INFORMATION MEASURES BASED TOPSIS METHOD FOR MULTICRITERIA DECISION MAKING PROBLEM IN INTUITIONISTIC FUZZY ENVIRONMENT

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Abstract. In the fuzzy set theory, information measures play a paramount role in several areas such as decision making, pattern recognition etc. In this paper, similarity measure based on cosine function and entropy measures based on logarithmic function for IFSs are proposed. Comparisons of proposed similarity and entropy measures with the existing ones are listed. Numerical results limpidly betoken the efficiency of these measures over others. An intuitionistic fuzzy weighted measures (IFWM) with TOPSIS method for multi-criteria decision making (MCDM) quandary is introduced to grade the alternatives. This approach is predicated on entropy and weighted similarity measures for IFSs. An authentic case study is discussed to rank the four organizations. To compare the different rankings, a portfolio selection problem is considered. Various portfolios have been constructed and analysed for their risk and return. It has been examined that if the portfolios are developed using the ranking obtained with proposed method, the return is increased with slight increment in risk.

1. Introduction

Fuzzy sets (FSs) developed by Zadeh [56], are characterized by a degree of membership and have extensively been applied in various fields of science and technology such as clustering analysis, group decision making, image processing etc. [57]. To avoid the drawbacks of FSs, Atanassov [1] introduced the concept of intuitionistic fuzzy set (IFS), which is a generalization of the notion of fuzzy set. The distinguishing characteristic of intuitionistic fuzzy set is that it assigns to each element a membership degree, a non membership degree and the hesitation degree. Various authors explored the topic of IFS and recognized some momentous conclusions. For instance, Grzegorzewski [6] examined the possible and necessary inclusion of IFS. Xu proposed an approach in group decision making based on intuitionistic preference relationships and developed clustering algorithm for IFS [48, 49]. Wang and Xin [12] initiated the axiomatic definition of distance measure in IFSs and applied it to pattern recognition.

Entropy is a significant idea to measure the uncertain information, which has been received more and more concentration in the past decades. In 1972, De Luca and Termini [4] first provided axiomatic structure for the entropy of fuzzy sets and

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defined entropy measure to describe the fuzziness degree of a fuzzy set. Afterwards, various authors developed different entropy measures for FSs [7, 8, 28, 30, 31, 37]. For the entropy measure of intuitionistic fuzzy set, Burillo and Bustince [2] introduced the concept of entropy of IFS in 1996. Later on, Szmidt and Kacprzyk [39] developed an entropy for intuitionistic fuzzy set by employing a geometric interpretation of IFS. Vlachos and Sergidias [41] proposed the intuitionistic fuzzy entropy and applied it to pattern recognition, medical diagnosis and image processing problems. Subsequently, various researchers also proposed different entropy formulae for IFSs [11, 25, 26, 27, 47, 53].

The similarity measure is a paramount concept in the fuzzy set theory. It describes the amount of homogeneous attribute of two fuzzy sets. Wang [43] first developed the doctrine of the similarity measure and introduced a computation procedure for it. Relevantly, Wang et al. [42] performed a relative study of similarity measures. Li and Cheng [17] initiated the notion of similarity measure of IFS and applied it to pattern recognition. Liang et al. [20] and Mitchell [32] enhanced the similarity measure of IFS initiated by Li and Cheng [17]. Further, many authors concentrated on the similarity measure of IFS [10, 12, 18, 20, 27, 45].

In multi-criteria decision making (MCDM) quandary, decision makers' rank or choose options after qualitative or quantitative assessment of a finite set of interdependent or independent criteria. Desirable alternative can be selected by providing preference information in terms of exact numerical value or interval. However, preference information in real life situation can be assessed in a qualitative way with vague or imprecise cognizance. In such situations, ambiguity caused by vague or imprecise preference information is an astronomically immense challenge for decision makers, which is a great motivation for researchers to elongate MCDM techniques in fuzzy environment. Decision making problems are initiated in various disciplines such as engineering, management, supply selection, portfolio analysis etc. MCDM is categorized as: multi objective decision making (MODM) and multi attribute decision making (MADM). MADM is employed to find the appropriate alternative among numerous alternatives versus multi-attribute but MODM is applied the decision problems in which the decision space is continuous [40].

In the last decades, a lot of methods have been developed to pact with group decision making under different environments [9, 21, 22, 33, 35, 36, 55]. From another point of view, according to the information about the weight vectors of the criteria, group decision-making problems fall into two categories: the one with the weight vector known and the one with the weight vector incompletely known which also contains the one with the weight vector completely unknown. For the first class, the research topics focus on investigating aggregation techniques, and various ones have been developed, such as the ordered weighted averaging operator [52], the ordered weighted geometric operator [50], etc. which are used to aggregate the original decision information. To obtain the relative value of the alternatives, many methods have also been developed based on the traditional methods, such as the TOPSIS method [3, 13, 14], the VIKOR method [36, 40], the ELECTRE method [5] and the PROMETHEE method [38]. For the second class, the research topics concern about the weight-determined methods, such as the maximizing deviation method

[46], the entropy method [54], the non linear model method [51] and the compromise ratio method [16]. Mousavi and Vahdani introduced a hierarchical group decision-making (IFHGDM) model for the cross-docking location selection problem based on intuitionistic fuzzy modified group complex proportional assessment and group order preference by similarity to ideal solution [34].

From the above investigation, we can find that determining weight vectors and aggregating the individual opinions are the necessary steps in many existing group decision-making approaches. To get a more reasonable result, the decision makers have to choose the most suitable one from a lot of aggregation techniques and weight-determined methods according to practical problems, which may take much time and the selected aggregation technique or method may be not the most suitable one due to the complexity of the problem or the limited knowledge of the decision makers.

Briefly inspired by the aforementioned works, cosine similarity measure for IFSs based on [28] is developed. Two new effective entropy measures for IFSs are introduced, which are generalized version of fuzzy entropy measures in [29]. TOPSIS method is extended for intuitionistic fuzzy environment by using construction theorem given by [15] and proposed similarity measure. To facilitate the relative importance of criteria, weights to each criterion have been determined by entropy method (using proposed intuitionistic fuzzy entropy measure). To obtain the optimal group decision matrix or the overall performances of the alternatives, the proposed models can avoid the steps of aggregating the individual opinions and the ones of determining the weight vectors of the decision makers or the criteria, which can provide the decision makers a simple but efficient method. Further, the proposed intuitionistic fuzzy TOPSIS method is applied to rank the four organizations based on some important criteria. Finally, the obtained ranking is implemented in portfolio selection problem to analyse the risk and return in different portfolios.

2. **Prerequisite**

In the present section, we discuss fundamental conceptions of fuzzy sets (FSs), intuitionistic fuzzy sets (IFSs), entropy and similarity measures for IFSs, and construction method of IFSs from FSs.

Definition 2.1. Let $X = \{x_1,\ x_2,\ ...,\ x_n\}$ be a finite universe of discourse and let $\tilde{A} \subset X$. Then $\tilde{A}$ is fuzzy set defined as [56]

$$\tilde{A} = \{(x_i, \mu_{\tilde{A}}(x_i)) : \mu_{\tilde{A}}(x_i) \in [0,1]\,; \forall x_i \in X\},$$

where $\mu_{\tilde{A}} : X \to [0,1]$ is membership function of $\tilde{A}$. The number $\mu_{\tilde{A}}(x_i)$ shows the degree of membership of $x_i \in X$ to $\tilde{A}$.

Definition 2.2. Let $X = \{x_1,\ x_2,\ ...,\ x_n\}$ be a finite universe of discourse, then an intuitionistic fuzzy set (IFS) $A \subset X$ is given by [1]

$$A = \{\langle x_i,\ \mu_A(x_i),\ \nu_A(x_i)\rangle\ :\ x_i \in X\},$$

where $\mu_A : X \to [0,\ 1]$ is the degree of membership and $\nu_A : X \to [0,\ 1]$ is the degree of non-membership of $x_i \in X$ to A, respectively, such that

$$0 \leq \mu_A(x_i) + \nu_A(x_i) \leq 1,\ \ \forall\, x_i \in X.$$

For IFS A in X, the intuitionistic index (or hesitancy degree) of an element $x_i \in X$ in A is defined as

$$\pi_A(x_i) = 1 - \mu_A(x_i) - \nu_A(x_i).$$

It is evident that $0 \leq \pi_A(x_i) \leq 1, \forall\, x_i \in X$ (Atanassov [1]). FSs can also be represented using the notation of IFSs as
$A = \{\langle x_i,\, \mu_A(x_i),\, 1 - \mu_A(x_i)\rangle \ :\ x_i \in X\}$, with $\pi_A(x_i) = 0,\ \forall\, x_i \in X$.
The complementary set A^c of A is defined as

$$A^c = \{\langle x_i,\ \nu_A(x_i),\ \mu_A(x_i)\rangle \ :\ x_i \in X\}.$$

Again, let $A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle | x_i \in X\}$ and $B = \{\langle x_i, \mu_B(x_i), \nu_B(x_i)\rangle | x_i \in X\}$ be two IFSs then operations on IFSs are defined as follows:

(1) $A \subseteq B$ iff $\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i),\ \forall x_i \in X$;
(2) $A = B$ iff $A \subseteq B$ and $B \subseteq A$;
(3) $A \cup B = \{\langle x_i,\ (\mu_A(x_i) \vee \mu_B(x_i)),\ (\mu_A(x_i) \wedge \mu_B(x_i))\rangle | x_i \in X\}$;
(4) $A \cap B = \{\langle x_i,\ (\mu_A(x_i) \wedge \mu_B(x_i)),\ (\mu_A(x_i) \vee \mu_B(x_i))\rangle | x_i \in X\}$.

Definition 2.3. A function $h : IFS(X) \to [0,\ 1]$ is said to be entropy for IFSs if it satisfies the following axioms: [39]

(P1). $h(A) = 0$ (minimum) iff A is a crisp set;
(P2). $h(A) = 1$ (maximum) iff $\mu_A(x_i) = \nu_A(x_i)$ for all $x_i \in X$;
(P3). $h(A) \leq h(B)$ and if A is less fuzzy than B, *i. e.*
$\mu_A(x_i) \leq \mu_B(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i)$ for $\mu_B(x_i) \leq \nu_B(x_i)$ or
$\mu_A(x_i) \geq \mu_B(x_i)$ and $\nu_A(x_i) \leq \nu_B(x_i)$ for $\mu_B(x_i) \geq \nu_B(x_i)$ for any $x_i \in X$;
(P4). $h(A) = h(A^c)$.

Definition 2.4. A real function $Sim : IFS(X) \times IFS(X) \to [0,\ 1]$ is said to be similarity measure for IFSs(X) if Sim satisfies the following postulates: [18]

(S1). $Sim\,(A,\ A^c) = 0$ if A is a crisp set;
(S2). $Sim\,(A, B) = 1 \Leftrightarrow A = B$;
(S3). $Sim\,(A, B) = Sim\,(B, A)$;
(S4). For all $A,\ B,\ C \in IFS\,(X)$, if $A \subseteq B \subseteq C$, then
$Sim\,(A,\ C) \leq Sim\,(A,\ B)$ and $Sim\,(A,\ C) \leq Sim\,(B,\ C)$.

Next, construction method for IFSs from FSs proposed by Jurio et al. [15] is discussed. Implication of the method presents on the fact that IFSs are built in such a way that the indeterminacy index for each element is fine-tuned beforehand. The method can be given as follows:

Definition 2.5. Let $\tilde{A} \in FS(X)$, where $FS(X)$ represents the set of all fuzzy sets in the universal set X and $\lambda,\ \pi : X \to [0,\ 1]$ be two mapping. Then [15]

$$I = \{\langle x_i,\ f\,(\mu_{\tilde{A}}(x_i),\ \pi(x_i),\ \lambda(x_i))\rangle :\ x_i \in X\}$$

is an Atanassov [1] IFS, where the mapping $f : [0,\ 1]^2 \times [0,\ 1] \to L$ given by

$$f\,(a,\ b,\ \lambda) = (f_\mu(a,\ b,\ \lambda),\ f_\nu(a,\ b,\ \lambda))$$

where $f_\mu(a,\ b,\ \lambda) = a(1-\lambda b)$, $f_\nu(a,\ b,\ \lambda) = 1 - a(1-\lambda b) - \lambda b$ and

$$L = \{(a,\ b) :\ (a,\ b)\ \in [0,\ 1]\ \times\ [0,\ 1] \text{ and } a + b \leq 1\}$$

satisfies the following conditions:

(C1). If $b_1 \leq b_2$, then $\pi\,(f(a,\ b_1,\ \lambda)) \leq \pi\,(f(a,\ b_2,\ \lambda))$ for all $a,\ \lambda \in [0,\ 1]$;
(C2). $f_\mu(a,\ b,\ \lambda) \leq x \leq 1 -\ f_\nu(a,\ b,\ \lambda)$ for all $a,\ \lambda \in [0,\ 1]$;
(C3). $f(a,\ 0,\ \lambda) = (a,\ 1-a)$;
(C4). $f(0,\ b,\ \lambda) = (0,\ 1-\lambda b)$;
(C5). $f(a,\ b,\ 0) = (a,\ 1-a)$;
(C6). $\pi\,(f(a,\ b,\ \lambda)) = \lambda b$.

Method for Transforming IFSs into FSs. Li et al. [19] introduced a method for transforming 'intuitionistic fuzzy sets (IFSs)' into 'fuzzy sets (FSs)' by distributing hesitation degree equally with membership degree and non membership degree, respectively.

Definition 2.6. Let $A \in IFS(X)$, then the fuzzy membership function $\mu_{\tilde{A}}(x)$ to $\tilde{A}$ ($\tilde{A}$ be the fuzzy set corresponding to intuitionistic fuzzy set A) is defined as [19]

$$\mu_{\tilde{A}}(x) = \mu_A(x) + \frac{\pi_A(x)}{2} = \frac{\mu_A(x) + 1 - \nu_A(x)}{2}.$$

3. Similarity Measure for IFSs

In current section, we define a new similarity measure for IFSs and compare it with the existing measures.
Let $A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle : x_i \in X\}, B = \{\langle x_i, \mu_B(x_i), \nu_B(x_i)\rangle : x_i \in X\} \subseteq X$, then we have

$$Sim\,(A,\ B) = \frac{1}{n}\sum_{i=1}^{n} \cos\left\{\frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{4\,(1 + |\pi_A(x_i) - \pi_B(x_i)|)}\pi\right\}. \qquad (1)$$

Theorem 3.1. *The function Sim, defined by equation(1), is a similarity measure for IFSs.*

Proof. It is easy to claim that $Sim\,(A,\ B)$ satisfies the condition (S1)-(S4). We, therefore, only have to prove (S4) for $Sim\,(A,\ B)$. Let $A \subset B \subset C$, then $\mu_A(x_i) \leq \mu_B(x_i) \leq \mu_C(x_i)$ and $\nu_A(x_i) \geq \nu_B(x_i) \geq \nu_C(x_i)$, $\forall\, x_i \in X$.
It follows that $|\mu_A(x_i) - \mu_B(x_i)| \leq |\mu_A(x_i) - \mu_C(x_i)|$ and $|\nu_A(x_i) - \nu_B(x_i)| \leq |\nu_A(x_i) - \nu_C(x_i)|$, $\forall\, x_i \in X$.

$$\frac{|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)|}{4\,(1 + |\pi_A(x_i) - \pi_B(x_i)|)}\pi$$

$$\leq \frac{|\mu_A(x_i) - \mu_C(x_i)| + |\nu_A(x_i) - \nu_C(x_i)|}{4\,(1 + |\pi_A(x_i) - \pi_C(x_i)|)}\pi,\ \ \forall\, x_i \in X$$

These imply

$$\cos\left\{\frac{|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|}{4\,(1+|\pi_A(x_i)-\pi_B(x_i)|)}\pi\right\}$$
$$\geq \cos\left\{\frac{|\mu_A(x_i)-\mu_C(x_i)|+|\nu_A(x_i)-\nu_C(x_i)|}{4\,(1+|\pi_A(x_i)-\pi_C(x_i)|)}\pi\right\},\ \forall\, x_i\in X.$$

Hence

$$Sim\,(A,\ B)\geq Sim\,(A,\ C).$$

Similarly, $Sim\,(B,\ C)\geq Sim\,(A,\ C)$.
From the above result, we obtain that Sim satisfies (S4). It is obvious that Sim satisfies (S1), (S2) and (S3). Therefore, Sim is similarity measure for $IFSs(X)$. □

Moreover, if $\omega=(\omega_1,\,\omega_2,\,...,\,\omega_n)^T$ such that $\omega_i\in[0,\,1]$ and $\sum_{i=1}^{n}\omega_i=1$ is a weight vector of the elements $x_i\in X$, $i=1\,(1)\,n$, then equation(1) becomes

$$S\,(A,\,B)=\sum_{i=1}^{n}\omega_i\ \cos\left\{\frac{|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|}{4\,(1+|\pi_A(x_i)-\pi_B(x_i)|)}\pi\right\},\qquad(2)$$

is a weighted similarity measure on IFSs.

Next, we illustrate the efficiency of similarity measure through comparison with some already existing similarity measures for IFSs.

Example 3.2. Let us consider that there are three patterns represented with IFSs in $X=\{x_1,x_2,\,x_3\}$. These patterns are as follows: [20]

$$A_1=\{\langle x_1,\,0.1,\,0.1\rangle,\,\langle x_2,\,0.5,\,0.1\rangle,\,\langle x_3,\,0.1,\,0.9\rangle\};$$
$$A_2=\{\langle x_1,\,0.5,\,0.5\rangle,\,\langle x_2,\,0.7,\,0.3\rangle,\,\langle x_3,\,0.0,\,0.8\rangle\};$$
$$A_3=\{\langle x_1,\,0.7,\,0.2\rangle,\,\langle x_2,\,0.1,\,0.8\rangle,\,\langle x_3,\,0.4,\,0.4\rangle\}.$$

Assume that a sample

$$B=\{\langle x_1,\,0.4,\,0.4\rangle,\,\langle x_2,\,0.6,\,0.2\rangle,\,\langle x_3,\,0.0,\,0.8\rangle\}$$

is given.

Here, we recall various existing similarity measures.
Burrilo and Bustince [2] developed similarity measure $S_E(A,\,B)$ induced by Euclidean distance measure

$$S_E(A,\ B)=1-\xi_E(A,\ B),\ \text{where}\ \xi_E(A,\ B)=d_E(A,\ B)/\sqrt{n}.\qquad(3)$$

Hung and Yang [10] generated exponential type similarity measure for IFSs

$$S_{HY}(A,\ B)=1-\frac{1-\exp\left\{\frac{1}{2}\sum_{i=1}^{n}\left(|\mu_A(x_i)-\mu_B(x_i)|+|\nu_A(x_i)-\nu_B(x_i)|\right)\right\}}{1-\exp\left(-n\right)}.\qquad(4)$$

Mishra et al. [27] introduced trigonometric similarity measure for IFSs

$$S_M\,(A,B)=1-\frac{1}{n}\sum_{i=1}^{n}\sin\left[\frac{\{\max\left(|\mu_A(x_i)-\mu_B(x_i)|,\ |\nu_A(x_i)-\nu_B(x_i)|\right)\}\,\pi}{2}\right].\qquad(5)$$

	A_1	A_2	A_3
	$0.980^{(1)}$	$0.994^{(1)}$	$0.818^{(1)}$
B	$0.809^{(3)}$	$0.918^{(3)}$	$0.580^{(3)}$
	$0.586^{(4)}$	$0.809^{(4)}$	$0.265^{(4)}$
	$0.744^{(5)}$	$0.896^{(5)}$	$0.293^{(5)}$

TABLE 1. Comparison of the Measure equation (1) with the Measures equations (3), (4) and (5)

These similarity measures satisfy the set of axiomatic requirements in Definition 2.4. The values of a similarity measures among A_1, A_2 and A_3 with B are listed in Table 1.

Accordance with results of Hung and Yang [12], the sample B belongs to the pattern A_2. According to the Table 1, similarity measures defined by equations (1), (3), (4) and (5) show the same classification accordance with the doctrine of maximum membership degree.

4. Entropy Measures for IFSs

Let $X = \{x_1, x_2, ..., x_n\}$ and $A = \{\langle x_i,\ \mu_A(x_i),\ \nu_A(x_i)\rangle :\ x_i \in X\} \in IFS(X)$. Then, from Definition 2.6, an intuitionistic fuzzy set can be transformed into a fuzzy set to structure an entropy measure of the intuitionistic fuzzy set by means of

$$\mu_{\tilde{A}}(x_i) = \left(\mu_A(x_i) + 1 - \nu_A(x_i)\right)/2.$$

Based on Mishra et al. [29], we propose entropy measure for IFS A as follows:

$$h_1(A) = -\frac{2}{n}\sum_{i=1}^{n}\left[\ln\left\{\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}e^{\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}} + \frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}e^{\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}}\right\} - 1\right]. \tag{6}$$

Theorem 4.1. *The function $h_1(A)$, defined by (6), is an entropy for IFSs.*

Proof. The mapping $h_1(\cdot)$ defined by (6) satisfies the condition (P1)-(P4) in Definition 2.3. Hence, it is valid measure for IFSs. □

Corollary 4.2. *The function $h_2(A)$, defined by*

$$h_2(A) = \frac{2}{n}\sum_{i=1}^{n}\ln\left[\left\{\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}e^{\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}} + \frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}e^{\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}}\right\}\right], \tag{7}$$

for $A = \{\langle x_i,\ \mu_A(x_i),\ \nu_A(x_i)\rangle :\ x_i \in X\} \in IFS(X)$, is an intuitionistic fuzzy entropy.

Remark 4.3. It is fascinating to observe that if an intuitionistic fuzzy set (IFS) is an ordinary fuzzy set (FS), i. e., for all $x_i \in X$, $\nu_A(x_i) = 1 - \mu_A(x_i)$, then the logarithmic intuitionistic fuzzy entropy measures defined by equations (6) and (7) reduce to logarithmic fuzzy entropy measures as proposed in [29].

Moreover, if $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$ such that $\omega_i \in [0, 1]$ and $\sum_{i=1}^{n} \omega_i = 1$ is a weight vector, then equations (6) and (7) become

$$h_1(A) = -2\sum_{i=1}^{n} \omega_i \left[\ln\left\{\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2} e^{\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}} + \frac{\nu_A(x_i)+1-\mu_A(x_i)}{2} e^{\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}}\right\} - 1\right], \tag{8}$$

$$h_2(A) = 2\sum_{i=1}^{n} \omega_i \ln\left[\left\{\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2} e^{\frac{\nu_A(x_i)+1-\mu_A(x_i)}{2}} + \frac{\nu_A(x_i)+1-\mu_A(x_i)}{2} e^{\frac{\mu_A(x_i)+1-\nu_A(x_i)}{2}}\right\}\right], \tag{9}$$

are weighted entropy measures for IFSs(X).

Proposition 4.4. *Let $A, B \in IFSs(X)$, where $A = \{\langle x_i, \mu_A(x_i), \nu_A(x_i)\rangle\}$, $B = \{\langle x_i, \mu_B(x_i), \nu_B(x_i)\rangle\}$ such that they satisfy for any $x_i \in X$ either $A \subseteq B$ or $B \subseteq A$, then we have*

(1) $h_1(A \cup B) + h_1(A \cap B) = h_1(A) + h_1(B)$;
(2) $h_2(A \cup B) + h_2(A \cap B) = h_2(A) + h_2(B)$.

Corollary 4.5. *For any $A \in IFS(X)$ and A^c is a complement of the intuitionistic fuzzy set A, then we obtain*

(1) $h_1(A) = h_1(A^c) = h_1(A \cup A^c) = h_1(A \cap A^c)$;
(2) $h_2(A) = h_2(A^c) = h_2(A \cup A^c) = h_2(A \cap A^c)$.

Next, the Example 4.6 shows that formulas (6) and (7) generate some counter-intuition cases:

Example 4.6. Let us consider the universe of discourse $X = \{x\}$ and evaluate the entropy measures for the following IFSs:

$$A_1 = \{\langle x, 0.2, 0.4\rangle\},\ A_2 = \{\langle x, 0.3, 0.4\rangle\},\ A_3 = \{\langle x, 0.4, 0.5\rangle\}\ A_4 = \{\langle x, 0.5, 0.5\rangle\}.$$

Here, we first evoke these entropy measures:
Vlachos and Sergiadis [41] proposed entropy E_{VS} according to cross entropy measure:

$$E_{VS}(A) = -\frac{1}{n\ln 2}\sum_{i=1}^{n} [\mu_A(x_i)\ln\mu_A(x_i) + \nu_A(x_i)\ln\nu_A(x_i) - (1-\pi_A(x_i))\ln(1-\pi_A(x_i)) - \pi_A(x_i)\ln 2]. \tag{10}$$

Wei and Zhang [44] developed entropy based on cosine function for IFSs:

$$E_{WZ}(A) = \frac{1}{n}\sum_{i=1}^{n} \cos\left\{\frac{(\mu_A(x_i)-\nu_A(x_i))\,\pi}{2\,(1+\pi_A(x_i))}\right\}. \tag{11}$$

Mishra et al. [27] constructed entropy based on sine function for IFSs:

$$h(A) = \frac{1}{n}\sum_{i=1}^{n} \sin\left\{\left(\frac{\mu_A(x_i)\wedge\nu_A(x_i)}{\mu_A(x_i)\vee\nu_A(x_i)}\right)\frac{\pi}{2}\right\}. \tag{12}$$

	A_1	A_2	A_3	A_4
h	0.7071	0.9239	0.9511	1
E_{VS}	0.9510	0.9897	0.9920	1
E_{WZ}	0.9749	**0.9927**	**0.9898**	1
h_1	0.9506	0.9876	0.9888	1
h_2	0.9696	0.9924	0.9944	1

TABLE 2. Comparison with Existing Entropy Measures

These entropy measures satisfy the set of requirements in Definition 2.3.
From Table 2, we observe that h, h_1, h_2 and E_{VS}, the closer membership and non membership degree, the greater its entropy. Particularly, when the membership degree and the non membership degree are equal, the entropy reaches the maximum value. The results obtained by using the entropy measures h, h_1, h_2 and E_{VS} are in accordance with our intuition. From Table 2, we have

$$h(A_1) < h(A_2) < h(A_3) < h(A_4),$$

$$h_1(A_1) < h_1(A_2) < h_1(A_3) < h_1(A_4),$$

$$h_2(A_1) < h_2(A_2) < h_2(A_3) < h_2(A_4)$$

and $E_{VS}(A_1) < E_{VS}(A_2) < E_{VS}(A_3) < E_{VS}(A_4)$, which are consistent with our intuition. But, from entropy E_{WZ}, we have

$$E_{WZ}(A_1) < E_{WZ}(A_2) > E_{WZ}(A_3) < E_{WZ}(A_4).$$

Obviously, the results obtained by using the entropy are not in accordance with our intuition.

5. An Algorithm for IFWM with TOPSIS

In numerous practical decision making problems, such as the supplier selection or selection of a partner for an enterprise in the field of supply chain management, military system efficiency evaluation, township development system and so on, decision makers usually need to provide their preferences over the alternatives. Consider that the socio-economic situation becomes more complex, the preference information provided by decision makers is usually imprecise, that is, there may be hesitation or uncertainty about preferences because a decision should be made under time pressure and lack of knowledge or data or the decision makers have limited attention and information processing capacities. In such cases, it is appropriate and convenient to express the decision makers' preferences in intuitionistic fuzzy sets (IFSs). The fundamental characteristic of the IFS is that the values of its membership function, non membership function and hesitancy function are the exact numbers. Therefore, it is necessary and interesting to pay attention to the group decision making problems with intuitionistic preference information.

Here, we have extended a similarity measure in the technique for order preference based on similarity to the ideal solution (TOPSIS) approach by measuring the similarity of each alternative to positive and negative ideal intuitionistic fuzzy numbers (IFNs) and applied the similarity measure between IFNs to the decision-making

process to increase the ability of the process to account for risks in a variable, complex, and uncertain environment.

In the following section, algorithm for intuitionistic fuzzy weighted measures (IFWM) with TOPSIS method is presented. Let $E = \{E_1, E_2, ..., E_m\}$ be a set of m alternatives and the best one from E according to the criterion set $G = \{G_1, G_2, ..., G_n\}$ which include n criteria is to be chosen:

Step 1: Construction of decision matrix for IFSs
In this step, intuitionistic fuzzy decision matrix $D = [d_{ij}]_{n \times m}$ of intuitionistic fuzzy value $d_{ij} = (\mu_{ij}, \nu_{ij})$ is created, where μ_{ij} and ν_{ij} are the degrees of membership and non-membership of the alternatives E_j satisfying the criterion G_i. The intuitionistic index $\pi_{ij} = 1 - \mu_{ij} - \nu_{ij}$ denotes the decision maker's hesitation of the alternatives E_j with respect to criterion G_i. This step include the following sub steps:

(1) Define the universe of discourse for each alternative against each criterion by taking their real numerical values.
(2) Construct the proper fuzzy sets for each criterion.
(3) Apply the method given by Jurio et al. [15] to construct IFSs from FSs obtained in Step 2.

Step 2: Obtain information and normalized information matrix
From equation (6), compute the information of each intuitionistic fuzzy value in the intuitionistic fuzzy judgement matrix and get the information matrix of judgement matrix as $D = (h_{ij})_{n \times m}$, where $h_{ij} = h_1(\tilde{r}_{ij})$.

Normalize (Normalization is used to eliminate the units of criterion functions, so that all the criteria are dimensionless.) the information values in the above decision matrix by

$$\overline{h}_{ij} = \frac{h_{ij}}{\max h_{ij}},\ j = 1(1)m;\ i = 1(1)n. \tag{13}$$

And the normalized information matrix is expressed as $\overline{D} = \left(\overline{h}_{ij}\right)_{n \times m}$.

Step 3: Determination of weights of criteria
Compute the weight vector $\omega = (\omega_1, \omega_2, ..., \omega_n)^T$, where $\omega_i \geq 0$ and $\sum_{i=1}^{n} \omega_i = 1$, by applying the given formula

$$w_i = \frac{1 - \sum_{j=1}^{m} \overline{h}_{ij}}{n - \sum_{i=1}^{n} \sum_{j=1}^{m} \overline{h}_{ij}};\ i = 1(1)n;\ j = 1(1)m. \tag{14}$$

Step 4: Compute the positive-ideal solution (PIS) and negative-ideal solution (NIS)
In TOPSIS method, the PIS is the solution that maximizes the benefit criteria and minimizes the cost criteria, whereas the NIS is the solution that minimizes the benefit criteria and maximizes the cost criteria. The optimal alternative is the one with the shortest distance from the positive solution and the farthest distance from the negative solution. The estimation criteria can be characterized into two categories: benefit and cost. Let B and F be the collection of benefit and cost criteria, respectively. According to intuitionistic fuzzy theory and the doctrine of

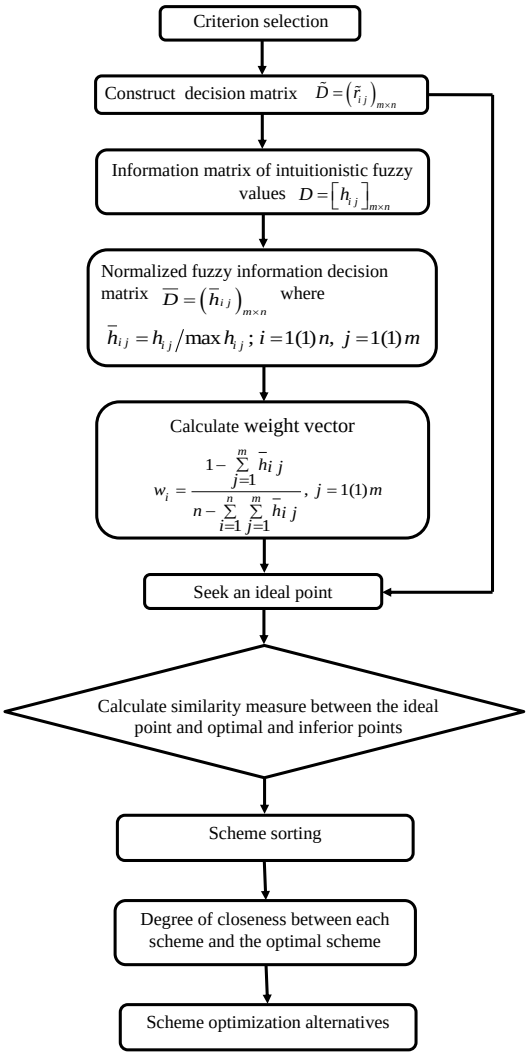


FIGURE 1. Implementation Flow Chart of Proposed IFWM TOPSIS Method

the conventional TOPSIS technique, positive-ideal E^+ and negative-ideal solution E^- can be defined as follows:

$$E^+ = \left\{ \left(G_j, \left\langle \max_i \mu_{ij} | \, j \in \mathrm{B}, \min_i \ \mu_{ij} | j \in F \right\rangle \right. \right. ,$$
$$\left. \left. \left\langle \min_i \ \nu_{ij} | \, j \in \mathrm{B}, \max_i \ \nu_{ij} | \, j \in F \right\rangle \right) : \ i = 1(1)n \right\}, \tag{15}$$
$$E^- = \left\{ \left(G_j, \left\langle \min_i \mu_{ij} | \, j \in \mathrm{B}, \max_i \ \mu_{ij} | j \in F \right\rangle \right. \right. ,$$

$$\left\langle \max_i \ \nu_{ij} | \, j \in \mathrm{B}, \min_i \ \nu_{ij} | \, j \in F \right\rangle \Big) : \ i = 1(1)n \Big\}, \tag{16}$$

where for each $j = 1(1)m$.

Step 5: Calculation of similarity measures from positive ideal solution (PIS) and negative ideal solution (NIS)
From equation (2), calculate the weighted similarity measure $S(E_j,\, E^+)$ among the options $E_j (j = 1(1)m)$ and the positive-ideal solution E^+, and the similarity measure $S(E_j,\, E^-)$ among the options $E_j (j = 1(1)m)$ and the negative-ideal solution E^-.

Step 6: Calculation of relative closeness coefficient (CC)
At last, relative closeness coefficient of each alternative with respect to intuitionistic fuzzy ideal solutions can be computed by using the following expression:

$$C_c(E_j) = \frac{S(E_j,\, E^+)}{S(E_j,\, E^+) + S(E_j,\, E^-)}, \quad j = 1(1)m. \tag{17}$$

Step 7: Desirable ranking
Choose the highest value, denoted by $C_c(E_k)$, among the values $C_c(E_j),\ j = 1(1)m$. And hence E_k is the optimal choice.

5.1. **Case Study.** In the process of TOPSIS method, the performance ratings and the weights of the criteria are given as exact values. In real-world situation, because of incomplete or non obtainable information, the attributes (data) are often not so deterministic; therefore they usually are fuzzy/imprecise. Therefore, some researches try to use the TOPSIS method for fuzzy/imprecise data.

In the financial system, assessment and evaluation of the organization based on some financial criteria are very significant. In the present section, proposed intuitionistic fuzzy TOPSIS method is applied in the ranking of four organizations, Bajaj Steel (e_1), HDFC Bank (e_2), Tata Steel (e_3) and Infotech Enterprises (e_4) using real data.

Four alternatives $\{e_1,\, e_2,\, e_3,\, e_4\}$ are assessed for their performance on the basis of following five inter-independent criteria $\{g_1,\, g_2,\, g_3,\, g_4,\, g_5\}$ such as g_1 : Earnings per share (EPS); g_2 : Face value; g_3 : P/C (Put-Call) ratio; g_4 : Dividend; g_5 : P/E (Price-to-earning) ratio. First two criteria belong to benefit criteria, i.e., high value indicates good growth prospects and last three belong to non-beneficial criteria, i.e., low value indicates good growth prospects. Therefore, $g_1,\, g_2 \in B$ and $g_3,\, g_4,\, g_5 \in F$. Actual numerical values of these five criteria of the four alternatives are retrieved from [14] from 21.9.2012 to 27.9.2012 and average of this information is placed in the table 3.

Step 1: Crisp numerical values given in Table 3 are fuzzified by defining the following fuzzy sets for each criteria and fuzzy decision matrix (Table 4) is obtained.

$$\begin{aligned}
A_1 &= 0.2846/20.5 + 0.75/10 + 0.234/2.21 + 0.23/2 + 0.24/4.8,\\
A_2 &= 0.318/23.31 + 0.248/2 + 0.766/24.7 + 0.097/0.67 + 0.76/27,\\
A_3 &= 0.759/60.06 + 0.75/10 + 0.315/5.65 + 0.32/2.92 + 0.278/6.5,\\
A_4 &= 0.241/16.86 + 0.437/5 + 0.41/9.7 + 0.155/1.25 + 0.394/11.4.
\end{aligned}$$

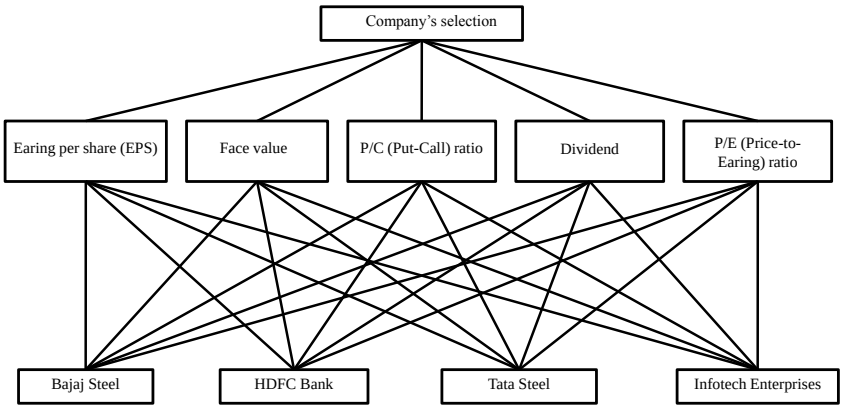


FIGURE 2. The Hierarchical Structure

Alternatives	g_1	g_2	g_3	g_4	g_5
e_1	20.50	10.00	2.21	2.00	4.80
e_2	23.31	2.00	24.7	0.67	27.00
e_3	60.06	10.00	5.65	2.92	6.50
e_4	16.86	5.00	9.70	1.25	11.40

TABLE 3. Average of Actual Numerical Value Criteria

	g_1	g_2	g_3	g_4	g_5
e_1	0.2846	0.75	0.234	0.23	0.24
e_2	0.318	0.248	0.766	0.097	0.76
e_3	0.759	0.75	0.315	0.32	0.278
e_4	0.241	0.437	0.41	0.155	0.394

TABLE 4. Fuzzy Decision Matrix

	g_1	g_2	g_3	g_4	g_5
e_1	(0.23, 0.587)	(0.61, 0.2)	(0.192, 0.63)	(0.22, 0.75)	(0.196, 0.62)
e_2	(0.26, 0.554)	(0.2, 0.61)	(0.63, 0.192)	(0.094, 0.875)	(0.62, 0.196)
e_3	(0.62, 0.197)	(0.61, 0.2)	(0.259, 0.56)	(0.31, 0.66)	(0.227, 0.59)
e_4	(0.197, 0.62)	(0.36, 0.454)	(0.337, 0.484)	(0.15, 0.82)	(0.322, 0.50)

TABLE 5. Intuitionistic Fuzzy Decision Matrix

The intuitionistic fuzzy decision matrix (Table 5) is constructed from the fuzzy decision matrix given by Table 4, and construction theorem given by Definition 2.5.

Alternatives	E_j^+	E_j^-
e_1	0.9586	0.8585
e_2	0.8240	0.9915
e_3	0.9910	0.8290
e_4	0.9275	0.9430

TABLE 6. Intuitionistic Similarity Measures

Step 2: By using the formula given by equation(6), we can evaluate the information measure of each intuitionistic fuzzy value in the above judgement matrix and get the following information matrix:

$$D = (h_{ij})_{4\times 5} = \begin{bmatrix} 0.8460 & 0.7990 & 0.7720 & 0.6734 & 0.7857 \\ 0.8944 & 0.7990 & 0.7720 & 0.3414 & 0.7857 \\ 0.7866 & 0.7990 & 0.8895 & 0.8518 & 0.8410 \\ 0.7866 & 0.9890 & 0.9731 & 0.4980 & 0.9607 \end{bmatrix}.$$

Using equation (13), the information matrix is transform into the normalized information matrix:

$$\overline{D} = \left(\overline{h}_{ij}\right)_{4\times 5} = \begin{bmatrix} 1.0000 & 0.9444 & 0.9125 & 0.7960 & 0.9287 \\ 1.0000 & 0.8933 & 0.8631 & 0.3817 & 0.8785 \\ 0.8843 & 0.8983 & 1.0000 & 0.9576 & 0.9455 \\ 0.7953 & 1.0000 & 0.9839 & 0.5035 & 0.9714 \end{bmatrix}.$$

Step 3: Compute criterion weight vector, utilize the normalized information matrix and by using the equation (14), we have

$$w_i = \frac{1 - \sum_{j=1}^{m} \overline{h}_{ij}}{n - \sum_{i=1}^{n}\sum_{j=1}^{m} \overline{h}_{ij}};\ i = 1(1)n;\ j = 1(1)m.$$

Thus, the weight vector of all the decision attributes is obtained as

$$W = (0.214,\ 0.218,\ 0.220,\ 0.131,\ 0.217)^T.$$

Step 4: Intuitionistic fuzzy positive and negative-ideal solutions are evaluated by using equations (15) and (16) are given by

$$E^+ = \{(0.62,\ 0.197),\ (0.61,\ 0.2),\ (0.192,\ 0.63),\ (0.094,\ 0.875),\ (0.196,\ 0.62)\},$$

$$E^- = \{(0.197,\ 0.62),\ (0.2,\ 0.61),\ (0.63,\ 0.192),\ (0.31,\ 0.66),\ (0.62,\ 0.196)\}.$$

Step 5: Intuitionistic fuzzy separation measure of each alternative from the positive and negative-ideal solution are computed by using equation (2) and are given by Table 6.

Step 6: Relative closeness coefficients are evaluated using equation (17) and are given in Table 7.

Step 7: Here, we obtain the following ordering as $e_3 \succ e_1 \succ e_4 \succ e_2$. Thus the most desirable ranking is e_3.

Alternatives	e_1	e_2	e_3	e_4
g_i	0.5275	0.4539	0.5445	0.4958

TABLE 7. Relative Closeness Coefficients

Method	Ranking	Best alternative
TOPSIS method proposed by [13]	$e_3 \succ e_4 \succ e_1 \succ e_2$	e_3
Fuzzy TOPSIS proposed by [9]	$e_3 \succ e_1 \succ e_4 \succ e_2$	e_3
Intuitionistic fuzzy TOPSIS proposed by [45]	$e_3 \succ e_1 \succ e_4 \succ e_2$	e_3
Weighted coefficient correlation method proposed by [54]	$e_3 \succ e_4 \succ e_1 \succ e_2$	e_3
Proposed IFWM with TOPSIS method	$e_3 \succ e_1 \succ e_4 \succ e_2$	e_3

TABLE 8. Comparison of Ranking Order of Alternatives with Different Existing Methods

5.2. **Discussion and Comparisons.** In order to handle the uncertain decision making problems with crisp or fuzzy data, proposed method is simple and efficient to use and provides the optimal alternative to decision makers under intuitionistic fuzzy environment. To validate the effectiveness and applicability of the proposed method, this paper chooses well-known and widely used methods as comparative benchmarks to facilitate the comparative analysis: the TOPSIS method, fuzzy TOPSIS method, intuitionistic fuzzy TOPSIS method and weighted coefficient correlation method.

Accordance with the descending order of relative closeness coefficients values of four alternatives are ranked as $e_3 \succ e_1 \succ e_4 \succ e_2$. The ranking of these four alternatives is also obtained by the TOPSIS method [13], fuzzy TOPSIS method [9], intuitionistic fuzzy TOPSIS method [45], weighted coefficient correlation method [54] and proposed intuitionistic fuzzy weighted measures (IFWM) with TOPSIS method and are shown in Table 8.

It is found that there is no conflict in the preference ordering of all the alternatives by fuzzy TOPSIS method, intuitionistic fuzzy TOPSIS method [45] and proposed method. There is only one distinction is found in deciding the preference ordering of e_1 and e_4.

In this subsection, for the purpose of comparison we evaluate the score value (function) of each intuitionistic fuzzy number of decision matrix, IF PIS and IF NIS. Subsequently, performances of companies with respect to each criterion are computed and revealed in Figure 3. As can be seen from Figure 3, company 3 (Tata Steel) performs relatively better than the other three companies under most of the criteria, and is closer to IF PIS than other companies. Alternatively, company 2 (HDFC Bank) performs relatively worse than the other three companies under most of the criteria and is closer to IF NIS than the other companies. Finally, it is found that there is no conflict in the preference ordering of all the alternatives obtained by proposed method and score values revealed in Figure 3. To review, company 3 should be selected for cooperation.

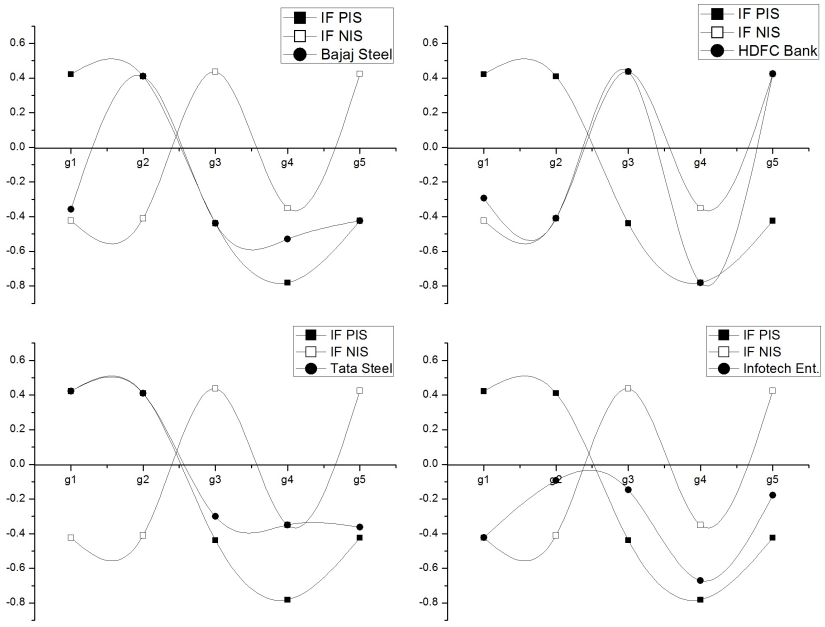


FIGURE 3. Comparison Between Each Company and IF PIS and IF NIS

6. **Application of IF TOPSIS in Portfolio Selection**

To evaluate the performance of proposed intuitionistic fuzzy TOPSIS with fuzzy TOPSIS [9] and TOPSIS method [13], a portfolio selection problem is considered. Fundamental conceptions related to portfolio and portfolio analysis are also discussed in present section.

Here, Some fundamental concepts associated to portfolio analysis specified by Markowitz [23, 24] are discussed.

Definition 6.1. An asset is a valuable economic entity owned by an individual or corporation from which the future economic benefits are expected to flow to the owner.

Definition 6.2. Return on asset is a pointer of gain/loss in the investment of an asset in financial market. If present cost of an asset is $\eta(0)$ and after T time period, the asset is sold off at cost $\eta(T)$, then return r, on asset for T time period is given by

$$r = \frac{\eta(T) - \eta(0)}{\eta(0)}. \tag{18}$$

Definition 6.3. Risk is defined as the degree of uncertainty of return on an asset. It signifies the possibility of loss in the investment. The risk can be either zero (risk free asset) or positive (risky asset). There are two kinds of risk associated with risky asset (i) systematic risk; (ii) unsystematic risk.

Date of observation	Bajaj Steel (e_1)	HDFC Bank (e_2)	Tata Steel (e_3)	Infotech Enterprises (e_4)
22/09/2012	92.50	607	396	196.50
23/09/2012	92.50	607	396	196.50
24/09/2012	93.60	626	410	195
25/09/2012	103.30	634	406.05	194
26/09/2012	101.05	639.25	398.55	193
27/09/2012	101	634.60	396.90	193
28/09/2012	100	631	400.05	191

TABLE 9. Assets Worth (in Rs.) at Different Equal Intervals

Definition 6.4. A portfolio is a bundle of two or more individual assets or securities, e_1, e_2, ..., e_n (say) denoted by an ordered $n-$tuple $(u_1, u_2, ..., u_n)$, where $u_i \in R$ is the number of units of the asset $e_i\,(i = 1(1)n)$ owned by the investor.

Definition 6.5. Let $\xi_i(0)$ and $\xi_i(t)$ be the values of i^{th} asset at time $T = 0$ and $T = t$, respectively. The value of portfolio $(u_1, u_2, ..., u_n)$ at time $T = 0$ and $T = t$, is denoted by $\xi(0)$ and $\xi(t)$ and defined as

$$\xi(0) = \sum_{i=1}^{n} u_i\,\xi_i(0) \text{ and } \xi(t) = \sum_{i=1}^{n} u_i\,\xi_i(t). \tag{19}$$

Definition 6.6. The weight w_i of the asset e_i is the percentage of the value of the asset in the portfolio $(u_1, u_2, ..., u_n)$ at time $t = 0,\ i.\ e.$

$$w_i = \frac{u_i\,\xi_i(0)}{\sum_{i=1}^{n} u_i\,\xi_i(0)};\ \ (i = 1(1)n) \text{ such that } \sum_{i=1}^{n} w_i = 1,\ \forall\, w_i > 0. \tag{20}$$

6.1. **Portfolio Testing.** Here, different portfolios with the assets of companies (e_1, e_2, e_3, e_4) are constructed. Additionally, the examined portfolios are analysed for their risk and return.

For scrutinize the portfolio the assets worth of four companies (e_1, e_2, e_3, e_4) are occupied form [14] at different days and are given in Table 9. Consider total number of units of assets cannot exceed by 100 in the portfolio $(u_1, u_2, ..., u_n)$. According to our ranking $e_3 \succ e_1 \succ e_4 \succ e_2$, we take $u_1 = 30, u_2 = 10, u_3 = 40$ and $u_4 = 20$, where u_i denotes the number of units of assets of a company e_i.

By using equation (20), weight of each company corresponding to each observation are obtained and are located in Table 10. To facilitate the ranking proposed by TOPSIS method [13], i. e., $e_3 \succ e_4 \succ e_1 \succ e_2$, choose $u_1 = 20,\ u_2 = 10,\ u_3 = 40$ and $u_4 = 30$, and using equation (20), the weight of the asset of each company on different observations are evaluated and are placed in Table 11. Ranking obtained by fuzzy TOPSIS method is same as given by our proposed method. Subsequently, the weight matrix of the assets by fuzzy TOPSIS method will be same as given in Table 10. Return series is fashioned with the help of weight matrix of organizations. Return series is the matrix of order $n \times m$, where n is number of observations and m is number of assets with equally spaced incremental return observations. The first row is the oldest observation, and the last row is the most recent. The return series can be formed by multiplying the assets prices with corresponding weights.

Date of observation	Bajaj Steel (e_1)	HDFC Bank (e_2)	Tata Steel (e_3)	Infotech Enterprises (e_4)
22/09/2012	0.097	0.212	0.554	0.137
23/09/2012	0.097	0.212	0.554	0.137
24/09/2012	0.096	0.213	0.558	0.133
25/09/2012	0.106	0.214	0.549	0.131
26/09/2012	0.104	0.219	0.545	0.132
27/09/2012	0.104	0.218	0.545	0.133
28/09/2012	0.103	0.217	0.549	0.131

TABLE 10. Weight Matrix of the Assets by Proposed Method

Date of observation	Bajaj Steel (e_1)	HDFC Bank (e_2)	Tata Steel (e_3)	Infotech Enterprises (e_4)
22/09/2012	0.062	0.205	0.534	0.199
23/09/2012	0.062	0.205	0.534	0.199
24/09/2012	0.061	0.206	0.540	0.193
25/09/2012	0.068	0.208	0.533	0.191
26/09/2012	0.067	0.212	0.529	0.192
27/09/2012	0.067	0.211	0.529	0.193
28/09/2012	0.066	0.210	0.533	0.191

TABLE 11. Weight Matrix of the Assets by TOPSIS Method

Matrices A, B and C represent the return series for proposed, fuzzy TOPSIS and TOPSIS method, respectively.

$$A = \begin{bmatrix} 8.9725 & 128.7 & 219.4 & 26.92 \\ 8.9725 & 128.7 & 219.4 & 26.92 \\ 8.9856 & 133.3 & 228.78 & 25.93 \\ 10.9498 & 135.68 & 222.92 & 25.414 \\ 10.5092 & 139.99 & 217.21 & 25.47 \\ 10.5 & 138.34 & 216.3 & 25.7 \\ 10.3 & 136.937 & 219.63 & 25.02 \end{bmatrix},$$

$$B = \begin{bmatrix} 8.9725 & 128.7 & 219.4 & 26.92 \\ 8.9725 & 128.7 & 219.4 & 26.92 \\ 8.9856 & 133.3 & 228.78 & 25.93 \\ 10.9498 & 135.68 & 222.92 & 25.414 \\ 10.5092 & 139.99 & 217.21 & 25.47 \\ 10.5 & 138.34 & 216.3 & 25.7 \\ 10.3 & 136.937 & 219.63 & 25.02 \end{bmatrix},$$

$$C = \begin{bmatrix} 5.735 & 124.43 & 211.46 & 39.10 \\ 5.735 & 124.43 & 211.46 & 39.10 \\ 5.709 & 128.956 & 221.4 & 37.635 \\ 7.024 & 131.872 & 216.42 & 37.054 \\ 6.770 & 135.521 & 210.83 & 37.056 \\ 6.767 & 133.901 & 209.96 & 37.249 \\ 6.6 & 132.51 & 213.23 & 36.481 \end{bmatrix}.$$

Using return series, portfolios are constructed. Risk and return associated with each portfolio are placed in Table 12.

Portfolio	By proposed method and fuzzy TOPSIS method		By TOPSIS method	
	Portfolio risk	Portfolio return	Portfolio risk	Portfolio return
Portfolio 1	0.1374	35.594	0.1326	34.2186
Portfolio 2	0.3594	64.7725	0.2127	64.5829
Portfolio 3	0.8594	95.7662	0.7047	94.7472
Portfolio 4	1.4144	216.947	1.3698	214.7882
Portfolio 5	1.9691	157.9331	2.0303	153.6759
Portfolio 6	2.5285	189.3186	2.7348	182.9400
Portfolio 7	3.8904	220.500	3.7953	213.8043
	Coefficient of correlation $(\rho) = 0.842882$		Coefficient of correlation $(\rho) = 0.833941$	

TABLE 12. Portfolio Risk and Return

7. **Conclusion**

For the period of modern days, how to determine suitable alternative in the decision-making process has become a key strategic consideration. In general, many real-world MCDM problems take place in a complex environment and usually adhere to imprecise data and uncertainty. As a result, the decision makers are unable (or unwilling) to express their judgement on the alternatives with exact and crisp values, and the evaluations are very often expressed in linguistic terms. The IFS theory is adequate for dealing with the vagueness of decision makers' judgement over alternatives with respect to attributes.

In this paper, we have developed new similarity measure based on cosine function and entropy measures based on logarithmic function for IFSs. Numerical comparisons have discussed to show the effectiveness of proposed information measures over the existing ones. An intuitionistic fuzzy weighted measures (IFWM) based TOPSIS method for MCDM quandary has proposed. An authentic case study has studied to rank four organizations based on five attributes utilizing the proposed method.

The proposed method has firstly used the construction theorem given by [15] to generate the intuitionistic fuzzy decision matrix from FSs and the weight of each criterion has calculated using intuitionistic fuzzy entropy measure. Then, the IF PIS and the IF NIS are estimated. Based on the proposed weighted similarity measures, the relative closeness coefficients of each alternative to the IF PIS have obtained and ranked the alternative according to the relative closeness, and selected the most desirable one(s). Further, the rankings obtained by proposed method have compared with the already existing methods in Table 8. Comparisons reveal the authenticity of the proposed method over existing one. Reliability of the proposed methods has also enhanced by calculating the weight vector, which was previously assumed by few of the authors.

Finally, a portfolio selection problem has taken. By retrieving the stock worth of four companies occupied in the study from [14] and considering the different rankings, different portfolios are generated and examined for their risk and returns. Table 12 depicts the return of different portfolios associated with risk and coefficient of correlation between them. As there is no conflict in the ranking of alternatives with the fuzzy TOPSIS method and proposed intuitionistic fuzzy TOPSIS method, return of each portfolio and involved risk are same. But, when the proposed method is compared with the TOPSIS method, we found that the return of each portfolio is higher with slightly high risk involved in each portfolio. Coefficient of correlation $(\rho) = 0.842882$ is also confirmed the better relationship between portfolio risk and return in different portfolios constructed using the ranking obtained by proposed method. Thus, the proposed method is very flexible. Using this method not only enables us to determine outranking order of alternatives but also assess and rate the alternatives and the decision makers. Apparently, our approach is straightforward and has less loss of information, and can be applied easily to various managerial decision making problems under intuitionistic fuzzy environment.

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