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Interval number ranking method considering multiple decision attitudes

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Abstract

Many interval number ranking methods cannot represent the different attitudes of decision makers with different risk appetites. Therefore, interval numbers are expressed in the Rectangular Coordinate System (RCS). After mining the interval numbers in the RCS, the Symmetry Axis Compensation Factor, which is known as λ, was introduced, and the Equivalent Function of the Goal Interval Number (GIN) was deduced. Thus, the interval number ranking method considering symmetry axis compensation was defined along with its application procedures. Additionally, the feasibility and effectiveness of this method were verified through examples. This method is intuitive and simple and can represent multiple attitudes of decision makers with different risk appetites.

Keywords: Decision attitude, interval numbers, ranking method.

1 Introduction

In decision-making science, evaluation values are not always real numbers. Using interval numbers to represent evaluation values in the decision-making processes more accurately represents the reality of uncertainty and is more consistent with the fuzzy human mind than using real numbers [2, 19, 26]. Ranking interval numbers is key in decision-making approaches that use interval numbers to represent evaluation values [10, 25].

Since interval numbers were first proposed by Dwyer [5], many scholars have investigated methods for ranking interval numbers. Ishibuchi and Tanaka [12] defined the weak preference order relationship of two interval numbers in linear programming. Kundu [15] claimed that selecting the least (or most) preferred item of two interval numbers can be achieved using a function. Dubois and Prade [3, 4], Nakahara *et al.* [21], Xu and Da [28], Xu [27], Wang *et al.* [24], Fan and Liu [6] and Jiang *et al.* [14] significantly improved this approach by defining possibility degree functions to calculate the advantage possibility degree when comparing two interval numbers.

These methods are not appropriate for comparing interval numbers with equal symmetry axes (also called midpoints) mainly because they can represent only one decision-maker's attitude (most are moderate risk enthusiast/avoider), which is insufficient for describing decision makers that have different risk preferences.

However, in reality, two interval numbers could be regarded as different comparison results because of expected psychological characteristics among people. In general, interval numbers with better symmetry and shorter lengths are superior. However, how these two elements coordinate to influence the advantage degree of interval numbers is complicated. When two interval numbers $\widetilde{a}$ and $\widetilde{b}$ are compared, if the symmetry axis of $\widetilde{a}$ is slightly lower than that of $\widetilde{b}$ and the length of $\widetilde{a}$ is shorter than that of $\widetilde{b}$, then $\widetilde{a}$ may be equal to or superior to $\widetilde{b}$. The relative advantage of the interval number could be offset by the advantage of the shorter length, thereby compensating for the lower symmetry axis. For example, the interval number [40,50] has a lower symmetry axis than [20,80], and as a result, [40,50] might be regarded as equal to the other interval because of the shorter length for a given risk tolerance level and superior or inferior for some other risk tolerance levels.

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The methods for ranking fuzzy numbers considering different risk preferences were widely researched, such as in the studies by Jain [13], Baldwin and Guild [1], Liou and Wang [17], Lee-Kwang and Lee [16], and Huynh *et al.* [11]. Furthermore, Hashemi *et al.* [9] and Ghatee [8] defined two parameters for solving interval linear programming problems with different risk preferences. However, few scholars considered designing factors to represent different risk appetites during the processes of ranking interval numbers. Based on the studies by Ishibuchi and Tanaka [12] and Kundu [15], Sengupta and Pal [23], a factor with a range of [0,1] was defined to reflect the degrees of the decision makers' pessimism/optimism and the method for calculating this factor was presented. However, this ranking method had an obvious limitation in that the rank was difficult to achieve when comparing a group of interval numbers. Zhang *et al.* [30] and Liu *et al.* [18] proposed an index describing decision makers' attitudes and reported that the interval decision matrix could be transformed into the decision matrix using this attitude index. Thus, the ranking of interval numbers can be achieved using different attitude indexes. However, this index was provided without its calculation method, and the actual meaning was unclear. Ruan *et al.* [22] defined the optimism degrees of emergency decision makers and the relevant calculation methods, but both were associated with problems in Liu *et al.* [18] and Sengupta and Pal [23]: the index was designed without a basis, and ranking a group of interval numbers was difficult. The typical related works are summarized in Table 1.

Studies on ranking interval numbers	Contributions	Able to rank a group of interval numbers conveniently	Able to represent multiple decision attitudes	The factors have actual meaning	Calculation burden
Ishibuchi and Tanaka [12]	Defined weak preference order relation	No	No	-	-
Kundu [15]	Defined a leftness order relation to measure the advantage possibility degree	Yes	No	-	Small
Nakahara *et al.* [21], Fan and Liu [6]	Defined an advantage possibility function	Yes	No	-	Small
Hashemi *et al.* [9]	Defined parameters describing different decision-maker attitudes	No	Yes	No	Large
Sengupta and Pal [23]	Defined an acceptability index	No	Yes	No	Large
Zhang *et al.* [30] and Liu *et al.* [18]	Defined an index describing different decision-maker attitudes	Yes	Yes	No	Small
Ye *et al.* [29]	Expressed interval numbers in the RCS and defined an advantage possibility function based on this expression	Yes	No	-	Small
Our work	Defined a symmetry axis compensation factor	Yes	Yes	Yes	Small

Table 1: Related works on ranking interval numbers.

In solving the aforementioned problems, including issues with representing multiple decision-maker attitudes in numerous interval number ranking methods, the actual meanings of the factors were unclear, and it was difficult to compare a group of interval numbers. To express the relationships of interval numbers in the RCS [30], the Symmetry Axis Compensation Factor λ and the Equivalent Function of GIN were defined. Then, an interval number ranking method considering symmetry axis compensation was proposed that was able to reflect multiple attitudes of decision makers with different risk appetites and that allowed simple and feasible ranking.

The main contributions of this work are summarized as follows: (i) The Symmetry Axis Compensation Factor

λ was defined using the actual interval numbers after expressing them in the RCS. (ii) Then, the interval number ranking method considering symmetry axis compensation was used to describe multiple attitudes of decision makers with different risk appetites in a simple and feasible manner.

The remainder of this manuscript is organized as follows: Section 2 introduces basic knowledge about interval numbers and describes the method of expressing them in the RCS. In addition, the Symmetry Axis Compensation Factor and the Equivalent Function of GIN are also defined. In Section 3, the method of ranking interval numbers is presented, with a particular focus on the design principles of this method considering symmetry axis compensation and its application procedures. In Section 4, the effectiveness of the method in comparing a group of interval numbers is verified through a well-designed example. In Section 5, the effectiveness of this method is further verified through a decision-making example. The conclusions are drawn in Section 6, and recommendations for future studies are given.

2 Preliminary definitions

2.1 The basic definitions of interval numbers

Definition 2.1. [16, 20] *Let* $\widetilde{a} = \left[a^L, a^U\right] = \{a|a^L \leq a \leq a^U,\ \ a^L,\ a^U \in R\}$ *be an interval number, where* a^L *and* a^U *are the upper and lower limits of* $\widetilde{a}$*, respectively. Specifically, if* $a^L = a^U$*, then* $\widetilde{a}$ *degenerates into a real number (where* $\widetilde{a}$ *is also called the degenerate interval number).*

Definition 2.2. [29, 7] *Let* $\widetilde{a} = \left[a^L, a^U\right]$*; then,* $l^+(\widetilde{a})$ *and* $l^-(\widetilde{a})$ *are defined as the symmetry axis and the length of the interval number* $\widetilde{a}$*, respectively,* i.e., $l^+(\widetilde{a}) = (a^L + a^U)/2$, $l^-(\widetilde{a}) = a^U - a^L$.

Definition 2.3. [12, 24] *Let* $\widetilde{a} = \left[a^L, a^U\right]$ *and* $\widetilde{b} = \left[b^L, b^U\right]$*; define* $P(\widetilde{a} \succ \widetilde{b})$ *as the advantage degree of* $\widetilde{a}$ *compared with* $\widetilde{b}$*:* $P\left(\widetilde{a} \succ \widetilde{b}\right) \in [0, 1]$ *and* $P\left(\widetilde{a} \succ \widetilde{b}\right) + P(\widetilde{b} \succ \widetilde{a})$*=1. If* $P\left(\widetilde{a} \succ \widetilde{b}\right) > 0.5$*, then* $\widetilde{a} \succ \widetilde{b}$*; if* $P\left(\widetilde{a} \succ \widetilde{b}\right) = 0.5$*, then* $\widetilde{a} = \widetilde{b}$*; and if* $P\left(\widetilde{a} \succ \widetilde{b}\right) < 0.5$*, then* $\widetilde{b} \succ \widetilde{a}$*.*

Definition 2.4. [6, 29] *Let* $\widetilde{a} = \left[a^L, a^U\right]$ *and* $\widetilde{b} = \left[b^L, b^U\right]$*; if* $a^L \geq b^U$*, then* $P(\widetilde{a} \succ \widetilde{b}) = 1$ *and* $P(\widetilde{b} \succ \widetilde{a})$*=0. Thus,* $\widetilde{a}$ *and* $\widetilde{b}$ *are separate interval numbers.*

Definition 2.5. [29] *Let* $\{\widetilde{a}_1, \widetilde{a}_2, \ldots, \widetilde{a}_n\}$ *be a group of interval numbers, and suppose that* $\widetilde{a}_m$ *is one of them. If* $a_m^U = max(a_1^U, a_2^U, \ldots, a_n^U)$*, then define* $\widetilde{a}_m = [a_m^L, a_m^U]$ *as the GIN of the group of interval numbers. In particular, if* $\max\left(a_1^U, a_2^U, \ldots, a_n^U\right) = a_c^U = a_d^U = \ldots = a_k^U$*,* $a_i^L = \max\left(a_c^L, a_d^L, \ldots, a_k^L\right)$*, then the GIN is* $\widetilde{a}_i$*.*

2.2 Analyzing interval numbers in the RCS

In decision-making science, the upper and lower limits of the interval number evaluation values are always positive real numbers. Therefore, positive interval numbers and no degeneration are found for only $\widetilde{a} = \left[a^L, a^U\right] = \{a|a^L \leq a \leq a^U,\ a^L < a^U,\ a^L, a^U \in R^+\}$. Interval numbers can be expressed in the RCS as shown in Figure 1. More details about the regularity of interval numbers after expressing them in the RCS can be found in Ye *et al.* [30]. Brief descriptions of the figure are provided below:

(1) The upper and lower limits of the interval numbers are expressed by the y-axis and x-axis of the RCS, respectively.

(2) Suppose that $\widetilde{a} = \left[a^L, a^U\right]$ and is the GIN of a group of interval numbers; then, it is easy to obtain the arbitrary interval number $\widetilde{a}_*$ of the group corresponding to the point (a_*^L, a_*^U) that can be expressed as the triangle area bounded by lines $y = a^U$, $x = 0$ and $y = x$.

(3) When determining the GIN, three zones were formed in the RCS by five lines: $y = a^U$, $x = 0$, $y = x$, $y = a^L$ and the line of the Equivalent Function of GIN (introduced below). These three zones were named Zones ①, ② and ③.

2.3 The symmetry axis compensation factor

Because the advantage of a shorter length compensates for the disadvantage of a lower symmetry axis, the Symmetry Axis Compensation Factor was defined for expressing these numbers in the RCS (Figure 2) as follows:

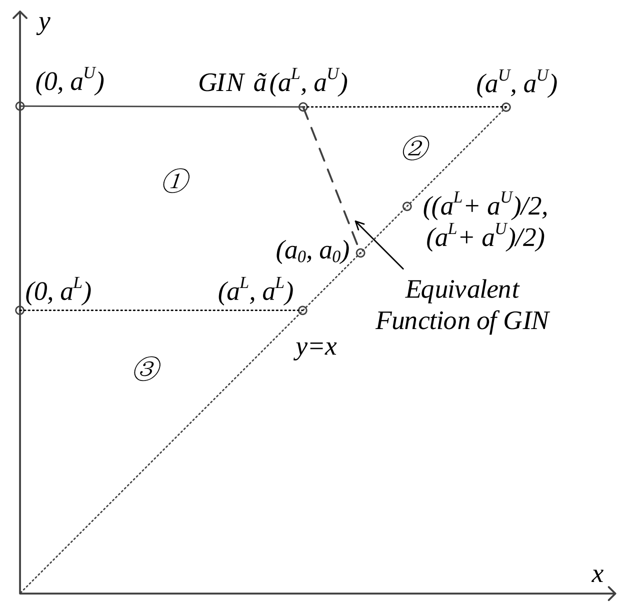

Figure 1: Expressing interval numbers in the RCS

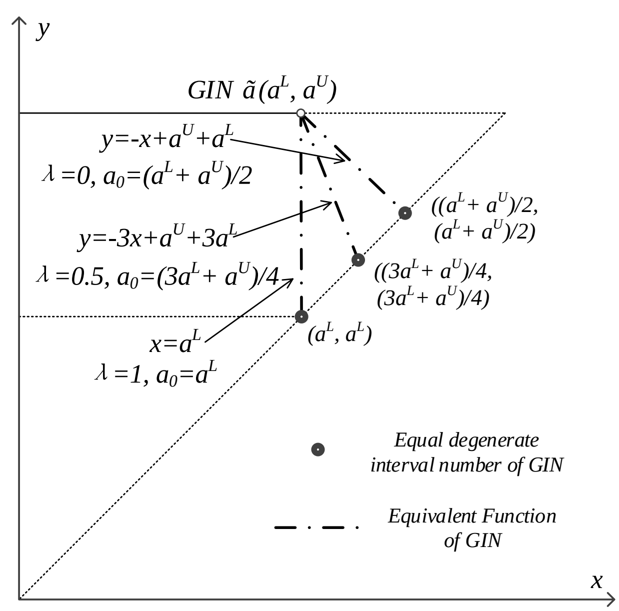

Figure 2: Relations of λ and the equivalent function of GIN in the RCS

Definition 2.6. *Let an arbitrary interval number $\widetilde{a}_* = [a_*^L, a_*^U]$; let λ be the Symmetry Axis Compensation Factor, the calculation method of which is shown in Eq. (1); and let $\widetilde{a}_0 = [a_0,\ a_0]$ be the equal degenerate interval number of $\widetilde{a}_*$. The range of a_0 is $[a_*^L, (a_*^L + a_*^U)/2]$; thus, the corresponding range of λ is* [0,1].

$$\lambda = (a_*^U + a_*^L - 2a_0)/(a_*^U - a_*^L) = 2(l^+(\tilde{a}) - a_0)/l^-(\tilde{a}) \tag{1}$$

Definition 2.7. *Let an arbitrary interval number $\widetilde{a}_* = [a_*^L, a_*^U]$ and the equal symmetry axis degenerate interval number be a_*^1, where $a_*^1 = [(a_*^L + a_*^U)/2, (a_*^L + a_*^U)/2]$. Then, the equal lower-limit degenerate interval number is*

a_*^2, *where* $a_*^2 = [a_*^L, a_*^L]$. *If someone believes that* $a_*^1 = \widetilde{a}_*$, *then he or she is an extreme risk avoider; in contrast, if someone believes* $a_*^2 = \widetilde{a}_*$, *then he or she is an extreme risk enthusiast. Connecting the corresponding points of* a_*^1 *and* a_*^2 *into a segment in the RCS reveals that the corresponding interval number of the segment midpoint is* $a_*^{'}$, $a_*^{'} = [(3a_*^L/4 + a_*^U/4), (3a_*^L/4 + a_*^U/4)]$. *If someone believes* $a_*^{'} = \widetilde{a}_*$, *then he or she is a moderate risk enthusiast/avoider.*

Taking an interval number $\widetilde{a}_* = [20, 80]$ for example, then $a_*^1 = [50, 50]$, $a_*^2 = [20, 20]$ and $a_*^{'} = [35, 35]$. If someone believes $[50, 50] = [20, 80]$, $[20, 20] = [20, 80]$ or $[35, 35] = [20, 80]$, then he or she is an extreme risk avoider, an extreme risk enthusiast or a moderate risk enthusiast/avoider, respectively.

2.4 The equivalent function of GIN

Based on the above definitions, the GIN is the target when ranking a group of interval numbers. Define a point on the line $y = x$ (the set of corresponding points of degenerate interval numbers) that corresponds to the degenerate interval number equal to GIN in the RCS. Then, connect this point and the corresponding point of GIN into a segment. The corresponding function of the segment is the Equivalent Function of GIN, *i.e.*, the segment is the set of corresponding points of the interval numbers that are equal to GIN. In other words, the advantage of the corresponding interval number of an arbitrary point on the segment to GIN is 0.5.

Definition 2.8. *Let* $\widetilde{a}$ *be the GIN of a group of interval numbers,* $\widetilde{a} = [a^L, a^U]$, *where the Symmetry Axis Compensation Factor is* λ, *and the equal corresponding degenerate interval number of GIN is* $\widetilde{a}_0 = [a_0,\ a_0]$. *From Eq. (1), the calculation method of* a_0 *is shown in Eq. (2). Then, the corresponding Equivalent Function of GIN is* $y = (\lambda+1)/(\lambda-1)x + a^U - (\lambda+1)/(\lambda-1)a^L$ *(when* $\lambda \neq 1$*). Specifically, when* $\lambda = 1$ *and* $\widetilde{a}_0 = [a^L, a^L]$, *the corresponding Equivalent Function of GIN is* $x = a^L$. *The relations of* λ *and the Equivalent Function of GIN in the RCS are shown in Figure 2.*

$$a_0 = (a^U + a^L)/2 - (a^U - a^L)\lambda/2 = l^+(\tilde{a}) - \lambda l^-(\tilde{a})/2 \tag{2}$$

3 Interval number ranking method

3.1 The design principles of the advantage degree function of interval numbers

Figure 1 shows that the interval numbers in Zone ③ are inferior to the GIN because their upper interval numbers are less than the lower limit of the GIN. The degree of advantage of the interval numbers in Zones ① and ② relative to the GIN must be calculated. The points of the interval numbers in Zone ① are below the corresponding line of the Equivalent Function of GIN; thus, it is apparent that the advantage degrees of the interval numbers in this zone relative to the GIN are less than 0.5. Similarly, the points of the interval numbers in Zone ② are above the corresponding line of the Equivalent Function of GIN; therefore, the advantage degrees of the interval numbers in this zone relative to the GIN exceed 0.5.

If the corresponding points of the interval numbers in Zone ① are close to the corresponding line of the Equivalent Function of GIN, their advantage degrees relative to the GIN are close to 0.5 from the left side, whereas if their corresponding points are far from the line, their advantage degrees relative to the GIN are close to 0. Similarly, if the corresponding points of the interval numbers in Zone ② are close to the corresponding line of the Equivalent Function of GIN, their advantage degrees relative to the GIN are close to 0.5 from the right side, whereas if their corresponding points are far from the line, their advantage degrees relative to the GIN are close to 1.

Considering the different characteristics of the interval numbers at the limit positions in Zones ① and ② and the variation rules of the distances from different interval numbers to the Equivalent Function of GIN in the RCS, an advantage degree function of interval numbers considering symmetry axis compensation was designed based on the principles of Limit and Piecewise functions; the design method is shown in Figure 3.

1) When $\lambda \neq 1$, let the distances from the farthest points on both sides to the corresponding line of the Equivalent Function of GIN $y = (\lambda+1)/(\lambda-1)x + a^U - (\lambda+1)/(\lambda-1)a^L$ in the RCS be d_{1max}^- and d_{1max}^+, respectively, *i.e.*, point $(0, a^L)$ on the left side of the line in Zone ① and point (a^U, a^U) on the right side of the line in Zone ②, respectively. Then, d_{1max}^- and d_{1max}^+ are as follows:

$$d_{1\max}^- = \frac{|\frac{\lambda+1}{\lambda-1} \cdot 0 - a^L + a^U - \frac{\lambda+1}{\lambda-1} \cdot a^L|}{\sqrt{\left(\frac{\lambda+1}{\lambda-1}\right)^2 + (-1)^2}} = \frac{a^U - \frac{2\lambda}{\lambda-1} a^L}{\sqrt{\left(\frac{\lambda+1}{\lambda-1}\right)^2 + 1}}$$

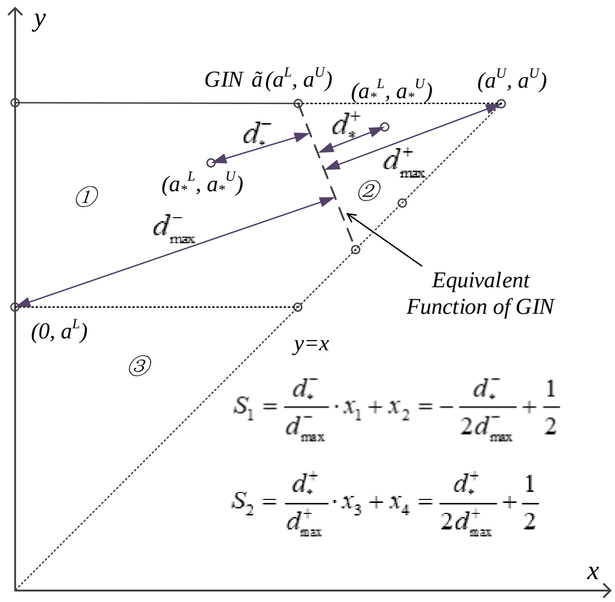

Figure 3: Design method of an advantage degree function of interval numbers considering symmetry axis compensation in the RCS

$$d^+_{1\max} = \frac{\left|\frac{\lambda+1}{\lambda-1}\cdot a^U - a^U + a^U - \frac{\lambda+1}{\lambda-1}\cdot a^L\right|}{\sqrt{\left(\frac{\lambda+1}{\lambda-1}\right)^2 + (-1)^2}} = \frac{\frac{\lambda+1}{1-\lambda}(a^U - a^L)}{\sqrt{\left(\frac{\lambda+1}{\lambda-1}\right)^2 + 1}}$$

For the corresponding point (a_*^L, a_*^U) of an arbitrary interval number in Zone ①, the distance d^-_{1*} between it and the line $y = (\lambda+1)/(\lambda-1)x + a^U - (\lambda+1)/(\lambda-1)a^L$ (line y) is

$$d^-_{1*} = \frac{\left|\frac{\lambda+1}{\lambda-1}\cdot a_*^L - a_*^U + a^U - \frac{\lambda+1}{\lambda-1}\cdot a^L\right|}{\sqrt{\left(\frac{\lambda+1}{\lambda-1}\right)^2 + (-1)^2}} = \frac{\frac{\lambda+1}{\lambda-1}a_*^L - a_*^U + a^U - \frac{\lambda+1}{\lambda-1}a^L}{\sqrt{\left(\frac{\lambda+1}{\lambda-1}\right)^2 + 1}}$$

For the corresponding point (a_*^L, a_*^U) of an arbitrary interval number in Zone ②, the distance d^+_{1*} between it and line y is

$$d^+_{1*} = \frac{\left|\frac{\lambda+1}{\lambda-1}\cdot a_*^L - a_*^U + a^U - \frac{\lambda+1}{\lambda-1}\cdot a^L\right|}{\sqrt{\left(\frac{\lambda+1}{\lambda-1}\right)^2 + (-1)^2}} = \frac{\frac{\lambda+1}{1-\lambda}a_*^L + a_*^U - a^U - \frac{\lambda+1}{1-\lambda}a^L}{\sqrt{\left(\frac{\lambda+1}{\lambda-1}\right)^2 + 1}}$$

Let the advantage degree of the arbitrary interval number (a_*^L, a_*^U) in Zones ① and ② to the GIN be S_1 and S_2, respectively. Then, when $\lambda \neq 1$, we obtain

$$S_1 = \frac{d^-_{1*}}{d^-_{1max}} \cdot x_1 + x_2 = \frac{\frac{\lambda+1}{\lambda-1}a_*^L - a_*^U + a^U - \frac{\lambda+1}{\lambda-1}a^L}{a^U - \frac{2\lambda}{\lambda-1}a^L} \cdot x_1 + x_2 \tag{3}$$

$$S_2 = \frac{d^-_{1*}}{d^-_{1max}} \cdot x_3 + x_4 = \frac{\frac{\lambda+1}{1-\lambda}a_*^L + a_*^U - a^U - \frac{\lambda+1}{1-\lambda}a^L}{\frac{\lambda+1}{1-\lambda}(a^U - a^L)} \cdot x_3 + x_4 \tag{4}$$

When the point is farther from line y on its left side and closer to point $(0, a^L)$, the advantage degree of the corresponding interval number relative to the GIN decreases. Let the advantage degree of the corresponding interval number of point $(0, a^L)$ be 0; thus,

$$\lim_{\substack{a_*^L \to 0 \\ a_*^U \to a^L}} S_1 = \lim_{\substack{a_*^L \to 0 \\ a_*^U \to a^L}} \left(\frac{d^-_{1*}}{d^-_{1\max}} \cdot x_1 + x_2\right) = 0 \Longrightarrow \lim_{\substack{a_*^L \to 0 \\ a_*^U \to a^L}} \left(\frac{\frac{\lambda+1}{\lambda-1}a_*^L - a_*^U + a^U - \frac{\lambda+1}{\lambda-1}a^L}{a^U - \frac{2\lambda}{\lambda-1}a^L} \cdot x_1 + x_2\right) = x_1 + x_2 = 0$$

When the point is farther from line y on its right side and closer to point (a^U, a^U), the advantage degree of the corresponding interval number relative to the GIN increases. Let the advantage degree of the corresponding interval number of point (a^U, a^U) be 1; thus,

$$\lim_{\substack{a_*^L \to a^U \\ a_*^U \to a^U}} S_2 = \lim_{\substack{a_*^L \to a^U \\ a_*^U \to a^U}} \left(\frac{d_{1*}^+}{d_{1\max}^+} \cdot x_3 + x_4 \right) = 1 \Longrightarrow \lim_{\substack{a_*^L \to a^U \\ a_*^U \to a^U}} \left(\frac{\frac{\lambda+1}{1-\lambda} a_*^L + a_*^U - a^U - \frac{\lambda+1}{1-\lambda} a^L}{\frac{\lambda+1}{1-\lambda}(a^U - a^L)} \cdot x_3 + x_4 \right) = x_3 + x_4 = 1$$

For the corresponding point (a_*^L, a_*^U) of an arbitrary interval number on line y, the distance between this point and the line is 0.

When the point is closer to line y, the corresponding interval number is closer to the GIN; thus, the degree of advantage is closer to 0.5, *i.e.*, both the left and right limits of the advantage degrees of the interval numbers when the corresponding points are close to the line of the Equivalent Function of GIN. Therefore,

$$\lim_{d_{1*}^- \to 0} S_1 = \lim_{d_{1*}^- \to 0} \left(\frac{d_{1*}^-}{d_{1max}^-} \cdot x_1 + x_2\right) = 0.5 \Longrightarrow \lim_{d_{1*}^- \to 0} \frac{d_{1*}^-}{a^U - \frac{2\lambda}{\lambda-1} a^L} \cdot x_1 + x_2 = 0 \cdot x_1 + x_2 = 0.5$$

$$\lim_{d_{1*}^- \to 0} S_2 = \lim_{d_{1*}^- \to 0} \left(\frac{d_{1*}^+}{d_{1\max}^+} \cdot x_3 + x_4 \right) = 0.5 \Longrightarrow \lim_{d_{1*}^+ \to 0} \frac{d_{1*}^+}{\frac{\lambda+1}{1-\lambda}(a^U - a^L)} \cdot x_3 + x_4 = 0 \cdot x_3 + x_4 = 0.5$$

where x_1, x_2, x_3 and x_4 are unknown constants, and their values are calculated to be -1/2, 1/2, 1/2 and 1/2, respectively.

Substitute the values of x_1, x_2, x_3 and x_4 into Eq. (3) and Eq. (4); then, the Advantage Degree Function of Interval Numbers Considering Symmetry Axis Compensation $S(\tilde{a}_* \succ \tilde{a})$ of non-separated interval numbers when $\lambda \neq 1$ is defined as follows:

$$S(\tilde{a}_* \succ \tilde{a}) = \begin{cases} \dfrac{(1-\lambda)a_*^U + (1+\lambda)a_*^L - (1-\lambda)a^L}{2\left[(1-\lambda)a^U + 2\lambda a^L\right]} & (1-\lambda)a_*^U < -(\lambda+1)a_*^L + (1-\lambda)a^U + (\lambda+1)a^L \\ 0.5 & (1-\lambda)a_*^U = -(\lambda+1)a_*^L + (1-\lambda)a^U + (\lambda+1)a^L \quad a_*^U > a^L \quad \lambda \neq 1 \\ \dfrac{\frac{1-\lambda}{2} a_*^U + \frac{1+\lambda}{2} a_*^L + \lambda a^U - (1+\lambda)a^L}{(1+\lambda)(a^U - a^L)} & (1-\lambda)a_*^U > -(\lambda+1)a_*^L + (1-\lambda)a^U + (\lambda+1)a^L \end{cases} \tag{5}$$

2) When $\lambda = 1$, the corresponding line of the Equivalent Function of GIN is $x = a^L$. Similarly, the distance d_{1max}^- from the farthest point on the left side of the line to the line and the distance d_{1max}^+ from the farthest point on the right side of the line to the line are as follows:

$d_{1\max}^- = a^L$, $d_{1\max}^+ = a^U - a^L$.

For the corresponding point (a_*^L, a_*^U) of an arbitrary interval number in Zone ①, the distance d_{1*}^- between this point and the line is $d_{1*}^- = a^L - a_*^L$.

For the corresponding point (a_*^L, a_*^U) of an arbitrary interval number in Zone ②, the distance d_{1*}^+ between this point and the line is $d_{1*}^+ = a_*^L - a^L$.

Let the advantage degrees of the arbitrary interval number (a_*^L, a_*^U) in Zones ① and ② to the GIN be $S_1^{'}$ and $S_2^{'}$, respectively. Then, when $\lambda = 0$, it is easy to obtain the following:

$$S_1' = \frac{d_{1*}^-}{d_{1max}^-} \cdot x_5 + x_6 = \frac{a^L - a_*^L}{a^L} \cdot x_5 + x_6 \tag{6}$$

$$S_2' = \frac{d_{1*}^+}{d_{1\max}^+} \cdot x_7 + x_8 = \frac{a_*^L - a^L}{a^U - a^L} \cdot x_7 + x_8 \tag{7}$$

Similarly, the values of x_5, x_6, x_7 and x_8 are calculated as -1/2, 1/2, 1/2 and 1/2, which are equal to x_1, x_2, x_3 and x_4, respectively. Furthermore, the function has the same form when $\lambda \neq 1$ after substituting into Eq. (6) and Eq. (7), *i.e.*, the function defined when $\lambda \neq 1$ is also applicable when $\lambda = 1$.

3.2 Advantage degree function of interval numbers considering symmetry axis compensation

Proving that the function $S(\widetilde{a}_* \succ \widetilde{a})$ in Eq. (5) is continuous when its value is 0.5 is easy. The Advantage Degree Function of Interval Numbers Considering Symmetry Axis Compensation S can be defined as Eq. (8) after simplifying Eq. (5).

$$S\left(\tilde{a}_* \succ \tilde{a}\right)=\begin{cases} \dfrac{(1-\lambda)a_*^U+(1+\lambda)a_*^L-(1-\lambda)a^L}{2\left[(1-\lambda)a^U+2\lambda a^L\right]} & (1-\lambda)a_*^U \leq -(\lambda+1)a_*^L+(1-\lambda)a^U+(\lambda+1)a^L \quad a_*^U>a^L \\ \dfrac{\frac{1-\lambda}{2}a_*^U+\frac{1+\lambda}{2}a_*^L+\lambda a^U-(1+\lambda)a^L}{(1+\lambda)\left(a^U-a^L\right)} & (1-\lambda)a_*^U \geq -(\lambda+1)a_*^L+(1-\lambda)a^U+(\lambda+1)a^L \quad a_*^U>a^L \\ 0 & a_*^U \leq a^L \end{cases} \tag{8}$$

When two interval numbers are compared, the shorter length of one could compensate for its own disadvantageous symmetry axis, thereby forming a new set of interval numbers that are equal to the GIN in the RCS, *i.e.*, the corresponding line of the Equivalent Function of GIN. The relative advantage degrees of interval numbers can be obtained after analyzing the relationships between the distances from the corresponding points of non-separated interval numbers to the line and the distances from the points at limit positions to the line.

3.3 Procedures of the ranking method

Correspondingly, the procedures involved in the ranking method can be summarized as follows:

(i) Select the GIN $\widetilde{a}$ from a group of interval numbers by using the method introduced in Definition 5.

(ii) Determine the Symmetry Axis Compensation Factor λ and calculate the equal corresponding degenerate interval number of GIN $\widetilde{a}_0$ and the corresponding Equivalent Function of GIN by using the method introduced in Definition 8.

(iii) Calculate the Advantage Degree Function of Interval Numbers Considering Symmetry Axis Compensation S with the Symmetry Axis Compensation Factor λ using Eq. (8).

(iv) Analyze the relationships of the arbitrary interval number (in the group) $\widetilde{a}_*$ and GIN $\widetilde{a}$. If $a_*^U \leq a^L$, then $S(\widetilde{a}_* \succ \widetilde{a}) = 0$. If $a_*^U > a^L$, calculate the advantage degrees of the interval numbers relative to the GIN using Eq. (8) and rank the interval numbers according to the advantage degrees.

(v) If two or more $S(\widetilde{a}_* \succ \widetilde{a}) = 0$, repeat Procedures (i), (ii), (iii) and (iv) until all interval numbers of the group are ranked.

The detailed procedures for ranking interval numbers are described in Figure 4.

4 Verification examples for the ranking method

Example 4.1. *Let* $\widetilde{a}_1$, $\widetilde{a}_2$, $\widetilde{a}_3$, $\widetilde{a}_4$, $\widetilde{a}_5$, $\widetilde{a}_6$, $\widetilde{a}_7$, $\widetilde{a}_8$, $\widetilde{a}_9$, *and* $\widetilde{a}_{10}$ *be a group of interval numbers:* $\widetilde{a}_1 = [20, 80]$, $\widetilde{a}_2 = [40, 60]$, $\widetilde{a}_3 = [5, 70]$, $\widetilde{a}_4 = [10, 20]$, $\widetilde{a}_5 = [10, 40]$, $\widetilde{a}_6 = [40, 70]$, $\widetilde{a}_7 = [5, 15]$, $\widetilde{a}_8 = [10, 70]$, $\widetilde{a}_9 = [30, 50]$ *and* $\widetilde{a}_{10} = [50, 50]$. *Attempt to rank all the interval numbers.*

Use the ranking interval number method introduced in Section 3.3.

(i) Compare the upper limits of the interval numbers, $a_1^U = max\left(a_1^U, a_2^U, \ldots, a_{10}^U\right) = 80$; the GIN is $\widetilde{a}_1$.

(ii) Determine the Symmetry Axis Compensation Factor $\lambda = 0.5$; then, the equal corresponding degenerate interval number of GIN $\widetilde{a}_0 = [35, 35]$, and the corresponding Equivalent Function of GIN is $y = -3x + 140$.

(iii) Substitute $\lambda = 0.5$ into Eq. (8). Thus, the corresponding advantage degree function of the interval numbers considering symmetry axis compensation is

$$S\left(\tilde{a}_* \succ \tilde{a}\right)=\begin{cases} \left(a_*^U+3a_*^L-20\right)/240 & a_*^U \leq -3a_*^L+140 \quad a_*^U>20 \\ \left(a_*^U+3a_*^L+40\right)/360 & a_*^U \geq -3a_*^L+140 \quad a_*^U>20 \\ 0 & a_*^U \leq 20 \end{cases} \tag{9}$$

(iv) Because $a_4^U \leq a_1^L$ and $a_7^U \leq a_1^L$, $S\left(\widetilde{a}_4 \succ \widetilde{a}_1\right) = 0 = S\left(\widetilde{a}_7 \succ \widetilde{a}_1\right)$.

Calculate the advantage degrees of $\widetilde{a}_2$, $\widetilde{a}_3$, $\widetilde{a}_5$, $\widetilde{a}_6$, $\widetilde{a}_8$, $\widetilde{a}_9$, and $\widetilde{a}_{10}$ relative to $\widetilde{a}_1$(GIN) using Eq. (9); the results are as follows:

$S\left(\widetilde{a}_2 \succ \widetilde{a}_1\right) = 0.611$, $S\left(\widetilde{a}_3 \succ \widetilde{a}_1\right) = 0.271$,
$S\left(\widetilde{a}_5 \succ \widetilde{a}_1\right) = 0.204$, $S\left(\widetilde{a}_6 \succ \widetilde{a}_1\right) = 0.639$,

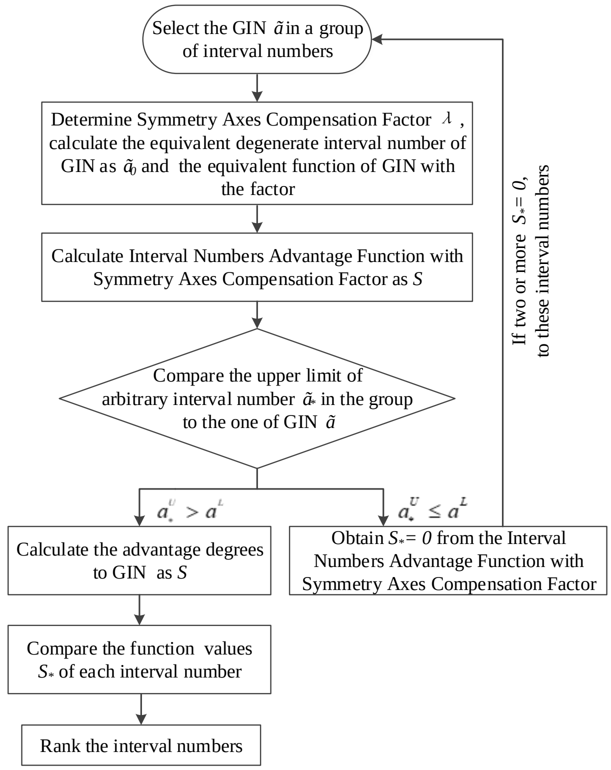

Figure 4: Procedures for ranking interval numbers

$S(\tilde{a}_8 \succ \tilde{a}_1) = 0.333$, $S(\tilde{a}_9 \succ \tilde{a}_1) = 0.500$, $S(\tilde{a}_{10} \succ \tilde{a}_1) = 0.667$.

$S(\tilde{a}_{10} \succ \tilde{a}_1) > S(\tilde{a}_6 \succ \tilde{a}_1) > S(\tilde{a}_2 \succ \tilde{a}_1) > 0.5 = S(\tilde{a}_9 \succ \tilde{a}_1) > S(\tilde{a}_8 \succ \tilde{a}_1) > S(\tilde{a}_3 \succ \tilde{a}_1) > S(\tilde{a}_5 \succ \tilde{a}_1)$, then $\tilde{a}_{10} \succ \tilde{a}_6 \succ \tilde{a}_2 \succ \tilde{a}_1 = \tilde{a}_9 \succ \tilde{a}_8 \succ \tilde{a}_3 \succ \tilde{a}_5$.

(v) Because $S(\tilde{a}_4 \succ \tilde{a}_1) = 0 = S(\tilde{a}_7 \succ \tilde{a}_1)$, Procedures (i), (ii), (iii) and (iv) are repeated, and $\tilde{a}_4$ and $\tilde{a}_7$ can be ranked as $\tilde{a}_4 \succ \tilde{a}_7$. Thus, the final rank of all ten interval numbers is $\tilde{a}_{10} \succ \tilde{a}_6 \succ \tilde{a}_2 \succ \tilde{a}_1 = \tilde{a}_9 \succ \tilde{a}_8 \succ \tilde{a}_3 \succ \tilde{a}_5 \succ \tilde{a}_4 \succ \tilde{a}_7$.

In addition, when $\lambda = 0$, the corresponding advantage degree function after simplifying the function in Eq. (8) is

$$S(\tilde{a}_* \succ \tilde{a}) = \begin{cases} \left(a_*^U + a_*^L - a^L\right) / \left(2a^U\right) & a_*^U \le -a_*^L + a^U + a^L \quad a_*^U > a^L \\ \left(a_*^U/2 + a_*^L/2 - a^L\right) / \left(a^U - a^L\right) & a_*^U \ge -a_*^L + a^U + a^L \quad a_*^U > a^L \\ 0, & a_*^U \le a^L \end{cases} \tag{10}$$

Eq. (10) uses the same advantage degree and restriction conditions calculation methods as the Advantage Degree Function of Interval Numbers Symmetry Axis in Ye *et al.* [29].

Furthermore, the method introduced above can be applied for varying values of λ, such as 0, 0.25, 0.5, 0.75 and 1. The advantage degrees of the ten interval numbers with varying λ are shown in Table 2.

Symmetry Axis Compensation Factor λ	Equal degenerate interval number $\tilde{a}_0$	Advantage degrees $S(\tilde{a}_* \succ \tilde{a})$									
		$\tilde{a}_1$ [20,80]	$\tilde{a}_2$ [40,60]	$\tilde{a}_3$ [5,70]	$\tilde{a}_4$ [10,20]	$\tilde{a}_5$ [10,40]	$\tilde{a}_6$ [40,70]	$\tilde{a}_7$ [5,15]	$\tilde{a}_8$ [10,70]	$\tilde{a}_9$ [30,50]	$\tilde{a}_{10}$ [50,50]
0	[50,50]	0.500	0.500	0.344	-	0.188	0.583	-	0.375	0.375	0.500
0.25	[42.5,42.5]	0.500	0.567	0.313	-	0.196	0.617	-	0.357	0.429	0.600
0.5	[35,35]	0.500	0.611	0.271	-	0.208	0.639	-	0.333	0.500	0.667
0.75	[27.5,27.5]	0.500	0.643	0.213	-	0.225	0.655	-	0.300	0.548	0.714
1	[20,20]	0.500	0.667	0.125	-	0.250	0.667	-	0.250	0.583	0.750

Table 2: The advantage degrees of interval numbers with different λ (GIN $\tilde{a} = [20, 80]$).

More importantly, the different values of λ in the interval numbers ranking method considering symmetry axis compensation can reflect multiple decision attitudes of decision makers with different risk appetites and allow for the same interval numbers to have different advantage degrees under different risk levels, making this ranking method flexible and adaptable.

5 An example of applying the proposed method

The initial seven mining methods are proposed according to the local experience and the specific conditions of one mine. Eight evaluation indexes were selected to determine the final mining method. The weights and values of the evaluation indexes are listed in Table 3 and Table 4, respectively.

Evaluation indexes	G_1	G_2	G_3	G_4	G_5	G_6	G_7	G_8
Weights	0.145	0.076	0.078	0.188	0.146	0.141	0.133	0.094

Table 3: Interval number weights of the evaluation indexes.

Evaluation indexes	Method X_1	Method X_2	Method X_3	Method X_4	Method X_5	Method X_6	Method X_7
Total taxation profit $(G_1)/(\text{RMB¥}\cdot Mt^{-1})$	[215,232]	[205,230]	[285,310]	[270,305]	[270,290]	[280,300]	[280,300]
Coefficient of loss $(G_2)/\%$	[8,10]	[11,13]	[8,10]	[8,11]	[8,10]	[9,12]	[8,10]
Boulder frequency $(G_3)/\%$	[5,10]	[10,20]	[5,8]	[8,15]	[6,10]	[7,11]	[5,8]
Miningratio $(G_4)/(\text{m}\cdot Mt^{-1})$	[40,44]	[30,35]	[26,30]	[31,36]	[31,35]	[35,38]	[28,32]
Mining safety (G_5)	[0.9,1]	[0.8,0.9]	[0.7,0.8]	[0.6,0.9]	[0.7,0.9]	[0.9,1]	[0.8,0.9]
Ventilation condition (G_6)	[0.5,0.7]	[0.8,0.9]	[0.8,1]	[0.7,1]	[0.7,0.9]	[0.9,1]	[0.9,1]
Technical difficulty (G_7)	[0.9,1]	[0.8,0.9]	[0.7,0.9]	[0.7,0.9]	[0.7,0.9]	[0.9,1]	[0.7,0.9]
Environment protection (G_8)	[0.9,1]	[0.9,1]	[0.9,1]	[0.7,0.9]	[0.9,1]	[0.9,1]	[0.8,0.9]

Table 4: Interval number values of the evaluation indexes.

G_1, G_5, G_6, G_7, and G_8 are income indexes, and G_2, G_3, and G_4 are cost indexes. Suppose that the interval number $[a_{ij}^L, a_{ij}^U]$ is the value of the evaluation index G_j of Method X_i and that $[a_{ij}^{L*}, a_{ij}^{U*}]$ is the dimensionalized value of this index. The values of each evaluation index can be made dimensionless *(0, 1)* using Eq.(11). The dimensionalized interval number values of the evaluation indexes are shown in Table 5. Suppose that $\widetilde{A}_i$=$[A_i^L, A_i^U]$ includes the comprehensive values of Method X_i and that w_j is the weight of evaluation index G_j. Use Eq.(12), the comprehensive interval number values of each method can be calculated, and the results are shown in Table 6.

$$\begin{cases} a_{ij}^{L*} = a_{ij}^{L}/max\left(a_{ij}^{U}\right), \quad a_{ij}^{U*} = a_{ij}^{U}/max\left(a_{ij}^{U}\right) & \text{income index} \\ a_{ij}^{L*} = min\left(a_{ij}^{L}\right)/a_{ij}^{U}, \quad a_{ij}^{U*} = min\left(a_{ij}^{L}\right)/a_{ij}^{L} & \text{cost index} \end{cases} \tag{11}$$

$$\begin{cases} A_i^L = \sum_{j=1}^{n} a_{ij}^{L^*}\cdot w_j \\ A_i^U = \sum_{j=1}^{n} a_{ij}^{U^*}\cdot w_j \end{cases} \tag{12}$$

By integrating the method for ranking the comprehensive interval number values, the procedures are as follows:

(i) $A_3^U = max\left(A_1^U, A_2^U, \ldots, A_7^U\right) = 0.9585$, so the GIN is $\widetilde{A}_3$.

(ii) Suppose that there are five people with different risk preferences and that the corresponding Symmetry Axis Compensation Factors λ are 0, 0.25, 0.5, 0.75 and 1.

Evaluation indexes	Method X_1	Method X_2	Method X_3	Method X_4	Method X_5	Method X_6	Method X_7
G_1	[0.694,0.748]	[0.661,0.742]	[0.919,1]	[0.871,0.984]	[0.871,0.935]	[0.903,0.968]	[0.903,0.968]
G_2	[0.8,1]	[0.615,0.727]	[0.8,1]	[0.727,1]	[0.8,1]	[0.667,0.889]	[0.8,1]
G_3	[0.5,1]	[0.25,0.5]	[0.625,1]	[0.333,0.625]	[0.5,0.833]	[0.455,0.714]	[0.625,1]
G_4	[0.591,0.650]	[0.743,0.867]	[0.867,1]	[0.722,0.839]	[0.743,0.839]	[0.684,0.743]	[0.813,0.929]
G_5	[0.9,1]	[0.8,0.9]	[0.7,0.8]	[0.6,0.9]	[0.7,0.9]	[0.9,1]	[0.8,0.9]
G_6	[0.5,0.7]	[0.8,0.9]	[0.8,1]	[0.7,1]	[0.7,0.9]	[0.9,1]	[0.9,1]
G_7	[0.9,1]	[0.8,0.9]	[0.7,0.9]	[0.7,0.9]	[0.7,0.9]	[0.9,1]	[0.7,0.9]
G_8	[0.9,1]	[0.9,1]	[0.9,1]	[0.7,0.9]	[0.9,1]	[0.9,1]	[0.8,0.9]

Table 5: Dimensionalized interval number values of the evaluation indexes.

Method	X_1	X_2	X_3	X_4	X_5	X_6	X_7
Comprehensive interval number values	[0.7177, 0.8564]	[0.7224, 0.8368]	[0.7985, 0.9585]	[0.6885, 0.9018]	[0.7443, 0.9063]	[0.8083, 0.9172]	[0.8280, 0.9550]

Table 6: Comprehensive interval number values of each method.

(iii) Substitute λ and $\widetilde{A}_1, \ldots, \widetilde{A}_7$ into Eq. (8); thus, the corresponding advantage degrees of $\widetilde{A}_1, \ldots, \widetilde{A}_7$ to $\widetilde{A}_3$ (GIN) are presented in Figure 5.

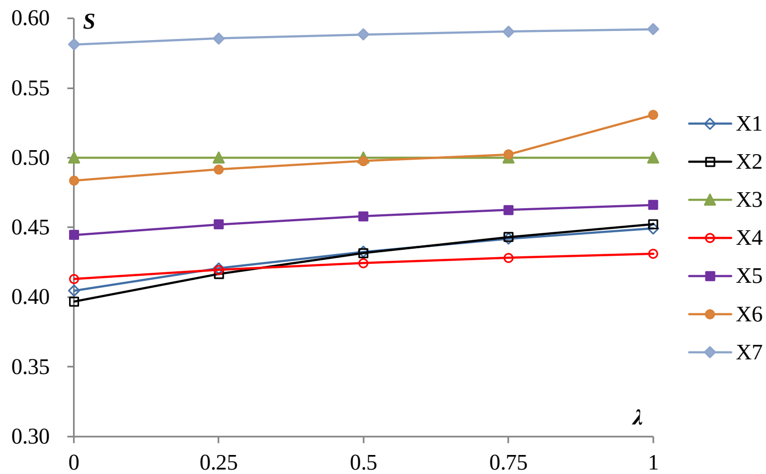

Figure 5: Advantage degrees of methods with different values of λ

Figure 5 shows that the rankings of the seven methods may be different because of the variation of λ. For example, when $\lambda = 0.25$ or 0.75, the rank is $X_7 \succ X_3 \succ X_6 \succ X_5 \succ X_4 \succ X_1 \succ X_2$ and $X_7 \succ X_6 \succ X_3 \succ X_5 \succ X_2 \succ X_1 \succ X_4$, respectively. The different rank results in this example show that the interval number ranking method considering symmetry axis compensation can reflect multiple decision attitudes of decision makers with different risk preferences.

6 Conclusions

By expressing the interval numbers in the RCS, the Symmetry Axis Compensation Factor λ was defined, and their actual meanings were analyzed. Then, the interval number ranking method considering symmetry axis compensation and its application procedures were outlined, and the feasibility and effectiveness of this method were verified using an example. This method can reflect multiple attitudes of decision makers with different risk appetites, making it flexible and adaptable.

The ranking procedures and results of the examples show that the interval number ranking method considering multiple decision attitudes is feasible, simple and effective. In addition, as a practical method, interval numbers can be ranked with different decision attitudes.

Additional studies will be performed in the future. For example, Table 2 and Figure 5 show that if the GIN is determined, the increasing or decreasing trends of the advantage degrees of different interval numbers as λ varies are different.

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Compliance with Ethical Standards

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