INTRODUCTION TO A SIMPLE YET EFFECTIVE TWO-DIMENSIONAL FUZZY SMOOTHING FILTER

H. SHAKOURI G., M. ARABI AND M. B. MENHAJ

Abstract. Annihilation or reduction of each kind of noise blended in correct data signals is a field that has attracted many researchers. It is a fact that fuzzy theory presents full capability in this field. Fuzzy filters are often strong in smoothing corrupted signals, whereas they have simple structures. In this paper, a new powerful yet simple fuzzy procedure is introduced for sharpness reduction in two-dimensional signals. It is indeed an extension of our previously published one-dimensional fuzzy smoothing filter. This procedure has been designed for annihilation of all unknown noises in two-dimensional corrupted signals, although works the best for impulse noise. The proposed method looks for *sharp points* in the corrupted signal and then smoothes them out by *sharing* their values with eight (or more) neighboring point values. Preservation of correct data in the corrupted signal is an important advantage of this method. To obtain experimental results of the proposed procedure, both color and black & white images are used as the most common two-dimensional signals, and the results are compared with several other filters recently cited in the literature. Experimental results exhibit a high capability of our method in both numerical measures and visual inspection, preserving its simplicity. Finally, application of the proposed filter to socio-economic fields is presented using a demographic mixed data set to better illustrate original motivation for this idea.

1. Introduction

Signal processing is discussed as one of the most important fields that still attracts many researchers. Filter designing is one of the main branches of signal processing [27, 16, 23, 22]. There exist various methods of signal filtering for the purpose of noise reduction so that more useful information can be extracted from corrupted signals. Among these methods, fuzzy logic has performed an important role in nonlinear filtering. In fact, fuzzy logic capability in simplification has been considered as the key point of fuzzy theory in designing nonlinear filters [35, 3, 9].

Among one-dimensional signals, speech signal processing has first attracted researchers to apply fuzzy logic [27]. Image processing, in which smoothing methods are often applied to, is a common application for fuzzy computations. Fuzzy edge

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detection and fuzzy image enhancement [15, 10, 12, 33, 1] by impulse noise annihilation [24] or additive noise reduction using fuzzy filtering are interesting fields in image processing that fuzzy logic is very influential [24, 20].

Applying fuzzy theory to medical image processing is also an especial active field of research among all image processing fields [34, 25]. Besides image and voice processing, fuzzy theory expands its usage in many various fields such as language processing, bio informatics, traffic smoothing in network communication, etc [4, 11, 5, 8]. Among applications of fuzzy logic in other fields of science, one may refer to socio-economic and human sciences which may be affected the most [19, 30]. Indeed, reduction of uncertainty in demographic and economic data was the main motivation for the authors of [30].

Various filters are used in order to reduce noise effects [21]. In these methods, information in uncorrupted parts of image is used to modify corrupted parts. Median filters and weighted mean filters are two recognized groups of the early generations of filters. However, fuzzy versions of these filters like Weighted Fuzzy Mean (WFM) and Fuzzy Median Filter (FMF) have achieved a high improvement [7, 17, 32].

Adaptive filters make a distinct class of filters. Many types of adaptive filters can also be found in the literature [26, 14]. A modified adaptive version of WFM, named AWFM, estimates impulsive noise amplitude in infected parts, and has been very successful with highly stable outputs [7].

Impulse noise, additive noise and multiplicative noise are three main types of noise that destroy two-dimensional signals. Impulse noise corrupts signal partially and some parts of signal remain without being affected, while additive noise and multiplicative noise impact on the entire signal. Fixed value impulse noise and random value impulse noise are two types of impulse noise. The main purpose of the method proposed in this paper is to attenuate all types of impulse noises. However, it has led to notable results in reduction of other kinds of noise as well. In particular, the method is successful in filtering an especial kind of *mixed* data.

Primitive idea of designing our proposed method is based on the one-dimensional signal smoothing filter called Fuzzy Smoothing Filter (FSF) in [29], where the method is introduced and its time domain and frequency domain properties are analyzed. The method is applied to smooth out sharpness of socio-economic signals as well as the sound as one of the most common one-dimensional signals. The merit of the approach in [7] associated with plausible results persuades authors to generalize the idea to two-dimensional problems in order to make it suitable for more practical image processing applications.

The proposed Two-Dimensional Fuzzy Smoothing filter (TDFS) is established through three distinguished stages. At the first stage, sharpness of signal is calculated at all signal points. Second, a fuzzy set named "*sharp points*" is formed. Finally, sharp points are smoothed by sharing their values with the neighboring point values. The smoothing process is performed based on the sharp points membership values designed properly in the fuzzy set.

To remove or reduce impulse noise effects, all sharp points must be processed. In order to obtain sharpness index of the signal in the first stage of TDFS method, we have to calculate angle of signal in all of its points. Having distinguished sharp

points in the second stage, *association* portion of neighboring points are calculated for their restoration in the third stage. In short, our proposed method is an extended version of the FSF method, meaning that the one-dimensional signal smoothing has been generalized to two-dimensional signal smoothing.

The method is applied to several benchmarks commonly used in papers and the results are compared to several recent methods in the literature. Our achievements show a high capacity in this simple, yet capable method to remove salt & pepper noises and considerably reducing other kinds as well.

In the next section, we briefly review the basic concepts and the kind of fuzzy rule that will be used by the proposed filter. In, section 3 we introduce the proposed two-dimensional smoothing filter in detail. Some modifications to the method that improves its efficiency will be given in section 4. Section 5 contains the simulation results, where sample images are corrupted and filtered by several methods to compare the results. Because the main motive for deriving such a filter was to apply in socio-economic research fields that the authors are interested, we will introduce an interesting application of the proposed smoothing filter to demographic data in section 6. Finally, section 7 concludes the paper.

2. The Single Fuzzy Rule of the Proposed Method

It is easy to demonstrate by simulation that the more noise added in a signal, the more average sharpness of the signal. This is illustrated in Figure 1, where the average sharpness of an infected sinusoidal signal, $x_n(t)$, is plotted against the amplitude of an additive normal white noise: $x_n(t) = x(t) + \alpha n(t)$. Besides impulse and Gaussian noises that commonly found in sensors or data transfers, an especial kind of infection is defined in [29], where we named the infected data by mixed data. A sample mixed data is also depicted in the same figure, namely $x_m(t)$. It is seen that both the infected signals almost have the same kind of distortion leading to coarse and sharpness. Sharpness of a signal will be defined later on.

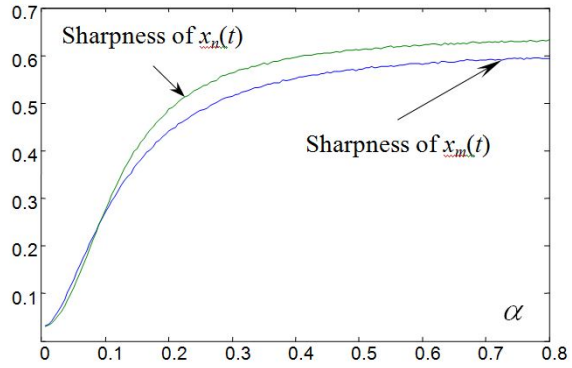
The fuzzy filter reduces average sharpness of noisy signals, while preserving the original information as much as possible. Similar idea has been utilized in several works [25, 29]. To extend these advantages in image processing, we generalized the Fuzzy Smoothing filter applications in reducing noise effects from one-dimensional signals to two-dimensional signals. Before introducing the proposed method for two-dimensional signals in detail, it is useful to explain the idea behind this method briefly. In this respect, our previous idea for smoothing one-dimensional signals is described below shortly.

2.1. A Review of the Proposed Fuzzy Smoothing Filters for One-dimensional Signals.

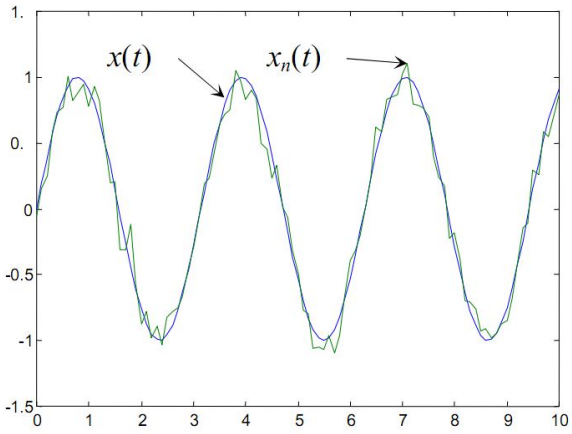
Let us denote an input signal to a smoothing filter by $x(t)$ and the output signal, i.e. the smoothed version of $x(t)$, by $y(t)$. Further, let the smoothing process be expressed by

$$y(t) = S_{FS}\{x(t)\} \tag{1}$$

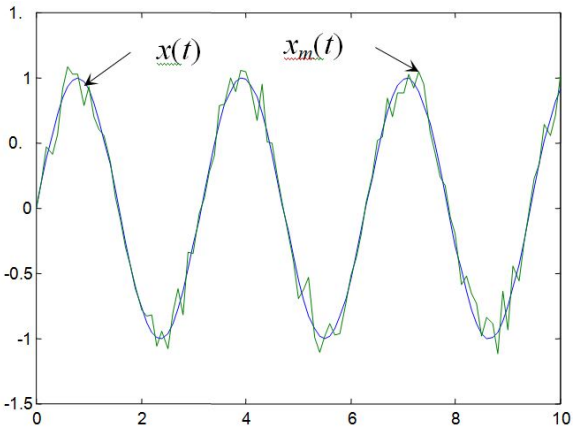
where $S_{FS}\{.\}$ simply represents the following single rule:
R_1: If sharpness of $x(t)$ at point t is *high*, then *share* its value with its neighboring points at $t-1$ and $t+1$.



(a) Average Sharpness of a Sinusoidal Signal Infected by Two Kinds of Noises as a Function of Noise Amplitude



(b) A Sample Signal Infected by an Impulse Noise



(c) Sample Mixed Data

FIGURE 1. Sharpness Increase Along with an Additive Noise Amplitude Increase in a Sinusoidal Signal

The first step for determining sharpness of each point is to calculate the angle of the discrete signal at that point. It is obvious that in smoothing the sharp points, there is no difference between negative and positive angles; only the absolute values are considered. As an appropriate index to measure sharpness at a certain sample k, namely s_k, we consider the cosine of the angle. Thus:

$$s_k = \cos(\theta_k) + 1 \tag{2}$$

$$\theta_k = \cos^{-1}\left(\bar{v}_{k-1,k}.\bar{v}_{k+1,k}\right) \tag{3}$$

$$v_{i,j} = \left[\gamma\left(t_j - t_i\right), x_j - x_i\right]; \bar{v}_{i,j} = v_{i,j} / \left\|v_{i,j}\right\| \tag{4}$$

where $\bar{v}_{i,j}$ is a normalized 2-vector from the ith point to the jth point. Here, $\gamma = (constant)\ E\{\Delta x(t)\}$ is a normalizing factor in order to make the angles independent of the axes scale. However, for the sake of simplicity, this design parameter will be replaced by 1 hereafter. It is plain truth that the sharpness defined above can vary from zero (for $\theta = 180°$) to 2 (for $\theta = 0°$). Moreover, $E\{.\}$ denotes the mean operator, and Δ stands for the difference operator.

In the primary method developed in [29], the points are categorized according to their sharpness with a simple set of membership functions which define a linguistic variable "*sharpness*" in the fuzzy rule R_1. However, in the modified method, signal processing is done only for "*very sharp*" points specified by:

$$s_k > \sigma_0 \tag{5}$$

where σ_0 is an adjustable constant. To deal with the fuzzy rule, R_1, instead of *sharing* the values, we replace the linguistic variable "*association*" with "*exclusion*". Then, instead of using usual membership functions, we employ a parameter named ρ_0 which controls the correction process at each point. Indeed, this parameter determines the amount of exclusion needed for the kth point in smoothing process. Thus, the output of filter can be calculated by:

$$y\left(t_k\right) = \left[x(t_{k-1}) + \rho_k x(t_k) + x(t_{k+1})\right] / \left(2 + \rho_k\right) \tag{6}$$

with

$$\rho_k = \rho_0; \forall s_k > \sigma_0 \tag{7}$$

In each step, the values of neighboring points are locally adjusted to keep the overall integral of the signal curve constant. Therefore, data values at t_{k-1} and t_{k+1} should be modified by the following:

$$\begin{aligned} y\left(t_{k-1}\right) &= x(t_{k-1}) + \left[x(t_k) - y(t_k)\right]/2 \\ y\left(t_{k+1}\right) &= x(t_{k+1}) + \left[x(t_k) - y(t_k)\right]/2 \end{aligned} \tag{8}$$

This modification will not be needed if it is known that the signal is infected by an impulse noise and it is not indeed a mixed one. However, where we have a mixed data, since the sharpness is calculated by the two neighboring points, modifications are also done in those two neighboring points in order to keep the integral constant. We conclude that the window width of the filter could be set to $ww = 3$. Although as the window width increases the resulting filter becomes more complex, this simple filter has shown adequate efficiency for one-dimensional

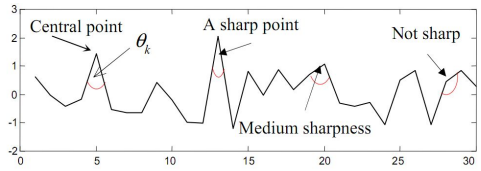


FIGURE 2. A One-dimensional Signal, Distinct Central Point and its Two Neighboring Points. Examples of Not, Medium and Very Sharp Points are Given

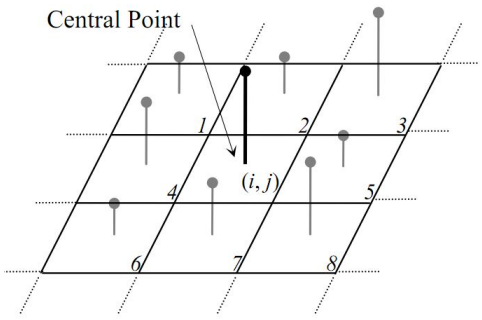


FIGURE 3. Central Point (i, j) and its Eight Neighboring Points in a Two-Dimensional Digital Signal

signals. Calculations in (8) clearly will lead to new uneven points in a signal; hence, it is necessary to repeat the process until there is no uneven point left comparing to the critical sharpness defined by (5).

2.2. **Differences Between One-dimensional and Two-dimensional Signals.** Generally, signals can be categorized based on their dimension. Sounds and images are the most common signals belonging to one-dimensional and two-dimensional signals, respectively. Before extending the aforementioned method, and introducing the proposed method in detail, it is useful to explain two-dimensional signals and their differences with one-dimensional ones from sharpness point of view. The main difference is that in one-dimensional signals there are two neighboring points close to each distinct point, while in two-dimensional signals there are 8 neighboring points. In one-dimensional signals, each central distinct point and its two neighboring points form a distinctive angle. In fact this angle reveals sharpness or smoothness of signal at the central point. Figure 2 illustrates an angle in a typical one-dimensional signal, along with three examples for *not sharp*, *very sharp*, and *medium sharp* points.

However, in two-dimensional signals, central point and its eight neighboring points together create a 3×3 square window. Figure 3 illustrates how a central point and its eight neighboring points are associated together.

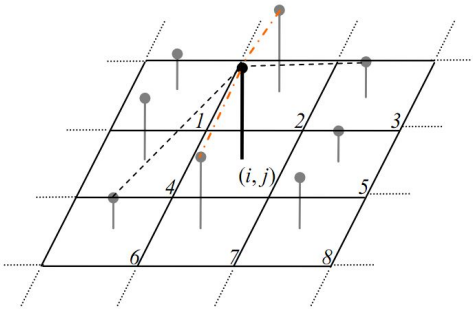


FIGURE 4. Replacement of the 2nd with the 3rd and the 7th with the 8th Neighboring Points and its Effect on Sharpness of the Central Point Compared to Figure 3

Clearly, border points in a (one) two-dimensional signal have less than (two) eight neighboring points. However, assuming that these points can be smoothed by a one-dimensional filter prior to entering the two-dimensional filtering process, this paper ignores further discussion on this matter. Since in a two-dimensional signal there are numerous corners and also displacement in neighboring points cause changes in sharpness of the central point, it is more complicated to assign distinct angles for distinct central points. Figure 4 illustrates how displacement of neighboring points has an effect on the central point sharpness in two-dimensional signal, and consequently sequence of the points is important in calculation of the angles.

3. The Two-dimensional Fuzzy Smoothing Filter

Our proposed method called Two-dimensional Fuzzy Smoothing filter (TDFS) is designed in order to smooth two-dimensional signals corrupted by impulsive noise applying fuzzy rules. The method is based on the assumption that the signal is mixed, such that data of some points is blended with the neighboring points. Therefore, in this algorithm, portions of the signal are used to modify the corrupted parts. In other words, the distinguished corrupted parts are modified by their neighboring points. In brief, the algorithm can be summarized in three steps. Since in two-dimensional signals, corrupted parts have complicated characteristics besides their sharpness, as the first step, our attempts focus on finding a criterion to detect parts of signals infected by noise where data is mixed. In the next step, we define a fuzzy set consisting of corruption amounts in affected points. For the fact that different points in a signal are not equally corrupted, such a fuzzy set helps us to be more accurate in modification. In the last step of our proposed method, association portion of the neighboring points to correct the central point is calculated. This is based on corrupted points membership degree in the predefined fuzzy set. This way we can restore the corrupted parts.

Assuming that $x_0(t, \acute{t})$ is the original two-dimensional signal, $x(t, \acute{t})$ is the corrupted signal and $y(t, \acute{t})$ is the filtered signal, the proposed method is briefly represented by:

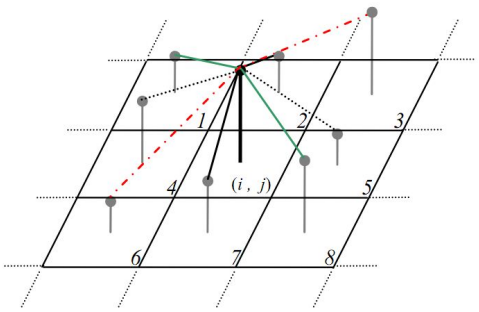


FIGURE 5. A Central Point and its Four Diagonal Angles

$$y\left(t,t'\right) = S_{TDFS}\left\{x\left(t,\acute{t}\right)\right\} \tag{9}$$

where $S_{TDFS}\{.\}$ presents the process (operator) of smoothing. Similar to R_1, $S_{TDFS}\{.\}$ can be summarized in the following single fuzzy rule:

$$\begin{aligned} R_2 : &\text{If the overall sharpness of } x(t,\acute{t}), \text{at point } (i,j) \text{is high ,} \\ &\text{then share its value with its neighboring points.} \end{aligned} \tag{10}$$

As seen in the above rule, the degree of sharpness of signal must be calculated in all its points. The first and second stage of the proposed method focus on calculating sharpness index over all points of $x(t,\acute{t}) = x_{ij}$ in order to form "*sharp*" points fuzzy set. According to "then" part of the fuzzy rule R_2, in the third stage of proposed method, we calculate the degree of association of neighboring points for modification of the distinct central point.

3.1. **Signal Angle Calculation.** The amount of sharpness of a signal at a central point depends on angles between the point and its neighboring points. There are so many angles that can be considered as a measure for sharpness of a point in a two-dimensional signal. Hence, sharpness calculation in each point is more challenging rather than that of a one-dimensional signal. To come up with this problem, we will use just four diagonal angles to obtain an index to measure sharpness at a distinct central point denoted (i,j), instead of all $28(=\sum_{1}^{7} i)$ possibly computable angles. In fact, these four diagonal angles suffice to present sharpness value of a specified central point properly. Figure 5 illustrates the four diagonal angles for the central point (i,j).

Assuming that x_{ij} refers to the signal value at the central point (i,j), one can easily calculate the four diagonal angles through substituting two-dimension signal values for $x(.)$, replacing tk by i and j in (3). Below, (3) is replaced for each point (i,j) by dividing the angle into two sub-angles, considering different scales for pixel dimensions in images:

$$\theta_{ij}^{mn} = \pm\tan^{-1}\left[\frac{d^m}{x_{ij}-x_{ij}^m}\right] \pm \tan^{-1}\left[\frac{d^n}{x_{ij}-x_{ij}^n}\right]; m = 1,\ldots,4; n = 9-m \tag{11}$$

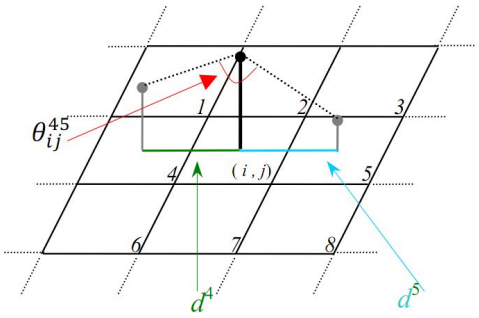


FIGURE 6. Calculation of the Virtual Distances d^4, d^5

where appropriate signs should be applied based on the change of directions. In this formula, the superscript m, which belongs to $\{1, 2, 3, 4\}$, indicates the four first neighboring points and the superscript n indicates the four last neighboring points. Therefore, x^m and x^n present color information (main colors in colored image or gray level in uncolored image) of neighboring points. Moreover, d^m and d^n denote distances between the central point and its neighboring points regarding to each of vertical and horizontal axis, respectively. In fact, the se distances are assumed to be different. Even if the pixels are assumed to be square, clearly the diagonal distances would be $\sqrt{2}$ times bigger than the off diagonal distances:

$$d^1 = d^3 = d^6 = d^8 = \sqrt{2}d^2 \tag{12}$$

However, without lose of generality, hereafter we assume that distances are adjustable, while:

$$d^n = d^m; m = 1, \ldots, 4; n = 9 - m \tag{13}$$

This capacity hidden in the degree of freedom in this method, which improves remarkably the power of our proposed TDFS method in signal smoothing. A sample angle, θ_{ij}^{45}, and its two corresponding virtual distances, $d^4 = d^5$, are illustrated in Figure 6. After obtaining four diagonal angles for a distinct point of the corrupted signal, $x(t, \acute{t})$, it is necessary to assign only one distinct angle, which represents its angle. For this purpose, we use the average of aforementioned four diagonal angles as the signal angle at each distinct point:

$$\theta_{ij} = \frac{1}{4} \sum_{m=1}^{4} \theta_{ij}^{mn}; n = 9 - m \tag{14}$$

where θ_{ij}^{mn} indicates the four diagonal angles at the central point (i, j) computed by (11). The effect of using all the eight recognizable angles can be investigated in future studies. Noting that (11) is the most complicated part of the algorithm, it would double the computation time at this step.

3.2. **Sharpness Index Calculation.** According to the "if" part of the fuzzy rule R_2, in the first stage of our proposed method, sharpness at each x_{ij} must be calculated for all (corrupted) points, i.e. s_{ij} must be calculated for all points. As mentioned before, since in this method, the sharpness index in each central point demonstrates noise effects in that point; the point with higher sharpness index

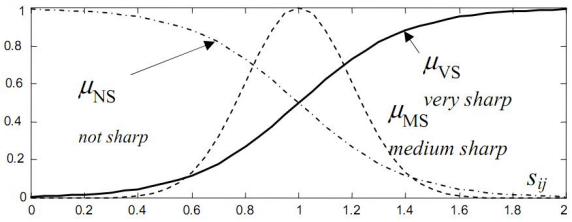


FIGURE 7. Sample Membership Functions for Linguistic Variables of *not*, *medium* and *very sharp*

probably must have fewer uncorrupted information. Indeed, the underlying idea is a conjecture that the original signal could have relatively smooth variations which have become rougher by noise attack. However, this presumption is generally acceptable for any image, except in sharp edges. Herein, similar to the MFS filter in [29] we use the cosine of angle to define the sharpness index of the signal in each distinct point:

$$s_{ij} = \cos\theta_{ij} + 1 \tag{15}$$

Therefore, for angles less than 45° sharpness index tends to 2 (very sharp points), for angles more than 135° sharpness index is very small and tends to zero, and for angles about 180° meaning smooth or non sharp points. For angles about 90° , sharpness index tends to 1 (medium sharp point). In fact sharpness index assigns memberships value to each point of "*very sharp*", "*not sharp*" and "*medium sharp*" fuzzy term sets. Moreover, it can help to consider a threshold for corrupted points which must be smoothed. In addition, *association* membership grade of the neighboring points can be calculated by the sharpness index. Figure 7 illustrates membership functions for the three mentioned fuzzy term sets, namely μ_{NS}, μ_{MS} and μ_{VS}, respectively. Nevertheless, we only use the "*very sharp*" fuzzy set. As a result, the fuzzy rule R_2 used to design the smoothing filter will be rewritten as:

R_2: If the central point is a "*very sharp*" point, then the neighboring points "*association*" portion to restore the central point gets "*higher*".

Figure 8 demonstrates sharpness index calculated for a sample image by (15). Evidently, sharpness of smooth areas is almost zero, appearing in dark. It is seen that the sharpness in the edges is higher than smooth areas; therefore it seems that combination of the proposed model with an edge detection algorithm can further help to improve the results. However, although the overall angle computed by (14) may deteriorate edge clarity, simplicity of this method is more preferable rather than its perfectness.

3.3. **Association Coefficient Calculation.** In this section, we show how the neighboring points association portion in central point restoration should be calculated. According to the then part in the aforementioned fuzzy rule R_2, it is necessary to calculate association portion of the neighboring points for corrupted central point restoration $x(t, \acute{t})$. In this method, neighboring points association portion depends on the sharpness index of central points. To be more concrete, the

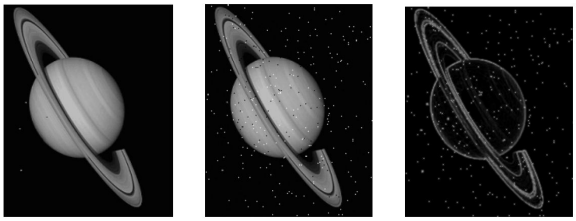


FIGURE 8. Sample Image (Saturn with 1% salt & paper noise) and its Corresponding Sharpness Index

greater sharpness index in central points, the more neighbors association portion in the process of central points restoration. Thus, in points with smaller sharpness index, neighbors association portion is less or even zero.

Considering Figure 7, it can be inferred that sharp points have association coefficients equal to 1, indicating full association. For non sharp points, association of neighboring points in central point restoration is equal to 0. It means there is no need to employ association of neighboring points in central point restoration. In order to obtain smoothed signal, we use the following expression for each point (i, j). Let us denote the corrected values of x_{ij} by y_{ij}. The smoothing process is then, actually a simple weighted average as

$$y_{ij} = (1 - \eta_{ij})x_{ij} + \frac{1}{8}\eta_{ij}(x_{i-1,j-1} + x_{i-1,j} + x_{i-1,j+1} + x_{i,j-1} + x_{i,j+1} + x_{i+1,j-1} + x_{i+1,j} + x_{i+1,j+1}) \quad (16)$$

where η_{ij} is association coefficient of neighboring points for restoration of the central point. In this section, the coefficient is considered to be equal for all the 8 neighboring points. Eq. (16) indicates that at sharp points with higher association coefficient, the neighboring points association portion becomes greater in central points restoration. For the simplification purpose, (16) can be rewritten as:

$$y_{ij} = \frac{1}{8 + \rho_{ij}}(x_{i-1,j-1} + x_{i-1,j} + x_{i-1,j+1} + x_{i,j-1} + \rho_{ij}x_{ij} + x_{i,j+1} + x_{i+1,j-1} + x_{i+1,j} + x_{i+1,j+1}) \quad (17)$$

where $\rho_{ij} = 8(\rho_{ij}^{-1} - 1)$ is called *exclusion* coefficient that ranges in the interval $[0, \infty)$ and provides more flexibility to define continuous nonlinear membership functions. Like the sharpness, it represents a linguistic variable that can be defined in the entire range; however, we only use "*not exclusive*" fuzzy term set. For sharp points, while s_{ij} approaches 2, the association coefficient η_{ij} reaches 1 and the exclusion coefficient ρ_{ij} tends to 0. In such a case, neighboring points have more portions in restoration of the filtered point meaning *full* association. Conversely, as $s_{ij} \to 0$, $\eta_{ij} \to 0$ and $\rho_{ij} \to \infty$, which means *full* exclusion and the central points are supposed to have the most correct information. Figure 9 illustrates two typical membership functions for the "*not exclusive*" fuzzy term set. Based on (17), the fuzzy rule R_2 can be rewritten as: R_3: If the (i, j)th point of $x(t, \acute{t})$ is very sharp,

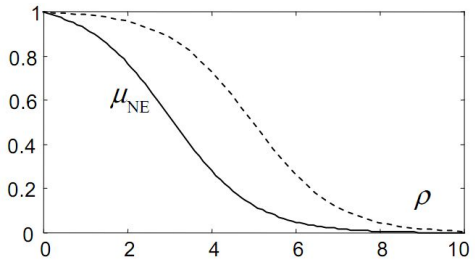


FIGURE 9. Two Typical Membership Functions Defined for the Linguistic Variable of Exclusion and "*not exclusive*" Fuzzy Set

then its value in $y(t, \acute{t})$ will be not exclusive. In a mathematical notation, R_3 can be represented by (19):

$$R_3 : s_{ij} \tilde{\in} \text{VS} \rightarrow \rho_{ij} \tilde{\in} \text{NE}; i = 2, 3, \ldots, N_1 - 1; j = 2, 3, \ldots, N_2 - 1 \tag{18}$$

where $\tilde{\in}$ means inclusion in a fuzzy set and N_1, N_2 represent the size of signal in each dimension. Moreover, "NE" and "VS" represent "*not exclusive*" and "*very sharp*" fuzzy term sets, respectively.

3.4. **Iterative Fuzzy Smoothing.** Now, if we rewrite (9) as:

$$y_{ij} = S_{\text{TDFS}} \{x_{ij}\}; i = 2, 3, \ldots, N_1 - 1; j = 2, 3, \ldots, N_2 - 1 \tag{19}$$

then the output of our proposed fuzzy filter is obtained by (17). Obviously, while running the algorithm, it is possible to generate new sharp points during the process of restoration of corrupted points. Therefore, once (19) is calculated for all points, the signal modification can be repeated frequently as (20), until there is no sharp point included in the "*very sharp*" fuzzy set.

$$y_{ij}^{(k+1)} = S_{\text{TDFS}} \left\{ y_{ij}^{(k)} \right\}; i = 2, 3, \ldots, N_1 - 1; j = 2, 3, \ldots, N_2 - 1; k = 1, 2, \ldots \tag{20}$$

where $y_{ij}^{(1)}$ is the same as y_{ij} obtained by (19). It is clear that if the membership function for "*very sharp*" fuzzy set, μ_{VS}, is defined over all values in the range of (similar to what appeared in Figure 7), iteration would never stop. To prevent such a problem, a termination criterion should be predefined for this endless process. By measuring the variations between two iterations k and $k+1$ one can obtain a proper stopping criterion to terminate the process.

Up to this point, the association coefficient is assumed to be equal for all eight neighboring points. However, since the eight angles between the central point and its neighbors are not equal, sharpness indexes are not equal for all the neighboring points. Dissolving this problem by applying separate association portions is discussed in the next section.

4. Introducing the TDMFS Filter As a Modified and Simplified TDFS

In this section, we will discuss some modifying algorithms to improve the efficiency of the smoothing filter. These modifications are formulated in particular for two dimensional signals and represented as an iterative process. This modified filter, which is named Two Dimensional Modified Fuzzy Smoothing filter (TDMFS), greatly improves the previous results.

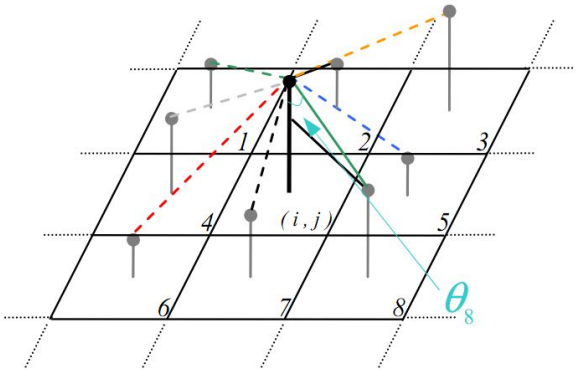


FIGURE 10. Eight Angles for Central Point with Eight Neighboring Points. θ_8 is Shown as an Example

4.1. **Applying Separate Association Portions.** As discussed before, the four angles θ_{ij}^{mn} for each point can be found separately by (11). It is obvious that the related sharpness coefficients calculated for these angles are not equal. It means that sharpness of the signal at each point (i, j) is different in direction of each neighboring point. To consider this point, the portion of each neighboring point must be calculated individually in order to restore the central point. Consequently, it is necessary to calculate eight sharpness indices individually for the neighboring points. Based on the related sharpness indices, the neighboring points association portion in restoration could be obtained separately. Therefore, the expression given in (17) has to be modified and the association coefficient must be presented separately for each of the eight neighboring points.

$$y_{ij} = 1/8(\eta_{ij}^1 x_{i-1,j-1} + \eta_{ij}^2 x_{i-1,j} + \eta_{ij}^3 x_{i-1,j+1} + \eta_{ij}^4 x_{i,j-1} + \eta_{ij}^5 x_{i,j+1} + \eta_{ij}^6 x_{i+1,j-1} + \eta_{ij}^7 x_{i+1,j} + \eta_{ij}^8 x_{i+1,j+1}) + \left(1 - \frac{\sum_1^8 \eta_{ij}^m}{8}\right) x_{ij} \tag{21}$$

where η_{ij}^m is the association coefficient for the mth neighbor of the point (i, j); it is calculated distinctly based on the sharpness due to each one of the neighboring points. The neighboring angles are easily calculated by:

$$\theta_{ij}^m = \tan^{-1}\left(\frac{d^m}{|x_{ij} - x_{ij}^m|}\right); m = 1, 2, \ldots, 8 \tag{22}$$

It is clear from Figure 10 that these eight angles are about half of the previously defined four angles in (11). Therefore, we have to redefine the sharpness index and replace (2) with

$$s_{ij}^m = \cos\left(2\theta_{ij}^m\right) + 1 \tag{23}$$

4.2. **Modification of Sharpness Membership Function.** The speed of algorithms has been always noticed by researchers. Image processing is also a time consuming task that proper simplification techniques can help its efficiency. However, the TDFS filter is an iterative algorithm that needs a criterion to control its

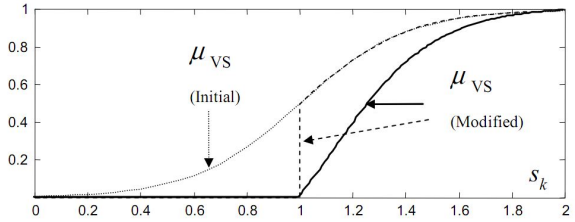


FIGURE 11. Modified Membership Function
(Compared to that of Figure 7)

continuation. In this regard, filtering is applied for points that have sharpness index more than a presumed threshold, σ_0. In this way, many (all) non-sharp points will be remained without any change; this speeds up the algorithm. Thus, (18) is replaced by:

$$\begin{gathered} R_4 : s_{ij}^m \tilde{\in} VS \rightarrow \eta_{ij}^m \tilde{\in} HA; \{\forall (i,j) | s_{ij}^m > \sigma_0\} \\ m = 1, \cdots, 8; i = 2, 3, \cdots, N_1 - 1; j = 2, 3, \cdots, N_2 - 1 \end{gathered} \tag{24}$$

where HA indicates the fuzzy term set of "*highly associative*". This modification will improve the power of proposed method in preserving correct data. Each time the algorithm is iterated, only a portion of the entire signal proportional to the percentage of the corrupted points is processed. Hence, it must be iterated until all sharpness indexes become less than the given threshold, σ_0. Indeed, the threshold contributes as a criterion for controlling the iterations. This threshold, normally set to 1, can be determined according to the sharpest edges in the signal. Figure 11 illustrates the modified membership function used in the proposed TDMFS filter. It is straightforward to show that the R_4 fuzzy rule is indeed a nonlinear relationship between s_{ij}^m and η_{ij}^m, which simply maps the sharpness index of each neighboring point to its association portion. This map, which is indeed replaced for $\mu_{HA}^{(-1)}(\mu_V S(s_{ij}^m))$, can be represented by a saturation function such as:

$$\eta_{ij}^m = \frac{1}{1 + \exp\left(-\alpha \left| s_{ij}^m \right| - 1\right)} \tag{25}$$

where α is an adjusting parameter. The higher is α, the faster is the growth of η with respect to the growth of the sharpness index. A set of sample saturation curves according to (25) with different αs is illustrated in Figure 12. It is better to pick $\alpha = 10$ to better distinguish sharpness.

Based on the fuzzy rule R_4, this function should be evaluated only for points with $s > \sigma_0$, which is normally set to one by default. Therefore, in practice these nonlinear functions should be cut off to zero for $s > \sigma_0$.

4.3. **Modification by Sorting or Applying Variable Virtual Distance.** The sharper are the angles around a point being processed, the higher is its priority to be processed. Therefore, if the pixels are sorted according to their average sharpness, our fuzzy filter would be much more efficient, while the process is started from the sharper pixels. Instead, one may use a variable virtual distance. This modification is explained in this section. According to (11), in calculation of angles between a central point and its neighboring points a virtual distance is considered

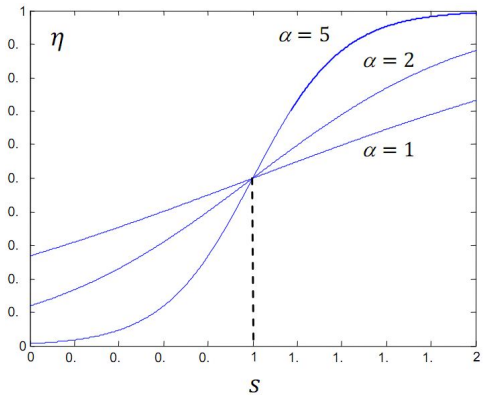


FIGURE 12. Sample Saturation Functions to Map Sharpness Index to Association Portion

between each two points of the signal. In addition, relations between the virtual distances are described in (12) . However, an equal distance is considered for signal angle calculation by (13) and thereafter. As mentioned, virtual distances can be considered as adjustable parameters. Indeed, variation of the virtual distance leads to changes in scaling the angle or the sharpness index. Therefore, we may find various sharpness indexes for distinct central points with equal sharpness. Since, in this method corrupted points are restored by correct (less corrupted) points, the more is the number of uncorrupted points, the higher becomes the accuracy of modification of the corrupted point value. Hence, as the algorithm progresses according to (20), the output signal becomes more accurate. Now, we employ a variable virtual distance to lessen sensitivity of the filter to the sharp points at each iteration of the smoothing process. This way, after a few iterations, merely very sharp points are included in the process of smoothing. In order to implement this idea, we indeed apply a fuzzy rule like: "If the signal is highly corrupted, then virtual distances are considered to be smaller". However, a simple adaptation rule suffices to realize the idea. Suppose, as defined in (26), the virtual distances change exponentially during the iterations from a given initial value,

$$d^{(k+1)} = e^{-\lambda k} d^{(k)}; k = 0, 1, 2, \dots \tag{26}$$

where the parameter $\lambda > 0$ adjusts the change rate and is constant. Evidently, $d^{(0)}$ should be chosen based on the magnitude of the two-dimensional signal. However, when an image is being filtered (with the highest magnitude of 255), we usually choose it in the range from 10 to 50. Experimental results show that for lower noise densities it would be better to set d to higher values. Figure 13 represents performance of the filter, measured by the well-known measure of the *PSNR* for different noise densities added to a test image, i.e. Baboon. Detailed results of this test case are given in the next section. Employment of (26) starting by higher $d^{(0)}$ means that we first smooth out very sharp points at the beginning. Then, by improving the correct data, the smoothing process continues with decreasing the virtual distances. Therefore, points with smaller sharpness index are processed.

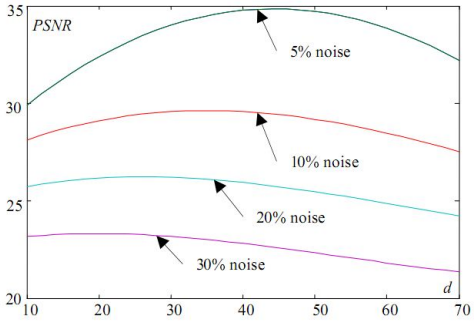


FIGURE 13. Performance of TDMFS with Respect to the Distance Parameter d (Constant) for Various Noise Densities

The results with decreasing virtual distances are better than those with constant virtual distances. Actually, variable virtual distances help to smooth corrupted points from very sharp points to less sharp points in order.

As a substitution for variable virtual distance, it is possible to sort the corrupted points by their sharpness index and perform signal modification from very sharp points. However, in two dimensional signals, unlike to one-dimensional, it is time consuming to sort all data; therefore the variable virtual distance is a proper equivalent.

5. **Simulation Results**

The proposed method has been implemented [29] for one-dimensional signals specifically for socio-economical data and its performance is already shown [30]. In this paper, we apply the filter to images represented by two-dimensional signals to elaborate its performance. An uncolored image like "I" can be considered as two-dimensional array

$$I = [I(i,j)]_{N_1 \times N_2} \tag{27}$$

where $I(i,j)$ presents gray level of pixel (i,j). For black & white images with 256 gray level, $I(i,j)$ belongs to the interval $[0,255]$.

Although the designed simple fuzzy filter is originally a grayscale digital filter, it can also be easily employed for colored images. There are a few differences between color images and black & white images. Color images are three-dimensional arrays, the first two dimensions of which specify size of the image and the third dimension involves a color component, say R, G and B, which stand for Red, Green and Blue, respectively:

$$I_C = [I_C(i,j,k)]_{N_1 \times N_2 \times 3} \tag{28}$$

There are different classes of vector based color filters to avoid probable artifacts [18, 2]; however, we have used a simple method to gain speed while saving acceptable visual results, although it may lead to suboptimal estimates of the original image. In order to apply TDMFS to colored images, we employ three two-dimensional matrixes, namely I_R, I_G and I_B; each concerning a color component. Each matrix is then processed individually. Therefore, color image is considered as three separate

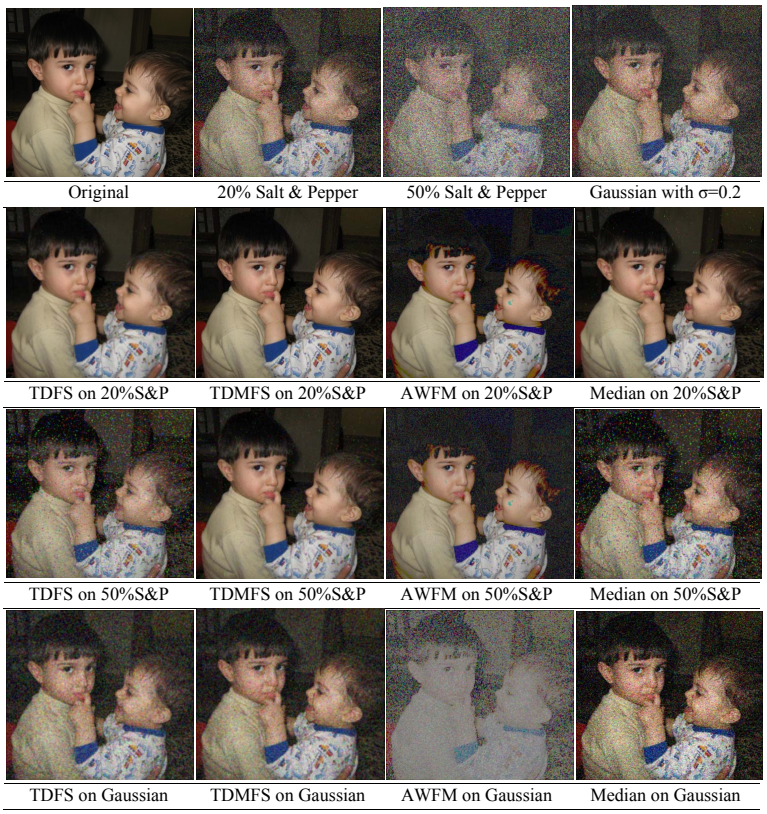


FIGURE 14. Results of Applying TDFS, TDMFS, AWFM and Median Filters on a Sample Image Infected by 20% and 50% Salt & Paper (Impulse) and Gaussian (std=0.05) Noises

two-dimensional signals and the algorithm of TDMFS filter is iteratively applied for each color component separately.

Both visual and numerical measures can be used to assess the algorithm. First, let us visually compare the results to see its capability in preserving the original data. Figure 14 contains filtering results of a sample 700×640 color image infected by different impulse and Gaussian noises. It seems that the Gaussian noise is better filtered by the Median, while the TDMFS filter has led to the best results on the Salt & Pepper noise.

Although there are several proxies to represent quality and goodness of the results, to obtain a numerical measure to quantify filter performance, we use the well known PSNR criterion. It is defined by:

$$PSNR = 10\log_{10}\frac{S^2}{MSE} \tag{29}$$

where MSE is the Mean Squared Error and is calculated by comparing the original signal $x(i,j)$ with the filtered one $y(i,j)$.

Noise Density	5%	10%	20%	30%	5%	10%	20%	30%	40%
Filter Type	*PSNR* (Lena)				*PSNR* (Baboon)				
No filter (Noisy Image)	21.8	18.6	15.6	13.9	21.5	18.4	15.5	13.7	10.3
TDMFS ($\sigma_0 = 1$)	37.4	34.4	31.6	29.7	41.49	37.83	32.76	28.84	24.29
TDFS	37.1	33.4	29.8	26.8	39.08	35.66	29.10	25.56	19.95
AWFM (2000)	31.6	31.3	30.6	29.4	25.0	24.4	23.5	22.2	-
HAF (1999)	29.7	29.4	29.0	28.5	24.6	24.4	24.2	23.9	-
PWLFIRE	37.2	30.6	22.9	18.2	37.7	29.3	22.3	17.9	24.13*
DSFIRE	35.9	34.2	30.8	25.9	33.2	30.7	27.6	24.2	-
SSFCF	29.6	28.3	24.6	20.5	24.8	24.0	21.9	19.0	-
FMF	36.1	33.1	28.6	23.8	32.0	27.7	24.2	21.0	-
FSM (2008)	-	-	37.2	-	-	-	-	-	24.18
FTSCF (2006)	51.4	45.7	39.8	-	50.6	48.2	42.0	37.5	-
TSM	37.0	33.4	28.1	23.1	28.3	26.8	23.8	21.1	-

* All numbers in this row are quoted from [36], except for the last column which is from [35].

TABLE 1. Comparison of Different Filters Applied to Lenas and Baboons Noisy Images with Various Impulse Noise Densities Based on PSNR Criterion

$$MSE = \frac{\sum_{i=1}^{N_1} \sum_{j=1}^{N_2} \left(x\left(i,j\right) - y\left(i,j\right)\right)^2}{N_1 \times N_2} \tag{30}$$

In addition, the parameter $S = 255$ is the maximum amplitude of the original signal. Rather than the Mean Absolute Error (MAE) or the MSE, $PSNR$ which is being used commonly, makes a better sense to measure enhancements and compare the results. Other measures, such as the Quality index (Q) and the Structural Similarity Index (SSI) which represent perceptual image quality, may better index the quality of image, but they are not available for the chosen filters outputs listed below.

In order to compare the results obtained by the proposed TDFS and TDMFS filters with those of other methods, we have applied the filter to commonly used benchmark images. The famous colored image of Lena (256×256) and the gray scale Baboon (512×512) are chosen for this purpose.

Table.1 contains $PSNR$ concerning to all the noisy images as well as the filtered images. Besides TDFS and TDMFS, we have results obtained by AWFM (Adaptive Weighted Fuzzy Mean) [21], HAF (Histogram-Adaptive Filter) [36], PWL (Piece-wise Linear) FIRE, DSFIRE, SFCF, FMF (Fuzzy Median Filter), FSM (Fuzzy Switching Median) [35], FTSCF (Fuzzy Two-Step Color Filter) [28] and Tri-State Median filter (TSM) in the table, where the original image is infected by a salt & pepper noise with different densities from 5% to 30% and 40%. Most results in the table are quoted from [28]. It is worthy to mention that for each case of noise density, various noises should be added several different times with the same density in order to obtain the PSNR value almost equal to that of listed in [13] and [28]. This is necessary to make the comparison admissible.

Figure 15 demonstrates the original, infected and filtered images. Results obtained by TDMFS are satisfactory considering the numerical criterion, as well as visual inspection. It should be mentioned that results for FSM and FTSCF, which are merely given for noisy images with more than $PSNR = 12$ dB, seem much better and more promising. However, FSM uses a triple fuzzy rule each combined of two precedents, and hence sounds to be a more time consuming filter compared

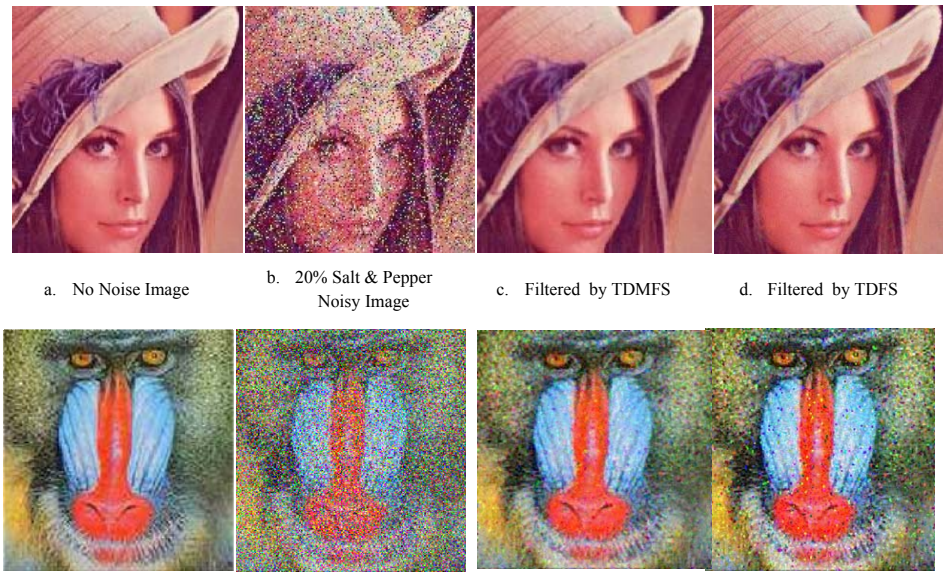


FIGURE 15. Results Obtained by TDMFS & TDFS Methods for Impulse Noise Reduction from Lena and Baboon Noisy Images Infected by 20% and 40% Salt & Pepper Noise, Respectively

to our single fuzzy rule. FTSCF is also a two-step highly complex filter with five groups of complicated fuzzy rules, and again seems much more time consuming. Certainly, comparison of calculation time can be done if only all the procedures of these filters are implemented optimally in a common framework, which is not concerned here. The capability of TDMFS is made further transparent with the second test image. The results evidently show successfulness and out performance of the proposed filter.

We have to mention that there are so many fuzzy filters in the literature which may present high performance but are not listed in this paper for three reasons: a) this paper is not a literature review paper, b) it does not claim superiority of the proposed method to all other methods, and c) comparing all methods by implementing all by one software is seriously difficult and practically impossible.

6. **Application of the Filter: Processing Demographic Data**

Dealing with statistical errors is a serious challenge for scientists, especially in socio-economic fields of study. Similar to its one-dimensional version, the filter can be applied to socio-economic data in order to reduce error impacts. In practice, every two-dimensional function of the form $z = f(x, y)$, where z is expected to be a smooth function of its two independent arguments, x and y, is subject to the application of our proposed filter. There are many socio-economic variables that can be introduced as a two-variable function, such as population, households income, price levels, energy consumption in income groups, etc.

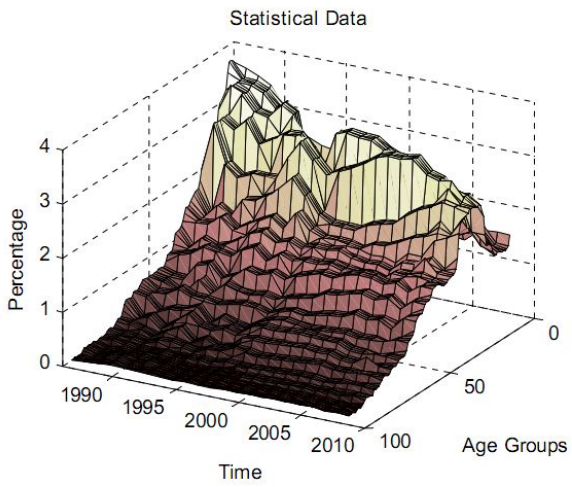


FIGURE 16. Statistical Data for Percentage of Irans Population Age Groups

In this section we will apply the fuzzy smoothing filter to demographic data, which is an awful area for investigation on statistical data, particularly in developing countries. One reason is that the country lacks a well designed people are not well trained about importance of statistical data and do not trust on the organizers of any survey to give precise information. Moreover, the standards for statistics collection are not well considered by the questioners and/or other staff involved with the related project.

Irans population is measured every five year through a national census. The total population can be divided to age groups. In this way, we will have the population matrix as samples of a two-dimensional function of the form:

$$p_{t,x} = f\,(t, x) \tag{31}$$

where x denotes each age, say from zero ages (newly born babies up to under one year babies) to 95 years old people and more, and t is the time in which the population is measured or estimated. Such a two-dimensional statistics make a matrix like an image. Clearly, in a normal condition, we suppose p to be a smooth function without any unexpected jumps. Even catastrophic events, such as war, earthquake, flood, etc. cannot affect the population of a large country (such as Iran with more than 70 million) as usual. However, there are enough causes that actually lead to inaccurate data, such that the percentage of population in each age group is an uneven function. Figure 16 shows an image built of the data recorded for p in percents, while x is sorted to cover all age groups in the interval $[0, 95]$, and t includes the years between 1986 and 2008. Data are gathered from various references such as reports published by [6] and [31]. Let us denote the data matrix by $p_{t,x}$ and the smoothed matrix by:

$$q_{t,x} = S_{\mathrm{FS}}\,\{p_{t,x}\} \tag{32}$$

with t related to the rows of the matrix and x related to the columns. In the

following some of the reasons justifying to have such a mixed data are listed:

- Real measurements are done through census every five years. Therefore, data available for the years between two censuses are just collected by limited surveys, which cover about 0.1% of the total population.
- Statistics of different years are published in different formats. For example, data of 2006 census is available for all ages separately, while for 1986, 1991 and 1996 censuses the percentages are given for 5-year age groups: $[0-4]$, $[5-9]$, etc.
- Information collected for the years between 1997 and 2000 are merely estimated by some researchers, and no survey is available. Moreover, age groups in 1997 data are arranged in 10-year as: $[0-9]$, $[10-19]$, etc.
- Data published for the surveys are arranged such that we have the percentage of new babies (zero years old) and then data is given in 5-year age groups starting from 1: $[1-5]$, $[6-10]$, etc.
- Ignoring immigration to the country, data in the matrix cannot increase diagonally, while it is observed in several points. It should be mentioned that immigration never could have such an influence on the total population of an age group, although in some years the total number of all immigrants has been considerable, but distributed in all age groups.
- The ending age group differs with different censuses or surveys. For example, the 2006 census includes all over-95 years old people, while for the other censuses this is not the case.

Therefore, it is necessary to compensate for the effect of such problems in data by an intelligent processing method. It is shown in [29] that our fuzzy smoothing filter can be successfully applied to smoothing mixed socio-economic data. The method is now being employed to smooth a two-dimensional population data which is expected to appear without sharp points.

6.1. **Smoothing with Smoothed Birth Rates.** At the first step, we smooth out sharpness in the signal of the Birth Rate, which is approximately equal to the first column. In fact, neglecting death rate of the zero-year age group, the corresponding column (behind the surface shown in Figure 16) is the same as birth rate. We apply our Modified Fuzzy Smoothing filter to do so, by setting the critical sharpness value to $\sigma_0 = 0.2$. The result is depicted in Figure 17, accompanying the original mixed data. The algorithm may be modified such that the exact points (e.g. data samples of the years in which the census are done) are fixed and not processed. However, this assumption is ignored here.

The same smoothing process is done for age group of over-95 years old percentages. Moreover, we have to smooth the two other edges of the surface, i.e. data of years 1986 and 2008, for the algorithm cannot include the boarder.

Then the surface is smoothed by the algorithm with two different thresholds of $\sigma_0 = 0.5$ and $\sigma_0 = 0.2$. The results are given in Figure 18. The average sharpness is given in Table 2, in company with the error norm generated by smoothing. Although it is not always the case, it is seen that both the sharpness and error norm are reduced by decreasing the threshold, where we have attained less than

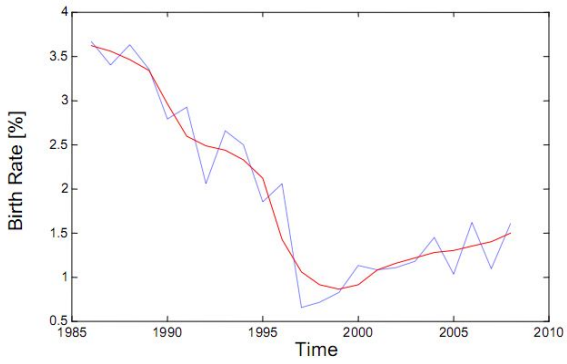


FIGURE 17. Smoothed Percentage for the Zero-year Age Group, Which is Approximately the Birth Rate

Surface	Original Data	Smoothed by $\sigma_0 = 0.5$	Smoothed by $\sigma_0 = 0.2$
Average Sharpness	0.0503	0.0251	0.0103
Error Norm [%]	0.015	0.795	0.484

TABLE 2. Results Obtained by Smoothing Data of the Population Percentages

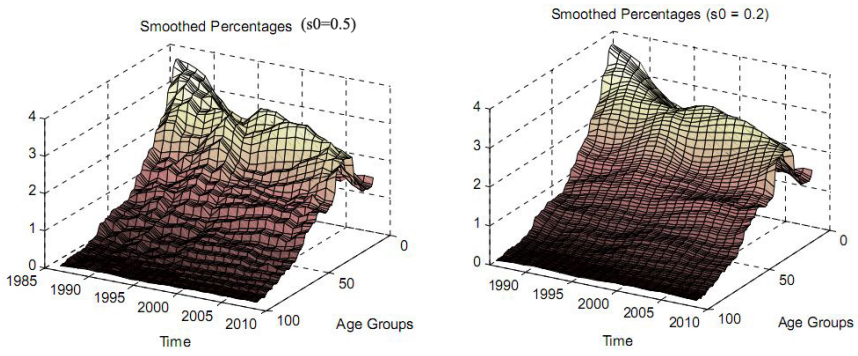


FIGURE 18. Smoothed Data with $\sigma_0 = 0.5$, and $\sigma_0 = 0.2$

0.5% error. In the next subsection error is clearly defined and it will be discussed that how the error can be reduced even more.

The most interesting point is that the fuzzy filter has revealed the generation waves moving from the left back corner to the right front; while it was not so transparent in the original statistical data, smoothing has shed light on its sub-diagonal animation.

6.2. **Integration Error Correction while Smoothing.** Let us first explain what exactly is aimed by the meaning of error in this case study. It is clearly known that the sum of data for each year should be 100%. Therefore, deviation from 100% for each row is counted as error, shown in Figure 19, and the total error is the norm of

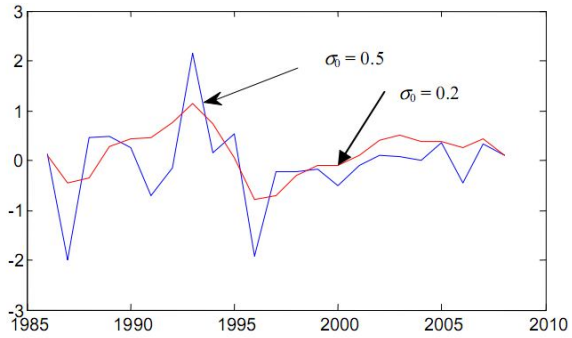


FIGURE 19. Error in Smoothed Surface for Each Column [%]

error signals. As it could be expected, the error has been annihilating along with the time, for the fact that statistics have been improving in precise and richness.

Knowing that integral of $p_{t,x} = f(t,x)$ on x has a predefined value of 100%, the process can be modified such that provides this desirable property. Hence, for each t, we have:

$$\sum_{x=0}^{95} p_{t,x} \cong \int_{0}^{\infty} f\,(t,x)\,dx = 100\,[\%] \tag{33}$$

Recall that based on the nature of mixed data, defined in [29], similar to modifications done by (8), we can modify the processed signal at each iteration as:

$$q_{t,x} = S_{\mathrm{FS}}\left\{p_{t,x}\right\} \tag{34}$$

$$q_{t,x-1}^{(k+1)} = q_{t,x-1}^{(k)} + \left(p_{t,x} - q_{t,x}^{(k)}\right)/2 \tag{35}$$

$$q_{t,x+1}^{(k+1)} = q_{t,x+1}^{(k)} + \left(p_{t,x} - q_{t,x}^{(k)}\right)/2 \tag{36}$$

where the superscript (k) means the iteration number. Since the percent error is less than 1%, these modifications are ignored in the above example.

7. Conclusions

In this paper, a simple yet effective method for smoothing of two-dimensional signals was presented. This method is a generalization of a one-dimensional signal smoothing filter which was developed before by the authors. In the proposed method, corrupted points are modified by sharing its neighboring points values. This method is designed based on a single fuzzy rule to remove/reduce all kinds of impulse noises in two-dimensional signals. A sharpness index is defined that presents corruption of the image points. Three steps are considered for implementation of the filter: first, sharpness index for all point is calculated; next, "very sharp" points are verified and their memberships to fuzzy sets are determined. Finally, "association portion" of points in neighborhood of the points belonging to the "very sharp" fuzzy set is calculated and used to modify them based on their sharpness index.

Simulation results easily approved effectiveness of TDMFS in comparison with several other filters cited in the literature. Saving its simplicity, both numerical and

visual criteria indicate excellent power of TDMFS filter for impulse noise reduction in both black & white and colored images. It should be noticed that there is always a tradeoff between speed and accuracy of computational algorithms. Although the proposed method is less accurate compared to some of the recently cited filters, it is efficient with respect to its simplicity and speed. It should be noted that the fastness of the proposed method is deduced here just by means of its simplicity. Otherwise, all the compared methods should be implemented within the same computer with optimized algorithms and codes professionally. Moreover, employing a piecewise linear fuzzy membership for the term "*very sharp*" we would speed up more the proposed filter. Optimization of the membership function and other parameters of the filter is an open problem and needs further research.

Various applications can be found for the filter, particularly in demographic data, where mixed data are frequently observed, among which population is chosen. The fuzzy smoothing filter successfully has shown the inherent dynamics of the population due to generation movements, while it was appearing as a very coarse signal full of many inconsistencies without any capability to represent the dynamic behavior.

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H. Shakouri G., School of Industrial and Systems Engineering, College of Engineering, University of Tehran

E-mail address: hshakouri@ut.ac.ir

M. Arabi, Electrical Engineering Department, Shahed University

E-mail address: mojtaba.m.arabi@gmail.com

M. B. Menhaj*, Department of Electrical Engineering, Amirkabir University of Technology

E-mail address: menhaj@aut.ac.ir

*Corresponding author