COMMON FIXED POINT THEOREMS IN MODIFIED INTUITIONISTIC FUZZY METRIC SPACES

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Abstract. In this paper, we introduce a new class of implicit functions and also common property (E.A) in modified intuitionistic fuzzy metric spaces and utilize the same to prove some common fixed point theorems in modified intuitionistic fuzzy metric spaces besides discussing related results and illustrative examples. We are not aware of any paper dealing with such implicit functions in modified intuitionistic fuzzy metric spaces.

1. Introduction and Preliminaries

The theory of fuzzy sets was initiated by Zadeh [32]. In the last four decades, like all other aspects of Mathematics, various authors have introduced the concept of fuzzy metric in several ways. George and Veeramani [11] modified the concept of fuzzy metric space due to Kramosil and Michalek [20] and defined a Hausdorff topology on modified fuzzy metric space which is often used in current researches. Grabiec [12] extended classical fixed point theorems of Banach and Edelstein to complete and compact fuzzy metric spaces respectively. For the work of this kind, one can also consult [2, 5, 29, 30].

Atanassov [6] introduced and studied the concept of intuitionistic fuzzy set as a noted generalization of fuzzy set which has inspired intense research activity around this newly introduced notion. Recently Park [23], using the idea of intuitionistic fuzzy sets, defined intuitionistic fuzzy metric spaces as a generalization of fuzzy metric spaces due to George and Veeramani [11] and also proved some basic results which include Baire's theorem and uniform limit theorem besides some other core results. Thereafter, Saadati and Park [26] defined precompact sets in intuitionistic fuzzy metric spaces and proved that any subset of an intuitionistic fuzzy metric space is compact if and only if it is precompact and complete. They also defined topologically complete intuitionistic fuzzy metrizable spaces and proved that any G_δ set in a complete intuitionistic fuzzy metric space is a topologically complete intuitionistic fuzzy metrizable space and vice versa. For more relevant work, one can be referred to [13, 23, 25, 26].

One of the most important problems in fuzzy topology is to introduce appropriate concepts of intuitionistic fuzzy metric and intuitionistic fuzzy norm. These

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problems were investigated by Park [23] and Saadati and Park [26] respectively by introducing the notions of intuitionistic fuzzy metric and fuzzy norm. Intuitionistic fuzzy metric can be employed in modeling some phenomena wherein it is necessary to study the relationship between two probability functions as explained in [13]. Concretely speaking, it finds an application to two-slit experiment as the foundation of E-infinity theory in high energy physics were studied by El Naschie in [8, 9]. Gregori et al. [13] pointed out that topologies generated by fuzzy metric and intuitionistic fuzzy metric coincide. In view of this observation, Saadati et al. [27] modified the notion of intuitionistic fuzzy metric and defined the notion of modified intuitionistic fuzzy metric spaces with the help of continuous t-representable.

Lemma 1.1. [7]. *Consider the set L^* and operation $\leq_{L^*}$ defined by*

$$L^* = \{(x_1, x_2) : (x_1, x_2) \in [0,1]^2 \text{ and } x_1 + x_2 \leq 1\},$$

$(x_1, x_2) \leq_{L^*} (y_1, y_2) \iff x_1 \leq y_1$ *and* $x_2 \geq y_2\ \forall\ (x_1, x_2), (y_1, y_2) \in L^*$. *Then* $(L^*, \leq_{L^*})$ *is a complete lattice.*

Definition 1.2. [6]. An intuitionistic fuzzy set $\mathcal{A}_{\zeta,\eta}$ in a universe U is an object $\mathcal{A}_{\zeta\eta} = \{(\zeta_{\mathcal{A}}(u), \eta_{\mathcal{A}}(u)) | u \in U\}$, where, for all $u \in U$, $\zeta_{\mathcal{A}}(u) \in [0,1]$ and $\eta_{\mathcal{A}}(u) \in [0,1]$ are respectively called the membership degree and the non-membership degree of u in $\mathcal{A}_{\zeta,\eta}$ and furthermore $\zeta_{\mathcal{A}}(u) + \eta_{\mathcal{A}}(u) \leq 1$.

For every $z_i = (x_i, y_i) \in L^*$ and $c_i \in [0,1]$ such that $\sum_{j=1}^n c_j = 1$, it is easy to verify that

$$c_1(x_1, y_1) + \cdots + c_n(x_n, y_n) = \sum_{j=1}^n c_j(x_j, y_j) = \left(\sum_{j=1}^n c_j x_j, \sum_{j=1}^n c_j y_j\right) \in L^*.$$

We denote its units by $0_{L^*} = (0,1)$ and $1_{L^*} = (1,0)$. Classically, a triangular norm $* = T$ on $[0,1]$ is defined as an increasing, commutative and associative mapping $T : [0,1]^2 \to [0,1]$ satisfying $T(1,x) = 1 * x = x$, for all $x \in [0,1]$. A triangular co-norm $S = \diamondsuit$ is defined as an increasing, commutative and associative mapping $S : [0,1]^2 \to [0,1]$ satisfying $S(0,x) = 0 \diamondsuit x = x$, for all $x \in [0,1]$. These definitions can be straightforwardly extended to $(L^*, \leq_{L^*})$ as follows:

Definition 1.3. [7] A triangular norm (t-norm) on L^* is a mapping $\mathcal{T} : L^* \times L^* \to L^*$ satisfying the following conditions:

(I) $\mathcal{T}(x, 1_{L^*}) = x$,
(II) $\mathcal{T}(x,y) = \mathcal{T}(y,x)$,
(III) $\mathcal{T}(x, \mathcal{T}(y,z)) = \mathcal{T}(\mathcal{T}(x,y), z)$,
(IV) $x \leq_{L^*} x'$ and $y \leq_{L^*} y' \Rightarrow \mathcal{T}(x,y) \leq_{L^*} \mathcal{T}(x', y')$,
for all x, y, z, x' and $y' \in L^*$.

Definition 1.4. [7] A continuous t-norm $\mathcal{T}$ on L^* is called continuous t-representable if and only if there exist a continuous t-norm $*$ and a continuous t-conorm $\diamondsuit$ on $[0,1]$ such that, for all $x = (x_1, x_2),\ y = (y_1, y_2) \in L^*$,

$$\mathcal{T}(x,y) = (x_1 * y_1, x_2 \diamondsuit y_2).$$

Now, we define a sequence $\{\mathcal{T}^n\}$ recursively by $\{\mathcal{T}^1 = \mathcal{T}\}$ and

$$\mathcal{T}^n(x^{(1)}, \cdots, x^{(n+1)}) = \mathcal{T}(\mathcal{T}^{n-1}(x^{(1)}, \cdots, x^{(n)}), x^{(n+1)})$$

for $n \geq 2$ and $x^{(i)} \in L^*$.

Definition 1.5. [27] Let M, N be fuzzy sets from $X^2 \times (0, \infty)$ to $[0, 1]$ such that $M(x, y, t) + N(x, y, t) \leq 1$ for all $x, y \in X$ and $t > 0$. The 3-tuple $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is said to be a modified intuitionistic fuzzy metric space (in short, modified IFMS) if X is an arbitrary non-empty set, $\mathcal{T}$ is a continuous t-representable and $\mathcal{M}_{M,N}$ is an intuitionistic fuzzy set from $X^2 \times (0, \infty) \to L^*$ satisfying the following conditions (for every $x, y \in X$ and $t, s > 0$):

(I) $\mathcal{M}_{M,N}(x, y, t) >_{L^*} 0_{L^*}$,
(II) $\mathcal{M}_{M,N}(x, y, t) = 1_{L^*}$ if and only if $x = y$,
(III) $\mathcal{M}_{M,N}(x, y, t) = \mathcal{M}_{M,N}(y, x, t)$,
(IV) $\mathcal{M}_{M,N}(x, y, t + s) \geq_{L^*} \mathcal{T}(\mathcal{M}_{M,N}(x, y, t), \mathcal{M}_{M,N}(x, y, s))$,
(V) $\mathcal{M}_{M,N}(x, y, .) : (0, \infty,) \to L^*$ is continuous.

In this case $\mathcal{M}_{M,N}$ is called a modified intuitionistic fuzzy metric. Here,

$$\mathcal{M}_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t)).$$

Remark 1.6. [30] In an intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T}), M(x, y, .)$ is non-decreasing and $N(x, y, .)$ is non-increasing for all $x, y \in X$.

Example 1.7. Let (X, d) be a metric space. Denote $\mathcal{T}(a, b) = (a_1b_1, \min\{a_2 + b_2, 1\})$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2) \in L^*$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$. Then an intuitionistic fuzzy metric can be defined as

$$\mathcal{M}_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t)) = \left(\frac{ht^n}{ht^n + md(x, y)}, \frac{md(x, y)}{ht^n + md(x, y)}\right)$$

for all $h, m, n, t \in \mathbb{R}^+$ so that $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is a modified IFMS.

Remark 1.8. Let $X = \mathbb{N}$. Denote $\mathcal{T}(a, b) = (\max\{0, a_1 + b_1 - 1\}, a_2 + b_2 - a_2b_2))$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2) \in L^*$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$. Then $\mathcal{M}_{M,N}(x, y, t)$ defined as (for all $x, y \in X$):

$$\mathcal{M}_{M,N}(x, y, t) = (M(x, y, t), N(x, y, t)) = \begin{cases} \left(\frac{x}{y}, \frac{y-x}{y}\right) & \text{if } x \leq y \\ \left(\frac{y}{x}, \frac{x-y}{x}\right) & \text{if } y \leq x \end{cases}$$

is an intuitionistic fuzzy metric so that $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is a modified IFMS.

Definition 1.9. [7] A negator on L^* is a decreasing mapping $\mathcal{N} : L^* \to L^*$ satisfying $\mathcal{N}(0_{L^*}) = 1_{L^*}$ and $\mathcal{N}(1_{L^*}) = 0_{L^*}$. A negator on $[0, 1]$ is a decreasing mapping $N : [0, 1] \to [0, 1]$ satisfying $N(0) = 1$ and $N(1) = 0$. In what follows, N_s denotes the standard negator on $[0, 1]$ defined as $N_s(x) = 1 - x$ for all $x \in [0, 1]$.

Definition 1.10. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a modified IFMS. For $t > 0$, define the open ball $B(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$, as

$$B(x, r, t) = \{y \in X : \mathcal{M}_{M,N}(x, y, t) >_{L^*} (N_s(r), r)\}.$$

A subset $A \subseteq X$ is called open if for each $x \in A$, there exist $t > 0$ and $0 < r < 1$ such that $B(x, r, t) \subset A$. Let $\tau_{\mathcal{M}_{M,N}}$ denote the family of all open subsets of X. Then, $\tau_{\mathcal{M}_{M,N}}$ is called the topology induced by modified intuitionistic fuzzy metric $\mathcal{M}_{M,N}$.

Notice that this topology is Hausdorff (see Remark 3.3 and Theorem 3.5 in [23]).

Definition 1.11. A sequence $\{x_n\}$ in a modified IFMS $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is called a Cauchy sequence if for each $0 < \epsilon < 1$ and $t > 0$, there exists $n_0 \in \mathbb{N}$ such that

$$\mathcal{M}_{M,N}(x_n, y_m, t) >_{L^*} (N_s(\epsilon), \epsilon)$$

and for each $n, m \geq n_0$ where N_s is the standard negator. The sequence $\{x_n\}$ is said to be convergent to $x \in X$ in the intuitionistic fuzzy metric space $(X, \mathcal{M}_{M,N}, \mathcal{T})$ and is generally denoted by $x_n \to^{\mathcal{M}_{M,N}} x$ if $\mathcal{M}_{M,N}(x_n, x, t) \to 1_{L^*}$ whenever $n \to \infty$ for every $t > 0$. An IFMS is said to be complete if and only if every Cauchy sequence is convergent.

Lemma 1.12. [27] *Let $\mathcal{M}_{M,N}$ be an intuitionistic fuzzy metric. Then, for any $t > 0$, $\mathcal{M}_{M,N}(x, y, t)$ is non-decreasing with respect to t, in $(L^*, \leq_{L^*})$, for all $x, y \in X$.*

Definition 1.13. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a modified IFMS. Then $\mathcal{M}_{M,N}$ is said to be continuous on $X \times X \times (0, \infty)$ if

$$\lim_{n\to\infty} \mathcal{M}_{M,N}(x_n, y_n, t_n) = \mathcal{M}_{M,N}(x, y, t),$$

whenever a sequence $\{(x_n, y_n, t_n)\}$ in $X \times X \times (0, \infty)$ converges to a point $(x, y, t) \in X \times X \times (0, \infty)$, i.e.
$\lim_{n\to\infty} \mathcal{M}_{M,N}(x_n, x, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(y_n, y, t) = 1_{L^*}$ and $\lim_{n\to\infty} \mathcal{M}_{M,N}(x, y, t_n) = \mathcal{M}_{M,N}(x, y, t)$.

Lemma 1.14. [27] *Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a modified IFMS. Then $\mathcal{M}_{M,N}$ is a continuous function on $X \times X \times (0, \infty)$.*

Definition 1.15. Let f and g be mappings from a modified IFMS $(X, \mathcal{M}_{M,N}, \mathcal{T})$ into itself. Then the mappings are said to be compatible if

$$\lim_{n\to\infty} \mathcal{M}_{M,N}(fgx_n, gfx_n, t) = 1_{L^*},\ \forall t > 0$$

whenever $\{x_n\}$ is a sequence in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x \in X.$$

Definition 1.16. Let f and g be mappings from a modified IFMS $(X, \mathcal{M}_{M,N}, \mathcal{T})$ into itself. Then the mappings are said to be noncompatible if there exists at least one sequence $\{x_n\}$ in X such that $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = x \in X$ but $\lim_{n\to\infty} \mathcal{M}_{M,N}(fgx_n, gfx_n, t) \neq 1_{L^*}$ or nonexistent for at least one $t > 0$.

Definition 1.17. Let f and g be self mappings of a nonempty set X. Then the mappings are said to be weak compatible if they commute at their coincidence point, i.e. $fx = gx$ implies that $fgx = gfx$.

Remark 1.18. Every pair of compatible self mappings f and g of a modified IFMS $(X, \mathcal{M}_{M,N}, \mathcal{T})$ is weak compatible. But the converse is not true as it can be seen in the following example.

Example 1.19. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a modified IFMS, where $X = [0, 2]$ and $\mathcal{M}_{M,N}(x, y, t) = \left(\frac{t}{t+|x-y|}, \frac{|x-y|}{t+|x-y|}\right)$ for all $t > 0$ and $x, y \in X$, and $\mathcal{T}(a, b) = (a_1 b_1, \min\{a_2 + b_2, 1\})$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2) \in L^*$. Define self-mappings f and g on X as follows:

$$f(x) = \begin{cases} 2 & \text{if } 0 \leq x \leq 1 \\ \frac{x}{2} & \text{if } 1 < x \leq 2 \end{cases}, \quad g(x) = \begin{cases} 2 & \text{if } x = 1 \\ \frac{x+3}{5} & \text{if } x \neq 1 \end{cases}.$$

Then we have $g1 = f1 = 2$ and $g2 = f2 = 1$. Also $gf1 = fg1 = 1$ and $gf2 = fg2 = 2$. Thus the pair (f, g) is weak compatible. Moreover, $fx_n = 1 - \frac{1}{4n}$ and $gx_n = 1 - \frac{1}{10n}$. Thus $fx_n \to 1,\ gx_n \to 1$. Further $gfx_n = \frac{4}{5} - \frac{1}{20n},\ fgx_n = 2$. Now

$$\lim_{n\to\infty} \mathcal{M}_{M,N}(fgx_n, gfx_n, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}\Big(2, \frac{4}{5} - \frac{1}{20n}, t\Big) = \left(\frac{t}{t + \frac{6}{5}}, \frac{\frac{6}{5}}{t + \frac{6}{5}}\right) \neq 1_{L^*}$$

$\forall t > 0$. Hence the pair (f, g) is not compatible.

Motivated by Aamri and Moutawakil [1], we define the following:

Definition 1.20. Let f and g be two self mappings of a modified IFMS $(X, \mathcal{M}_{M,N}, \mathcal{T})$. We say that f and g have the property (E.A) if there exists a sequence $\{x_n\}$ in X such that for all $t > 0$

$$\lim_{n\to\infty} \mathcal{M}_{M,N}(fx_n, u, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(gx_n, u, t) = 1_{L^*}$$

for some $u \in X$ and $t > 0$.

Example 1.21. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a modified IFMS, where $X = \mathbb{R}$ and $\mathcal{M}_{M,N}(x, y, t) = \left(\frac{t}{t+|x-y|}, \frac{|x-y|}{t+|x-y|)}\right)$ for all $t > 0$ and $x, y \in X$. Define self-mappings f and g on X as follows:

$$fx = 2x + 1 \text{ and } gx = x + 2.$$

Consider the sequence $\{x_n = 1 + \frac{1}{n},\ n = 1, 2, \cdots\}$. Thus we have

$$\lim_{n\to\infty} \mathcal{M}_{M,N}(fx_n, 3, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(gx_n, 3, t) = 1_{L^*}$$

for every $t > 0$. Then f and g share the property (E.A).

In the next example, we show that there exists a pair of mappings which do not share the property (E.A).

Example 1.22. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be a modified IFMS, where $X = \mathbb{R}$ and $\mathcal{M}_{M,N}(x, y, t) = \left(\frac{t}{t+|x-y|}, \frac{|x-y|}{t+|x-y|)}\right)$ for all $t > 0$ and $x, y \in X$. Define self-mappings f and g on X as $fx = x + 1$ and $gx = x + 2$. Suppose there exists a sequence $\{x_n\}$ such that

$$\lim_{n\to\infty} \mathcal{M}_{M,N}(fx_n, u, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(gx_n, u, t) = 1_{L^*}$$

for some $u \in X$. Then

$$\lim_{n\to\infty} \mathcal{M}_{M,N}(fx_n, u, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(x_n+1, u, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(x_n, u-1, t) = 1_{L^*}$$

$$\lim_{n\to\infty} \mathcal{M}_{M,N}(gx_n, u, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(x_n+2, u, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(x_n, u-2, t) = 1_{L^*},$$

which show that $x_n \to u-1$ and $x_n \to u-2$, which is a contradiction. Hence f and g do not share the property (E.A).

Motivated by Liu et al. [21], we also define the following:

Definition 1.23. Two pairs (f, S) and (g, T) of self mappings of a modified IFMS $(X, \mathcal{M}_{M,N}, \mathcal{T})$ are said to satisfy the common property (E.A) if there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} \mathcal{M}_{M,N}(fx_n, u, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(Sx_n, u, t) = \lim_{n\to\infty} \mathcal{M}_{M,N}(gy_n, u, t)$$
$$= \lim_{n\to\infty} \mathcal{M}_{M,N}(Ty_n, u, t) = 1_{L^*}$$

for some $u \in X$ and $t > 0$.

Definition 1.24. [17] Two finite families of self mappings $\{f_i\}_{i=1}^m$ and $\{g_k\}_{k=1}^n$ of a set X are said to be pairwise commuting if:

(i) $f_i f_j = f_j f_i \quad i, j \in \{1, 2, \ldots, m\}$,
(ii) $g_k g_l = g_l g_k \quad k, l \in \{1, 2, \ldots, n\}$,
(iii) $f_i g_k = g_k f_i \quad i \in \{1, 2, \ldots, m\}$ and $k \in \{1, 2, \ldots, n\}$.

The purpose of this paper is to introduce a new class of implicit functions and common property $(E.A)$ in modified IFMS and utilize the same to prove some common fixed point theorems in modified IFMS.

2. Implicit Relations

Motivated by Ali and Imdad [4], we introduce a new class of implicit functions as follows:

Let Ψ be the set of all upper semi-continuous functions $F(t_1, t_2, \cdots, t_6) : (L^*)^6 \to L^*$, satisfying the following conditions (for all $u, \mathbf{0}, \mathbf{1} \in L^*$ where $u = (u_1, u_2), \mathbf{0} = 0_{L^*} = (0, 1)$ and $\mathbf{1} = 1_{L^*} = (1, 0)$):

$(F_1) : F(u, \mathbf{1}, u, \mathbf{1}, \mathbf{1}, u) <_{L^*} \mathbf{0}$, for all $u >_{L^*} \mathbf{0}$,

$(F_2) : F(u, \mathbf{1}, \mathbf{1}, u, u, \mathbf{1}) <_{L^*} \mathbf{0}$, for all $u >_{L^*} \mathbf{0}$,

$(F_3) : F(u, u, \mathbf{1}, \mathbf{1}, u, u) <_{L^*} \mathbf{0}$, for all $u >_{L^*} \mathbf{0}$.

The following examples satisfy $(F_1), (F_2)$ and (F_3).

Example 2.1. Define $F(t_1, t_2, \cdots, t_6) : L^{*6} \to L^*$ as

$$F(t_1, t_2, \cdots, t_6) = t_1 - a \min\{t_2, t_3, t_4, t_5, t_6\}, \text{ where } a > 1.$$

Example 2.2. Define $F(t_1, t_2, \cdots, t_6) : L^{*6} \to L^*$ as

$$F(t_1, t_2, \cdots, t_6) = t_1^2 - c_1 \min\{t_2^2, t_3^2, t_4^2\} - c_2 \min\{t_3t_6, t_4t_5\},$$

where $c_1, c_2 > 0,\ c_1 + c_2 > 1$ and $c_1 \geq 1$.

Example 2.3. Define $F(t_1, t_2, \cdots, t_6) : L^{*6} \to L^*$ as

$$F(t_1, t_2, \cdots, t_6) = t_1^3 - a \min\{t_1^2 t_2, t_1 t_3 t_4, t_5^2 t_6, t_5 t_6^2\},$$

where $a > 1$.

Example 2.4. Define $F(t_1, t_2, \cdots, t_6) : L^{*6} \to L^*$ as

$$F(t_1, t_2, \cdots, t_6) = t_1^3 - a\frac{t_3^2 t_4^2 + t_5^2 t_6^2}{t_2 + t_3 + t_4},$$

where $a \geq \frac{3}{2}$.

Example 2.5. Define $F(t_1, t_2, \cdots, t_6) : L^{*6} \to L^*$ as

$$F(t_1, t_2, \cdots, t_6) = (1 + pt_2)t_1 - p \min\{t_3t_4, t_5t_6\} - \psi(\min\{t_2, t_3, t_4, t_5, t_6\}),$$

where $p \geq 0$ and $\psi : L^* \to L^*$ is a continuous function such that $\psi(t) >_{L^*} t$ for all $t \in L^* \backslash \{\mathbf{0}, \mathbf{1}\}$.

Example 2.6. Define $F(t_1, t_2, \cdots, t_6) : L^{*6} \to L^*$ as

$$F(t_1, t_2, \cdots, t_6) = t_1^2 - a\frac{t_2^2 + t_3^2 + t_4^2}{t_5 + t_6},$$

where $a \geq 2$.

Example 2.7. Define $F(t_1, t_2, \cdots, t_6) : L^{*6} \to L^*$ as

$$F(t_1, t_2, \cdots, t_6) = t_1 - \psi(\min\{t_2, t_3, t_4, t_5, t_6\}),$$

where $\psi : L^* \to L^*$ is a continuous function such that $\psi(t) >_{L^*} t$ for all $t \in L^* \backslash \{\mathbf{0}, \mathbf{1}\}$.

Example 2.8. Define $F(t_1, t_2, \cdots, t_6) : L^{*6} \to L^*$ as

$$F(t_1, t_2, \cdots, t_6) = t_1^3 - a\frac{t_3^2 t_4^2}{t_2 + t_5 + t_6},$$

where $a \geq 3$.

Example 2.9. Define $F(t_1, t_2, \cdots, t_6) : L^{*6} \to L^*$ as

$$F(t_1, t_2, \cdots, t_6) = t_1^2 - a \min\{t_2^2, t_3^2, t_4^2\} - b\frac{t_5}{t_5 + t_6},$$

where $a \geq 1$ and $b > 0$.

Example 2.10. Define $F(t_1, t_2, \cdots, t_6) : L^{*6} \to L^*$ as

$$F(t_1, t_2, \cdots, t_6) = t_1^2 - a \min\{t_2^2, t_5^2, t_6^2\} - b\frac{t_3}{t_3 + t_4},$$

where $a \geq 1$ and $b > 0$.

Example 2.11. Define $F(t_1, t_2, \cdots, t_6) : L^{*6} \to L^*$ as

$$F(t_1, t_2, \cdots, t_6) = t_1 - a_1t_2 - a_2t_3 - a_3t_4 - a_4t_5 - a_5t_6,$$

where $a_1, a_2, a_3, a_4, a_5 > 0,\ a_2 + a_5 \geq 1,\ a_3 + a_4 \geq 1$ and $a_1 + a_4 + a_5 \geq 1$.

3. **Results**

We begin with the following lemma.

Lemma 3.1. *Let f, g, S and T be self mappings of a modified IFMS $(X, \mathcal{M}_{M,N}, \mathcal{T})$ satisfying the following conditions:*

(I) *the pair (f, S) (or (g, T)) satisfies the property (E.A),*
(II) *$f(X) \subset T(X)$ (or $g(X) \subset S(X)$),*
(III) *$g(y_n)$ converges for every sequence y_n in X whenever $T(y_n)$ converges (or $f(x_n)$ converges for every sequence x_n in X whenever $S(x_n)$ converges),*
(IV) *for all $x, y \in X,\ F \in \Psi$*

$$\left.\begin{array}{l} F(\mathcal{M}_{M,N}(fx, gy, t), \mathcal{M}_{M,N}(Sx, Ty, t), \mathcal{M}_{M,N}(fx, Sx, t), \mathcal{M}_{M,N}(gy, Ty, t), \\ \mathcal{M}_{M,N}(Sx, gy, t), \mathcal{M}_{M,N}(fx, Ty, t)) \geq_{L^*} \mathbf{0} \end{array}\right\}, \quad (1)$$

then the pairs (f, S) and (g, T) share the common property (E.A).

Proof. Suppose that the pair (f, S) enjoys the property (E.A), there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Sx_n = z, \text{ for some } z \in X,$$

i.e. $\lim_{n\to\infty} \mathcal{M}_{M,N}(fx_n, Sx_n, t) = \mathbf{1}$. Since $f(X) \subset T(X)$, for each x_n there exists y_n in X such that $fx_n = Ty_n$. Therefore, $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Ty_n = z$. Thus in all we have $fx_n \to z,\ Sx_n \to z$ and $Ty_n \to z$. Moreover in view of (III), $\{gy_n\}$ also converges. Now we assert that $\lim_{n\to\infty} \mathcal{M}_{M,N}(gy_n, z, t) = \mathbf{1}$. If not, then using inequality (1), we have

$$F(\mathcal{M}_{M,N}(fx_n, gy_n, t), \mathcal{M}_{M,N}(Sx_n, Ty_n, t), \mathcal{M}_{M,N}(fx_n, Sx_n, t), \mathcal{M}_{M,N}(gy_n, Ty_n, t),$$
$$\mathcal{M}_{M,N}(Sx_n, gy_n, t), \mathcal{M}_{M,N}(fx_n, Ty_n, t)) \geq_{L^*} \mathbf{0}$$

which on making $n \to \infty$, reduces to

$$F(\lim_{n\to\infty} \mathcal{M}_{M,N}(Ty_n, gy_n, t), \mathbf{1}, \mathbf{1}, \lim_{n\to\infty} \mathcal{M}_{M,N}(gy_n, Ty_n, t),$$
$$\lim_{n\to\infty} \mathcal{M}_{M,N}(Ty_n, gy_n, t), \mathbf{1}) \geq_{L^*} \mathbf{0}$$

which is a contradiction to F_2. Hence $\lim_{n\to\infty} \mathcal{M}_{M,N}(gy_n, Ty_n, t) = \mathbf{1}$, i.e. $\lim_{n\to\infty} gy_n = z$, which shows that the pairs (f, S) and (g, T) share the common property (E.A). □

Our next result is a common fixed point theorem via the common property (E.A).

Theorem 3.2. *Let f, g, S and T be self mappings of a modified IFMS $(X, \mathcal{M}_{M,N}, \mathcal{T})$ satisfying the condition (1). Suppose that*

(I) *the pairs (f, S) and (g, T) share the common property (E.A) and*
(II) *$S(X)$ and $T(X)$ are closed subsets of X.*

Then the pairs (f, S) and (g, T) have a coincidence point. Moreover, f, g, S and T have a unique common fixed point in X provided both the pairs (f, S) and (g, T) are weakly compatible.

Proof. Since the pairs (f, S) and (g, T) share the common property (E.A), there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} Ty_n = z, \text{ for some } z \in X.$$

As $S(X)$ is a closed subset of X, $\lim_{n\to\infty} Sx_n = z \in S(X)$ and hence there exists a point $u \in X$ such that $Su = z$. Now we assert that $\mathcal{M}_{M,N}(fu, z, t) = \mathbf{1}$. If not, then using inequality (1), we have

$$F(\mathcal{M}_{M,N}(fu, gy_n, t), \mathcal{M}_{M,N}(Su, Ty_n, t), \mathcal{M}_{M,N}(fu, Su, t), \mathcal{M}_{M,N}(gy_n, Ty_n, t), \mathcal{M}_{M,N}(Su, gy_n, t), \mathcal{M}_{M,N}(fu, Ty_n, t)) \geq_{L^*} \mathbf{0}$$

which on making $n \to \infty$, give rise to

$$F(\mathcal{M}_{M,N}(fu, z, t), \mathbf{1}, \mathcal{M}_{M,N}(fu, z, t), \mathbf{1}, \mathbf{1}, \mathcal{M}_{M,N}(fu, z, t)) \geq_{L^*} \mathbf{0}$$

a contradiction to F_1 so that $\mathcal{M}_{M,N}(fu, z, t) = \mathbf{1}$, i.e. $fu = z = Su$. Hence u is a coincidence point of the pair (f, S).

Since $T(X)$ is a closed subset of X, $\lim_{n\to\infty} Ty_n = z \in T(X)$ and hence there exists a point $w \in X$ such that $Tw = z$. Now we assert that $\mathcal{M}_{M,N}(gw, z, t) = \mathbf{1}$. If not, then using inequality (1), we have

$$F(\mathcal{M}_{M,N}(fx_n, gw, t), \mathcal{M}_{M,N}(Sx_n, Tw, t), M(fx_n, Sx_n, t), \mathcal{M}_{M,N}(gw, Tw, t), \mathcal{M}_{M,N}(Sx_n, gw, t), \mathcal{M}_{M,N}(fx_n, Tw, t)) \geq_{L^*} \mathbf{0}$$

which on making $n \to \infty$, give rise to

$$F(\mathcal{M}_{M,N}(z, gw, t), \mathbf{1}, \mathbf{1}, \mathcal{M}_{M,N}(gw, z, t), \mathcal{M}_{M,N}(z, gw, t), \mathbf{1}) \geq_{L^*} \mathbf{0}$$

a contradiction to F_2, so that $\mathcal{M}_{M,N}(gw, z, t) = \mathbf{1}$, i.e. $gw = z = Tw$. This shows that w is a coincidence point of the pair (g, T).

Since $fu = Su$ and the pair (f, S) is weakly compatible, therefore $fz = fSu = Sfu = Sz$. Now we need to show that z is a common fixed point of the pair (f, S). To accomplish this, we assert that $\mathcal{M}_{M,N}(fz, z, t) = \mathbf{1}$. If not, then using inequality (1), we have

$$F(\mathcal{M}_{M,N}(fz, gw, t), \mathcal{M}_{M,N}(Sz, Tw, t), \mathcal{M}_{M,N}(fz, Sz, t), \mathcal{M}_{M,N}(gw, Tw, t), \mathcal{M}_{M,N}(Sz, gw, t), \mathcal{M}_{M,N}(fz, Tw, t)) \geq_{L^*} \mathbf{0}$$

or

$$F(\mathcal{M}_{M,N}(fz, z, t), \mathcal{M}_{M,N}(fz, z, t), \mathbf{1}, \mathbf{1}, \mathcal{M}_{M,N}(fz, z, t), \mathcal{M}_{M,N}(fz, z, t)) \geq_{L^*} \mathbf{0}$$

which is a contradiction to F_3, so that $\mathcal{M}_{M,N}(fz, z, t) = \mathbf{1}$ implying thereby $fz = z$ which shows that z is a common fixed point of the pair (f, S).

Also $gw = Tw$ and the pair (g, T) is weakly compatible, therefore $gz = gTw = Tgw = Tz$. Next, we have to show that z is a common fixed point of the pair

(g,T). To do this, we assert that $M(gz,z,t) = \mathbf{1}$. If not, then using inequality (1), we have

$$F(\mathcal{M}_{M,N}(fu,gz,t), \mathcal{M}_{M,N}(Su,Tz,t), \mathcal{M}_{M,N}(fu,Su,t), \mathcal{M}_{M,N}(gz,Tz,t), \mathcal{M}_{M,N}(Su,gz,t), \mathcal{M}_{M,N}(fu,Tz,t)) \geq_{L^*} \mathbf{0}$$

or

$$F(\mathcal{M}_{M,N}(z,gz,t), \mathcal{M}_{M,N}(z,gz,t), \mathbf{1}, \mathbf{1}, \mathcal{M}_{M,N}(z,gz,t), \mathcal{M}_{M,N}(z,gz,t)) \geq_{L^*} \mathbf{0}$$

which is a contradiction to F_3 so that $\mathcal{M}_{M,N}(gz,z,t) = \mathbf{1}$, i.e. $gz = z$ which shows that z is a common fixed point of the pair (g,T). Hence z is a common fixed point of f, g, S and T. Uniqueness of the common fixed point is an easy consequence of the inequality (1) (in view of condition F_3). □

Remark 3.3. Theorem 3.2 extends relevant results of Ali and Imdad [4], Imdad and Ali [15, 16] and other results to intuitionistic fuzzy metric space.

Theorem 3.4. *The conclusions of Theorem 3.2 remain true if the condition (II) of Theorem 3.2 be replaced by the following.*

(II') $\overline{f(X)} \subset T(X)$ *and* $\overline{g(X)} \subset S(X)$.

As a corollary of Theorem 3.4, we can have the following result which is also a variant of Theorem 3.2.

Corollary 3.5. *The conclusions of Theorems 3.2 and 3.4 remain true if the conditions (II) and* (II') *are replaced by the following.*

(II'') *$f(X)$ and $g(X)$ are closed subsets of X provided that $f(X) \subset T(X)$ and $g(X) \subset S(X)$.*

Theorem 3.6. *Let f, g, S and T be self mappings of a modified IFMS $(X, \mathcal{M}_{M,N}, \mathcal{T})$ satisfying the conditions (I-IV) of Lemma 3.1. Suppose that*

(V) $S(X)$ (or $T(X)$) is a closed subset of X.

Then the pairs (f,S) and (g,T) have a coincidence point. Moreover, f, g, S and T have a unique common fixed point in X provided that the pairs (f,S) and (g,T) are weakly compatible.

Proof. In view of Lemma 3.1, the pairs (f,S) and (g,T) share the common property (E.A), i.e. there exist two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$\lim_{n\to\infty} fx_n = \lim_{n\to\infty} Sx_n = \lim_{n\to\infty} gy_n = \lim_{n\to\infty} Ty_n = z, \text{ for some } z \in X.$$

As $S(X)$ is a closed subset of X, on the lines of Theorem 3.2, one can show that the pair (f,S) has a coincidence point, say u, i.e. $fu = Su$. Since $fu \in f(X)$ and $f(X) \subset T(X)$, there exists $w \in X$ such that $fu = Tw$. Now we assert that $\mathcal{M}_{M,N}(gw,z,t) = \mathbf{1}$. If not, then using inequality (1), we have

$$F(\mathcal{M}_{M,N}(fx_n,gw,t), \mathcal{M}_{M,N}(Sx_n,Tw,t), \mathcal{M}_{M,N}(fx_n,Sx_n,t), \mathcal{M}_{M,N}(gw,Tw,t), \mathcal{M}_{M,N}(Sx_n,gw,t), \mathcal{M}_{M,N}(fx_n,Tw,t)) \geq_{L^*} \mathbf{0}$$

which on making $n \to \infty$, reduces to

$$F(\mathcal{M}_{M,N}(z, gw, t), \mathbf{1}, \mathbf{1}, \mathcal{M}_{M,N}(gw, z, t), \mathcal{M}_{M,N}(z, gw, t), \mathbf{1}) \geq_{L^*} \mathbf{0}$$

which is a contradiction to F_2, hence $\mathcal{M}_{M,N}(gw, z, t) = \mathbf{1}$, i.e. $gw = z = Tw$. Therefore, w is a coincidence point of the pair (g, T). The rest of the proof can be completed on the lines of Theorem 3.2. □

By choosing f, g, S and T suitably, one can deduce result for a pair of mappings.

Corollary 3.7. *Let f and S be self mappings of a modified IFMS $(X, \mathcal{M}_{M,N}, \mathcal{T})$ satisfying the following conditions:*

(I) *the pair (f, S) satisfies the property (E.A),*
(II) *$S(X)$ is a closed subset of X and*
(III) *for all $x, y \in X$, $F \in \Psi$ and $t > 0$*

$$F(\mathcal{M}_{M,N}(fx, fy, t), \mathcal{M}_{M,N}(Sx, Sy, t), \mathcal{M}_{M,N}(fx, Sx, t), \mathcal{M}_{M,N}(fy, Sy, t),$$
$$\mathcal{M}_{M,N}(Sx, fy, t), \mathcal{M}_{M,N}(fx, Sy, t)) \geq_{L^*} \mathbf{0}.$$

Then the pair (f, S) has a coincidence point. Moreover, f and S have a unique common fixed point in X provided that the pair (f, S) is weakly compatible.

Remark 3.8. the above corollary extends and generalizes certain relevant results involving pair of mappings from the existing literature (e.g. [15, 16]).

Corollary 3.9. *The conclusions of Theorem 3.2 remain true if inequality* (1) *is replaced by one of the following conditions. For all $x, y \in X$ and $t > 0$,*

(a_1)

$$\mathcal{M}_{M,N}(fx, gy, t) \geq_{L^*} a \min\{\mathcal{M}_{M,N}(Sx, Ty, t), \mathcal{M}_{M,N}(fx, Sx, t), \mathcal{M}_{M,N}(gy, Sy, t),$$
$$\mathcal{M}_{M,N}(Sx, gy, t), \mathcal{M}_{M,N}(fx, Ty, t), \} \text{ where } a > 1.$$

(a_2)

$$\mathcal{M}_{M,N}(fx, gy, t)^2 \geq_{L^*} c_1 \min\{\mathcal{M}_{M,N}(Sx, Ty, t)^2, \mathcal{M}_{M,N}(fx, Sx, t)^2, \mathcal{M}_{M,N}(gy, Ty, t)^2\}$$
$$+c_2 \min\{\mathcal{M}_{M,N}(gy, Ty, t)\mathcal{M}_{M,N}(Sx, gy, t), \mathcal{M}_{M,N}(fx, Sx, t)\mathcal{M}_{M,N}(fx, Ty, t)\},$$

where $c_1, c_2 > 0$, $c_1 + c_2 \geq 1$ *and* $c_1 \geq 1$.

(a_3)

$$\mathcal{M}_{M,N}(fx, gy, t)^3 \geq_{L^*} a \ \min\{\mathcal{M}_{M,N}(fx, gy, t)^2\mathcal{M}_{M,N}(Sx, Ty, t),$$
$$\mathcal{M}_{M,N}(fx, gy, t)\mathcal{M}_{M,N}(fx, Sx, t)\mathcal{M}_{M,N}(gy, Ty, t),$$
$$\mathcal{M}_{M,N}(fx, Ty, t)\mathcal{M}_{M,N}(Sx, gy, t)^2, \mathcal{M}_{M,N}(fx, Ty, t)^2\mathcal{M}_{M,N}(Sx, gy, t)\},$$

where $a > 1$.

(a_4)

$$\mathcal{M}_{M,N}(fx, gy, t)^3$$
$$\geq_{L^*} a \ \frac{\mathcal{M}_{M,N}(gy, Ty, t)^2\mathcal{M}_{M,N}(fx, Sx, t)^2 + \mathcal{M}_{M,N}(fx, Ty, t)^2\mathcal{M}_{M,N}(Sx, gy, t)^2}{\mathcal{M}_{M,N}(Sx, Ty, t) + \mathcal{M}_{M,N}(gy, Ty, t) + \mathcal{M}_{M,N}(fx, Sx, t)},$$

where $a > \frac{3}{2}$.

(a_5)

$$(1+p\mathcal{M}_{M,N}(Sx,Ty,t))\mathcal{M}_{M,N}(fx,gy,t) \geq_{L^*} p\min\{\mathcal{M}_{M,N}(fx,Sx,t)\mathcal{M}_{M,N}(gy,Ty,t), \mathcal{M}_{M,N}(Sx,gy,t)\mathcal{M}_{M,N}(fx,Ty,t)\} + \psi(\min\{\mathcal{M}_{M,N}(Sx,Ty,t), \mathcal{M}_{M,N}(gy,Ty,t), \mathcal{M}_{M,N}(fx,Sx,t), \mathcal{M}_{M,N}(fx,Ty,t), \mathcal{M}_{M,N}(Sx,gy,t)\}),$$

where $p \geq 0$ *and* $\psi : L^* \to L^*$ *is a continuous function such that* $\psi(t) >_{L^*} t$ *for all* $t \in L^*\backslash\{\mathbf{0},\mathbf{1}\}$.

(a_6)

$$\mathcal{M}_{M,N}(fx,gy,t)^2 \geq_{L^*} a\ \frac{\mathcal{M}_{M,N}(Sx,Ty,t)^2 + \mathcal{M}_{M,N}(gy,Ty,t)^2 + \mathcal{M}_{M,N}(fx,Sx,t)^2}{\mathcal{M}_{M,N}(fx,Ty,t) + \mathcal{M}_{M,N}(Sx,gy,t)},$$

where $a > 2$.

(a_7)

$$\mathcal{M}_{M,N}(fx,gy,t) \geq_{L^*} \psi(\min\{\mathcal{M}_{M,N}(Sx,Ty,t), \mathcal{M}_{M,N}(gy,Ty,t), \mathcal{M}_{M,N}(fx,Sx,t), \mathcal{M}_{M,N}(fx,Ty,t), \mathcal{M}_{M,N}(Sx,gy,t)\}),$$

where $\psi : L^* \to L^*$ *is a continuous function such that* $\psi(t) >_{L^*} t$ *for all* $t \in L^*\backslash\{\mathbf{0},\mathbf{1}\}$.

(a_8)

$$\mathcal{M}_{M,N}(fx,gy,t)^3 \geq_{L^*} a\frac{\mathcal{M}_{M,N}(gy,Ty,t)^2\mathcal{M}_{M,N}(fx,Sx,t)^2}{\mathcal{M}_{M,N}(Sx,Ty,t) + \mathcal{M}_{M,N}(fx,Ty,t) + \mathcal{M}_{M,N}(Sx,gy,t)},$$

where $a \geq 3$.

(a_9)

$$\mathcal{M}_{M,N}(fx,gy,t)^2 \geq_{L^*} a\min\{\mathcal{M}_{M,N}(Sx,Ty,t)^2, \mathcal{M}_{M,N}(gy,Ty,t)^2, \mathcal{M}_{M,N}(fx,Sx,t)^2\} + b\frac{\mathcal{M}_{M,N}(Sx,gy,t)}{\mathcal{M}_{M,N}(Sx,gy,t) + \mathcal{M}_{M,N}(fx,Ty,t)},$$

where $a \geq 1$ *and* $b > 0$.

(a_{10})

$$\mathcal{M}_{M,N}(fx,gy,t)^2 \geq_{L^*} a\min\{\mathcal{M}_{M,N}(Sx,Ty,t)^2, \mathcal{M}_{M,N}(fx,Ty,t)^2, \mathcal{M}_{M,N}(gy,Sx,t)^2\} + b\frac{\mathcal{M}_{M,N}(fx,Sx,t)}{\mathcal{M}_{M,N}(fx,Sx,t) + \mathcal{M}_{M,N}(gy,Ty,t)},$$

where $a \geq 1$ *and* $b > 0$,

(a_{11})

$$\mathcal{M}_{M,N}(fx,gy,t) \geq_{L^*} a_1\mathcal{M}_{M,N}(Sx,Ty,t) + a_2\mathcal{M}_{M,N}(fx,Sx,t) + a_3\mathcal{M}_{M,N}(gy,Ty,t) + a_4\mathcal{M}_{M,N}(gy,Sx,t) + a_5\mathcal{M}_{M,N}(fx,Ty,t),$$

where $a_1, a_2, a_3, a_4, a_5 > 0$, $a_2 + a_5 \geq 1$, $a_3 + a_4 \geq 1$ *and* $a_1 + a_4 + a_5 \geq 1$.

Proof. The proof follows from Theorem 3.2 and Examples 2.1–2.11. □

Remark 3.10. Corollaries corresponding to contraction conditions $(a_1 - a_{11})$ are new results as these results never require conditions on the containment of ranges of involved mappings as employed by earlier authors. Some contraction conditions embodied in the above corollary are well known, and extend and generalize corresponding relevant results (e.g., [3, 4, 15, 16, 22, 25, 30, 31]).

As an application of Theorem 3.2, we can have the following result for four finite families of self mappings.

Theorem 3.11. *Let* $\{f_1, f_2, \cdots, f_m\}$,$\{g_1, g_2, \cdots, g_p\}$,$\{S_1, S_2, \cdots, S_n\}$ *and* $\{T_1, T_2, \cdots, T_q\}$ *be four finite families of self mappings of a modified IFMS* $(X, \mathcal{M}_{M,N}, \mathcal{T})$ *with* $f = f_1 f_2 \cdots f_m$, $g = g_1 g_2 \cdots g_p$, $S = S_1 S_2 \cdots S_n$ *and* $T = T_1 T_2 \cdots T_q$ *satisfying inequality* (1) *and the pairs* (f, S) *and* (g, T) *share the common property (E.A). If* $S(X)$ *and* $T(X)$ *are closed subsets of* X, *then*

(I) the pairs (f, S) *and* (g, T) *have coincidence point each.*

Moreover, f_i, S_k, g_r *and* T_t *have a unique common fixed point provided that the pairs of families* $(\{f_i\}, \{S_k\})$ *and* $(\{g_r\}, \{T_t\})$ *commute pairwise where* $i \in \{1, \ldots, m\}, k \in \{1, \ldots, n\}, r \in \{1, \ldots, p\}$ *and* $t \in \{1, \ldots, q\}$.

Proof. Proof follows on the lines of the corresponding result contained in Imdad et al. [17]. □

By setting $f_1 = f_2 = \cdots = f_m = G,\ g_1 = g_2 = \cdots = g_p = H,\ S_1 = S_2 = \cdots = S_n = I$ and $T_1 = T_2 = \cdots = T_q = J$ in Theorem 3.11, we deduce the following:

Corollary 3.12. *Let* G, H, I *and* J *be four self mappings of a modified IFMS* $(X, \mathcal{M}_{M,N}, \mathcal{T})$, *pairs* (G^m, I^n) *and* (H^p, J^q) *share the common property (E.A) and satisfying the condition for all* $x, y \in X$, $F \in \Psi$ *and* $t > 0$

$$F(\mathcal{M}_{M,N}(G^m x, H^p y, t), \mathcal{M}_{M,N}(I^n x, J^q y, t), \mathcal{M}_{M,N}(G^m x, I^n x, t), \mathcal{M}_{M,N}(H^p y, J^q y, t),$$
$$\mathcal{M}_{M,N}(G^m x, J^q y, t), \mathcal{M}_{M,N}(I^n x, H^p y, t)) \geq_{L^*} \mathbf{0}$$

where m, n, p *and* q *are positive integers. If* $I^n(X)$ *and* $J^q(X)$ *are closed subsets of* X, *then* G, H, I *and* J *have a unique common fixed point provided that* $GI = IG$ *and* $HJ = JH$.

Finally, we conclude this paper with the following two examples. Example 3.13 demonstrates Theorem 3.2 besides exhibiting it's superiority over earlier relevant results (e.g. [15, 16]).

Example 3.13. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be an intuitionistic fuzzy metric space, wherein $X = (0, 128)$, $\mathcal{T}(a, b) = (a_1 b_1, \min\{a_2 + b_2, 1\})$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2) \in L^*$ with

$$\mathcal{M}_{M,N}(x, y, t) = \left(\frac{t}{t + |x - y|}, \frac{|x - y|}{t + |x - y|)} \right).$$

Define self mappings f, g, S and T on X by

$$f(x) = \begin{cases} 2x^4 - 1 & \text{if } 1 \leq x \leq 2 \\ 7 & \text{if otherwise} \end{cases},\ g(x) = \begin{cases} 2x^6 - 1 & \text{if } 1 \leq x \leq 2 \\ 3 & \text{if otherwise} \end{cases},$$

$$S(x) = \begin{cases} x^2 & \text{if } 1 \leq x \leq 2 \\ 2 & \text{if otherwise} \end{cases} \text{ and } T(x) = \begin{cases} x^3 & \text{if } 1 \leq x \leq 2 \\ 4 & \text{if otherwise} \end{cases}.$$

Define $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \psi[\min\{t_2, t_3, t_4, t_5, t_6\}]$ where $\psi(s) >_{L^*} s$ for all $s \in L^* \backslash \{\mathbf{0}, \mathbf{1}\}$ and $F \in \Psi$.
Now, for all $x, y \in X$ and $t > 0$, we have

$$\psi\Big[\min\{\mathcal{M}_{M,N}(Sx, Ty, t), \mathcal{M}_{M,N}(fx, Sx, t), \mathcal{M}_{M,N}(gy, Ty, t), \mathcal{M}_{M,N}(Sx, gy, t),$$
$$\mathcal{M}_{M,N}(fx, Ty, t)\}\Big] \leq_{L^*} \mathcal{M}_{M,N}(fx, gy, t)$$

which demonstrates the verification of the esteemed implicit function. The remaining requirements of Theorem 3.2 can be easily verified. Notice that 1 is the unique common fixed point of f, g, S and T. This example cannot be used in the context of similar results contained in [15, 16] as those results require condition on containments amongst the ranges of the mappings. Notice that $f(X) = [1, 31] \not\subset [1, 8] = T(X)$ whereas $g(X) = [1, 127] \not\subset [1, 4] = S(X)$.

Next example shows an instance wherein Corollary 3.12 is applicable but Theorem 3.2 is not.

Example 3.14. Let $(X, \mathcal{M}_{M,N}, \mathcal{T})$ be an intuitionistic fuzzy metric space, wherein $X = [0, 1]$, $\mathcal{T}(a, b) = (a_1 b_1, \min\{a_2 + b_2, 1\})$ for all $a = (a_1, a_2)$ and $b = (b_1, b_2) \in L^*$ with

$$\mathcal{M}_{M,N}(x, y, t) = \left(\frac{t}{t + |x - y|}, \frac{|x - y|}{t + |x - y|)}\right).$$

Define self mappings f, g, S and T on X by

$$f(x) = \begin{cases} 1 & \text{if } x \in [0,1] \cap Q \\ \frac{1}{2} & \text{if } x \notin [0,1] \cap Q \end{cases}, \quad g(x) = \begin{cases} 1 & \text{if } x \in [0,1] \cap Q \\ \frac{1}{4} & \text{if } x \notin [0,1] \cap Q \end{cases},$$

$$S(x) = \begin{cases} 1 & \text{if } x = 1 \\ 0 & \text{if } x \in [0,1) \end{cases} \text{ and } T(x) = \begin{cases} 1 & \text{if } x = 1 \\ \frac{1}{3} & \text{if } x \in [0,1) \end{cases}.$$

Then $f^2(X) = \{1\} \subset \{\frac{1}{3}, 1\} = T^2(X)$ and $g^2(X) = \{1\} \subset \{0, 1\} = S^2(X)$. Define $F(t_1, t_2, t_3, t_4, t_5, t_6) = t_1 - \psi[\min\{t_2, t_3, t_4, t_5, t_6\}]$ where $\psi(s) = \sqrt{s}$ for all $s \in L^* \backslash \{\mathbf{0}, \mathbf{1}\}$ and $F \in \Psi$.
Now, for all $x, y \in X$ and $t > 0$, we have

$$\psi\Big[\min\big\{\mathcal{M}_{M,N}(S^2x, T^2y, t), \mathcal{M}_{M,N}(f^2x, S^2x, t), \mathcal{M}_{M,N}(g^2y, T^2y, t),$$
$$\mathcal{M}_{M,N}(S^2x, g^2y, t), \mathcal{M}_{M,N}(f^2x, T^2y, t)\big\}\Big]$$
$$\leq_{L^*} \mathbf{1} = \mathcal{M}_{M,N}(1, 1, t) = \mathcal{M}_{M,N}(f^2x, g^2y, t)$$

which demonstrates the verification of the esteemed implicit function. The remaining requirements of Corollary 3.12 can be easily verified. Notice that 1 is the unique common fixed point of f, g, S and T.

However this implicit function does not hold for the maps f, g, S and T in the respect of Theorem 3.2. Otherwise, with $x = 0$ and $y = \frac{1}{\sqrt{2}}$, we get

$$\psi\Big[\min\big\{\mathcal{M}_{M,N}(Sx, Ty, t), \mathcal{M}_{M,N}(fx, Sx, t), \mathcal{M}_{M,N}(gy, Ty, t),$$
$$\mathcal{M}_{M,N}(Sx, gy, t), \mathcal{M}_{M,N}(fx, Ty, t)\big\}\Big]$$
$$= \psi\Big[\min\big\{\mathcal{M}_{M,N}(0, \frac{1}{3}, t), \mathcal{M}_{M,N}(1, 0, t), \mathcal{M}_{M,N}(\frac{1}{4}, \frac{1}{3}, t), \mathcal{M}_{M,N}(0, \frac{1}{4}, t),$$
$$\mathcal{M}_{M,N}(1, \frac{1}{3}, t)\big\}\Big] \leq_{L^*} \mathcal{M}_{M,N}(1, \frac{1}{4}, t) = \mathcal{M}_{M,N}(fx, gy, t)$$

which is not true for all $t > 0$ (e.g. $t = \frac{1}{2}$). Thus Corollary 3.12 is a partial generalization of Theorem 3.2 and can be situationally useful.

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