# SOME CONDITIONS UNDER WHICH SLOW OSCILLATION OF A SEQUENCE OF FUZZY NUMBERS FOLLOWS FROM CESÀRO SUMMABILITY OF ITS GENERATOR SEQUENCE

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Abstract. Let $(u_n)$ be a sequence of fuzzy numbers. We recover the slow oscillation of $(u_n)$ of fuzzy numbers from the Cesàro summability of its generator sequence and some additional conditions imposed on $(u_n)$. Further, fuzzy analogues of some well known classical Tauberian theorems for Cesàro summability method are established as particular cases.

## 1. **Introduction**

Zadeh [21] introduced the concept of the fuzzy set. Matloka [8] introduced the bounded and convergent sequences of fuzzy numbers and proved that every convergent sequence is bounded. Nanda [11] studied the spaces of bounded and convergent sequences of fuzzy numbers and proved that every Cauchy sequence of fuzzy numbers is convergent.

Subrahmanyam [13] proved that if a sequence of fuzzy numbers converges, then it is Cesàro summable to the same limit. However, the converse of this statement is not always true. Convergence of a sequence follows from its Cesàro convergence under certain condition which is called a Tauberian condition.

Subrahmanyam [13] established a Tauberian theorem for Cesàro summability method for the sequence of fuzzy real numbers and obtained fuzzy analogues of some classical Tauberian theorems. Later, Tripathy and Baruah [15] introduced Nörlund and Riesz means of fuzzy real numbers and proved a fuzzy analogue of a Tauberian theorem for Riesz summability method for sequences of fuzzy numbers. Recenty, Talo and Çakan [14] have established the fuzzy analogue of a Tauberian theorem for Cesàro summability method due to Móricz [9]. The works on fuzzy sequences were further investigated by Tripathy and Borgohain [18], Dutta and Tripathy [5], Tripathy and Dutta [20], Tripathy and Debnath [19] and many others.

A generalization of convergence is the concept of the statistical convergence independently introduced by Fast [6] and Schoenberg [12]. The idea of statistical summability $(C, 1)$ for real numbers was introduced by Móricz [10]. The statistical convergence for sequences of fuzzy real numbers has been studied in [1, 2, 3, 16, 17] and many others.

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In this paper we recover the slow oscillation of a sequence from the Cesàro summability of its generator sequence with some additional conditions. Further, we have fuzzy analogues of some classical Tauberian theorems for Cesàro summability method.

## 2. Preliminaries

Let $D$ denote the set of all closed and bounded intervals $A = [\underline{A}, \overline{A}]$ on the real line $\mathbb{R}$.

For $A, B \in D$, we define

$$d(A,B) = \max\{|\underline{A} - \underline{B}|, |\overline{A} - \overline{B}|\}$$

where $A = [\underline{A}, \overline{A}]$ and $B = [\underline{B}, \overline{B}]$. It is known that $(D, d)$ is a complete metric space.

A function $u : \mathbb{R} \to [0,1]$ which is normal and fuzzy convex is called a fuzzy number. Let $E^1$ denote the set of all fuzzy numbers those are upper semi-continuous and have a compact support.

For $0 < \alpha \leq 1$, the $\alpha$-level set of a fuzzy number $u$ denoted by $u^\alpha$ is defined by

$$u^\alpha = \{t \in \mathbb{R} : u(t) \geq \alpha\}.$$

$u^0$ is defined as the closure of the set $\{t \in \mathbb{R} : u(t) > 0\}$.

We define $D : E^1 \times E^1 \to \mathbb{R}_+ \cup \{0\}$ by

$$D(u,v) = \sup_{0 \leq \alpha \leq 1} d(u^\alpha, v^\alpha).$$

It is straightforward to see that the mapping $D$ is a metric on $E^1$. The additive identity in $E^1$ is denoted by $\overline{0}$.

The following properties ([4]) of $D$ are needed in the sequel.

(i) $D(u+w, v+w) = D(u,v)$ for all $u, v, w \in E^1$

(ii) $D(ku, kv) = |k| D(u,v)$ for all $u, v \in E^1, k \in \mathbb{R}$

(iii) $D(u+v, w+z) \leq D(u,w) + D(v,z)$ for all $u, v, w, z \in E^1$

A sequence $u = (u_n)$ of fuzzy numbers is a function $u$ from the set $\mathbb{N}$ of all positive integer into $E^1$.

A sequence $(u_n)$ of fuzzy numbers is said to be convergent to $\ell$, written as $\lim_{n\to\infty} u_n = \ell$, if for every $\epsilon > 0$ there exists a positive integer $N_0$ such that

$$D(u_n, \ell) < \epsilon$$

for $n > N_0$.

The arithmetic means $\sigma_n^{(1)}(u)$ of $(u_n)$ are defined by $\sigma_n^{(1)}(u) = \frac{1}{n+1}\sum_{j=0}^{n} u_j$ for all $n \in \mathbb{N} \cup \{0\}$. A sequence $(u_n)$ of fuzzy numbers is said to be Cesàro summable to $\ell$ if $D(\sigma_n^{(1)}, \ell) \to 0$ as $n \to \infty$.

De la Vallée Poussin means of $u = (u_n)$ are defined by, for $\lambda > 1$ and sufficiently large $n$,

$$\tau^{>}_{n,\lambda_n}(u) = \frac{1}{\lambda_n - n}\sum_{k=n+1}^{\lambda_n} u_k$$

and, for $0 < \lambda < 1$ and sufficiently large $n$,

$$\tau^{<}_{n,\lambda_n}(u) = \frac{1}{n-\lambda_n} \sum_{k=\lambda_n+1}^{n} u_k,$$

where $\lambda_n$ denotes the integer part of the product $\lambda n$, in symbol $\lambda_n := [\lambda n]$.

The classical control modulo of the oscillatory behavior of $(u_n)$ is denoted by $\omega_n^{(0)}(u) = n\Delta u_n$, where $\Delta u_n = \begin{cases} u_n - u_{n-1} & , n \in \mathbb{N} \\ u_0 & , n = 0 \end{cases}$.

The general control modulo of the oscillatory behavior of order $m$ of $(u_n)$ is denoted by $\omega_n^{(m)}(u)$ and defined by

$$\omega_n^{(m)}(u) = \omega_n^{(m-1)}(u) - \sigma_n^{(1)}(\omega^{(m-1)}(u)) \quad (m \in \mathbb{N} \text{ and } n \in \mathbb{N} \cup \{0\}).$$

For a sequence $u = (u_n)$ of fuzzy numbers, we define $(n\Delta)_m u_n = (n\Delta)_{m-1}((n\Delta)u_n) = n\Delta((n\Delta)_{m-1}u_n)$ for $m \in \mathbb{N}$ and $n \in \mathbb{N} \cup \{0\}$, where $(n\Delta)_0 u_n = u_n$ and $(n\Delta)_1 u_n = n\Delta u_n$. It is known that $\omega_n^{(m)}(u) = (n\Delta)_m v_n^{(m-1)}$, where $v_n^{(m)} = \sigma_n^{(1)}(v^{(m-1)})$ for $m \in \mathbb{N}$.

A sequence $(u_n)$ is said to be slowly oscillating if

$$\lim_{\lambda \to 1^+} \limsup_{n \to \infty} \max_{n < k \leq \lambda_n} D(u_k, u_n) = 0.$$

It is clear from the definition of the slow oscillation that every convergent sequence $(u_n)$ is slowly oscillating.

For any $m \in \mathbb{N} \cup \{0\}$, the identity

$$\sigma_n^{(m)}(u) - \sigma_n^{(m+1)}(u) = v_n^{(m)}, \tag{1}$$

where

$$v_n^{(m)} = \begin{cases} \dfrac{1}{n+1} \displaystyle\sum_{j=0}^{n} v_j^{(m-1)} & , m \geq 1 \\ \dfrac{1}{n+1} \displaystyle\sum_{j=0}^{n} j(u_j - u_{j-1}) & , m = 0, \end{cases}$$

will be used frequently. The identity (1) for $m = 0$ is known as the Kronecker identity. Throughout this paper, a sequence term with a negative index will be assumed zero fuzzy number for any sequence $(u_n)$ of fuzzy numbers.

By the expression of $\sigma_n^{(m+1)}(u)$ in the form

$$\sigma_n^{(m+1)}(u) = u_0 + \sum_{j=1}^{n} \frac{v_j^{(m)}}{j}, \tag{2}$$

we may express (1) as

$$\sigma_n^{(m)}(u) = v_n^{(m)} + \sum_{j=1}^{n} \frac{v_j^{(m)}}{j} + u_0. \tag{3}$$

We say from the expression (3) that $(\sigma_n^{(m)}(u))$ is regularly generated by $(v_n^{(m)})$ and $(v_n^{(m)})$ is called a generator of $(\sigma_n^{(m)}(u))$.

## 3. **Main Theorem**

We prove the following extended Tauberian theorem which gives a sufficient condition for recovering the slow oscillation of a sequence from Cesàro summability of its generator sequence.

**Theorem 3.1.** *For a sequence $(u_n)$ of fuzzy numbers let there exist a nonnegative sequence $m = (m_n)$ such that*

$$D(\omega_n^{(2)}(u), 0) = O(m_n) \tag{4}$$

*is satisfied for all $m \in \mathbb{N} \cup \{0\}$. Then, $(u_n)$ is slowly oscillating if $(v_n^{(0)})$ is Cesàro summable to zero and one of the following conditions are satisfied:*

$$(\lambda - 1) \limsup_{n\to\infty} \tau^{>}_{n,\lambda_n}(m) = o(1), \;\; \lambda \to 1^+, \tag{5}$$

$$(1 - \lambda) \limsup_{n\to\infty} \tau^{<}_{n,\lambda_n}(m) = o(1), \;\; \lambda \to 1^-. \tag{6}$$

## 4. **Auxiliary Results**

We need the following lemmas for establishing Theorem 3.1.

**Lemma 4.1.** [14] *Let $u = (u_n)$ be a sequence of fuzzy numbers. Then,*
*(i) For $\lambda > 1$ and sufficiently large $n$,*

$$D(\tau^{>}_{n,\lambda_n}(u), \sigma_n(u)) \leq \frac{\lambda_n + 1}{\lambda_n - n} D(\sigma_{\lambda_n}(u), \sigma_n(u)).$$

*(ii) For $0 < \lambda < 1$ and sufficiently large $n$,*

$$D(\tau^{<}_{n,\lambda_n}(u), \sigma_n(u)) \leq \frac{\lambda_n + 1}{n - \lambda_n} D(\sigma_n(u), \sigma_{\lambda_n}(u)).$$

The next lemma states that if a sequence $(u_n)$ of fuzzy numbers is Cesàro summable to a fuzzy number $\ell$, then de la Vallée Poussin means of $(u_n)$ converges to $\ell$.

**Lemma 4.2.** [14] *If $u = (u_n)$ is Cesàro summable to a fuzzy number $\ell$, then*
*(i)* $\lim_{n\to\infty} \tau^{>}_{n,\lambda_n}(u) = \ell$,
*(ii)* $\lim_{n\to\infty} \tau^{<}_{n,\lambda_n}(u) = \ell$.

**Lemma 4.3.** *For a sequence $u = (u_n)$ of fuzzy numbers, let there exist a nonnegative sequence $m = (m_n)$ such that*

$$D(u_n, u_{n-1}) \leq \frac{m_n}{n}.$$

*Then*
*(i)* $D(\tau^{>}_{n,\lambda_n}(u), u_n) \leq \dfrac{\lambda_n - n}{n} \tau^{>}_{n,\lambda_n}(m)$
*(ii)* $D(\tau^{<}_{n,\lambda_n}(u), u_n) \leq \dfrac{n - \lambda_n}{\lambda_n} \tau^{<}_{n,\lambda_n}(m)$.

*Proof.* (i) Since $D(u_n,u_{n-1}) \le \frac{m_n}{n}$ $(n\in\mathbb{N})$, we have

$$D(u_k,u_n) \le \sum_{j=n+1}^{k} D(u_j,u_{j-1}) \le \sum_{j=n+1}^{k} \frac{m_j}{j}. \tag{7}$$

Hence

$$\begin{aligned} D(\tau^{>}_{n,\lambda_n}(u),u_n) &= D\left(\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n} u_k, u_n\right)\\ &= D\left(\frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n} u_k, \frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n} u_n\right)\\ &= \frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n} D(u_k,u_n) \le \frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}\sum_{j=n+1}^{k} D(u_j,u_{j-1})\\ &\le \frac{1}{\lambda_n-n}\sum_{k=n+1}^{\lambda_n}\sum_{j=n+1}^{k}\frac{m_j}{j} \le \frac{1}{n(\lambda_n-n)}\sum_{k=n+1}^{\lambda_n}\sum_{j=n+1}^{k} m_j\\ &= \frac{\lambda_n-n}{n}\tau^{>}_{n,\lambda_n}(m), \end{aligned}$$

which completes the proof of (i).

(ii) Since $D(u_n,u_{n-1}) \le \frac{m_n}{n}$ $(n\in\mathbb{N})$, we have

$$D(u_k,u_n) \le \sum_{j=\lambda_n+1}^{k} D(u_j,u_{j-1}) \le \sum_{j=\lambda_n+1}^{k} \frac{m_j}{j}. \tag{8}$$

Hence

$$\begin{aligned} D(\tau^{<}_{n,\lambda_n}(u),u_n) &= D\left(\frac{1}{n-\lambda_n}\sum_{k=\lambda_n+1}^{n} u_k, u_n\right)\\ &= D\left(\frac{1}{n-\lambda_n}\sum_{k=\lambda_n+1}^{n} u_k, \frac{1}{n-\lambda_n}\sum_{k=\lambda_n+1}^{n} u_n\right)\\ &= \frac{1}{n-\lambda_n}\sum_{k=\lambda_n+1}^{n} D(u_k,u_n) \le \frac{1}{n-\lambda_n}\sum_{k=\lambda_n+1}^{n}\sum_{j=\lambda_n+1}^{k} D(u_j,u_{j-1})\\ &\le \frac{1}{n-\lambda_n}\sum_{k=\lambda_n+1}^{n}\sum_{j=\lambda_n+1}^{k}\frac{m_j}{j} \le \frac{1}{\lambda_n(n-\lambda_n)}\sum_{k=\lambda_n+1}^{n}\sum_{j=\lambda_n+1}^{k} m_j\\ &= \frac{n-\lambda_n}{\lambda_n}\tau^{<}_{n,\lambda_n}(m), \end{aligned}$$

which completes the proof of (ii). □

## 5. **Proof of Theorem 3.1**

*Proof.* Since $(v_n^{(0)})$ is Cesàro summable to $\overline{0}$ , we have $\lim_{n\to\infty} n\Delta v_n^{(2)} = \overline{0}$ . Under the assumptions of Theorem 3.1 we prove that $\lim_{n\to\infty} n\Delta v_n^{(1)} = \overline{0}$. Let $t_n := n\Delta v_n^{(1)}$. Then $\sigma_n(t) = n\Delta v_n^{(2)}$. By triangle inequality, we have

$$D(t_n,\sigma_n(t)) \le D(t_n,\tau^{>}_{n,\lambda_n}(t)) + D(\tau^{>}_{n,\lambda_n}(t),\sigma_n(t)). \tag{9}$$

Since

$$D(t_n, \tau^{>}_{n,\lambda_n}(t)) \leq \frac{\lambda_n - n}{n} \tau^{>}_{n,\lambda_n}(m) \tag{10}$$

by Lemma 4.3 (i), we have

$$D(t_n, \sigma_n(t)) \leq \frac{\lambda_n - n}{n} \tau^{>}_{n,\lambda_n}(m) + D(\tau^{>}_{n,\lambda_n}(t), \sigma_n(t)). \tag{11}$$

Taking lim sup of both sides of (11), we get

$$\limsup_{n\to\infty} D(t_n, \sigma_n(t)) \leq \limsup_{n\to\infty} \left(\frac{\lambda_n - n}{n} \tau^{>}_{n,\lambda_n}(m)\right) + \limsup_{n\to\infty} D(\tau^{>}_{n,\lambda_n}(t), \sigma_n(t)). \tag{12}$$

Since the second term on the right-hand side of (12) vanishes by Lemma 4.1 (i), we deduce that

$$\begin{aligned}
\limsup_{n\to\infty} D(t_n, \sigma_n(t)) &\leq \limsup_{n\to\infty} \left(\frac{\lambda_n - n}{n} \tau^{>}_{n,\lambda_n}(m)\right) \\
&\leq \limsup_{n\to\infty} \left(\frac{\lambda_n - n}{n}\right) \limsup_{n\to\infty} \tau^{>}_{n,\lambda_n}(m) \\
&= (\lambda - 1) \limsup_{n\to\infty} \tau^{>}_{n,\lambda_n}(m).
\end{aligned}$$

Hence, it follows by (5) that

$$\limsup_{n\to\infty} D(t_n, \sigma_n(t)) = 0. \tag{13}$$

Similarly, by triangle inequality, we have

$$D(t_n, \sigma_n(t)) \leq D(t_n, \tau^{<}_{n,\lambda_n}(t)) + D(\tau^{<}_{n,\lambda_n}(t), \sigma_n(t)). \tag{14}$$

Since

$$D(t_n, \tau^{<}_{n,\lambda_n}(t)) \leq \frac{n - \lambda_n}{\lambda_n} \tau^{<}_{n,\lambda_n}(m) \tag{15}$$

by Lemma 4.3 (ii), we have

$$D(t_n, \sigma_n(t)) \leq \frac{n - \lambda_n}{\lambda_n} \tau^{<}_{n,\lambda_n}(m) + D(\tau^{<}_{n,\lambda_n}(t), \sigma_n(t)). \tag{16}$$

Taking lim sup of both sides of (16), we get

$$\limsup_{n\to\infty} D(t_n, \sigma_n(t)) \leq \limsup_{n\to\infty} \left(\frac{n - \lambda_n}{\lambda_n} \tau^{<}_{n,\lambda_n}(m)\right) + \limsup_{n\to\infty} D(\tau^{<}_{n,\lambda_n}(t), \sigma_n(t)). \tag{17}$$

Since the second term on the right-hand side of (17) vanishes by Lemma 4.1 (ii), we deduce that

$$\begin{aligned}
\limsup_{n\to\infty} D(t_n, \sigma_n(t)) &\leq \limsup_{n\to\infty} \left(\frac{n - \lambda_n}{\lambda_n} \tau^{<}_{n,\lambda_n}(m)\right) \\
&\leq \limsup_{n\to\infty} \left(\frac{n - \lambda_n}{\lambda_n}\right) \limsup_{n\to\infty} \tau^{<}_{n,\lambda_n}(m) \\
&= \frac{1 - \lambda}{\lambda} \limsup_{n\to\infty} \tau^{<}_{n,\lambda_n}(m).
\end{aligned}$$

Hence, it follows by (6) that

$$\limsup_{n\to\infty} D(t_n, \sigma_n(t)) = 0. \tag{18}$$

Since $(\sigma_n(t))$ converges to $\overline{0}$, it follows from (13) or (18) that $(t_n) = (n\Delta v_n^{(1)})$ converges to $\overline{0}$. Cesàro summability of $(v_n^{(0)})$ to $\overline{0}$ implies that $(v_n^{(0)})$ converges to $\overline{0}$.

From the representation $u_n = v_n^{(0)} + \sum_{k=1}^{n} \frac{v_k^{(0)}}{k} + u_0$, it follows that $(u_n)$ is slowly oscillating. □

## 6. Corollaries

**Corollary 6.1.** *For a sequence $(u_n)$ of fuzzy numbers let there exist a nonnegative sequence $m = (m_n)$ such that (4) is satisfied. Then, $(u_n)$ converges to $\ell$ if $(u_n)$ is Cesàro summable to $\ell$ and one of the conditions (5) or (6) is satisfied.*

*Proof.* Assume that $(u_n)$ is Cesàro summable to $\ell$. It follows by (1) that $(v_n^{(0)})$ is Cesàro summable to $\overline{0}$. By Theorem 3.1 $(v_n^{(0)})$ converges to $\overline{0}$. Again by (1) $(u_n)$ converges to $\ell$. □

The proof of the following corollary is straightforward.

**Corollary 6.2.** *For a sequence $(u_n)$ of fuzzy numbers let there exist a nonnegative sequence $m = (m_n)$ such that (4) is satisfied, where $(\sigma_n^{(1)}(m))$ is bounded and slowly oscillating. Then, $(u_n)$ converges to $\ell$ if $(u_n)$ is Cesàro summable to $\ell$.*

The following corollary is a fuzzy analogue of the classical Tauberian theorem for Cesàro summability method proved by Hardy [7].

**Corollary 6.3.** *If $(u_n)$ is Cesàro summable to $\ell$ and $D(\omega_n^{(0)}(u), \overline{0}) = O(1)$, then $(u_n)$ converges to $\ell$.*

*Proof.* Assume that $(u_n)$ is Cesàro summable to $\ell$. It follows by (1) that $(v_n^{(0)})$ is Cesàro summable to $\overline{0}$. It is easy to show that the conditions (4), (5) and (6) with $m_n = 1$ for all $n \in \mathbb{N}$ are satisfied. □

For the other proof of Corollary 6.3, we refer to [13].

**Corollary 6.4.** *If $(u_n)$ is Cesàro summable to $\ell$ and $D(\omega_n^{(1)}(u), \overline{0}) = O(1)$, then $(u_n)$ converges to $\ell$.*

*Proof.* The proof is similar to that of Corollary 6.3. □

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## References

[1] Y. Altın, M. Mursaleen and H. Altınok, *Statistical summability (C,1) for sequences of fuzzy real numbers and a Tauberian theorem*, J. Intell. Fuzzy Syst., **21(6)** (2010), 379–384.

[2] H. Altınok and M. Mursaleen, $\Delta$-*Statistically boundedness for sequences of fuzzy numbers*, Taiwanese J. Math., **15(5)** (2011), 2081–2093.

[3] R. Çolak, Y. Altın and M. Mursaleen, *On some sets of difference sequences of fuzzy numbers*, Soft Comput., **15(4)** (2011), 787–793
[4] D. Dubois and H. Prade, *Fuzzy numbers: An overview, Analysis of Fuzzy Information, vol. 1*, Mathematical Logic, CRC Press, Boca, FL, 1987, 3–39.
[5] A. J. Dutta and B. C. Tripathy, *On I-acceleration convergence of sequences of fuzzy real numbers*, Math. Model. Anal., **17(4)** (2012), 549–557.
[6] H. Fast, *Sur la convergence statistique*, Colloq. Math, **2** (1951), 241–244.
[7] G. H. Hardy, *Divergent Series*, 2nd ed., New York, NY, 1991.
[8] M. Matloka, *Sequences of fuzzy numbers*, BUSEFAL, **28** (1986), 28–37.
[9] F. Móricz, *Necessary and sufficient Tauberian conditions, under which convergence follows from summability* $(C, 1)$, Bull. London Math. Soc., **26** (1994), 288–294.
[10] F. Móricz, *Ordinary convergence follows from statistical summability* $(C, 1)$ *in the case of slowly decreasing or oscillating sequences*, Colloq. Math., **99(2)** (2004), 207–219.
[11] S. Nanda, *On sequences of fuzzy numbers*, Fuzzy Sets and Systems, **33(1)** (1989), 123–126.
[12] I. J. Schoenberg, *The integrability of certain functions and related summability methods*, Amer. Math. Monthly, **66** (1959) 361–375.
[13] P. V. Subrahmanyam, *Cesàro summability of fuzzy real numbers*, J. Anal., **7** (1999), 159–168.
[14] Ö. Talo and C. Çakan, *On the Cesàro convergence of sequences of fuzzy numbers*, Appl. Math. Lett., **25** (2012), 676–681.
[15] B. C. Tripathy and A. Baruah, *Nörlund and Riesz mean of sequences of fuzzy real number*, Appl. Math. Lett., **23(5)** (2010), 651–655.
[16] B. C. Tripathy and A. Baruah, *Lacunary statistically convergent and lacunary strongly convergent generalized difference sequences of fuzzy real numbers*, Kyungpook Math. J., **50(4)** (2010), 565–574.
[17] B. C. Tripathy, A. Baruah, M. Et and M. Güngör, *On almost statistical convergence of new type of generalized difference sequence of fuzzy numbers*, Iran. J. Sci. Technol. Trans. A Sci., **36(2)** (2012), 147–155.
[18] B. C. Tripathy and S. Borgohain, *Some classes of difference sequence spaces of fuzzy real numbers defined by Orlicz function*, Adv. Fuzzy Syst., Art. ID 216414, (2011), 6.
[19] B. C. Tripathy and S. Debnath, *On generalized difference sequence spaces of fuzzy numbers*, Acta Scientiarum Technology, **35(1)** (2013), 117–121.
[20] B. C. Tripathy and A. J. Dutta, *Lacunary bounded variation sequence of fuzzy real numbers*, J. Intell. Fuzzy Systems, **24(1)** (2013), 185–189.
[21] L. A. Zadeh, Fuzzy sets, *Information and Control*, **8** (1965), 338–353.

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