POTENTIAL ENERGY BASED STABILITY ANALYSIS OF FUZZY LINGUISTIC SYSTEMS

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ABSTRACT. This paper presents the basic concepts of stability in fuzzy linguistic models. The authors have proposed a criterion for BIBO stability analysis of fuzzy linguistic models associated to linear time invariant systems [25]-[28]. This paper presents the basic concepts of stability in the general nonlinear and linear systems. This stability analysis method is verified using a benchmark system analysis.

1. **Introduction**

For many real-world systems and processes, a mathematical description in the form of differential/difference equations or other conventional model is either infeasible or impracticable due to complexity involved, the time consideration, and the intrinsic nature of information incompleteness. For finding a practical way studies have been done to analyze fuzzy control systems [32]-[12]. Tanaka and et al [30] proposed a method for stability analysis of TSK model by finding a common Lyapunov function, Percup and et al [19] used the center of manifold theory for fuzzy system stability analysis. The authors of this paper proposed a sufficient condition for stability of TSK fuzzy model [22], Farinawata [14] and Linder [4] separately worked on robust stability controller design. Many other efforts, which have been done in the TSK stability analysis, are found in [21]. They use classical approaches for stability analysis of fuzzy systems. Unfortunately these approaches conflict with the simplicity idea, which was the main aim of Zadeh when presenting his fuzzy system approach. For simplicity in system analysis and humanity interface, a linguistic model is offered [35]. Some authors have done some incomplete researches on this area [8]-[34]. Recently Margaliot and Langholz [15]-[17] have proposed some approaches for linguistic nonlinear systems. The basic idea is using crisp equation of Lyapunov for the stability analysis in their approaches. Furuhashi et. al [8]-[34] have proposed a definition for equilibrium in fuzzy linguistic model. In their definition some things seems not to be quite right [23]. Until now the stability analysis of linguistic systems is an open problem [12]. Here we will extend a stability analysis method [24] to fuzzy linguistic systems associated with a class of applied nonlinear systems. Section 2 proposes some preliminary definitions in computation with word. In section 3 a theorem for stability analysis of crisp system is proposed. Section 4 presents the linguistic stability concept and includes a benchmark problem. Finally section 5

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includes a conclusion.

2. Definitions in Linguistic Calculus

In this section we propose some definitions. These definitions are used in the next sections in order to define equilibrium and stability concepts.

Definition 2.1. *Center of Linguistic Value:* consider, as an example, the following linguistic value membership functions:

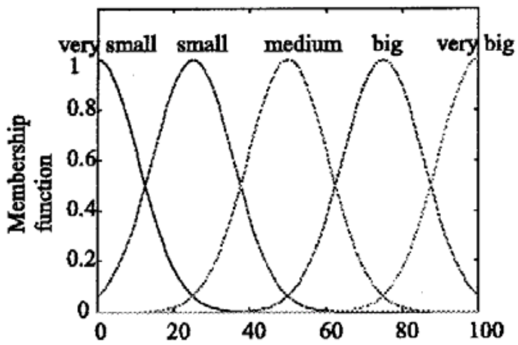

FIGURE 1. Gaussian membership functions

The linguistic values are $\{Ling1, Ling2, Ling3, Ling4,....\}$. The X_{cent} is the center of linguistic value $LingM$ if and only if

(1) $$\max_x \mu_{lingM}(x) = \mu_{lingM}(X_{cent})$$

Note 1. If there is an finite interval (i.g. $[a,b]$) where $\mu_{lingM}(x)=1$, then the mean point of interval is defined as center of linguistic value.

Note 2. If there is an infinite interval (i.g. $[a,\infty)$) where $\mu_{lingM}(x)=1$, then the finite boundary of interval is defined as center of linguistic value. □

Definition 2.2. *Linguistic Neighborhood:*

Consider a set of fuzzy sets as

(2) $$Set = \{A, B, C,...\}$$

Where $A, B, C,...$ are the fuzzy sets. Sort the center of linguistic values of these fuzzy sets increasingly. Associate the sorted sequence with N_m [1]. Thus the first term is 1 and its associated fuzzy set is labeled $ling1$, the second term is 2 and its associated fuzzy set is labeled $ling2$,…

[1] N_m is the first *m* terms of natural number sequence.

The $\zeta \in Z^{+}$ neighborhood of $lingR$ is shown by

(3) $$\Omega_{\zeta}^{L.N.}(lingR) = \{lingj | R-\zeta \leq j \leq R+\zeta\}$$ □

Example 2.1. Consider fuzzy sets with the following membership functions.

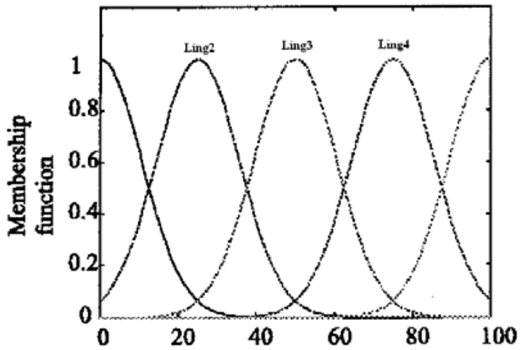

FIGURE 2. A sample of membership functions

Then

(4) $$\Omega_{\zeta=1}^{L.N.}(ling3) = \{ling2, ling3, ling4\}$$ □

Definition 2.3. *Linguistic Deleted Neighborhood*:
The Linguistic Neighborhood of $lingR$ which does not contain $lingR$, is called Linguistic Deleted Neighborhood of $lingR$. It is shown as

(5) $$\Omega_{\zeta}^{L.D.N..}(lingR) = \{lingj | R-\zeta \leq j \leq R+\zeta,\ j \neq r\}$$ □

Definition 2.4. *Fuzzy equilibrium subset:*
Consider a rule base that describes the dynamic relation of the state variables of a system without exogenous input as follows:

(6)
$$\begin{aligned} &R^{i}: If \quad x(k) \quad is \quad Ling_{i}H \quad then \quad x(k+1) \quad is \quad Ling_{i}E \\ &i = 1,...,m \end{aligned}$$

If there is a rule in which $Ling_{i}H$ and $Ling_{i}E$ are linguistically equal, then $Ling_{i}H \equiv Ling_{i}E \equiv Ling_{i}eq$ is called a fuzzy equilibrium subset.

Definition 2.5. *Equilibrium halo:*
Consider $P=\{lingP_1, lingP_2, lingP_3,.., lingP_N\}$ as a set of all fuzzy equilibrium subsets. Then $lingP_i$ is called equilibrium halo if for initial condition $x(k_0)=lingP_i$ the following rule
if x(k) is lingP$_i$ then x(k+1) is lingP$_i$.
is fired with highest degree of firing, among the other rules, for all the $k>k_0$. □

3. A Necessary and Sufficient Condition for Stability Analysis of a Class of Applicable Mechanical Systems [24]

This section reviews some definitions about the potential energy and conservative forces in the pure mechanical systems. We have observed that the work done against a gravitational or an elastic force depends only on the net change of position and not on the particular path followed in reaching the new position. The Force fields with this characteristic are called *conservative force fields* which posses an important mathematical property and will be illustrated as follows. Consider a force field as shown in fig.1, where the force F is a function of its coordinates.

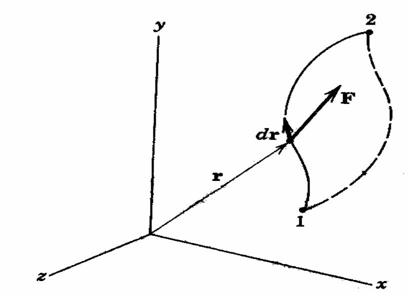

FIGURE 3. The sample path for describing the potential energy

The work done by the force F during the displacement dr of its point of application is

(7) $$dU = F.dr$$

The total work done along its path form 1 to 2 is

(8) $$U=\int_1^2 F.dr=\int_1^2 \left(F_x dx+F_y dy+F_z dz\right)$$

The integral $\int F.dr$ is a line integral dependent, in general, upon the particular path followed between any two points 1 and 2 in space. If, however, $F.dr$ is an exact differential $-dV$ of some scalar function V of the coordinates, then

(9) $$U = \int_{V_1}^{V_2} -dV = -(V_2 - V_1)$$

which depends only on the end points of the motion and which is thus *independent* of the path followed. The minus sign in front of dV is arbitrary but is chosen to agree with the customary designation of the sign of potential energy changes in the earth's gravity field. If V exist, the differential change in V becomes

(10) $$dV = \frac{\partial V}{\partial x}dx + \frac{\partial V}{\partial y}dy + \frac{\partial V}{\partial z}dz$$

Comparing it with

(11) $$-dV = F.dr = F_x dx + F_y dy + F_z dz$$

yields

(12) $$F_x = -\frac{\partial V}{\partial x} \text{ and } F_y = -\frac{\partial V}{\partial y} \text{ and } F_z = -\frac{\partial V}{\partial z}$$

So the force may also be written as the vector

(13) $$F = -\nabla V$$

where the symbol ∇ stands for the vector operator "del" which is defined as follows

(14) $$\nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$$

The quantity V is known as the *potential function*, and the expression ∇V is known as the *gradient of the potential function.* When the force components are differentiable from a potential function as described above, the force is said to be *conservative*, and the work done by the force between any two points is independent of path followed.
In the following we propose the necessary and sufficient conditions for asymptotic stability of some applied mechanical systems.

Theorem 3.1. *Consider a mechanical system which is perturbed with a force and starts to move at initial time. There is no exogenous input. Suppose all the forces are conservative forces except the frictional torque, and x_{eq} is system's isolated equilibrium point. The equilibrium state x_{eq} is asymptotic stable if and only if there is a symmetric deleted neighborhood Ω_{SDNE} of x_{eq} such that*

(15) $$\forall x \in \Omega_{SDNE} : \frac{dE_{potential}(x)}{dx_i} \times \frac{|x_i|}{x_i} > 0$$

and $$\frac{dE_{potential}(x_{eq})}{dx_i} = 0 \text{ i=1,2,3}$$

where the $E_{potential}$ is the potential energy of the system. □

Note1. Without loss in generality x_{eq} is chosen as the origin of coordinates and x_{eq} is supposed to be the reference of system's potential energy, i.e.

(16) $$E_{potential}\left(x_{eq}\right) = 0$$

Note2. In Newtonian dynamic systems the set of two numbers, the position and velocity at time t_0, is qualified to be called the state of the system at time t_0 [2]. In the other word $\left[x \quad y \quad z \quad \dot{x} \quad \dot{y} \quad \dot{z}\right]^T$ is a state vector. This concept is used in the theorem's proof.

Note 3. It is supposed that $E_{potential}$ is directional derivable on Ω_{SDNE}.

Note 4. It is supposed that the perturbed system at its initial condition has potential energy but does not have kinetic energy.

Proof. see [24]. □

4. A Necessary and Sufficient Condition for Stability Analysis of a Class of Linguistic Fuzzy Models

This section uses Theorem 3.1. and proposes a necessary and sufficient condition for stability of linguistic models associated with a class of linear and/or nonlinear mechanical systems.

Theorem 4.1. *Consider a system without exogenous inputs and all applied forces (except the frictions forces) are conservative forces. An equilibrium halo is asymptotic stable if and only if it has a symmetric linguistic deleted neighborhood such that the potential energy of system is greater or equal to its potential energy.*

Proof. In the previous section we saw the necessary and sufficient condition for asymptotic stability of an equilibrium point of the given system.
The following are equivalent.

1- $\exists\Omega_{SDNE} \quad ;\forall x \in \Omega_{SDNE} : \dfrac{x_i}{|x_i|}\dfrac{dE_{potential}(x)}{dx_i} > 0$.

2-A local minimum of $E_{potential}(x)$ is $E_{potential}(x_{eq})$.

Consider an arbitrary direction as x_i. If the minimum potential energy is at x_{eq} then in a given neighborhood of x_{eq} we have:

If $\quad 0 = x_{eq} > x_i \quad$ then

$$E_{potential}(x_{eq}) < E_{potential}(x_i)$$

If $0 = x_{eq} < x_i$ then

$$E_{potential}(x_{eq}) < E_{potential}(x_i)$$

Thus

(17) $$\frac{\partial E_{potential}(x)}{\partial x_i}\Big|_{x > x_{eq} = 0} > 0$$

(18) $$\frac{\partial E_{potential}(x)}{\partial x_i}\Big|_{x < x_{eq} = 0} < 0$$

Therefore

$$x_i > x_{eq} = 0 \Rightarrow \frac{\partial E_{potential}(x)}{\partial x_i}\frac{x_i}{|x_i|} > 0$$
$$x_i < x_{eq} = 0 \Rightarrow \frac{\partial E_{potential}(x)}{\partial x_i}\frac{x_i}{|x_i|} > 0$$

It means that if a local minimum of $E_{potential}(x)$ is $E_{potential}(x_{eq})$, then

$$\exists \Omega_{SDNE} \quad ; \forall x \in \Omega_{SDNE} : \frac{x_i}{|x_i|}\frac{dE_{potential}(x)}{dx_i} > 0 \, .$$

On the other hand, if

$$\exists \Omega_{SDNE} \quad ; \forall x \in \Omega_{SDNE} : \frac{x_i}{|x_i|}\frac{dE_{potential}(x)}{dx_i} > 0$$

it is obvious that a local minimum of $E_{potential}(x)$ is $E_{potential}(x_{eq})$.

Hence Theorem 3.1. can be restated as follows.
"The equilibrium point of the system given in Theorem1 is asymptotic stable if and only if a local minimum of potential energy is in x_{eq} ".

However crisp model concepts can be extended in the fuzzy model. The equilibrium point in a crisp model is associated with the equilibrium halo in fuzzy model; the crisp neighborhood is associated with a linguistic neighborhood and so on. Using the above idea it is concluded that "In the system given in Theorem 3.1., an equilibrium halo is asymptotic stable if and only if there is a symmetric linguistic deleted neighborhood of equilibrium halo such that potential energy of the system there is greater than or equal to the potential energy of equilibrium halo". □

Example 3.1. Consider a pendulum system.

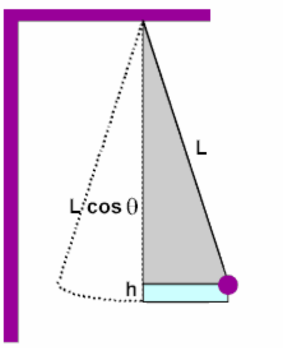

FIGURE 4. A pendulum

The rules of equilibrium subsets is as follows

$$X(k)=\begin{bmatrix} Ling19\\ Ling4\\ Ling13\\ Ling14\\ Ling11\\ Ling10\\ Ling9\\ Ling8\\ Ling10\\ Ling11\\ Ling9\\ Ling1\\ Ling16\\ Ling7\\ Ling6\\ Ling12 \end{bmatrix}, \qquad Z(k)=\begin{bmatrix} Ling10\\ Ling10\\ Ling10\\ Ling10\\ Ling11\\ Ling11\\ Ling11\\ Ling10\\ Ling10\\ Ling10\\ Ling10\\ Ling10\\ Ling10\\ Ling10\\ Ling10\\ Ling10 \end{bmatrix},$$

where the $\boldsymbol{Z}$ and X are the coordinates of the system. The equilibrium halo of system will be obtained as

X(k)= $Ling10$ and Z(k) = $Ling10$

The system's potential energy rule base of equilibrium halo is as follows:
If X(k) is $Ling10$ and Z(k) is $Ling10$ then $E_{Potential}$ is $Ling10$.

Using the system's potential energy rule base, there is a deleted neighborhood, with potential energy greater than or equal to the equilibrium halo potential energy.

If X(k) is $Ling9$ and Z(k) is $Ling11$, then $E_{Potential}$ is $Ling11$.

If X(k) is $Ling9$ and Z(k) is $Ling10$, then $E_{Potential}$ is $Ling10$.

If X(k) is $Ling11$ and Z(k) is $Ling10$, then $E_{Potential}$ is $Ling10$.

If X(k) is $Ling11$ and Z(k) is $Ling11$, then $E_{Potential}$ is $Ling11$.

Using Theorem 4.1., the equilibrium halo of system is asymptotic stable. This analysis coincides with the classic stability analysis.

5. **Conclusion**

This paper has presented the basic concepts of stability analysis for the fuzzy linguistic models; also this paper presents the necessary and sufficient condition for stability analysis of a class of applied mechanical nonlinear systems. Finally the proposed stability analysis method is verified using a benchmark problem.

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