INDIRECT ADAPTIVE INTERVAL TYPE-2 FUZZY PI SLIDING MODE CONTROL FOR A CLASS OF UNCERTAIN NONLINEAR SYSTEMS

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Abstract. Controller design remains an elusive and challenging problem for uncertain nonlinear dynamics. Interval type-2 fuzzy logic systems (IT2FLS) in comparison with type-1 fuzzy logic systems claim to effectively handle system uncertainties especially in the presence of disturbances and noises, but lack a formal mechanism to guarantee performance. In contrast, adaptive sliding mode control (ASMC) provides a robust mechanism to provide system stability against parameter changes and uncertainties, but suffers from chattering phenomenon. In this paper, a stable indirect adaptive interval type-2 fuzzy PI sliding mode controller (AIT2FSMC) is investigated for a class of nonlinear systems in the presence of system's unmodeled dynamics and external disturbances. The added Proportional Integral (PI) structure is used to further attenuate the chattering problem that is common in sliding mode control systems. The interval type-2 fuzzy adaptation law adjusts the consequent parameters of the rules based on a Lyapunov synthesis approach. Mathematical analysis proves the closed loop asymptotic stability, while benefiting from human expert knowledge to improve transient response of the system. Application to two nonlinear systems verifies the robustness of the proposed AIT2FSMC approach in the presence of uncertainties and bounded external disturbances, especially when disturbances have fast changes and large magnitudes.

1. Introduction

Uncertainty is one of the most challenging obstacles in controlling nonlinear systems. Generally, there are two kinds of uncertainties. One is caused by a lack of information about the internal structure and parameters of a system, and the other is caused by external disturbances and noise. Sliding mode control (SMC) is a common strategy that aims to address uncertainty in plant dynamics and to reach a robust discontinuous nonlinear feedback control technique that asymptotically convergences to the sliding surface. But the basic SMC suffers from a major drawback of chattering [29] by exciting undesirable high-frequency dynamics through its switching mechanism. A common approach to resolve this chattering phenomenon is by introducing a small boundary layer around the sliding surface. However, asymptotic convergence of the resulting state trajectory is no longer guaranteed

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to the sliding surface [18]. Another way to eliminate chattering is introducing a proportional integral (PI) control term [2, 11] to approximate the discontinuous control strategy.

A great source of uncertainty is parameter drift. In classical control approaches, this is often addressed by adaptive control strategies. In this paradigm, an adaptation law is introduced which adjusts the parameters of the controller against system uncertainties and disturbances. It should be noted that adaptive control generally guarantees parameter convergence only if parameter changes are slow enough [3]. To address this problem, Shahnazi and Akbarzadeh in 2008 [28] introduced an indirect PI Adaptive fuzzy controller which could efficiently reject fast and large disturbances.

In recent decades, various other techniques based on fuzzy logic have similarly emerged for control of nonlinear systems [33, 36, 38]. Fuzzy logic is often promoted as a tool for addressing uncertainty by information disambiguation and for using human expert knowledge in complement to mathematical knowledge. This is mainly useful for employing fuzzy knowledge-based control to deal with systems whose dynamics are not so well understood, their models cannot be so conveniently established, or their mathematical analysis is difficult [34].

Therefore, hybrid combinations of the SMC, fuzzy logic, and adaptive control can be an attractive alternative for designing robust control systems with high degrees of non-linearity and uncertainties. In [2], a discrete-time adaptive fuzzy sliding mode controller (AFSMC) was proposed for an anti-lock braking system that was robust and stable against system uncertainties. Due to strong uncertainties in braking system, their controller aimed to least depend on the mathematical models. Later, Ho et al. [12] devised an indirect adaptive control strategy by using PI control term, which approximated the discontinuous control to minimize chattering. This method not only attenuated chattering but also guaranteed asymptotic stability. In 2006, Akbarzadeh-T and Shahnazi [1] proposed an approach to design direct adaptive fuzzy PI sliding mode control based on a fuzzy basis function framework and Lyapunov synthesis. Their approach required the uncertainties and disturbances to be bounded but did not require this bound to be known. In 2006, Chiang and Yang [7] developed a new approach to robust adaptive fuzzy sliding mode controllers which could guarantee the robust stability of the system where the dead-zone in the actuators was unknown. In [40], an AFSMC for a class of non-affine nonlinear systems was proposed where fuzzy tuning schemes based on neural networks were used to improve performance and reduce chattering in the sliding surface. In 2007, Wai [26] proposed an AFSMC and successfully demonstrated its application to an indirect field-oriented induction motor drive for tracking periodic commands. Moreover, he utilized fuzzy inference mechanism in order to implement a fuzzy hitting control law to remove the chattering phenomenon. In [9], fuzzy systems as universal approximators and SMC were used to control a class of under-actuated systems. They used an adaptive law to improve the approximation accuracy and guarantee the tracking performance. In 2010, Tie-Shan et al. [30] introduced a novel robust adaptive Takagi-Sugeno fuzzy tracking control for a Class of nonlinear MIMO Systems. Their method reduced the number of parameters updated online for each

subsystem to one. Tsung-Chih et al. [32] proposed an AFSMC for synchronization of two different uncertain fractional-order time-delay chaotic systems. They used Lyapunov stability criterion to show that the free parameters of the AFSMC can be tuned online by output-feedback-control law and adaptive law. More recently in 2013, Bessa et al. [4] used a combination of AFSMC and the close return method for the stabilization of unstable periodic orbits in a chaotic pendulum. However, the available information which is used to construct the rules in a fuzzy logic system (FLS) can itself be uncertain [17, 24]. This is while a solid majority of the existing works in this area, such as the above mentioned researches, have only benefited from the type-1 fuzzy construction. In contrast to Type-1 FLS, Type-2 FLSs can handle rule uncertainties.

Type-2 fuzzy sets as an extension of type-1 fuzzy sets were first introduced by Zadeh [39] in 1975. Basic concepts of type-2 fuzzy sets and systems were advanced and well established in [15-17, 23-25]. Karnik and Mendel [16] introduced five different kinds of type reduction methods which are extended versions of type-1 defuzzification methods. In 2000, Qilian and Mendel [25] proposed an efficient and simplified method for computing the input and antecedent operations for interval type-2 fuzzy logic systems (IT2FLS) using the concept of upper and lower Membership functions (MF). Karnik and Mendel [15] developed the centroid of an interval type-2 fuzzy set (IT2FS), not only for an IT2FS and IT2FLSs but also for general type-2 FSs and introduced an algorithm for its computation. Mendel [22] described important advances for both general and interval type-2 fuzzy sets and systems in 2007. Because of the calculation complexity especially in the type reduction, use of IT2FLS is still controversial. Seplveda et al. [27] showed that using adequate hardware implementation, IT2FLS can be efficiently utilized in applications that require high speed processing. Thus, the type-2 FLS has been successfully applied to several fuzzy controller designs [6, 8, 10, 20, 37]. For instance, Castillo et al. [5] used Ant Colony Optimization (ACO) and Particle Swarm Optimization (PSO) to optimize the membership functions parameters of a fuzzy logic controller in order to find the optimal intelligent controller for an autonomous wheeled mobile robot. They claimed ACO outperforms PSO and GA, while PSO outperforms GA in their case study. In 2013, Maldonado et al. [21] presented PSO as an optimization method for control parameters, authors showed in this case study that PSO was faster in runtime when compared to GA. A direct adaptive interval type-2 fuzzy-neural network sliding mode control for SISO nonlinear systems is proposed by Lin [19]. He used Lyapunov stability criterion to achieve asymptotic stability of the closed loop system without prior knowledge on the upper bound of the lumped uncertainty. Moreover, an indirect adaptive interval type-2 fuzzy sliding mode control for Synchronization of chaotic systems was proposed in [31] which they do not have the ability to reject fast disturbances.

In this paper, we propose a hybrid combination of the SMC, IT2FLS and adaptive control to deal with structure, rules and parameters uncertainties. We use IT2FLS to approximate the unknown nonlinear terms where both antecedent and consequent sets are interval type-2 fuzzy sets. The proposed controller combines

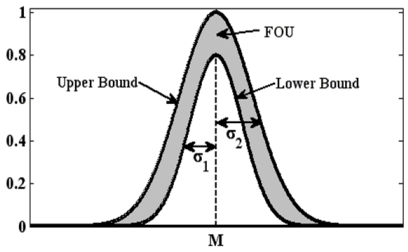


FIGURE 1. Interval Type-2 MF

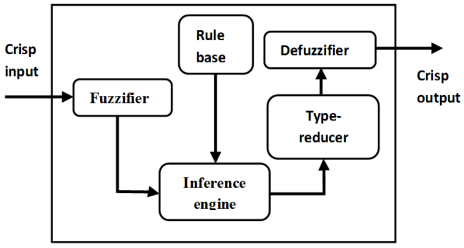


FIGURE 2. Structure of an IT2FLS

the advantages of IT2FLS and adaptive sliding mode controller to cope with uncertainties. The main contribution of this paper are: 1) using a PI term to eliminate chattering and increasing convergence rate of the controller near the sliding surface, 2) using Lyapunov synthesis to guarantee asymptotic stability and obtaining adaptation laws, 3) using two nonlinear systems to illustrate the robustness of the proposed approach in dealing with various disturbances, and finally 4) showing that the use of human expert knowledge can further improve the performance.

This paper is organized as follows. Section 2 presents a brief introduction to the IT2FLSs. Section 3 formulates the class of nonlinear systems under consideration here and describes the assumptions based on the theory of SMC. The proposed controller is presented in Section 4. Simulation example to demonstrate the performance of the proposed method is provided in Section 5. Section 6 provides the conclusions of the proposed design methodology.

2. **Interval Type-2 Fuzzy Logic Systems**

A type-2 fuzzy set in a universal set X is denoted as $\tilde{A}$ and can be characterized in the following form:

$$\tilde{A} = \int_{x \in X} \mu_{\tilde{A}}(x)/x \tag{1}$$

$$\mu_{\tilde{A}}(x) = \int_{v \epsilon J_x} f_x(v)/v \,, \;\; J_x \epsilon [0,1] \tag{2}$$

where $\mu_{\tilde{A}}(x)$ is called a secondary MF and $f_x(v)$ is called secondary grade; J_x is the domain of the secondary MF which is called primary membership; v is a fuzzy set in [0, 1]. When $f_x(v) = 1$ for $\forall\ v \epsilon J_x$, then secondary membership functions are interval sets and resulting FS can be called an interval type-2 fuzzy set which is rewritten as bellow:

$$\tilde{A} = \int_{x \in X} \mu_{\tilde{A}}(x)/x = \int_{x \epsilon X} [\int_{v \epsilon J_x} 1/v]/x \tag{3}$$

Equation (3) implies that IT2FSs demonstrate a uniform uncertainty in the primary membership. Because of calculation simplicity, especially in the type reduction, many researchers use interval type-2 fuzzy sets [8, 14, 19, 37, 31] instead of general type-2 fuzzy sets. An IT2FS is described by its lower $\underline{\mu}_{\tilde{A}}(x)$ and upper $\overline{\mu}_{\tilde{A}}(x)$ MFs.

Nomenclature

$\tilde{A}$	Type-2 fuzzy set	sgn	sign function
C	sliding surface coefficients	u	control input
COS	center of set type reduction	V	Lyapunov function
D	bound of disturbance	v	a fuzzy set
$d(t)$	external disturbance	X	vector space
E	error vector	x_i	i^{th} input of the IT2FLS
F^j	j^{th} firing set	Y	output of the IT2FLS
$\tilde{F}_i^l$	i^{th} antecedent set	X_d	desired state vector
f and g	system nonlinear functions	$\mu_{\tilde{A}}(x)$	secondary MF
$f_x(u)$	secondary grade	$\underline{\mu}_{\tilde{A}}(x)$	lower MF
FOU	footprint of uncertainty	$\overline{\mu}_{\tilde{A}}(x)$	upper MF
$\tilde{G}^l$	consequent set	δ	standard division
J_x	primary membership	θ_l^j	left-end point of j^{th} consequent set
KM	Karnik-Mendel	θ_r^j	right-end point of j^{th} consequent set
K_I	Integral gain	θ_p	PI controller adaptation parameter vector
K_P	Proportional gain	η	a positive small constant
M	number of rules	φ	thickness of the boundary layer
Pr	projection operator	γ	a positive constant
$s(E)$	sliding surface	ω	minimum approximation error

For an IT2FS, the footprint of uncertainty (FOU) is described in terms of lower and upper MFs as:

$$FOU\ \left(\tilde{A}\right) = \bigcup_{x\epsilon X} [\underline{\mu}_{\tilde{A}}(x),\ \overline{\mu}_{\tilde{A}}(x)] \tag{4}$$

Figure 1 shows a type-2 fuzzy MF with its FOU, upper and lower bounds and standard division $(\delta_1,\ \delta_2)$. Generally, an IT2FLS is composed of a fuzzifier; fuzzy rule base; fuzzy inference engine; type reducer and defuzzifier. Figure 2 shows the general structure of an IT2FLS and described in following.

2.1. **Fuzzifier.** The fuzzifier maps a real-valued variable into fuzzy sets. These fuzzy sets, in general, can be type-1 or type-2 fuzzy sets. In this paper, we simply use the singleton fuzzifier which its output is a single point of a unity membership grade.

2.2. **Rule Base.** The knowledge of experts will be placed in this part, which consists of a set of fuzzy IF-THEN rules. The j^{th} rule in the IT2FLS can be written as bellow:

$$R^j : If\ x_1\ is\ \tilde{F}_1^j\ and\ x_2\ is\ \tilde{F}_2^j\ and\ x_n\ is\ \tilde{F}_n^j\ then\ y\ is\ \tilde{G}^j \quad j = 1, 2 \dots M \tag{5}$$

where M shows the number of rules; x_i $(i = 1, 2, \dots, n)$ and y are the input and output of the IT2FLS, respectively; $\tilde{F}_i^l$ and $\tilde{G}^l$ are the type-1 or type-2 antecedent and the consequent sets, respectively.

2.3. **Inference Engine.** In IT2FLS, the inference engine combines rules and gives a mapping from input IT2FSs to output IT2FSs. By performing input and antecedent operations, the firing set is generally obtained as follows:

$$F^j(X) = \prod_{i=1}^{n} \mu_{\tilde{F}_i^j}(x_i) \tag{6}$$

where product t-norm is assumed. More specifically, since we are concerned with IT2FSs here, the firing input sets are defined based on their upper and lower MFs, as follows:

$$F^j(X) = [\underline{f}^j(X), \overline{f}^j(X)] \tag{7}$$

$$\underline{f}^j(X) = \underline{\mu}_{\tilde{F}_1^j} * \underline{\mu}_{\tilde{F}_2^j} * \ldots * \underline{\mu}_{\tilde{F}_n^j} \tag{8}$$

$$\overline{f}^j(X) = \overline{\mu}_{\tilde{F}_1^j} * \overline{\mu}_{\tilde{F}_2^j} * \ldots * \overline{\mu}_{\tilde{F}_n^j} \tag{9}$$

where $\underline{f}^j(X)$ and $\overline{f}^j(X)$ are the j^{th} lower and upper MFs, respectively, and * denote t-norm.

2.4. **Type Reducer and Defuzzifier.** Since the output of the inference engine is a type-2 fuzzy set, a type-reducer is needed before defuzzification to convert type-2 fuzzy sets into type-1. Type reducer was first proposed by Karnik and Mendel [16, 17]. In [16] five different type reduction methods are described which are based on computing the centroid of an IT2FS. Among the type reductions, center of sets (COS) is widely used, because it can be computed more easily by the Karnik-Mendel (KM) iterative algorithm [17]. Since we only use IT2FSs here, Y_{cos} is an interval set that is determined with its left-end point y_l, and right-end point y_r. The COS type reducer can be expressed as:

$$Y_{cos}[y_l, y_r] = \int_{\theta^1} \ldots \int_{\theta^M} \int_{f^1} \ldots \int_{f^M} 1 / \frac{\sum_{j=1}^{M} f^j \theta^j}{\sum_{j=1}^{M} f^j} \tag{10}$$

where $f^j \in F^j = [\underline{f}^j(X), \overline{f}^j(X)]$ and θ^j is the centroid of j^{th} consequent set. In general, there are no closed form formula for computing y_l and y_r. However, Mendel and Karnik introduced two algorithms for calculating these two end points in [15] that is now known as the KM iterative algorithms. If we use singleton fuzzifier, product inference engine and COS type reducer, y_l and y_r can be illustrated as follows:

$$y_l = \frac{\sum_{j=1}^{M} f_l^j \theta_l^j}{\sum_{j=1}^{M} f_l^j} = \theta_l^T \xi_l \tag{11}$$

where θ_l^j is the left-end point of j^{th} consequent set, $\theta_l = [\theta_l^1, \ldots, \theta_l^M]^T$, $\xi_l^j = \frac{f_l^j}{\sum_{j=1}^{M} f_l^j}$ and $\xi_l = [\xi_l^1, \ldots, \xi_l^M]^T$ and:

$$y_r = \frac{\sum_{j=1}^{M} f_r^j \theta_r^j}{\sum_{j=1}^{M} f_r^j} = \theta_r^T \xi_r \tag{12}$$

where θ_r^j is the right-end point of j^{th} consequent set, $\theta_r = [\theta_r^1, \ldots, \theta_r^M]^T$, $\xi_r^j = \frac{f_r^j}{\sum_{j=1}^{M} f_r^j}$ and $\xi_r = [\xi_r^1, \ldots, \xi_r^M]^T$.

For calculating y_l, without loss of generality, assume θ_l^j are arranged in ascending order i.e. $\theta_l^1 \leq \theta_l^2 \leq \ldots \leq \theta_l^M$. After calculating θ_l and θ_r using the KM algorithms in [15], we have the flowchart in Figure 3 that shows the calculating of y_l.

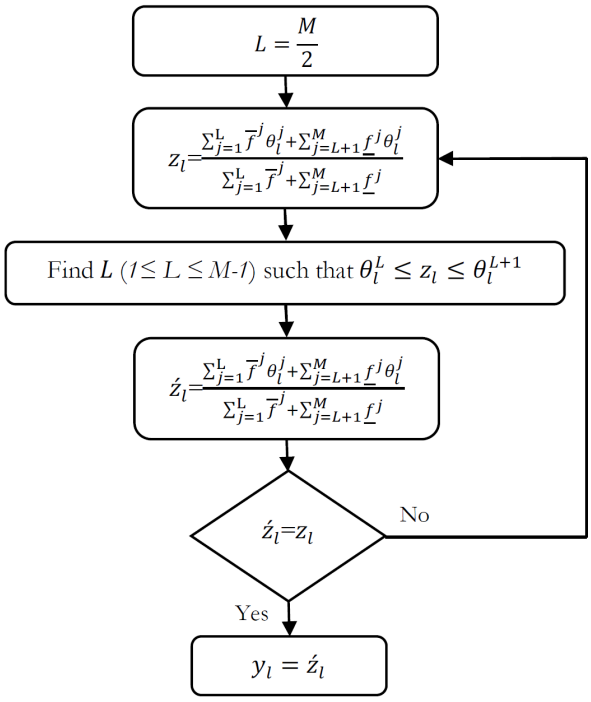


FIGURE 3. Flowchart of Calculating Left End Point

Computing y_r is similar to y_l; only z'_r is computed by $f_r^j = \underline{f}^j$ for j$\leq R$ and $f_r^j = \overline{f}^j$ for $R > j$. Now, we defuzzify the set obtained from type reducer by using the average of y_l and y_r [25], as below:

$$y = \frac{y_r + y_l}{2} = \frac{1}{2}(\theta_l^T \xi_l + \theta_r^T \xi_r) \tag{13}$$

3. Sliding Mode Control

Consider a general class of SISO nonlinear system as:

$$x^{(n)} = f(X,t) + g(X,t)\,u + d(t)$$
$$y = x \tag{14}$$

where f and g are unknown bounded nonlinear functions, where this bounds is also unknown; $d(t)$ is the bounded external disturbance with known bound D; $u \epsilon R$ and $y \epsilon R$ are input and output of the system, respectively; and $X = [x, \dot{x}, , x^{(n-1)}]$ is the state vector of the system which is assumed to be available for measurement. In other words we have following assumptions:

Assumption 1. We assume external disturbance to be bounded by a known constant D, i.e. :

$$|d(t)| \leq D \tag{15}$$

Assumption 2. System (14) is controllable, i.e. $g(X,t) \neq 0$. Without loss of generality, we assume $g(X,t) > 0$, i.e. can be negative and the control can be similarly derived.

The control objective is to determine a feedback control $u(X)$ such that the state X of the system follows the desired state vector $X_d = [x_d, \dot{x}_d, \ldots, x_d^{(n-1)}]$ in the presence of disturbances and uncertainties; therefore the tracking error is as follows:

$$E = X - X_d = [e, \dot{e}, \ldots, e^{(n-1)}]^T \tag{16}$$

Then for system (14) a sliding surface is defined in the error state space as:

$$s(E) = C^T E = e^{(n-1)} + c_{n-1}e^{(n-2)} + \ldots + c_1 e \tag{17}$$

where the coefficients $C = [c_1, \ldots, c_{(n-2)}, c_{(n-1)}, c_n = 1]^T \epsilon R^n$ are chosen such that the polynomial $h(\lambda) = \lambda^{(n-1)} + c_{(n-1)}\lambda^{(n-2)} + \ldots + c_1$($\lambda$ is a Laplace operator) is Hurwitz. If the initial condition $E(0) = 0$, the tracking problem can be considered such that state error vector $E(t)$ remains on the sliding surface $s(E) = 0$ for all $t \geq 0$. A sufficient condition for achieving this objective is given in [29] as:

$$\frac{1}{2}\frac{d}{dt}s^2(E) \leq -\eta|s|, \quad \eta > 0 \tag{18}$$

where η is a positive constant. After simplifying, sliding condition of (18) can be rewritten as:

$$\dot{s} \leq -\eta sgn(s) \tag{19}$$

where:

$$sgn(s) = \begin{cases} 1 & for \quad s > 0 \\ 0 & for \quad s = 0 \\ -1 & for \quad s < 0 \end{cases} \tag{20}$$

By taking the time derivative of (17), we obtain:

$$\dot{s} = \sum_{i=1}^{n-1} c_i e^{(i)} + x^{(n)} - x_d^{(n)} = \sum_{i=1}^{n-1} c_i e^{(i)} + f(X,t) + g(X,t)u + d(t) - x_d^{(n)} \tag{21}$$

Substituting (21) into (19), sliding condition can be re-expressed as:

$$\sum_{i=1}^{n-1} c_i e^{(i)} + f(X,t) + g(X,t)u + d(t) - x_d^{(n)} \leq -\eta sgn(s) \tag{22}$$

Now the control problem is to obtain the optimal control input such that the sliding condition (22) is guaranteed. If $f(X,t)$, $g(X,t)$ and $d(t)$ were known, we could design the optimal sliding mode control law as below:

$$u^* \leq \frac{1}{g(X,t)}\left[-\sum_{i=1}^{n-1} c_i e^{(i)} - f(X,t) - d(t) + x_d^{(n)} - \eta sgn(s)\right] \tag{23}$$

therefore the control input is:

$$u^* = \frac{1}{g(X,t)}\left[-\sum_{i=1}^{n-1} c_i e^{(i)} - f(X,t) - d(t) + x_d^{(n)} - \eta_\Delta sgn(s)\right] \tag{24}$$

where $\eta_\Delta \geq \eta > 0$.

4. Adaptive Sliding Mode Control Based on IT2FLS

In obtaining (24), we assumed $f(X,t)$, $g(X,t)$ and $d(t)$ to be well known. In reality, however, these are unknown. Hence, we replace $f(X,t)$ and $g(X,t)$ by their interval type-2 fuzzy approximation,$\hat{f}(X|\theta_f)$ and $\hat{g}(X|\theta_g)$ which are in the form (13). Since the sliding control law (24) is discontinuous across the sliding surface and can lead to chattering, we employ a PI control term to eliminate the chattering where PI controller is in the form of:

$$P = K_P s + K_I \int s\,dt = \theta_p^T \psi(\mathrm{Z}) \tag{25}$$

where, $\theta_p = [K_P, K_I]^T$is an adaptation parameter vector, and $\psi(\mathrm{Z}) = [\mathrm{z}_1, \mathrm{z}_2] = [\mathrm{s}, \int s\,dt]^{\mathrm{T}}$. By applying the PI term, near the sliding surface, we introduce an input control as:

$$\hat{u}_p(\mathrm{Z}|\theta_p) = \begin{cases} \hat{u}_1 = \theta_p^T \psi(Z) & if\ |s| < \varphi \\ \hat{u}_2 = (D + \eta_\Delta) sgn(s) & if\ |s| \geq \varphi \end{cases} \tag{26}$$

where φ is the thickness of the boundary layer. Selecting φ is a tradeoff between reducing chattering versus increasing the speed of convergence without affecting stability of the resulting closed loop system. Increasing the boundary layer will affect the closed loop stability and decreasing the thickness of the boundary layer will lead to chattering. If the state trajectory of the system is outside of the boundary layer, then sliding control $\hat{u}_2 = (D + \eta_\Delta) sgn(s)$ forces the trajectory toward the boundary layer; and if the state trajectory enters the boundary layer, the adaptive PI control $\hat{u}_1 = \theta_p^T \psi(Z)$ begins operating. The resulting control input is as follow:

$$u = \frac{1}{\hat{g}(X|\theta_g)} \left[-\sum_{i=1}^{n-1} c_i e^{(i)} - \hat{f}(X|\theta_f) + x_d^{(n)} - \hat{u}_p(Z|\theta_p) \right] \tag{27}$$

$$\hat{f}(X|\theta_f) = \frac{\hat{f}_l + \hat{f}_r}{2} = \frac{1}{2}\left(\theta_{fl}^T \xi_l + \theta_{fr}^T \xi_r\right) = \theta_f^T \xi_f \tag{28}$$

$$\hat{g}(X|\theta_g) = \frac{\hat{g}_l + \hat{g}_r}{2} = \frac{1}{2}\left(\theta_{gl}^T \xi_l + \theta_{gr}^T \xi_r\right) = \theta_g^T \xi_g \tag{29}$$

Theorem 4.1. *The nonlinear system in (14), with the control law u in (27) and according to the bounded disturbance in assumption(1) and controllability in assumption(2), will be stable and the tracking error will asymptotically converge to zero if the interval type-2 fuzzy adaptive laws are chosen as:*

$$\dot{\theta}_{fl} = \gamma_1 s \xi_{fl} \tag{30}$$

$$\dot{\theta}_{fr} = \gamma_2 s \xi_{fr} \tag{31}$$

$$\dot{\theta}_{gl} = \gamma_3 s \xi_{gl} u \tag{32}$$

$$\dot{\theta}_{gr} = \gamma_4 s \xi_{gr} u \tag{33}$$

$$\dot{\theta}_p = \gamma_5 s \psi \tag{34}$$

Proof. Define the optimal parameters of fuzzy systems as:

$$\theta_f^* = arg \quad min\ _{\theta_f \epsilon \Omega_f} [\sup_{X \epsilon R^n} |\hat{f}(X|\theta_f) - f(X,t)|] \tag{35}$$

$$\theta_g^* = arg \quad min \ _{\theta_g \epsilon \Omega_g} [\sup_{X \epsilon R^n} |\hat{g}\left(X|\theta_g\right) - g\left(X,t\right)|] \tag{36}$$

$$\theta_p^* = arg \quad min \ _{\theta_p \epsilon \Omega_p} [\sup_{Z \epsilon R^2} |\hat{u}_p(Z|\theta_p) - \hat{u}_2|] \tag{37}$$

where Ω_f, Ω_g and Ω_p are constant sets for θ_f, θ_g and θ_p respectively, and are defined as :

$$\Omega_f = \left\{\theta_f \epsilon R^n \middle| \quad |\theta_f| \leq M_f \right\} \tag{38}$$

$$\Omega_g = \{\theta_g \epsilon R^n | \ 0 < \epsilon < |\theta_g| \leq M_g \} \tag{39}$$

$$\Omega_p = \left\{\theta_p \epsilon R^2 \middle| \quad |\theta_p| \leq M_p \right\} \tag{40}$$

where M_f, M_g and M_p are positive constants. Now we can define the minimum approximation error as:

$$\omega = \left[f\left(X,t\right) - \hat{f}\left(X|\theta_f^*\right)\right] + \left[g\left(X,t\right) - \hat{g}\left(X|\theta_g^*\right)\right] u \tag{41}$$

Then from substituting (27) into (21), derivative of the sliding surface is:

$$\begin{aligned}
\dot{s} &= \sum_{i=1}^{n-1} c_i e^{(i)} + f\left(X,t\right) + g\left(X,t\right) u + d\left(t\right) - x_d^{(n)} \\
&= f\left(X,t\right) - \hat{f}\left(X|\theta_f\right) + \left(g\left(X,t\right) - \hat{g}\left(X|\theta_g\right)\right) u + d\left(t\right) - \hat{u}_p(Z|\theta_p) \\
&= \left[\hat{f}\left(X|\theta_f^*\right) - \hat{f}\left(X|\theta_f\right)\right] + [\hat{g}\left(X|\theta_g^*\right) - \hat{g}\left(X|\theta_g\right)]u + d\left(t\right) \\
&+ [\hat{u}_p\left(Z\theta_p^*\right) - \hat{u}_p\left(Z\theta_p\right)] + \omega - \hat{u}_p(Z|\theta_p^*) \\
&= \left(\theta_f^{*T}\xi_f - \theta_f^T\xi_f\right) + \left(\theta_g^{*T}\xi_g - \theta_g^T\xi_g\right) u + \left(\theta_p^{*T}\psi - \theta_p^T\psi\right) + d(t) - \hat{u}_p(Z|\theta_p^*) \ + \omega \\
&= \varnothing_f^T\xi_f + \varnothing_g^T\xi_f u + \varnothing_p^T\psi + \ d(t) \ - \hat{u}_p(Z|\theta_p^*) \ + \omega \\
&= \frac{1}{2}(\varnothing_{fl}^T\xi_{fl} + \varnothing_{fr}^T\xi_{fr}) + \frac{1}{2}(\varnothing_{gl}^T\xi_{gl} + \varnothing_{gr}^T\xi_{gr})u + \varnothing_p^T\psi + d(t) - \hat{u}_p(Z|\theta_p^*) + \omega
\end{aligned} \tag{42}$$

where $\varnothing_f = \theta_f^* - \theta_f$, $\varnothing_g = \theta_g^* - \theta_g$ and $\varnothing_p = \theta_p^* - \theta_p$.
Now the Lyapunov function is defined as:

$$V = \frac{1}{2}s^2 + \frac{1}{4\gamma_1}\varnothing_{fl}^T\varnothing_{fl} + \frac{1}{4\gamma_2}\varnothing_{fr}^T\varnothing_{fr} + \frac{1}{4\gamma_3}\varnothing_{gl}^T\varnothing_{gl} + \frac{1}{4\gamma_4}\varnothing_{gr}^T\varnothing_{gr} + \frac{1}{2\gamma_5}\varnothing_p^T\varnothing_p \tag{43}$$

where γ_1 ,γ_2, γ_3 ,γ_4 and γ_5 are positive constants. The time derivative of V is:

$$\begin{aligned}
\dot{V} &= s\dot{s} + \frac{1}{2\gamma_1}\varnothing_{fl}^T\dot{\varnothing}_{fl} + \frac{1}{2\gamma_2}\varnothing_{fr}^T\dot{\varnothing}_{fr} + \frac{1}{2\gamma_3}\varnothing_{gl}^T\dot{\varnothing}_{gl} + \frac{1}{2\gamma_4}\varnothing_{gr}^T\dot{\varnothing}_{gr} + \frac{1}{\gamma_5}\varnothing_p^T\dot{\varnothing}_p \\
&= s\left(\frac{1}{2}(\varnothing_{fl}^T\xi_{fl} + \varnothing_{fr}^T\xi_{fr}) + \frac{1}{2}(\varnothing_{gl}^T\xi_{gl} + \varnothing_{gr}^T\xi_{gr})u + \varnothing_p^T\psi + d(t) - \hat{u}_p(Z|\theta_p^*) + \omega\right) \\
&+ \frac{1}{2\gamma_1}\varnothing_{fl}^T\dot{\varnothing}_{fl} + \frac{1}{2\gamma_2}\varnothing_{fr}^T\dot{\varnothing}_{fr} + \frac{1}{2\gamma_3}\varnothing_{gl}^T\dot{\varnothing}_{gl} + \frac{1}{2\gamma_4}\varnothing_{gr}^T\dot{\varnothing}_{gr} + \frac{1}{\gamma_5}\varnothing_p^T\dot{\varnothing}_p \\
&= \frac{1}{2\gamma_1}\varnothing_{fl}^T\left(\dot{\varnothing}_{fl} + \gamma_1 s\xi_{fl}\right) + \frac{1}{2\gamma_2}\varnothing_{fr}^T\left(\dot{\varnothing}_{fr} + \gamma_2 s\xi_{fr}\right) + \frac{1}{2\gamma_3}\varnothing_{gl}^T\left(\dot{\varnothing}_{gl} + \gamma_3 s\xi_{gl}u\right) \\
&+ \ \frac{1}{2\gamma_4}\varnothing_{gr}^T\left(\dot{\varnothing}_{gr} + \gamma_4 s\xi_{gr}u\right) + \frac{1}{\gamma_5}\varnothing_p^T\left(\dot{\varnothing}_p + \gamma_5 s\psi\right) + sd\left(t\right) - s\hat{u}_p\left(Z\theta_p^*\right) + s\omega \\
&\leq \frac{1}{2\gamma_1}\varnothing_{fl}^T\left(\dot{\varnothing}_{fl} + \gamma_1 s\xi_{fl}\right) + \frac{1}{2\gamma_2}\varnothing_{fr}^T\left(\dot{\varnothing}_{fr} + \gamma_2 s\xi_{fr}\right) \\
&+ \frac{1}{2\gamma_3}\varnothing_{gl}^T\left(\dot{\varnothing}_{gl} + \gamma_3 s\xi_{gl}u\right) + \frac{1}{2\gamma_4}\varnothing_{gr}^T\left(\dot{\varnothing}_{gr} + \gamma_4 s\xi_{gr}u\right) \\
&+ \frac{1}{\gamma_5}\varnothing_p^T\left(\dot{\varnothing}_p + \gamma_5 s\psi\right) + sd\left(t\right) - s\left(\eta_\Delta + D\right) sgn\left(s\right) + s\omega
\end{aligned} \tag{44}$$

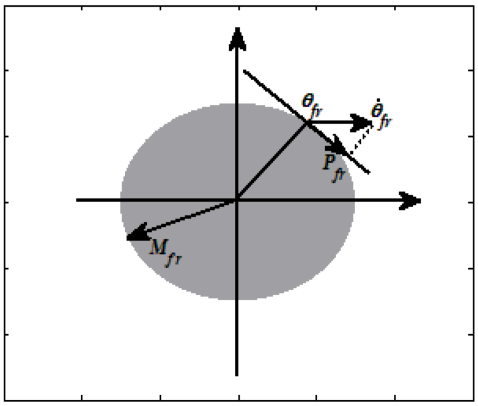


FIGURE 4. Projection Algorithm

where $\dot{\varnothing}_{fr} = -\dot{\theta}_{fr}$, $\dot{\varnothing}_{fl} = -\dot{\theta}_{fl}$, $\dot{\varnothing}_{gr} = -\dot{\theta}_{gr}$ *and* $\dot{\varnothing}_{gl} = -\dot{\theta}_{gl}$. Substituting (30)-(34) into (44) we have:

$$\begin{aligned}\dot{V} \leq\ & sd(t) - s(\eta_\Delta + D)\, sgn(s) + s\omega = sd(t) - |s|\,(\eta_\Delta + D) + s\omega \\ \leq\ & -|s|\,(\eta_\Delta) + s\omega\ . \end{aligned} \tag{45}$$

Because of the universal approximation functionality of fuzzy systems, the term $s\omega$ will be very small. Then, we have:

$$\dot{V} \leq\ -|s|\,(\eta_\Delta) \leq 0 \tag{46}$$

Since $C = [c_1, , c_{(n-2)}, c_{(n-1)}, c_n = 1]^T$ are chosen such that the polynomial $h(\lambda) = \sum_{i=1}^{n} c_i \lambda^{(i-1)}$ is a Hurwitz, we can anticipate that $\lim_{t\to\infty} |e(t)|$ =0; therefore $\lim_{t\to\infty} |s(e)| = 0$. Hence, the system is stable and the error asymptotically converges to zero. The proof is complete. □

While the parameters of the controller are within the desired and reasonable regions (38)-(40), the stability of the system is guaranteed by the Lyapunov approach. One of the ways to guarantee the parameters within the regions is the projection algorithm [34, 35]. If the parameter vector is on the boundary constraint and moving toward the outside, projection algorithm projects the gradient of the parameter vector onto the supporting hyper-plane; see Figure 4. Then the adaptive laws (30)-(34) can be modified as follows:

$$\dot{\theta}_{fl} = \begin{cases} \gamma_1 s\xi_{fl} & if\ \ (|\theta_{fl}| < M_{fl}) \quad or\ \ (|\theta_{fl}| = M_{fl}\ and\ s\theta_{fl}^T \xi_{fl} \leq 0) \\ Pr_{fl}\left(\gamma_1 s\xi_{fl}\right) & if\ \ (|\theta_{fl}| \geq M_{fl} \quad and \quad s\theta_{fl}^T \xi_{fl} > 0) \end{cases} \tag{47}$$

$$\dot{\theta}_{fr} = \begin{cases} \gamma_2 s\xi_{fr} & if\ \ (|\theta_{fr}| < M_{fr}) \quad or\ \ (|\theta_{fr}| = M_{fr}\ and\ s\theta_{fr}^T \xi_{fr} \leq 0) \\ Pr_{fr}\left(\gamma_2 s\xi_{fr}\right) & if\ \ (|\theta_{fr}| \geq M_{fr}\ \ and\ \ s\theta_{fr}^T \xi_{fr} > 0) \end{cases} \tag{48}$$

$$\dot{\theta}_{gl} = \begin{cases} \gamma_3 s\xi_{gl}\ u & if\ \ (|\theta_{gl}| < M_{gl}) \quad or\ \ (|\theta_{gl}| = M_{gl}\ and\ s\theta_{gl}^T \xi_{gl} u \leq 0) \\ Pr_{gl}\left(\gamma_3 s\xi_{gl}\ u\right) & if\ \ (|\theta_{gl}| \geq M_{gl} \quad and \quad s\theta_{gl}^T \xi_{gl} u > 0) \end{cases} \tag{49}$$

$$\dot{\theta}_{gr} = \begin{cases} \gamma_4 s\xi_{gr}\ u & if\ \ (|\theta_{gr}| < M_{gr}) \quad or\ \ (|\theta_{gr}| = M_{gr}\ and\ s\theta_{gr}^T \xi_{gr} u \leq 0) \\ Pr_{gr}\left(\gamma_4 s\xi_{gr}\ u\right) & if\ (|\theta_{gr}| \geq M_{gr}\ \ and\ \ s\theta_{gr}^T \xi_{gr} u > 0) \end{cases} \tag{50}$$

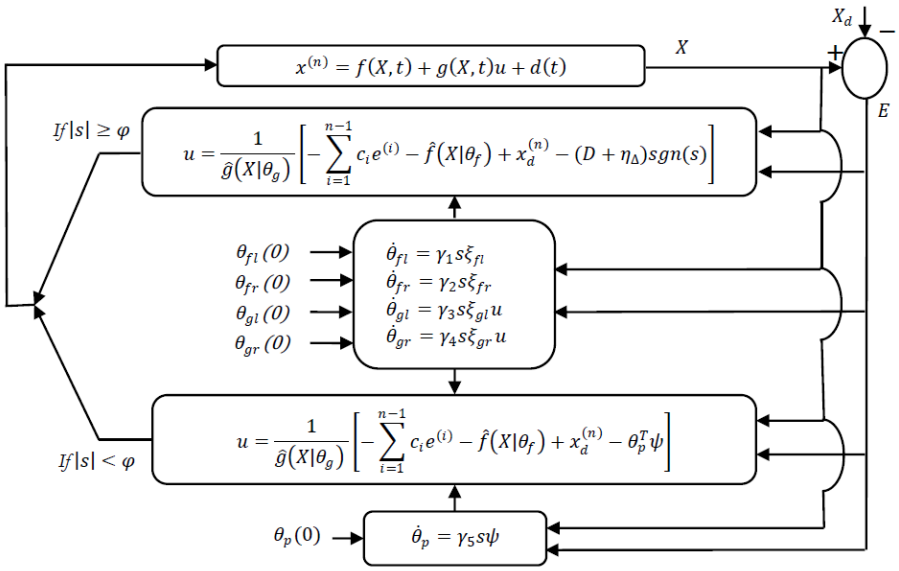


FIGURE 5. Overall Scheme of the Proposed Controller

$$\dot{\theta}_p = \begin{cases} \gamma_5 s\psi(Z) & if\ (|\theta_p| < M_p)\ \ or\ \ (|\theta_p| = M_p\ and\ \gamma_5 s\psi(\underline{z}) < 0) \\ Pr_p\ (\gamma_5 s\psi(Z)) & if\ (|\theta_p| \geq M_p\ \ and\ \ \gamma_5 s\psi(\underline{z}) > 0) \end{cases} \tag{51}$$

where M_{fl} , M_{fr} , M_{gl} , M_{gr} *and* M_p are positive constants and the projection operators Pr_{fl} , Pr_{fr} , Pr_{gl} , Pr_{gr} are defined as :

$$Pr_{fl}\ \left(\gamma_1 s\xi_{fl}\ \right) = \gamma_1 s\xi_{fl}\ - \gamma_1 s\frac{\theta_{fl}\theta_{fl}^T\xi_{fl}}{|\theta_{fl}|^2} \tag{52}$$

$$Pr_{fr}\ \left(\gamma_2 s\xi_{fr}\ \right) = \gamma_2 s\xi_{fr}\ - \gamma_2 s\frac{\theta_{fr}\theta_{fr}^T\xi_{fr}}{|\theta_{fr}|^2} \tag{53}$$

$$Pr_{gl}\ \left(\gamma_3 s\xi_{gl}\ u\right) = \gamma_3 s\xi_{gl}u - \gamma_3 s\frac{\theta_{gl}\theta_{gl}^T\xi_{gl}u}{|\theta_{gl}|^2} \tag{54}$$

$$Pr_{gr}\ \left(\gamma_4 s\xi_{gr}\ u\right) = \gamma_4 s\xi_{gr}u - \gamma_4 s\frac{\theta_{gr}\theta_{gr}^T\xi_{gr}u}{|\theta_{gr}|^2} \tag{55}$$

$$Pr_p\left(\gamma_5 s\psi\right) = \gamma_5 s\psi - \gamma_5 s\frac{\theta_p\theta_p^T\psi}{|\theta_p|^2} \tag{56}$$

Then, the overall scheme of the proposed method is shown in Figure 5.

5. **Simulation Examples**

In this section, we apply the proposed adaptive interval type-2 fuzzy PI sliding mode controller on two examples. First is a 1^{st} order nonlinear system that should reach a constant state. Second is a Duffing forced oscillator that should track a sine-wave trajectory.

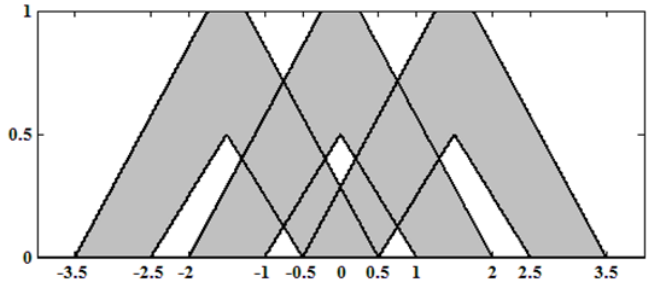


FIGURE 6. Triangular Like Membership Functions

Example 5.1. Consider a first order nonlinear system as follows:

$$\dot{x} = \frac{1 - e^{-x}}{1 + e^{x}} + u(t) + d(t) \quad , \qquad d(t) = \sin(t) + 3 \tag{57}$$

where $u(t)$ is the control input signal and $d(t)$ is a bounded disturbance with known bound D=5. Assume $K_P(0) = 40$, $K_I(0) = 15$, $\eta_\Delta = 0.1$, $\varphi = 0.05$ and the desired trajectory is $x_d = 0$. Three triangular MFs over interval [-3, 3] are chosen as in Figure 6. Let the learning rate $\gamma_1 = 7$, $\gamma_2 = 9$, $\gamma_5 = 20$,and set $M_{fl} = M_{fr} = M_p = 20$.
The results are simulated for the initial state $x(0) = 1$.The integral of the square of system error over a fixed interval of time (ISE) is used as a performance measure to show small but steady error accumulation. Figure 7 shows the system response and the integral squared error (ISE) for conventional PID controller, referenced adaptive fuzzy PI sliding mode control in [13] and the proposed controller where initial consequent parameters $\theta_{fl}(0)$ and $\theta_{fr}(0)$ are chosen uniformly over interval [-2.5, 0.5] and [-2, 1], respectively. The PID controller coefficients K_P, K_D and K_I are set to 50, 1, and 40, respectively. These simulations show that the proposed method based on IT2FSs have a better capability to cope with the uncertainty and disturbances. Specifically, a higher speed of convergence is shown for the proposed controller in comparison with the classical PID controller and the adaptive fuzzy PI sliding mode controller in [13]. Benefiting from human expert knowledge, initial consequent parameters $\theta_{fl}(0)$ and $\theta_{fr}(0)$ are then chosen [-2.5, 1, 3] and [-2, 1, 4], respectively. As Figure 7 illustrates, initializing by human expert knowledge further improves the transient performance.

Example 5.2. Consider the Duffing forced-oscillation system in the form of:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -0.1x_2 - x_1^3 + 12cost + u(t) + d(t) \end{aligned} \tag{58}$$

where $d(t)$ is an unknown disturbance with known bound D=6. The desired trajectory is $x_d = \sin(t)$. Set $K_P(0) = 40$, $K_I(0) = 80$, $\eta_\Delta = 0.1$, $\varphi = 0.1$ *and* $M_{fl} = M_{fr} = M_p = 500$. Let the learning rate $\gamma_1 = 75$, $\gamma_2 = 85$, $\gamma_5 = 15$. Choose the sliding surface as $s = \dot{e} + 2e$. The results are simulated for the initial states $x_1(0) = 1$, $x_2(0) = -1$. The membership functions for system states x_1 and x_2

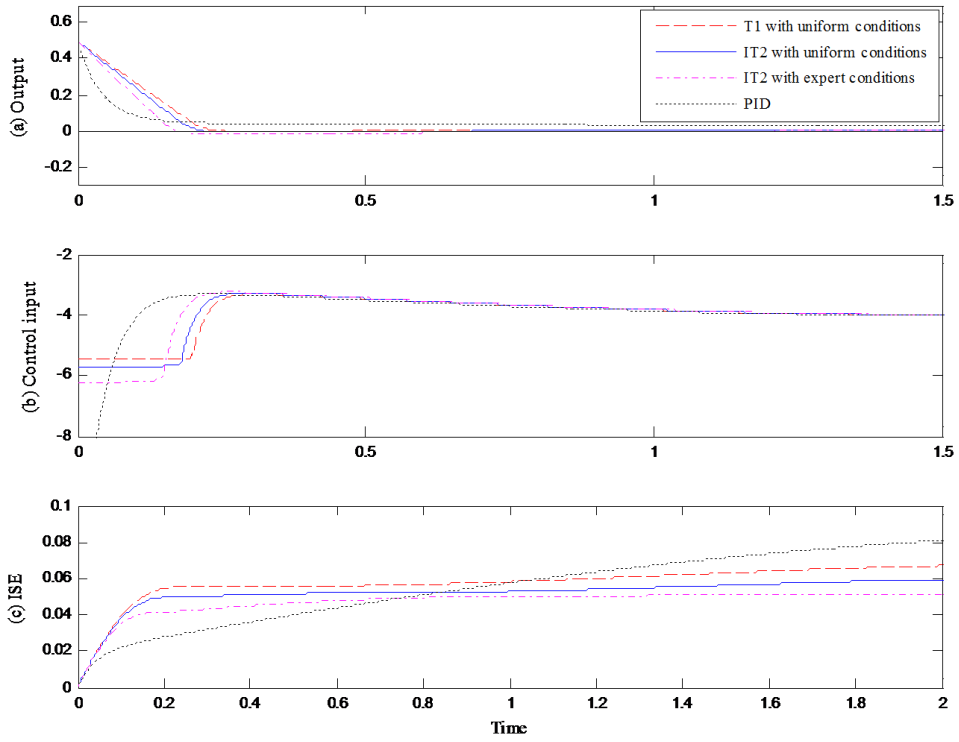


FIGURE 7. The Controller Based on T1FSs in Reference [13] (Dashed), the Proposed Controller Based on IT2FSs with Uniform (Solid) and with Human Expert (Dot-Dash) Initial Consequent Parameters, Conventional PID Controller (Dot): (a) System Output; (b) Control Input Signal; (c) Integral Squared Error (ISE)

are chosen as in Figure 6. Then there are 9 rules to approximate the system function. In order to study the robustness of the proposed control scheme against fast changes, a sinusoidal disturbance like $d(t) = a\cos(bt)$ is selected, where a and b are the gain and frequency of the disturbance respectively. The initial consequent parameters $\theta_{fl}(0)$ and $\theta_{fr}(0)$ are chosen uniformly over interval [-2, 0] and [0, 2], respectively; and then in each case, we use human expert knowledge to set initial consequent parameters in order to improve the performance. Three cases of slow, fast and very fast disturbances are considered; and in all the cases, robustness of the proposed controller is compared with the referenced adaptive fuzzy PI sliding mode controller based on T1FSs in [13] and the conventional PID controller on small and large magnitude disturbances. The PID controller coefficients K_p, K_D and K_I are set to 100, 2, and 40, respectively.

1) Slow Disturbance with $d(t)=a\cos(t)$. Figures 8 and 9, show system response, control input signal and ISE for the proposed controller based on IT2FSs,

controller	Cos (t)	5Cos(t)	Cos(10t)	5Cos(10t)	Cos(50t)	5Cos(50t)
PID	1.52	1.82	1.52	3.02	1.47	1.48
T1 with uniform conditions	1.04	1.46	0.97	1.18	0.97	1.16
IT2 with uniform conditions	0.90	1.04	0.84	0.94	0.87	0.89
IT2 with expert conditions	0.58	0.69	0.55	0.58	0.56	0.57

TABLE 1. Integral Squared Error (ISE) of the Controllers with Different Disturbance Magnitudes and Frequencies

the referenced adaptive fuzzy PI sliding mode control in [13] and conventional PID controller, under small, $a = 1$ and large, $a = 5$ magnitude of disturbances. From the simulation results it can be seen that the proposed method based on interval type-2 has a better response against the controller based on T1FSs in [13] and PID controller, and has capability to better cope with large magnitude of uncertainty. In addition, using human expert knowledge can improve the performance. Table 1 numerically confirms the effectiveness of the proposed controller in comparison with the PID and the adaptive type-1 fuzzy PI sliding mode controller in [13]. As shown here, boosting the magnitude of disturbance increases ISE by more than 0.4 and 0.3 for the T1 and PID controllers, respectively; while ISE increases by less than 0.15 for the proposed IT2 controller.

2) Fast Disturbance with $d(t)=a\ \cos(10t)$. Figures 10 and 11, show time response and ISE for the proposed controller based on IT2FSs, the referenced controller in [13] and PID controller with $d(t) = \cos(10t)$ and $d(t) = 5\cos(10t)$, respectively. It can be seen the PIAFSMC based on IT2FSs has a more smooth control input signal and handles fast and large disturbances better than the controller based on T1FSs and PID controller. In addition, using human expert knowledge in setting initial values for $\theta_{fl}(0)$ and $\theta_{fr}(0)$ further improves the ISE and performance as numerically confirmed in Table 1. In other words, boosting the magnitude of the disturbance increases the ISE for the IT2 controller by less than 0.1, whereas it is increased by more than 0.2 and 1.5 for the T1 and PID controller, respectively.

3) Very Fast Disturbance with $d(t)=a\ \cos(50t)$. Figures 12 and 13, show system response and ISE for the proposed controller, the referenced controller in [13] and the conventional PID controller with $d(t) = \cos(50t)$ and $d(t) = 5\cos(50t)$, respectively. From the simulation results, it can be seen that the proposed controller based on IT2FSs has a more smooth control input signal, better transient response and can handle fast and large disturbances better than the controller based on T1FSs and PID controller. Moreover, using human expert knowledge significantly improves the transient performance. The numerical data in Table 1 confirms that, when amplitude of the disturbance increases, the ISE for the proposed controller based on uniform and expert conditions also yield to smaller change in comparison with the other two competing controllers.

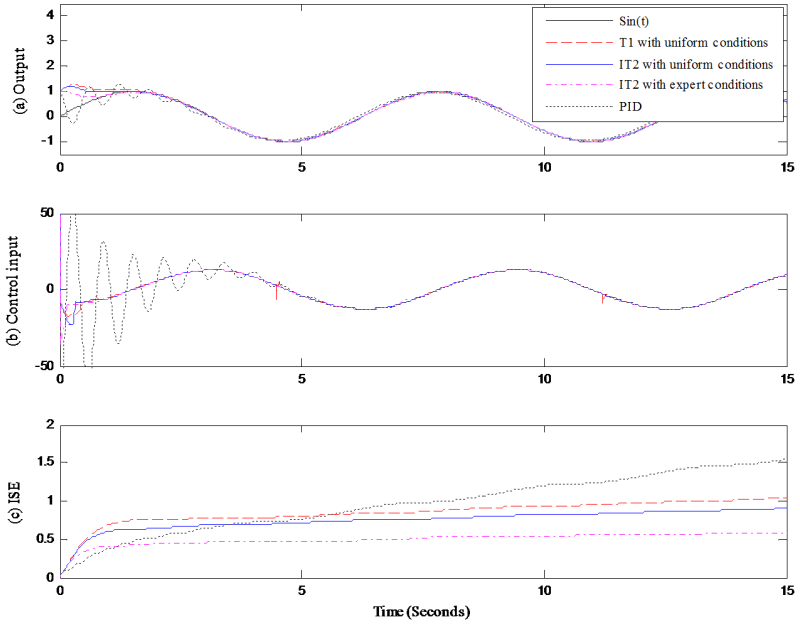


FIGURE 8. $d(t) = cos(t)$: (a) System Output; (b) Control Input, (c) ISE

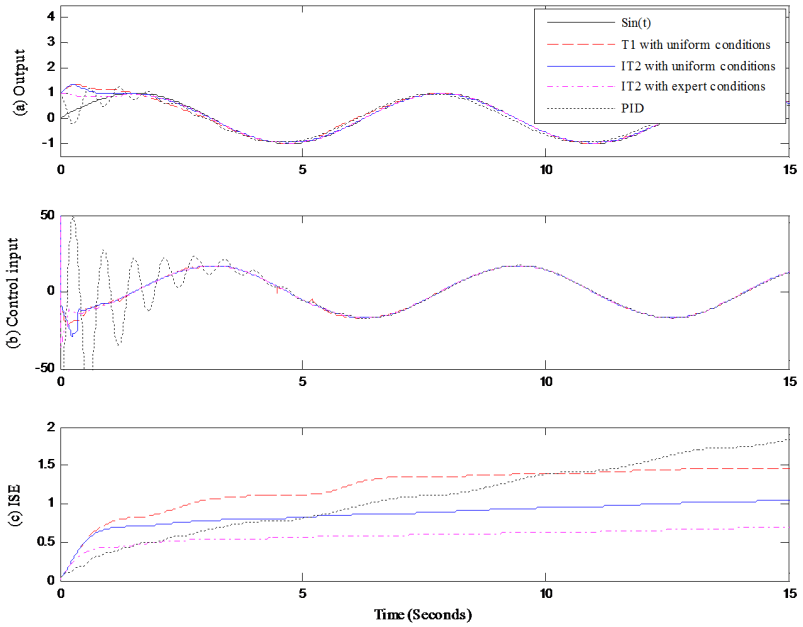


FIGURE 9. $d(t) = 5cos(t)$: (a) System Output; (b) Control Input, (c) ISE

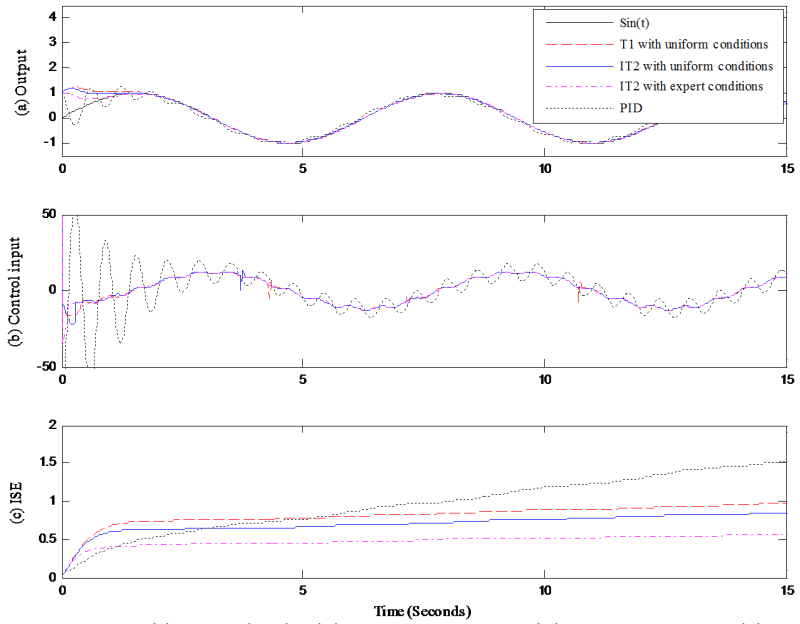


FIGURE 10. $d(t) = cos(10t)$: (a) System Output; (b) Control Input, (c) ISE

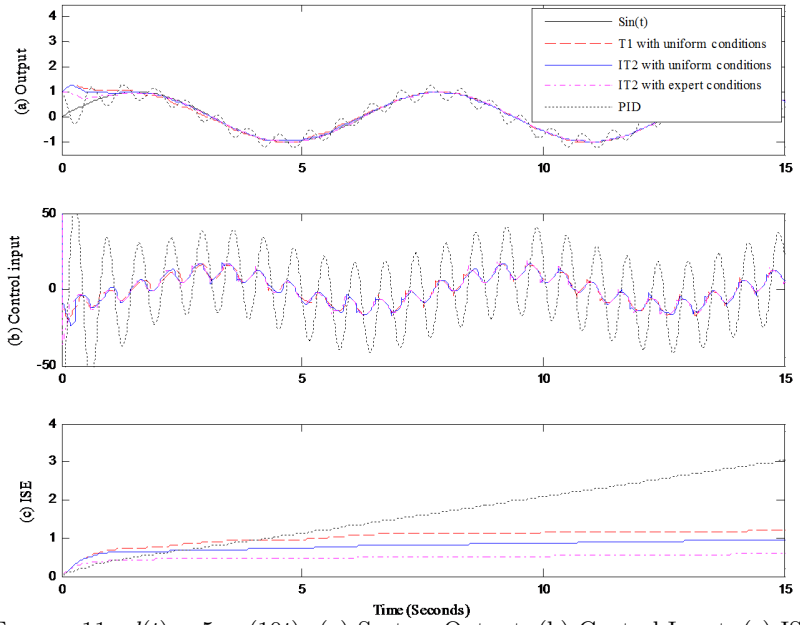


FIGURE 11. $d(t) = 5cos(10t)$: (a) System Output; (b) Control Input, (c) ISE

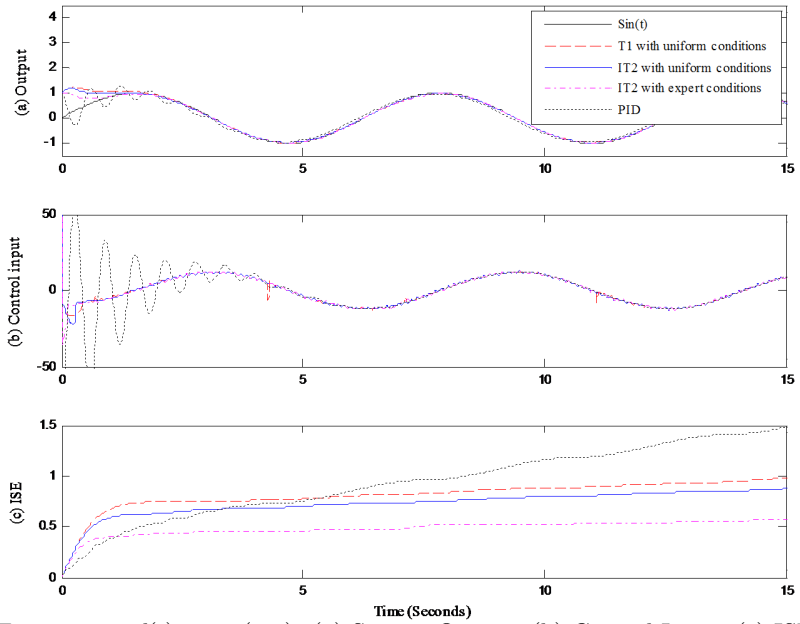


FIGURE 12. $d(t) = cos(50t)$: (a) System Output; (b) Control Input, (c) ISE

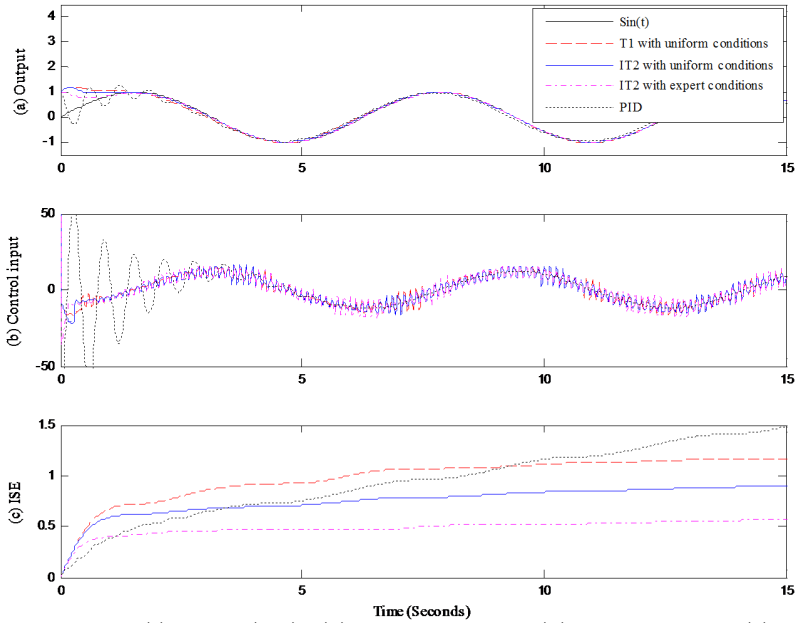


FIGURE 13. $d(t) = 5cos(50t)$: (a) System Output; (b) Control Input, (c) ISE

6. **Conclusions**

The problem of control in unstructured environments is an ever remaining challenge of control system technology. To date, various many paradigms have been

proposed such as adaptive control, nonlinear sliding mode control, as well as type-2 fuzzy mathematics. Each of these paradigms carries its own merits and yet has its own fallacies. For instance, the adaptive control provides a venue to handle parameter variations but only if it is slow enough, sliding mode control allows handling unknown system models that have a certain structure but introduces chattering, and type-2 mathematics attends to the issue of deterministic uncertainty but it is not clear how its parameters should be determined and can be computationally expensive. Here, we propose a hybrid paradigm that aims to take benefit from each of the above mentioned paradigms in a synergistic fashion. The proposed indirect adaptive interval type-2 fuzzy PI sliding mode controller uses type-2 fuzzy logic and adapts consequent parameters to approximate the unknown nonlinear system functions. The use of interval type-2 helps to minimize the added computational burden, and hence renders the overall system to be more practically applicable. To eliminate the chattering problem, a PI controller is designed where its parameters are also adjustable. Finally, since the backbone of the proposed approach is based on sliding mode control, Lyapunov analysis is applied to prove asymptotic stability of the proposed approach. Application to two benchmark control problems shows that the controller achieves good performance, especially when human expert knowledge is used to initialize consequent parameters. More specifically, the proposed controller has better and a more smooth response against a competing controller based on T1FSs and a conventional PID controller, especially when the disturbance has faster changes and larger magnitudes.

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