DEFUZZIFICATION METHOD FOR RANKING FUZZY NUMBERS BASED ON CENTER OF GRAVITY

T. ALLAHVIRANLOO AND R. SANEIFARD

Abstract. Ranking fuzzy numbers plays a very important role in decision making and some other fuzzy application systems. Many different methods have been proposed to deal with ranking fuzzy numbers. Constructing ranking indexes based on the centroid of fuzzy numbers is an important case. But some weaknesses are found in these indexes. The purpose of this paper is to give a new ranking index to rank various fuzzy numbers effectively. Finally, several numerical examples following the procedure indicate the ranking results to be valid.

1. Introduction

A pervasive and controversial issue in the field of theoretical and applied research has been devoted to the role of fuzzy system theory in decision making process and other fuzzy application systems. It is acknowledged that fuzzy system theory has been improved steadily and has essential consequence for decision making process in engineering, economics, and social systems, to name a few (Zadeh, 1965). A growing body of research has concerned with fuzzy multi-attribute decision making problem especially in regard to show how ranking fuzzy numbers are not the total order relations under the ordinary meaning, but the partial order under the lattice structure. This implies that one significant aspect of fuzzy decision problems is the significance of ranking theories of fuzzy numbers. To date, substantial body of research on fuzzy system theory has examined different methods of ranking fuzzy numbers in different contexts. However, which types of methods are more influential is still substantiated. Among the existing ranking methods, centroid-index methods are considered as an important part of decision making problems. In that way the main concern of the present study is to investigate some drawbacks of the existing centroid ranking methods. As a result, conducting investigation on the possible drawbacks of the existing centroid ranking method (i.e., they cannot correctly rank fuzzy numbers in some situations), seems a fruitful area. Accordingly the present study aims to explore a novel technique for ordering fuzzy numbers (normal/non-normal/trapezoidal/general) to deal. The present paper also uses some examples to compare the proposed method with other ranking techniques.

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2. The Centroid Formulae For Fuzzy Numbers

A fuzzy number is a convex fuzzy subset of the real line R and is completely defined by its membership function. Let A be a fuzzy number, whose membership function $f_A(x)$ can generally be defined as in [5, 14, 15, 16, 18, 19, 25, 27],

$$f_A(x) = \begin{cases} f_A^L(x) & when \quad a \leq x < b, \\ \omega & when \quad b \leq x < c, \\ f_A^R(x) & when \quad c \leq x < d, \\ 0 & otherwise. \end{cases} \tag{1}$$

Where $0 \leq \omega \leq 1$ is a constant, $f_A^L : [a,b] \to [0,\omega]$ and $f_A^R : [c,d] \to [0,\omega]$ are two strictly monotonically and continuous mappings from R to closed interval $[0,\omega]$. If $\omega = 1$, then A is a normal fuzzy number; otherwise it is said to be a non-normal fuzzy number. If the membership function $f_A(x)$ is piecewise linear, then A is referred to as a trapezoidal fuzzy number and is usually denoted by $A = (a,b,c,d;\omega)$. In particular, when $b = c$, the trapezoidal fuzzy number is reduced to a triangular fuzzy number.
Since $f_A^L(x)$ and $f_A^R(x)$ are both strictly monotonicall and continuous functions, their inverse functions exist and should also be continuous and strictly monotonicall. Let $g_A^L : [0,\omega] \to [a,b]$ and $g_A^R : [0,\omega] \to [c,d]$ be the inverse functions of f_A^L and f_A^R , respectively. Then $g_A^L(y)$ and $g_A^R(y)$ should be integrable on the closed interval $[0,\omega]$. In other words, both $\int_0^\omega g_A^L(y)dy$ and $\int_0^\omega g_A^R(y)dy$ should exist. In the case of trapezoidal fuzzy number, the inverse function $g_A^L(y)$ and $g_A^R(y)$ can be analytically expressed as:

$$g_A^L(y) = a + \frac{(b-a)y}{\omega}, \quad 0 \leq y \leq \omega. \tag{2}$$

$$g_A^R(y) = d - \frac{(d-c)y}{\omega}, \quad 0 \leq y \leq \omega. \tag{3}$$

In order to determine the centroid point $(\overline{x}_0, \overline{y}_0)$ of a fuzzy number A, Wang et al. [30] provided the following centroid formulae:

$$\overline{x}_0(A) = \frac{\int_a^b x f_A^L(x)dx + \int_b^c (x\omega)dx + \int_c^d x f_A^R(x)dx}{\int_a^b f_A^L(x)dx + \int_b^c (\omega)dx + \int_c^d f_A^R(x)dx}, \tag{4}$$

$$\overline{y}_0(A) = \frac{\int_0^\omega y(g_A^R(y) - g_A^L(y))dy}{\int_0^\omega (g_A^R(y) - g_A^L(y))dy}. \tag{5}$$

Consider a general trapezoidal fuzzy number $A = [a,b,c,d;\omega]$, whose membership function is defined as

$$f_A(x) = \begin{cases} \frac{\omega(x-a)}{b-a} & when \quad a \leq x < b, \\ \omega & when \quad b \leq x < c, \\ \frac{\omega(d-x)}{d-c} & when \quad c \leq x < d, \\ 0 & otherwise. \end{cases} \tag{6}$$

For the present trapezoidal fuzzy number, the following figures demonstrate the result of the analysis derived from formulae 4 and 5,

$$\overline{x}_0(A) = \frac{1}{3}[a+b+c+d-\frac{dc-ab}{(d+c)-(a+b)}], \tag{7}$$

$$\overline{y}_0(A) = \omega.\frac{1}{3}[1+\frac{c-b}{(d+c)-(a+b)}]. \tag{8}$$

The ranking value R(A) of the fuzzy number A is defined as follows [11]:

$$R(A) = \sqrt{\overline{x}_0^2(A)+\overline{y}_0^2(A)}. \tag{9}$$

The larger the value of $R(A)$, the better the ranking of A. In [13], the authors presented a centroid-index ranking method for ordering fuzzy numbers. The centroid point of fuzzy number A, is $(\overline{x}_A, \overline{y}_A)$ where $\overline{x}_A$ and $\overline{y}_A$ are the same as formula 2 and 3 in [13]. The ranking value $S(A)$ of the fuzzy number A is defined as follows:

$$S(A) = \overline{x}_A\overline{y}_A. \tag{10}$$

The larger the value $S(A)$, the better the ranking of A. According to Zhao [32], the criterions provided by Cheng and Chu were not sufficient to promote some fuzzy numbers ranking and proposed another index to improve the first two methods. Based on Wang [30] concept, a revision of Cheng and Chu's method was presented. The results of the study showed that, these indexes have some obvious shortcomings for some fuzzy numbers ranking accordingly. To compensate for these shortcomings, a new index of ranking fuzzy numbers was explored in this study.

3. **A New Ranking Method For Fuzzy Numbers**

Considering the limitations of the previous studies and the existing gaps, we present a new approach for ranking fuzzy numbers based on the distance method. The method not only considers the centroid point of a fuzzy number, but also considers the maximum crisp value of fuzzy numbers.

For ranking fuzzy numbers, this study firstly defines a maximum crisp value $\mu_{\tau_{\max}}(x)$ to be the benchmark and its characteristic function $\mu_{\tau_{max}}(x)$ is as follows:

$$\mu_{\tau_{\max}}(x) = \begin{cases} 1, \ when \ x = \tau_{\max}, \\ 0, \ when \ x \neq \tau_{\max}. \end{cases} \tag{11}$$

When ranking n fuzzy numbers $A_1, A_2, \cdots, A_n$, the maximum crisp value $\tau_{\max}$ is defined as:

$$\tau_{\max} = max\{x | x \in Domain(A_1, A_2, \cdots, A_n)\}. \tag{12}$$

The advantages of the definition of maximum crisp value are two-fold: first, the maximum crisp values will be obtained by themselves, and secondly is it is easy to be computed.

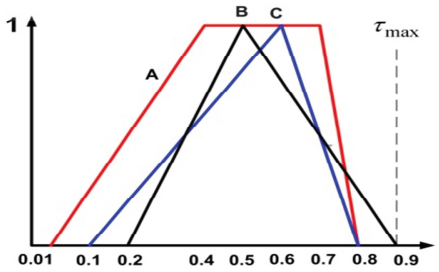


FIGURE 1. Fuzzy Numbers A, B, C and $\tau_{\max}$

Fn	The membership functions	The inverse functions
A	$\mu_A(x) = \begin{cases} 2.5x - 0.25, & 0.01 \le x \le 0.4, \\ 1, & 0.40 \le x \le 0.7, \\ -10x + 8, & 0.70 \le x \le 0.8. \end{cases}$	$g_A(x) = \begin{cases} L_A(x) = 0.30x + 0.01, \\ R_A(x) = -0.1x + 0.8. \end{cases}$
B	$\mu_B(x) = \begin{cases} 3.3x - 0.6, & 0.2 \le x \le 0.5, \\ 1, & x = 0.5, \\ -2.5x + 2.25, & 0.5 \le x \le 0.9. \end{cases}$	$g_B(x) = \begin{cases} L_B(x) = 0.30x + 0.20, \\ R_B(x) = -0.4x + 0.9. \end{cases}$
C	$\mu_C(x) = \begin{cases} 2x - 0.2, & 0.1 \le x \le 0.6, \\ 1, & x = 0.6, \\ -5x + 4, & 0.6 \le x \le 0.8. \end{cases}$	$g_C(x) = \begin{cases} L_C(x) = 0.50x + 0.10, \\ R_C(x) = -0.2x + 0.8. \end{cases}$

TABLE 1. Fuzzy Numbers A, B and C

Example 3.1. Three fuzzy numbers A, B and C have been illustrated by Chen [9] and their membership functions are shown in Table 3.1. The inverse functions calculated by equations 2.2 and 2.3 are also shown in this Table. The fuzzy numbers and the minimum crisp value are illustrated in Figure 1. By the equations 2.2, 2.3 and 3.12, we obtain τ_{max} as follows:

$$\tau_{\max} = max\{x | x \in Domain(A, B, C)\}$$
$$= max\{0.01, 0.1, 0.2, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\} = 0.9,$$

and

$$g_{max}(x) = \begin{cases} g^L_{max}(x) = 0.9, \\ g^R_{max}(x) = 0.9. \end{cases}$$

Assume that there are n fuzzy numbers $A_1, A_2, \cdots, A_n$. The proposed method for ranking fuzzy numbers $A_1, A_2, \cdots, A_n$ is now presented as follows:

Step 1: Use formulas 2.4 and 2.5 to calculate the centroid point $(\overline{x}_{0_{A_j}}, \overline{y}_{0_{A_j}})$ of each fuzzy numbers A_j, where $1 \le j \le n$.

Step 2: Calculate the maximum crisp value $\tau_{\max}$ of all fuzzy numbers A_j, where $1 \le j \le n$.

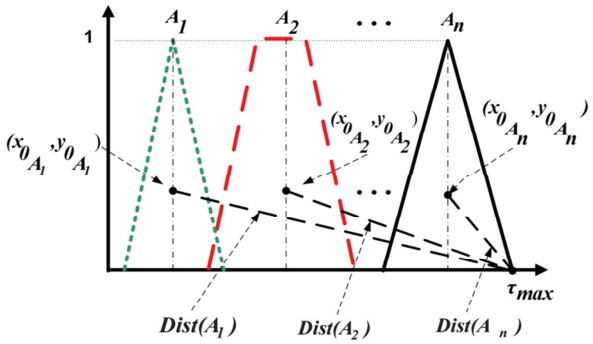


FIGURE 2. The Distance Between $(\overline{x}_{0_{A_j}}, \overline{y}_{0_{A_j}})$ and $(\tau_{\max}, 0)$

Step 3: Use the point $(\overline{x}_{0_{A_j}}, \overline{y}_{0_{A_j}})$ to calculate the ranking value $Dist(A_j)$ of the fuzzy numbers A_j, where $1 \leq j \leq n$, as follows:

$$Dist(A_j) = \sqrt{(\overline{x}_{0_{A_j}} - \tau_{\max})^2 + (\overline{y}_{0_{A_j}} - 0)^2} = \sqrt{(\overline{x}_{0_{A_j}} - \tau_{\max})^2 + (\overline{y}_{0_{A_j}})^2}. \quad (13)$$

From formula 13, we can see that $Dist(A_j)$ can be considered as the Euclidean distance between the point $(\overline{x}_{0_{A_j}}, \overline{y}_{0_{A_j}})$ and the point $(\tau_{\max}, 0)$, as shown in Figure 2. From this Figure, we can see that the smaller the value of $Dist(A_j)$, the better the ranking of A_j, where $1 \leq j \leq n$.

Let A be a fuzzy number characterized by 1 and $Dist(A)$ be the Euclidean distance between two points $(\overline{x}_{0_{A_j}}, \overline{y}_{0_{A_j}})$ and $(\tau_{\max}, 0)$ of it self.

Since the aim of the article is to approximate a fuzzy number by a scalar value, the researchers have to use an operator $Dist : F \to R$ (A space of all fuzzy numbers will be denoted by F) which transforms fuzzy numbers to a family of real line. Operator $Dist$ is a crisp approximation operator. Since all the above defuzzification can be used as a crisp approximation of a fuzzy number, the result value is used to rank the fuzzy numbers. Thus, $Dist$ is used to rank fuzzy numbers. The smaller $Dist$, the larger fuzzy number.

Let $A_1, A_2 \in F$ be two arbitrary fuzzy numbers. Define the ranking of A_1 and A_2 by $Dist$ on F as follows:

(1) $Dist(A_1) < Dist(A_2)$ if only if $A_1 \succ A_2$,

(2) $Dist(A_1) > Dist(A_2)$ if only if $A_1 \prec A_2$,

(3) $Dist(A_1) = Dist(A_2)$ if only if $A_1 \sim A_2$.

Then, this article formulates the order $\succeq$ and $\preceq$ as $A_1 \succeq A_2$ if and only if $A_1 \succ A_2$ or $A_1 \sim A_2$, $A_1 \preceq A_2$ if and only if $A_1 \prec A_2$ or $A_1 \sim A_2$. The new ranking index can sort many different fuzzy numbers simultaneously. In addition, the calculation is simple, and the index also satisfies the common properties of ranking fuzzy numbers:

(a) Transitivity of the order relation, that is if $A_1 \preceq A_2$ and $A_2 \preceq A_3$, then we should have $A_1 \preceq A_3$.

(b) Compatibility of addition, that is if there is $A_1 \preceq A_2$ on $\{A_1, A_2\}$, then there

is $A_1 + A_3 \preceq A_2 + A_3$ on $\{A_1 + A_3, A_2 + A_3\}$.
Some properties of the new index are given as follows:

Proposition 3.2. *Let $A \in F$, for all $\in [0, \omega]$,*

(1) If $inf\ supp(A) \geq 0 (inf_{y\in[0,\omega]}\ g_A^L(y) \geq 0)$, then $Dist(A) \geq 0$.
(2) If $sup\ supp(A) \leq 0 (sup_{y\in[0,\omega]}\ g_A^R(y) \leq 0)$, then $Dist(A) \geq 0$.

Proposition 3.3. *Let $A_1, A_2 \in F$,*

(1) If $A_1 \preceq A_2$, then $-A_2 \preceq -A_1$.
(2) If $A_1 \preceq (\prec)A_2$, for any arbitrary fuzzy number A_3, then $A_1 + A_3 \preceq (\prec)A_2 + A_3$.
(3) Let $A_1 = (a_1, b_1, c_1, d_1; \omega)$ and $A_2 = (a_2, b_2, c_2, d_2; \omega)$ are arbitrary fuzzy numbers. If sup $supp(A_1) \leq (<)inf\ supp(A_2)$,then $A_1 \preceq (\prec)A_2$.

Proof. (1), (2). It is easy to obtain the conclusion from 3.13.
(3). If *sup* $supp(A_1) \leq (<)inf\ supp(A_2)$, then $d_1 \leq a_2 (d_1 < a_2)$. For $\forall y \in [0, \omega]$, obviously, $\overline{x}_{0_{A_1}} \leq (<)\overline{x}_{0_{A_2}}, \overline{y}_{0_{A_1}} \leq (<)\overline{y}_{0_{A_2}}$, so we have $Dist(A_1) \geq (>)Dist(A_2)$ from 3.13. Thus $A_1 \preceq (\prec)A_2$. According to the method of ranking fuzzy numbers and the algorithm of ranking numbers, the corresponding examples are given as follows. □

4. **Examples**

In this section, we compare the proposed method with others in [2, 4, 13, 17, 21, 23, 29].

Example 4.1. Consider the data used in [1], i.e. three fuzzy numbers, $A = (5, 6, 7)$, $B = (5.9, 6, 7)$, $C = (6, 6, 7)$, as shown in Figure 3. According to the equation 3.13, the ranking index values are obtained i.e. $Dist(A) = 1.05$, $Dist(B) = 0.77$ and $Dist(C) = 0.74$. Accordingly, the ranking order of fuzzy numbers is $A \succ B \succ C$. However, by Chu and Tsao's approach [13], the ranking order is $A \succ C \succ B$. Meanwhile, using CV index proposed [11], the ranking order is $C \succ B \succ A$. From Figure 3, it is easy to see that the ranking results obtained by the existing approaches [13, 12] are unreasonable and are not consistent with human intuition. On the other hand, in [1, 24, 26], the ranking result is $A \succ B \succ C$, which is the same as the one obtained by the writers approach. However, their approach in the computation procedure is more simple.

Example 4.2. Consider the following set: $A = (1, 2, 5)$, $B = (0, 3, 4)$ and $C = (2, 2.5, 3)$,(see Figure 4). By using this new approach, $Dist(A) = 2.35$, $Dist(B) = 2.68$ and $Dist(C) = 2.52$. Hence, the ranking order is $B \succ C \succ A$ too. It seems that, the result obtained by "Distance Minimization" method is unreasonable. To compare with some of the other methods in [13, 20, 22] , the readers can refer to Table 4.2.

Example 4.3. The *Dist* values of 12 examples are shown in Figure 5. Table 4.3 shows the ranking results. From this Table, the main findings and *Dist* with some advantages are as follows:

1. In examples L, K, some methods use complicated and normalized process to rank and they can't obtain consistent results. However, their proposed method is

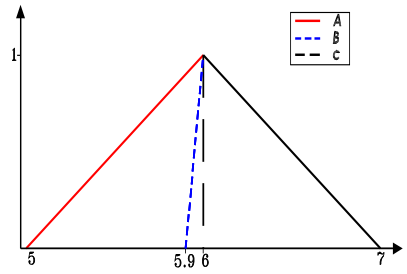


FIGURE 3

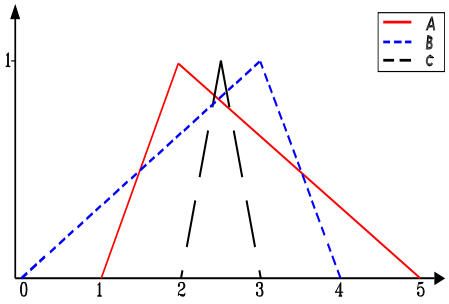


FIGURE 4

FN	New method	Sign distance p=2	Distance Minimization	Chu-Tsao	CV index
A	2.35	3.91	2.5	0.74	0.32
B	2.68	3.91	2.5	0.74	0.36
C	2.52	3.55	2.5	0.75	0.08
Results	$B \prec C \prec A$	$C \prec A \sim B$	$C \sim A \sim B$	$A \sim B \prec C$	$B \prec A \prec C$

TABLE 2. Comparative Results of Example 4.2

more suitable for ranking any kind of fuzzy number without normalization process.

2. For fuzzy numbers with the same mean (Examples B, I), Yager [31], Kerre [29], Bass and Kwakernaak [7] have not been able to obtain their orderings. Chang's method [8] has been able to rank their orderings, but Chang's results violate the smaller spread, the higher ranking order. In Examples B, I, it obviously shows that the researchers proposed method can rank instantly and their results comply with intuition of human being.

3. In Examples C, D, L we can see that the method of Kerre [29], Bass and Kwakernaak [7] have many limitations on triangle, trapezoid, non-normalized fuzzy numbers and so on.

New method	Yager [31]	Kerre [29]	Chang [8]
$A_1 \prec A_2$	$A_1 \prec A_2$	$A_1 \prec A_2$	$A_1 \prec A_2$
$B_1 \prec B_2$	$B_1 \sim B_2$	$B_1 \prec B_2$	$B_1 \succ B_2$
$C_1 \prec C_2 \prec C_3$	$C_1 \prec C_2 \prec C_3$	$C_1 \sim C_2 \prec C_3$	$C_1 \prec C_2 \prec C_3$
$D_1 \prec D_2 \prec D_3$	$D_1 \prec D_2 \prec D_3$	$D_2 \prec D_1 \prec D_3$	$D_1 \prec D_2 \prec D_3$
$E_1 \succ E_2$	$E_1 \succ E_2$	$E_1 \succ E_2$	$E_1 \succ E_2$
$F_1 \prec F_2$	$F_1 \prec F_2$	$F_1 \prec F_2$	$F_1 \succ F_2$
$G_1 \prec G_2 \prec G_3$	$G_1 \prec G_2 \prec G_3$	$G_1 \prec G_2 \prec G_3$	$G_1 \prec G_2 \prec G_3$
$H_1 \prec H_2 \prec H_3$	$H_1 \prec H_2 \prec H_3$	$H_1 \prec H_2 \prec H_3$	$H_1 \prec H_2 \prec H_3$
$I_1 \succ I_2$	$I_1 \sim I_2$	$I_1 \sim I_2$	$I_1 \succ I_2$
$J_1 \prec J_2 \prec J_3$	$J_1 \prec J_2 \prec J_3$	$J_1 \prec J_2 \prec J_3$	$J_1 \prec J_2 \prec J_3$
$K_1 \prec K_2$	$K_1 \prec K_2$	$K_1 \prec K_2$	$K_1 \prec K_2$
$L_1 \prec L_2$	$L_1 \prec L_2$	$L_1 \prec L_2$	$L_1 \succ L_2$

TABLE 3. The Comparison with Different Ranking Approaches of Example 4.3

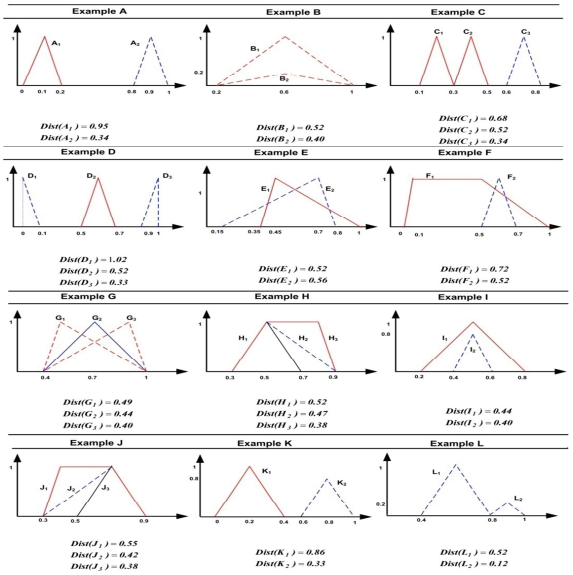


FIGURE 5

4. The proposed method can be used for ranking fuzzy numbers and crisp values. But Yager [31] has not been able to handle the crisp value problem.

5. Kerre's method [29] would favor a fuzzy number with smaller area measurement, regardless the relative location on the X-axis. The results are against their intuition in examples C, D. From Table 4.3 their proposed ranking method can correct the problem.

Example	FN's	Tseng and Klein	Kerre [29]	Lee and Li Proportional	Bass and Kwakernaak	Chen and Lu $\beta =$ 1 0.5 0
A	A_1	0.5	0.95	0.62	0.57	-0.04 0.005 0.05
	A_2	0.5	0.95	0.6	0.53	
B	B_1	0.87	0.99	0.8	0.56	0.1 0.3 0.5
	B_2	0.13	0.54	0.5	0.19	0.3 0.3 0.3
C	C_1	0.87	1.0	0.7	0.56	0.3 0.3 0.3
	C_2	0.13	0.55	0.4	0.19	
D	D_1	0.47	0.89	0.50	0.44	0.05 -0.03 -0.1
	D_2	0.53	0.95	0.53	0.48	
E	E_1	0.49	0.45	0.5	0.36	0.0 0.0 0.0
	E_2	0.51	0.96	0.50	0.39	
F	F_1	0.56	0.93	0.55	0.40	0.1 0.1 0.1
	F_2	0.44	0.87	0.45	0.36	
G	G_1	0.50	0.90	0.50	0.38	-0.1 0.0 0.1
	G_2	0.50	0.90	0.50	0.38	
H	H_1	0.52	1.00	0.40	0.29	0.02 0.02 0.02
	H_2	0.48	0.98	0.39	0.28	
I	I_1	0.56	1.0	0.60	0.33	0.0 0.025 0.050
	I_2	0.44	0.95	0.58	0.29	
J	J_1	0.64	1.0	0.60	0.38	0.0 0.075 0.150
	J_2	0.36	0.85	0.52	0.29	
K	K_1	0.58	1.0	0.58	0.38	0.0 0.05 0.1
	K_2	0.42	0.90	0.52	0.33	
L	L_1	0.52	1.0	0.60	0.57	0.07 0.05 0.00
	L_2	0.48	0.96	0.60	0.44	

TABLE 4. The Results of Comparison Using Tseng's (1989) Examples

Example 4.4. The other examples in Figure (4.4) are all positive fuzzy numbers, so they can be ranked by other methods. In this case, examples A, I, J, K and L of Tseng and Klein [28] are chosen to explain the results. They use other methods to explain the results of these methods and Table 4.4 shows the outcomes. We can easily see that most experimental results are consistent with other methods (Examples B, C, D, E, H, I, J and K). According to Table 4.4 the results of the methods are reconciled with those of other methods except for example A. In this example, Tseng, Klein [28] and Kerre [29] two fuzzy numbers are the same, but Baldwin and Guilds [6] is not, both agree that A_1 is larger than A_2 and their difference is very small. Because of the different β the Chen and Lu [10] approach from, we may get the results of ranking in reverse. Roughly, there is not much difference in the authors, method and theirs.

All the above examples show that this method is more consistent with institution than the previous ranking methods.

5. Conclusion

The current study suggests a new method for ranking fuzzy numbers. Consequently the proposed method considers the centroid points and maximum crisp value of fuzzy numbers to ranking fuzzy numbers. Therefore, the present study builds on previous studies to shed more light on the consideration of centroid methods.

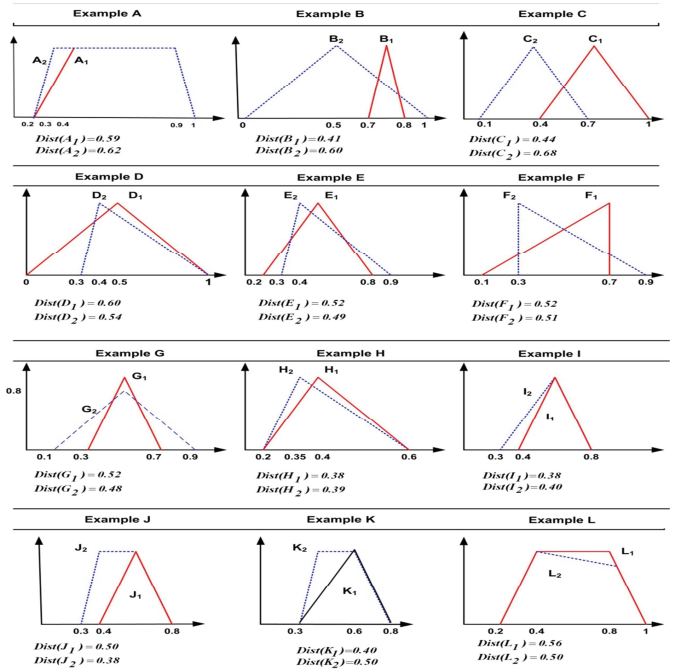


FIGURE 6

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Tofigh Allahviranloo*, Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran
E-mail address: allahviranloo@yahoo.com

Rahim Saneifard, Department of Mathematics, Science and Research Branch, Islamic Azad University, Tehran, Iran
E-mail address: srsaneeifard@yahoo.com

*Corresponding author