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A CONSTRAINED SOLID TSP IN FUZZY ENVIRONMENT: TWO HEURISTIC APPROACHES

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Abstract. A solid travelling salesman problem (STSP) is a travelling salesman problem (TSP) where the salesman visits all the cities only once in his tour using different conveyances to travel from one city to another. Costs and environmental effect factors for travelling between the cities using different conveyances are different. Goal of the problem is to find a complete tour with minimum cost that damages the environment least. An ant colony optimization (ACO) algorithm is developed to solve the problem. Performance of the algorithm for the problem is compared with another soft computing algorithm, Genetic Algorithm(GA). Problems are solved with crisp as well as fuzzy costs. For fuzzy cost and environmental effect factors, cost function as well as environment constraints become fuzzy. As optimization of a fuzzy objective function is not well defined, fuzzy possibility approach is used to get optimal decision. To test the efficiency of the algorithm, the problem is solved considering only one conveyance facility ignoring the environmental effect constraint, i.e., a classical two dimensional TSP (taking standard data sets from TSPLIB for solving the problem). Different numerical examples are used for illustration.

1. Introduction

The TSP consists of a complete graph of N vertices whose edges represent distances/costs between the nodes associated with it. The goal is to find the shortest possible tour through the vertices (nodes/cities) so that each vertex is visited exactly once except the starting vertex. Tour ends at starting vertex. This problem is known to be NP-hard and cannot be solved exactly in polynomial time [17, 18]. Different types of TSPs have been solved by the researchers during last two decades. Among these, TSP with time windows [12], stochastic TSP [3, 19], double TSP [32], asymmetric TSP [23], TSP with precedence constraints [26], etc are worth mentioning. In TSP with precedence constraints, there exists an order in which the vertices should be visited. In asymmetric TSP, cost of travelling from vertex (node/city) v_i to v_j is not equal to the cost of travelling from vertex v_j to v_i. In stochastic TSP, each vertex is visited with a given probability and goal is to minimize the expected distance/cost of a priori tour. In TSP with time windows, each vertex is visited within a specified time windows.

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In all the above literature it is implicitly assumed that travelling cost from one node to another is fixed, i.e., crisp in nature. But travelling cost from one node to another depends on the conveyance used for travelling. Also it varies slightly depending on the availability of the conveyance, condition of the road, etc., though its value normally lies in an interval. Due to this reason it is better to model the costs of a TSP as fuzzy numbers. It is less error prone as these estimations are based on experts opinion. Also problem should be modelled in such a way that salesman can visit one city to another using different conveyances. Though it is normally practised by salesmen, no researcher has considered TSP with different conveyance facilities i.e. solid TSP. Again environment is differently adversely effected in travelling through different conveyances. So goal should be such that the tour should damage the environment least, i.e., it should be maximum echo-friendly. Till now, none has considered this constraint even in classical TSP. But now-a-days, this environment friendly travel, that is tour using gas filed vehicles, etc, is given maximum importance.

Removing the above shortcomings, here a TSP is considered where salesman can use different conveyances to travel from one city to another. Cost and environmental friendliness for travelling using different conveyances from one city to another are different. The salesman should maintain a minimum environmental friendliness to damage the environment in his tour up to the maximum permitted limit. Crisp as well as fuzzy costs and echo-friendliness coefficients are used in different problems. A fuzzy possibility based approach is used to make optimal decision for fuzzy costs and environmental effect factors (in positive sense). As three dimensional cost and echo-friendliness coefficients are required to represent the problem, the problem is named as Solid TSP.

In the existing literature besides exact methods, meta-heuristics, local search and hybrid algorithms of optimization and searching approaches are applied to solve TSPs. Exact methods include cutting plane, LP relaxation [6], branch and bound [30], branch and cut [31], dynamic programming [11] etc. However very small size problems can be solved by exact methods. On the other hand large size problems are solved using heuristics like Simulated Annealing [5], Local Search [14], Hybrid Algorithm [12], Tabu search [15], Genetic Algorithm (GA) [26, 28], Ant Colony Optimization (ACO)[2]. Heuristic algorithms establish their efficiency for solving different real life problems in imprecise environment[16, 20, 27, 35]. The present problem under investigation is more complicated for its imprecise costs and constraints on environmental effect. Fuzziness of the cost and environmental effect leads to fuzzy total-cost with a fuzzy constraint. As optimization of fuzzy objective is not well defined and hence it is very difficult to find optimal paths for the stated problem. Due to this complexity, a fuzzy possibility/necessity based approach is proposed to transfer fuzzy objective into an equivalent crisp objective (see section-3). An ACO is developed to find optimal path of such a realistic solid TSP. A GA with cyclic crossover, two-point mutation is also used to find optimal paths. Results through two approaches are compared to check the validity of ACO for the problem. Finally problem is illustrated with numerical examples. The novelty of the paper is the introduction of solid TSP for the first time in the field of TSP and

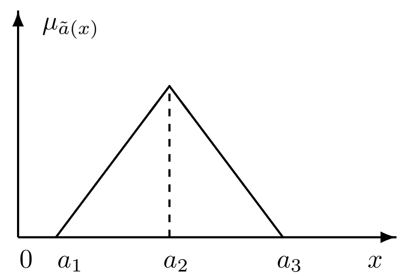

FIGURE 1. Triangular Fuzzy Number $a = (a_1, a_2, a_3)$

the consideration of the environmental effect during the tour. The problem is also solved considering only one conveyance facility ignoring environmental effect(i.e. classical TSP) using standard data set from the literature (taking data set from TSPLIB), which establishes that the proposed approach is sufficiently powerful to solve the considered general problem.

2. Mathematical Prerequisite

Let $\tilde{a}$ and $\tilde{b}$ be two fuzzy numbers with membership functions $\mu_{\tilde{a}}(x)$ and $\mu_{\tilde{b}}(x)$ respectively. Then according to [9, 36],

$$pos(\tilde{a} * \tilde{b}) = sup\{min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)), x, y \in \Re, x * y\} \tag{1}$$

where the abbreviation pos represents possibility, $*$ is any one of the relations $>, <, =, \leq, \geq$ and $\Re$ represents set of real numbers.

$$nes(\tilde{a} * \tilde{b}) = 1 - pos(\overline{\tilde{a} * \tilde{b}}) \tag{2}$$

where the abbreviation nes represents necessity.

If $\tilde{a}$, $\tilde{b} \subseteq \Re$ and $\tilde{c} = f(\tilde{a}, \tilde{b})$ where $f : \Re \times \Re \to \Re$ is a binary operation then membership function $\mu_{\tilde{c}}$ of $\tilde{c}$ is defined as in [37]

$$\text{For each } z \in \Re,\ \mu_{\tilde{c}}(z) = sup\{min(\mu_{\tilde{a}}(x), \mu_{\tilde{b}}(y)), x, y \in \Re \text{ and } z = f(x, y)\} \tag{3}$$

Triangular fuzzy number(TFN): A TFN $\tilde{a} = (a_1, a_2, a_3)$ (cf. Fig-1) has three parameters a_1, a_2, a_3 where $a_1 < a_2 < a_3$ and is characterized by the membership function $\mu_{\tilde{a}}$, given by

$$\mu_{\tilde{a}}(x) = \begin{cases} \frac{x-a_1}{a_2-a_1} & \text{for } a_1 \leq x \leq a_2 \\ \frac{a_3-x}{a_3-a_2} & \text{for } a_2 \leq x \leq a_3 \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

According to the above definitions following lemmas can easily be derived (see e.g. [21, 22]).

Lemma 2.1. *If $\tilde{a} = (a_1, a_2, a_3)$ be a TFN with $0 < a_1$ and b a crisp number, then $pos(\tilde{a} < b) \geq \alpha$ iff $\dfrac{b - a_1}{a_2 - a_1} \geq \alpha$.*

Lemma 2.2. *If $\tilde{a} = (a_1, a_2, a_3)$ be a TFN with $0 < a_1$ and b a crisp number then $nes(\tilde{a} < b) \geq \alpha$ iff $\frac{a_3 - b}{a_3 - a_2} \leq 1 - \alpha$.*

Lemma 2.3. *If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be TFNs with $0 < a_1$ and $0 < b_1$, then $pos(\tilde{a} > \tilde{b}) \geq \alpha$ iff $\frac{a_3 - b_1}{a_3 - a_2 + b_2 - b_1} \geq \alpha$.*

Lemma 2.4. *If $\tilde{a} = (a_1, a_2, a_3)$ and $\tilde{b} = (b_1, b_2, b_3)$ be TFNs with $0 < a_1$ and $0 < b_1$ then $nes(\tilde{a} > \tilde{b}) \geq \alpha$ iff $\frac{b_3 - a_1}{a_2 - a_1 + b_3 - b_2} \leq 1 - \alpha$.*

3. Problem Definition

3.1. **General TSP with Environmental Effect Constraints.** In a classical two-dimensional travelling salesman problem, a salesman has to travel N cities using minimum cost. In his tour salesman starts from a city, visits all the cities exactly once and comes to the starting city using minimum cost. Here a conveyances echo-friendliness in travelling from one city to another is considered. The salesman should choice such a path in which a minimum environmental damage is ensured, on the other hand, maximum environment conservation is maintained. Let $c(i, j)$ be the cost for travelling from i-th city to j-th city and $s(i, j)$ be the travel comfort in travelling from i-th city to j-th city. Then the problem can be mathematically formulated as:

$$\left\{\begin{array}{ll} \text{Minimize} \quad Z = \sum_{i=1}^{N} \sum_{j=1}^{N} t_{ij} c(i, j) \\ \text{subject to} \quad \sum_{i=1}^{N} t_{ij} = 1 \ \text{ for } j = 1, 2, ..., N \\ \qquad \sum_{j=1}^{N} t_{ij} = 1 \ \text{ for } i = 1, 2, ..., N \\ \qquad \sum_{i=1}^{N} \sum_{j=1}^{N} t_{ij} s(i, j) \geq s_{min} \end{array}\right. \tag{5}$$

where $t_{ij} = 1$ if the salesman travels from city-i to city-j, otherwise $t_{ij} = 0$ and s_{min} is the minimum environmental effect level that should be maintained by the salesman. Let$(x_1,\ x_2,\ ...,x_N,x_1)$ be a complete tour of a salesman, where $x_i \in \{1, 2, ..N\}$, for $i = 1, 2, ..., N$ and all x_i are distinct. Then the above problem reduces to

$$\left\{\begin{array}{l} \text{Determine a complete tour} \quad (x_1, x_2, ..., x_N, x_1) \\ \text{to minimize} \quad Z = \sum_{i=1}^{N-1} c(x_i, x_{i+1}) + c(x_N, x_1) \\ \text{subject to} \quad \sum_{i=1}^{N-1} s(x_i, x_{i+1}) + s(x_N, x_1) \geq s_{min} \end{array}\right. \tag{6}$$

3.2. **Classical Fuzzy TSP with Environmental Effect Constraints.** In the above problem if costs and environmental effect factors be fuzzy numbers, i.e, $\tilde{c}(i, j)$

and $\tilde{s}(i,j)$ respectively, and environmental effect limit s_{min} and also fuzzy number $\tilde{s}_{min}$, then the above problem reduces to:

$$\begin{cases} \text{Determine a complete tour} \quad (x_1, x_2, ..., x_N, x_1) \\ \text{to minimize} \quad \tilde{Z} = \sum_{i=1}^{N-1} \tilde{c}(x_i, x_{i+1}) + \tilde{c}(x_N, x_1) \\ \text{subject to} \quad \sum_{i=1}^{N-1} \tilde{s}(x_i, x_{i+1}) + \tilde{s}(x_N, x_1) \geq \tilde{s}_{min} \end{cases} \tag{7}$$

As minimization of fuzzy objective is not well defined, the problem can be equivalently treated as

$$\begin{cases} \text{Determine a complete tour} \quad (x_1, x_2, ..., x_N, x_1) \text{ to minimize} \quad F \\ \text{subject to} \quad \sum_{i=1}^{N-1} \tilde{c}(x_i, x_{i+1}) + \tilde{c}(x_N, x_1) < F \\ \sum_{i=1}^{N-1} \tilde{s}(x_i, x_{i+1}) + \tilde{s}(x_N, x_1) \geq \tilde{s}_{min} \end{cases} \tag{8}$$

where F is any crisp parameter. It is clear that minimization of F implies minimization of fuzzy objective $\tilde{Z}$. For this reason the above maintained approach is used to treat fuzzy objective $\tilde{Z}$.

Again minimization of fuzzy constraints are not well defined. So following section-2 the above problem can be rewritten in optimistic and pessimistic sense by (9) and (10) respectively.

$$\begin{cases} \text{Determine a complete tour} \quad (x_1, x_2, ..., x_N, x_1) \text{ to minimize} \quad F \\ \text{subject to} \quad Pos(\sum_{i=1}^{N-1} \tilde{c}(x_i, x_{i+1}) + \tilde{c}(x_N, x_1) < F) \geq \alpha_1 \\ Pos(\sum_{i=1}^{N-1} \tilde{s}(x_i, x_{i+1}) + \tilde{s}(x_N, x_1) \geq \tilde{s}_{min}) \geq \beta_1 \end{cases} \tag{9}$$

$$\begin{cases} \text{Determine a complete tour} \quad (x_1, x_2, ..., x_N, x_1) \text{ to minimize} \quad F \\ \text{subject to} \quad Nes(\sum_{i=1}^{N-1} \tilde{c}(x_i, x_{i+1}) + \tilde{c}(x_N, x_1) < F) \geq \alpha_2 \\ Nes(\sum_{i=1}^{N-1} \tilde{s}(x_i, x_{i+1}) + \tilde{s}(x_N, x_1) \geq \tilde{s}_{min}) \geq \beta_2 \end{cases} \tag{10}$$

where α_1, β_1 and α_2, β_2 of equation (9) and (10) are predefined levels of possibility and necessity respectively which are entirely determined by the salesman. Their significance is discussed at the end of this subsection.

If we consider the fuzzy numbers as TFNs, $\tilde{c}(i,j) = (c(i,j)_1,\ c(i,j)_2,\ c(i,j)_3)$, $\tilde{s}(i,j) = (s(i,j)_1,\ s(i,j)_2,\ s(i,j)_3)$ and $\tilde{s}_{min} = (s_1, s_2, s_3)$, following section-2 the above problems (9) and (10) reduces to (11) and (12) respectively.

$$\begin{cases} \text{Determine a complete tour} \quad (x_1, x_2, ..., x_N, x_1) \text{ to minimize} \quad F \\ \text{subject to} \quad \dfrac{F - C_1}{C_2 - C_1} \geq \alpha_1 \\ \dfrac{S_3 - s_1}{S_3 - S_2 + s_2 - s_1} \geq \beta_1 \end{cases} \tag{11}$$

$$\text{where } C_j = \sum_{i=1}^{N-1} c(x_i, x_{i+1})_j + c(x_N, x_1)_j, \; j = 1, 2, 3.$$

$$\text{and } S_j = \sum_{i=1}^{N-1} s(x_i, x_{i+1})_j + s(x_N, x_1)_j, \; j = 1, 2, 3.$$

$$\begin{cases} \text{Determine a complete tour } (x_1, x_2, ..., x_N, x_1) \text{ to minimize } F \\ \text{subject to } \dfrac{C_3 - F}{C_3 - C_2} \leq 1 - \alpha_2 \\ \dfrac{s_3 - S_1}{S_2 - S_1 + s_3 - s_2} \leq 1 - \beta_2 \end{cases} \tag{12}$$

Which are equivalent to (13) and (14) respectively.

$$\begin{cases} \text{Determine a complete tour } (x_1, x_2, ..., x_N, x_1) \\ \text{to minimize } C_1 + \alpha_1(C_2 - C_1) \\ \text{subject to } \dfrac{S_3 - s_1}{S_3 - S_2 + s_2 - s_1} \geq \beta_1 \end{cases} \tag{13}$$

$$\begin{cases} \text{Determine a complete tour } (x_1, x_2, ..., x_N, x_1) \\ \text{to minimize } C_3 - (1 - \alpha_2)(C_3 - C_2) \\ \text{subject to } \dfrac{s_3 - S_1}{S_2 - S_1 + s_3 - s_2} \leq 1 - \beta_2 \end{cases} \tag{14}$$

If the salesman is most optimistic then he will choice value of α_1, β_1 nearly 0 and in that case minimum possible cost function (C_1) is minimized assuming maximum possible environmental echo-friendliness of the tour (S_3) reaches the minimum possible environmental effect requirement (s_1). On the other hand if the salesman is least optimistic then he/she will choice values of α_1, $\beta_1$nearly 1 and in that case most feasible cost function (C_2) is minimized assuming most feasible environmental friendliness or travel comfort of the tour (S_2) reaches the most feasible environmental effect requirement (s_2). Pessimistic salesman will go for necessity approach. If he is most pessimistic, he will choice values of α_2, β_2 nearly 1 and in that case maximum possible cost function (C_3) is minimized assuming minimum possible environmental effect of the tour (S_1) reaches the maximum possible environmental effect requirement (s_3). On the other hand if the salesman is least pessimistic then he/she will choice value of α_2, β_2 nearly 0 and in that case most feasible cost function (C_2) is minimized assuming most feasible environmental effect of the tour (S_2) reaches the most feasible environmental effect requirement (s_2).

3.3. **Solid TSP with Environmental Effect Constraints.** In a solid travelling salesman problem, a salesman has to travel N cities by choosing any one of the M conveyances available using minimum cost. In his tour salesman starts from a city, visits all the cities exactly once using suitable conveyances available at the cities and comes to the starting city using minimum cost. Environmental factor in travelling from one city to another using different conveyances are different. The salesman should choice such a path and conveyances such that a minimum environmental effect is maintained. Let $c(i,j,k)$ be the cost of travelling from i-th city to j-th city using k-th type conveyance and $s(i,j,k)$ be the environmental effect in travelling from i-th city to j-th using k-th type conveyance. Then the salesman has to determine a complete tour $(x_1,\ x_2,\ ...,x_N,x_1)$ in which a particular or different combinations of conveyance types $(v_1,\ v_2,\ ...,v_N)$ is to be used for the tour, where $x_i \in \{1,2,..N\}$ for $i = 1,2,...,N$, $v_i \in \{1,2,..M\}$ for $i = 1,2,...,N$ and all x_i's are distinct. Then the problem can be mathematically formulated as:

$$\left\{\begin{array}{l} \text{Determine a complete tour} \quad (x_1,x_2,...,x_N,x_1) \\ \text{and corresponding conveyance types} \ (v_1,v_2,...,v_N) \\ \text{to minimize} \quad Z = \sum_{i=1}^{N-1} c(x_i,x_{i+1},v_i) + c(x_N,x_1,v_N) \\ \text{subject to} \quad \sum_{i=1}^{N-1} s(x_i,x_{i+1},v_i) + s(x_N,x_1,v_N) \geq s_{min} \end{array}\right. \tag{15}$$

where s_{min} is the minimum environment friendliness travel comfort that should be maintained by the salesman.

3.4. **Solid TSP with Fuzzy Costs and Environmental Effect Constraints.** In the above problem if costs and environmental effect factors are fuzzy numbers, i.e, $\tilde{c}(i,j,k)$ and $\tilde{s}(i,j,k)$ respectively and environmental effect limit s_{min} also fuzzy number $\tilde{s}_{min}$, the above problem reduces to:

$$\left\{\begin{array}{l} \text{Determine a complete tour} \quad (x_1,x_2,...,x_N,x_1) \\ \text{and corresponding conveyance} \quad (v_1,v_2,...,v_N) \\ \text{to minimize} \quad Z = \sum_{i=1}^{N-1} \tilde{c}(x_i,x_{i+1},v_i) + \tilde{c}(x_N,x_1,v_N) \\ \text{subject to} \quad \sum_{i=1}^{N-1} \tilde{s}(x_i,x_{i+1},v_i) + \tilde{s}(x_N,x_1,v_N) \geq \tilde{s}_{min} \end{array}\right. \tag{16}$$

As minimization of fuzzy objective as well as fuzzy constraints are not well defined following section-2 (as discussed in section-3.2) the above problem can be rewritten in optimistic and pessimistic sense by (17) and (18) respectively as

$$\left\{\begin{array}{l} \text{Determine a complete tour} \quad (x_1,x_2,...,x_N,x_1) \\ \text{and corresponding conveyance} \ (v_1,v_2,...,v_N) \ \text{to minimize} \quad Z \\ \text{subject to} \quad Pos(\sum_{i=1}^{N-1} \tilde{c}(x_i,x_{i+1},v_i) + \tilde{c}(x_N,x_1,v_N) < Z) \geq \alpha_3 \\ \qquad Pos(\sum_{i=1}^{N-1} \tilde{s}(x_i,x_{i+1},v_i) + \tilde{s}(x_N,x_1,v_N) \geq \tilde{s}_{min}) \geq \beta_3 \end{array}\right. \tag{17}$$

$$\left\{\begin{array}{l} \text{Determine a complete tour} \quad (x_1,x_2,...,x_N,x_1) \\ \text{and corresponding conveyance} \ (v_1,v_2,...,v_N) \ \text{to minimize} \quad Z \\ \text{subject to} \quad Nes(\sum_{i=1}^{N-1} \tilde{c}(x_i,x_{i+1},v_i) + \tilde{c}(x_N,x_1,v_N) < Z) \geq \alpha_4 \\ \qquad Nes(\sum_{i=1}^{N-1} \tilde{s}(x_i,x_{i+1},v_i) + \tilde{s}(x_N,x_1,v_N) \geq \tilde{s}_{min}) \geq \beta_4 \end{array}\right. \tag{18}$$

where α_3, β_3 and α_4, β_4 are predefined levels of possibility and necessity respectively which are entirely determined by the salesman. The meaning of α_3, β_3, α_4 and β_4 are the same as α_2, β_2, α_3 and β_3 of section-3.2, respectively.
If we consider the fuzzy numbers as TFNs, $\tilde{c}(i,j,k) = (c(i,j,k)_1, c(i,j,k)_2, c(i,j,k)_3)$, $\tilde{s}(i,j,k) = (s(i,j,k)_1, s(i,j,k)_2, s(i,j,k)_3)$ and $\tilde{s}_{min} = (s_1, s_2, s_3)$, then following section-2 the above problems (17), (18) reduces to (19), (20) respectively as below:

$$\left\{\begin{array}{c} \text{Determine a complete tour } (x_1, x_2, ..., x_N, x_1) \\ \text{and corresponding conveyance types } (v_1, v_2, ..., v_N) \text{ to minimize } Z \\ \text{subject to } \dfrac{Z - F_1}{F_2 - F_1} \geq \alpha_3 \\ \dfrac{G_3 - s_1}{G_3 - G_2 + s_2 - s_1} \geq \beta_3 \end{array}\right. \tag{19}$$

$$\text{where } F_j = \sum_{i=1}^{N-1} c(x_i, x_{i+1}, v_i)_j + c(x_N, x_1, v_N)_j, \ j = 1, 2, 3.$$

$$\text{and } G_j = \sum_{i=1}^{N-1} s(x_i, x_{i+1}, v_i)_j + s(x_N, x_1, v_N)_j, \ j = 1, 2, 3.$$

$$\left\{\begin{array}{c} \text{Determine a complete tour } (x_1, x_2, ..., x_N, x_1) \\ \text{and corresponding conveyance } (v_1, v_2, ..., v_N) \text{ to minimize } Z \\ \text{subject to } \dfrac{F_3 - Z}{F_3 - F_2} \leq 1 - \alpha_4 \\ \dfrac{s_3 - G_1}{G_2 - G_1 + s_3 - s_2} \leq 1 - \beta_4 \end{array}\right. \tag{20}$$

which are equivalent to (21) and (22) respectively.

$$\left\{\begin{array}{c} \text{Determine a complete tour } (x_1, x_2, ..., x_N, x_1) \\ \text{and corresponding conveyance } (v_1, v_2, ..., v_N) \\ \text{to minimize } F_1 + \alpha_3(F_2 - F_1) \\ \text{subject to } \dfrac{G_3 - s_1}{G_3 - G_2 + s_2 - s_1} \geq \beta_3 \end{array}\right. \tag{21}$$

$$\left\{\begin{array}{c} \text{Determine a complete tour } (x_1, x_2, ..., x_N, x_1) \\ \text{and corresponding conveyance } (v_1, v_2, ..., v_N) \\ \text{to minimize } F_3 - (1 - \alpha_4)(F_3 - F_2) \\ \text{subject to } \dfrac{s_3 - G_1}{G_2 - G_1 + s_3 - s_2} \leq 1 - \beta_4 \end{array}\right. \tag{22}$$

4. **Ant Colony Optimization Algorithm**

Observing the foraging behavior of ants that their ability in finding the shortest path between their nest and a food sources, Dorigo [7] first developed an algorithmic model to find solution of combinatorial optimization problem in 1992. Since then, research in the development of ant based algorithms has get momentum, resulting in a large number of algorithms and applications [10]. The principle of these methods is based on the way ants search for food and find their way back to the nest. During trips of ants a chemical trail called pheromone is left on the ground. The role of pheromone is to guide the other ants towards the target point. For one ant, the path is chosen according to the quantity of pheromone. Presently successful ACO algorithms are available [1, 2, 4, 25, 33] to solve different combinatorial optimization problems. Here, to solve our problems (both crisp and fuzzy) basic ACO [8] algorithm is little modified and is presented below. In the algorithm τ_{ij} is the amount of pheromone left on the path joining the nodes i and j. For STSP τ_{ijv} is the amount of pheromone left on the path of conveyance v joining the nodes i and j. Path[k] represents the path of k-th ant. Path[k][1] represents starting node, Path[k][2] represents second node to be visited and so on. V[k] represents the conveyance used from different nodes by k-th ant. V[k][i] represents conveyance used by k-th ant to travel from node Path[k][i] to Path[k][$i+1$], for $i = 1, 2, ...$ $N-1$, and V[k][N] to travel from node Path[k][N] to Path[k][1].

```
Begin
Set iteration counter t = 0 and maximum iteration number M_0.
Place n ants at node 1.
For i = 1, 2, ...,N do
    For j = 1, 2,...,N do
        Initialize τ_ij [τ_ijv ∀v for STSP]
    End For.
End For.
While (t ≤ M_0) do
    For each ant k = 1, 2, ..., n do //construct path of k-th ant
      DO
        Set Path[k][1] = 1
        Set NODE={2, ..., N}
        Repeat
            Let the present position of the ant be node i and l nodes
            has already been visited including the starting node, i.e.,
            Path[k][l]
            = i
            For TSP select next node j from NODE, depending upon
            the pheromone τ_ij left on the link (i,j). For STSP next
            node j is selected for visit by the the traveller using con-
            veyance v (i.e., V[k][i] = v) depending upon the pheromone
            τ_ijv. Roulette-Wheel selection process [24] is used for this
            selection in both the cases.
            Add link (i, j) to Path[k], i.e., set Path[k][l + 1] = j.
            NODE=NODE-{j}
        Until NODE=φ [φ is NULL set]
      While (Path[k] does not satisfy the environmental effect constraint)
    End For.
    For i = 1, 2, ... ,N do
        For j = 1, 2, ... ,N do
```

Evaporate τ_{ij} $[\tau_{ijv}, \forall v$ for STSP]
End For.
End For.
For each ant $k = 1, 2, \ldots ,n$ do
For $l = 1, 2, \ldots\ N - 1$ do
i =PATH[k][l]
j =PATH[k][$l + 1$]
Update τ_{ij} [$\tau_{ijV[k][l]}$ for STSP]
End For.
i =PATH[k][N]
j =PATH[k][1]
Update τ_{ij} [$\tau_{ijV[k][N]}$ for STSP]
End For.
$t = t + 1$
End While
Output: path with minimum cost
End

4.1. **ACO Procedures for the Proposed TSPs.**

4.1.1. **Representation.** An integer variable n is used to represent number of ants in the system. A two dimensional integer array PATH[n][N] is used to represent paths of different ants. PATH[k] represents path of k-th ant. Path[k][1] represents starting node, Path[k][2] represents second node to be visited and so on and PATH[k][N] is the last node visited by the ant-k. For STSP another two dimensional integer array V[n][N] is used to represent different conveyances used by different ants to travel from different nodes. V[k][i] represents conveyance used by k-th ant to travel from i-th node (PATH[k][i]) to $(i + 1)$-th node (PATH[k][$i + 1$]).

4.1.2. **Pheromone Initialization.** As aim of a TSP is to minimize the cost, it is assumed that initial value of $\tau_{ij} = 1/C(i, j)$. Similarly for STSP it is assumed that $\tau_{ijk} = 1/C(i, j, k)$.

4.1.3. **Path Selection.** Let recent position of the ant-$k$be node-i and it is the l-th node visited by the ant, i.e., PATH[k][l] $= i$. Then next node $j \in$NODE is selected by the ant with a probability p_{ij} given by the formula [10]

$$p_{ij} = \frac{T_{ij}^{\alpha}}{\sum\limits_{j \in NODE} T_{ij}^{\alpha}}$$

where α is a positive constant used to amplify the influence of pheromone concentrations and NODE is the set of nodes not visited by the ant. For selection of path using above probability Roulette-Wheel selection process [24] is used. Once a node is selected it is discarded from NODE.

For STSP next node $j \in$NODE is selected for visiting by the traveller using conveyances v (i.e., V[k][i] $= v$) with a probability p_{ijv} given by the formula

$$p_{ijv} = \frac{T_{ijv}^{\alpha}}{\sum\limits_{j \in NODE} \sum\limits_{v=1}^{M} T_{ijv}^{\alpha}}$$

where α is a positive constant as described above. In this case Roulette-Wheel selection process is also used. Here, to solve the problem α is taken 1.6.

4.1.4. **Pheromone Evaporation.** For evaporation of pheromone the following formulas are used

$$\tau_{ij} = (1-\rho)\tau_{ij}, \;\; \tau_{ijk} = (1-\rho)\tau_{ijk}$$

where $\rho \in [0,1]$. The constant ρ, specifies the rate at which pheromone evaporates, causing ants to forget previous decisions. Here, to solve the problem ρ is taken 0.2.

4.1.5. **Pheromone Updating.** Once all ants have constructed their complete tour pheromone is increased on the paths through which the ants move. If COST(k) be the cost of PATH[k], then for this path $\tau_{PATH[k][i]PATH[k][i+1]}$ is increased by 1/COST(k). As our aim is to minimize the cost this formula is used. Similarly for STSP if COST(k) be the cost of (PATH[k], V[k]), $\tau_{PATH[k][i]PATH[k][i+1]V[k][i]}$ is increased by 1/COST(k).

4.1.6. **Implementation.** With the above function and values the algorithm is implemented using C-programming language.

5. Genetic Algorithm

Genetic Algorithms are exhaustive search algorithms based on the mechanics of natural selection and genesis (crossover, mutation etc.) and have been developed by Holland, his colleagues and students at the University of Michigan [13]. Because of its generality and other advantages over conventional optimization methods it has been successfully applied to different decision making problems [24].

In natural genesis, we know that chromosomes are the main carriers of hereditary information from parent to offspring and that genes, which present hereditary factors, are lined up in chromosomes. At the time of reproduction, crossover and mutation take place among the chromosomes of parents. In this way hereditary factors of parents are mixed-up and carried over to their offsprings. Again Darwinian principle states that only the fittest animals can survive in nature. So a pair of fittest parents normally reproduces a better offspring.

The above-mentioned phenomenon is followed to create a genetic algorithm for an optimization problem. Here potential solutions of the problem are analogous with the chromosomes and chromosome of better offspring with the better solution of the problem. Crossover and mutation happen among a set of potential solutions to get a new set of solutions and it continues until terminating conditions are encountered. Here a GA is used to solve our TSP in both the crisp and fuzzy environment. The proposed GA and its procedures are presented below.

GA Algorithm

1. Begin
2. Initialize max generation number M_0, population size(pop_size), probability of crossover(p_c) & mutation (p_m).
3. Set iteration counter $t = 0$.
2. Randomly generate initial population $p(t)$
3. Evaluate initial population $p(t)$
4. While $t \leq M_0$ do
   a. $t \leftarrow t + 1$.
   b. Select $p(t)$ from $p(t-1)$.
   c. Alter (crossover and mutate) $p(t)$.
   d. Evaluate $p(t)$.
7. End While
8. print optimum result
9. End

5.1. **GA procedures for the proposed TSPs.**

5.1.1. **Representation.** Here a complete tour on N cities represents a solution. So a 'N dimensional integer vector' $X_i = (x_{i1}, x_{i2}, ..., x_{iN})$ is used to represent a solution, where x_{i1}, x_{i2}, ... , x_{iN} represent N consecutive cities in a tour. For solid TSP another integer vector $V_i = (v_{i1}, v_{i2}, ..., v_{iN})$ is used to represent the conveyances types used travel between different cities. Here v_{ij} represents the conveyance (an integer) used to travel from city x_{ij} to $x_{i(j+1)}$ for j=1, 2, ..., N-1 and v_{iN} represents the conveyance type used to travel from city x_{iN} to x_{i1}.

5.1.2. **Initialization.** pop_size number of such solutions $X_i = (x_{i1}, x_{i2}, ...x_{iN})$, $i = 1, 2, ...,$ pop_size, are randomly generated by random number generator, such that each solution satisfies the constraints of the problem. A separate sub_function check_constrint(X_i) is used for this purpose. For STSP another integer vector $V_i = (v_{i1}, v_{i2}, ..., v_{iN})$ is randomly generated corresponding to the solution X_i, to represent the conveyances types used to travel between different cities. So in that case (X_i, V_i) represent a solution.

5.1.3. **Evaluation Process.** To find fitness of a solution X_i [(X_i, V_i) for STSP], following two steps are used-

- Calculate objective function value OBJ_i for the solution X_i [(X_i, V_i) for STSP].
- As the problems are minimization type take MVAL-OBJ_i as fitness, FIT_i, of X_i [(X_i, V_i) for STSP], where MVAL is a sufficiently large value to make the fitness positive.

(d)Roulette-wheel selection process for mating pool: The following steps are followed for selection of $p(t)$ from $p(t-1)$ [24]:

(i) Find total fitness of the population, $p(t-1)$, $F_{tot} = \sum_{j=1}^{pop\_size} FIT_j$
(ii) Calculate the probability of selection p_i of each solution X_i [(X_i, V_i) for STSP] by the formula $p_i = \frac{FIT_i}{F_{tot}}$.

(iii) Calculate the cumulative probability q_i for each solution X_i [(X_i, V_i) for STSP] by the formula $q_i = \sum_{k=1}^{i} p_k$

(iv) Generate a random number r from the range [0,1].

(v) If $r < q_1$ then select X_1 [(X_1, V_1) for STSP] otherwise select $X_j (2 \leq j \leq n)$ [(X_j, V_j) for STSP] where $q_{j-1} \leq r < q_j$.

(vi) Repeat step (iv) and (v) pop_size times to select pop_size solutions for mating pool. Clearly one solution may be selected more than once.

(vii) Selected solution set is denoted by $p(t)$ in the proposed GA algorithm.

5.1.4. **Crossover.**

(i) Selection for crossover: For each solution of $p(t)$ generate a random number r from the range [0,1]. If $r < p_c$ then the solution is taken for crossover.

(ii) Crossover process: For simple TSP cyclic crossover process [29] is used. The cyclic crossover focuses on subsets of cities that occupy the same subset of positions in both parents. Then, these cities are copied from the first parent to the offspring (at the same positions), and the remaining positions are filled with the cities of the second parent. In this way, the position of each city is inherited from one of the two parents. However, many edges can be broken in the process, because the initial subset of cities is not necessarily located at consecutive positions in the parent tours. To illustrate the process let us consider a TSP consisting of 9 cities and consider two parents PR_1, PR_2 as below:

$$PR_1 : 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \quad 8 \quad 9$$

$$PR_2 : 3 \quad 4 \quad 5 \quad 1 \quad 2 \quad 9 \quad 8 \quad 7 \quad 6$$

Let CH_1, CH_2 be two children born after crossover. The mechanism of birth of CH_1, CH_2 using cycle crossover is explained by the following steps:

- Randomly generate an integer in the range [1 ... 9]. Let it be 3.
- As $PR_1[3] = 3$, 3-rd element of CH_1 is 3, i.e., $CH_1[3] = 3$.
- PR_2, is then searched to check the presence of element 3 and it has been found in the 1-st position. Then 1-st element of CH_1 is selected from the 1-st element of PR_1, i.e., $CH_1[1] = PR_1[1] = 1$.
- PR_2, is again searched for the presence of element 1 and it has occurred at the 4-th position. Thus 4-th element of PR_1 has been copied as the 4-st element of CH_1, i.e., $CH_1[4] = PR_1[4] = 4$. Similarly, following are obtained:

$$CH_1[2] = PR_1[2] = 2, \quad CH_1[5] = PR_1[5] = 5$$

This completes one cycle because 5 is seen to be present at the 3-rd position of PR_2 and the corresponding 3-rd position element of PR_1 is 3, which has already been selected as the starting element of the cycle.

– The remaining elements of CH_1 are selected directly from PR_2 as follows:

$$CH_1[6] = PR_2[6] = 9, \quad CH_1[7] = PR_1[7] = 8$$

$$CH_1[8] = PR_2[8] = 7, \quad CH_1[9] = PR_1[9] = 6$$

– Final forms of CH_1 and CH_2 are as below:

$$CH_1 : 1 \;\; 2 \;\; 3 \;\; 4 \;\; 5 \;\; 9 \;\; 8 \;\; 7 \;\; 6$$

$$CH_2 : 3 \;\; 4 \;\; 5 \;\; 1 \;\; 2 \;\; 6 \;\; 7 \;\; 8 \;\; 9$$

If CH_1 satisfies the constraint of the problem then PR_1 is replaced by CH_1. Similarly if CH_2 satisfies the constraint of the problem then PR_2 is replaced by CH_2.

For STSP to made crossover on two parents (PR_1, V_1), (PR_2, V_2), the same procedure is followed by PR_1 and PR_2 to obtain CH_1 and CH_2. To keep randomness in selection of conveyances conveyance sets V_1 and V_2 remains unchanged, i.e., resultant child after crossover becomes (CH_1, V_1), (CH_2, V_2). If (CH_1, V_1) satisfies the constraint of the problem then (PR_1, V_1) is replaced by (CH_1, V_1). Similarly if (CH_2, V_2) satisfies the constraint of the problem then (PR_2, V_2) is replaced by (CH_2, V_2).

5.1.5. **Mutation.**

(i) Selection for mutation: For each solution of $p(t)$ generate a random number r from the range [0,1]. If $r < p_m$ then the solution is taken for mutation.

(ii) Mutation process: To mutate a solution $X = (x_1, x_2, ...x_N)$ of TSP select two random integers i, j in the range $[1, N]$. Then interchange x_i, x_j to get child solution. New solution, if satisfies the constraint of the problem, replaces the parent solution.

For STSP to mutate a solution (X,V), where $X = (x_1, x_2, ...x_N)$, $V = (v_1, v_2, ...v_N)$ at first an integer is randomly selected in the range [1,2]. If 1 is selected then another two random integers i, j are selected in the range $[1, N]$. Then interchange x_i, x_j to get child solution. If 2 is selected then another two random integers i and j are selected in the range $[1, N]$ and $[1, M]$ respectively. Value of v_i is replaced by j to get child solution. If child solution satisfies the constraint of the problem then it replaces the parent solution.

5.1.6. **Implementation.** With the above function and values the algorithm is implemented using C-programming language.

6. **Numerical Illustration**

6.1. **Classical TSP with Environmental Effect Constraints.** This problem is illustrated for ten cities (N=10). Cost and environmental effect matrices are presented in Tables- 1 and 2 respectively. Optimal paths are obtained using both ACO and GA process without and with different minimum environmental effects s_{min}

i/j	1	2	3	4	5	6	7	8	9	10
1	∞	25	28	32	20	6	35	37	40	30
2	37	∞	20	28	35	40	30	42	28	4
3	42	28	∞	30	25	35	9	32	40	30
4	28	30	7	∞	20	25	30	35	22	37
5	37	22	35	30	∞	20	25	30	9	28
6	25	30	25	8	28	∞	32	40	32	30
7	28	25	30	22	37	40	∞	10	32	20
8	20	5	32	40	35	25	40	∞	22	37
9	30	40	35	25	20	22	37	32	∞	28
10	28	30	28	20	11	32	37	40	30	∞

Table 1. Cost Matrix $c(i, j)$ for Ten Cities ($N = 10$)

i/j	1	2	3	4	5	6	7	8	9	10
1	-	.30	.40	.50	.23	.51	.70	.60	.56	.55
2	.51	-	.60	.56	.55	.33	.44	.45	.52	.30
3	.56	.55	-	.42	.33	.36	.50	.61	.55	.70
4	.51	.70	.60	-	.55	.33	.56	.55	.41	.42
5	.20	.30	.40	.50	-	.41	.42	.33	.36	.50
6	.51	.70	.60	.56	.55	-	.44	.45	.52	.30
7	.56	.55	.41	.42	.33	.36	-	.61	.55	.70
8	.51	.70	.60	.56	.55	.33	.56	-	.41	.42
9	.30	.36	.39	.42	.51	.62	.29	.63	-	.45
10	.42	.51	.62	.29	.63	.71	.38	.28	.48	-

Table 2. Environmental Effect Matrix $s(i, j)$ for Ten Cities ($N = 10$)

Method	x[1],x[2],...,x[9],x[10]	cost
ACO/GA	1,6,4,3,7,8,2,10,5,9	99

Table 3. Optimal Solution for Classical TSP without Environment Effect Constraints

s_{min}	Method	x[1],x[2],...,x[9],x[10]	cost
5.10	ACO/GA	3,7,2,10,5,9,8,9,6,4	131
5.20	ACO/GA	8,2,10,5,9,6,1,4,3,7	134
5.40	ACO/GA	6,4,3,7,1,8,2,10,5,9	140
5.50	ACO/GA	9,7,2,1,6,4,3,7,10,5	144

Table 4. Optimal Solution for Classical TSP with Environment Effect Constraints

i/j	1	2	3	4	5
1	∞	25,25.5,26	28,28.5,29	32,32.5,33	20,20.5,21
2	37,37.5,38	∞	20,20.5,21	28,28.5,29	35,35.5,36
3	42,42.5,43	28,28.5,29	∞	30,30.5,31	25,25.5,26
4	28,28.5,29	30,30.5,31	7,7.5,8	∞	20,20.5,21
5	37,37.5,38	22,22.5,23	35,35.5,36	30,30.5,31	∞
6	25,25.5,26	30,30.5,31	25,25.5,26	8,8.5,9	28,28.5,29
7	28,28.5,29	25,25.5,26	30,30.5,31	22,22.5,23	37,37.5,38
8	20,20.5,21	5,5.5,6	32,32.5,33	40,40.5,41	35,35.5,36
9	30,30.5,31	40,40.5,41	35,35.5,36	25,25.5,26	20,20.5,21
10	28,28.5,29	30,30.5,31	28,28.5,29	20,20.5,21	11,11.5,12
i/j	6	7	8	9	10
1	6,6.5,6.9	35,35.5,36	37,37.5,38	40,40.5,41	30,30.5,31
2	40,40.5,41	30,30.5,31	42,42.5,43	28,28.5,29	4,4.5,5
3	35,35.5,36	9,9.5,10	32,32.5,33	40,40.5,41	30,30.5,31
4	25,25.5,26	30,30.5,31	35,35.5,36	22,22.5,23	37,37.5,38
5	20,20.5,21	25,25.5,26	30,30.5,31	9,9.5,10	28,28.5,29
6	∞	32,32.5,33	40,40.5,41	32,32.5,33	30,30.5,31
7	40,40.5,41	∞	10,10.5,11	32,32.5,33	20,20.5,21
8	25,25.5,26	40,40.5,41	∞	22,22.5,23	37,37.5,38
9	22,22.5,23	37,37.5,38	32,32.5,33	∞	28,28.5,29
10	32,32.5,33	37,37.5,38	40,40.5,41	30,30.5,31	∞

TABLE 5. Fuzzy Cost Matrix $\tilde{c}$(i,j) for Ten Cities

and presented in Tables-3 and 4 respectively. It is observed that results obtained via both the processes are the same for e cases without and with environment effect. It is also observed that cost of the tour increases with increment of S_{min}, which agrees with reality.

6.2. **Classical Fuzzy TSP with Environmental Effect Constraints.** This problem is also illustrated for ten cities ($N = 10$). Cost and environmental effect matrices are presented in Tables-5 and 6 respectively. Other assumed parametric values are $\widetilde{S_{min}} = (5.15, 5.60, 6.0)$, $\alpha_1 = 0.9$, $\alpha_2 = 0.1$.

Optimal paths for both the optimistic DM (ODM) and pessimistic DM (PDM), i.e., solutions of problems (13) and (14) respectively are obtained using both ACO and GA processes due to different values of β_1 and β_2 and are presented in Table-8. In this case it is also observed that results obtained via both processes are almost the same. It is also observed that total cost increases by increasing β_1, β_2 in both pessimistic and optimistic cases. All these are our expectations, rigid constraint increases the total cost. The results without taking any constraint into account are also obtained and presented in Table-7. In this table objective values (OBJV) i.e. C_1+α_1*$(C_2 - C_1)$ are also given, which are different in pessimistic and optimistic

i/j	1	2	3	4	5	6	7	8	9	10
1	-	.3,.35,.38	.4,.45,5	.5,.55,.6	.23,.25,.29	.51,.6,.62	.7,.75,.78	.6,.65,.7	.56,.59,.6	.55,.56,.57
2	.51,.55,.58	-	.6,.65,.69	.8,.85,.88	.55,.58,.6	.33,.36,.38	.44,.48,.5	.45,.48,.51	.52,.55,.58	.3,.35,.4
3	.56,.6,.65	.55,.58,.6	-	.42,.44,.45	.33,.35,.38	.36,.38,.4	50,.55,.58	.61,.65,.7	.55,.58,.58	.7,.75,.8
4	.51,.55,.58	.7,.75,.77	.6,.7,.73	-	.55,.58,.61	.33,.36,.38	.56,.58,.62	.55,.56,.57	.41,.45,.48	.42,.45,.48
5	.51,.52,.53	.7,.71,.72	.6,.62,.63	.56,.57,.58	-	.33,.34,.35	.56,.57,.58	.55,.56,.57	.41,.42,.43	.42,.43,.44
6	.20,.25,.28	.30,.35,.38	.40,.45,50	.50,.55,.60	.23,.25,.29	-	.7,.75,.78	.6,.65,.7	.56,.59,.6	.55,.56,.57
7	.51,.55,.58	.7,.75,.78	.6,.65,.69	.8,.85,.88,	.55,.58,.6	.33,.36,.38	-	.45,.48,.51	.52,.55,.58	.3,.35,.4
8	.56,.6,.65	.55,.58,.6	.41,.45,.5	.42,.44,.45	.33,.35,.38	.36,.38,.4	50,.55,.58	-	.55,.58,.58	.7,.75,.8
9	.51,.55,.58	.7,.75,.77	.6,.7,.73	.56,.57,.58	.55,.58,.61	.33,.36,.38	.56,.58,.62	.55,.56,.57	-	.42,.45,.48
10	.51,.52,.53	.7,.71,.72	.6,.62,.63	.56,.57,.58	.55,.56,.57	.33,.34,.35	.56,.57,.58	.55,.56,.57	.41,.42,.43	-

TABLE 6. Fuzzy Environmental Effect Matrix $\tilde{s}$(i,j) for Ten Cities

Method	DM	x[1],x[2],...,x[9],x[10]	$\widetilde{cost}$	OBJV
ACO/	PDM	4,3,7,8,2,10,5,9,1,6	99.0,104.0,108.0	107.59
GA	ODM	7,8,2,10,5,9,4,3,1,6	99.0,104.0,108.0	103.50

TABLE 7. Optimal Solution for Classical Fuzzy TSP without Environment Effect Constraints

models. For the present parametric values it is observed that the optimum cost of both models are the same as the following approaches ACO and GA.

6.3. **Solid TSP with Crisp Cost and Environmental Effect Constraints.** This problem is illustrated for ten cities (N=10) and three types of conveyances(M=3). Cost matrices are presented in Table-9, 10, 11 respectively. Similarly environmental effect matrices for different conveyances are given in Table-12, 13, 14 respectively.

Optimal paths along with the selected conveyances are obtained due to different S_{min} using both ACO and GA processes and presented in Table-16. It is observed that results obtained via both processes are almost the same. In this table, as

β_1/β_2	Method	DM	x[1],x[2],...,x[9],x[10]	$\widetilde{cost}$
.2	ACO	PDM	6,7,4,3,8,2,10,5,9,1	158.0,164.0,167.3
		ODM	1,6,4,3,7,8,2,10,5,9	99.0,104.0,108.9
	GA	PDM	6,7,4,3,8,2,10,5,9,1	158.0,164.0,167.3
		ODM	1,6,4,3,7,8,2,10,5,9	99.0,104.0,108.9
.6	ACO	PDM	1,6,10,5,9,2,4,3,7,8	170.0,175.00,179.0
		ODM	9,8,1,6,4,3,7,2,10,5	131.0,136.0,140.9
	GA	PDM	1,6,7,8,2,4,3,10,5,9	168.0,173.00,177.9
		ODM	9,8,1,6,4,3,7,2,10,5	131.0,136.0,140.9
.8	ACO	PDM	1,6,7,4,3,10,5,9,8,2	191.0,196.00,200.8
		ODM	9,8,1,6,4,3,7,2,10,5	131.0,136.0,140.9
	GA	PDM	1,6,7,4,3,10,5,9,8,2	191.0,196.00,200.8
		ODM	9,8,1,6,4,3,7,2,10,5	131.0,136.0,140.9

Table 8. Optimal Solution for Classical Fuzzy TSP with Environment Effect Constraints

i/j	1	2	3	4	5	6	7	8	9	10
1	∞	25	28	32	20	6	35	37	40	30
2	37	∞	20	28	35	40	30	42	28	4
3	42	28	∞	30	25	35	9	32	40	30
4	28	30	7	∞	20	25	30	35	22	37
5	37	22	35	30	∞	20	25	30	9	28
6	25	30	25	8	28	∞	32	40	32	30
7	28	25	30	22	37	40	∞	10	32	20
8	20	5	32	40	35	25	40	∞	22	37
9	30	40	35	25	20	22	37	32	∞	28
10	28	30	28	20	11	32	37	40	30	∞

Table 9. Cost Matrix c(i,j,1) for Ten Cities Via First Conveyances

i/j	1	2	3	4	5	6	7	8	9	10
1	∞	25.5	28.5	32.5	20.5	6.5	35.5	37.5	40.5	30.5
2	37.5	∞	20.5	28.5	35.5	40.5	30.5	42.5	28.5	4.5
3	42.5	28.5	∞	30.5	25.5	35.5	9.5	32.5	40.5	30.5
4	28.5	30.5	7.5	∞	20.5	25.5	30.5	35.5	22.5	37.5
5	37.5	22.5	35.5	30.5	∞	20.5	25.5	30.5	9.5	28.5
6	25.5	30.5	25.5	8.5	28.5	∞	32.5	40.5	32.5	30.5
7	28.5	25.5	30.5	22.5	37.5	40.5	∞	10.5	32.5	20.5
8	20.5	5.5	32.5	40.5	35.5	25.5	40.5	∞	22.5	37.5
9	30.5	40.5	35.5	25.5	20.5	22.5	37.5	32.55	∞	28.5
10	28.5	30.5	28.5	20.5	11.5	32.5	37.5	40.5	30.5	∞

Table 10. Cost Matrix c(i,j,2) for Ten Cities Via Second Conveyances

i/j	1	2	3	4	5	6	7	8	9	10
1	∞	26	29	33	21	6.9	36	38	41	31
2	37	∞	20	28	35	40	30	42	28	4
3	42	28	∞	30	25	35	9	32	40	30
4	28	30	7	∞	20	25	30	35	22	37
5	37	22	35	30	∞	20	25	30	9	28
6	25	30	25	8	28	∞	32	40	32	30
7	28	25	30	22	37	40	∞	10	32	20
8	20	5	32	40	35	25	40	∞	22	37
9	30	40	35	25	20	22	37	32	∞	28
10	28	30	28	20	11	32	37	40	30	∞

Table 11. Cost Matrix c(i,j,3) for Ten Cities Via Third Conveyances

i/j	1	2	3	4	5	6	7	8	9	10
1	∞	.3	.4	.5	.23	.51	.7	.6	.56	.55
2	.51	∞	.6	.8	.55	.33	.44	.45	.52	.3
3	.56	.55	∞	.42	.33	.36	.50	.61	.55	.7
4	.51	.7	.6	∞	.55	.33	.56	.55	.41	.42
5	.51	.7	.6	.56	∞	.33	.56	.55	.41	.42
6	.2	.3	.4	.5	.23	∞	.7	.6	.56	.55
7	.51	.7	.6	.8	.55	.33	∞	.45	.52	.3
8	.56	.55	.41	.42	.33	.36	.50	∞	.55	.7
9	.51	.7	.6	.56	.55	.33	.56	.55	∞	.42
10	.51	.7	.6	.8	.55	.33	.56	.55	.41	∞

Table 12. Environmental Effect Matrix s(i,j,1) for Ten Cities Via First Conveyances

i/j	1	2	3	4	5	6	7	8	9	10
1	∞	.35	.45	.55	.25	.6	.75	.65	.59	.56
2	.55	∞	.65	.85	.58	.36	.48	.48	.55	.35
3	.6	.58	∞	.44	.35	.38	.55	.65	.58	.75
4	.55	.75	.7	∞	.58	.36	.58	.56	.45	.45
5	.52	.71	.62	.57	∞	.34	.57	.56	.42	.43
6	.25	.35	.45	.55	.25	∞	.75	.65	.59	.56
7	.55	.75	.65	.85	.58	.36	∞	.48	.55	.35
8	.6	.58	.45	.44	.35	.38	.55	∞	.58	.75
9	.55	.75	.7	.57	.58	.36	.58	.56	∞	.45
10	.52	.71	.62	.57	.56	.34	.57	.56	.42	∞

Table 13. Environmental Effect Matrix s(i,j,2) for Ten Cities Via Second Conveyances

i/j	1	2	3	4	5	6	7	8	9	10
1	∞	.38	.5	.6	.29	.62	.78	.7	.6	.57
2	.58	∞	.69	.88	.6	.38	.5	..51	.58	.5
3	.65	.6	∞	.45	.38	.4	.58	.7	.58	.8
4	.58	.77	.73	∞	.61	.38	.62	.57	.48	.48
5	.53	.72	.63	.58	∞	.35	.58	.57	.43	.44
6	.28	.38	.5	.6	.29	∞	.78	.7	.6	.57
7	.58	.78	.69	.88	.6	.38	∞	.51	.58	.4
8	.65	.6	.5	.45	.38	.4	.58	∞	.58	.8
9	.58	.77	.73	.58	.61	.38	.62	.57	∞	.48
10	.53	.72	.63	.58	.57	.35	.58	.57	.43	∞

Table 14. Environmental Effect Matrix s(i,j,3) for Ten Cities Via Third Conveyances

Method	(x[1],v[1]),(x[2],v[2]),...,(x[10],v[10])	*cost*
ACO	1,1;6,1;4,1;3,1;7,3;8,1;2,1;10,2;5,2;9,3	102.5
GA	5,1;4,3;3,2;7,1;8,1;2,3;10,2;5,1;9,2;1,3	104.0

Table 15. Optimal Solution for Solid TSP without Environment Effect Constraints

S_{min}	Method	x[1],v[1]; x[2],v[2];...;x[10],v[10]	*cost*
5.6	ACO	1,3; 6,3; 4,2; 3,2; 7,2; 2,3; 10,1; 5,1; 9,3; 8,2	136.3
	GA	5,1; 9,1; 8,3; 1,2; 6,1; 4,3; 3,1; 7,2; 2,3; 10,1	135.6
5.7	ACO	1,3; 6,3; 4,3; 3,3; 7,2; 2,3; 10,1; 5,1; 9,3; 8,3	138.3
	GA	1,1; 6,2; 4,3; 3,2; 7,3; 2,3; 10,3; 5,1; 9,1; 8,3	137.5
5.8	ACO	1,1; 6,3; 7,1; 4,3; 3,2; 8,2; 2,1; 10,3; 5,3; 9,1	163.3
	GA	1,1; 6,2; 7,1; 4,2; 3,2; 8,3; 2,1; 10,3; 5,3; 9,1	162.7

Table 16. Optimal Solution for Solid TSP with Crisp Cost and Environment Effect Constraints

expected, the results of two processes increase with the increase of S_{min}. The results without taking constrains are also obtained and presented in Table-15.

6.4. **Solid TSP with Fuzzy Costs and Environmental Effect Constraints.** This problem is also illustrated for ten cities (N=10) and three types of conveyances(M=3). Cost matrix for M=1, M=2 and M=3 are presented in Table-17, 18 and 19 respectively. Similarly environmental effect matrix for different conveyances are given in Table-20, 21 and 22 respectively. Other assumed parametric values are $\tilde{S_{min}} = (5.6, 5.7, 5.8)$, $\alpha_3 = 0.9$, $\alpha_4 = 0.1$. Optimal paths along with the selected conveyances are obtained using both ACO and GA process for both the optimistic and pessimistic DMs due to different values of β_3, β_4 and are presented in Table-24. It is observed that in few cases ACO gives slightly better results than

i/j	1	2	3	4	5
1	-	25,25.2,25.4	28,28.2,28.5	32,32.2,32.5	20,20.2,20.5
2	37,37.2,37.5	-	20,20.2,20.5	28,28.2,28.5	35,35.2,35.5
3	42,42.2,42.5	28,28.2,28.5	-	30,30.2,30.5	25,25.2,25.4
4	28,28.2,28.5	30,30.2,30.5	7,7.2,7.5	-	20,20.2,20.5
5	37,37.2,37.5	22,22.2,22.	35,35.2,35.5	30,30.2,30.5	-
6	25,25.2,25.4	30,30.2,30.5	25,25.2,25.4	8,8.2,8.4	28,28.2,28.5
7	28,28.2,28.5	25,25.2,25.4	30,30.2,30.5	22,22.2,22.5	37,37.2,37.5
8	20,20.2,20.5	5,5.2,5.5	32,32.2,32.5	40,40.2,40.5	35,35.2,35.5
9	30,30.2,30.5	40,40.2,40.5	35,35.2,35.5	25,25.2,25.4	20,20.2,20.5
10	28,28.2,28.5	30,30.2,30.5	28,28.2,28.5	20,20.2,20.5	11,11.2,11.4
i/j	6	7	8	9	10
1	6,6.2,6.5	35,35.2,35.5	37,37.2,37.5	40,40.2,40.5	30,30.2,30.5
2	40,40.2,40.5	30,30.2,30.5	42,42.2,42.5	28,28.2,28.5	4,4.2,4.5
3	35,35.2,35.5	9,9.2,9.5	32,32.2,32.5	40,40.2,40.5	30,30.2,30.5
4	25,25.2,25.4	30,30.2,30.5	35,35.2,35.5	22,22.2,22.5	37,37.2,37.5
5	20,20.2,20.5	25,25.2,25.4	30,30.2,30.5	9,9.2,9.5	28,28.2,28.5
6	-	32,32.2,32.5	40,40.2,40.5	32,32.2,32.5	30,30.2,30.5
7	40,40.2,40.5	-	10,10.2,10.5	32,32.2,32.5	20,20.2,20.5
8	25,25.2,25.4	40,40.2,40.5	-	22,22.2,22.5	37,37.1,37.4
9	22,22.2,22.5	37,37.1,37.4	32,32.2,32.5	-	28,28.2,28.5
10	32,32.2,32.5	37,37.1,37.4	40,40.2,40.5	30,30.2,30.5	-

TABLE 17. Fuzzy Cost Matrix $\tilde{c}(i, j, 1)$ for Ten Cities Via First Conveyances

i/j	1	2	3	4	5
1	-	25.5,25.6,25.8	28.5,28.6,28.8	32.5,32.6,32.8	20.5,20.6,20.8
2	37.5,37.6,37.8	-	20.5,20.6,20.8	28.5,28.6,28.8	35.5,35.6,35.8
3	42.5,42.6,42.8	28.5,28.6,28.8	-	30.5,30.2,30.5	25.5,25.6,25.8
4	28.5,28.6,28.8	30.5,30.2,30.5	7.5,7.6,7.8	-	20.5,20.6,20.8
5	37.5,37.6,37.8	22.5,22.6,22.8	35.5,35.6,35.	30.5,30.2,30.5	-
6	25.5,25.6,25.8	30.5,30.2,30.5	25.5,25.6,25.8	8.5,8.6,8.8	28.5,28.6,28.8
7	28.5,28.6,28.8	25.5,25.6,25.8	30.5,30.2,30.5	22.5,22.6,22.8	37.5,37.6,37.8
8	20.5,20.6,20.8	5.5,5.6,5.8	32.5,32.6,32.8	40.5,40.6,40.8	35.5,35.6,35.8
9	30.5,30.2,30.5	40.5,40.6,40.8	35.5,35.6,35.8	25.5,25.6,25.8	20.5,20.6,20.8
10	28.5,28.6,28.8	30.5,30.2,30.5	28.5,28.6,28.8	20.5,20.6,20.8	11.5,11.6,11.8
i/j	6	7	8	9	10
1	6.5,6.6,6.8	35.5,35.6,35.8	37.5,37.6,37.8	40.5,40.6,40.8	30.5,30.2,30.5
2	40.5,40.6,40.8	30.5,30.2,30.5	42.5,42.6,42.8	28.5,28.6,28.8	4.5,4.6,4.8
3	35.5,35.6,35.8	9.5,9.6,9.8	32.5,32.6,32.8	40.5,40.6,40.8	30.5,30.2,30.5
4	25.5,25.6,25.8	30.5,30.2,30.5	35.5,35.6,35.8	22.5,22.6,22.8	37.5,37.6,37.8
5	20.5,20.6,20.8	25.5,25.6,25.8	30.5,30.2,30.5	9.5,9.6,9.8	28.5,28.6,28.8
6	-	32.5,32.6,32.8	40.5,40.6,40.8	32.5,32.6,32.8	30.5,30.2,30.5
7	40.5,40.6,40.8	-	10.5,10.6,10.8	32.5,32.6,32.8	20.5,20.6,20.8
8	25.5,25.6,25.8	40.5,40.6,40.8	-	22.5,22.6,22.8	37.5,37.6,37.7
9	22.5,22.6,22.8	37.5,37.6,37.7	32.5,32.6,32.8	-	28.5,28.6,28.8
10	32.5,32.6,32.8	37.5,37.6,37.7	40.5,40.6,40.8	30.5,30.2,30.5	-

TABLE 18. Fuzzy Cost Matrix $\tilde{c}(i, j, 2)$ for Ten Cities Via Second Conveyances

i/j	1	2	3	4	5
1	-	26,26.5,26.8	29,29.3,29.8	33,33.2,3.5	21,21.3,21.6
2	38,38.2,38.5	-	21,21.3,21.6	29,29.3,29.8	36,36.2,36.6
3	43,43.2,43.5	29,29.3,29.8	-	31,31.2,31.5	26,26.5,26.8
4	29,29.3,29.8	31,31.2,31.5	8,8.2,8.5	-	21,21.3,21.6
5	38,38.2,38.	23,23.2,23.6	36,36.2,36.6	31,31.2,31.5	-
6	26,26.5,26.8	31,31.2,31.5	26,26.5,26.5	9,9.2,9.5	29,29.3,29.8
7	29,29.3,29.8	26,26.5,26.5	31,31.2,31.5	23,23.2,23.6	38,38.2,38.5
8	21,21.3,21.6	6,6.3,6.5	33,33.2,3.5	41,41.2,41.5	36,36.2,36.6
9	31,31.2,31.5	41,41.2,41.5	36,36.2,36.6	26,26.5,26.5	21,21.3,21.6
10	29,29.3,29.8	31,31.2,31.5	29,29.3,29.8	21,21.3,21.6	12,12.3,12.5

i/j	6	7	8	9	10
1	6.9,7.0,7.3	36,36.2,36.6	38,38.2,38.5	41,41.2,41.5	31,31.2,31.5
2	41,41.2,41.5	31,31.2,31.5	43,43.2,43.5	29,29.3,29.8	5,5.2,5.5
3	36,36.2,36.6	10,10.2,10.5	33,33.2,3.5	41,41.2,41.5	31,31.2,31.5
4	26,26.5,26.8	31,31.2,31.5	36,36.2,36.6	23,23.2,23.6	38,38.2,38.5
5	21,21.3,21.6	26,26.5,26.8	31,31.2,31.5	10,10.2,10.5	29,29.3,29.8
6	-	33,33.2,3.5	41,41.2,41.5	33,33.2,3.5	31,31.2,31.5
7	41,41.2,41.5	-	11,11.2,11.5	33,33.2,3.5	21,21.3,21.6
8	26,26.5,26.5	41,41.2,41.5	-	23,23.2,23.6	38,38.1,38.2
9	23,23.2,23.6	38,38.1,38.2	33,33.2,3.5	-	29,29.3,29.8
10	33,33.2,33.5	38,38.1,38.2	41,41.2,41.5	31,31.2,31.5	-

TABLE 19. Fuzzy Cost Matrix $\tilde{c}(i,j,3)$ for Ten Cities Via Second Conveyances

i/j	1	2	3	4	5
1	∞	.3,.31,.32	.4,.41,.42	.5,.51,.52	.23,.24,.25
2	.51,.55,.58	∞	.41,.45,.5	.44,.48,.5	.51,.55,.58
3	.2,.21,.22	.3,.31,.32	∞	.5,.51,.52	.23,.24,.25
4	.51,.55,.58	.56,.6,.65	.41,.45,.5	∞	.51,.55,.58
5	.2,.21,.22	.3,.31,.32	.4,.41,.42	.5,.51,.52	∞
6	.51,.55,.58	.56,.6,.65	.41,.45,.5	.44,.48,.5	.51,.55,.58
7	.2,.21,.22	.3,.31,.32	.4,.41,.42	.5,.51,.52	.23,.24,.25
8	.51,.55,.58	.56,.6,.65	.41,.45,.5	.44,.48,.5	.51,.55,.58
9	.2,.21,.22	.3,.31,.32	.4,.41,.42	.5,.51,.52	.23,.24,.25
10	.51,.55,.58	.56,.6,.65	.41,.45,.5	.44,.48,.5	.51,.55,.58

i/j	6	7	8	9	10
1	.51,.52,.53	.7,.71,.72	.6,.61,.62	.56,.57,.58	.55,.56,.57
2	.51,.55,.58	.33,.36,.38	.36,.38,.4	.51,.55,.58	.41,.45,.5
3	.51,.52,.53	.7,.71,.72	.6,.61,.62	.56,.57,.58	.55,.56,.57
4	.51,.55,.58	.33,.36,.38	.36,.38,.4	.51,.55,.58	.41,.45,.5
5	.51,.52,.53	.7,.71,.72	.6,.61,.62	.56,.57,.58	.55,.56,.57
6	∞	.33,.36,.38	.36,.38,.4	.51,.55,.58	.41,.45,.5
7	.51,.52,.53	∞	.6,.61,.62	.56,.57,.58	.55,.56,.57
8	.51,.55,.58	.33,.36,.38	∞	.51,.55,.58	.41,.45,.5
9	.51,.52,.53	.7,.71,.72	.6,.61,.62	∞	.55,.56,.57
10	.51,.55,.58	.33,.36,.38	.36,.38,.4	.51,.55,.58	∞

TABLE 20. Fuzzy Environmental Effect Matrix $\tilde{S}(i,j,1)$ for Ten Cities Using First Conveyances

GA and some other cases GA gives better result than ACO. As before the result increases with the increase of β_3, β_4. The results without taking any constrains are presented in Table-23.

6.5. **TSPLIB Data Set.** In this section some standard data sets are used to test the effectiveness of our algorithm for solving the proposed problem. For this purpose some standard benchmark TSPs from TSPLIB are used. These standard TSPs are classical, but our requirement is STSP. A STSP with single conveyance facility, without environmental effect constraint is the same as a classical TSP. For this reason our algorithms are tested for TSPs from TSPLIB, taking only one conveyance facility and ignoring the environmental effects. Our observed results for different problems are presented in Table-25. The same problems were solved by Wang et al. [34] using Self Adaptive Genetic Algorithm (SAGA) and SGA. Their

i/j	1	2	3	4	5
1	∞	.35,.36,.37	.45,.46,.47	.55,.56,.57	.25,.26,.27
2	.7,.75,.78	∞	.42,.44,.45	.45,.48,.51	.7,.75,.77
3	.25,.26,.27	.35,.36,.37	∞	.55,.56,.57	.25,.26,.27
4	.7,.75,.78	.55,.58,.6	.42,.44,.45	∞	.7,.75,.77
5	.25,.26,.27	.35,.36,.37	.45,.46,.47	.55,.56,.57	∞
6	.7,.75,.78	.55,.58,.6	.42,.44,.45	.45,.48,.51	.7,.75,.77
7	.25,.26,.27	.35,.36,.37	.45,.46,.47	.55,.56,.57	.25,.26,.27
8	.7,.75,.78	.55,.58,.6	.42,.44,.45	.45,.48,.51	.7,.75,.77
9	.25,.26,.27	.35,.36,.37	.45,.46,.47	.55,.56,.57	.25,.26,.27
10	.7,.75,.78	.55,.58,.6	.42,.44,.45	.45,.48,.51	.7,.75,.77

i/j	6	7	8	9	10
1	.6,.61,.62	.75,.76,.77	.65,.66,.67	.59,.6,.61	.56,.57,.58
2	.7,.75,.77	.44,.48,.5	.50,.55,.58	.6,.7,.73	.50,.55,.58
3	.6,.61,.62	.75,.76,.77	.65,.66,.67	.59,.6,.61	.56,.57,.58
4	.7,.75,.77	.44,.48,.5	.50,.55,.58	.6,.7,.73	50,.55,.58
5	.6,.61,.62	.75,.76,.77	.65,.66,.67	.59,.6,.61	.56,.57,.58
6	∞	.44,.48,.5	.50,.55,.58	.6,.7,.73	.50,.55,.58
7	.6,.61,.62	∞	.65,.66,.67	.59,.6,.61	.56,.57,.58
8	.7,.75,.77	.44,.48,.5	∞	.6,.7,.73	.50,.55,.58
9	.6,.61,.62	.75,.76,.77	.65,.66,.67	∞	.56,.57,.58
10	.7,.75,.77	.44,.48,.5	.50,.55,.58	.6,.7,.73	∞

Table 21. Fuzzy Environmental Effect Matrix $\tilde{S}(i,j,2)$ for Ten Cities Using Second Conveyances

i/j	1	2	3	4	5
1	∞	.38,.39,.40	.5,.51,.52	.6,.61,.62	.29,.30,.31
2	.6,.65,.69	∞	.33,.35,.38	.52,.55,.58	.6,.7,.73
3	.28,.29,.29	.38,.39,.40	∞	.6,.61,.62	.29,.30,.31
4	.6,.65,.69	.41,.45,.5	.33,.35,.38	∞	.6,.7,.73
5	.27,.28,.29	.38,.39,.40	.5,.51,.52	.6,.61,.62	∞
6	.6,.65,.69	.41,.45,.5	.33,.35,.38	.52,.55,.58	.6,.7,.73
7	.28,.29,.30	.38,.39,.40	.5,.51,.52	.6,.61,.62	.29,.30,.31
8	.6,.65,.69	.41,.45,.5	.33,.35,.38	.52,.55,.58	.6,.7,.73
9	.28,.29,.29	.38,.39,.40	.5,.51,.52	.6,.61,.62	.29,.30,.31
10	.6,.65,.69	.41,.45,.5	.33,.35,.38	.52,.55,.58	.6,.7,.73

i/j	6	7	8	9	10
1	.62,.63,.64	.78,.79,.8	.7,.71,.72	.6,.61,.62	.57,.58,.6
2	.6,.7,.73	.45,.48,.51	.61,.65,.7	.7,.75,.77	.61,.65,.7
3	.62,.63,.64	.78,.79,.8	.7,.71,.72	.6,.61,.62	.57,.58,.6
4	.6,.7,.73	.45,.48,.51	.61,.65,.7	.7,.75,.77	.61,.65,.7
5	.62,.63,.64	.78,.79,.80	.70,.71,.72	.60,.61,.62	.57,.58,.6
6	∞	.45,.48,.51	.61,.65,.7	.7,.75,.77	.61,.65,.7
7	.62,.63,.64	∞	.7,.71,.72	.6,.61,.62	.57,.58,.6
8	.6,.7,.73	.45,.48,.51	∞	.7,.75,.77	.61,.65,.7
9	.62,.63,.64	.78,.79,.8	.7,.71,.72	∞	.57,.58,.6
10	.6,.7,.73	.45,.48,.51	.61,.65,.7	.7,.75,.77	∞

Table 22. Fuzzy Environmental Effect Matrix $\tilde{S}(i,j,3)$ for Ten Cities Using Third Conveyances

Method	DM	x[1],v[1];x[2],v[2];...;x[10],v[10]	$\widetilde{cost}$
ACO	PDM	1,2;6,2;4,1;3,1;7,3;8,2;2,1;10,1;5,2;9,3	103.5, 105.6, 108.1
	ODM	1,1;6,1;4,1;3,1;7,3;8,3;2,1;10,3;5,2;9,3	103.5, 104.0, 107.0
GA	PDM	1,2;6,2;4,2;3,1;7,1;8,3;2,1;10,2;5,1;9,3	103.0, 104.7, 107.8
	ODM	3,3;7,2;8,2;2,2;10,1;5,1;9,3;1,1;6,2;4,1	103.0, 104.6, 107.1

Table 23. Optimal Solution for Solid TSP without Environment Effect Constraints

results along with the optimum results of the problems are also presented in Table-25, to check the efficiency of our algorithms. It is observed that our algorithms give better results than that of either SAGA or SGA using less number of iterations. Best tours obtained using our ACO for these problems are presented in Appendix-A.

The above maintained standard benchmark TSPs from TSPLIB are redefined for the proposed Solid TSP, and results for different problems are presented in Table-26. For this purpose three types of conveyance are used for each problem.

β_3/β_4	Method	DM	x[1],v[1];x[2],v[2];...;x[10],v[10]	$\widetilde{cost}$
.2	ACO	PDM	1,1;2,1;10,2;5,2;9,1;6,3;4,1;3,2;7,2;8,2	128.5,130.1,132.4
		ODM	1,1;6,1;4,1;3,1;7,2;8,1;2,2;10,2;5,1;9,3	101.5,103.1,105.8
	GA	PDM	1,1;2,1;10,2;5,2;9,1;6,3;4,1;3,2;7,2;8,2	128.5,130.1,132.4
		ODM	2,2;10,1;5,2;9,1;1,3;6,1;4,1;3,2;7,2;8,1	101.4,103.0,105.5
.6	ACO	PDM	1,1;6,2;5,3;9,1;4,2;3,1;7,1;8,1;2,1;10,1	133.3,134.3,136.2
		ODM	1,2;6,3;4,1;3,1;7,2;8,1;2,2;10,2;5,2;9,3	103.5,105.0,107.5
	GA	PDM	3,2;7,3;8,2;1,3;2,3;10,1;5,3;9,2;6,2;4,1	131.0,132.9,135.4
		ODM	10,2;5,1;9,1;1,2;6,3;4,2;3,3;7,1;8,1;2,2	103.0,104.6,107.25
.8	ACO	PDM	1,2;6,1;2,2;10,2;5,1;9,1;4,1;3,3;7,2;8,1	134.0,135.5,138.1
		ODM	1,2;6,3;4,1;3,1;7,3;8,1;2,2;10,2;5,2;9,3	103.64,105.2,107.8
	GA	PDM	10,1;1,2;6,2;5,1;9,3;4,1;3,1;7,1;8,1;2,2	133.5,135.5,137.9
		ODM	4,1;3,2;7,1;8,1;2,2;10,2;5,3;9,2;1,3;6,3	103.9,105.0,107.0

TABLE 24. Optimal Solution for Solid TSP with Fuzzy Cost and Environment Effect Constraints with $\widetilde{S_{min}} = (5.6, 5.7, 5.8)$

Problem Name	Size	Optimal by TSPLIB	Method	Generation	Cost of best tour
br17	17	39	ACO	500	39
			GA	500	39
			SAGA	1000	39
ftv33	34	1286	ACO	1000	1343
			GA	1000	1349
			SAGA	4000	1453
ftv55	56	1608	ACO	1000	1635
			GA	1000	1663
			SAGA	4000	1672
ry48p	48	14422	ACO	1000	14915
			GA	1000	14915
			SGA	4000	15024
ft70	70	38673	ACO	1000	39702
			GA	1000	39702
			SGA	4000	41056

TABLE 25. Results of TSPLIB without Environmental Effect for Standard Five Cases

Problem Name (Redefined)	Size	S_{min}	Method	Generation	Cost of Best Tour
Redefined br17	17	11.0	ACO	500	102.85
			GA	500	102.85
Redefined ftv33	34	23.0	ACO	1000	1340.02
			GA	1000	1343.80
Redefined ftv55	56	36.5	ACO	1000	1567.03
			GA	1000	1569.37
Redefined ry48p	48	34.5	ACO	1000	15139.04
			GA	1000	15372.70
Redefined ft70	70	50.0	ACO	1000	49283.38
			GA	1000	49346.46

TABLE 26. Results of Solid TSP Using Redefined TSPLIB Data Base with Environmental Effect for Standard Five Cases in Crisp Environment

If c_{ij} is actual cost of travel from i-th city to j-th city for a benchmark TSP, then c_{ij1}, c_{ij2} and c_{ij3} are taken as cost of corresponding Solid TSP to travel from i-th city to j-th city using first, second and third type of conveyance respectively where $c_{ij1} = c_{ij} * \xi_{ij1}$, $c_{ij2} = c_{ij}$ and $c_{ij3} = c_{ij} * \xi_{ij3}$ where ξ_{ij1} and ξ_{ij3} are two randomly generated data in the range$(1, 5)$ and $*$ is the binary operation randomly selected from the set$\{+, -\}$. Finally environmental effect due to different types of conveyances are randomly generated in the range$(.4, .9)$.

Problem Name	$\widetilde{S_{min}}$	Meth -od	Gene- ration	Cost of $\widetilde{\text{Best}}$ Tour	Obje- ctive
Redefined $br17$	8.0,9.0,9.1	ACO	500	97.33,103.00,107.72	102.43
		GA	500	97.33,103.00,107.72	102.43
Redefined ftv33	21.0,22.5,22.8	ACO	1000	1328.87,1339.54,1349.70	1338.47
		GA	1000	1330.57,1341.04,1350.30	1339.99
Redefined ftv55	36.0,36.5,36.8	ACO	1000	1648.78,1665.96,1684.88	1664.25
		GA	1000	1660.62,1677.22,1695.48	1675.56
Redefined ry48p	32.0,33.0,33.8	ACO	1000	16620.44,16635.09,16649.03	16633.63
		GA	1000	16650.96,16664.97,16679.29	16663.57
Redefined ft70	43.6,48.7,55.8	ACO	1000	43425.03,43445.75,43467.57	43443.68
		GA	1000	43425.03,43445.75,43467.57	43443.68

Table 27. Results of Solid TSP Using Redefined TSPLIB Data Base with Environmental Effect for Standard Five Cases in Fuzzy Environment

Redefined Bench Mark TSPs for Solid TSP are again redefined for Solid TSP in fuzzy environment and results are presented in Table-27. Here fuzzy cost and data are taken as TFN type. Mid value of these fuzzy data are taken as corresponding data of crisp problem. Then a random number is generated in the range $(.1, .5)$ and is subtracted from mid value to get left component of the data. Similarly another number is generated in the range $(.1, .5)$ and is added to mid value to get right component. For the generation of data of fuzzy environment effect procedure is the same as fuzzy cost, but the random data is generated in the range $(.05, .08)$.

7. Conclusion

Here for the first time, a constrained solid TSP is modelled where the salesman can choose suitable conveyance for the tour from one city to another against different environmental effects. Goal of the traveller is to visit all the cities with a minimum cost such that a maximum environmental effect(in positive sense) is maintained, but does not go down below a prescribe quantity. To deal with the problem in fuzzy environment, an approach is proposed using possibility/necessity measure of fuzzy constraints. To solve such a real life problem in both crisp and fuzzy environments, here an ant colony algorithm is proposed and its performance is studied and compared against another soft computing algorithm GA. Numerical studies shows that both the algorithms ACO and GA are suitable for solving the proposed problem.

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Appendix A: Path of Best tour for different data set of TSPLIB

Problem: br17 for 17 Cities: By proposed ACO, path is 9, 12, 10, 7, 8, 16, 4, 3, 6,15, 5, 14, 11, 0, 2, 13, 1 and cost=39. By proposed GA, path is 9, 12, 10, 7, 8, 16, 4, 3, 6, 15, 5, 14, 11, 0, 2, 13, 1 and cost=39

Problem: ftv33 for 34 Cities: By proposed ACO, path is 0, 13, 12, 14, 15, 16, 1, 29, 26, 22, 27, 28, 25, 24, 23, 19, 20, 21, 31, 18, 17, 11, 8, 10, 9, 32, 7, 4, 6, 5, 30, 33, 2, 0 and cost=1343. By proposed GA, path is 0, 13, 25, 26, 16, 31, 30, 22, 8, 17, 7, 15, 29, 24, 14, 27, 20, 6, 10, 19, 4, 3, 28, 23, 12, 21, 9, 32, 18, 2, 5, 11, 0 and cost=1349.

Problem: ftv55 for 56 Cities: By proposed ACO, path is 0, 33, 2, 13, 35, 4, 6, 5, 47, 48, 31, 46, 29, 26, 25, 24, 42, 22, 41, 21, 50, 23, 54, 27, 49, 43, 44, 28, 53, 45, 30, 55, 34, 1, 3, 7, 32, 8, 36, 9, 37, 11, 19, 20, 40, 18, 39, 38, 10, 51, 14, 12, 15, 16, 17, 0 and cost=1635. By proposed GA path is 0, 33, 2, 13, 35, 4, 6, 5, 47, 48, 31, 46, 55, 34, 1, 3, 7, 32, 8, 36, 9, 37, 11, 19, 18, 39, 38, 10, 51, 14, 12, 15, 16, 17, 52, 26, 25,24 , 42, 21, 22, 20, 40, 41, 50, 23, 54, 27, 49, 43, 44, 28, 53, 45, 30, 0 and cost=1663.

Problem: ry48p for 48 Cities: By proposed ACO, path is 0, 8, 39, 14, 11, 32, 45, 10, 22, 12, 24, 13, 2, 21, 15, 40, 33, 28, 1, 3, 25, 34, 44, 23, 9, 41, 4, 47, 38, 31, 20, 46, 19, 16, 42, 29, 5, 26, 18, 36, 27, 17, 35, 6, 43, 30, 37, 0 and cost=14915. By proposed GA, path is 0, 8, 39, 14, 11, 32, 45, 10, 22, 12, 24, 13, 2, 21, 15, 40, 33, 28, 1, 3, 25, 34, 44, 23, 9, 41, 4, 47, 38, 31, 20, 46, 19, 16, 42, 29, 5, 26, 18, 36, 27, 17, 35, 6, 43, 30, 37, 0 and cost=14915

Problem: ft70 for 70 Cities: By proposed ACO, path is 0, 1, 60, 59, 57, 44, 48, 43, 49, 47, 33, 31, 30, 46, 45, 62, 61, 58, 27, 22, 21, 24, 56, 53, 20, 18, 19, 25, 35, 36, 42, 41, 40, 38, 39, 37, 69, 67, 63, 66, 64, 65, 68, 34, 32, 29, 12, 14, 10, 9, 7, 3, 5, 6, 28, 54, 52, 13, 8, 11, 2, 4, 23, 26, 55, 51, 50, 17, 16, 0 and cost=39702. By proposed GA path is 0, 1, 60, 59, 57, 44, 48, 43, 49, 47, 33, 31, 30, 46, 45, 62, 61, 58, 27, 22, 21, 24, 56, 53, 20, 18, 19, 25, 35, 36, 42, 41, 40, 38, 39, 37, 69, 67, 63, 66, 64, 65,68, 34, 32,29, 12, 14, 10, 9, 7, 3, 5, 6, 28, 54, 52, 13, 8, 11, 2, 4, 23, 26, 55, 51, 50, 17, 16, 0 and cost=39702.

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