DESIGNING A MODEL OF INTUITIONISTIC FUZZY VIKOR IN MULTI-ATTRIBUTE GROUP DECISION-MAKING PROBLEMS

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Abstract. Multiple attributes group decision making (MAGDM) is regarded as the process of determining the best feasible solution by a group of experts or decision makers according to the attributes that represent different effects. In assessing the performance of each alternative with respect to each attribute and the relative importance of the selected attributes, quantitative/qualitative evaluations are often required to handle uncertainty, imprecise and inadequate information, which are well suited to represent with fuzzy values. This paper develops a VIKOR method based on intuitionistic fuzzy sets with multi-judges and multi-attributes in real-life situations. Intuitionistic fuzzy weighted averaging (IFWA) operator is used to aggregate individual judgments of experts to rate the importance of attributes and alternatives. Then, an intuitionistic ranking index is introduced to obtain a compromise solution to solve MAGDM problems. For application and validation, this paper presents two application examples and solves the practical portfolio selection and material handling selection problems to verify the proposed method. Finally, the intuitionistic fuzzy VIKOR method is compared with the existing intuitionistic fuzzy MAGDM method for two application examples, and their computational results are discussed.

1. Introduction

Multiple attributes group decision making (MAGDM) often involves a group of experts or decision makers (DMs) subjective judgments and preferences, including qualitative alternatives ratings and the weight of attributes. These ratings and weights are usually difficult to be judged precisely because of the existence of uncertainty and vagueness. However, it can be easily provided by linguistic variables, such as unimportant, medium, important and very important, which are fuzzy in nature [28]. In fact, under many conditions in real-life situations, crisp values are insufficient or inadequate to model MAGDM problems [16, 34]. In fact, human judgments including preferences are vague in nature, and it may not be appropriate to represent them by accurate numerical values. A more realistic approach to model human judgments can be using linguistic variables [29, 30, 32, 35, 36, 37, 46, 47, 48], that is, to suppose that the ratings of alternatives and weights of the attributes in the MAGDM problem are assessed by means of linguistic variables by a group of DMs.

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The use of linguistic information has implied the necessity of operating with linguistic variables for the decision making problems. Yager [54] introduced different computing schemes to operate with linguistic information. Computing with words (CWWs) is regarded as the methodology for the reasoning, computing and decision making whose information is expressed in natural language, i.e., the objects of computation are words or sentences in natural language [33, 55, 56, 57]. Hence, it has emulated human cognitive processes in order to enhance the solving processes of complex decision problems to handle uncertainty. Different symbolic linguistic computing models have been developed and applied as computational basis to the CWW in linguistic decision making [19, 20, 24, 25, 26].

Herrera and Herrera-Viedma [24] presented three steps for solving a multi-criteria decision making problem under linguistic information, including (1) the choice of the linguistic term set with its semantic in order to express the linguistic performance values versus all the criteria, (2) the choice of the aggregation operator of linguistic information in order to aggregate the linguistic performance values, and (3) the choice of the best options. Herrera and Martnez [26] applied the linguistic information, expressed by means of 2-tuples, which were composed of a linguistic term and a numeric value evaluated in $(-0.5, 0.5)$. Then, the model was extended for the CWWs without any loss of information. Also, several classical aggregation operators were developed to cope with the 2-tuple linguistic model. Xu [51] defined some operational laws of linguistic variables, and extended some new aggregation operators, including linguistic geometric averaging (LGA) operator, linguistic weighted geometric averaging (LWGA) operator, linguistic ordered weighted geometric averaging (LOWGA) operator and linguistic hybrid geometric averaging (LHGA) operator, for group decision making with linguistic preference relations. Wang and Hao [49] focused on a (proportional) 2-tuple fuzzy linguistic representation model for the CWWs, which was based on the concept of symbolic proportion to provide a space to allow the continuous interpolation of a sequence of ordered linguistic labels. Herrera et al. [25] proposed a fuzzy linguistic methodology in order to cope with unbalanced linguistic term sets. Alonso et al. [1] introduced a procedure to estimate missing preference values when dealing with incomplete fuzzy linguistic preference relations evaluated by a two-tuple fuzzy linguistic approach. Dong et al. [22] developed an optimization model to compute the numerical scale of the linguistic term set. Furthermore, they presented the desired properties of the optimization model. Li [30] utilized the concept of extended linguistic variables to develop a methodology for solving MAGDM problems under linguistic assessments.

To make a decision analysis under a fuzzy environment, classic VIKOR method (Vlse Kriterijumska Optimizacija I Kompromisno Resenje in Serbian, meaning multiple criteria optimization and compromise solution) proposed by Opricovic [38] has been widely applied by many researchers to deal with MAGDM problems [e.g., [13, 41]]. The VIKOR method is an effective and well-known multi-attributes decision analysis method capable of obtaining a compromise solution for complex systems. This method began with the form of Lp-metric, which was utilized as an aggregating function in a compromise programming method and developed into the multi-attributes measure for compromise ranking. In the VIKOR method, the positive ideal solution (PIS) and negative ideal solution (NIS) are constructed by which the relative closeness of each alternative is obtained through calculating the relative importance of the distance to both the PIS and NIS simultaneously. In MAGDM problems, attributes are often in conflict in the group of decision making process; therefore, finding a compromise solution is preferred rather than an optimal solution. For this purpose, the VIKOR introduces the multi-attributes ranking index based on the particular measure of closeness to the ideal solution. The main feather of this method is to determine a compromise solution that supports the maximum group utility for the majority and a minimum of individual regret for the opponent [39, 40]. Consequently, the alternatives in the group decision making process are evaluated according to their relative closeness to these fuzzy ideal solutions.

In recent years, some researchers focused on the VIKOR method to solve MAGDM problems. For instance, Tzeng et al. [45] selected a set of compromise solutions by the VIKOR method for a restaurant location in Taipei. Tong et al. [44] used the VIKOR method to optimize a multiple response process, which considers both the mean and the variation of quality losses regarding several multiple responses, and ensures a small variation in quality losses among the responses. Bykzkan and Ruan [13] solved the MADM problem using the fuzzy VIKOR method for assessing the software development projects. Chang and Hsu [14] applied the VIKOR method to determine the best feasible solution with respect to the selected criteria, including geographical and meteorological factors, and obtain the priority ranking of land-use restrictions strategies in southern Taiwan. Chen and Wang [15] proposed a fuzzy VIKOR method to provide a more efficient delivery approach for assessing possible suppliers/vendors in IS/IT outsourcing projects. Sanayei et al. [41] presented a group decision making process for supplier selection with the VIKOR under a fuzzy environment. In this study, linguistic values are used to evaluate the ratings and weights for the factors, which linguistic ratings are expressed in trapezoidal or triangular fuzzy numbers. There are necessary steps to apply the VIKOR method involving numerical measures of the relative importance of attributes and the rating of each alternative with respect to these attributes. However, determining exact data may be difficult to be precisely since human judgments are often vague under many conditions in real world applications. Hence, an extension of the VIKOR to a fuzzy environment is a natural generalization of VIKOR method. Sometimes available information is not suitable and sufficient for the crisp definition of a degree of membership for certain elements. There may be some hesitation degree between membership and non-membership [10]. In view that there are numerous decision making situations where due to insufficiency in information availability, intuitionistic fuzzy sets (IFSs) introduced by Atanassov [3] with ill-known membership grades are appropriate to deal with such problems. IFSs have been found to be particularly useful to deal with uncertainty. Recently, many studies have been investigated the IFS theory and applied it to various industrial fields, including decision making [10, 11, 43], logic programming [3], pattern recognition [27, 59] and robotic systems [23]. Atanassov [27] has recorded more than many researchers on the IFSs. Most of the researchers have discussed important issues related to the theory or applications of the IFSs.

Over the last decades, the IFS has been widely used in the decision making problems. Szmidt and Kacprzyk [43] presented some solution concepts including the intuitionistic fuzzy core and consensus winner in the group decision making process with intuitionistic fuzzy preference relations, and provided a method to aggregate the individual intuitionistic fuzzy preference relations into a social fuzzy preference relation on the basis of fuzzy majority equated with a fuzzy linguistic quantifier. Atanassov et al. [10] developed an intuitionistic fuzzy interpretation of multi-person multi-criteria decision making. Xu and Yager [53] proposed some geometric aggregation operators based on intuitionistic fuzzy sets, and applied them to multiple attributes decision making (MADM) problems. Lin et al. [31] constructed a new model by linear programming for handling fuzzy MADM problems based on IFSs. Xu [52] extended some new intuitionistic fuzzy aggregation operators including the intuitionistic fuzzy weighted averaging operator and intuitionistic fuzzy ordered weighted averaging operator for aggregating intuitionistic fuzzy information. Boran et al. [11] presented a multi-attributes intuitionistic fuzzy group decision making based on TOPSIS method for supplier selection in which DMs knowledge was vague and imprecise. In this paper, an extension of VIKOR method is presented for handling fuzzy MAGDM problems based on the IFS, where the characteristics of the alternatives and attributes are represented by the IFS. The proposed intuitionistic fuzzy VIKOR method utilizes the truth-membership function and non-truth-membership function to indicate the degrees of satisfiability and non-satisfiability of each alternative with respect to a set of attributes, respectively. Also, it allows each attribute to have the degrees of membership and non-membership to the fuzzy concept importance. In the extended version of the VIKOR, linguistic variables are applied to capture fuzziness in decision information and the group decision making process by means of an intuitionistic fuzzy decision matrix. In the group decision making process, aggregation of DMs judgments is very important to appropriately perform the assessment process. Hence, intuitionistic fuzzy weighted averaging (IFWA) operator is used to aggregate all individual DMs judgments for rating the alternatives with respect to the selected attributes and the relative importance of these attributes. Then, an intuitionistic ranking index is introduced, which is developed from the concept of the measure of closeness to the ideal solution. The compromise ranking method is based on the concept that the chosen alternative should have the distance from the ideal solution as short as possible and the distance from the negative deal solution as far as possible concurrently. The proposed intuitionistic fuzzy method can provide a useful way to efficiently help the DMs to make his/her decision in the group decision making process. Furthermore, a comparative analysis is demonstrated with two application examples in the portfolio selection and material handling selection problems, illustrating differences between proposed intuitionistic fuzzy VIKOR method and the existing intuitionistic fuzzy MAGDM method in the literature.

The rest of paper is organized as follows. A brief overview of intuitionistic fuzzy sets is given in Section 2. Section 3 presents the procedure of proposed intuitionsitic fuzzy VIKOR method for MAGDM problems. In Section 4, two application examples are demonstrated through the selection problems to highlight the applicability of proposed intuitionsitic group decision making method. A comparison is made between the proposed method and the existing intuitionistic fuzzy method, and the discussion of results is given. Finally, some concluding remarks are provided in Section 6.

2. **Intuitionistic Fuzzy Sets**

Let X be a universe of discourse. Concept of fuzzy set introduced by Zadeh [58]:

$$F = \{(x, \mu_F(x)|x \in X)\}, \tag{1}$$

whose basic component is only a membership degree $\mu_F(x)$ with the non-membership degree being $1 - \mu_F(x)$. However, in real-life situations when a person is asked to express his/her preference degree to an object, there usually exists an uncertainty or hesitation about the degree, and there is no means to incorporate the uncertainty or hesitation in a fuzzy set [21]. To solve this issue, Atanassov [3, 4, 5] generalized Zadehs fuzzy set to IFS by adding an uncertainty (or hesitation) degree. IFS is defined as follows. IFS A in a finite set X can be written as:

$$A = \{(x, \mu_A(x), v_A(x)|x \in X)\}, \tag{2}$$

where $\mu_A(x), v_A(x) : X \to [0, 1]$ are membership function and non-membership function, respectively, such that

$$0 \leq \mu_A(x) + v_A(x) \leq 1. \tag{3}$$

A third parameter of IFS is $positiveimplicative_A(x)$, known as the intuitionistic fuzzy index or hesitation degree of whether x belongs to A or not

$$positiveimplicative_A(x) = 1 - \mu_A(x) - v_A(x). \tag{4}$$

It is obviously seen that for every $x \in X$:

$$0 \leq positiveimplicative_A(x) \leq 1. \tag{5}$$

If the $positiveimplicative_A(x)$ is small, knowledge about x is more certain. If $positiveimplicative_A(x)$ is great, knowledge about x is more uncertain. Obviously, when $\mu_A(x) = 1 - v_A(x)$ for all elements of the universe, the ordinary fuzzy set concept is recovered [3, 4, 5, 6]. Let A and B denote two IFSs of the universe of discourse X, where $A = \{(x, \mu_A(x), v_A(x)|x \in X)\}$, $B = \{(x, \mu_B(x), v_B(x)|x \in X)\}$. The following expressions are defined.

Definition 2.1. $A \leq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \geq v_B(x)$ for all $x \in X$ [12].

Definition 2.2. $A \preceq B$ if and only if $\mu_A(x) \leq \mu_B(x)$ and $v_A(x) \leq v_B(x)$ for all $x \in X$. In addition, $A \geq B$ if and only if $B \leq A$; $A \succeq B$ if and only if $B \preceq A$ [12].

Definition 2.3. Let A and B be two IFSs. Addition operation of them can be defined as [5, 9]:

$$A + B = \left\{ (x, \mu_A(x) + \mu_B(x) - \mu_A(x)\mu_B(x), v_A(x)v_B(x)|x \in X) \right\}. \tag{6}$$

Definition 2.4. Let A and B be two IFSs. Multiplication operation of them can be defined as [5, 9].

$$A \cdot B = \left\{ (x, \mu_A(x) \cdot \mu_B(x), v_A(x) + v_B(x) - v_A(x) \cdot v_B(x)|x\imath X) \right\}. \tag{7}$$

Definition 2.5. Let A and B be two IFSs. Subtraction operation of them can be defined as [17].

$$A - B = \left\{ \left(x, \frac{\mu_A(x) - \mu_B(x)}{1 - \mu_B(x)}, \frac{v_A(x)}{v_B(x)} \right) |x \in X \right\}. \tag{8}$$

Condition: $A \geq B$, $\mu_B(x) \neq 1$, $v_B(x) \neq 0$ and $\mu_A(x)v_B(x) - \mu_B(x)v_A(x) \leq v_B(x) - v_A(x)$.

Definition 2.6. Let A and B be two IFSs. Division operation of them can be defined as [17]:

$$\frac{A}{B} = \left\{ (x, \left(\frac{\mu_A(x)}{\mu_B(x)}, \frac{v_A(x) - v_B(x)}{1 - v_B(x)} \right) |x \in X \right\}. \tag{9}$$

Condition: $A \leq B$, $\mu_B(x) \neq 0$, $v_B(x) \neq 1$ and $\mu_A(x)v_B(x) - \mu_B(x)v_A(x) \geq \mu_A(x) - \mu_B(x)$.

Definition 2.7. The IFS nA for any positive integer n as follows [18]:

$$nA = \left\{ (x, \mu_n A(x), v_n A(x))|x \in X \right\}, \tag{10}$$

where $\mu_{nA}(x) = 1 - (1 - \mu_A(x))^n, v_n A(x) = [v_A(x)]^n$.

3. Proposed Intuitionistic Fuzzy VIKOR Method

Let A be a set of alternatives and let C be a set of attributes, where

$$A = \{A_1, A_2, \ldots, A_m\}, \quad C = \{C_1, C_2, \ldots, C_n\}. \tag{11}$$

Assume that the characteristics of the alternative A_i are presented by the IFS shown as follows:

$$A_i = \{(C_1, \mu_{i1}, v_{i1}), (C_2, \mu_{i2}, v_{i2}), \cdots, (C_n, \mu_{in}, v_{in})\}, \quad i = 1, 2, \ldots, m, \tag{12}$$

where μ_{ij} indicates the degree to which the alternative A_i satisfies attribute C_j, v_{ij} indicates the degree to which the alternative A_i does not satisfy attribute $C_j(\mu_{ij}, v_{ij})$, $i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$. The flowchart of proposed fuzzy group decision making method is illustrated in Figure 1, and the procedure for the proposed intuitionistic fuzzy VIKOR method is given as follow.

Step 1: Determine the weights of DMs.

Assume that decision group contains l DMs. The importance of the DMs is regarded as linguistic terms expressed in intuitionistic fuzzy values.

Let $D_k = [\mu_k, v_k, positiveimplicative_k]$ be an intuitionistic fuzzy value for rating of kth DM. Then the weight of kth DM can be computed by definitions (3) and (6) as follows [11]:

$$\lambda_k = \frac{\left(\mu_k + positiveimplicative_k(\frac{\mu_k}{\mu_k + v_k})\right)}{\sum_{k=1}^{l}\left(\mu_k + positiveimplicative_k(\frac{\mu_k}{\mu_k + v_k})\right)}, \tag{13}$$

and $\sum_{k=1}^{l} \lambda_k = 1$.

Step 2: Construct aggregated intuitionistic fuzzy decision matrix based on the opinions of DMs. Let $R^{(k)} = (r_{ij}^{(k)})_{m\times n}$ is an intuitionistic fuzzy decision matrix of each DM. $\lambda = \{\lambda_1, \lambda_2, \dots, \lambda_l\}$ is the weight of each DM and $\sum_{k=1}^{l} \lambda_k = 1$, $\lambda_k \in [0, 1]$. In the group decision making process, all the individual decision opinions need to be fused into a group opinion to construct aggregated intuitionistic fuzzy decision matrix. In order to do that, IFWA operator proposed by Xu [52] can be used as follows, by applying definitions (3) and (7). $R = (r_{ij})_{m\times n}$, where

$$\begin{aligned} r_{ij} &= IFWA_\lambda(r_{ij}^{(1)}, r_{ij}^{(2)}, \dots, r_{ij}^{(l)}) = \lambda_1 r_{ij}^{(1)} \oplus \lambda_2 r_{ij}^{(2)} \oplus \dots \oplus \lambda_l r_{ij}^{(l)} \\ &= \left[1 - \prod_{k=1}^{l}(1-\mu_{ij}^{(k)})^{(\lambda_k)}, \prod_{k=1}^{l}(v_{ij}^{(k)})^{(\lambda_k)}, \prod_{k=1}^{l}(1-\mu_{ij}^{(k)})^{\lambda_k} - \prod_{k=1}^{l}(v_{ij}^{(k)})^{(\lambda_k)}\right]. \end{aligned} \tag{14}$$

Here, $r_{ij} = \left(\mu_{A_i}(x_j), v_{A_i}(x_j), positiveimplicative_{A_i}(x_j)\right)$, $i = 1, 2, \dots, m$; $j = 1, 2, \dots, n$.

The aggregated intuitionistic fuzzy decision matrix can be defined as follows:

$$R = \begin{bmatrix} ((\mu_{A_1}(x_1), v_{A_1}(x_1), positiveimplicative_{A_1}(x_1)), (\mu_{A_1}(x_n), v_{A_1}(x_n), positiveimplicative_{A_1}(x_n)) \\ ((\mu_{A_2}(x_1), v_{A_2}(x_1), positiveimplicative_{A_1}(x_1)), (\mu_{A_2}(x_n), v_{A_1}(x_n), positiveimplicative_{A_2}(x_n)) \\ \\ ((\mu_{A_m}(x_1), v_{A_m}(x_1), positiveimplicative_{A_m}(x_1)), (\mu_{A_m}(x_n), v_{A_m}(x_n), positiveimplicative_{A_m}(x_n)) \end{bmatrix}$$

Step 3: Determine the weights of attributes.

All attributes may not be assumed to be equal importance.W represents a set of grades of importance. In order to obtain W, all the individual DM opinions for the importance of each attribute need to be fused.

Let $w_j^{(k)} = [\mu_j^{(k)}, v_j^{(k)}, positiveimplicative_j^{(k)}]$ be an intuitionistic fuzzy value assigned to attribute C_j by the kth DM. Then the weights of the attributes can be calculated by using IFWA operator and, applying definitions (3) and (7) as follows:

$$\begin{aligned} w_j &= IFWA_\lambda(w_j^{(1)}, w_j^{(2)}, \dots, w_j^{(l)}) = \lambda_1 w_j^{(1)} \oplus \lambda_2 w_j^{(2)} \oplus \dots \oplus \lambda_l w_j^{(l)} \\ &= \left[1 - \prod_{k=1}^{l}(1-\mu_j^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(v_j^{(k)})^{\lambda_k}, \prod_{k=1}^{l}(1-\mu_j^{(k)})^{\lambda_k} - \prod_{k=1}^{l}(v_j^{(k)})^{(\lambda_k)}\right], \\ W &= [w_1, w_2, , w_j]. \end{aligned} \tag{15}$$

Here, $w_j = [\mu_j, v_j, positiveimplicative_j]$, $j = 1, 2, \dots, n$.

Step 4: Determine intuitionistic fuzzy PIS (r_j^+) and intuitionistic fuzzy NIS (r_j^-) for all attributes.

Let J_1 and J_2 be benefit attribute and cost attribute, respectively. Then r_j^+ and r_j^- are obtained as follows:

$$r_{j^+} = \Big(\mu_{r_j^+}(x_j), v_{r_j^+}(x_j)\Big), \quad r_j^- = \Big(\mu_{r_j^-}(x_j), v_{r_j^-}(x_j)\Big). \tag{16}$$

Where

$$\begin{aligned} \mu_{r_j^+}(x_j) &= \Big((\max_i \mu_{r_j}(x_j)|j \in J_1), (\min_i \mu_{r_j}(x_j)|j \in J_2)\Big), \\ v_{r_j^+}(x_j) &= \Big((\min_i v_{r_j}(x_j)|j \in J_1), (\max_i v_{r_j}(x_j)|j \in J_2)\Big) \\ \mu_{r_j^+}(x_j) &= \Big((\min_i \mu_{r_j}(x_j)|j \in J_1), (\max_i \mu_{r_j}(x_j)|j \in J_2)\Big) \\ v_{r_j^-}(x_j) &= \Big((\max_i v_{r_j}(x_j)|j \in J_1), (\min_i v_{r_j}(x_j)|j \in J_2)\Big). \end{aligned} \tag{17}$$

Step 5: Compute the intuitionistic fuzzy average group score value $S_i = [\mu_i, v_i, \mathit{positiveimplicative}_i]$ and intuitionistic fuzzy worst group score value $R_j = [\mu_i, v_i, \mathit{positiveimplicative}_i]$ for alternative $A_i (i = 1, 2, \ldots, m)$, respectively, with respect to definitions (3) to (7) as follows:

$$\begin{aligned} S_i &= \textstyle\sum_{j=1}^n w_j \frac{(r_{j^+} - r_{ij})}{(r_{j^+} - r_{j^-})}, \qquad S_i \in [0,1] \\ R_i &= \max_j [w_j \frac{(r_{j^+} - r_{ij})}{(r_{j^+} - r_{j^-})}], \quad R_i \in [0,1] \end{aligned} \tag{18}$$

Here, w_j are the relative importance weights of the attributes set by the DM. The smaller values of S_i and R_i correspond to the better average and the worse group scores for the alternative A_i, respectively.

Step 6: Calculate the $Q_i = [\mu_i, v_i, \mathit{positiveimplicative}_i]$ values for $i = 1, 2, \ldots, m$ with respect to definitions (3), (5), (6) and (7) as follows:

$$Q_i = \frac{\varphi(S_i - S^+)}{(S^- - S^+)} + \frac{(1-\varphi)(R_i - R^+)}{(R^- - R^+)}, \tag{19}$$

where $S^+ = \Big((\min_i \mu_{S_i}(x)), (\max_i v_{S_i}(x)\Big), S^- = \Big((\max_i \mu_{S_i}(x)), \min_i v_{S_i}(x))\Big)$, where $R^+ = \Big((\min_i \mu_{R_i}(x)), (\max_i v_{R_i}(x)\Big), R^- = \Big((\max_i \mu_{R_i}(x)), \min_i v_{R_i}(x))\Big)$. and φ is the weight of the decision making strategy (the majority of attributes or the maximum group utility). The compromise can be selected with voting by majority ($\varphi > 0.5$), with consensus ($\varphi = 0.5$), or with veto ($\varphi > 0.5$).

Step 7: Rank the alternatives by sorting each S, R and Q values in an increasing order with respect to the definitions (1) and (2). The result is a set of three ranking lists denoted as $S_{[\cdot]}$, $R_{[\cdot]}$ and $Q_{[\cdot]}$.

Step 8: Propose the alternative A_1 corresponding to $Q_{[1]}$. (The smallest among Q_i values) as a compromise solution if

C1 : The alternative A_1 has an acceptable advantage; in other words, $Q_{[2]} - Q_{[1]} \geq DQ$ where $DQ = 1/(m-1)$, and m is the number of alternatives.

C2 : The alternative A_1 is stable within the decision making process; in other words, it is also the best ranked in $S_{[\cdot]}$ or $R_{[\cdot]}$.

If one of the above conditions is not satisfied, then a set of compromise solutions is proposed, which consists of:

- Alternatives A_1 and A_2 if only the condition C2 is not satisfied, or
- Alternatives A_1 and $A_2, \ldots, A_m$ if the condition C1 is not satisfied; A_m is determined by the relation $Q(A_m) - Q(A_1) < DQ$ for maximum m (the positions of these alternatives are in closeness).

FIGURE 1. Flowchart of Proposed Intuitionistic Fuzzy VIKOR Method for MAGDM Problems

4. Application of the Proposed Method in Solving Problems

4.1. Illustrative Example 1.

4.1.1. **Implementation.** In this section, the implementation of the proposed intuitionistic fuzzy VIKOR method is explained for the portfolio selection in a stock exchange (SE). A comprehensive analysis has been performed on a pharmaceutical sector in the SE, and five companies (i.e., alternatives) have been regarded for the process of asset allocation. A committee of three DMs (DM_1, DM_2 and DM_3) has been formed to conduct the interview and to select the most suitable alternative. Six attributes are considered, including market share (C_1), sales to assets ratio (C_2), mean profit (C_3), liquidity (C_4), $P/E(C_5)$ and assets (C_6). Linguistic variables utilize for the ratings of attributes, and the DMs express in Table 1. Also, the importance degree of the DMs and the weight of attributes are provided in the group decision making process in Tables 2 and 3, respectively. Then three DMs express the linguistic variables illustrated in Table 4 to evaluate the importance of the six attributes, given in Table 5.

Linguistic variables	Intuitionistic fuzzy values
Very important (VI)	$[0.90, 0.10]$
Important (I)	$[0.75, 0.20]$
Medium (M)	$[0.50, 0.45]$
Unimportant (UI)	$[0.35, 0.60]$
Very unimportant (VUI)	$[0.10, 0.90]$

Table 1. Linguistic Variables for Rating the Importance of Attributes and Decision Makers

	DM_1	DM_2	DM_3
Linguistic variables	Important	Medium	Very important
Weight	0.356	0.238	0.406

Table 2. Importance of Decision Makers for Portfolio Selection Problem

	C_1	C_2	C_3	C_4	C_5	C_6
DM_1	M	VUI	VI	M	UI	VUI
DM_2	M	VUI	VI	I	UI	VUI
DM_3	UI	UI	I	M	VUI	VUI

Table 3. Weights of the Attributes for Portfolio Selection Problem

After rating each alternative with respect to each attribute by three DMs, the aggregated intuitionistic fuzzy decision matrix and the weight of attributes are obtained based on their judgments in Tables 6 and 7. It is worth to note that in the related literature, in order to propose or utilize an intuitionistic fuzzy aggregation operator, two functions including score function and accuracy function should be taken into consideration. However, in some special situations, i.e., if the score values of two intuitionistic fuzzy values are equal, then it is impossible to know which one is better. In such cases, the score function is not suitable for the comparison of two intuitionistic fuzzy values. Moreover, since this situation can occur for some decision making problems in real-life applications, hence, some other measures are needed. One of these new methods is intuitionistic fuzzy weighted averaging

Linguistic variables	Intuitionistic fuzzy values
Extremely good (EG)/extremely high (EH)	[1.00, 0.00]
Very very good (VVG)/very very high (VVH)	[0.90, 0.10]
Very good (VG)/very high (VH)	[0.80, 0.10]
Good (G)/high (H)	[0.70, 0.20]
Medium good (MG)/medium high (MH)	[0.60, 0.30]
Fair (F)/medium (M)	[0.50, 0.40]
Medium bad (MB)/medium low (ML)	[0.40, 0.50]
Bad (B)/low (L)	[0.25, 0.60]
Very bad (VB)/very low (VL)	[0.10, 0.75]
Very very bad (VVB)/very very low (VVL)	[0.10, 0.90]

Table 4. Linguistic Variables for the Rating of Alternatives

Attributes	Alternatives	DM_1	DM_2	DM_3
	A_1	H	H	MH
	A_2	VG	G	VG
C_1	A_3	VG	VG	VG
	A_4	VH	VH	H
	A_5	F	F	MG
	A_1	MG	MG	G
	A_2	MB	MB	MB
C_2	A_3	VVG	VG	VG
	A_4	VVG	VG	VG
	A_5	MB	F	F
	A_1	G	G	VG
	A_2	VG	G	VG
C_3	A_3	VG	G	G
	A_4	VG	G	G
	A_5	G	MG	MG
	A_1	H	H	H
	A_2	MB	F	MB
C_4	A_3	VH	H	H
	A_4	H	MH	MH
	A_5	M	MH	M
	A_1	MG	MG	MG
	A_2	MH	MH	M
C_5	A_3	VG	G	VG
	A_4	G	G	F
	A_5	MB	F	MB
	A_1	MG	G	MG
	A_2	MH	M	MH
C_6	A_3	MB	MB	MB
	A_4	VVG	VG	VG
	A_5	MH	MH	M

Table 5. Ratings of the Alternatives for Portfolio Selection Problem

(IFWA) operator proposed by Xu [52]. Further, in recent years the IFWA operator has been widely applied to the complex decision making problems in the related literature [e.g., [11, 50, 60]].

	C_1	C_2	C_3
A_1	(0.663,0.236,0.101)	(0.644,0.254,0.101)	(0.746,0.151,0.104)
A_2	(0.780,0.118,0.102)	(0.4, 0.5, 0.1)	(0.78,0.118,0.102)
A_3	(0.8, 0.1, 0.1)	(0.844,0.1,0.056)	(0.74,0.156,0.103)
A_4	(0.764,0.133,0.103)	(0.844,0.1,0.056)	(0.74,0.156,0.103)
A_5	(0.543,0.356,0.101)	(0.466,0.433,0.1)	(0.639,0.26,0.101)
Weights	(0.252, 0.748, 0)	(0.535,0.41,0.055)	(0.576,0.371,0.053)

Table 6. Aggregated Intuitionistic Fuzzy Decision Matrix and Weight of Attributes for Portfolio Selection Problem

	C_4	C_5	C_6
A_1	(0.7, 0.2, 0.1)	(0.6, 0.3, 0.1)	(0.626,0.272,0.101)
A_2	(0.425,0.474,0.1)	(0.562,0.337,0.101)	(0.578,0.321,0.101)
A_3	(0.74,0.156,0.103)	(0.78,0.118,0.102)	(0.4, 0.5, 0.1)
A_4	(0.639,0.26,0.101)	(0.631,0.265,0.104)	(0.844,0.1,0.056)
A_5	(0.526,0.374,0.101)	(0.425,0.474,0.1)	(0.562,0.337,0.101)
Weights	(0.855,0.133,0.013)	(0.211,0.763,0.025)	(0.444,0.506,0.05)

Table 7. Aggregated Intuitionistic Fuzzy Decision Matrix and Weight of Attributes for Portfolio Selection Problem

S_j, R_j and Q_j values are calculated by selecting $\varphi = 0.5$ (consensus) and computational results are shown in Table 7. Given these results, we observe that the condition C_2 is satisfied whereas C_1 not. Thus the alternative 2 (A_2) has not an acceptable advantage ($Q_{[2]}Q_{[1]} = 0.166 \leq DQ = 0.25$). On the other hand, we observe that the alternative 2 is stable within the decision making process. Also, it is the best ranked in S_j and R_j. Finally, the ranking of five alternatives with respect to six criteria is as follows: $\mathcal{A}_3 \succ \mathcal{A}_4 \succ \mathcal{A}_1 \succ \mathcal{A}_2 \succ \mathcal{A}_5$.

Alternatives	S_j	R_j	Q_j
A_1	(0.830,0.109,0.062)	(0.390,0.523,0.087)	(0.462,0.429,0.109)
A_2	(0.861,0.097,0.041)	(0.576,0.371,0.053)	(0.628,0.316,0.056)
A_3	(0.501,0.457,0.043)	(0.333,0.610,0.057)	(0, 1, 0)
A_4	(0.731,0.179,0.090)	(0.350,0.564,0.086)	(0.286,0.587,0.127)
A_5	(0.988,0.008,0.005)	(0.855,0.133,0.013)	(1, 0, 0)

Table 8. Values S_j, R_j and Q_j by the Proposed Intuitionistic Fuzzy VIKOR for Portfolio Selection Problem

4.1.2. **Discussion**. The value of the weight has a central role in the ranking of alternatives in the group decision making process. A sensitivity analysis can be conducted by setting systematically φ to some values between 0 and 1 and by tracking the changes in the ranking. The results of such an analysis based on values are given in Table 8. It is observed that the preference order ranking of alternatives is not changed, and the results of proposed intuitionistic fuzzy VIKOR method are stable through the group decision making process.

Q_j with different values of V							Preference order ranking
φ		Q_j	A_1	A_2	A_3	A_4	A_5
	A_1	(0.195,0.719,0.086)					
	A_2	(0.503,0.456,0.041)					
$\varphi = 0.1$	A_3	(0, 1, 0)	3	4	2	1	5
	A_4	(0.09,0.829,0.081)					
	A_5	(1, 0, 0)					
	A_1	(0.342,0.556,0.102)					
	A_2	(0.57,0.379,0.051)					
$\varphi = 0.3$	A_3	(1, 0, 0)	3	4	2	1	5
	A_4	(0.194,0.698,0.108)					
	A_5	(0, 1, 0)					
	A_1	(0.462,0.429,0.109)					
	A_2	(0.628,0.316,0.056)					
$\varphi = 0.5$	A_3	(0, 1, 0)	3	4	2	1	5
	A_4	(0.286,0.587,0.127)					
	A_5	(1, 0, 0)					
	A_1	(0.561,0.332,0.107)					
	A_2	(0.678,0.263,0.059)					
$\varphi = 0.7$	A_3	(0, 1, 0)	3	4	2	1	5
	A_4	(0.368,0.495,0.137)					
	A_5	(1, 0, 0)					
	A_1	(0.641,0.256,0.103)					
	A_2	(0.721,0.219,0.06)					
$\varphi = 0.9$	A_3	(0, 1, 0)	3	4	2	1	5
	A_4	(0.44,0.416,0.144)					
	A_5	(1, 0, 0)					

TABLE 9. Different Values of Q_j and Preference Order Ranking by Proposed Intuitionistic Fuzzy VIKOR for Portfolio Selection Problem

To demonstrate the effectiveness of the proposed intuitionistic fuzzy VIKOR, it is compared to the intuitionistic fuzzy TOPSIS method presented by Boran et al. [11]. Computational results for intuitionistic fuzzy TOPSIS are given in Table 10 according to separation measures and the relative closeness coefficient of each alternative. By considering Tables 8 and 10, it is found that the ranking of five alternatives are equivalent, in which A_3 is the first rank and A5 is the fifth rank with respect to six selected attributes in the portfolio selection problem. The fuzzy TOPSIS is one of the MAGDM methods that utilized a compromise solution. This technique should satisfy the closest to the PIS and the farthest from the NIS. The final decision is reached to implement the multiple goal group decisions by computing the comparative distance from the ideal solutions. The intuitionistic fuzzy TOPSIS set $p = 2$ to offer the largest max-min variance in order to find the solution. In contrast, proposed intuitionistic fuzzy VIKOR set $p = 1$ and $p = \infty$ to find the two distances and to set weight φ and $1 - \varphi$ to justify all possible combinations. Performing on the priority analysis by using the fuzzy TOPSIS, the minority bias is not considered and the majority opinions are represented.

Alternatives	S^+	S^-	C_i^*	Preference order ranking
A_1	0.058	0.096	0.623	3
A_2	0.104	0.077	0.425	4
A_3	0.046	0.134	0.746	1
A_4	0.045	0.108	0.705	2
A_5	0.127	0.031	0.193	5

TABLE 10. Computational Results for the Intuitionistic Fuzzy TOPSIS in Portfolio Selection Problem

4.2. **Illustrative example 2.**

4.2.1. **Implementation**. To further demonstrate the proposed method, another illustrative example from Amiri-Aref and Javadian [2] is presented for the material handling selection problem. This selection can be obtained according to four conflicting attributes, including investment cost (C_1), operation time (C_2), expansion possibility (C_3), closeness to market demand (C_4), $P/E(C_5)$ and assets (C_6). Moreover, five alternatives have been regarded for the process of asset allocation. A group of three DMs ($\mathrm{DM}_1, \mathrm{DM}_2 \mathrm{and} \mathrm{DM}_3$) is arranged to evaluate and select the appropriate candidate. The importance degree of the DMs and the weight of attributes are provided in the group decision making process in Tables 11 and 12, respectively. Then three DMs express the linguistic variables illustrated in Table 4 to evaluate the importance of the four attributes, given in Table 13.

	DM_1	DM_2	DM_3
Linguistic variables	Important	Medium	Unimportant
Weights	0.469	0.313	0.219

Table 11. Importance of Decision Makers for Material Handling Selection Problem

	C_1	C_2	C_3	C_4
DM_1	M	UI	I	UI
DM_2	M	VUI	VI	I
DM_3	VI	M	I	M

Table 12. Weights of the Attributes for Material Handling Selection Problem

After rating each alternative with respect to each attribute by three DMs, the aggregated intuitionistic fuzzy decision matrix and the weight of attributes are obtained based on their judgments in Table 14.

S_j, R_j and Q_j values are calculated by selecting $\varphi = 0.5$ (consensus) and computational results are shown in Table 15. the ranking of five alternatives with respect to four criteria is as follows: $\mathcal{A}_4 \succ \mathcal{A}_3 \succ \mathcal{A}_1 \succ \mathcal{A}_5 \succ \mathcal{A}_2$.

4.2.2. **Discussion**. A sensitivity analysis can be conducted by setting systematically φ to some values between 0 and 1 and by tracking the changes in the ranking. The results of such an analysis based on values are given in Table 16. It is observed that the preference order ranking of alternatives is changed. Hence, the proposed method can provide various solutions (i.e., different order ranking) by considering different value of φ depending on specific applications in the material handling problem. In the following, we utilize the intuitionistic fuzzy TOPSIS method presented by Boran et al. [11] for the purpose of ranking the potential alternatives and comparing the results in this application example. Computational results for

Attributes	Alternatives	DM_1	DM_2	DM_3
	A_1	VB	B	VVB
	A_2	MG	G	G
C_1	A_3	VG	G	VVG
	A_4	G	VG	VVG
	A_5	F	MB	MG
	A_1	B	F	MB
	A_2	F	MB	MB
C_2	A_3	VVG	VG	VG
	A_4	MG	G	G
	A_5	MB	F	B
	A_1	H	H	H
	A_2	B	VB	MB
C_3	A_3	MB	MB	F
	A_4	VG	G	VVG
	A_5	MB	MG	MG
	A_1	H	MG	H
	A_2	MB	F	F
C_4	A_3	VH	H	MH
	A_4	H	MH	VH
	A_5	MB	MH	MB

Table 13. Ratings of the Alternatives for Material Handling Selection Problem

	C_1	C_2	C_3	C_4
A_1	(0.15,0.728,0.122)	(0.371,0.508,0.121)	(0.7,0.2,0.1)	(0.672, 0.227, 0.101)
A_2	(0.657,0.242,0.101)	(0.449, 0.450, 0.101)	(0.244,0.618,0.138)	(0.482,0.417,0.101)
A_3	(0.805, 0.124, 0.071)	(0.856,0.1,0.044)	(0.424,0.476,0.1)	(0.736,0.158,0.106)
A_4	(0.793,0.138,0.069)	(0.657,0.242,0.101)	(0.805,0.124,0.071)	(0.7,0.195,0.105)
A_5	(0.496,0.402,0.102)	(0.405,0.485,0.11)	(0.517,0.381,0.102)	(0.472,0.426,0.102)
Weights	(0.649, 0.323, 0.028)	(0.321,0.639,0.04)	(0.813,0.161,0.026)	(0.545,0.399,0.056)

Table 14. Aggregated Intuitionistic Fuzzy Decision Matrix and Weight of Attributes for Material Handling Selection Problem

Alternatives	S_j	R_j	Q_j
A_1	(0.884,0.088,0.028)	(0.649,0.323,0.028)	(0.806,0.183,0.011)
A_2	(0.962,0.026,0.012)	(0.813,0.161,0.026)	(1,0,0)
A_3	(0.725,0.224,0.051)	(0.725,0.224,0.051)	(0.751, 0.203, 0.046)
A_4	(0.375,0.552,0.073)	(0.241,0.736,0.023)	(0,1,0)
A_5	(0.948,0.032,0.02)	(0.854,0.292,0.054)	(0.919, 0.055, 0.026)

Table 15. The Values S_j, R_j and Q_j and by the Proposed Intuitionistic Fuzzy VIKOR for Material Handling Selection Problem

intuitionistic fuzzy TOPSIS are given in Table 17 according to separation measures and the relative closeness coefficient of each alternative.

The proposed fuzzy VIKOR may be used in the multiple goal decision making for several reasons as follows:

Q_j with different values of V							Preference order ranking
φ		Q_j	A_1	A_2	A_3	A_4	A_5
$\varphi = 0.1$	A_1	(0.735,0.259,0.006)	3	5	2	1	4
	A_2	(1, 0, 0)					
	A_3	(0.83, 0.124, 0.046)					
	A_4	(0,1,0)					
	A_5	(0.783, 0.171, 0.046)					
$\varphi = 0.3$	A_1	(0.773,0.218,0.009)	3	5	2	1	4
	A_2	(1, 0, 0)					
	A_3	(0.794, 0.158, 0.048)					
	A_4	(0,1,0)					
	A_5	(0.867, 0.097, 0.036)					
$\varphi = 0.5$	A_1	(0.806,0.183,0.011)	3	5	2	1	4
	A_2	(1, 0, 0)					
	A_3	(0.751, 0.203, 0.046)					
	A_4	(0,1,0)					
	A_5	(0.919, 0.055, 0.026)					
$\varphi = 0.7$	A_1	(0.833,0.154,0.012)	3	5	2	1	4
	A_2	(1, 0, 0)					
	A_3	(0.698, 0.259, 0.043)					
	A_4	(0,1,0)					
	A_5	(0.951, 0.031, 0.018)					
$\varphi = 0.9$	A_1	(0.857,0.13,0.013)	3	5	2	1	4
	A_2	(1, 0, 0)					
	A_3	(0.633, 0.332, 0.035)					
	A_4	(0,1,0)					
	A_5	(0.97, 0.018, 0.012)					

TABLE 16. Different Values of Q_j and Preference Order Ranking by Proposed Intuitionistic Fuzzy VIKOR for Material Handling Selection Problem

Alternatives	S^+	S^-	C_i^*	Preference order ranking
A_1	0.184	0.155	0.456	4
A_2	0.199	0.135	0.404	5
A_3	0.124	0.199	0.617	2
A_4	0.026	0.253	0.908	1
A_5	0.149	0.125	0.457	3

TABLE 17. Computational Results for the Intuitionistic Fuzzy TOPSIS in Material Handling Selection Problem

- The proposed method helps to find the final decision index as does the intuitionistic fuzzy TOPSIS, but the proposed fuzzy VIKOR takes the side effects into consideration.
- Proposed intuitionistic fuzzy VIKOR method utilizes relative distances and different calculating modes and weights in order to formulate an overall target minority injury that is smallest by considering the side effect in situation.

Comparing the proposed intuitionistic fuzzy VIKOR with the intuitionistic fuzzy TOPSIS illustrates that there are three main differences as follows. First, the fuzzy TOPSIS offers majority rule while proposed fuzzy VIKOR offers the smallest injures. Second, the fuzzy TOPSIS regards the weight of attributes in the distance calculation while proposed fuzzy VIKOR regards the weight in the final value. Furthermore, the ranking of alternatives will be almost identical if the processes apply the same weight between two intuitionistic fuzzy TOPSIS and VIKOR methods.

Third, in the proposed fuzzy VIKOR the defuzzification is avoided, and the main intuitionistic fuzzy operations are utilized unlike the existing intuitionistic fuzzy TOPSIS.

Furthermore, the advantages of proposed intuitionistic fuzzy VIKOR in comparison with the traditional VIKOR method are described from different aspects, including the performance rating of alternatives, the weight of attributes, the relative importance of experts and the aggregation of experts judgments along with the impreciseness and vagueness in the information, as follows:

In classical VIKOR method to present a compromise solution for a decision problem, the performance ratings of alternatives and the weights of attributes are provided by numerical values. Indeed, this classical method is only appropriate for the decision problems with the crisp evaluation information, and it cannot solve the discrete multi-attributes problems under uncertainty with non-commensurable and conflicting attributes. In practice, it is difficult to precisely provide the exact data as experts' judgments are often vague under numerous real-life situations and conditions along with the complex structure of the decision problem and conflicting nature of the evaluation attributes. It is worth to mention that as the complexity of the decision problem under multi-attributes increases, impreciseness and vagueness in the related data to solve the problem increases. Also, the relative importance of experts or DMs should be regarded in the process of group decision making because of their different backgrounds and experiences; therefore, the proposed intuitionistic fuzzy VIKOR method is more applicable and suitable to handle uncertainty in the real-life applications. In numerous practical situations, available information is not appropriate and sufficient for the exact definition regarding the degree of membership in order to introduce a certain element. In fact, some hesitation degree has existed between membership and non-membership.

To solve numerous real-life complex problems by considering insufficiency in information availability and the vagueness inherent in the DMs' opinions, the proposed intuitionistic fuzzy VIKOR with ill-known membership grades are appropriate. In this regard, the method is particularly useful to deal with uncertainty because the alternatives can be generated and their feasibility can be examined by the group of experts and intuitionistic fuzzy compromise solution approach. Also, the values of performance ratings, weights of attributes and the relative importance of experts are considered as linguistic variables that are converted into intuitionistic fuzzy numbers. The proposed method as an extension of the classical VIKOR is introduced to deal with MAGDM problems under intuitionistic fuzzy environment. In the presented method, the aggregation of DMs judgments under uncertainty is taken into account in the group decision making process. Then, an intuitionistic fuzzy ranking index, to maximize the group utility and to minimize the individual regret simultaneously, is extended based on the concept of the measure of closeness to the ideal solutions for the complex multi-attributes decision problems. Finally, the outcomes of the proposed method are recognized as effective and efficient decision aids to obtain the reliable ranked ordering of alternatives under uncertainty as illustrated with two real-life application examples of portfolio and material handling selection problems.

5. **Conclusions**

Multiple attributes group decision making (MAGDM) problems cope with uncertain and insufficient information, and the fuzzy set theory is a suitable way to deal with these conditions. Being a generalization of the fuzzy set, the intuitionistic fuzzy set allows decision makers (DMs) to provide an additional possibility in order to represent imperfect knowledge. In fact, it lets DMs utilize more flexible ways to simulate real-life decision situations by using the truth-membership function and non-truth-membership function in order to indicate the degrees of satisfiability and non-satisfiability. This paper presents a group decision making method for assessment of the alternatives in the MAGDM problems based on intuitionistic fuzzy VIKOR. Proposed decision method is more adequate to deal with uncertainty than conventional approaches. In the assessment process, the ratings of each alternative with respect to each attribute and the weights of each criterion are provided as linguistic variables, characterized by intuitionistic fuzzy values with multi-judges. First, intuitionistic fuzzy averaging operator is utilized to aggregate judgments of DMs. Second, an intuitionistic ranking index is presented based on the incorporated efficient fuzzy approach and concepts of positive ideal and negative ideal solutions to solve decision making problems by a group of experts in the real-life environment. The relative distance of the alternatives is then obtained. It can satisfy the closest to the intuitionistic fuzzy positive ideal solution and the farthest from the intuitionistic fuzzy negative ideal solution simultaneously, and finally the alternatives are ranked based on the main intuitionistic fuzzy operations. To highlight the applicability and suitability of proposed intuitionistic fuzzy VIKOR method, two application examples were presented through the practical portfolio selection and material handling selection problems. Moreover, the comparison was made between the proposed method and the existing intuitionistic fuzzy decision making method. Their computational results, similarities and differences were discussed through this MAGDM problem. Although the proposed intuitionistic fuzzy VIKOR method in this paper is demonstrated by two portfolio selection and material handling selection problems in the group decision making process, it can also be applied to other MAGDM problems, such as plant location, supplier selection and other strategy selection problems.

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