

ADAPTIVE FUZZY TRACKING CONTROL FOR A CLASS OF NONLINEAR SYSTEMS WITH UNKNOWN DISTRIBUTED TIME-VARYING DELAYS AND UNKNOWN CONTROL DIRECTIONS

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ABSTRACT. In this paper, an adaptive fuzzy control scheme is proposed for a class of perturbed strict-feedback nonlinear systems with unknown discrete and distributed time-varying delays, and the proposed design method does not require a priori knowledge of the signs of the control gains. Based on the backstepping technique, the adaptive fuzzy controller is constructed. The main contributions of the paper are that (i) by constructing appropriate Lyapunov functionals and using the Nussbaum functions, the adaptive tracking control problem is solved for the strict-feedback unknown nonlinear systems with the unknown discrete and distributed time-varying delays and the unknown control directions (ii) the number of adaptive parameters is independent of the number of rules of fuzzy logic systems and system state variables, which reduces the computation burden greatly. It is proven that the proposed controller guarantees that all the signals in the closed-loop system are bounded and the system output converges to a small neighborhood of the desired reference signal. Finally, an example is used to show the effectiveness of the proposed approach.

1. Introduction

Over the past few decades, considerable attention has been paid to fuzzy control and it has emerged as a promising way to handle nonlinear control problems and some fuzzy control techniques have been developed, for examples, [11, 12] and the references therein. Moreover, approximation-based adaptive fuzzy control has emerged as a popular and convenient tool in analysis and synthesis of complex and ill-defined systems, to which the application of conventional control techniques is not straightforward or feasible. The main idea of such a control methodology is to employ fuzzy logic systems to approximate the unknown nonlinearities in dynamic systems and the adaptive backstepping technique to construct the controllers. Following such an idea, some adaptive fuzzy control schemes were proposed for nonlinear systems [5, 8, 22, 23, 25, 26, 27, 35]. Moreover, some adaptive neural control methods have been developed for nonlinear systems, such as [6, 18, 20].

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It is well known that time delays are frequently encountered in real engineering systems, and the existence of time delays usually becomes the source of instability and degrading performance of systems. Therefore, the stability analysis and controller synthesis of nonlinear systems with time-delay are important. Recently, several fuzzy adaptive control schemes have been reported that combined the Lyapunov- Krasovskii functional and the FLS with the backstepping technique for nonlinear systems with constant or time-varying delays [2, 4, 30], in which Lyapunov- Krasovskii functionals are employed to compensate the time-delay terms, the FLS are used to approximate the unknown functions, and an adaptive fuzzy controller is constructed recursively, and in [7, 13], the systems with time delays were investigated by using the neural network control approach. However, it is worth noting that the results in [2, 4, 7, 13, 30] were obtained in the context of continuous systems with constant or time-varying delays. When the number of summands in a system equation is increased and the differences between neighboring argument values are decreased, systems with distributed delays will arise. Therefore, distributed delay systems have been an attractive research topic in the past years. For example, via different approaches, the authors in [25, 27] have investigated the robust H_∞ state feedback and output feedback controller design problem for uncertain distributed delay systems, respectively. The H_∞ filtering for uncertain distributed delay systems, either delay-independent or delay-dependent, have been considered in [33, 34, 36]. Nevertheless, the problem of adaptive fuzzy tracking control by using Lyapunov- Krasovskii functionals and the backstepping technique for uncertain systems with distributed time-varying delays is still open and remains unsolved, which motivates the present study.

On the other hand, a common weakness of the early fuzzy control methods is that the number of adaptation parameters depends on the number of the fuzzy rule bases. With an increase of fuzzy rules, the number of parameters to be estimated will increase significantly. As a result, the online learning time becomes prohibitively large. To avoid this disadvantage, authors in [4, 21, 29, 30] considered the norm of the ideal weighting vector in fuzzy logic systems as the estimation parameter instead of the elements of weighting vector. Therefore, the number of the online adaptive parameters is not more than the order of the original system. Thus, it is worth studying to devise the controller containing less adaptive parameters for nonlinear system with unknown discrete and distributed time-varying delays.

Another challenging problem for practical systems is that the control direction may be unknown. When the signs of control directions are unknown, the control problem becomes much more difficult, because in this case, we cannot decide the direction along which the control operates. In this case, Nussbaum-gain technique is an effective way to deal with it, for details see [1, 3, 9, 14, 15, 19, 37, 38], and the references therein. However, if the system is with time-varying delays, especially distributed time-varying delays, how to design the controller is a challenging problem. To the authors' knowledge, up to now, there is no result reported on this problem.

Motivated by the above observations, in this paper, the problem of output tracking is investigated for unknown strict-feedback nonlinear systems with unknown

control directions via fuzzy control method, in which the discrete and distributed time-varying delays appear in states. Under approximate Assumptions, the appropriate Lyapunov- Krasovskii functionals are constructed to compensate the unknown time-varying delays terms. Then, the FLS are employed to approximate the unknown nonlinear functions and Nussbaum-type functions are used to detect the high-frequency gain signs. Finally, the adaptive backstepping approach is utilized to construct the fuzzy controller. The main contributions of the paper are listed as follows.

(i) By combining the appropriate Lyapunov- Krasovskii functionals and the Nussbaum-type functions, the unknown control direction problem is solved for the strict-feedback nonlinear system with unknown discrete and distributed time-varying delays.

(ii) The number of adaptive parameters is independent of the number of rules of fuzzy logic systems and system state variables, which reduces the computation burden greatly.

The proposed controller guarantees the boundedness of all the signals in the closed loop system and gets a good tracking performance. Simulation results are provided to show the effectiveness of the proposed approach.

2. Problem Formulation and Preliminaries

2.1. Problem Formulation. Consider the following uncertain system with time-varying distributed delays

$$\begin{cases} \dot{x}_i = g_i(\bar{x}_i) x_{i+1} + f_i(\bar{x}_i) + h_i(\bar{x}_i(t), \bar{x}_i(t - d_{i1}(t))) \\ \quad + \int_{t-d_{i2}(t)}^t m_i(\bar{x}_i(s)) ds + \omega_i(t, x(t)), \\ \dot{x}_n = g_n(x) u + f_n(x) + h_n(x(t), x(t - d_{n1}(t))) \\ \quad + \int_{t-d_{n2}(t)}^t m_n(x(s)) ds + \omega_n(t, x(t)), \\ y(t) = x_1(t), \end{cases} \quad (1)$$

where $x = [x_1, \dots, x_n]^T \in R^n$, $u \in R$ and $y \in R$ denote state variable, the control input and the output of the system (1), respectively. For $1 \leq i \leq n - 1$, $\bar{x}_i(t - d_{i1}(t)) = [x_1(t - d_{i1}(t)), \dots, x_i(t - d_{i1}(t))]^T$ and $\bar{x}_i = [x_1, \dots, x_i]^T$. Functions $f_i(\cdot)$, $g_i(\cdot)$, $m_i(\cdot)$ and $h_i(\cdot)$ are unknown smooth functions. $d_{i1}(t)$ and $d_{i2}(t)$ represent unknown discrete and unknown distributed delays of system, respectively. $\omega_i(t, x(t))$ stand for unknown external disturbance inputs.

Control objectives: Utilizing the fuzzy logic systems and the backstepping technique to determine a fuzzy controller and parameter adaptive laws for system (1) such that the system output $y(t)$ tracks a desired reference signal $y_d(t)$, while all the signals remain bounded. To the end, define a vector function as $\bar{y}_d^T = [y_d, \dots, y_d^{(i)}]$, $i = 1, \dots, n$, where $y_d^{(i)}$ is the i th time derivative of y_d .

Now, the following Assumptions are introduced.

Assumption 2.1. $0 \leq d_{i1}(t) \leq d_1$ and $0 \leq d_{i2}(t) \leq d_2$. Furthermore, their derivations satisfy $\dot{d}_{i1}(t) \leq d_1^* < 1$ and $\dot{d}_{i2}(t) \leq d_2^* < 1$, respectively.

Remark 2.2. In this paper, it should be emphasized that the constants d_1 , d_2 , d_1^* and d_2^* are introduced only for the stability analysis and they are not used in the controller design, thus they can be unknown in the controller design procedure.

Assumption 2.3. For $i = 1, \dots, n$, the following inequality holds

$$|h_i(\bar{x}_i, \bar{x}_i(t - d_{i1}(t)))|^2 \leq h_{i1}(\bar{x}_i) + h_{i2}(\bar{x}_i(t - d_{i1}(t))), \quad (2)$$

where $h_{i1}(\bar{x}_i) \geq 0$ and $h_{i2}(\bar{x}_i(t - d_{i1}(t))) \geq 0$ are unknown smooth functions.

Assumption 2.4. For $1 \leq i \leq n$, there exist unknown positive smooth functions $\rho_i(\bar{x}_i)$ such that $|\omega_i(t, x)| \leq \rho_i(\bar{x}_i)$.

Assumption 2.5. The desired trajectory vectors \bar{y}_{di} are known and continuous, and $\bar{y}_{di} \in \Omega_{di} \subset R^{i+1}$ with Ω_{di} being known compact sets.

Assumption 2.6. For $1 \leq i \leq n$, the sign of $g_i(\bar{x}_i) \neq 0$ is unknown, and there exist b and c such that $0 < b \leq |g_i(\bar{x}_i)| \leq c < \infty$.

Remark 2.7. The existence of nonlinear distributed time-varying delays terms $\int_{t-d_{i2}(t)}^t m_i(\bar{x}_i(s)) ds$ makes controller design more difficult than the case without distributed time-varying delays terms [14, 15, 21, 29, 30]. During the controller design process, the discrete and distributed time-varying delays are required to satisfy Assumption 2.1. That is to say they are bounded and their derivations are less than constant one. Practically speaking, Assumption 2.1 is common and relaxed, for example [1, 2, 7, 36] and the references therein. On the other hand, Assumption 2.3 is imposed on functions $h_i(\bar{x}_i, \bar{x}_i(t - d_{i1}(t)))$, which means that they satisfy separation principle [17], and the Assumption is very common. It should be emphasized that in Assumption 2.6 the constants b and c are introduced only for the purpose of analysis and are not used to design controllers. Therefore, they can be unknown as well.

2.2. Preliminaries. Before proposing our main results, we introduce the knowledge about Nussbaum-type function and fuzzy logic system.

Definition 2.8. [21] Any continuous even function $N(\zeta) : R \rightarrow R$ is called a Nussbaum-type if it has the following properties:

$$\limsup_{s \rightarrow \infty} \frac{1}{s} \int_0^s N(\zeta) d\zeta = +\infty, \quad \liminf_{s \rightarrow \infty} \frac{1}{s} \int_0^s N(\zeta) d\zeta = -\infty.$$

Many functions such as $\zeta^2 \cos(\zeta)$, $\exp(\zeta^2) \cos(\frac{\pi}{2} \zeta)$ can serve as Nussbaum-type functions. Throughout this paper, we will choose even function $\exp(\zeta^2) \cos(\frac{\pi}{2} \zeta)$ as Nussbaum-type function to carry out the controller design and the simulation.

Lemma 2.9. [10] *If $N(\zeta)$ is a Nussbaum-type function, then:*

(1) *Given an arbitrary bounded function $g(\cdot) : R^n \rightarrow R$, and $|g(\cdot)| \in [\varepsilon_0, g_0]$ in which ε_0 is an arbitrary positive constant and g_0 is an unknown positive constant with $\varepsilon_0 < g_0 < +\infty$, then, $g(\cdot)N(\zeta)$ is also a Nussbaum-type function.*

(2) *Given an arbitrary function $c(\cdot) \in [-\varepsilon_0, \varepsilon_0] \subset R$, then, $N(\zeta) + c(\cdot)$ is also a Nussbaum-type function.*

Lemma 2.10. [21] *Let $V(t) \geq 0$ and $\zeta_i(t), i = 1, 2 \dots p$ be smooth functions defined on $[0, t_f)$, and $N(\zeta_i(t)), i = 1, 2 \dots p$ be even smooth Nussbaum-type functions. If the following inequality holds:*

$$V(t) \leq c_0 + e^{-Dt} \sum_{i=1}^p \int_0^t N(\zeta_i(\tau)) \dot{\zeta}_i(\tau) e^{D\tau} d\tau, \quad (3)$$

where p is bounded integer, D is positive constant and c_0 represents some suitable constant, then, $V(t), \zeta_i(t), i = 1, \dots, p$ and $\int_0^t N(\zeta_i(\tau)) \dot{\zeta}_i(\tau) d\tau$ must be bounded on $[0, t_f)$.

Lemma 2.9 has been proven in [10] (See pp. 510) and lemma 2.10 has been proven in [21] (See pp. 2750-2752, Appendix A).

Lemma 2.10 can be easily extended to case where $t_f = +\infty$ due to Proposition 2.11 given below. Consider [3, 14]

$$\dot{x}(t) = F(x(t)), \quad (4)$$

with $x(0) = x_0$, where $z \mapsto F(z) \subset R^N$ is upper semicontinuous on R^N with non-empty convex and compact values. It is well known that the initial-value problem has a solution and that every solution can be maximally extended.

Proposition 2.11. [24] *If $x : [0, t_f) \rightarrow R^N$ is a bounded maximal solution of (4), then $t_f = +\infty$.*

In this paper, the following rules are used to develop the adaptive fuzzy controller

R^l : if x_1 is F_1^l and x_2 is F_2^l and x_n is F_n^l , then y is $G^l, l = 1, 2, \dots, N$,

where $x = [x_1, \dots, x_n]^T$ and y are the FLS input and output, respectively. Fuzzy sets F_i^l and G^l , associated with the membership function $\mu_{F_i^l}(x_i)$ and $\mu_{G^l}(y)$, respectively. N is the rules number. Through singleton function, center average defuzzification, the FLS can be expressed as follows

$$y(x) = \frac{\sum_{l=1}^N \Phi_l \prod_{i=1}^n \mu_{F_i^l}(x_i)}{\sum_{l=1}^N \prod_{i=1}^n \mu_{F_i^l}(x_i)}, \quad (5)$$

where $\Phi_l = \arg \sup_{y \in R} \mu_{G^l}(y)$. Let fuzzy basis function

$$\xi_l = \frac{\prod_{i=1}^n \mu_{F_i^l}(x_i)}{\sum_{l=1}^N \prod_{i=1}^n \mu_{F_i^l}(x_i)}. \quad (6)$$

Denoting $\phi = [\Phi_1, \dots, \Phi_N]^T$ and $\xi(x) = [\xi_1(x), \dots, \xi_N(x)]^T$. Then, the fuzzy logic system (5) can be rewritten as

$$y(x) = \phi^T \xi(x). \quad (7)$$

Our first choice for the membership function is the Gaussian function $\mu_{F_i^l}(x_i) = \exp\left(-\frac{1}{2} \left(\frac{x_i - a_i^l}{\sigma_i^l}\right)^2\right)$, where σ_i^l and a_i^l are fixed parameters. It has been proven that when the membership functions are chosen as Gaussian functions, the above fuzzy logic system is capable of uniformly approximating any continuous nonlinear function over a compact set with any degree of accuracy. This property is shown by the following lemma.

Lemma 2.12. [28] *Let $f(x)$ be a continuous function defined on compact set Ω . Then, for any constant $\varepsilon > 0$, there exists an FLS (7) such that*

$$\sup_{x \in \Omega} |f(x) - \phi^T \xi(x)| \leq \varepsilon. \quad (8)$$

Lemma 2.13. [16] *For any constant matrix $M > 0$, any scalars c and d with $c < d$, and a vector function $\omega(s) : [c, d] \rightarrow R^n$ such that the integrals concerned as well defined, then the following holds*

$$\left(\int_c^d \omega^T(s) ds \right) M \left(\int_c^d \omega(s) ds \right) \leq (d-c) \int_c^d \omega^T(s) M \omega(s) ds. \quad (9)$$

3. Controller Design

In this section, we will use the recursive backstepping technique to develop the adaptive fuzzy tracking control laws as follows

$$\alpha_i = N(\zeta_i) M_i, \quad \dot{\zeta}_i = e_i M_i, M_i = k_i e_i + \frac{\vartheta_i \hat{\theta}_i \xi_i^T(Z_i) \xi_i(Z_i) e_i}{2\eta_i^2}, \quad (10)$$

$$\dot{\hat{\theta}}_i = \vartheta_i \xi_i^T(Z_i) \xi_i(Z_i) e_i^2 - \sigma_i \hat{\theta}_i, \quad (11)$$

where $1 \leq i \leq n$, $\vartheta_i > 0$ and $\sigma_i > 0$ are designed parameters. $\theta_i = \|\phi_i\|^2$ and ϕ_i is an unknown weight parameter vector and will be specified later. $\hat{\theta}_i$ is the estimation of θ_i and the estimation error is $\tilde{\theta}_i = \hat{\theta}_i - \theta_i$. $e_i = x_i - \alpha_{i-1}$, α_0 is equal to y_d , the control gain k_i satisfy $k_i > \frac{1}{\lambda_i}$ with λ_i being a positive design parameter, $\xi_i(Z_i)$ is a fuzzy basis function vector with Z_i being the input vector. Note that when $i = n$, α_n is the true control input $u(t)$.

Now, we propose the following backstepping-based design procedure.

Step1: Define tracking error as $e_1 = x_1 - y_d$. Then, its time derivative is given by

$$\begin{aligned} \dot{e}_1 = \dot{x}_1 - \dot{y}_d = f_1(x_1) + h_1(x_1, x_1(t-d_{11}(t))) + g_1(x_1) x_2 \\ + \int_{t-d_{12}(t)}^t m_1(x_1(s)) ds + \omega_1(t, x(t)) - \dot{y}_d. \end{aligned} \quad (12)$$

Choose Lyapunov-Krasovskii functional candidate as

$$V_{e_1} = \frac{1}{2} e_1^2 + \Lambda_1, \quad (13)$$

where

$$\begin{aligned} \Lambda_1 = \frac{d_2 e^{-\gamma(t-d_2)}}{2(1-d_2^*)} \int_{t-d_{12}(t)}^t \int_{\tau}^t e^{\gamma s} m_1^2(x_1(s)) ds d\tau \\ + \frac{e^{-\gamma(t-d_1)}}{2(1-d_1^*)} \int_{t-d_{11}(t)}^t e^{\gamma s} h_{12}(x_1(s)) ds \end{aligned}$$

and γ is a positive constant.

Differentiating V_{e_1} and then using (12) give that

$$\begin{aligned} \dot{V}_{e_1} = e_1 \dot{e}_1 + \frac{e^{\gamma d_1}}{2(1-d_1^*)} h_{12}(x_1) + \frac{d_{12}(t) d_2 e^{\gamma d_2}}{2(1-d_2^*)} m_1^2(x_1) \\ - \frac{1-d_{12}(t)}{2(1-d_2^*)} d_2 \int_{t-d_{12}(t)}^t e^{-\gamma(t-d_2-s)} m_1^2(x_1(s)) ds \\ - \frac{1-d_{11}(t)}{2(1-d_1^*)} e^{\gamma(d_1-d_{11}(t))} h_{12}(x_1(t-d_{11}(t))) - \gamma \Lambda_1. \end{aligned} \quad (14)$$

By Assumptions 2.1, 2.3-2.4 and the triangular inequality, the following inequalities can be obtained

$$\begin{aligned} e_1 h_1(x_1, x_1(t-d_{11}(t))) &\leq \frac{1}{2}e_1^2 + \frac{1}{2}h_{12}(x_1(t-d_{11}(t))) + \frac{1}{2}h_{11}(x_1), \\ -\frac{1-d_{12}(t)}{2(1-d_2^*)} &\leq -\frac{1}{2}, e_1 \omega_1(t, x) \leq \frac{e_1^2 \varphi_1^2(x_1)}{2a_{11}^2} + \frac{a_{11}^2}{2}, \\ -\frac{1-d_{11}(t)}{2(1-d_1^*)} e^{\gamma(d_1-d_{11}(t))} &\leq -\frac{1}{2}, \quad e^{-\gamma(t-d_2-s)} \geq 1, \quad s \in [t-d_{12}(t), t], \end{aligned} \quad (15)$$

we can also get the following inequalities by using Assumption 2.1, the triangular inequality and Lemma 2.13

$$\begin{aligned} &\frac{-(1-d_{12}(t))d_2}{2(1-d_2^*)} \int_{t-d_{12}(t)}^t e^{-\gamma(t-d_2-s)} m_1^2(x_1(s)) ds \\ &\leq -\frac{d_2}{2} \int_{t-d_{12}(t)}^t m_1^2(x_1(s)) ds, \\ e_1 \int_{t-d_{12}(t)}^t m_1(x_1(s)) ds &\leq \frac{1}{2}e_1^2 + \frac{1}{2} \left[\int_{t-d_{12}(t)}^t m_1(x_1(s)) ds \right]^2 \\ &\leq \frac{1}{2}e_1^2 + \frac{1}{2}d_2 \left[\int_{t-d_{12}(t)}^t m_1^2(x_1(s)) ds \right], \end{aligned} \quad (16)$$

where $a_{11} > 0$ is a design constant. Then, substituting $x_2 = e_2 + \alpha_1$ into (14) and using (15)-(16) yield that

$$\dot{V}_{e_1} \leq e_1 \left[f_1(\bar{x}_1) + g_1(\bar{x}_1)(e_2 + \alpha_1) - \dot{y}_d + e_1 + \frac{e_1 \varphi_1^2(x_1)}{2a_{11}^2} + \frac{H_{11}}{e_1} \right] + \frac{a_{11}^2}{2} - \gamma \Lambda_1,$$

where

$$H_{11} = \frac{1}{2}h_{11}(x_1) + \frac{1}{2(1-d_1^*)} e^{\gamma d_1} h_{12}(x_1) + \frac{d_{12}(t)d_2 e^{\gamma d_2}}{2(1-d_2^*)} m_1^2(x_1),$$

furthermore, it follows from $d_{12}(t) \leq d_2$ that

$$H_{11} \leq H_1 = \frac{1}{2}h_{11}(x_1) + \frac{1}{2(1-d_1^*)} e^{\gamma d_1} h_{12}(x_1) + \frac{d_2^2 e^{\gamma d_2}}{2(1-d_2^*)} m_1^2(x_1). \quad (17)$$

Then, we have

$$\begin{aligned} \dot{V}_{e_1} &\leq e_1 \left[f_1(x_1) + g_1(x_1)(e_2 + \alpha_1) - \dot{y}_d + e_1 + \frac{e_1 \varphi_1^2(x_1)}{2a_{11}^2} + \frac{H_1}{e_1} \right] \\ &\quad + \frac{a_{11}^2}{2} - \gamma \Lambda_1 \end{aligned} \quad (18)$$

Notice that in (18) $\frac{H_1}{e_1}$ is discontinuous at $e_1 = 0$. Therefore, it can not be approximated by the FLS. Similarly to [30], we introduce hyperbolic tangent function $\tanh(\frac{e_1}{\nu_1})$ to deal with the term. Define

$$\bar{f}_1(Z_1) = f_1(x_1) - \dot{y}_d + e_1 + \frac{e_1 \varphi_1^2(x_1)}{2a_{11}^2} + \frac{16}{e_1} \tanh^2\left(\frac{e_1}{\nu_1}\right) H_1, \quad (19)$$

where ν_1 is a positive design parameter, $Z_1 = [e_1, \bar{y}_{d1}^T]^T \in \Omega_{Z_1} \subset R^3$, and Ω_{Z_1} being some known compact set in R^3 . Note that $\lim_{e_1 \rightarrow 0} \frac{16}{e_1} \tanh^2\left(\frac{e_1}{\nu_1}\right) H_1$ exists, thus, the nonlinear function $\bar{f}_1(Z_1)$ can be approximated by an FLS $\phi_1^T \xi_1(Z_1)$ such that

$$\bar{f}_1(Z_1) = \phi_1^T \xi_1(Z_1) + \delta_1(Z_1). \quad (20)$$

Consequently, substituting (19) and (20) into (18) produces that

$$\begin{aligned} \dot{V}_{e_1} &\leq g_1(x_1)(e_2 + \alpha_1) e_1 + e_1 \left[\phi_1^T \xi_1(Z_1) + \delta_1(Z_1) \right] \\ &\quad + \left(1 - 16 \tanh^2\left(\frac{e_1}{\nu_1}\right) \right) H_1 + \frac{a_{11}^2}{2} - \gamma \Lambda_1. \end{aligned} \quad (21)$$

Let ε_1 be the upper bound of the fuzzy approximation error $\delta_1(Z_1)$ and according to the definition of θ_1 , the following inequality can be obtained

$$\begin{aligned} & e_1 \phi_1^T \xi_1(Z_1) + e_1 \delta_1(Z_1) \\ & \leq \frac{\vartheta_1 \theta_1}{2\eta_1^2} \xi_1^T(Z_1) \xi_1(Z_1) e_1^2 + \frac{\eta_1^2}{2\vartheta_1} + \frac{e_1^2}{2\lambda_1} + \frac{\lambda_1 \varepsilon_1^2}{2}. \end{aligned} \quad (22)$$

Then, combining (21) with (22) yields that

$$\begin{aligned} \dot{V}_{e_1} & \leq g_1(x_1) (e_2 + \alpha_1) e_1 + \frac{\vartheta_1 \theta_1}{2\eta_1^2} \xi_1^T \xi_1 e_1^2 + \frac{e_1^2}{2\lambda_1} \\ & \quad + q_1 + \left(1 - 16 \tanh^2\left(\frac{e_1}{\nu_1}\right)\right) H_1 - \gamma \Lambda_1 \end{aligned} \quad (23)$$

where $q_1 = \frac{a_{11}^2}{2} + \frac{\lambda_1 \varepsilon_1^2}{2} + \frac{\eta_1^2}{2\vartheta_1}$ with η_1 and λ_1 being positive design parameters.

Now, choosing the virtual control α_1 in (10) and substituting it into (23) yield that

$$\begin{aligned} \dot{V}_{e_1} & \leq g_1(x_1) e_2 e_1 + (g_1(x_1) N(\zeta_1) + 1) \dot{\zeta}_1 - k_1 e_1^2 + \frac{e_1^2}{2\lambda_1} + q_1 - \gamma \Lambda_1 \\ & \quad - \frac{\vartheta_1}{2\eta_1^2} \hat{\theta}_1 \xi_1^T(Z_1) \xi_1(Z_1) e_1^2 + \frac{\vartheta_1}{2\eta_1^2} \theta_1 \xi_1^T \xi_1 e_1^2 + \left(1 - 16 \tanh^2\left(\frac{e_1}{\nu_1}\right)\right) H_1, \end{aligned} \quad (24)$$

according to the definition of $\tilde{\theta}_1$, (24) becomes

$$\begin{aligned} \dot{V}_{e_1} & \leq g_1(x_1) e_2 e_1 + (g_1(x_1) N(\zeta_1) + 1) \dot{\zeta}_1 - e_1^2 \left(k_1 - \frac{1}{2\lambda_1}\right) \\ & \quad - \frac{\vartheta_1}{2\eta_1^2} \tilde{\theta}_1 \xi_1^T(Z_1) \xi_1(Z_1) e_1^2 + \left(1 - 16 \tanh^2\left(\frac{e_1}{\nu_1}\right)\right) H_1 + q_1 - \gamma \Lambda_1, \end{aligned} \quad (25)$$

where $g_1(x_1) e_2 e_1$ will be canceled in the next step.

Remark 3.1. In (13), we introduce the appropriate Lyapunov-Krasovskii functionals $\frac{e^{-\gamma(t-d_1)}}{2(1-d_1^*)} \int_{t-d_{11}(t)}^t e^{\gamma s} h_{12}(x_1(s)) ds$ and $\frac{e^{-\gamma(t-d_2)}}{2(1-d_2^*)} d_2 \int_{t-d_{12}(t)}^t \int_{\tau}^t e^{\gamma s} m_1^2(x_1(s)) ds d\tau$ to deal with the discrete and distributed time-varying delays terms, moreover, by employing them, we successfully construct the memoryless controller, and the stability analysis will be given later. In addition, it is worth pointing out that after disposing the time-delay terms we can get the equation (17) which is handled by the FLS in (20), thus, in this paper the time delays and the upper bounds of them do not need to be known.

Step i ($1 \leq i \leq n$): Considering $e_i = x_i - \alpha_{i-1}$, where α_{i-1} defined in (10), the dynamics of e_i -subsystem is given by

$$\begin{aligned} \dot{e}_i & = \dot{x}_i - \dot{\alpha}_{i-1} = f_i(\bar{x}_i) + g_i(\bar{x}_i) x_{i+1} + h_i(\bar{x}_i, \bar{x}_i(t-d_{i1}(t))) \\ & \quad + \int_{t-d_{i2}(t)}^t m_i(\bar{x}_i(s)) ds + \omega_i(t) - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (h_j(\bar{x}_j, \bar{x}_j(t-d_{j1}(t)))) \\ & \quad + \int_{t-d_{j2}(t)}^t m_j(\bar{x}_j(s)) ds + \omega_j(t, x) - W_i, \end{aligned}$$

where $W_i = \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} (f_j(\bar{x}_j) + g_j(\bar{x}_j) x_{j+1}) + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j + \frac{\partial \alpha_{i-1}}{\partial \bar{y}_{d(i-1)}} \dot{\bar{y}}_{d(i-1)}$. We

choose the Lyapunov-Krasovskii functional as

$$V_{e_i} = \frac{1}{2} e_i^2 + \Lambda_i,$$

where

$$\begin{aligned} \Lambda_i & = \frac{d_2 e^{-\gamma(t-d_2)}}{2(1-d_2^*)} \sum_{j=1}^i \int_{t-d_{j2}(t)}^t \int_{\tau}^t e^{\gamma s} m_j^2(\bar{x}_j(s)) ds d\tau \\ & \quad + \frac{e^{-\gamma(t-d_1)}}{2(1-d_1^*)} \sum_{j=1}^i \int_{t-d_{j1}(t)}^t e^{\gamma s} h_{j2}(\bar{x}_j(s)) ds. \end{aligned}$$

The time derivative of V_{e_i} is given by

$$\begin{aligned} \dot{V}_{e_i} &= e_i \dot{e}_i + \frac{e^{\gamma d_1}}{2(1-d_1^*)} \sum_{j=1}^i h_{j2}(\bar{x}_j) + \frac{d_2 e^{\gamma d_2}}{2(1-d_2^*)} \sum_{j=1}^i d_{j2}(t) m_j^2(\bar{x}_j) \\ &\quad - \sum_{j=1}^i \frac{1-d_{j1}(t)}{2(1-d_1^*)} e^{\gamma(d_1-d_{j1}(t))} h_{j2}(\bar{x}_j(t-d_{j1}(t))) \\ &\quad - \sum_{j=1}^i \int_{t-d_{j2}(t)}^t \frac{d_2(1-d_{j2}(t))}{2(1-d_2^*)} e^{-\gamma(t-d_2-s)} m_j^2(\bar{x}_j(s)) ds - \gamma \Lambda_i. \end{aligned} \quad (26)$$

By applying Assumptions 2.1, 2.3-2.4 and the triangular inequality, we have

$$\begin{aligned} e_i h_i(\bar{x}_i, \bar{x}_i(t-d_{i1}(t))) &\leq \frac{e_i^2}{2} + \frac{1}{2} h_{i2}(\bar{x}_i(t-d_{i1}(t))) + \frac{1}{2} h_{i1}(\bar{x}_i), \\ -e_i \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \omega_j(t, x) &\leq \frac{e_i^2}{2} \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \frac{\varphi_j}{a_{ij}} \right)^2 + \sum_{j=1}^{i-1} \frac{a_{ij}^2}{2}, \\ -e_i \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} h_j(\bar{x}_j, \bar{x}_j(t-d_{j1}(t))) &\leq \frac{e_i^2}{2} \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \right)^2 \\ &\quad + \frac{1}{2} \sum_{j=1}^{i-1} h_{j2}(\bar{x}_j(t-d_{j1}(t))) + \frac{1}{2} \sum_{j=1}^{i-1} h_{j1}(\bar{x}_j), \\ -\frac{(1-d_{j2}(t))e^{\gamma(d_2-d_{j2}(t))}}{2(1-d_2^*)} &\leq -\frac{1}{2}, e^{-\gamma(t-d_2-s)} \geq 1, \quad s \in [t-d_{j2}(t), t], \\ e_i \omega_i(t, x) &\leq \frac{e_i^2 \varphi_i^2(\bar{x}_i)}{2a_{ii}^2} + \frac{a_{ii}^2}{2}, \quad -\frac{(1-d_{j1}(t))e^{\gamma(d_1-d_{j1}(t))}}{2(1-d_1^*)} < -\frac{1}{2}, \end{aligned} \quad (27)$$

then, by using Assumption 2.1, the triangular inequality and Lemma 2.13, we have

$$\begin{aligned} &-\frac{(1-d_{j2}(t))d_2}{2(1-d_2^*)} \int_{t-d_{j2}(t)}^t e^{-\gamma(t-d_2-s)} m_j^2(\bar{x}_j(s)) ds \\ &\leq -\frac{1}{2} d_2 \int_{t-d_{j2}(t)}^t m_j^2(\bar{x}_j(s)) ds, \\ e_i \int_{t-d_{i2}(t)}^t m_i(\bar{x}_i(s)) ds &\leq \frac{1}{2} e_i^2 + \frac{1}{2} \left[\int_{t-d_{i2}(t)}^t m_i(\bar{x}_i(s)) ds \right]^2 \\ &\leq \frac{1}{2} e_i^2 + \frac{1}{2} d_2 \left[\int_{t-d_{i2}(t)}^t m_i^2(\bar{x}_i(s)) ds \right], \quad e_i \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \int_{t-d_{j2}(t)}^t m_j(\bar{x}_j(s)) ds \\ &\leq \frac{e_i^2}{2} \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \right)^2 + \sum_{j=1}^{i-1} \frac{1}{2} \left(\int_{t-d_{j2}(t)}^t m_j(\bar{x}_j(s)) ds \right)^2, \end{aligned} \quad (28)$$

where $a_{ij} > 0$ are design parameters. Consequently, substituting (27) and (28) into (26) produces that

$$\begin{aligned} \dot{V}_{e_i} &\leq g_i(\bar{x}_i) e_i x_{i+1} + e_i \left(f_i(\bar{x}_i) + e_i \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \right)^2 + \frac{e_i}{2} \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \frac{\varphi_j}{a_{ij}} \right)^2 \right) \\ &\quad + e_i + g_{i-1} e_{i-1} - W_i + \frac{H_{i1}}{e_i} + \frac{e_i \varphi_i^2(\bar{x}_i)}{2a_{ii}^2} + \sum_{j=1}^i \frac{a_{ij}^2}{2} - g_{i-1} e_{i-1} e_i - \gamma \Lambda_i, \end{aligned} \quad (29)$$

where

$$H_{i1} = \frac{1}{2} \sum_{j=1}^i h_{j1}(\bar{x}_j) + \frac{1}{2(1-d_1^*)} e^{\gamma d_1} \sum_{j=1}^i h_{j2}(\bar{x}_j) + \frac{d_2 e^{\gamma d_2}}{2(1-d_2^*)} \sum_{j=1}^i d_{j2}(t) m_j^2(\bar{x}_j)$$

Furthermore, in the light of $d_{j2}(t) \leq d_2$, we get

$$H_{i1} \leq H_i = \frac{1}{2} \sum_{j=1}^i h_{j1}(\bar{x}_j) + \frac{1}{2(1-d_1^*)} e^{\gamma d_1} \sum_{j=1}^i h_{j2}(\bar{x}_j) + \frac{d_2^2 e^{\gamma d_2}}{2(1-d_2^*)} \sum_{j=1}^i m_j^2(\bar{x}_j).$$

Then, similar to step 1, (29) can be rewritten as

$$\begin{aligned} \dot{V}_{e_i} &\leq g_i(\bar{x}_i) e_i x_{i+1} + e_i \bar{f}_i(Z_i) + \sum_{j=1}^i \frac{a_{ij}^2}{2} - g_{i-1}(\bar{x}_{i-1}) e_{i-1} e_i \\ &\quad + \left(1 - 16 \tanh^2 \left(\frac{e_i}{\nu_i} \right) \right) H_i - \gamma \Lambda_i, \end{aligned} \quad (30)$$

where $\bar{f}_i(Z_i)$ is defined as

$$\begin{aligned} \bar{f}_i(Z_i) &= f_i(\bar{x}_i) + g_{i-1}(\bar{x}_{i-1})e_{i-1} + e_i + e_i \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \right)^2 + \\ &\quad \frac{e_i}{2} \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j} \frac{\varphi_j}{a_{ij}} \right)^2 - W_i + \frac{e_i \varphi_i^2(\bar{x}_i)}{2a_{ii}^2} + \frac{16}{e_i} \tanh^2 \left(\frac{e_i}{\nu_i} \right) H_i, \end{aligned}$$

next, we will approximate $\bar{f}_i(Z_i)$ by using the FLS, now define

$$\phi_i^T \xi_i + \delta_i(Z_i) = \bar{f}_i(Z_i), \quad (31)$$

where $Z_i = [\bar{e}_i^T, \bar{y}_{di}^T]^T \in \Omega_{Z_i} \subset R^{2i+1}$ with $\bar{e}_i = [e_1, e_2, \dots, e_i]^T$ and Ω_{Z_i} being some known compact set. Consequently, by substituting (31) into (30), one can get

$$\begin{aligned} \dot{V}_{e_i} &\leq g_i(\bar{x}_i) e_i x_{i+1} + e_i (\phi_i^T \xi_i + \delta_i(Z_i)) + \sum_{j=1}^i \frac{a_{ij}^2}{2} \\ &\quad - g_{i-1}(\bar{x}_{i-1}) e_{i-1} e_i + \left(1 - 16 \tanh^2 \left(\frac{e_i}{\nu_i} \right) \right) H_i - \gamma \Lambda_i, \end{aligned} \quad (32)$$

according to the inequalities similar to (22), the above formula becomes

$$\begin{aligned} \dot{V}_{e_i} &\leq g_i(\bar{x}_i) e_{i+1} e_i + g_i(\bar{x}_i) e_i \alpha_i + \frac{\vartheta_i \theta_i}{2\eta_i^2} \xi_i^T(Z_i) \xi_i(Z_i) e_i^2 + q_i + \frac{e_i^2}{2\lambda_i} \\ &\quad + \left(1 - 16 \tanh^2 \left(\frac{e_i}{\nu_i} \right) \right) H_i - g_{i-1}(\bar{x}_{i-1}) e_{i-1} e_i - \gamma \Lambda_i, \end{aligned} \quad (33)$$

where $q_i = \sum_{j=1}^i \frac{a_{ij}^2}{2} + \frac{\lambda_i \varepsilon_i^2}{2} + \frac{\eta_i^2}{2\vartheta_i}$. Now, substituting the virtual control defined α_i in (10) into (33) yields

$$\begin{aligned} \dot{V}_{e_i} &\leq g_i(\bar{x}_i) e_{i+1} e_i + (g_i(\bar{x}_i) N(\zeta_i) + 1) \dot{\zeta}_i - \frac{\vartheta_i \bar{\theta}_i}{2\eta_i^2} \xi_i^T(\bar{Z}_i) \xi_i(\bar{Z}_i) e_i^2 + q_i \\ &\quad - e_i^2 \left(k_i - \frac{1}{2\lambda_i} \right) + \left(1 - 16 \tanh^2 \left(\frac{e_i}{\nu_i} \right) \right) H_i - g_{i-1}(\bar{x}_{i-1}) e_{i-1} e_i - \gamma \Lambda_i. \end{aligned} \quad (34)$$

Step n: In this step, the true control $u(t)$ will be constructed. Considering $e_n = x_n - \alpha_{n-1}$, the dynamics of e_n -subsystem is given by

$$\begin{aligned} \dot{e}_n &= \dot{x}_n - \dot{\alpha}_{n-1} = f_n(\bar{x}_n) + g_n(\bar{x}_n) u + h_n(\bar{x}_n, \bar{x}_n(t - d_{n1}(t))) \\ &\quad - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} \left(h_j(\bar{x}_j, \bar{x}_j(t - d_{j1}(t))) + \int_{t-d_{j2}(t)}^t m_j(\bar{x}_j(s)) ds + \omega_j(t, x) \right) \\ &\quad + \int_{t-d_{n2}(t)}^t m_n(\bar{x}_n(s)) ds + \omega_n(t, x(t)) - W_n, \end{aligned} \quad (35)$$

where

$$W_n = \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j} (f_j(\bar{x}_j) + g_j(\bar{x}_j) x_{j+1}) + \frac{\partial \alpha_{n-1}}{\partial \bar{y}_{d(n-1)}} \dot{\bar{y}}_{d(n-1)} + \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}_j} \dot{\hat{\theta}}_j.$$

Then, choose the following Lyapunov-Krasovskii functional

$$V_{e_n} = \frac{1}{2} e_n^2 + \Lambda_n, \quad (36)$$

where

$$\begin{aligned} \Lambda_n &= \frac{e^{-\gamma(t-d_2)}}{2(1-d_2^*)} d_2 \sum_{j=1}^n \int_{t-d_{j2}(t)}^t \int_{\tau}^t e^{\gamma s} m_j^2(\bar{x}_j(s)) ds d\tau \\ &\quad + \frac{e^{-\gamma(t-d_1)}}{2(1-d_1^*)} \sum_{j=1}^n \int_{t-d_{j1}(t)}^t e^{\gamma s} h_{j2}(\bar{x}_j(s)) ds. \end{aligned}$$

Differentiating V_{e_n} yields that

$$\begin{aligned}\dot{V}_{e_n} &= e_n \dot{e}_n + \frac{e^{\gamma d_1}}{2(1-d_1^*)} \sum_{j=1}^n h_{j2}(\bar{x}_j) + \frac{d_2 e^{\gamma d_2}}{2(1-d_2^*)} \sum_{j=1}^n d_{j2}(t) m_j^2(\bar{x}_j) \\ &\quad - \sum_{j=1}^n \frac{1-d_{j1}(t)}{2(1-d_1^*)} e^{\gamma(d_1-d_{j1}(t))} h_{j2}(\bar{x}_j(t-d_{j1}(t))) \\ &\quad - \sum_{j=1}^n \int_{t-d_{j2}(t)}^t \frac{d_2(1-d_{j2}(t))}{2(1-d_2^*)} e^{-\gamma(t-d_2-s)} m_j^2(\bar{x}_j(s)) ds - \gamma \Lambda_n.\end{aligned}\quad (37)$$

Moreover, similar to step i , we have

$$\begin{aligned}\dot{V}_{e_n} &\leq g_n(\bar{x}_n) e_n u + e_n (f_n(\bar{x}_n) + g_{n-1}(\bar{x}_{n-1}) e_{n-1}) \\ &\quad + e_n + e_n \sum_{j=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} \right)^2 + \frac{e_n}{2} \sum_{j=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} \frac{\varphi_j}{a_{nj}} \right)^2 \\ &\quad - W_n + \frac{H_n}{e_n} + \frac{e_n \varphi_n^2(\bar{x}_n)}{2a_{nn}^2} + \sum_{j=1}^n \frac{a_{nj}^2}{2} - g_{n-1} e_{n-1} e_n - \gamma \Lambda_n,\end{aligned}\quad (38)$$

where

$$H_n = \frac{e^{\gamma d_1}}{2(1-d_1^*)} \sum_{j=1}^n h_{j2}(\bar{x}_j) + \frac{1}{2} \sum_{j=1}^n h_{j1}(\bar{x}_j) + \frac{d_2^2 e^{\gamma d_2}}{2(1-d_2^*)} \sum_{j=1}^n m_j^2(\bar{x}_j).$$

Now define

$$\begin{aligned}\phi_n^T \xi_n + \delta_n(Z_n) &= \bar{f}_n(Z_n) \\ &= f_n(\bar{x}_n) + g_{n-1}(\bar{x}_{n-1}) e_{n-1} + e_n + \frac{e_n \varphi_n^2(\bar{x}_n)}{2a_{nn}^2} - W_n + e_n \sum_{j=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} \right)^2 \\ &\quad + \frac{e_n}{2} \sum_{j=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j} \frac{\varphi_j(\bar{x}_j)}{a_{nj}} \right)^2 + \frac{16}{e_n} \tanh^2 \left(\frac{e_n}{\nu_n} \right) H_n,\end{aligned}$$

where $Z_n = [e^T, \bar{y}_{dn}^T]^T \in \Omega_{Z_n} \subset R^{2n+1}$ with $e = [e_1, e_2, \dots, e_n]^T$ and Ω_{Z_n} being some known compact set. Consequently, (38) becomes

$$\begin{aligned}\dot{V}_{e_n} &\leq g_n(\bar{x}_n) e_n u + e_n (\phi_n^T \xi_n + \delta_n(Z_n)) + \sum_{j=1}^n \frac{a_{nj}^2}{2} \\ &\quad - g_{n-1}(\bar{x}_{n-1}) e_{n-1} e_n + \left(1 - 16 \tanh^2 \left(\frac{e_n}{\nu_n} \right) \right) H_n - \gamma \Lambda_n.\end{aligned}\quad (39)$$

Combining (39) with similar to (22) produces

$$\begin{aligned}\dot{V}_{e_n} &\leq g_n(\bar{x}_n) e_n u - g_{n-1}(\bar{x}_{n-1}) e_{n-1} e_n + \frac{e_n^2}{2\lambda_n} + \frac{\vartheta_n \hat{\theta}_n}{2\eta_n^2} \xi_n^T \xi_n e_n^2 + q_n - \gamma \Lambda_n \\ &\quad + \left(1 - 16 \tanh^2 \left(\frac{e_n}{\nu_n} \right) \right) H_n - \frac{\gamma e^{-\gamma(t-d_1)}}{2(1-d_1^*)} \sum_{j=1}^n \int_{t-d_{j1}(t)}^t e^{\gamma \tau} h_{j2}(\bar{x}_j(s)) ds.\end{aligned}\quad (40)$$

Now, choosing the virtual control as

$$u = N(\zeta_n) M_n, \quad \dot{\zeta}_n = e_n M_n, \quad M_n = k_n e_n + \frac{\vartheta_n \hat{\theta}_n \xi_n^T(Z_n) \xi_n(Z_n) e_n}{2\eta_n^2},$$

we have

$$\begin{aligned}\dot{V}_{e_n} &\leq (g_n(\bar{x}_n) N(\zeta_n) + 1) \dot{\zeta}_n - \frac{\vartheta_n \hat{\theta}_n}{2\eta_n^2} \xi_n^T \xi_n e_n^2 - e_n^2 \left(k_n - \frac{1}{2\lambda_n} \right) \\ &\quad + \left(1 - 16 \tanh^2 \left(\frac{e_n}{\nu_n} \right) \right) H_n - g_{n-1}(\bar{x}_{n-1}) e_{n-1} e_n + q_n - \gamma \Lambda_n.\end{aligned}\quad (41)$$

Finally, choosing a Lyapunov function

$$V(t) = \sum_{j=1}^n V_{e_j} + \sum_{j=1}^n \frac{\hat{\theta}_j^2}{4\eta_j^2},$$

and using the adaptive laws defined in (11) give that

$$\begin{aligned} \dot{V}(t) \leq & \sum_{j=1}^n (g_j(\bar{x}_j) N(\zeta_j) + 1) \dot{\zeta}_j - \sum_{j=1}^n \frac{\sigma_j \bar{\theta}_j}{2\eta_j^2} \hat{\theta}_j + \sum_{j=1}^n q_j \\ & - \sum_{j=1}^n e_j^2 \left(k_j - \frac{1}{2\lambda_j} \right) + \sum_{j=1}^n \left(1 - 16 \tanh^2 \left(\frac{e_j}{\nu_j} \right) \right) H_j - \gamma \Lambda_n, \end{aligned} \quad (42)$$

where q_j is defined as $q_j = \sum_{k=1}^j \frac{a_{jk}^2}{2} + \frac{\lambda_j \epsilon_j^2}{2} + \frac{\eta_j^2}{2\vartheta_j}$. Next, we will give the following theorem.

Theorem 3.2. *Consider the closed-loop system consisting of system (1) under Assumptions 2.1, 2.3-2.6, the control laws (10) and the adaptive laws (11). Suppose that the packaged uncertain functions $\tilde{f}_i(Z_i), i = 1, 2, \dots, n$, can be approximated by fuzzy logic systems in the sense that approximation errors are bounded. Then, with bounded initial conditions $\hat{\theta}(t_0) \geq 0$ and $\zeta_i(t_0)$, for $1 \leq i \leq n$, the following properties are guaranteed:*

- (i) all the signals in the closed-loop system are bounded;
- (ii) the error signal e , in the mean square sense, eventually converges to the following set:

$$\Xi_\chi := \{e \in R^n \mid e_{r\chi} \leq \Pi_\chi\},$$

where $e = [e_1, \dots, e_n]^T$, $e_{r\chi} = \frac{1}{t} \int_0^t \|e(\tau)\|^2 d\tau$ and Π_χ will be given later, see (69).

Remark 3.3. In [29, 30], the strict-feedback adaptive fuzzy tracking control problem was investigated by the control directions known. In [21] the tracking control was addressed for nonlinear delay-free systems with unknown control directions. However, in this paper, the existence of the unknown discrete and distributed time-varying delays terms makes the controller design much more complex and difficult. For $i = 1, \dots, n$, by employing the Lyapunov-Krasovskii functionals terms $\frac{e^{-\gamma(t-d_1)}}{2(1-d_1^*)} \sum_{j=1}^i \int_{t-d_{j1}(t)}^t e^{\gamma\tau} h_{j2}(\bar{x}_j(\tau)) d\tau$ and $\frac{e^{-\gamma(t-d_2)}}{2(1-d_2^*)} d_2 \sum_{j=1}^i \int_{t-d_{j2}(t)}^t \int_s^t e^{\gamma\tau} m_j^2(\bar{x}_j(s)) d\tau ds$, we successfully construct the memoryless controller such that the system output y tracks a desired reference signal y_d , while all the signals in the closed-loop system remain bounded.

Before proving of this theorem, we first introduce the following lemma.

Lemma 3.4. [30] *For $1 \leq j \leq n$ and $\nu_j > 0$, consider the set Θ_{ν_j} given by $\Theta_{\nu_j} := \{e_j \mid |e_j| \leq 0.2554\nu_j\}$. Then, for $e_j \notin \Theta_{\nu_j}$, the inequalities $1 - 16 \tanh^2 \left(\frac{e_j}{\nu_j} \right) < 0$ are satisfied.*

Proof. Now, we assume that $\nu_j = \nu$, for $j = 1, \dots, n$, where $\nu > 0$ is an arbitrary small constant. Then, the set Θ_{ν_j} can be written as Θ_ν . Firstly, we need to prove that all the signals in the closed-loop system are boundedness.

(i) Boundedness of all signals in the closed-loop system can be proved by the following three cases.

Case 1: For $j = 1, \dots, n$, $e_j \in \Theta_\nu$. In this case, $|e_j| \leq 0.2554\nu$.

According to (10), it is obvious that $\hat{\theta}_j$ is bounded for bounded e_j . Further, $\tilde{\theta}_j$ is bounded as θ_j is a constant. Next, we need to prove the boundedness of all other signals in the closed-loop systems step by step.

Firstly, according to Assumption 2.5 one gets the boundedness of y_d . Since $e_1 = x_1 - y_d$ and y_d are bounded, the boundedness of x_1 is obtained. In addition, since H_1 is a continuous function of x_1 , we can conclude that H_1 is also bounded. Let \bar{H}_1 be the upper bound of H_1 , that is, to say $H_1 \leq \bar{H}_1$, where \bar{H}_1 is a positive constant. Now, considering the following function candidate

$$V_1 = V_{e_1} + \frac{\tilde{\theta}_1^2}{4\eta_1^2},$$

by using (25) and (11), one gets

$$\begin{aligned} \dot{V}_1 &\leq g_1(x_1) e_2 e_1 + (g_1(x_1) N(\zeta_1) + 1) \dot{\zeta}_1 + C_1 - \frac{\sigma_1 \tilde{\theta}_1^2}{4\eta_1^2} \\ &\quad - \left(k_1 - \frac{1}{2\lambda_1}\right) e_1^2 + (1 - 16 \tanh^2(\frac{e_1}{\nu})) H_1 - \gamma \Lambda_1, \end{aligned} \quad (43)$$

where $C_1 = \frac{\sigma_1 \tilde{\theta}_1^2}{4\eta_1^2} + q_1$. Moreover, according to $|e_j| \leq 0.2554\nu$ and Lemma 3.4, we have

$$g_1(x_1) e_2 e_1 \leq \frac{e_2^2}{2\lambda_1} + \frac{1}{2} \lambda_1 c^2 (0.2554\nu)^2, \quad 0 \leq (1 - 16 \tanh^2(\frac{e_1}{\nu})) \leq 1. \quad (44)$$

Following in view of $H_1 \leq \bar{H}_1$, we have

$$0 \leq (1 - 16 \tanh^2(e_1/\nu)) H_1 \leq \bar{H}_1. \quad (45)$$

Therefore, combining (43), (44) with (45) yields that

$$\dot{V}_1 \leq (g_1(x_1) N(\zeta_1) + 1) \dot{\zeta}_1 - \left(k_1 - \frac{1}{\lambda_1}\right) e_1^2 + \bar{K}_1 - \frac{\sigma_1 \tilde{\theta}_1^2}{4\eta_1^2} - \gamma \Lambda_1, \quad (46)$$

where $\bar{K}_1 = \bar{H}_1 + \lambda_1 c^2 (0.2554\nu)^2 / 2 + C_1$ and $k_1 > \frac{1}{\lambda_1}$, so we can deduce $k_1 - \frac{1}{\lambda_1} > 0$. Now, denoting $\psi_1 = \min(2k_1 - 2/\lambda_1, \sigma_1, \gamma)$, one gets

$$\dot{V}_1 \leq -\psi_1 V_1 + (g_1(x_1) N(\zeta_1) + 1) \dot{\zeta}_1 + \bar{K}_1, \quad (47)$$

multiplying both side by $e^{\psi_1 t}$, (47) can be expressed as

$$\frac{dV_1(t) e^{\psi_1 t}}{dt} \leq \bar{K}_1 e^{\psi_1 t} + e^{\psi_1 t} (g_1(x_1) N(\zeta_1) + 1) \dot{\zeta}_1. \quad (48)$$

Integrating (48) over $[0, t]$ shows that

$$\begin{aligned} 0 \leq V_1(t) &\leq \frac{\bar{K}_1}{\psi_1} \\ &\quad + \left[V_1(0) - \frac{\bar{K}_1}{\psi_1}\right] e^{-\psi_1 t} + e^{-\psi_1 t} \int_0^t (g_1(x_1) N(\zeta_1) + 1) \dot{\zeta}_1 e^{\psi_1 \tau} d\tau. \end{aligned} \quad (49)$$

Note that $0 < e^{-\psi_1 t} < 1$, we have $\left[V_1(0) - \frac{\bar{K}_1}{\psi_1}\right] e^{-\psi_1 t} \leq V_1(0)$. Then, (49) becomes

$$0 \leq V_1 \leq \frac{\bar{K}_1}{\psi_1} + V(0) + \int_0^t e^{-\psi_1(t-\tau)} (g_1(x_1) N(\zeta_1) + 1) \dot{\zeta}_1 d\tau. \quad (50)$$

From Assumption 2.6, we know that $g_1(x_1)$ is a bounded function. With the help of Lemma 2.9, it can be shown that $g_1(x_1)N(\zeta_1) + 1$ is Nussbaum-type function. Then, using Lemma 2.10, it can easily conclude that $V_1(t)$, $\zeta_1(t)$ and $\int_0^t (g_1(x_1)N(\zeta_1) + 1)\dot{\zeta}_1 d\tau$ must be bounded on $[0, t_f)$. Further, since $N(\zeta_1)$ is the continuous function of $\zeta_1(t)$, the boundedness of $N(\zeta_1)$ can be obtained. According to the definition of $V_1(t)$, boundedness of e_1 and $\hat{\theta}_1$ on $[0, t_f)$ is obtained. Moreover, $\tilde{\theta}_1$ is bounded as θ_1 is a constant. Furthermore, according to the boundedness of x_1 , e_1 , $\xi_1(Z_1)$ and $\hat{\theta}_1$, we can get that M_1 and α_1 are also bounded for any $t \in [0, t_f)$. Following in view of $e_2 = x_2 - \alpha_1$, the boundedness of x_2 is ensured for $t \in [0, t_f)$.

Secondly, considering the following function candidate

$$V_2 = V_{e_2} + \frac{\tilde{\theta}_2^2}{4\eta_2^2},$$

it can easily be obtained that

$$\begin{aligned} \dot{V}_2 \leq & g_2(\bar{x}_2)e_3e_2 - g_1(x_1)e_1e_2 + (g_2(\bar{x}_2)N(\zeta_2) + 1)\dot{\zeta}_2 \\ & + (1 - 16 \tanh^2(\frac{e_2}{\nu}))H_2 - \frac{\sigma_2\tilde{\theta}_2^2}{4\eta_2^2} + C_2 - \left(k_2 - \frac{1}{2\lambda_2}\right)e_2^2 - \gamma\Lambda_2, \end{aligned} \quad (51)$$

where $C_2 = q_2 + \frac{\sigma_2\tilde{\theta}_2^2}{4\eta_2^2}$ and

$$H_2 = \frac{d_2^2 e^{\gamma d_2}}{2(1-d_2^*)} \sum_{j=1}^2 m_j^2(\bar{x}_j) + \frac{1}{2} \sum_{j=1}^2 h_{j1}(\bar{x}_j) + \frac{e^{\gamma d_1}}{2(1-d_1^*)} \sum_{j=1}^2 h_{j2}(\bar{x}_j).$$

According to the boundedness of x_j , $1 \leq j \leq 2$ and the the continuity of H_2 , one gets that H_2 is also bounded. Then, the terms of $g_2(\bar{x}_2)e_3e_2 - g_1(x_1)e_1e_2$ and $(1 - 16 \tanh^2(e_2/\nu))H_2$ can be dealt with similar to (44) and (45). Similarly, let \bar{H}_2 be the upper bound of H_2 , where \bar{H}_2 is a positive constant. Then, we have

$$\begin{aligned} & g_2(\bar{x}_2)e_3e_2 - g_1(x_1)e_1e_2 \leq \\ & \frac{e_2^2}{4\lambda_2} + \lambda_2 c^2 (0.2554\nu)^2 + \frac{e_2^2}{4\lambda_2} + \lambda_2 c^2 (0.2554\nu)^2 = \frac{e_2^2}{2\lambda_2} + 2\lambda_2 c^2 (0.2554\nu)^2, \\ & 0 \leq (1 - 16 \tanh^2(\frac{e_2}{\nu}))H_2 \leq \bar{H}_2. \end{aligned} \quad (52)$$

Combining (51) with (52) yields that

$$\dot{V}_2 \leq (g_2(\bar{x}_2)N(\zeta_2) + 1)\dot{\zeta}_2 - \frac{\sigma_2\tilde{\theta}_2^2}{4\eta_2^2} + \bar{K}_2 - e_2^2 \left(k_2 - \frac{1}{\lambda_2}\right) - \gamma\Lambda_2, \quad (53)$$

where $\bar{K}_2 = \bar{H}_2 + 2\lambda_2 c^2 (0.2554\nu)^2 + C_2$. Then, similar to the process (46)-(50), we can prove that $V_2(t)$, $\hat{\theta}_2$, $\zeta_2(t)$, $\tilde{\theta}_2$, M_2 , α_2 , $\int_0^t (g_2(\bar{x}_2)N(\zeta_2) + 1)\dot{\zeta}_2 d\tau$ and x_3 are bounded on $[0, t_f)$. Furthermore, in view of $e_3 = x_3 - \alpha_2$ we get the boundedness of x_3 . Similarly, we can prove that all other signals in the closed-loop are bounded on $[0, t_f)$.

Case 2: For $j = 1, \dots, n$, $e_j \notin \Theta_\nu$. In this case, $|e_j| > 0.2554\nu$.

It follows from the definitions of H_j and Lemma 3.4 that $\sum_{j=1}^n (1 - 16 \tanh^2(\frac{e_j}{\nu}))H_j < 0$, then, combining (42) with the following inequality

$$-\sum_{j=1}^n \frac{\sigma_j \tilde{\theta}_j}{2\eta_j^2} \hat{\theta}_j \leq -\sum_{j=1}^n \frac{\sigma_j \tilde{\theta}_j^2}{4\eta_j^2} + \sum_{j=1}^n \frac{\sigma_j \theta_j^2}{4\eta_j^2}$$

yields that

$$\begin{aligned} \dot{V}(t) \leq & \sum_{j=1}^n (g_j(\bar{x}_j) N(\zeta_j) + 1) \dot{\zeta}_j - \sum_{j=1}^n \frac{\sigma_j \hat{\theta}_j^2}{4\eta_j^2} \\ & + \sum_{j=1}^n \frac{\sigma_j \theta_j^2}{4\eta_j^2} + \sum_{j=1}^n q_j - \sum_{j=1}^n e_j^2 \left(k_j - \frac{1}{2\lambda_j} \right) - \gamma \Lambda, \end{aligned} \quad (54)$$

where $\Lambda = \sum_{j=1}^n \Lambda_j$ and k_j satisfy $k_j > \frac{1}{\lambda_j}$, so we can deduce that $k_j - \frac{1}{2\lambda_j} > 0$. Now, denoting

$$\psi = \min(2k_1 - 1/\lambda_1, \dots, 2k_n - 1/\lambda_n, \sigma_1, \dots, \sigma_n, \gamma)$$

and $C = \sum_{j=1}^n \frac{\sigma_j \theta_j^2}{4\eta_j^2} + \sum_{j=1}^n q_j$, we have

$$\dot{V} \leq -\psi V + \sum_{j=1}^n (g_j N(\zeta_j) + 1) \dot{\zeta}_j + C, \quad (55)$$

Similar to (46)-(50), we have

$$\begin{aligned} 0 \leq V(t) & \leq \frac{C}{\psi} + \left[V(0) - \frac{C}{\psi} \right] e^{-\psi t} + \int_0^t e^{-\psi(t-\tau)} \sum_{j=1}^n (g_j(\bar{x}_j) N(\zeta_j) + 1) \dot{\zeta}_j e^{\psi\tau} d\tau \\ & \leq \frac{C}{\psi} + V(0) + \int_0^t e^{-\psi\tau} \sum_{j=1}^n (g_j(\bar{x}_j) N(\zeta_j) + 1) \dot{\zeta}_j e^{\psi\tau} d\tau. \end{aligned} \quad (56)$$

Next, according to the boundedness of $g_j(\bar{x}_j)$, $1 \leq j \leq n$ and Lemma 2.9, we know that $g_j(\bar{x}_j) N(\zeta_j) + 1$ are Nussbaum-type functions. Then, using Lemma 2.10, it can easily conclude that $V(t)$, $\zeta_j(t)$ and $\int_0^t (g_j(\bar{x}_j) N(\zeta_j) + 1) \dot{\zeta}_j d\tau$, $1 \leq j \leq n$ must be bounded on $[0, t_f)$. Further, the boundedness of $N(\zeta_j)$ can be obtained. Then, according to the definition of $V(t)$, boundedness of e_j , $j = 1, \dots, n$ and $\hat{\theta}_j$ on $[0, t_f)$ is obtained. In addition, $\hat{\theta}_j$ is bounded as θ_j is a constant. According to Assumption 2.5, $y_d, \dot{y}_d, \dots, y_d^{(n)}$ are all bounded. Consequently, it follows from $e_1 = x_1 - y_d$ that x_1 is also bounded, then, combining with (10) one gets that M_1 and α_1 are also bounded for any $t \in [0, t_f)$. Furthermore, since $e_2 = x_2 - \alpha_1$, the boundedness of x_2 is ensured for $t \in [0, t_f)$. Similarly, we can conclude that all the state variables in the closed-loop system are bounded on $[0, t_f)$.

Case 3: Some $e_m \in \Theta_\nu$, while some $e_j \notin \Theta_\nu$. Define Σ_M and Σ_J as the index sets of subsystems consisting of $e_m \in \Theta_\nu$ and $e_j \notin \Theta_\nu$, respectively, and then, for $j \in \Sigma_J$, choose the Lyapunov function candidate as

$$V_{\Sigma_J} = \sum_{j \in \Sigma_J} V_{e_j} + \sum_{j \in \Sigma_J} \frac{1}{4\eta_j^2} \hat{\theta}_j^2. \quad (57)$$

By the controller design process, we have

$$\begin{aligned} \dot{V}_{\Sigma_J} \leq & \sum_{j \in \Sigma_J} (g_j(\bar{x}_j) N(\zeta_j) + 1) \dot{\zeta}_j - \sum_{j \in \Sigma_J} \frac{\sigma_j \hat{\theta}_j^2}{4\eta_j^2} + \sum_{j \in \Sigma_J} \frac{\sigma_j \theta_j^2}{4\eta_j^2} \\ & + \sum_{j \in \Sigma_J} q_j - \sum_{j \in \Sigma_J} e_j^2 \left(k_j - \frac{1}{2\lambda_j} \right) + \sum_{j \in \Sigma_J} \left(1 - 16 \tanh^2 \left(\frac{e_j}{\nu} \right) \right) H_j \\ & - \gamma \sum_{j \in \Sigma_J} \Lambda_j + \sum_{j \in \Sigma_J} (g_j(\bar{x}_j) e_{j+1} e_j - g_{j-1}(\bar{x}_{j-1}) e_{j-1} e_j). \end{aligned} \quad (58)$$

It follows from the definitions of H_j and Lemma 3.4 that $\sum_{j \in \Sigma_J} \left(1 - 16 \tanh^2 \left(\frac{e_j}{\nu} \right) \right) H_j < 0$. Further, similar to the proof of Theorem 1 in [30], the last term of (58) can be expressed as

$$\begin{aligned} & \sum_{j \in \Sigma_J} (g_j(\bar{x}_j) e_{j+1} e_j - g_{j-1}(\bar{x}_{j-1}) e_{j-1} e_j) \\ & \leq \sum_{j \in \Sigma_J} \frac{e_j^2}{2\lambda_j} + \sum_{j-1 \in \Sigma_M, j+1 \in \Sigma_M} 2\lambda_j c^2 (0.2554\nu)^2. \end{aligned}$$

Now, let

$$C_{\Sigma_J} = \sum_{j-1 \in \Sigma_M, j+1 \in \Sigma_M} 2\lambda_j c^2 (0.2554\nu)^2 + \sum_{j \in \Sigma_J} \frac{\sigma_j \theta_j^2}{4\eta_j^2} + \sum_{j \in \Sigma_J} q_j.$$

Based on the above discussion, (58) can be rewritten as

$$\begin{aligned} \dot{V}_{\Sigma_J} &\leq \sum_{\Sigma_J} (g_j(\bar{x}_j) N(\zeta_j) + 1) \dot{\zeta}_j - \sum_{\Sigma_J} \frac{\sigma_j \tilde{\theta}_j^2}{4\eta_j^2} \\ &\quad - \sum_{\Sigma_J} e_j^2 \left(k_j - \frac{1}{\lambda_j} \right) + C_{\Sigma_J} - \gamma \sum_{j \in \Sigma_J} \Lambda_j. \end{aligned} \quad (59)$$

Now, denoting

$$\psi_{\Sigma_J} = \min(2k_j - 2/\lambda_j, \sigma_j, \gamma),$$

since $k_j > \frac{1}{\lambda_j}$, we get $k_j - \frac{1}{\lambda_j} > 0$, then, we have

$$\dot{V}_{\Sigma_J} \leq -\psi_{\Sigma_J} V_{\Sigma_J} + \sum_{\Sigma_J} (g_j(\bar{x}_j) N(\zeta_j) + 1) \dot{\zeta}_j + C_{\Sigma_J}. \quad (60)$$

Similar to **case 2**, it can be shown that V_{Σ_J} , e_j , $\zeta_j(t)$, $\tilde{\theta}_j$, $\int_0^t (g_j(\bar{x}_j) N(\zeta_j) + 1) \dot{\zeta}_j d\tau$ and $\tilde{\theta}_j$ are bounded on $[0, t_f)$ for $j \in \Sigma_J$. For $m \in \Sigma_M$, we know that e_m are bounded. Then, by combining the conclusions of $j \in \Sigma_J$ and the boundedness of e_m , similar to the proof process of **case 1**, we can obtain that x_m , $\int_0^t (g_m(\bar{x}_m) N(\zeta_m) + 1) \dot{\zeta}_m d\tau$ and $\zeta_m(t)$ are bounded on $[0, t_f)$.

In the light of the discussion for **cases 1-3**, we can conclude that all the signals in the closed-loop system are bounded on $[0, t_f)$. Furthermore, owing to the smoothness of the proposed controller, the closed-loop system admits a solution on its maximum interval of existence $[0, t_f)$. Therefore, according to Proposition 2.11, no finite time escape phenomenon may occur and thus t_f can be extended to $+\infty$ [3, 14, 37]. As an immediate result, all signals in the closed-loop system are bounded on $[0, +\infty)$.

This ends the proof of (i).

(ii) Similar to the proof of (i), the proof of (ii) is also divided into the following three cases.

Case 1: For $j = 1, \dots, n$, $e_j \in \Theta_\nu$. In this case, $|e_j| \leq 0.2554\nu$. The process of proof is similar to that of Theorem 1 in [30]. Let $\bar{\nu} = [\nu, \dots, \nu]^T$, we can obtain

$$e_{r\chi} = \frac{1}{t} \int_0^t \|e(\tau)\|^2 d\tau \leq (0.2554)^2 \|\bar{\nu}\|^2. \quad (61)$$

Case 2: For $j = 1, \dots, n$, $e_j \notin \Theta_\nu$. In this case, $|e_j| > 0.2554\nu$, by using the following inequalities and (54)

$$-\gamma\Lambda \leq 0, \quad \left(1 - 16 \tanh^2\left(\frac{e_j}{\nu}\right)\right) H_j \leq 0, \quad -\sum_{j=1}^n \frac{\sigma_j \tilde{\theta}_j^2}{4\eta_j^2} \leq 0,$$

we can deduce

$$\dot{V} \leq -\sum_{j=1}^n \left(k_j - \frac{1}{2\lambda_j}\right) e_j^2 + \sum_{j=1}^n (g_j(\bar{x}_j) N(\zeta_j) + 1) \dot{\zeta}_j + C. \quad (62)$$

Integrating (62) from 0 to t shows that

$$\begin{aligned} \frac{1}{t} (V(t) - V(0)) &\leq -\frac{1}{t} \sum_{j=1}^n \left(k_j - \frac{1}{2\lambda_j}\right) \int_0^t e_j^2(\tau) d\tau \\ &\quad + \frac{1}{t} \int_0^t \sum_{j=1}^n (g_j(\bar{x}_j) N(\zeta_j(\tau)) + 1) \dot{\zeta}_j(\tau) d\tau + C, \end{aligned}$$

defining $\bar{k} = \min \left[k_1 - \frac{1}{2\lambda_1}, \dots, k_n - \frac{1}{2\lambda_n} \right]$, we have

$$e_{r\chi} \triangleq \frac{1}{t} \int_0^t \|e(\tau)\|^2 d\tau \leq \frac{\frac{V(0)}{t} + \frac{1}{t} \int_0^t \sum_{j=1}^n (g_j(\bar{x}_j) N(\zeta_j(\tau) + 1) \dot{\zeta}_j(\tau) d\tau + C}{k}. \quad (63)$$

Using the result of (i), let Δ be the upper bound of $\int_0^t \sum_{j=1}^n (g_j(\bar{x}_j) N(\zeta_j(\tau) + 1) \dot{\zeta}_j(\tau) d\tau$, then one can get the following inequality

$$e_{r\chi} \leq \frac{V(0)/t + \Delta/t + C}{k}. \quad (64)$$

Case 3: Some $e_j \notin \Theta_\nu$, while some $e_m \in \Theta_\nu$. Considering the index set Σ_M of subsystem consisting of $e_m \in \Theta_\nu$ and according to **case 3** in the proof of (i), we have

$$e_{r\chi} |_{\Sigma_M} \triangleq \frac{1}{t} \int_0^t \|e_{\Sigma_M}(\tau)\|^2 d\tau \leq (0.2554)^2 \|\nu_{\Sigma_M}\|^2, \quad (65)$$

where $\|e_{\Sigma_M}\|^2 = \sum_{m \in \Sigma_M} e_m^2$ and $\|\nu_{\Sigma_M}\|^2 = \sum_{m \in \Sigma_M} \nu^2$. Further, considering the index set Σ_J of subsystem consisting of $e_j \notin \Theta_\nu$. Using (59) and the following inequalities

$$-\gamma \sum_{j \in \Sigma_J} \Lambda_j \leq 0, \quad -\sum_{j \in \Sigma_J} \frac{\sigma_j \bar{\theta}_j^2}{4\eta_j^2} \leq 0, \quad \sum_{j \in \Sigma_J} (1 - 16 \tanh^2(\frac{e_j}{\nu})) H_j \leq 0, \quad (66)$$

gives that

$$\dot{V}_{\Sigma_J} \leq -\sum_{j \in \Sigma_J} \left(k_j - \frac{1}{\lambda_j} \right) e_j^2 + \sum_{j \in \Sigma_J} (g_j(\bar{x}_j) N(\zeta_j) + 1) \dot{\zeta}_j + C_{\Sigma_J}.$$

Defining $\bar{k}_1 = \min \left[k_j - \frac{1}{\lambda_j} \right]$ with $j \in \Sigma_J$ and similar to the procedures from (62) to (64) yield that

$$e_{r\chi} |_{\Sigma_J} \triangleq \int_0^t \|e_{\Sigma_J}(\tau)\|^2 d\tau \leq \frac{V_{\Sigma_J}(0)/t + \Delta_J/t + C_{\Sigma_J}}{\bar{k}_1}, \quad (67)$$

where Δ_J is the upper bounded of $\int_0^t \sum_{j \in \Sigma_J} (g_j(\bar{x}_j) N(\zeta_j) + 1) \dot{\zeta}_j(\tau) d\tau$.

Because no finite-time escape phenomenon may happen, we can get

$$\begin{aligned} \lim_{t \rightarrow +\infty} \frac{V_{\Sigma_J}(0)/t + \Delta_J/t + C_{\Sigma_J}}{\bar{k}} &= \frac{C_{\Sigma_J}}{\bar{k}}, \\ \lim_{t \rightarrow +\infty} \frac{\bar{V}_n(0)/t + \Delta/t + C}{\bar{k}_1} &= \frac{C}{\bar{k}_1}, \end{aligned} \quad (68)$$

which means that e eventually converges to the following set:

$$\Xi_\chi := \{e \in R^n | e_{r\chi} \leq \Pi_\chi\},$$

where

$$\Pi_\chi = \max \left((0.2554)^2 \|\bar{\nu}\|^2, \frac{C}{\bar{k}}, (0.2554)^2 \|\nu_{\Sigma_M}\|^2 + \frac{C_{\Sigma_J}}{\bar{k}_1} \right). \quad (69)$$

This completes the proof of Theorem 3.2. \square

Remark 3.5. The boundedness e_i , $i = 1, \dots, n$ have been proved in the first part of the Theorem 3.2. Now, we assume that $0 \leq |e_i| \leq M_i$ are established, where M_i are the upper bounds of e_i , and denote the set Ω'_{Z_i} as $\Omega'_{Z_i} := \left\{ [\bar{y}_{di}^T, e_1, \dots, e_i]^T | \bar{y}_{di}^T \in \Omega_{di}, 0 \leq |e_i| \leq M_i \right\} \subset R^{2i+1}$. In the light of the requirement in Lemma 2.12, we know that if we choose Ω_{Z_i} large enough such that $\Omega'_{Z_i} \subset \Omega_{Z_i}$ is satisfied, then the above FLS which have been implemented are true.

4. Simulation

In this section, two numerical simulation examples are given to demonstrate the effectiveness of the proposed control method. One is a mathematically constructive system, and the other is a physically system—Brusselator model. In addition, in Remark 4.3, authors illustrate the advantages of the results in the paper compared with existing ones in literature.

Example 4.1. Consider the following unknown time-varying delay system:

$$\begin{cases} \dot{x}_1 = g_1(x_1(t))x_2 + f_1(x_1(t)) + \omega_1(t, x(t)) \\ \quad + h_1(x_1(t), x_1(t - d_{11}(t))) + \int_{t-d_{12}(t)}^t m_1(x_1(s)) ds, \\ \dot{x}_2 = g_2(x(t))u + f_2(x(t)) + \omega_2(t, x(t)) \\ \quad + h_2(x(t), x(t - d_{21}(t))) + \int_{t-d_{22}(t)}^t m_2(x(s)) ds, \\ y = x_1, \end{cases} \quad (70)$$

where $h_1 = 2 \cos(x_1) \sin(x_1^2(t - d_{11}(t)))$, $g_1(x_1) = 2 + x_1^2$, $f_1(x_1) = -e^{0.01x_1}x_1$, $\omega_1(t, x) = 0.7x_1 \cos(1.5t)$, $m_1(x_1) = \sin(x_1)$, $g_2(x) = 3 + \cos(x_1x_2)$, $f_2(x) = x_1^2x_2$, $h_2 = -0.2 \sin(x_1(t - d_{21}(t))) \cos(x_2)x_2(t - d_{21}(t))$, $m_2(x) = x_1 \exp(-x_2^2)$ and $\omega_2(t, x(t)) = -2(x_1^2 + x_2^2) \sin^3(t)$. In this simulation, we choose $d_{11}(t) = 0.2(1 + \sin(t))$, $d_{21}(t) = 1 - 0.5 \cos(t)$, $d_{12}(t) = 0.2(1 - \sin(t))$ and $d_{22}(t) = 1 + 0.5 \cos(t)$, the upper bounds of them are $d_1 = 0.4$ and $d_2 = 1.5$, and the derivations are given by $\dot{d}_{11}(t) = 0.2 \cos(t)$, $\dot{d}_{21}(t) = 0.5 \sin(t)$, $\dot{d}_{12}(t) = -0.2 \cos(t)$ and $\dot{d}_{22}(t) = -0.5 \sin(t)$. Thus, Assumption 2.1 is also satisfied. The initial states are chosen as $[x_1(0), x_2(0), \zeta_1(0), \zeta_2(0), \hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [3, 1, 1, 0.5, 0.2, 0.1]^T$.

The simulation objective is to apply the developed adaptive fuzzy controller such that (1) the boundedness of all the signals in the closed-loop system is guaranteed and (2) the system output y follows the reference signal y_d to a small neighborhood of zero where $y_d = 3 - 0.5 \sin(0.5t) + 0.5 \sin(t)$. Then, select the design parameters as $\vartheta_1 = 5$, $\vartheta_2 = 20$, $\eta_1 = 2$, $\eta_2 = 10$, $\sigma_1 = 0.002$, $\sigma_2 = 0.001$, $k_2 = 6$ and $k_1 = 0.5$. When $t \in [-d_i, 0]$, for $i = 1, 2$, choose $x_1(t) = 3$ and $x_2(t) = 1$. Membership functions are specified as $\mu_{F_i^l}(x_i) = \exp(-0.5(x_i + 6 - 2(l-1))^2)$, $l = 1, \dots, 7$. The control laws and the online adaptation laws for the system (70) are constructed as follows:

$$\begin{aligned} \alpha_1 &= N(\zeta_1)M_i, \quad \dot{\zeta}_1 = e_1M_1, \quad M_1 = k_1e_1 + \frac{\vartheta_1\hat{\theta}_1\xi_1^T(Z_1)\xi_1(Z_1)e_1}{2\eta_1^2}, \\ \dot{\hat{\theta}}_1 &= \vartheta_1\xi_1^T(Z_1)\xi_1(Z_1)e_1^2 - \sigma_1\hat{\theta}_1, \\ u &= N(\zeta_2)M_2, \quad \dot{\zeta}_2 = e_2M_2, \quad M_2 = k_2e_2 + \frac{\vartheta_2\hat{\theta}_2\xi_2^T(\bar{Z}_2)\xi_2(\bar{Z}_2)e_2}{2\eta_2^2}, \\ \dot{\hat{\theta}}_2 &= \vartheta_2\xi_2^T(Z_2)\xi_2(Z_2)e_2^2 - \sigma_2\hat{\theta}_2. \end{aligned}$$

Simulation results in Figures 1-6 show the effectiveness of the developed adaptive fuzzy control schemes for the system (70). From Figures 1 and 2 it can be seen that good tracking performance is obtained. The boundedness of u is illustrated in Figure 3. It can be concluded that the adaptive parameters $\hat{\theta}_1$ and $\hat{\theta}_2$ are also bounded from Figure 4. The variable x_2 is also bounded by Figure 5. We can deduce that parameters ζ_1 and ζ_2 are also bounded from Figure 6.

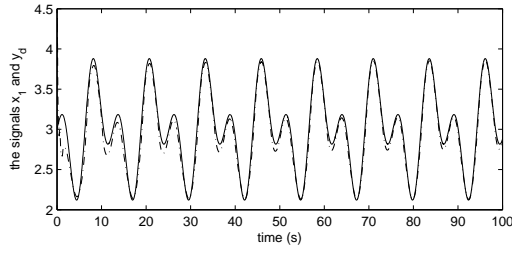


FIGURE 1. The output y (dashed line) and the Reference Signal y_d (solid line)

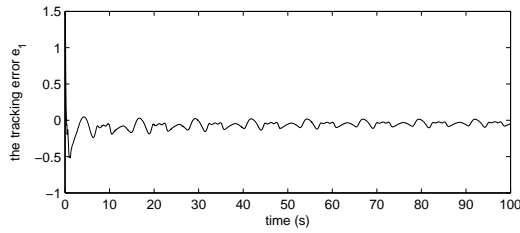


FIGURE 2. The Tracking Error e_1

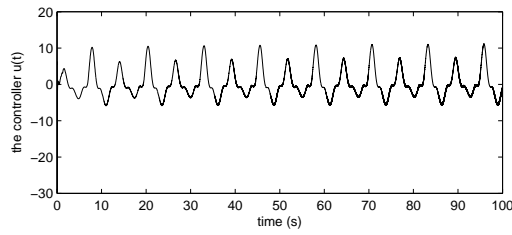


FIGURE 3. The Trajectory of Controller $u(t)$

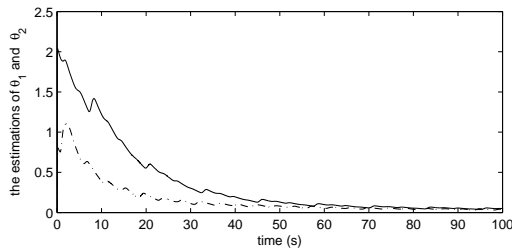
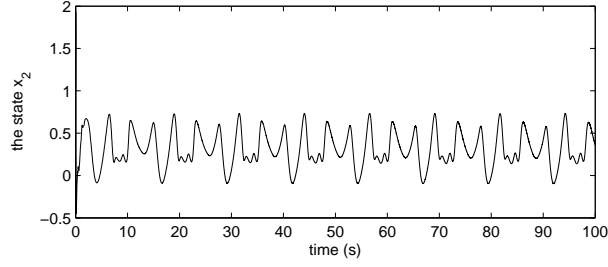
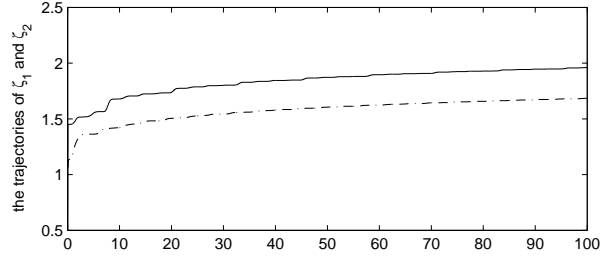


FIGURE 4. The curves of $\hat{\theta}_1$ (dashed line) and $\hat{\theta}_2$ (solid line)

Example 4.2. In [30], control problem for a controlled Brusselator model with external disturbances and constant delays is investigated via fuzzy approach with the control directions known. In this paper, the system will also be considered with the control directions unknown as in (71). Moreover, unlike [30], we will consider

FIGURE 5. The Trajectory of State x_2 FIGURE 6. The Parameters Curves of ζ_1 (dashed line) and ζ_2 (solid line)

the following system with discrete and distributed time-varying delays:

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 + h_1(x_1, x_1(t - d_1(t))) \\ \quad + \int_{t-d_{12}(t)}^t m_1(x_1(s)) ds + \omega_1(t, \bar{x}_2), \\ \dot{x}_2 = f_2(\bar{x}_2) + g_2(\bar{x}_2)u + h_2(\bar{x}_2, \bar{x}_2(t - d_2(t))) \\ \quad + \int_{t-d_{22}(t)}^t m_2(\bar{x}_2(s)) ds + \omega_2(t, \bar{x}_2), \\ y = x_1, \end{cases} \quad (71)$$

where x_1 and x_2 denote the concentrations of the reaction intermediates, u is the control input, $g_1(x_1) = x_1^2$, $g_2(x) = 2 + \cos(x_1)$, $f_1(x_1) = E - (F + 1)x_1$ and $f_2(\bar{x}_2) = E\bar{x}_1 - x_1^2\bar{x}_2$ with $E, F > 0$ being parameters which describe the supply of “reservoir” chemicals. $h_1(\cdot)$ and $h_2(\cdot)$ are the unknown time-varying delay functions, $\omega_1(t, \bar{x}_2)$ and $\omega_2(t, \bar{x}_2)$ are the external disturbance terms, which come from the modeling errors and other types of unknown nonlinearities in the practical chemical reactions. As stated in [30], it is assumed that $x_1 \neq 0$. In the simulation, we choose $E = 1$, $F = 3$, $\omega_1(t, \bar{x}_2) = 0.7x_1 \cos(1.5t)$, $\omega_2(t, \bar{x}_2) = (x_1^2 + x_2^2) \sin^3(t)$, $h_1 = -2 \cos(x_1) \sin(x_1(t - d_1(t)))$, $h_2 = -0.2e^{-0.5x_2(t-d_2(t))} \cos(x_1) \cos(x_1(t - d_2(t)))$, $m_1(x_1) = -\exp(-x_1)$, $m_2(\bar{x}_2) = x_2 \exp(-x_1^2)$, $d_{11}(t) = 0.2(1 - \sin(t))$, $d_{21}(t) = 1 - 0.5\cos(t)$, $d_{12} = 1 + 0.6 \cos(t)$ and $d_{22} = 0.4(1 - \cos(t))$, the upper bounds of them are $d_1 = 1.5$ and $d_2 = 1.6$. The control objective is to design an adaptive fuzzy controller such that all the signals in the closed-loop system remain bounded and the system output y follows the given reference signal $y_d = 3 + 0.5 \sin(t) + 0.5 \sin(1.5t)$. Then, we define fuzzy membership functions as follows: $\mu_{F_i^l}(x_i) = \exp(-0.5(x_i + 6 - 2(l - 1))^2)$, $l = 1, \dots, 7$. The control laws and the online adaptation laws for the system (71) are constructed as follows:

$$\begin{aligned}\alpha_1 &= N(\zeta_1) M_1, \quad \dot{\zeta}_1 = e_1 M_1, \quad M_1 = k_1 e_1 + \frac{\vartheta_1 \hat{\theta}_1 \xi_1^T(Z_1) \xi_1(Z_1) e_1}{2\eta_1^2}, \\ \dot{\hat{\theta}}_1 &= \vartheta_1 \xi_1^T(Z_1) \xi_1(Z_1) e_1^2 - \sigma_1 \hat{\theta}_1, \\ u &= N(\zeta_2) M_2, \quad \dot{\zeta}_2 = e_2 M_2, \quad M_2 = k_2 e_2 + \frac{\vartheta_2 \hat{\theta}_2 \xi_2^T(\bar{Z}_2) \xi_2(\bar{Z}_2) e_2}{2\eta_2^2}, \\ \dot{\hat{\theta}}_2 &= \vartheta_2 \xi_2^T(Z_2) \xi_2(Z_2) e_2^2 - \sigma_2 \hat{\theta}_2.\end{aligned}$$

In the simulation, the initial values are chosen as $[x_1(0), x_2(0), \zeta_1(0), \zeta_2(0), \hat{\theta}_1(0), \hat{\theta}_2(0)]^T = [3, 1, 1, 1.8, 0.1, 0.1]^T$ and when $t \in [-d_i, 0]$, for $i = 1, 2$, choose $x_1(t) = 3$ and $x_2(t) = 1$. Select the design parameters as $\vartheta_1 = \vartheta_2 = 2$, $\eta_1 = \eta_2 = 2$, $\sigma_1 = \sigma_2 = 0.001$, $k_1 = 0.5$ and $k_2 = 0.1$. The simulation results are shown by Figures 7-12. Apparently, the simulation results show that under the action of the suggested controller, a good tracking performance has been achieved.

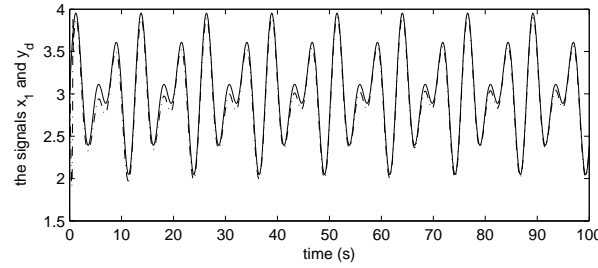


FIGURE 7. The Output y (dashed line) and the Reference Signal y_d (solid line)

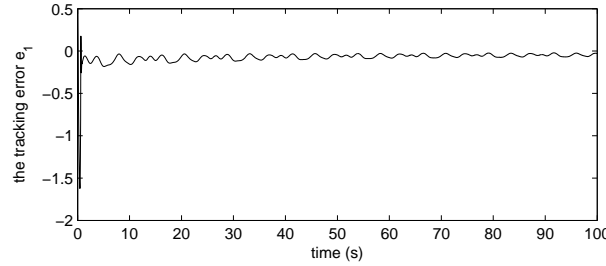
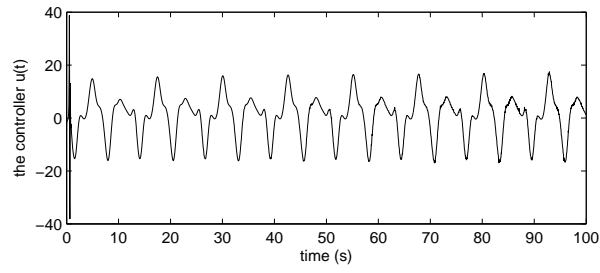
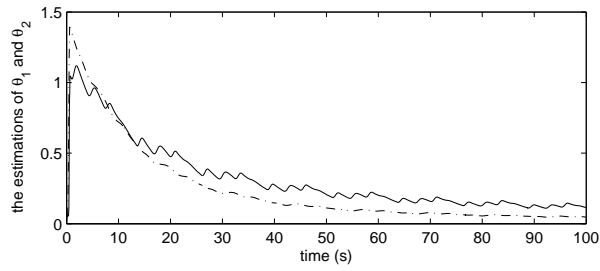
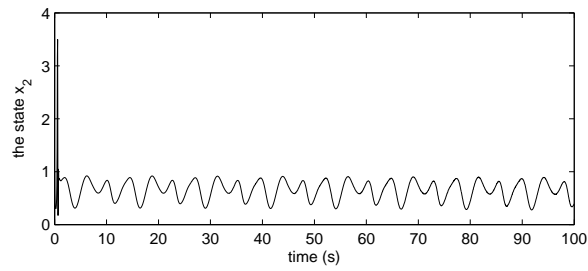
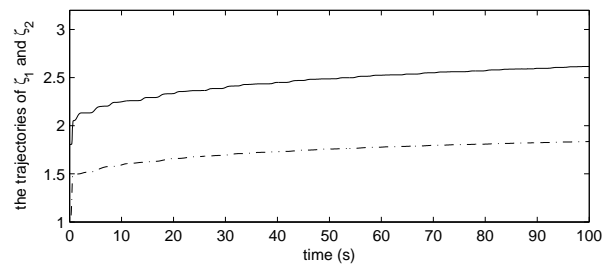


FIGURE 8. The Tracking Error e_1

Remark 4.3. Similar results have been proposed in [35] for SISO nonlinear delay-free systems, [30] for a class of perturbed strict-feedback nonlinear time-delay systems, [29] nonlinear time-delay systems with unknown virtual control coefficients and [5] for MIMO nonlinear delay-free systems. Unlike [5, 29, 30, 35], this paper mainly addresses the tracking control problem for nonlinear distributed time-varying delays systems with unknown control direction. Although the adaptive fuzzy controller has been constructed based on the similar design idea and the backstepping technique, the existence of nonlinear distributed time-varying delay terms and the unknown control directions make controller design more difficult, as

FIGURE 9. The Trajectory of Controller $u(t)$ FIGURE 10. The Curves of $\hat{\theta}_1$ (dashed line) and $\hat{\theta}_2$ (solid line)FIGURE 11. The Trajectory of State x_2 FIGURE 12. The Parameters Curves of ζ_1 (dashed line) and ζ_2 (solid line)

a result, the stability analysis of the closed-loop system in this paper is also much more complex than ones in [5, 29, 30, 35].

5. Conclusion

The stable adaptive fuzzy control scheme has been proposed for a class of perturbed strict-feedback nonlinear systems. Based on the appropriate Lyapunov-Krasovskii functionals, the Nussbaum-type functions, the FLS and the backstepping approach, the adaptive tracking controller is constructed. By using the Lyapunov stability analysis, we have proved that all the signals of the resulting closed-loop system are bounded and the tracking error converges to a small neighborhood of the origin. However, there are many further works to be studied in the future, such as how relax the Assumption 2.1, i.e. if \dot{d}_{i1} and \dot{d}_{i2} do not satisfy $\dot{d}_{i1}(t) \leq d_1^* < 1$ and $\dot{d}_{i2}(t) \leq d_2^* < 1$, how to design the controller for the nonlinear system (1) is a challenging problem and needs to be further investigated in the future.

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