FUZZY H_V-SUBSTRUCTURES IN A TWO DIMENSIONAL EUCLIDEAN VECTOR SPACE

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Abstract. In this paper, we study fuzzy substructures in connection with H_v-structures. The original idea comes from geometry, especially from the two dimensional Euclidean vector space. Using parameters, we obtain a large number of hyperstructures of the group-like or ring-like types. We connect, also, the mentioned hyperstructures with the theta-operations to obtain more strict hyperstructures, as H_v-groups or H_v-rings (the dual ones).

1. **Introduction**

A set H, equipped with at least one multivalued map $\cdot : H \times H \to \mathcal{P}(H)$ called hyperoperation, is said to be hyperstructure. Marty, in 1934, introduced the notion of a hypergroup as a hyperstructure $(H, \cdot)$ where, the following two axioms hold:

(i) $x \cdot H = H \cdot x = H$, $\forall x \in H$,
(ii) $(x \cdot y) \cdot z = x \cdot (y \cdot z)$, $\forall x, y, z \in H$.

Since then, many researchers have been worked on this area. Vougiouklis in 1990 introduced [12] the concept of H_v-structures which are generalizations of the classical hyperstructures. One can find, definitions and results on H_v-structures in the books [2], [13]. We recall some definitions from [13]:

Itemize. Let H be a set equipped with the hyperoperation $(\cdot)$, then the *weak associativity* is given by the relation

$$(x \cdot y) \cdot z \cap x \cdot (y \cdot z) \neq \emptyset, \qquad \forall x, y, z \in H.$$

The hyperstructure $(H, \cdot)$, is called *H_v-semigroup*, if the weak associativity is valid. The *H_v-group* is defined to be a H_v-semigroup, where the *reproductivity* axiom is valid, i.e. $x \cdot H = H \cdot x = H$, $\forall x \in H$.

The H_v-ring is defined to be the triple (H,+,·), where in both $(+)$ and $(\cdot)$ the weak associativity is valid, the weak distributivity of $(\cdot)$ with respect to $(+)$ is also valid, i.e.

$$x \cdot (y + z) \cap (x \cdot y + x \cdot z) \neq \emptyset, (x + y) \cdot z \cap (x \cdot z + y \cdot z) \neq \emptyset, \quad \forall x, y, z \in H$$

and $(+)$ is reproductive.

An H_v-ring $(R, +, \cdot, +)$ is called dual H_v-ring if the hyperstructure $(R, +, \cdot, +)$ is an H_v-ring, too [5]. In [15] a hyperoperation denoted by ∂ is defined as follows:

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Let $(H,\cdot)$ be a groupoid and $f\colon H\to H$ is any map. We define a hyperoperation (∂) , called *theta*-operation, on H as follows

$$x\partial y=\{f(x)\cdot y, x\cdot f(y)\},\quad \forall x,y\in H$$

or in case where $(\cdot)$ is hyperoperation or f is multivalued map we have

$$x\partial y=(f(x)\cdot y)\cup(x\cdot f(y)),\quad \forall x,y\in H.$$

A very useful proposition in this paper is the following one:

Proposition 1.1. [14] *If a hyperoperation* $(\cdot)$ *is weak associative or weak commutative then every greater hyperoperation than* $(\cdot)$*, will be weak associative or weak commutative. If a hyperoperation* $(\cdot)$ *is weak distributive with respect to* $(+)$*, then every greater hyperoperation than* $(\cdot)$ *or* $(+)$*, will be weak distributive with respect to* $(+)$*.*

For a categorical aspect of H_v-rings one can see [8].

The concept of a fuzzy subgroup of a group G was introduced in [11]. If G is a group and $m\colon G\to[0,1]$ is a fuzzy set, then m is called *fuzzy subgroup* of G if it satisfies:

(i) $\min\{m(x),m(y)\}\leqslant m(xy)$, $\forall x,y\in G$
(ii) $m(x)\leqslant m(x^{-1})$, $\forall x\in G$.

Davvaz in [3], [4] has given the following definitions:

Let H be a hypergroup (or H_v-group) and let m be a fuzzy subset of H. Then m is said to be a fuzzy subhypergroup (or fuzzy H_v-subgroup) of H, if the following axioms hold:

(i) $\min\{m(x),m(y)\}\leqslant \inf_{a\in x\cdot y}\{m(a)\}$, $\forall x,y\in H$
(ii) For all $x,a\in H$ there exists $y\in H$ such that $x\in a\cdot y$ and $\min\{m(a),m(x)\}\leqslant m(y)$.

Let $(R,+,\cdot)$ be an H_v-ring and m be a fuzzy subset of R. Then m is said to be a fuzzy H_v-subring of R, if the following axioms hold:

(i) $\min\{m(x),m(y)\}\leqslant inf_{a\in x+y}\{m(a)\}$, $\forall x,y\in R$
(ii) For all $x,a\in R$ there exists y∈R such that $x\in a+y$ and $\min\{m(a),m(x)\}\leqslant m(y)$
(iii) For all $x,a\in R$ there exists $z\in R$ such that $x\in z+a$ and $\min\{m(a),m(x)\}\leqslant m(z)$ and
(iv) $\min\{m(x),m(y)\}\leqslant inf_{a\in x\cdot y}\{m(a)\}$, $\forall x,y\in R$.

Let $(R,+,\cdot)$ be an H_v-ring and let m be a fuzzy subset of R. Then a fuzzy H_v-subring of R is said to be a *right (resp. left) fuzzy H_v-ideal* of R, if the following axiom hold:

$m(x)\leqslant inf_{a\in x\cdot y}\{m(a)\}$ (resp. $m(y)\leqslant inf_{a\in x\cdot y}\{m(a)\}$), $\forall x,y\in R$.

The concept of a *dual fuzzy H_v-subring* of a dual H_v-ring, is introduced in [7] as follows:

Let $(R,+,\cdot)$ be a dual H_v-ring and m be a fuzzy subset of R. Then m is said to be a *dual fuzzy H_v-subring* of R, if m is fuzzy H_v-subring of both H_v-rings $(R,+,\cdot)$ and $(R,\cdot,+)$.

2. A Geometric-like Hyperoperation

Let IR^2 be the two dimensional real vector space over the field of the real numbers IR, where we consider a coordinate system with origin O. We shall refer to the elements $X, Y, Z, \ldots, \ldots$ of the set IR^2 by their position vectors $x, y, z, \ldots$ respectively.

We consider an order in IR^2 according to the measure of the position vectors, i.e. $x \leqslant y$ if $|x| \leqslant |y|$ for every $x,\ y \in IR^2$.

Definition 2.1. In IR^2, for a given $\lambda \in [0,1]$, we define a commutative hyperoperation $(\circ)$, as follows: For every $x, y \in IR^2$

$$x \circ y = [\min\{x,y\}, \min\{x,y\} + \lambda \max\{x,y\})$$

which is a left closed, right opened segment.

For simplicity, let us assume that $\min\{x,y\} = x$, then the above hyperoperation is of the form

$$x \circ y = [x, x + \lambda y) = \{x + \mu y /\ \mu \in [0,\lambda)\ \}$$

.

Using the above hyperoperation into the plane, one can easily combine abstract algebraic properties with geometrical figures. The same procedure is appeared in [1]. From geometrical point of view, in that sense, the above hyperoperation is the side of the parallelogram with vertices O, X, Y, Z, where Z corresponds to the position vector $z = x + y$.

Proposition 2.2. *The hyperstructure $(IR^2, \circ)$ is a commutative H_v-group.*

Proof. For every $x \in IR^2$:

$$x \circ IR^2 = \Big(\bigcup_{x \leqslant r} (x \circ r)\Big) \cup \Big(\bigcup_{x > r'} (x \circ r')\Big) =$$
$$= \Big(\bigcup_{x \leqslant r} \{x + \mu r / \mu \in [0,\lambda)\}\Big) \cup \Big(\bigcup_{x > r'} \{r' + \mu x / \mu \in [0,\lambda)\}\Big) = IR^2 = IR^2 \circ x.$$

Now, let $x < y < z$. Then

$$(x \circ y) \circ z = \{x + \mu y / \mu \in [0,\lambda)\} \circ z = \Big(\bigcup_{\substack{w \in x \circ y \\ w \leqslant z}} (w \circ z)\Big) \cup \Big(\bigcup_{\substack{w \in x \circ y \\ w > z}} (w \circ z)\Big) =$$
$$= \Big(\bigcup_{\substack{w \in x \circ y \\ w \leqslant z}} \{w + \mu z \,/\, \mu \in [0,\lambda)\}\Big) \cup \Big(\bigcup_{\substack{w \in x \circ y \\ w > z}} \{z + \mu w \,/\, \mu \in [0,\lambda)\}\Big) =$$
$$= \{x + \mu' y + \mu z \,/ \mu \in [0,\lambda), \mu' \in [0,\lambda'] \text{ where } \lambda' < \lambda, z = x + \lambda' y\} \cup$$
$$\cup \{z + \mu x + \mu\mu'' y \,/ \mu \in [0,\lambda)\,,\ \mu'' \in (\lambda',\lambda) \text{ where } \lambda' < \lambda,\ z = x + \lambda' y\}. \quad (1)$$

Similarly, since $x < y + z$ we get the equation

$$x \circ (y \circ z) = \{x + \mu y + \mu^2\, z \,/ \mu \in [0,\lambda)\}. \quad (2)$$

By setting in (1) and (2) $\mu = \mu\prime = 0$ we get that $x, z \in (x \circ y) \circ z$ and $x \in x \circ (y \circ z)$. That means

$$(x \circ y) \circ z \cap x \circ (y \circ z) \neq \emptyset.$$

Checking the associativity, similarly, for the rest five cases, i.e.:

$$x < z < y,\ y < x < z,\ y < z < x,\ z < x < y,\ z < y < x$$

we get that

$$(x \circ y) \circ z \cap x \circ (y \circ z) \neq \emptyset,\ \forall x, y, z \in IR^2. \qquad \square$$

Now, we define a new hyperoperation $(\circ')$, depending on a parameter λ', similar to the hyperoperation $(\circ)$ which depends on the parameter λ, where $\lambda' \neq \lambda$, assuming for simplicity that $x < y$, as follows:

Definition 2.3. In IR^2, for a given $\lambda' \in [0, 1]$, we define a commutative hyperoperation $(\circ')$,

$$x \circ' y = [x,\, x + \lambda' y) = \{x + \mu' y \,/\, \mu' \in [0, \lambda')\}.$$

Proposition 2.4. *The hyperstructure $(IR^2, \circ, \circ')$ is a commutative dual H_v-ring.*

Proof. The only axiom we have to prove is that of the weak distributivity of $(\circ)$ with respect to $(\circ')$ and vice versa, since the rest axioms have been proven in Proposition 3.
Let $x < y < z$ then

$$x \circ (y \circ' z) = x \circ \{y + \mu' z \,/ \mu' \in [0, \lambda')\} = \bigcup_{w \in y \circ' z} (x \circ w) = \bigcup_{w \in y \circ' z} \{x + \mu w \,/\, \mu \in [0, \lambda)\}$$
$$= \{x + \mu y + \mu\mu' z / \,\mu \in [0, \lambda),\ \mu' \in [0, \lambda')\}. \tag{3}$$

$$(x \circ y) \circ' (x \circ z) = \{x + \mu y \,/\, \mu \in [0, \lambda)\} \circ' \{x + \mu z \,/\, \mu \in [0, \lambda)\} =$$
$$= \bigcup_{\substack{w \in x \circ y \\ w' \in x \circ z}} (w \circ' w') = \bigcup_{\substack{w \in x \circ y \\ w' \in x \circ z}} \{w + \mu' w' / \mu' \in [0, \lambda')\} =$$
$$= \{(1 + \mu')x + \mu y + \mu\mu' z \,/\, \mu \in [0, \lambda), \mu' \in [0, \lambda')\}. \tag{4}$$

Setting in (3) and (4) $\mu' = 0$, we get that

$$\{x + \mu y \,/\, \mu \in [0, \lambda)\} \subset [x \circ (y \circ' z)] \cap [(x \circ y) \circ' (x \circ z)] \tag{5}$$

Setting in (4) $\mu = 0$, then when $\mu' \neq 0$ we get that

$$\{(1 + \mu')x \,/\, \mu' \in [0, \lambda')\} \subset (x \circ y) \circ' (x \circ z) \tag{6}$$

but obviously

$$\{(1 + \mu')x \,/\, \mu' \in [0, \lambda')\} \not\subset x \circ (y \circ' z). \tag{7}$$

From (5), (6) and (7), we get that

$$(x \circ (y \circ' z)) \cap ((x \circ y) \circ' (x \circ z)) \neq \emptyset.$$

Similarly, it can be proved that

$$(x \circ' (y \circ z)) \cap ((x \circ' y) \circ (x \circ' z)) \neq \emptyset.$$

Checking, similarly, the distributivity for the rest cases of $x, y, z \in IR^2$ we can prove that, for every $x, y, z \in IR^2$

$$(x \circ (y \circ' z)) \cap ((x \circ y) \circ' (x \circ z)) \neq \emptyset, \quad (x \circ' (y \circ z)) \cap ((x \circ' y) \circ (x \circ' z)) \neq \emptyset. \quad \square$$

3. Constructing a Fuzzy Set

In order to use fuzzy sets, we define a map f from the set of positive real numbers to the closed interval $I = [0, 1]$, i.e. $f : IR_+ \to I$.

Construction 3.1. *Let Oxy be a Cartesian coordinate system. In the first quadrant, take the points $A\left(0, \frac{\sqrt{2}}{2}\right)$ and $B\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$, then obviously $|OB| = 1$.*

Let $C(c, 0)$ be the point on the x-axis, then CA intersects OB in $D(p, w)$. The equation of the line passes through the points C and D is of the form

$$\frac{x - p}{c - p} = \frac{y - w}{0 - w}$$

and since the point A belongs to the above line, we get that

$$2cw + \sqrt{2}p = \sqrt{2}c. \tag{8}$$

Obviously,

$$p = w. \tag{9}$$

From (8) and (9) that we get that $p = w = \frac{\sqrt{2}c}{2c+\sqrt{2}}$ and then

$$|OD| = \sqrt{2\left(\frac{\sqrt{2}c}{2c + \sqrt{2}}\right)^2} = \frac{c(2c - \sqrt{2})}{2c^2 - 1}.$$

That means, every $a \in IR_+$ is mapped to $0 \leqslant \frac{a(2a-\sqrt{2})}{2a^2-1} \leqslant 1$. Thus we have $|x| \leqslant |y| \Rightarrow m(x) \leqslant m(y)$, $\forall x, y \in IR^2$.

Therefore, the measure map, we defined, together with the map f constructed above, define a fuzzy set $m : IR^2 \to [0, 1]$.

4. Reducing IR^2 or Enlarging $(\circ)$

In this paper, our aim is to combine geometrical hyperoperations with fuzzy sets.

By taking $x, y \in IR^2$, $x < y$, such that x and y being into different quadrants of the Cartesian plane, we get

$$\inf_{a \in x \circ y} \{m(a)\} = m(w) \neq \min\{m(x), m(y)\} \tag{10}$$

where w is the position vector of the element W, such that $OW\perp\ XZ$ and Z corresponds to the position vector $z = x + y$.

Since the relation (4), is not the desirable one for studying fuzzy sets, we choose as a set, any of the four quadrants of the Cartesian plane, working on this, let us say $V = IR_+ \times IR_+ \subset IR^2$.

Obviously, for every $x, y \in V$, the angle $XOY < 90^0$ and that means that the angle (OXZ) = angle$(OYZ) > 90^0$, so from the obtuse-angle triangle OXZ (or OYZ) we get that $\max\{x, y\} < z = x + y$.

Let $A(a_1,\, a_2) \in V$ be any point with position vector a. Then we denote by G_a, the following set $G_a = \{(x, y) \in V/x \geqslant a_1 \text{ and } y \geqslant a_2\}$.

Now, take any $a \in V$ and $x \in V - G_a$, such that $a < x$. We shall try to find $y \in V$ such that $x \in a \circ y$.

Suppose, there exists $y \in$V, $a < y$ such that $x \in a \circ y = \{a + \mu y/\mu \in [0, \lambda)\}$, then there exists $\mu' \in [0, \lambda)$, such that $x = a + \mu' y$ and that means $X(a_1 + \mu' y_1$, $a_2 + \mu' y_2)$.

Since $X \in V - G_a$ we get, for example, $a_1 + \mu' y_1 > a_1$ and $a_2 + \mu' y_2 < a_2$.

These two inequalities leads to $y_1 > 0$ and $y_2 < 0$, so $y \notin V$, which is a contradiction.

Notice that, by taking any $a \in V$ and $x \in G_a$, such that $a < x$, the above inequalities are becoming $a_1 + \mu' y_1 \geqslant a_1$ and $a_2 + \mu' y_2 \geqslant a_2$, which means that y_1, $y_2 \geqslant 0$, so in this case, there exists $y \in V$ such that $x \in a \circ y$.

Going further, all these $y \in V$ are elements of the set $\{k(x - a) \,/\, k > \frac{1}{\lambda}\}$.
Indeed, let $y = k'(x - a)$, k' $> \frac{1}{\lambda}$, then

$$a \circ y = [a, a + \lambda y) = \{a + \mu y/\mu \in [0, \lambda)\} = \Big\{a + \mu k'(x - a)/\mu \in [0, \lambda), k' > \frac{1}{\lambda}\Big\},$$

but since $k' > \frac{1}{\lambda} \Rightarrow \frac{1}{k'} < \lambda$, by taking $\mu = \frac{1}{k'}$, we get that $x \in a \circ y$.

The above condition is very important for studying fuzzy sets, so in order to by-pass these difficulties there is two choices, either reduce the set or enlarge the hyperoperation.

For example, as we have seen, using the above notation one can get results relating to the concept of fuzzy H_v-subgroup of the set G_a.

The hyperoperation (∂) will be used to enlarge the hyperoperation $(\circ)$ and its necessity will be shown next.

5. **Fuzzy H_V-substructures**

Let $V = IR_+ \times IR_+ \subset IR^2$ be the first quadrant of the Cartesian plane with origin O. In the set V, define a commutative hyperoperation (∂) as follows:

Definition 5.1. Let f be the multivalued map, such that for a given $\lambda \in [0, 1]$

$$f\colon V \to V : x \to f(x) = [0, \lambda x).$$

Then, for every $x, y \in V$

$$x \partial y = (f(x) + y) \cup (x + f(y)) = [y, y + \lambda x) \cup [x, x + \lambda y).$$

From the above definition we get that $(x\partial y) \supset (x \circ y)$ for every $x, y \in V$. That means that $(\partial) > (\circ)$ and according to the Propositions 1 and 3 the hyperstructure (V, ∂) is an H_v-group.

Notice that in [9] Konguetsof defined a hyperoperation $(\perp)$ in a set $H \neq \emptyset$ as follows: For every $x, y \in H$, $x \perp y = \{x, y\}$, proving that $(H, \perp)$ is a hypergroup. The hyperoperation (∂) we defined above, is a generalization of the $(\perp)$, since for $\lambda = 0$, we get $x\partial y = \{x, y\} = x \perp y$.

In [16], it is proved that if $(G, \cdot)$ is a group and $f(x) = a$ constant map on G, then $(G, \partial)/\beta^*$ is singleton. In that case $f(x) = e$, and so $x\partial y = \{x, y\}$ is the smallest incidence hyperoperation.

In [6], a hyperstructure $(H, \square)$ with $x \square y \supset \{x, y\}$ for every $x, y \in H$ is called *containing H_v-group.* So, (V, ∂) is a containing H_v-group.

Proposition 5.2. *Consider the H_v-group (V, ∂) and let m be a fuzzy subset of V, such that $x \leqslant y \Rightarrow m(x) \leqslant m(y)$, $\forall x, y \in V$. Then m is a fuzzy H_v-subgroup of V.*

Proof. Let us consider the cases:

(i) $x \leqslant y$, then by Chapter 4, $x \leqslant y < x + y$
  Then, $\min\{m(x),\, m(y)\} = m(x)$ and $\inf\limits_{a \in x\partial y} \{m(a)\} = m(x)$.

(ii) $y \leqslant x$, then $y \leqslant x < x + y$
  Then, $\min\{m(x),\, m(y)\} = m(y)$ and $\inf\limits_{a \in x\partial y} \{m(a)\} = m(y)$.

In both cases, the following is valid:

$$\min\{m(x), m(y)\} \leqslant \inf_{a \in x\partial y} \{m(a)\}, \forall x, y \in V.$$

Now, let $x, a \in V$. Consider the cases:

(i) $x \leqslant a$, then $m(x) \leqslant m(a)$
  Since $a\partial x = [a, a + \lambda x) \cup [x, x + \lambda a)$ we get that $x \in a\partial x$.
  So, for $x = y$, there exists $y \in V$ such that $x \in a\partial y$ and $\min\{m(x), m(a)\} = m(x) \leqslant m(y)$.

(ii) $a \leqslant x$, then $m(a) \leqslant m(x)$, again $x \in a\partial x$, so for $x = y$, there exists $y \in V$ such that $x \in a\partial y$ and $\min\{m(x), m(a)\} = m(a) \leqslant m(x) \leqslant m(y)$. □

One can realise the necessity of the hyperoperation ∂, combining the above part of the proof together with the Chapter 4.

Now, as before and for a given $\lambda' \neq \lambda$, λ, $\lambda' \in [0, 1]$, we define the commutative hyperoperation (∂') as follows:

Definition 5.3. We define the hyperoperation (∂'), for every $x, y \in V$,

$$x\partial' y = [y, y + \lambda' x) \cup [x, x + \lambda' y).$$

From the above definition we get that $(x\partial' y) \supset (x \circ' y)$ for every $x, y \in V$. That means that $(\partial') > (\circ')$ and according to the Propositions 1 and 5 the hyperstructure (V, ∂, ∂') is a dual H_v-ring.

Proposition 5.4. *Let the dual H_v-ring (V, ∂, ∂'). Then m is a dual fuzzy H_v-subring of V.*

Proof. This follows easily, considering both the Proposition 8 and that the hyperoperations (∂) and (∂') are commutative. □

Proposition 5.5. *Let the dual H_v-ring (V, ∂, ∂'). Then the dual fuzzy H_v-subring of V, is either a right or a left fuzzy H_v-ideal of V.*

Proof. Let us consider the cases:

(i) $x \leqslant y$

Then, $\min\{m(x), m(y)\} = m(x)$ and $\inf\limits_{a \in x\partial y}\{m(a)\} = m(x)$, so

$$m(x) \leqslant \inf_{a \in x\partial y}\{m(a)\}, \forall x, y \in V$$

(ii) $y \leqslant x$

Then, $\min\{m(x), m(y)\} = m(y)$ and $\inf\limits_{a \in x\partial y}\{m(a)\} = m(y)$, so

$$m(y) \leqslant \inf_{a \in x\partial y}\{m(a)\}, \forall x, y \in V.$$

□

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