A novel fuzzy multi-criteria decision-making methodology based upon the spherical fuzzy sets with a real case study

Abit BALIN

Istanbul University, Department of Transportation and Logistics, Istanbul/Turkey
abitbalin@istanbul.edu.tr

Abstract

The choice of roll stabilization system is critical for many types of ships. For warships where operational activities are fast and the concept of time is very effective, determining the most appropriate of these systems is of particular importance. Some operations, such as the landing of the helicopter on board, are critical for naval ships. Unwanted rolling motion makes this difficult. In addition, the performance of the crew may be insufficient due to the effect of roll movement. Therefore, the determination of the most effective stabilizing device for naval ships was highly related to the rapid reduction of roll motion. With increasing technological studies, it became important which type of stabilizing system is more suitable for which type of naval ship. This study evaluates the relationship between criteria and alternatives and selects the most effective roll stabilizer system for naval ships according to expert opinion. Extension of TOPSIS method with interval-valued spherical fuzzy sets used to list the stabilizing systems alternatives for naval ships. When the obtained results were evaluated, the effect of the criteria on the alternative system types examined, Active Fin found to be the most functional alternative.

Keywords: Navigation Safety, Spherical Fuzzy Sets, Decision Making, TOPSIS.

1 Introduction

In maritime, stabilizing of wave-induced roll motion is an important issue. If the amplitude of this motion increases, all components on board are adversely affected. This is particularly important for naval ships that undertake important tasks. The choice of the most effective stabilizing system is critically important for naval ships, which must be as stable and fast as possible, given the structure and task of the naval ships [17]. In this context, the first passive system, called the bilge keel, was designed, and these system selections were further developed, and active stabilizing system applications were applied for different ship types [23]. The necessity of high maneuverability, especially in various operating conditions of naval ships, and having various balancing devices in the industry examined and evaluated by many researchers as a research subject.

Surendran et al. [20] proposed fin stabilizer system to increase the rolling movements of a frigate type warship in various sea states. Stafford and Osborne [15] evaluated stabilizer performance as a new program for operational type 23 Frigates. Swartz et al. [16] conducted a study examining the structural behavior of the high-speed littoral combat vessel ship in overseas operations. Perez and Blanke [22] investigated the development of various ship roll motion control systems with challenges related to their design and discussed the performance and applicability of these systems. Sutton et al. [18] increased operational efficiency by using fin stabilizers to reduce the rolling movement of a modern warship. Kim et al. [20] examined the roll damping characteristics of bilge keels as a balancing device and demonstrated the results of the study for three types of bilge keels. Zihnioglu et al. [4] examined the parametric model hydraulic system of a ship motion reduction active fin stabilizer system with fin and validated the simulation results with full-scale sea trials using a ship called Volcano71. Demirel et al. [12] evaluated the different stabilizing systems for a trawler type fishing vessel and suggested the most suitable one for this ship.

As mentioned above, roll stabilization systems have been studied as a scientific research subject for many years. It was undesirable for a ship to move unexpectedly, especially during the operation of the naval ships. Recently, with increasing technological studies, it has become important which kind of stabilizing system is more suitable for which ship type. This article reviews all balancing devices and proposes a new methodology for selecting the most suitable roll motion balancing system for naval ships. In addition to this point of view, it is important to evaluate the issue in different positions in the maritime sector and to make a decision with the data obtained interval-valued spherical fuzzy sets.

The rest of this paper has been organized as follows: In section 2 The information of spherical fuzzy sets methodology examined. In section 3 How interval-valued spherical fuzzy sets occurs explained in detail. In section 4 It is shown how extension of TOPSIS method with interval-valued spherical fuzzy sets method built and works. In section 5 perform analysis of all inputs and outputs of the real case application using spherical fuzzy sets. Finally, concluding remarks discussed in Section 6.

2 Spherical Fuzzy sets

Intuitive and Pythagorean fuzzy functions include membership, non-membership and hesitation parameters. These parameters could be calculated with $x_i = 1 - \mu_i - \nu_i$ and $\pi_i = \sqrt{1 - \mu_i - \nu_i}$ respectively. Neutrosophic membership functions defined by three parameters: truthiness, falsity and indeterminacy. These parameters must be between 0 and 3 as the total value, as well as the value of each parameter should be independently between 0 and 1. In spherical fuzzy sets, the squared sum of membership, non-membership and hesitation parameters must be between 0 and also 3 as the total value, as well as the value of each parameter should be independently between 0 and 1. $\tilde{A}_S$ is a spherical fuzzy set of the universe of discourse $U$ given by the equation given below.

$$\tilde{A}_S = \left\{ \left( u, \left( \left[ \mu_{A_S} (u), \mu_{A_S} (u) \right], \left[ v_{A_S} (u), \pi_{A_S} (u) \right], \left[ \nu_{A_S} (u), \nu_{A_S} (u) \right] \right) \right) | u \in U \right\}$$

where $\mu_{A_S} : U \rightarrow [0, 1]$, $\nu_{A_S} (u) : U \rightarrow [0, 1]$, $\pi_{A_S} (u) : U \rightarrow [0, 1]$ and $0 \leq \mu_{A_S} (u) \leq \mu_{A_S} (u) \leq 1, 0 \leq v_{A_S} (u) \leq v_{A_S} (u) \leq 1 \forall u \in U$ for each $u$, the numbers $\mu_{A_S} (u)$, $\nu_{A_S} (u)$ and $\pi_{A_S} (u)$ are the degree of membership, non-membership and hesitancy of $u$ to $A_S$, respectively.

Basic Operators were union, intersection, addition, multiplication, multiplication by a scalar, and power of $\tilde{A}_S$ respectively to be as follows.

$$\tilde{A}_S \cup \tilde{B}_S = \left\{ \max \left\{ \mu_{A_S}, \mu_{B_S} \right\}, \min \left\{ \nu_{A_S}, \nu_{B_S} \right\}, \min \left\{ 1 - \left( \left( \max \left\{ \mu_{A_S}, \mu_{B_S} \right\} \right) + \left( \min \left\{ \nu_{A_S}, \nu_{B_S} \right\} \right) \right) \right\}, \max \left\{ \pi_{A_S}, \pi_{B_S} \right\} \right\}$$

(2)

$$\tilde{A}_S \cap \tilde{B}_S = \left\{ \min \left\{ \mu_{A_S}, \mu_{B_S} \right\}, \max \left\{ \nu_{A_S}, \nu_{B_S} \right\}, \min \left\{ 1 - \left( \left( \min \left\{ \mu_{A_S}, \mu_{B_S} \right\} \right) + \left( \max \left\{ \nu_{A_S}, \nu_{B_S} \right\} \right) \right) \right\}, \min \left\{ \pi_{A_S}, \pi_{B_S} \right\} \right\}$$

(3)

$$\tilde{A}_S \oplus \tilde{B}_S = \left\{ \mu_{A_S}^2 + \mu_{B_S}^2 - \mu_{A_S}^2 \mu_{B_S}^2, \nu_{A_S}, \nu_{B_S}, \left( 1 - \mu_{B_S}^2 \right) \pi_{A_S}^2 + \left( 1 - \mu_{A_S}^2 \right) \pi_{B_S}^2 - \pi_{A_S}^2 \pi_{B_S}^2 \right\}^{1/2}$$

(4)
There are degrees of membership, non-membership and hesitation for each of \( \mu \) Fuzzy Set \( \tilde{a} \) on Interval-valued spherical fuzzy sets. Let \( \bar{\langle a,b \rangle} = \frac{a+b}{2} \) and consider some operations defined \( \{\bar{\langle a,b \rangle}, \bar{\langle c,d \rangle}, \bar{\langle e,f \rangle}\} \) where \( [a,b] \subset [0,1], [c,d] \subset [0,1], [e,f] \subset [0,1] \) and \( b^2 + d^2 + f^2 \leq 1 \).

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\[
\tilde{A}_S \otimes \tilde{B}_S = \left\{ \left( \mu_{\tilde{A}_S,\mu_{\tilde{B}_S}}, \left( \nu_{\tilde{A}_S}^2 + \nu_{\tilde{B}_S}^2 - \nu_{\tilde{A}_S}^2 \nu_{\tilde{B}_S}^2 \right)^{1/2}, -\nu_{\tilde{A}_S} \mu_{\tilde{B}_S} \right)^{1/2}, \left( 1 - \nu_{\tilde{B}_S}^2 \right) \pi_{\tilde{A}_S}^2 + \left( 1 - \nu_{\tilde{A}_S}^2 \right) \pi_{\tilde{B}_S}^2 - \pi_{\tilde{A}_S}^2 \pi_{\tilde{B}_S}^2 \right\}^{1/2}
\]

\[
\lambda \tilde{A}_S = \left\{ \left( 1 - \left( 1 - \mu_{\tilde{A}_S}^2 \right)^{1/\lambda}, \nu_{\tilde{A}_S}, \left( 1 - \mu_{\tilde{A}_S}^2 - \pi_{\tilde{A}_S}^2 \right)^{1/\lambda} \right\}^{1/2}, \left( 1 - \nu_{\tilde{A}_S}^2 \right) \left( 1 - \mu_{\tilde{A}_S}^2 - \pi_{\tilde{A}_S}^2 \right)^{1/\lambda} \right\}^{1/2}
\]

\[
\tilde{A}_S^\lambda = \left\{ \mu_{\tilde{A}_S}^\lambda, \left( 1 - \nu_{\tilde{A}_S}^2 \right)^{1/\lambda}, \left( 1 - \nu_{\tilde{A}_S}^2 - \nu_{\tilde{A}_S}^2 \right)^{1/\lambda} \right\}^{1/2}
\]

Spherical Weighted Arithmetic Mean (SWAM) and Spherical Weighted Geometric Mean (SWGM) according to, \( w = (w_1, w_2, \ldots, w_n); w_i \in [0,1]; \sum_{i=1}^n w_i = 1 \), SWAM and SWGM were defined as;

\[
SWAM_w (\tilde{A}_{S1}, \ldots, \tilde{A}_{Sn}) = w_1 \tilde{A}_{S1} + w_2 \tilde{A}_{S2} + \ldots + w_n \tilde{A}_{Sn} = \left\{ \left[ 1 - \prod_{i=1}^n \left( 1 - \mu_{\tilde{A}_{Si}}^2 \right)^{w_i} \right]^{1/2}, \left[ \prod_{i=1}^n \nu_{\tilde{A}_{Si}}^{w_i} \right] - \left[ \prod_{i=1}^n \left( 1 - \mu_{\tilde{A}_{Si}}^2 \right)^{w_i} \right] \right\}^{1/2}
\]

\[
SWGM_w (\tilde{A}_{S1}, \ldots, \tilde{A}_{Sn}) = \tilde{A}_{S1}^{w_1} + \tilde{A}_{S2}^{w_2} + \ldots + \tilde{A}_{Sn}^{w_n} = \left\{ \left[ \prod_{i=1}^n \mu_{\tilde{A}_{Si}}^{w_i} \right] - \left[ \prod_{i=1}^n \left( 1 - \mu_{\tilde{A}_{Si}}^2 \right)^{w_i} \right] \right\}^{1/2}
\]

Score function of sorting SFS were defined by;

\[
Score (\tilde{A}_S) = (\mu_{\tilde{A}_S} - \pi_{\tilde{A}_S})^2 - (v_{\tilde{A}_S} - \pi_{\tilde{A}_S})^2
\]

Accuracy function of sorting SFS were defined by;

\[
Accuracy (\tilde{A}_S) = \mu_{\tilde{A}_S}^2 + v_{\tilde{A}_S}^2 + \pi_{\tilde{A}_S}^2
\]

Note that: If it is \( \tilde{A}_S < \tilde{B}_S \) then it must be \( Score (\tilde{A}_S) < Score (\tilde{B}_S) \) or if \( Score (\tilde{A}_S) \) and \( Score (\tilde{B}_S) \) also have equality, \( Accuracy (\tilde{A}_S) < Accuracy (\tilde{B}_S) \) must be provided.

3 Interval-valued spherical fuzzy sets

This section gives the definition of interval valued spherical fuzzy sets (IVSFS) and informs distance measurement, arithmetic operations, aggregation and defuzzification operations [27, 9, 24, 25, 19]. An Interval-Valued Spherical Fuzzy Set \( \tilde{A}_S \) of the \( U \) discourse universe defined;

\[
\tilde{A}_S = \left\{ \left[ \left( \left[ u_{\tilde{A}_S}^L (u), v_{\tilde{A}_S}^L (u) \right], \left[ u_{\tilde{A}_S}^U (u), v_{\tilde{A}_S}^U (u) \right], \left[ \pi_{\tilde{A}_S}^L (u), \pi_{\tilde{A}_S}^U (u) \right] \right) \right] u \in U \}
\]

where \( 0 \leq \mu_{\tilde{A}_S}^L (u) \leq \mu_{\tilde{A}_S}^U (u) \leq 1, 0 \leq v_{\tilde{A}_S}^L (u) \leq v_{\tilde{A}_S}^U (u) \leq 1 \) and \( 0 \leq (\mu_{\tilde{A}_S}^U (u))^2 + (v_{\tilde{A}_S}^U (u))^2 + (\pi_{\tilde{A}_S}^U (u))^2 \leq 1 \).

There are degrees of membership, non-membership and hesitation for each of \( u \) to \( \tilde{A}_S \). For each \( u \in U \), if \( \mu_{\tilde{A}_S}^L (u) = \nu_{\tilde{A}_S}^L (u) = \nu_{\tilde{A}_S}^U (u) = \pi_{\tilde{A}_S}^L (u) \) then, IVSFS \( \tilde{A}_S \) reduced to a single valued SFS.

For an IVSFS \( \tilde{A}_S \), the pair \( \left( \mu_{\tilde{A}_S}^L (u), \mu_{\tilde{A}_S}^U (u) \right), \left[ \nu_{\tilde{A}_S}^L (u), \nu_{\tilde{A}_S}^U (u) \right], \left[ \pi_{\tilde{A}_S}^L (u), \pi_{\tilde{A}_S}^U (u) \right] \) called an interval-valued spherical fuzzy number. For convenience, the pair \( \left( \mu_{\tilde{A}_S}^L (u), \mu_{\tilde{A}_S}^U (u) \right), \left[ \nu_{\tilde{A}_S}^L (u), \nu_{\tilde{A}_S}^U (u) \right], \left[ \pi_{\tilde{A}_S}^L (u), \pi_{\tilde{A}_S}^U (u) \right] \) indicated by \( \tilde{a} = \langle [a,b], [c,d], [e,f] \rangle \) where \( [a,b] \subset [0,1], [c,d] \subset [0,1], [e,f] \subset [0,1] \) and \( b^2 + d^2 + f^2 \leq 1 \).

It is clear that \( \tilde{a}^* = \langle [1,1], [0,0], [0,0] \rangle \) was the largest IVSFS, \( \tilde{a}^- = \langle [0,0], [1,1], [0,0] \rangle \) was the smallest IVSFS, and \( \tilde{a}^{1/2} = \langle [0,0], [0,0], [1,1] \rangle \) was the value between the largest and smallest IVSFS. Considering some operations defined on Interval-valued spherical fuzzy sets. Let \( \tilde{a} = \langle [a,b], [c,d], [e,f] \rangle \) be a collection of Interval-valued Spherical Weighted
Arithmetic Mean (IVSWAM) and Spherical Geometric Mean (IVSWGM) according to, \( w \) tancy degrees assignment. So, collect DM assessments for weight criteria. Allow DMs to fill decision matrices us-

\[
IVSWAM_{\alpha}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \frac{1}{n} \sum_{i=1}^{n} \tilde{a}_i \oplus w_1 \tilde{a}_1 \oplus w_2 \tilde{a}_2 \oplus ....... \oplus w_n \tilde{a}_n
\]

\[
IVSWGM_{\alpha}(\tilde{a}_1, \tilde{a}_2, ..., \tilde{a}_n) = \hat{\alpha}_{1}^{\alpha} \hat{\alpha}_{2}^{\alpha} \oplus ....... \oplus \hat{\alpha}_{n}^{\alpha}
\]

\[
\left( \prod_{j=1}^{m} \left( 1 - c_j \right)^{\alpha} \right)^{1/2} \prod_{j=1}^{m} \left( 1 - d_j \right)^{\alpha} \]

\[
\left( \prod_{j=1}^{m} \left( 1 - c_j \right)^{\alpha} - \prod_{j=1}^{m} \left( 1 - d_j \right)^{\alpha} \right)^{1/2}
\]

\[
\left( \prod_{j=1}^{m} \left( 1 - d_j \right)^{\alpha} - \prod_{j=1}^{m} \left( 1 - b_j \right)^{\alpha} \right)^{1/2}
\]

\[
\left( \prod_{j=1}^{m} \left( 1 - b_j \right)^{\alpha} - \prod_{j=1}^{m} \left( 1 - a_j \right)^{\alpha} \right)^{1/2}
\]

The score function of IVSFS number \( \alpha \) determined as

\[
Score(\tilde{\alpha}) = S(\tilde{\alpha}) = \frac{a^2 + b^2 - c^2 - d^2 - (e/2)^2 - (f/2)^2}{2}
\]

where \( Score(\tilde{\alpha}) = S(\tilde{\alpha}) \in [-1, +1] \). Obviously, the larger the \( S(\tilde{\alpha}) \), the more preferable \( \alpha \) is. Specifically, \( S(\tilde{\alpha}) = 1 \) is then the largest in \( \tilde{\alpha} = \{(1,1), [0,0], [0,0]\} \); when \( \tilde{\alpha} = \{(0,0), [1,1], [0,0]\} \) \( \alpha \) is the smallest IVSFS number.

IVSFS number \( \alpha \)'s accuracy function determined as follows;

\[
Accuracy(\tilde{\alpha}) = H(\tilde{\alpha}) = \frac{a^2 + b^2 + c^2 + d^2 + (e/2)^2 + (f/2)^2}{2}
\]

where \( H(\tilde{\alpha}) \in [0,1] \) and where \( \tilde{\alpha}_1 < \tilde{\alpha}_2 \) must be either \( S(\tilde{\alpha}_1) < S(\tilde{\alpha}_2) \) or the two conditions \( S(\tilde{\alpha}_1) = S(\tilde{\alpha}_2) \) and \( H(\tilde{\alpha}_1) < H(\tilde{\alpha}_2) \) must be provided.

Step 1: The scale in Table 1 used for data input. If DMs do not prefer to use the given language terms, intermediate values could be used. This provides a large global volume for membership, non-membership, and hesitancy degrees assignment. So, collect DM assessments for weight criteria. Allow DMs to fill decision matrices using linguistic terms and allow DMs to fill decision matrices using linguistic terms. Creating a decision matrix: let \( C_j(X_i) = \left( [\mu_{ij}^L(u), \mu_{ij}^U(u)] , [v_{ij}^L(u), v_{ij}^U(u)] , [\pi_{ij}^L(u), \pi_{ij}^U(u)] \right) \) show the evaluation of each alternative \( X_i(i = 1, 2, ...., m) \)

### Table 1: Linguistic terms and their corresponding interval-valued spherical fuzzy numbers [9]

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>( \left( [\mu_{ij}^L(u), \mu_{ij}^U(u)] , [v_{ij}^L(u), v_{ij}^U(u)] , [\pi_{ij}^L(u), \pi_{ij}^U(u)] \right) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Absolutely more Importance (AMI)</td>
<td>(0.85,0.95,0.1,0.15,0.05,0.15)</td>
</tr>
<tr>
<td>Very High Importance (VHI)</td>
<td>(0.75,0.85,0.15,0.2,0.15,0.2)</td>
</tr>
<tr>
<td>High Importance (HI)</td>
<td>(0.65,0.75,0.2,0.25,0.2,0.25)</td>
</tr>
<tr>
<td>Slightly More Importance (SMI)</td>
<td>(0.55,0.65,0.25,0.3,0.25,0.3)</td>
</tr>
<tr>
<td>Equally Importance (EI)</td>
<td>(0.5,0.55,0.45,0.55,0.3,0.4)</td>
</tr>
<tr>
<td>Slightly Low Importance (SLI)</td>
<td>(0.25,0.35,0.55,0.65,0.25,0.3)</td>
</tr>
<tr>
<td>Low Importance (LI)</td>
<td>(0.2,0.25,0.65,0.75,0.2,0.25)</td>
</tr>
<tr>
<td>Very Low Importance (VLI)</td>
<td>(0.15,0.2,0.75,0.85,0.15,0.2)</td>
</tr>
<tr>
<td>Absolutely Low Importance (ALI)</td>
<td>(0.1,0.15,0.85,0.95,0.05,0.15)</td>
</tr>
</tbody>
</table>
with respect to each criterion $C_j (j = 1, 2, \ldots, n)$. Let $D = (C_j (X_i))_{m \times n}$ be a spherical fuzzy decision matrix. Then the decision matrix $D = (C_j (X_i))_{m \times n}$ created as in the following equation.

$$D = (C_j (X_i))_{m \times n} = \left[ \begin{array}{cccc} \left[ \left[ \mu_{L1}^{C_j}(u), \mu_{R1}^{C_j}(u) \right], \left[ v_{L1}^{C_j}(u), v_{R1}^{C_j}(u) \right], \left[ \pi_{L1}^{C_j}(u), \pi_{R1}^{C_j}(u) \right] \right] & \cdots & \left[ \left[ \mu_{L1}^{C_j}(u), \mu_{R1}^{C_j}(u) \right], \left[ v_{L1}^{C_j}(u), v_{R1}^{C_j}(u) \right], \left[ \pi_{L1}^{C_j}(u), \pi_{R1}^{C_j}(u) \right] \right] \\ \left[ \left[ \mu_{L2}^{C_j}(u), \mu_{R2}^{C_j}(u) \right], \left[ v_{L2}^{C_j}(u), v_{R2}^{C_j}(u) \right], \left[ \pi_{L2}^{C_j}(u), \pi_{R2}^{C_j}(u) \right] \right] & \cdots & \left[ \left[ \mu_{L2}^{C_j}(u), \mu_{R2}^{C_j}(u) \right], \left[ v_{L2}^{C_j}(u), v_{R2}^{C_j}(u) \right], \left[ \pi_{L2}^{C_j}(u), \pi_{R2}^{C_j}(u) \right] \right] \\ \vdots & \cdots & \vdots \\ \left[ \left[ \mu_{Lm}^{C_j}(u), \mu_{Rm}^{C_j}(u) \right], \left[ v_{Lm}^{C_j}(u), v_{Rm}^{C_j}(u) \right], \left[ \pi_{Lm}^{C_j}(u), \pi_{Rm}^{C_j}(u) \right] \right] & \cdots & \left[ \left[ \mu_{Lm}^{C_j}(u), \mu_{Rm}^{C_j}(u) \right], \left[ v_{Lm}^{C_j}(u), v_{Rm}^{C_j}(u) \right], \left[ \pi_{Lm}^{C_j}(u), \pi_{Rm}^{C_j}(u) \right] \right] \end{array} \right]$$

(18)

Step 2: The Interval Valued Spherical Weighted Arithmetic Mean (IVSWAM) operator used to generate the decision matrix. Aggregate the assessments on criteria weights and interval-valued spherical fuzzy decision matrices. Step 3: Create the weighted interval-valued spherical fuzzy decision matrix.

After determining the weights of the criteria and the scores of the alternatives, the weighted interval-valued spherical fuzzy decision matrix formed using the $D = (C_j (X_i))_{m \times n}$ equation;

$$D = (C_j (X_i))_{m \times n} = \left[ \begin{array}{cccc} \left[ \left[ \mu_{L1}^{C_j}(u), \mu_{R1}^{C_j}(u) \right], \left[ v_{L1}^{C_j}(u), v_{R1}^{C_j}(u) \right], \left[ \pi_{L1}^{C_j}(u), \pi_{R1}^{C_j}(u) \right] \right] & \cdots & \left[ \left[ \mu_{L1}^{C_j}(u), \mu_{R1}^{C_j}(u) \right], \left[ v_{L1}^{C_j}(u), v_{R1}^{C_j}(u) \right], \left[ \pi_{L1}^{C_j}(u), \pi_{R1}^{C_j}(u) \right] \right] \\ \left[ \left[ \mu_{L2}^{C_j}(u), \mu_{R2}^{C_j}(u) \right], \left[ v_{L2}^{C_j}(u), v_{R2}^{C_j}(u) \right], \left[ \pi_{L2}^{C_j}(u), \pi_{R2}^{C_j}(u) \right] \right] & \cdots & \left[ \left[ \mu_{L2}^{C_j}(u), \mu_{R2}^{C_j}(u) \right], \left[ v_{L2}^{C_j}(u), v_{R2}^{C_j}(u) \right], \left[ \pi_{L2}^{C_j}(u), \pi_{R2}^{C_j}(u) \right] \right] \\ \vdots & \cdots & \vdots \\ \left[ \left[ \mu_{Lm}^{C_j}(u), \mu_{Rm}^{C_j}(u) \right], \left[ v_{Lm}^{C_j}(u), v_{Rm}^{C_j}(u) \right], \left[ \pi_{Lm}^{C_j}(u), \pi_{Rm}^{C_j}(u) \right] \right] & \cdots & \left[ \left[ \mu_{Lm}^{C_j}(u), \mu_{Rm}^{C_j}(u) \right], \left[ v_{Lm}^{C_j}(u), v_{Rm}^{C_j}(u) \right], \left[ \pi_{Lm}^{C_j}(u), \pi_{Rm}^{C_j}(u) \right] \right] \end{array} \right]$$

(19)

Step 4: Defuzzify the weighted interval-valued spherical fuzzy decision matrix formed using the $S (C_j (X_{iw}))$ equation;

$$S (C_j (X_{iw})) = \frac{(\mu_{Lw}^{C_j}(u))^2 + (\mu_{Rw}^{C_j}(u))^2 - (v_{Lw}^{C_j}(u))^2 - (v_{Rw}^{C_j}(u))^2 - (\pi_{Lw}^{C_j}(u))^2 - (\pi_{Rw}^{C_j}(u))^2}{2}$$

(20)

Step 5: The Interval-valued Spherical Fuzzy Positive Ideal Solution (IVSF-PIS) and the Interval-valued Spherical Fuzzy Negative Ideal Solution (IVSF-NIS) determined based on the values obtained in step 4. The IVSF-PIS given by $X^*$:

$$X^* = \left\{ \left( C_j, \max_i < S(C_j (X_{iw})) \right) > j = 1, 2, \ldots, n \right\}$$

(21)

or

$$X^* = \{(C_1, [(\mu_{L1}^{C_j}, \mu_{R1}^{C_j}), (v_{L1}^{C_j}, v_{R1}^{C_j}), (\pi_{L1}^{C_j}, \pi_{R1}^{C_j})]), (C_2, [(\mu_{L2}^{C_j}, \mu_{R2}^{C_j}), (v_{L2}^{C_j}, v_{R2}^{C_j}), (\pi_{L2}^{C_j}, \pi_{R2}^{C_j})]), \ldots, (C_n, [(\mu_{Ln}^{C_j}, \mu_{Ln}^{C_j}), (v_{Ln}^{C_j}, v_{Ln}^{C_j}), (\pi_{Ln}^{C_j}, \pi_{Ln}^{C_j})])\}$$

(22)

The IVSF-NIS given by $X^-$:

$$X^- = \left\{ \left( C_j, \min_i < S(C_j (X_{iw})) \right) > j = 1, 2, \ldots, n \right\}$$

(23)

or

$$X^- = \{(C_1, [(\mu_{L1}^{C_j}, \mu_{R1}^{C_j}), (v_{L1}^{C_j}, v_{R1}^{C_j}), (\pi_{L1}^{C_j}, \pi_{R1}^{C_j})]), (C_2, [(\mu_{L2}^{C_j}, \mu_{R2}^{C_j}), (v_{L2}^{C_j}, v_{R2}^{C_j}), (\pi_{L2}^{C_j}, \pi_{R2}^{C_j})]), \ldots, (C_n, [(\mu_{Ln}^{C_j}, \mu_{Ln}^{C_j}), (v_{Ln}^{C_j}, v_{Ln}^{C_j}), (\pi_{Ln}^{C_j}, \pi_{Ln}^{C_j})])\}$$

(24)

Step 6: To calculate the distance between alternative $X_i$, IVSF-PIS and IVSF-NIS used, respectively. The normalized distance formula of Peng and Yang [21] used in this step. Distance to IVSF-PIS:

$$d(X_{ij}, X_j^+) = \frac{1}{n} \sum_{j=1}^{n} \left[ (u_{Lj}^{C_j})^2 - (u_{Lj})^2 \right] + \left[ (u_{Rj}^{C_j})^2 - (u_{Rj})^2 \right] + \left[ (v_{Lj}^{C_j})^2 - (v_{Lj})^2 \right] + \left[ (v_{Rj}^{C_j})^2 - (v_{Rj})^2 \right] + \left[ (\pi_{Lj}^{C_j})^2 - (\pi_{Lj})^2 \right] + \left[ (\pi_{Rj}^{C_j})^2 - (\pi_{Rj})^2 \right]$$

(25)

Distance to IVSF-NIS:

$$d(X_{ij}, X_j^-) = \frac{1}{n} \sum_{j=1}^{n} \left[ (u_{Lj}^{C_j})^2 - (u_{Lj})^2 \right] + \left[ (u_{Rj}^{C_j})^2 - (u_{Rj})^2 \right] + \left[ (v_{Lj}^{C_j})^2 - (v_{Lj})^2 \right] + \left[ (v_{Rj}^{C_j})^2 - (v_{Rj})^2 \right] + \left[ (\pi_{Lj}^{C_j})^2 - (\pi_{Lj})^2 \right] + \left[ (\pi_{Rj}^{C_j})^2 - (\pi_{Rj})^2 \right]$$

(26)

Step 7: Calculating the closeness ratio:

$$\text{Closeness Ratio}_{ij} = \frac{d(X_{ij}, X_j^+)}{d(X_{ij}, X_j^-) + d(X_{ij}, X_j^+)}$$

(27)
A real case application using extension of TOPSIS method with interval-valued spherical fuzzy sets

Proposed methodology is applied to the appropriate stabilization system selection problem for naval ship. The decision criteria for appropriate stabilization system selection may vary depending on the number of qualitative and quantitative elements. These criteria are directly effective in the performance of the operations shown in all operational zones and points where naval ships are employed. The technical knowledge of the selection of roll motion stabilization system as well as the expert opinions with high experience are highly effective. Twelve criteria were obtained after literature review and interviews with highly experienced experts.

Three decision makers (DM1, DM2 and DM3), engineers and academics experienced in stabilization systems, were included in the assessment process. The weights of these decision makers with different experience levels were determined as 0.3, 0.2 and 0.5, respectively. The specified criteria were presented in Table 2 with explanations. For this goal, four types of roll motion stabilization system (A1 Gyroscopic Roll Stabilizer, A2 Activated Fins, A3 Rudder Roll Stabilization and A4 Active Anti-roll tanks) were assessed. Figure 2 shows the relationship between criteria and alternatives according to the TOPSIS method.
A novel fuzzy multi-criteria decision-making methodology based upon the spherical fuzzy sets with a real case study

Figure 2: Hierarchical structure for selection of roll stabilizer systems.

Table 2: Definitions of defined criteria for roll stabilizer selection problem [8].

<table>
<thead>
<tr>
<th>No</th>
<th>Criteria</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>Initial cost</td>
<td>The total investment cost of all system equipments</td>
</tr>
<tr>
<td>C2</td>
<td>Cargo carrying Capacity</td>
<td>Whether or not carrying capacity is reduced</td>
</tr>
<tr>
<td>C3</td>
<td>Crew Performance</td>
<td>The flexibility in the mobility of crew</td>
</tr>
<tr>
<td>C4</td>
<td>Influence on Speed, Power and Resistance</td>
<td>The status on the performance of the ship</td>
</tr>
<tr>
<td>C5</td>
<td>Maintenance Requirements</td>
<td>Easy and cheap service and spare parts availability</td>
</tr>
<tr>
<td>C6</td>
<td>Roll Reduction</td>
<td>The effect of roll amplitudes</td>
</tr>
<tr>
<td>C7</td>
<td>Underwater Noise</td>
<td>Noise effect of stabilization systems</td>
</tr>
<tr>
<td>C8</td>
<td>Expensive Pieces of Equipments</td>
<td>Economic value of stabilizer parts</td>
</tr>
<tr>
<td>C9</td>
<td>Working on Low Speed Range</td>
<td>Low speed performance of stabilization system</td>
</tr>
<tr>
<td>C10</td>
<td>Working on High Speed Range</td>
<td>High speed performance of stabilization system</td>
</tr>
<tr>
<td>C11</td>
<td>Motion Limitations</td>
<td>Impact on the maneuvering ship</td>
</tr>
<tr>
<td>C12</td>
<td>Wave Conditions</td>
<td>Performance of stabilization system in various sea states</td>
</tr>
</tbody>
</table>

All evaluations given in Table 2 in the form of a decision matrix, which collected and combined using the IVSWGM operator, taking into consideration the weight of the decision-makers.
Table 3: Aggregated decision matrix.

<table>
<thead>
<tr>
<th></th>
<th>A1</th>
<th>A2</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>(0.62, 0.72)</td>
<td>(0.18, 0.25)</td>
</tr>
<tr>
<td>C2</td>
<td>(0.21, 0.28)</td>
<td>(0.83, 0.93)</td>
</tr>
<tr>
<td>C3</td>
<td>(0.39, 0.5)</td>
<td>(0.18, 0.23)</td>
</tr>
<tr>
<td>C4</td>
<td>(0.52, 0.6)</td>
<td>(0.22, 0.29)</td>
</tr>
<tr>
<td>C5</td>
<td>(0.37, 0.47)</td>
<td>(0.15, 0.2)</td>
</tr>
<tr>
<td>C6</td>
<td>(0.53, 0.61)</td>
<td>(0.22, 0.29)</td>
</tr>
<tr>
<td>C7</td>
<td>(0.24, 0.31)</td>
<td>(0.10, 0.15)</td>
</tr>
<tr>
<td>C8</td>
<td>(0.65, 0.76)</td>
<td>(0.13, 0.18)</td>
</tr>
<tr>
<td>C9</td>
<td>(0.11, 0.16)</td>
<td>(0.38, 0.49)</td>
</tr>
<tr>
<td>C10</td>
<td>(0.60, 0.7)</td>
<td>(0.35, 0.4)</td>
</tr>
<tr>
<td>C11</td>
<td>(0.32, 0.41)</td>
<td>(0.13, 0.19)</td>
</tr>
<tr>
<td>C12</td>
<td>(0.38, 0.47)</td>
<td>(0.36, 0.44)</td>
</tr>
</tbody>
</table>

The weight of each criteria obtained using the IVSWAM operator, expressing the importance of the criteria determined by the DMs, is given in Table 4.

Table 4: Aggregated criteria weight.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>(0.81, 0.91)</td>
</tr>
<tr>
<td>C2</td>
<td>(0.30, 0.35)</td>
</tr>
<tr>
<td>C3</td>
<td>(0.78, 0.88)</td>
</tr>
<tr>
<td>C4</td>
<td>(0.72, 0.82)</td>
</tr>
<tr>
<td>C5</td>
<td>(0.85, 0.95)</td>
</tr>
<tr>
<td>C6</td>
<td>(0.55, 0.63)</td>
</tr>
<tr>
<td>C7</td>
<td>(0.79, 0.89)</td>
</tr>
<tr>
<td>C8</td>
<td>(0.12, 0.17)</td>
</tr>
<tr>
<td>C9</td>
<td>(0.74, 0.84)</td>
</tr>
<tr>
<td>C10</td>
<td>(0.81, 0.91)</td>
</tr>
<tr>
<td>C11</td>
<td>(0.72, 0.82)</td>
</tr>
</tbody>
</table>

After determining the weights of the criteria and the rating of the alternatives, the spherical fuzzy decision matrix with the weighted range-value given in Table 5 was formed.
A novel fuzzy multi-criteria decision-making methodology based upon the spherical fuzzy sets with a real case study

Table 5: Weighted decision matrix.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>A1</th>
<th>A2</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>(0.08,0.12,0.82,0.92,0.16,0.2)</td>
<td>(0.02,0.04,0.9,0.97,0.13,0.14)</td>
</tr>
<tr>
<td>C2</td>
<td>(0.04,0.06,0.84,0.92,0.19,0.2)</td>
<td>(0.14,0.2,0.73,0.83,0.17,0.23)</td>
</tr>
<tr>
<td>C3</td>
<td>(0.23,0.32,0.54,0.65,0.32,0.39)</td>
<td>(0.1,0.15,0.74,0.84,0.25,0.28)</td>
</tr>
<tr>
<td>C4</td>
<td>(0.31,0.42,0.42,0.51,0.34,0.42)</td>
<td>(0.13,0.2,0.64,0.75,0.27,0.31)</td>
</tr>
<tr>
<td>C5</td>
<td>(0.08,0.15,0.7,0.8,0.27,0.3)</td>
<td>(0.03,0.06,0.85,0.93,0.19,0.2)</td>
</tr>
<tr>
<td>C6</td>
<td>(0.45,0.58,0.38,0.47,0.28,0.38)</td>
<td>(0.19,0.27,0.62,0.72,0.22,0.28)</td>
</tr>
<tr>
<td>C7</td>
<td>(0.17,0.26,0.6,0.71,0.26,0.31)</td>
<td>(0.07,0.12,0.85,0.95,0.1,0.16)</td>
</tr>
<tr>
<td>C8</td>
<td>(0.08,0.13,0.81,0.91,0.16,0.2)</td>
<td>(0.02,0.03,0.93,0.98,0.09,0.1)</td>
</tr>
<tr>
<td>C9</td>
<td>(0.01,0.02,0.96,0.99,0.05,0.07)</td>
<td>(0.04,0.07,0.88,0.96,0.14,0.15)</td>
</tr>
<tr>
<td>C10</td>
<td>(0.48,0.63,0.26,0.32,0.25,0.32)</td>
<td>(0.28,0.36,0.56,0.66,0.27,0.35)</td>
</tr>
<tr>
<td>C11</td>
<td>(0.19,0.28,0.56,0.66,0.32,0.38)</td>
<td>(0.08,0.13,0.8,0.9,0.18,0.21)</td>
</tr>
<tr>
<td>C12</td>
<td>(0.28,0.39,0.5,0.6,0.3,0.38)</td>
<td>(0.27,0.37,0.51,0.61,0.27,0.33)</td>
</tr>
</tbody>
</table>

Score function values according to Table 5 are obtained as in Table 6. The highest values represent PIS and the lowest values represent NIS.

Table 6: Score function values.

<table>
<thead>
<tr>
<th>Criteria</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>0.80</td>
<td>0.90</td>
<td>0.79</td>
<td>0.82</td>
</tr>
<tr>
<td>C2</td>
<td>0.82</td>
<td>0.68</td>
<td>0.82</td>
<td>0.91</td>
</tr>
<tr>
<td>C3</td>
<td>0.56</td>
<td>0.71</td>
<td>0.55</td>
<td>0.51</td>
</tr>
<tr>
<td>C4</td>
<td>0.50</td>
<td>0.60</td>
<td>0.45</td>
<td>0.43</td>
</tr>
<tr>
<td>C5</td>
<td>0.66</td>
<td>0.83</td>
<td>0.57</td>
<td>0.58</td>
</tr>
<tr>
<td>C6</td>
<td>0.56</td>
<td>0.57</td>
<td>0.72</td>
<td>0.51</td>
</tr>
<tr>
<td>C7</td>
<td>0.56</td>
<td>0.85</td>
<td>0.51</td>
<td>0.52</td>
</tr>
<tr>
<td>C8</td>
<td>0.79</td>
<td>0.93</td>
<td>0.78</td>
<td>0.79</td>
</tr>
<tr>
<td>C9</td>
<td>0.96</td>
<td>0.88</td>
<td>0.95</td>
<td>0.85</td>
</tr>
<tr>
<td>C10</td>
<td>0.48</td>
<td>0.58</td>
<td>0.57</td>
<td>0.52</td>
</tr>
<tr>
<td>C11</td>
<td>0.56</td>
<td>0.78</td>
<td>0.48</td>
<td>0.54</td>
</tr>
<tr>
<td>C12</td>
<td>0.54</td>
<td>0.50</td>
<td>0.52</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Table 7 shows the Interval-valued Spherical Fuzzy Positive Ideal Solution and Interval-valued Spherical Fuzzy Negative Ideal Solution corresponding to the best and worst scores obtained in Table 6.
Table 7: IVSF-PIS and IVSF-NIS.

<table>
<thead>
<tr>
<th></th>
<th>IVSF-PIS</th>
<th>IVSF-NIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>C1</td>
<td>([0.02,0.04], [0.9,0.97], [0.13,0.14])</td>
<td>([0.1,0.16], [0.81,0.92], [0.12,0.18])</td>
</tr>
<tr>
<td>C2</td>
<td>([0.10,0.15], [0.74,0.84], [0.25,0.28])</td>
<td>([0.31,0.42], [0.44,0.54], [0.32,0.4])</td>
</tr>
<tr>
<td>C3</td>
<td>([0.13,0.2], [0.64,0.75], [0.27,0.31])</td>
<td>([0.37,0.51], [0.31,0.38], [0.3,0.36])</td>
</tr>
<tr>
<td>C4</td>
<td>([0.03,0.06], [0.85,0.93], [0.19,0.2])</td>
<td>([0.16,0.25], [0.61,0.71], [0.26,0.31])</td>
</tr>
<tr>
<td>C5</td>
<td>([0.72,0.9], [0.14,0.21], [0.07,0.21])</td>
<td>([0.52,0.68], [0.24,0.3], [0.22,0.3])</td>
</tr>
<tr>
<td>C6</td>
<td>([0.07,0.12], [0.85,0.95], [0.1,0.16])</td>
<td>([0.4,0.52], [0.38,0.46], [0.3,0.39])</td>
</tr>
<tr>
<td>C7</td>
<td>([0.02,0.03], [0.96,0.99], [0.09,0.1])</td>
<td>([0.1,0.16], [0.86,0.95], [0.13,0.19])</td>
</tr>
<tr>
<td>C8</td>
<td>([0.01,0.02], [0.9,0.99], [0.05,0.07])</td>
<td>([0.06,0.11], [0.86,0.95], [0.13,0.16])</td>
</tr>
<tr>
<td>C9</td>
<td>([0.28,0.36], [0.56,0.66], [0.27,0.35])</td>
<td>([0.48,0.63], [0.26,0.32], [0.25,0.32])</td>
</tr>
<tr>
<td>C10</td>
<td>([0.08,0.13], [0.8,0.9], [0.18,0.21])</td>
<td>([0.48,0.63], [0.26,0.32], [0.25,0.32])</td>
</tr>
<tr>
<td>C11</td>
<td>([0.28,0.39], [0.5,0.6], [0.3,0.38])</td>
<td>([0.36,0.5], [0.41,0.5], [0.25,0.31])</td>
</tr>
</tbody>
</table>

As the next step of the methodology, the values giving the distances between alternative $X_i$ and the IVSF-PIS as well as IVSF-NIS and the closeness ratios values were calculated and ranked in Table 8 and Table 9, respectively. Table 8 and Table 9 demonstrate that Activated Fins chosen as the most appropriate alternatives with the 0.771 CR value as the common opinion of all subject-matter-experts, where Active Anti-roll tanks determined as the last option with the 0.187 CR value. Gyroscopic Roll Stabilizer and Rudder Roll Stabilization ranked as the second and third alternative with 0.398 and 0.223 CR value, respectively.

Table 8: Distances to interval-valued positive and negative ideal solutions and closeness ratio of each alternative.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>PIS</td>
<td>0.154</td>
<td>0.066</td>
<td>0.231</td>
<td>0.200</td>
</tr>
<tr>
<td>NIS</td>
<td>0.102</td>
<td>0.222</td>
<td>0.066</td>
<td>0.046</td>
</tr>
<tr>
<td>CR</td>
<td>0.398</td>
<td>0.771</td>
<td>0.223</td>
<td>0.187</td>
</tr>
</tbody>
</table>

Table 9: Ranking to interval-valued positive and negative ideal solutions and closeness ratio of each alternative.

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>A1</th>
<th>A2</th>
<th>A3</th>
<th>A4</th>
</tr>
</thead>
<tbody>
<tr>
<td>IV-PIS</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>IV-NIS</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>CR</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
</tbody>
</table>

IVSF-TOPSIS provides decision makers with a wider range of definitions to make their decisions. Considering the proximity ratio values of the proposed method, the best alternative is A2 and the overall ranking is A2 > A1 > A3 > A4.

6 Conclusions

It is known that the stabilizing system effect is of great importance when considering the performance of a ship in a seaway. In this context, a detailed investigation of the stabilizing system selection problem involving active and passive systems for naval ships has been evaluated in the light of different criteria. A multi-criteria decision-making method, namely interval-valued spherical fuzzy TOPSIS based on the novel theory has been proposed for selection of stabilization systems for naval ships. IVSF-TOPSIS provides decision makers with a wider range of definitions to make their decisions.

The objective of the present study is to determine the most effective stabilizing device for naval ships by means of the interval-valued spherical fuzzy TOPSIS. Experts evaluated in detail the roll stabilizers, which are alternatives to each other, taking into account many parameters. Therefore, the use of IVSF-TOPSIS makes the application more realistic and reliable. The proposed methodology for solving such problems seems to be functional.

When the effect of the criteria on the closeness ratio of the alternative system types examined, it was determined as a result of the common opinion of all experts that Activated Fin was the most functional alternative. In addition, it should be noted that the results of these assessments may change when expert changes and any changes in criteria are made.
In future studies, the choice of naval ship to be invested can be determined by increasing or decreasing the number of criteria by using their IVSF extensions of the multi-criteria decision-making methods AHP and VIKOR. Also, this approach can also be used by different ship construction companies.

References


