fuzzy event $\tilde{\gamma}$ built for the FDES $\tilde{G}$ of the example 2 is shown in Figure 2(b). Since the corresponding co-verifier is too large to show, we only present states where the first component is a boundary state.

4.2 Necessary and Sufficient Condition of Co-predictability for FDESs

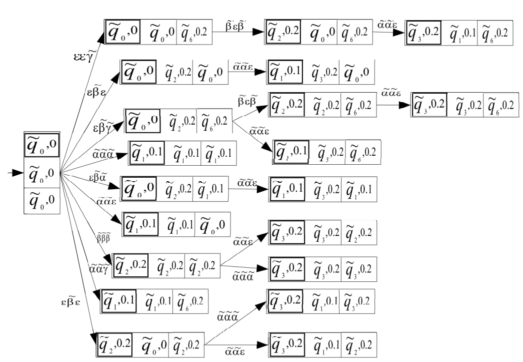
In this section we give the necessary and sufficient condition for decentralized fuzzy discrete event systems.

Definition 4.2. *Given an FDES $\tilde{G}$ and $\tilde{G}_{\tilde{\sigma}}$ the NFAP of the FDES $\tilde{G}$, the set of*

- *Non-indicator states are states of $\tilde{G}_{\tilde{\sigma}}$ from which a cycle in $\tilde{G}_{\tilde{\sigma}}$ can be reached;*
- *Boundary states are states of $\tilde{G}$ from which a fuzzy faulty event $\tilde{a}$ is enabled such that $\tilde{\Sigma}_f(\tilde{a}) \geq \tilde{\Sigma}_f(\tilde{\sigma})$.*
- *Unpredictable states are states of the fuzzy co-Verifier of the FDES $\tilde{G}$ where the first component is boundary state and the other components are Non-indicator states.*



(a)



(b)

Figure 2: (a) the NFAP the FDES $\tilde{G}$.(b) a part of the fuzzy Co-verifier with respect to fuzzy event $\tilde{\gamma}$.

Theorem 4.3. *Let $\tilde{G}$ be an FDES with m local prognosers $P_1, P_2 \cdots, P_m$ and $\tilde{G}_V$ its fuzzy co-verifier with respect to the fuzzy event $\tilde{\sigma} \in \tilde{E}$. Then the FDES $\tilde{G}$ is co-predictibale with respect to $\tilde{\sigma}$, if and only if, in fuzzy co-verifier with respect to the fuzzy event $\tilde{\sigma}$ there does not exist an accessible unpredictable state in the fuzzy co-verifier.*

Proof. For simplicity, we prove the theorem in the case of system with two local prognosers i.e. $m = 2$. In fact, we can extend the proof to the case where m is more than two.

Necessity: We prove the theorem by contradiction. Suppose that there is an accessible unpredictable state $\tilde{q}_v$ in the fuzzy co-verifier $\tilde{q}_v = \{\tilde{q}, \mu_1, \tilde{q}_{\tilde{\sigma},1}, \mu_2, \tilde{q}_{\tilde{\sigma},2}, \mu_3\}$. Suppose that $\tilde{a}_v = (\tilde{t}, \tilde{u}_1, \tilde{u}_2)$ and $\tilde{\delta}_V(\tilde{q}_{V,0}, \tilde{a}_v) = \tilde{q}_v = \{\tilde{q}, \mu_1, \tilde{q}_{\tilde{\sigma},1}, \mu_2, \tilde{q}_{\tilde{\sigma},2}, \mu_3\}$. From the definition of the fuzzy co-verifier we have:

$$\begin{cases} M_{\tilde{\sigma},1}(\tilde{t}) = M_{\tilde{\sigma},1}(\tilde{u}_1) \\ M_{\tilde{\sigma},2}(\tilde{t}) = M_{\tilde{\sigma},2}(\tilde{u}_2) \end{cases}$$

From the definition of the unpredictable state, the first component is boundary state and all others component are Non-indicator states.

Because $\tilde{q}_{\tilde{\sigma},1}$ and $\tilde{q}_{\tilde{\sigma},2}$ are Non-indicator states and they are states from the NFAP of the FDES $\tilde{G}$, then for any continuation of $\tilde{u}_1$ and $\tilde{u}_2$, we have:

- $\tilde{\Sigma}_f(\tilde{\sigma}) > \tilde{\Sigma}_f(\tilde{u}_1\tilde{v}_1)$ where $\|\tilde{v}_1\| > n$ $(\forall n \in \mathbb{N})$. (1)

- $\tilde{\Sigma}_f(\tilde{\sigma}) > \tilde{\Sigma}_f(\tilde{u}_2\tilde{v}_2)$ where $\|\tilde{v}_2\| > n$ $(\forall n \in \mathbb{N})$. (2)

Now, suppose that $\tilde{s} \in \tilde{\Psi}_{\tilde{\sigma}} : \tilde{s} = \tilde{t}\tilde{\sigma}_p : \tilde{\Sigma}_f(\tilde{\sigma}_p) \geq \tilde{\Sigma}_f(\tilde{\sigma})$ and $\tilde{w}_1 \in L_{\tilde{G}} : \tilde{w}_1 = \tilde{u}_1\tilde{v}_1$ and $\tilde{w}_2 \in L_{\tilde{G}} : \tilde{w}_2 = \tilde{u}_2\tilde{v}_2$, we have :

- $M_{\tilde{\sigma},1}(\tilde{u}_1) = M_{\tilde{\sigma},1}(\tilde{t}) \wedge \tilde{\Sigma}_f(\tilde{u}_1) < \tilde{\Sigma}_f(\tilde{\sigma}) \wedge \|\tilde{v}\| = n$ then $\tilde{w}_1 \in \Phi_{\tilde{\sigma},1}(\tilde{t}, n)$
- $M_{\tilde{\sigma},2}(\tilde{u}_2) = M_{\tilde{\sigma},2}(\tilde{t}) \wedge \tilde{\Sigma}_f(\tilde{u}_2) < \tilde{\Sigma}_f(\tilde{\sigma}) \wedge \|\tilde{v}\| = n$ then $\tilde{w}_2 \in \Phi_{\tilde{\sigma},2}(\tilde{t}, n)$

Now, because $\tilde{w}_1 \in \Phi_{\tilde{\sigma},1}(\tilde{t}, n)$ and $\tilde{w}_2 \in \Phi_{\tilde{\sigma},2}(\tilde{t}, n)$ and from (1) and (2) we find that $\exists \tilde{s} \in \tilde{\Psi}_{\tilde{\sigma}} : \tilde{s} = \tilde{t}\tilde{\sigma}_p$ and $\forall i \in \{1, 2\}$ where $\tilde{\Sigma}_f(\tilde{\sigma}) > min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{\sigma},i}(\tilde{t}, n)\}$, Therefore, by Definition 3.3, the fuzzy event $\tilde{\sigma}$ is not co-predictable in the FDES $\tilde{G}$.

Sufficiency: Suppose that the fuzzy event $\tilde{\sigma}$ is not co-predictable in the FDES $\tilde{G}$. From Definition 3.3 there is $\tilde{s} \in \tilde{\Psi}_{\tilde{\sigma}}$ and $\tilde{w}_1 \in \Phi_{\tilde{\sigma},1}(\tilde{t}, n)$ and $\tilde{w}_2 \in \Phi_{\tilde{\sigma},2}(\tilde{t}, n)$ where $\tilde{\Sigma}_f(\tilde{\sigma}) > \tilde{\Sigma}_f(\tilde{w}_1)$ and $\tilde{\Sigma}_f(\tilde{\sigma}) > \tilde{\Sigma}_f(\tilde{w}_2)$. Suppose that $\tilde{s} = \tilde{t}\tilde{\sigma}_p : \tilde{\Sigma}_f(\tilde{\sigma}_p) \geq \tilde{\Sigma}_f(\tilde{\sigma})$ and $\tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{\sigma})$.

- $\tilde{w}_1 \in \Phi_{\tilde{\sigma},1}(\tilde{t}, n)$ means that $\tilde{w}_1 = \tilde{u}_1\tilde{v}_1 \wedge M_{\tilde{\sigma},1}(\tilde{u}_1) = M_{\tilde{\sigma},1}(\tilde{t}) \wedge \tilde{\Sigma}_f(\tilde{u}_1) < \tilde{\Sigma}_f(\tilde{\sigma}) \wedge \|\tilde{v}_1\| = n$.
- $\tilde{w}_2 \in \Phi_{\tilde{\sigma},2}(\tilde{t}, n)$ means that $\tilde{w}_2 = \tilde{u}_2\tilde{v}_2 \wedge M_{\tilde{\sigma},2}(\tilde{u}_2) = M_{\tilde{\sigma},2}(\tilde{t}) \wedge \tilde{\Sigma}_f(\tilde{u}_2) < \tilde{\Sigma}_f(\tilde{\sigma}) \wedge \|\tilde{v}_2\| = n$

we have :

$$\begin{cases} M_{\tilde{\sigma},1}(\tilde{t}) = M_{\tilde{\sigma},1}(\tilde{u}_1) \\ M_{\tilde{\sigma},2}(\tilde{t}) = M_{\tilde{\sigma},2}(\tilde{u}_2) \end{cases}$$

Supoose that $\tilde{a}_v = (\tilde{t}, \tilde{u}_1, \tilde{u}_2)$ then $\tilde{\delta}_V(\tilde{q}_{V,0}, \tilde{a}_v) = \tilde{q}_v = \{\tilde{q}, \mu_1, \tilde{q}_{\tilde{\sigma},1}, \mu_2, \tilde{q}_{\tilde{\sigma},2}, \mu_3\}$. Because

- $\tilde{w}_1 \in \Phi_{\tilde{\sigma},1}(\tilde{t}, n)$ means that $\tilde{w}_1 = \tilde{u}_1\tilde{v}_1 \wedge M_{\tilde{\sigma},1}(\tilde{u}_1) = M_{\tilde{\sigma},1}(\tilde{t}) \wedge \tilde{\Sigma}_f(\tilde{u}_1) < \tilde{\Sigma}_f(\tilde{\sigma}) \wedge \|\tilde{v}_1\| > n(\forall n \in \mathbb{N})$ then $\tilde{q}_{\tilde{\sigma},1}$ is a Non-indicator state.

- $\tilde{w}_2 \in \Phi_{\tilde{\sigma},2}(\tilde{t}, n)$ means that $\tilde{w}_2 = \tilde{u}_2\tilde{v}_2 \wedge M_{\tilde{\sigma},2}(\tilde{u}_2) = M_{\tilde{\sigma},2}(\tilde{t}) \wedge \tilde{\Sigma}_f(\tilde{u}_2) < \tilde{\Sigma}_f(\tilde{\sigma}) \wedge \|\tilde{v}_2\| > n(\forall n \in \mathbb{N})$ then $\tilde{q}_{\tilde{\sigma},2}$ is also a Non-indicator state.

- $\tilde{\delta}(\tilde{q}_0, \tilde{t}) = \tilde{q}$ and we have $\tilde{s} \in \tilde{\Psi}_{\tilde{\sigma}}$ such that $\tilde{s} = \tilde{t}\tilde{\sigma}_p : \tilde{\Sigma}_f(\tilde{\sigma}_p) \geq \tilde{\Sigma}_f(\tilde{\sigma})$ and $\tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{\sigma})$ then the state $\tilde{q}$ is a boundary state because there is a fuzzy faulty event $\tilde{\sigma}_p$ which is enabled from the state $\tilde{q}$.

Now in the co-verifier there exists at least one accessible unpredictable state $\tilde{q}_v = \{\tilde{q}, \mu_1, \tilde{q}_{\tilde{\sigma},1}, \mu_2, \tilde{q}_{\tilde{\sigma},2}, \mu_3\}$ where the first component is a boundary state and all the other components are Non-indicator states in the fuzzy co-verifier of the FDES $\tilde{G}$ with respect to the fuzzy event $\tilde{\sigma}$. This means that the fuzzy event $\tilde{\sigma}$ is not co-predictable in the FDES $\tilde{G}$. □

Remark 4.4. *: The main complexity of the above verification of co-predictability for FDESs is in the construction of the fuzzy co-verifier. Since we can summarize the construction of the fuzzy co-verifier by the synchronous composition* $\tilde{G} \parallel NFAP_{\tilde{G}} \underbrace{\parallel NFAP_{\tilde{G}} \parallel \cdots \parallel NFAP_{\tilde{G}}}_{n\ times}$ *of the plant model* $\tilde{G}$ *with n+1 copies of Non-Faulty Accessible Part (NFAP) of* $\tilde{G}$ *, then the overall complexity of the construction of the fuzzy co-verifier is* $O(co-verifier) = O(|\tilde{G} \parallel NFAP_{\tilde{G}}|^{n+1})$, *which is polynomial with respect to the size of FDES being predicted, and is exponential with respect to the number of the local prognosis agents n.*

For the fuzzy event $\tilde{\gamma}$ of the FDES $\tilde{G}$ of the example 2. According to Step 1 and Step 2 in Section 4, the NFAP of the FDES $\tilde{G}$ and the fuzzy co-verifier are shown in Figure 2(a) and Figure 2(b), respectively.
Notice that in the NFAP there is two "Non-indicator states" $\tilde{q}_0$ and $\tilde{q}_6$ and two "boundary state" $\tilde{q}_1$ and $\tilde{q}_3$. And from the Co-verifier there does not exist an accessible unpredictable state. So according to the theorem 4.3 the failure event $\tilde{\gamma}$ is co-predictable in the FDES $\tilde{G}$.

5 Illustrative Examples

In this section, some examples are given to illustrate the proposed approach. In particular, we show that it works for both crisp DESs and FDESs.

Example 5.1. *In this example we show that the proposed approach can be used to deal with the problem of the decentralized prognosis for crisp DESs. The following case verify this view. A crisp DES can be considered a special case of FDES, if the states and events are completely observable or completely unobservable then they takes two values 0 or 1. Consider the crisp DES* G_1 *described in Figure 3, where* θ *is supposed to be the faulty event. Suppose that the DES* G_1 *is observed with two prognosers P1 and P2 given in Table 3 and Table4 respectively.*

Now, let us prove that the failure event θ *in* G_1 *is co-predictable by using the decentralized prognosis approach for FDESs presented in Section 3 and Section 4.*

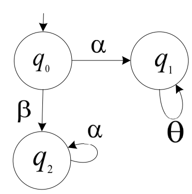


Figure 3: Crisp Discrete-event System G_1

Event	$\tilde{\alpha}$	$\tilde{\beta}$	$\tilde{\theta}$	ϵ
Observability	1	0	0	0
Faulty	0	0	1	0

Table 3: prognoser P1

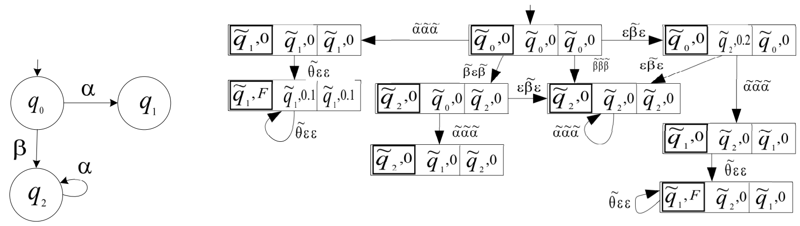
Event	$\tilde{\alpha}$	$\tilde{\beta}$	$\tilde{\theta}$	ϵ
Observability	1	1	0	0
Faulty	0	0	1	0

Table 4: prognoser P2

As we have already said the crisp DES G_1 *can be considered a special case of FDES with the fuzzy states*$q_0 = [1, 0, 0]$, $q_1 = [0, 1, 0]$, $q_2 = [0, 0, 1]$, *and events*

$$\alpha = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \beta = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \qquad \theta = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

According to the two Steps given in Section 4, the NFAP of the DES G_1 *and the fuzzy co-verifier with respect to event* θ *in crisp DES* G_1 *are shown in Figure 4(a). and Figure 4(b), respectively.*



(a) (b)

Figure 4: (a) the NFAP of the Crisp DES G_1.(b) the fuzzy Co-verifier with respect to event θ.

Notice that in the NFAP of the fuzzy event θ there is two "Non-indicator states" q_0 and q_2 and one "boundary state" q_1. From the co-verifier based on θ there does not exist an accessible unpredictable state. So according to the theorem 4.3 the failure event $\tilde{\theta}$ is co-predictable in the FDES $G1$.

Example 5.2. *Treatment process on an animal*

Consider the treatment process of an animal, modeled by the fuzzy DES $\tilde{G}_3$ depicted in Figure 5 (this example is inspired from [25]).

This animal becoming sick with a new disease. The drugs Theophylline, Ipratropium Bromide, Erythromycin Ethyl succinate, and Dopamine are denoted by fuzzy events $\tilde{\alpha}$,$\tilde{\beta}$, $\tilde{\gamma}$ and $\tilde{\theta}$, respectively. The doctor believes that these drugs may be useful for the treatment of the disease. A state in this fuzzy DES is denoted by a vector $\tilde{q} = (a_1, a_2, a_3)$ which means that the health state of the animal can belong in the same time to "good", "fair" and "poor" with respective membership degrees a_1, a_2 and a_3. The initial state is $\tilde{q}_0 = [0.9, 0.1, 0]$ and the other states calculated using max-min operation (see Definition 2.1) are: $\tilde{q}_1 = [0.4, 0.9, 0.4]$, $\tilde{q}_2 = [0.9, 0.4, 0.4]$, $\tilde{q}_3 = [0.4, 0.9, 0.4]$, $\tilde{q}_4 = [0.9, 0.9, 0.4]$, $\tilde{q}_5 = [0.5, 0.1, 0]$, $\tilde{q}_6 = [0.5, 0.4, 0.4]$ and $\tilde{q}_7 = [0.5, 0.4, 0.4]$.

Indeed, because each treatment can lead from a state to several states with respective membership degrees, we cannot know exactly at what point the animals condition has evolved from one state to another state. Hence, a fuzzy event is modeled by a 3×3 matrix.

$$\tilde{\sigma} = \begin{array}{c} . \\ good \\ fair \\ poor \end{array} \begin{array}{c} \begin{array}{ccc} good & fair & poor \end{array} \\ \begin{bmatrix} \tilde{a}_{11} & \tilde{a}_{12} & \tilde{a}_{13} \\ \tilde{a}_{21} & \tilde{a}_{22} & \tilde{a}_{23} \\ \tilde{a}_{31} & \tilde{a}_{32} & \tilde{a}_{33} \end{bmatrix} \end{array}$$

In this matrix each $\tilde{a}_{ij} \in [0, 1]$ represents the likelihood of the animals condition changing from the state i to the state j after the occurrence of the event $\tilde{\sigma}$.

Remark 5.3. *In fact, the fuzzy sets are defined empirically according to the knowledge obtained from specialists and experts of the studied system. To generate these possibilities/memberships, semi-Gaussian functions are used to produce the gradual changes of the possibilities. (For more detail about estimating the membership degrees the reader is referred to e.g.* [40, 21]*).*

Some potential side effects of the disease treatment such as high white blood cell (WBCs) count are undesired. Suppose that high WBCs count represents an undesired failure in this model. The problem of the prognosis in FDES $\tilde{G}_3$ is how to predict this failure in time. We consider that the drug events are given by the doctors experience as

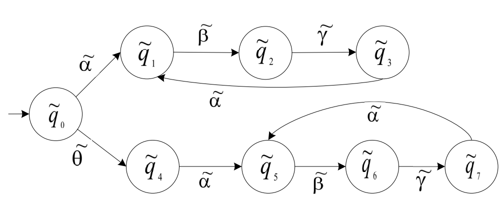


Figure 5: Treatment process [25] modeled by FDES $\tilde{G}_3$.

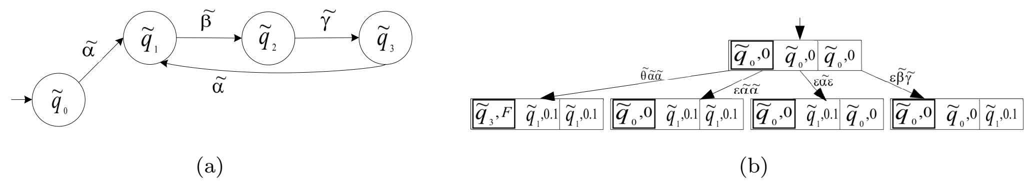


Figure 6: (a) The NFAP of the FDES $\tilde{G}3$.(b) A part of the fuzzy co-verifier with respect to fuzzy event $\tilde{\theta}$.

$$\tilde{\alpha} = \begin{pmatrix} 0.4 & 0.9 & 0.4 \\ 0 & 0.4 & 0.4 \\ 0 & 0 & 0.4 \end{pmatrix} \qquad \tilde{\beta} = \begin{pmatrix} 0.4 & 0 & 0 \\ 0.9 & 0.4 & 0 \\ 0.4 & 0.4 & 0.4 \end{pmatrix}$$

$$\tilde{\gamma} = \begin{pmatrix} 0.9 & 0.9 & 0.4 \\ 0 & 0.4 & 0.4 \\ 0 & 0 & 0.4 \end{pmatrix} \qquad \tilde{\theta} = \begin{pmatrix} 0.5 & 0 & 0 \\ 0.1 & 0.1 & 0 \\ 0.1 & 0.1 & 0.1 \end{pmatrix}$$

Now suppose that there are two therapeutic approaches for the treatment, each approach possesses an observability degree and a possibility of failure occurring for each drug events of the approach. Suppose that the observability degrees and the failure possibilities of the events are defined as follows:

Event	$\tilde{\alpha}$	$\tilde{\beta}$	γ	$\tilde{\theta}$
Observability	0.2	0.4	0.3	0.3
Faulty	0.1	0.2	0.3	0.4

Table 5: Therapeutic group 1

Event	$\tilde{\alpha}$	$\tilde{\beta}$	γ	$\tilde{\theta}$
Observability	0.3	0.4	0.2	0.6
Faulty	0.1	0.2	0.3	0.4

Table 6: Therapeutic group 2

- *For the fuzzy event $\tilde{\theta}$ the NFAP is shown in Figure 6 (a). A part of the fuzzy co-verifier with respect to $\tilde{\theta}$ is shown in Figure 6(b). From the FDES $\tilde{G}3$ there is one boundary state $\tilde{q}_0$ and from the NFAP there is four "Non-indicator states" $\{\tilde{q}_0, \tilde{q}_1, \tilde{q}_2, \tilde{q}_3\}$. From the fuzzy co-verifier with respect to $\tilde{\theta}$ there is several accessible unpredictable states $\{[\tilde{q}_0, 0, \tilde{q}_0, 0, \tilde{q}_0, 0], [\tilde{q}_0, 0, \tilde{q}_1, 0.1, \tilde{q}_0, 0], \cdots\}$. Therefore, using the theorem 4.3 the fuzzy event $\tilde{\theta}$ is not copredictable in the FDES $\tilde{G}3$. So the failure event high WBCs count in this example is not co-predictable using the two therapeutic groups.*

Example 5.4. *Let us consider a plant modeled by the fuzzy discrete event system $\tilde{G}4$ shown in Figure 7 (a) (this example is inspired from [26]). Suppose that the system $\tilde{G}4$ is observed with two prognosers P_1 and P_2 given in Table 7 and Table 8, respectively. Now, let us prove that the failure fuzzy event $\tilde{h}$ in $\tilde{G}4$ is co-predictable by using the fuzzy*

Event	$\tilde{a}$	$\tilde{b}$	$\tilde{c}$	$\tilde{h}$	ϵ
Observability	0.8	0.6	0.1	0.2	0
Faulty	0.5	0.2	0.3	0.6	0

Table 7: prognoser P1

Event	$\tilde{a}$	$\tilde{b}$	$\tilde{c}$	$\tilde{h}$	ϵ
Observability	0.4	0.7	0.2	0.3	0
Faulty	0.5	0.2	0.3	0.6	0

Table 8: prognoser P2

copredictability definition and the co-verifier.

A. **Using fuzzy co-predictability definition:**

- *The set of the faulty traces with respect to the prognoser* P_1 *is* $\tilde{\Psi}_{\tilde{h}} = \{\tilde{a}\tilde{b}\tilde{c}\tilde{h}, \tilde{b}\tilde{a}\tilde{b}\tilde{a}\tilde{h}\}$.

  – *For* $\tilde{s} = \tilde{a}\tilde{b}\tilde{c}\tilde{h}$, $\tilde{t} = MaxPref(\tilde{s}) = \tilde{a}\tilde{b}\tilde{c}$. *We have:*

$$\begin{cases} \tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{h}) \\ \Phi_{\tilde{h},1}(\tilde{t},\infty) = \{\tilde{a}\tilde{b}\tilde{c}\tilde{h}(\tilde{c}\tilde{h}\tilde{b})^*\} \end{cases}$$

  By Definition 3.3, the fuzzy event $\tilde{h}$ *is predictable in the FDES* $\tilde{G}4$ *with respect to the prognoser* P_1 *if the inequation (10) holds:*

$$\tilde{\Sigma}_f(\tilde{h}) \leq min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{h},1}(\tilde{t},n)\}$$

  Note that : $\tilde{\Sigma}_f(\tilde{h}) = 0.6$ *and* $min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{h},1}(\tilde{t},4)\} = min\{0.6\}$ *Then* $(\exists n_0 = 4)(\forall n > n_0)$, $\tilde{\Sigma}_f(\tilde{h}) \leq min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{h},1}(\tilde{t},4)\}$.
  Thus, the fuzzy string $\tilde{s} = \tilde{a}\tilde{b}\tilde{c}\tilde{h}$ *is predictable in the FDES* $\tilde{G}4$ *with respect to the prognoser* P_1.

  – *For* $\tilde{s} = \tilde{b}\tilde{a}\tilde{b}\tilde{a}\tilde{h}$, $\tilde{t} = MaxPref(\tilde{s}) = \tilde{b}\tilde{a}\tilde{b}\tilde{a}$. *We have:*

$$\begin{cases} \tilde{\Sigma}_f(\tilde{t}) < \tilde{\Sigma}_f(\tilde{h}) \\ \Phi_{\tilde{h},1}(\tilde{t},\infty) = \{\tilde{b}\tilde{a}\tilde{b}\tilde{a}\tilde{h}\tilde{c}(\tilde{c})^*, \tilde{b}\tilde{c}\tilde{a}\tilde{c}(\tilde{c})^*\} \end{cases}$$

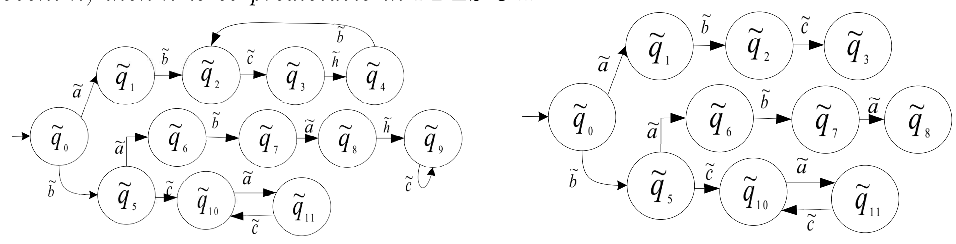
  By Definition 3.3, the fuzzy event $\tilde{h}$ *is predictable in the FDES* $\tilde{G}4$ *with respect to the prognoser* P_1 *if the inequation (10) holds:*

$$\tilde{\Sigma}_f(\tilde{h}) \leq min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{h},1}(\tilde{t},n)\}$$

  Note that : $\tilde{\Sigma}_f(\tilde{h}) = 0.6$ *and* $min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{h},1}(\tilde{t},5)\} = min\{0.6, 0.5\}$, *then* $(\forall n > n_0)$ $\tilde{\Sigma}_f(\tilde{h}) \leq min\{\tilde{\Sigma}_f(\tilde{w}) : \tilde{w} \in \Phi_{\tilde{h},1}(\tilde{t},n)\}$. *Thus, the fuzzy string* $\tilde{s} = \tilde{b}\tilde{a}\tilde{b}\tilde{a}\tilde{h}$ *is not predictable in the FDES* $\tilde{G}4$ *with respect to the prognoser* P_1. *So because there exists a string that is not predictable in FDES* $\tilde{G}4$ *with respect to the prognoser* P_1, *the fuzzy event* $\tilde{h}$ *is not predictable.*

  – *A similar method to that used for the prognoser* P_1 *is used for the faulty strings with respect of the prognoser* P_2.
  We obtain that all faulty traces are predictable with respect to the prognoser P_2, *which means that the fuzzy event* $\tilde{h}$ *in the FDES* $\tilde{G}4$ *is predictable with respect to the prognoser* P_2.

Now, we have found that: The fuzzy event $\tilde{h}$ *is not predictable with respect to the prognoser* P_1 *but it is predictable with respect to the prognoser* P_2. *Because for any faulty trace in the FDES* $\tilde{G}4$ *there is a prognoser (*P_1 *or* P_2*) that can predict the fuzzy event* $\tilde{h}$, *then* $\tilde{h}$ *is co-predictable in FDES* $\tilde{G}4$.



(a) (b)

Figure 7: (a) The FDES $\tilde{G}4$.(b) The NFAP of the FDES $\tilde{G}4$ with respect to fuzzy event $\tilde{h}$.

B. **Using the co-verifier:** *According to Step 1 and Step 2 presented in Section V, the NFAP of the FDES* $\tilde{G}4$ *and the fuzzy co-verifier with respect to the fuzzy event* $\tilde{h}$ *are shown in Figure 7 (b), and Figure 8, respectively. Notice that in the NFAP there is four "Non-indicator states"* $\tilde{q}_0, \tilde{q}_5, \tilde{q}_{10}$ *and* $\tilde{q}_{11}$ *and two "boundary states"* $\tilde{q}_3$ *and* $\tilde{q}_8$. *From the co-verifier in Figure 8, there does not exist an accessible unpredictable state. So, according to Theorem 4.3 the failure event* $\tilde{h}$ *is co-predictable in the FDES* $\tilde{G}4$.

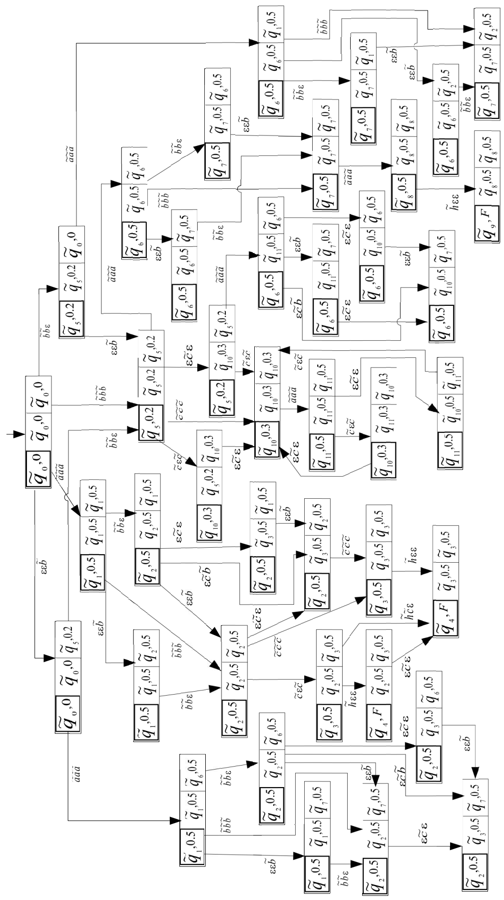


Figure 8: The fuzzy co-verifier with respect to fuzzy event $\tilde{G}_4$.

6 Conclusions

In this paper, we have extended the existing work on the problem of decentralized failure prognosis to fuzzy discrete event systems framework. The notion of co-predictability was introduced under the FDES settings, and the decentralized failure prognosis verification is performed by constructing a fuzzy co-verifier from a given fuzzy system. Especially, a necessary and sufficient condition needed for the co-predictability of FDESs was provided. In addition, to illustrate the effectiveness of the proposed approach some examples are given, and showed that it can be used to deal with the decentralized failure prognosis problem for crisp DESs as well as FDESs. As future work, a natural further issue worthy of consideration is to generalize our prognosis approach to the framework of distributed fuzzy DESs. Finally, another interesting future issue that we plan to tackle is to study predictability of patterns in fuzzy DESs where a failure is no longer a single event but rather a pattern, i.e., a set of sequences of events that are considered as "abnormal".

Acknowledgement

The authors wish to express their appreciation for several excellent suggestions for improvements in this paper made by the referees.

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