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A NEW MULTIPLE CRITERIA DECISION-MAKING METHOD BASED ON BIPOLAR FUZZY SOFT GRAPHS

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Abstract. In this research study, we present a novel frame work for handling bipolar fuzzy soft information by combining bipolar fuzzy soft sets with graphs. We introduce several basic notions concerning bipolar fuzzy soft graphs and investigate some related properties. We also consider the application of the bipolar fuzzy soft graphs. In particular, three efficient algorithms are developed to solve multiple criteria decision-making problems regarding social network, investment in shares and detection of bipolar disorder in children.

1. **Introduction**

Mathematical modeling, analysis and computing of problems with uncertainty is one of the hottest areas in interdisciplinary research, involving applied mathematics, computational intelligence and decision sciences. It is worth noting that uncertainty arises from various domains has very different nature and cannot be captured within a single mathematical framework. Molodtsov's soft sets [26] provide us a new way of coping with uncertainty from the viewpoint of parameterization. It has been revealed that soft sets have potential applications in several fields, including the smoothness of functions, game theory, operations research, Riemann integration, probability theory, and measurement theory [26]. Ali *et al.* [12] proposed some new operations in soft set theory. Maji *et al.* [21] introduced fuzzy soft sets by combining fuzzy sets and soft sets. From then on, many interesting applications of fuzzy soft set theory have been explored by some researchers. Roy and Maji [29] gave some applications of fuzzy soft sets. Feng *et al.* [16] combined soft sets with rough sets and fuzzy sets, obtaining three types of hybrid models: rough soft sets, soft rough sets, and soft-rough fuzzy sets.
In 1994, Zhang [40] initiated the concept of bipolar fuzzy sets. Bipolar fuzzy sets are an extension of fuzzy sets whose membership degree range is [-1,1]. In a bipolar fuzzy set, the membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree (0,1] of an element indicates that the element somewhat satisfies the property, and the membership degree [-1,0) of an element indicates that the element somewhat satisfies the implicit counter-property. The idea which lies behind such description is connected with

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the existence of "bipolar information" (e.g., positive information and negative information) about the given set. Positive information represents what is granted to be possible, while negative information represents what is considered to be impossible. Actually, a wide variety of human decision making is based on double-sided or bipolar judgmental thinking on a positive side and a negative side. For instance, cooperation and competition, friendship and hostility, common interests and conflict of interests, effect and side effect, likelihood and unlikelihood, feedforward and feedback, and so forth are often the two sides in decision and coordination. Thus bipolar fuzzy sets indeed have potential impacts on many fields, including artificial intelligence, computer science, information science, cognitive science, decision science, management science, economics, neural science, quantum computing, medical science, and social science. In recent years bipolar fuzzy sets seem to have been studied and applied a bit enthusiastically and a bit increasingly. Akram [8] discussed the concept of bipolar fuzzy soft Lie algebras. Abdullah *et al.* [1] introduced the concepts of operations on bipolar fuzzy soft sets.
Fuzzy graph theory is finding an increasing number of applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. Fuzzy models are becoming useful because of their aim in reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems. Kaufmann's initial definition of a fuzzy graph [20] was based on Zadeh's fuzzy relations [38]. Rosenfeld [28] described the structure of fuzzy graphs obtaining analogs of several graph theoretical concepts. Bhattacharya [14] gave some remarks on fuzzy graphs. Mordeson and Peng [24] discussed some operations on fuzzy graphs. The complement of a fuzzy graph was defined by Mordeson and Peng [24] and further studied by Sunita and Vijayakumar [35]. Akram [3, 4] worked on certain notions of bipolar fuzzy graphs. Further theory of bipolar fuzzy graphs was developed in [27, 30, 33, 34, 36]. Akram and Nawaz [5, 6, 7] first dealt with soft graphs and fuzzy soft graphs. Shahzadi and Akram [31, 32] considered intuitionistic fuzzy soft graphs with applications. In this research paper, we present a novel frame work for handling bipolar fuzzy soft information by combining the theory of bipolar fuzzy soft sets with graphs. We introduce certain notions of bipolar fuzzy soft graphs. We investigate some of their properties. We present several applications of the bipolar fuzzy soft graphs in a multiple criteria decision-making problem. We also develop algorithms in each multiple criteria decision-making problem.
For other notations, terminologies and applications not mentioned in the paper, the readers are referred to [2, 9, 10, 11, 15-19, 22-25, 2 37, 39].

2. **Preliminaries**

In this section, we review some basic definitions that will be used in the sequel. Let X be a non-empty set. Let $\widetilde{X^2}$ denotes the collection of all $2-$elements subsets of X. A pair $G^* = (X, E)$, where $E \subseteq \widetilde{X^2}$ is called a *graph.*

Definition 2.1. [38, 39] A *fuzzy set* μ in a universe X is a mapping $\mu : X \to [0, 1]$. A *fuzzy relation* on X is a fuzzy set ν in $X \times X$.

Definition 2.2. [39] Let μ be a fuzzy set in X and ν fuzzy relation on X. We call ν a fuzzy relation on μ if $\nu(x,y) \leq \min\{\mu(x), \mu(y)\}$ $\forall x, y \in X$.

Definition 2.3. [20] A *fuzzy graph* on a nonempty set X is a pair $G = (\mu, \lambda)$, where μ and λ are fuzzy sets on X and $\widetilde{X}^2$, respectively such that $\lambda(xy) \leq \min\{\mu(x), \mu(y)\}$ for all x, $y \in X$. Note that λ is a fuzzy relation on μ, and $\lambda(xy) = 0$ for all $xy \in \widetilde{X}^2 - E$.

Definition 2.4. [40] A bipolar fuzzy set on a nonempty set X is an object of the form $C = \{(x, \mu_C^p(x), \mu_C^n(x)) : x \in X\}$ where, $\mu_C^p : X \to [0,1]$ and $\mu_C^n : X \to [-1, 0]$ are mappings.
The positive membership degree $\mu_C^p(x)$ denotes the truth or satisfaction degree of an element x to a certain property corresponding to bipolar fuzzy set C and $\mu_C^n(x)$ represents the satisfaction degree of element x to some counter property of bipolar fuzzy set C. If $\mu_C^n(x) \neq 0$ and $\mu_C^p(x) = 0$, it is the situation that x is not satisfying the property of C but satisfying the counter property to C. If $\mu_C^p(x) \neq 0$ and $\mu_C^n(x) = 0$, it is the case when x has only positive satisfaction for C. It is possible for x to be such that $\mu_C^p(x) \neq 0$ and $\mu_C^n(x) \neq 0$ when x satisfies the property of C as well as its counter property in some part of X.

Definition 2.5. [3] A bipolar fuzzy graph on a nonempty set X is a pair $G = (C, D)$ where $C = (\mu_C^p, \mu_C^n)$ is a bipolar fuzzy set on X and $D = (\mu_D^p, \mu_D^n)$ is a bipolar fuzzy relation in X such that

$$\mu_D^p(xy) \leq \mu_C^p(x) \wedge \mu_C^p(y) \text{ and } \mu_D^n(xy) \geq \mu_C^n(x) \vee \mu_C^n(y) \text{ for all } x, y \in X.$$

Note that D is a bipolar fuzzy relation on C, and $\mu_D^p(xy) > 0$, $\mu_D^n(xy) < 0$ for $xy \in X \times X$, $\mu_D^p(xy) = \mu_D^n(xy) = 0$ for $xy \in \widetilde{X}^2 - E$.
Soft set theory was proposed by Molodtsov in 1999. Let U be the universe of discourse and P the universe of all possible parameters related to the objects in U. Each parameter is a word or a sentence. In most cases, parameters are considered to be attributes, characteristics or properties of objects in U. The pair (U, P) is also known as a soft universe. The power set of U is denoted by $\mathscr{P}(U)$.

Definition 2.6. [26] A pair $\mathfrak{S} = (\phi, A)$ is called a *soft set* over U, where $A \subseteq P$ is a parameter set and $\phi : A \to \mathscr{P}(U)$ is a set-valued mapping, called the *approximate function* of the soft set $\mathfrak{S}$. In other words, a *soft set* over U is a parameterized family of subsets of U. For any $\epsilon \in A$, $\phi(\epsilon)$ may be considered as set of ϵ-approximate elements of soft set (ϕ, A).

In other words, a soft set over U is a parameterized family of subsets of universe U. Thus a soft set over U can be written in a set of ordered pairs

$$(\phi, A) = \{(x, \phi(x)) : x \in A, \phi(x) \in \mathscr{P}(U)\}.$$

Let U be an initial universe and P be a set of parameters. Let $\mathcal{P}(U)$ denote the set of all bipolar fuzzy sets of U and A, B, C be nonempty subsets of P.

Definition 2.7. [1, 8] A pair (F, A) is called a bipolar fuzzy soft set over U, where F is a mapping given by $F : A \to \mathcal{P}(U)$. Thus, a bipolar fuzzy soft set over U gives a parameterized family of bipolar fuzzy subsets of the universe U. For any $e \in A$, $F(e) = \{(x, \mu^P_{F(e)}(x), \mu^N_{F(e)}(x)) : x \in U\}$. (U, P) is the collection of all bipolar fuzzy soft sets on U with attributes from P and is said to be *bipolar fuzzy soft class*.

Definition 2.8. [1] The complement of a bipolar fuzzy soft set (F, A) is denoted as $(F, A)^c$ and is defined by $(F, A)^c = \{(x, 1 - \mu^P_{F(e)}(x), -1 - \mu^N_{F(e)}(x)) : x \in U\}$.

Definition 2.9. [1, 8] Let U be an initial universe and P the set of all parameters. $\mathcal{P}(U)$ denotes the set of all bipolar fuzzy sets of U. Let A be a subset of P. A pair (F, A) is called a bipolar fuzzy soft set over U, where bipolar fuzzy approximation function is given by $F = (F^P, F^N) : A \to \mathcal{P}(U)$.

Let $\mathcal{P}(V)$ denotes the set of all bipolar fuzzy sets of V and $\mathcal{P}(E)$ denotes the set of all bipolar fuzzy sets of E.

3. Bipolar Fuzzy Soft Graphs

Definition 3.1. A bipolar fuzzy soft graph on a nonempty set V is an ordered 3-tuple $G = (\phi, \psi, A)$ such that

(a) A is a nonempty set of parameters,
(b) (ϕ, A) is a bipolar fuzzy soft set over V,
(c) (ψ, A) is a bipolar fuzzy soft relation on V, i.e., $\psi : A \to \mathcal{P}(V \times V)$, where $\mathcal{P}(V \times V)$ is bipolar fuzzy power set,
(d) $(\phi(a), \psi(a))$ is a bipolar fuzzy graph for all $a \in A$.

That is,
$\psi^P(a)(xy) \leq \min(\phi^P(a)(x), \phi^P(a)(y))$,

$\psi^N(a)(xy) \geq \max(\phi^N(a)(x), \phi^N(a)(y)),\ \ \forall\ x, y \in V$.

The bipolar fuzzy graph $(\phi(a), \psi(a))$ is denoted by $H(a)$. Note that $\psi^P(e)(uv) = \psi^N(e)(uv) = 0,\ \ \forall uv \in V \times V - E, e \in A$. (ϕ, A) is called a bipolar fuzzy soft vertex and (ψ, A) is called a bipolar fuzzy soft edge. Thus, $((\phi, A), (\psi, A))$ is called a bipolar fuzzy soft graph if

$$\psi^P(e)(uv) \leq \min(\phi^P(e)(u), \phi^P(e)(v)),\ \psi^N(e)(uv) \geq \max(\phi^N(e)(u), \phi^N(e)(v)),$$

$\forall\ e \in A,\ u, v \in V$. In other words, a bipolar fuzzy soft graph is a parameterized family of bipolar fuzzy graphs.

Throughout this paper, G^* will be used as a crisp undirected graph, and G a bipolar fuzzy soft graph.

Example 3.2. Consider a crisp undirected graph $G^* = (V, E)$ such that $V = \{u_1, u_2, u_3, u_4\}$ and $E = \{u_1u_2, u_2u_3, u_3u_1, u_1u_4, u_2u_4, u_3u_4\}$. Let $A = \{e_1, e_2\}$ be a set of parameters and let (ϕ, A) be a bipolar fuzzy soft set over V with bipolar fuzzy approximation function $\phi : A \to \mathcal{P}(V)$ defined by
$\phi(e_1) = \{(u_1, 0.3, -0.4), (u_2, 0.5, -0.2), (u_3, 0.7, -0.1), (u_4, 0.9, 0.0)\}$,
$\phi(e_2) = \{(u_1, 0.7, -0.1), (u_2, 0.3, -0.4), (u_3, 0.4, -0.4), (u_4, 0.6, -0.2)\}$.
Let (ψ, A) be a bipolar fuzzy soft set over E with bipolar fuzzy approximation function $\psi : A \to \mathcal{P}(E)$ defined by

$\psi(e_1) = \{(u_1u_2, 0.2, -0.1), (u_2u_3, 0.4, -0.1), (u_1u_3, 0.3, -0.1), (u_2u_4, 0.2, 0.0), (u_3u_4, 0.4, 0.0)\}$,
$\psi(e_2) = \{(u_1u_2, 0.2, -0.1), (u_2u_3, 0.2, -0.3), (u_1u_3, 0.4, -0.1), (u_1u_4, 0.5, -0.1), (u_2u_4, 0.1, -0.2)\}$.
Clearly, $H(e_1) = (\phi(e_1), \psi(e_1))$ and $H(e_2) = (\phi(e_2), \psi(e_2))$ are bipolar fuzzy graphs corresponding to the parameters e_1 and e_2, respectively as shown in Figure 1. Hence

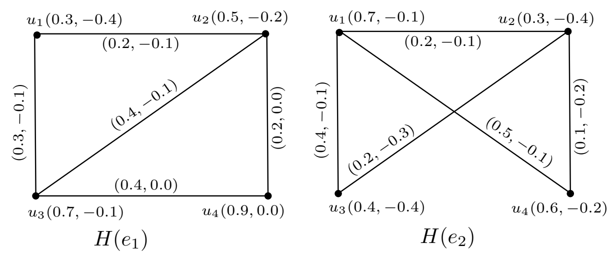

FIGURE 1. Bipolar Fuzzy Soft Graph $G = \{H(e_1), H(e_2)\}$

$G = \{H(e_1), H(e_2)\}$ is a bipolar fuzzy soft graph of G^*. Tabular representation of a bipolar fuzzy soft graph is given in Table 1.

ϕ	u_1	u_2	u_3	u_4
e_1	$(0.3, -0.4)$	$(0.5, -0.2)$	$(0.7, -0.1)$	$(0.9, 0.0)$
e_2	$(0.7, -0.1)$	$(0.3, -0.4)$	$(0.4, -0.4)$	$(0.6, -0.2)$

ψ	u_1u_2	u_2u_3	u_1u_3	u_1u_4	u_2u_4	u_3u_4
e_1	$(0.2, -0.1)$	$(0.4, -0.1)$	$(0.3, -0.1)$	$(0.0, 0.0)$	$(0.2, 0.0)$	$(0.4, 0.0)$
e_2	$(0.2, -0.1)$	$(0.2, -0.3)$	$(0.4, -0.1)$	$(0.5, -0.1)$	$(0.1, -0.2)$	$(0.0, 0.0)$

TABLE 1. Tabular Representation of a Bipolar Fuzzy Soft Graph

Proposition 3.3. *Cartesian product, composition, lexicographic product, cross product, and strong product of two bipolar fuzzy soft graphs are bipolar fuzzy soft graphs.*

Proof. By using similar arguments as used in the proofs of Propositions 2.1-2.5 of [31], the proofs are straightforward. □

Definition 3.4. A bipolar fuzzy soft graph G is a *complete bipolar fuzzy soft graph* if $B(a)$ is a complete bipolar fuzzy graph for all $a \in A$, i.e.,
$\psi^P(a)(st) = \min\{\phi^P(a)(s), \phi^P(a)(t)\}$ and

$\psi^N(a)(st) = \max\{\phi^N(a)(s), \phi^N(a)(t)\}\ \forall\ u, v \in Y, a \in A$.

Definition 3.5. A bipolar fuzzy soft graph G is a *strong bipolar fuzzy soft graph* if $B(a)$ is a strong bipolar fuzzy graph for all $a \in A$.

Proposition 3.6. *If G_1 and G_2 are strong bipolar fuzzy soft graphs, then $G_1 \ltimes G_2$, $G_1[G_2]$ and $G_1 \tilde{+} G_2$ are strong bipolar fuzzy soft graphs.*

Proof. By using similar arguments as used in [31], it is easy to prove this proposition. □

Example 3.7. Let $A = \{e_1, e_2\}$ and $B = \{e_1\}$ be the parameter sets. Let $G_1 = \{B_1(e_1), B_1(e_2)\}$ and $G_2 = \{B_2(e_1)\}$ be two strong bipolar fuzzy soft graphs as shown in Figure 2.

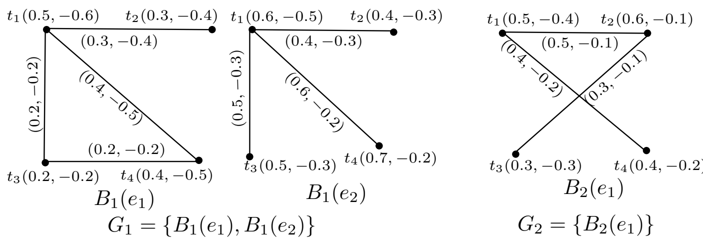

FIGURE 2. Bipolar Fuzzy Soft Graphs G_1 and G_2

Definition 3.8. Let $G = (\phi, \psi, A)$ be a bipolar fuzzy soft graph. Then G is called *an isolated bipolar fuzzy soft graph* if $\psi^P(e)(st) = 0 = \psi^N(e)(st)$ for all $(s,t) \in Y \times V, e \in A$.

Definition 3.9. The *μ-complement* of a bipolar fuzzy soft graph $G = (\phi, \psi, A)$, denoted by $G^\mu = (\phi^\mu, \psi^\mu, A^\mu)$, is defined by

(i) $\overline{A}^\mu = A$,
(ii) $\overline{\phi^\mu}(e)(s) = \phi(e)(s)$ for all $e \in A$ and $u \in Y$,
(iii) $\overline{\psi^{P\mu}}(e)(st) = \begin{cases} 0, & \text{if } \psi^P(e)(st) = 0, \\ \phi^P(e)(s) \wedge \phi^P(e)(t) - \psi^P(e)(st), & \text{if } \psi^P(e)(st) > 0, \end{cases}$

$$\overline{\psi^{N\mu}(e)}(st) = \begin{cases} 0, & \text{if } \psi^N(e)(st) = 0, \\ \phi^N(e)(s) \vee \phi^N(e)(t) - \psi^N(e)(st), & \text{if } \psi^N(e)(st) < 0. \end{cases}$$

Example 3.10. Consider an undirected graph $G^* = (Y, E)$, where $Y = \{t_1, t_2, t_3, t_4\}$ and $E = \{t_1t_2, t_2t_3, t_1t_3, t_1t_4\}$ Let $A = \{e_1, e_2\}$ and let (ϕ, A) be a bipolar fuzzy soft set over Y with its approximate function $\phi : A \to \mathcal{P}(V)$ given by
$\phi(e_1) = \{(t_1, 0.5, -0.3), (t_2, 0.4, -0.5), (t_3, 0.4, -0.2), (t_4, 0.3, -0.1)\}$,
$\phi(e_2) = \{(t_1, 0.4, -0.5), (t_2, 0.6, -0.3), (t_3, 0.4, -0.4), (t_4, 0.7, -0.2)\}$.
Let (ψ, A) be a bipolar fuzzy soft set over E with its approximate function $\psi : A \to \mathcal{P}(E)$ given by
$\psi(e_1) = \{(t_1t_2, 0.3, -0.3), (t_2t_3, 0.2, -0.1), (t_1t_4, 0.3, -0.1)\}$,
$\psi(e_2) = \{(t_1t_2, 0.3, -0.2), (t_1t_3, 0.2, -0.3), (t_1t_4, 0.3, -0.1)\}$.
By routine calculations, it is easy to see that $B(e_1)$ and $B(e_2)$ are bipolar fuzzy graphs corresponding to the parameters e_1 and e_2, respectively as shown in Figure 3. Hence $G = \{B(e_1), B(e_2)\}$ is a bipolar fuzzy soft graph. Now, the $\mu-$complement of bipolar fuzzy soft graph G is the $\mu-$complement of bipolar fuzzy graphs $B(e_1)$ and $B(e_2)$ which are shown in Figure 4.

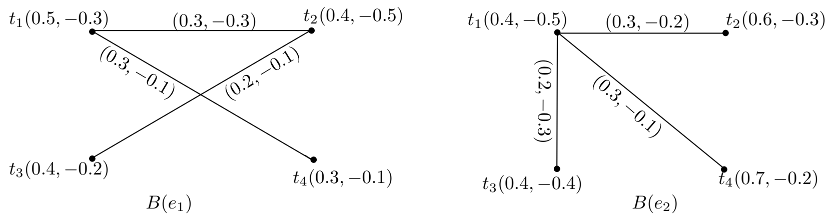

FIGURE 3. $G = \{B(e_1) = (\phi(e_1), \psi(e_1)), B(e_2) = (\phi(e_2), \psi(e_2))\}$

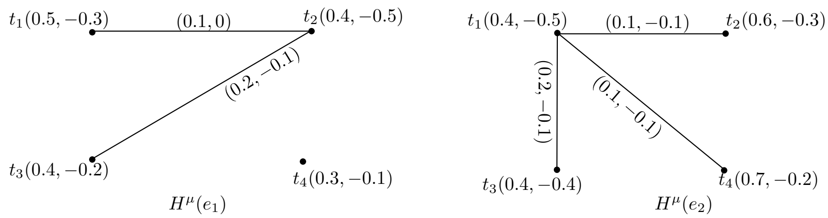

FIGURE 4. $G^\mu = \{H^\mu(e_1) = (\phi^\mu(e_1), \psi^\mu(e_1)), H^\mu(e_2) = (\phi^\mu(e_2), \psi^\mu(e_2))\}$

Theorem 3.11. *The complement of a strong bipolar fuzzy soft graph is strong bipolar fuzzy soft graph.*

Proof. Let $G = (\phi, \psi, A)$ be a strong bipolar fuzzy soft graph. By definition 3.5, we have

$$\begin{aligned}
\overline{\psi^P}(e)(st) &= \phi^P(e)(s) \wedge \phi^P(e)(t) - \psi^P(e)(st) \ \forall u, v \in Y, e \in A. \\
&= \phi^P(e)(s) \wedge \phi^P(e)(t) - \phi^P(e)(s) \wedge \phi^P(e)(t), \ \psi^P(e)(st) > 0 \\
&\quad\ \phi^P(e)(s) \wedge \phi^P(e)(t), \ \psi^P(e)(st) = 0 \\
&= 0, \ \psi^P(e)(st) > 0 \\
&\quad\ \phi^P(e)(s) \wedge \phi^P(e)(t), \ \psi^P(e)(st) = 0, \\
\overline{\psi^N}(e)(st) &= \phi^N(e)(s) \vee \phi^N(e)(t) - \psi^N(e)(st) \ \forall u, v \in Y, e \in A. \\
&= \phi^N(e)(s) \vee \phi^N(e)(t) - \phi^N(e)(s) \vee \phi^N(e)(t), \ \psi^N(e)(st) < 0 \\
&\quad\ \phi^N(e)(s) \vee \phi^N(e)(t), \ \psi^N(e)(st) = 0 \\
&= 0, \ \psi^N(e)(st) < 0 \\
&\quad\ \phi^N(e)(s) \vee \phi^N(e)(t), \ \psi^N(e)(st) = 0
\end{aligned}$$

$\overline{\psi^P}(e)(st) = \phi^P(e)(s) \wedge \phi^P(e)(t)$ and $\overline{\psi^N}(e)(st) = \phi^N(e)(s) \vee \phi^N(e)(t)$, $\overline{\psi^P}(e)(st) = 0 = \overline{\psi^N}(e)(st)$ for all $u, v \in Y$. This completes the proof. □

Theorem 3.12. *A bipolar fuzzy soft graph is an isolated bipolar fuzzy soft graph if and only if its complement is complete bipolar fuzzy soft graph.*

Proof. Let $G = (\phi, \psi, A)$ be a bipolar fuzzy soft graph and $\overline{G} = (\overline{\phi}, \overline{\psi}, \overline{A})$. Since G is an isolated bipolar fuzzy soft graph,

$$\psi^P(e)(st) = 0 = \psi^N(e)(st) \ \forall (s, t) \in Y \times V, e \in A. \tag{1}$$

By definition 3.5, we have

$$\begin{aligned}\overline{\psi^P}(e)(st) &= \phi^P(e)(s) \wedge \phi^P(e)(t) - \psi^P(e)(st),\\ \overline{\psi^N}(e)(st) &= \phi^N(e)(s) \vee \phi^N(e)(t) - \psi^N(e)(st)\end{aligned}$$

for all $(s,t) \in Y \times V, e \in A$.
It follows that

$$\begin{aligned}\overline{\psi^P}(e)(st) &= \phi^P(e)(s) \wedge \phi^P(e)(t),\\ \overline{\psi^N}(e)(st) &= \phi^N(e)(s) \vee \phi^N(e)(t)\end{aligned}$$

for all $(s,t) \in Y \times V, e \in A$. Hence $\overline{G}$ is complete bipolar fuzzy soft graph.

Conversely, let $\overline{G}$ be a complete bipolar fuzzy soft graph. Then,

$$\begin{aligned}\overline{\psi^P}(e)(st) &= \phi^P(e)(s) \wedge \phi^P(e)(t),\\ \overline{\psi^N}(e)(st) &= \phi^N(e)(s) \vee \phi^N(e)(t)\end{aligned}$$

for all $(s,t) \in Y \times V, e \in A$.
By definition 3.5, we have

$$\begin{aligned}\psi^P(e)(st) &= \phi^P(e)(s) \wedge \phi^P(e)(t) - \overline{\psi^P}(e)(st)\\ &= \overline{\psi^P}(e)(st) - \overline{\psi^P}(e)(st)\\ &= 0\\ \psi^N(e)(st) &= \phi^N(e)(s) \vee \phi^N(e)(t) - \overline{\psi^P}(e)(st)\\ &= \overline{\psi^N}(e)(st) - \overline{\psi^N}(e)(st)\\ &= 0\end{aligned}$$

for all $(s,t) \in Y \times V, e \in A$. Hence G is an isolated bipolar fuzzy soft graph. □

Theorem 3.13. $\overline{G} = (\overline{\phi}, \overline{\psi}, \overline{A})$ *has isolated nodes if and only if* G *is a strong bipolar fuzzy soft graph.*

Proof. Using similar method as used in the proof of Theorem 3.2, the proof is straightforward. □

The concepts of degree, total degree, edge regular and totally regular of bipolar fuzzy graphs are defined in [4].

Definition 3.14. Let G be a bipolar fuzzy soft graph of G^*. G is called a *regular bipolar fuzzy soft graph* if $H(e)$ is a regular bipolar fuzzy graph for all $e \in A$. If $H(e)$ is a regular bipolar fuzzy graph of degree $(r, \acute{r})$ for all $e \in A$, then G is a $(r, \acute{r})-$regular bipolar fuzzy soft graph.

Example 3.15. Consider a simple graph $G^* = (V, E)$, where $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_2v_3, v_3v_4, v_3v_1, v_1v_4\}$. Let $A = \{e_1, e_2\}$ be a set of parameters. Let $G = (H, A)$ be a bipolar fuzzy soft graph, where bipolar fuzzy graphs $H(e_1)$ and $H(e_2)$ corresponding to the parameters e_1 and e_2 are defined as follows:
$H(e_1) = \big(\{(v_1, 0.5, -0.4), (v_2, 0.5, -0.4), (v_3, 0.6, -0.5), (v_4, 0.6, -0.6)\},$
$\{(v_1v_2, 0.4, -0.3), (v_2v_4, 0.5, -0.4), (v_3v_4, 0.4, -0.3), (v_4v_1, 0.5, -0.4)\}\big),$
$H(e_2) = \big(\{(v_1, 0.7, -0.5), (v_2, 0.5, -0.4), (v_3, 0.4, -0.6), (v_4, 0.6, -0.5)\},$

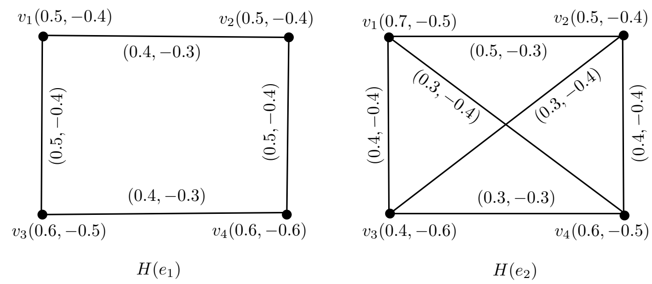

FIGURE 5. Regular Bipolar Fuzzy Soft Graph $G = \{H(e_1), H(e_2)$

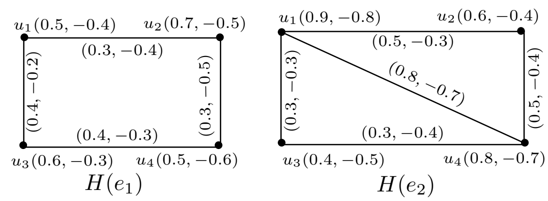

FIGURE 6. Edge Regular Bipolar Fuzzy Soft Graph $G = \{H(e_1), H(e_2))\}$

$\{(v_1v_2, 0.5, -0.3), (v_2v_3, 0.3, -0.4), (v_3v_4, 0.3, -0.3), (v_4v_1, 0.3, -0.4),$
$(v_2v_4, 0.4, -0.4), (v_1v_3, 0.4, 0.4)\}\big)$. Clearly, bipolar fuzzy graphs $H(e_1)$ and $H(e_2)$ corresponding to the parameter e_1 and e_2 are regular bipolar fuzzy graphs. Hence G is a regular bipolar fuzzy soft graph.

Definition 3.16. Let G be a bipolar fuzzy soft graph of G^*. G is said to be a *totally regular bipolar fuzzy soft graph* if $H(e)$ is a totally regular bipolar fuzzy graph for all $e \in A$. If $H(e)$ is a totally regular bipolar fuzzy graph of total degree $(t, \acute{t})$ for all $e \in A$, then G is a $(t, \acute{t})-$totally regular bipolar fuzzy soft graph.

Definition 3.17. Let $G = (\phi, \psi, A)$ be a bipolar fuzzy soft graph of G^*. G is said to be an *edge regular bipolar fuzzy soft graph* if $H(e)$ is an edge regular bipolar fuzzy graph for all $e \in A$. If $H(e)$ is an edge regular bipolar fuzzy graph of degree $(l, \acute{l})$ for all $e \in A$, then G is a $(l, \acute{l})-$edge regular bipolar fuzzy soft graph.

Example 3.18. Consider a simple graph $G^* = (V, E)$ such that $V = \{u_1, u_2, u_3, u_4\}$ and $E = \{u_1u_2, u_1u_3, u_1u_4, u_2u_4, u_3u_4\}$. Let $A = \{e_1, e_2\}$ be set of parameters. Let $G = (H, A)$ be an edge regular bipolar fuzzy soft graph, where edge regular bipolar fuzzy graphs are defined as follows:
$H(e_1) = \big(\{(u_1, 0.5, -0.4), (u_2, 0.7, -0.5), (u_3, 0.6, -0.3), (u_4, 0.5, -0.6)\},$
$\{(u_1u_2, 0.3, -0.4), (u_2u_4, 0.3, -0.5), (u_3u_4, 0.4, -0.3), (u_1u_3, 0.4, -0.2)\}\big),$
$H(e_2) = \big(\{(u_1, 0.9, -0.8), (u_2, 0.6, -0.4), (u_3, 0.4, -0.5), (u_4, 0.8, -0.7)\},$
$\{(u_1u_2, 0.5, -0.3), (u_1u_4, 0.8, -0.7), (u_1u_3, 0.3, -0.3), (u_2u_4, 0.5, -0.4),$
$(u_3u_4, 0.3, -0.4)\}\big).$

Definition 3.19. Let $G = (\phi, \psi, A)$ be a bipolar fuzzy soft graph of G^*. G is said to be a *totally edge regular bipolar fuzzy soft graph* if $H(e)$ is a totally edge regular bipolar fuzzy graph for all $e \in A$. If $H(e)$ is a totally edge regular bipolar fuzzy graph of total edge degree $(t_1, \acute{t_1})$ for all $e \in A$, then G is a $(t_1, \acute{t_1})-$totally edge regular bipolar fuzzy soft graph.

Definition 3.20. Let $G = (\phi, \psi, A)$ be a bipolar fuzzy soft graph of G^*. The *maximum edge degree* of G is $\Delta_E(G) = (\Delta^P(G), \Delta^N(G))$, where
$\Delta^P(G) = \vee\{d^P(v_iv_j)(e_k) \mid v_iv_j \in E,\ e_k \in A\}$ and
$\Delta^N(G) = \wedge\{d^N(v_iv_j)(e_k) \mid v_iv_j \in E, e_k \in A\}$.

Definition 3.21. Let $G = (\phi, \psi, A)$ be a bipolar fuzzy soft graph of G^*. The *minimum edge degree* of G is $\delta_E(G) = (\delta^P(G), \delta^N(G))$, where
$\delta^P(G) = \wedge\{d^P(v_iv_j)(e_k) \mid v_iv_j \in E,\ e_k \in A\}$ and
$\delta^N(G) = \vee\{d^N(v_iv_j)(e_k) \mid v_iv_j \in E,\ e_k \in A\}$.

Theorem 3.22. *Let $G = (\phi, \psi, A)$ be a bipolar fuzzy soft graph on a cycle $G^* = (V, E)$. Then*
$$\sum_{e_k \in A} \sum_{v_i \in V} d_G(u_i)(e_k) = \sum_{e_k \in A} \sum_{u_iu_j \in E} d_G(u_iu_j)(e_k).$$

Proof. Using similar method as used in the proof of Theorem 4.9 of [32], the proof is straightforward. □

Theorem 3.23. *Let $G = (\phi, \psi, A)$ be a bipolar fuzzy soft graph on a crisp graph G^*. If ψ is a constant function then G is edge regular bipolar fuzzy soft graph if and only if G^* is edge regular.*

Proof. Using similar method as in Theorem 4.10 of [32], the proof is straightforward. □

Theorem 3.24. *Let $G = (\phi, \psi, A)$ be a regular bipolar fuzzy soft graph, then G is edge regular bipolar fuzzy soft graph if and only if ψ is a constant function.*

Proof. By using similar arguments as used in proof of Theorem 4.11 of [32], it is easy to prove it. □

4. Applications to Multiple Criteria Decision-Making Problems

Many practical problems can be represented by graphs. In this section, we present applications of the bipolar fuzzy soft graphs to multiple criteria decision-making problems. Bipolar fuzzy graphs equipped with a parametrization tool are more suitable for decision-making problems.

1. Decision-making problem in social network
In the study of human group behavior in a network, it is noticed that individual ideas have considerable influence on others. A bipolar fuzzy soft graph can be used to model such behavior. We present an Algorithm 4.1 for most appropriate selection of an object in a social parameterized network.

Algorithm 4.1.

1. *Input the set of parameters $e_1, e_2, \ldots, e_k$.*
2. *Input the bipolar fuzzy soft sets (ϕ, A) and (ψ, A).*
3.. *Input the bipolar fuzzy graphs $H(e_1), H(e_2), \ldots, H(e_k)$.*

4. *Calculate the score values of bipolar fuzzy graphs* $H(e_1), H(e_2), \ldots, H(e_k)$ *using formula*

$$S_{ij} := (\mu^P)_j + (\mu^N)_j * d_j, \qquad d_j = \sqrt{(\mu^P)_j^2 + (\mu^N)_j^2}, \tag{2}$$

where d_j *is the distance between membership poles of edge in* $H(e_k)$*.*
5. *Compute the choice values of* $C_p = \sum_j S_{ij}$ *for all* $i = 1, 2, \ldots, n$ *and* $p = 1, 2, \ldots, k$*.*
6. *The decision is* S_i *if* $S_i = \max_{i=1}^{n}\{\min_{p=1}^{k} C_p\}$*.*
7. *IF* i *has more than one value THEN any one of* S_i *may be chosen.*

Let $P = \{$"hard working", "committed", "cooperative", "team spirit", "creative", "ambitious"$\}$ be a set of attributes or be the set of parameters. In bipolar fuzzy soft graph $G = (\phi, \psi, A)$, vertices represent employees and their positive membership degrees while the negative membership degrees represent the implicit counter-property value corresponding to the given parameters e_1 ="hard working", e_2 ="team spirit", and e_3 ="creative" and edges represent the influential relation of employees corresponding to the parameters, then we can find out the most perfect or valuable employee within the network. We now develop a bipolar fuzzy soft model to find out the most valuable employee in the department of industry with respect to different attributes of employees, including hard working, team spirit, creative.

Name	Designation
Suleman	Chief engineer
Rehman	Exective engineer
Adeel	Additional director
Imran	Joint director
Kamran	Deputy development officer
Bashir	Deputy information officer
Naeem	Supporting staff
Akbar	Operation manager
Zain	Accountant
Tariq	Sale supervisor

TABLE 2. Name of Employees in an Industry and Their Designations

Consider an undirected graph with vertex set V ={Suleman, Rehman, Adeel, Imran, Kamran, Bashir, Naeem, Akbar, Zain, Tariq}. Let $A = \{e_1, e_2, e_3\} \subseteq P$ be set of attributes. A bipolar fuzzy soft graph $G = \{H(e) = (\phi(e), \psi(e)) : \ e \in A\}$ is shown in Figure 7. The score values of bipolar fuzzy graphs using formula 2 and choice values are given in Tables 3, 4 and 5.

The decision value is $S_i = \max_i(\min_k C_k)$

$= \max_{i=1}^{10}\{0.35, 0.373, 0.2, 0.376, 0.33, 0.715, 0.31, 0.3, 0.38, 0.24\} = 0.715.$

Clearly, Bashir is more combative employee having maximum score value is 0.175 scored by Bashir in the industry.

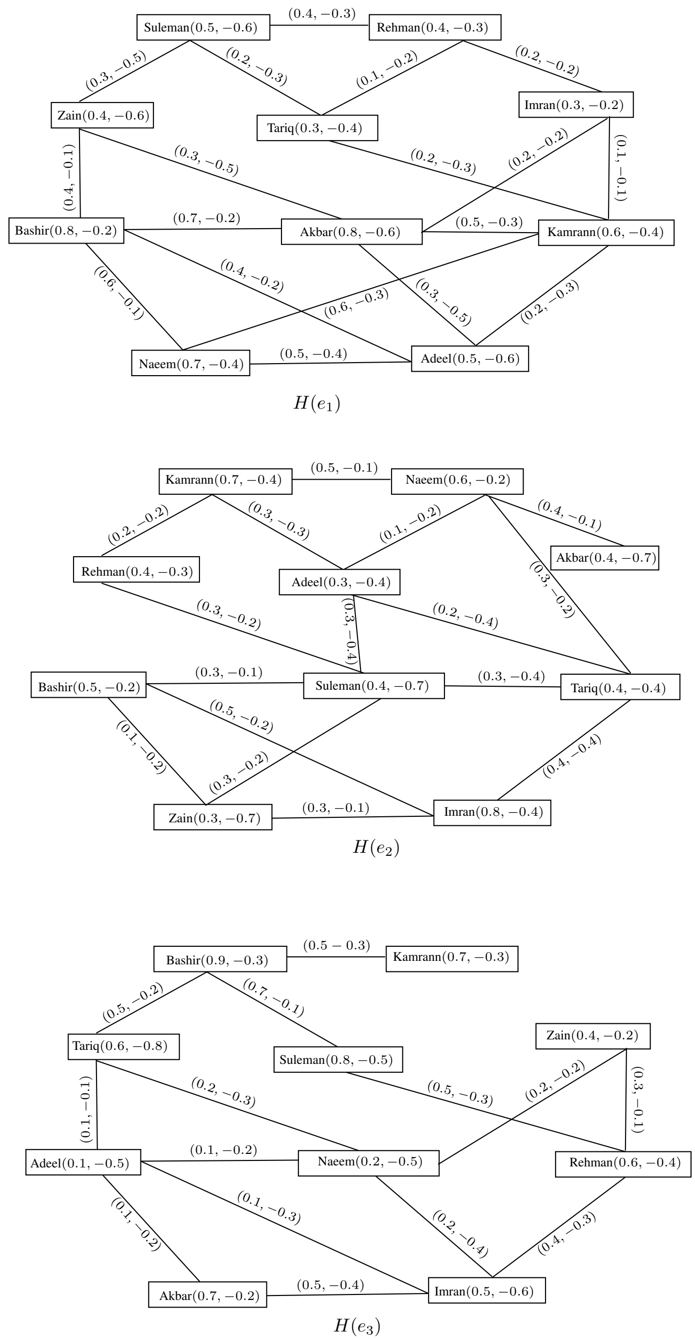

FIGURE 7. $G = \{H(e_1), H(e_2), H(e_3)\}$

	Suleman	Rehman	Adeel	Imran	Kamran	Bashir	Naeem	Akbar	Zain	Tariq	C_1
Suleman	0	0.25	0	0	0	0	0	0	0.01	0.09	0.35
Rehman	0.25	0	0	0.143	0	0	0	0	0	0.06	0.453
Adeel	0	0	0	0	0.09	0.31	0.243	0.01	0	0	0.653
Imran	0	0.143	0	0	0.09	0	0	0.143	0	0	0.376
Kamran	0	0	0.09	0.09	0	0	0.39	0.33	0	0.09	0.99
Bashir	0	0	0.31	0	0	0	0.54	0.55	0.36	0	1.76
Naeem	0	0	0.243	0	0.39	0.54	0	0	0	0	1.173
Akbar	0	0	0.01	0.143	0.33	0.55	0	0	0.01	0	1.0
Zain	0.01	0	0	0	0	0.36	0	0.01	0	0	0.38
Tariq	0.09	0.06	0	0	0.09	0	0	0	0	0	0.24

TABLE 3. Score Values and Choice Values of $H(e_1)$

	Suleman	Rehman	Adeel	Imran	Kamran	Bashir	Naeem	Akbar	Zain	Tariq	C_2
Suleman	0	0.23	0.1	0	0	0.27	0	0	0.23	0.1	0.93
Rehman	0.23	0	0	0	0.143	0	0	0	0	0	0.373
Adeel	0.1	0	0	0	0.172	0	0.055	0	0	0.02	0.347
Imran	0	0	0	0	0	0.39	0	0	0.27	0.174	0.834
Kamran	0	0.143	0.172	0	0	0	0.449	0	0	0	0.764
Bashir	0.27	0	0	0.39	0	0	0	0	0.055	0	0.715
Naeem	0	0	0.055	0	0.449	0	0	0.36	0	0	0.864
Akbar	0	0	0	0	0	0	0.36	0	0	0.23	0.59
Zain	0.23	0	0	0.27	0	0.055	0	0	0	0	0.555
Tariq	0.1	0	0.02	0.174	0	0	0.23	0	0	0	0.524

TABLE 4. Score Values and Choice Values of $H(e_2)$

	Suleman	Rehman	Adeel	Imran	Kamran	Bashir	Naeem	Akbar	Zain	Tariq	C_3
Suleman	0	0.33	0	0	0	0.63	0	0	0	0	0.96
Rehman	0.33	0	0	0.25	0	0	0	0	0.27	0	0.85
Adeel	0	0	0	0.005	0	0	0.055	0.055	0	0.09	0.2
Imran	0	0.25	0.005	0	0	0	0.02	0.243	0	0	0.518
Kamran	0	0	0	0	0	0.33	0	0	0	0	0.33
Bashir	0.63	0	0	0	0.33	0	0	0	0	0.39	1.35
Naeem	0	0	0.055	0.02	0	0	0	0	0.143	0.09	0.31
Akbar	0	0	0.055	0.243	0	0	0	0	0	0	0.3
Zain	0	0.27	0	0	0	0	0.143	0	0	0	0.413
Tariq	0	0	0.09	0	0	0.39	0.09	0	0	0	0.57

TABLE 5. Score Values and Choice Values of $H(e_3)$

2. Decision-making problem for investment in shares

Shares are tiny fractional units of a company and one of the major investment phenomenon in business market. They carry huge amount of risk but on the other hand they can return highest benefits. Therefore, the major problem is to decide which company's shares are right for investment. Business market has bipolar behavior, i.e., profit and loss, value of shares increase and decrease at the same time. These are parametric factors of shares in market. This phenomenon can be discussed using bipolar fuzzy soft graphs. We discuss here a multi-criteria decision making problem for investment in shares if there are multiple alternative companies. Assume that there are five interconnected companies C_1, C_2, C_3, C_4 and C_5 selling their shares in market. These companies can be represented by a bipolar fuzzy soft graph with respect to parameters e_1 =profit and e_2 =value of shares, in which vertices represent companies and edges represent relationship between companies. The positive degree of membership of vertices in $H(e_1)$ represents profit and negative degree of membership represents loss. The positive degree of membership of vertices in $H(e_2)$ represents percentage increase in the value of shares and negative degree of membership represents percentage decrease. The bipolar fuzzy soft graph is shown in Figure 8. The method for selection of investment in a company in presented in Algorithm 4.2.

Algorithm 4.2.

1. *Input the set of parameters* $e_1, e_2, \ldots, e_k$.
2. *Input the bipolar fuzzy soft set of companies* $C_1, C_2, \ldots, C_n$.
3. *Construct the table of membership values of companies with respect to k parameters.*
4. *Determine the average values using formula 3. This formula determines the average membership value with respect to all parameters. It calculates a single value that represents the bipolar behavior of each company among the other group of companies. Under this formula, the company's expected strength can be calculated to know its actual power in the market.*

$$\eta_i = (\sum_{j=1}^{k} C_j^P - \prod_{j=1}^{k} C_j^P, -\prod_{j=1}^{k} |C_j^N|), \qquad i = 1, 2, \ldots, n. \tag{3}$$

5. *Choose a weight vector* $\boldsymbol{w} = (w_1, w_2, \ldots, w_n)$ *and multiply each average value with corresponding weight value as,*

$$\xi_i = \eta_i w_i, \qquad i = 1, 2, \ldots, n.$$

6. *Calculate the normalized value of each company using formula 4,*

$$\gamma_i = \sqrt{(\xi_i^P)^2 + (1 - \xi_i^N)^2}. \tag{4}$$

7. *Choose a company with maximum normalized value for investment.*

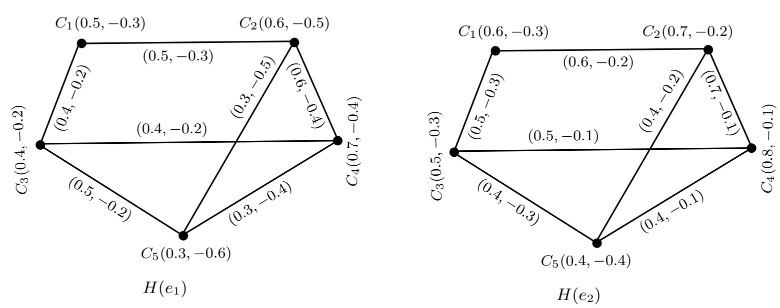

FIGURE 8. Bipolar Fuzzy Soft Graph of Companies

We now apply Algorithm 4.2 to select a company for investment in shares. The membership values of the companies with respect to parameters e_1 and e_2 are given in Table 6 and the average values using formula 3 are given Table 7.

Company	e_1	e_2
C_1	$(0.5, -0.3)$	$(0.6, -0.3)$
C_2	$(0.6, -0.5)$	$(0.7, -0.2)$
C_3	$(0.4, -0.2)$	$(0.5, -0.3)$
C_4	$(0.7, -0.4)$	$(0.8, -0.1)$
C_5	$(0.3, -0.6)$	$(0.4, -0.4)$

Table 6. Membership Values of Companies

Company	η_i
C_1	$(0.8, -0.09)$
C_2	$(0.88, -0.10)$
C_3	$(0.7, -0.06)$
C_4	$(0.94, -0.04)$
C_5	$(0.58, -0.24)$

Table 7. Average Values

With respect to weight vector $\boldsymbol{w} = (0.6, 0.7, 0.5, 0.4, 0.3)$, the weighted average values are given in Table 8 and normalized values are given in Table 9.

Company	ξ_i
C_1	$(0.48, -0.054)$
C_2	$(0.616, -0.07)$
C_3	$(0.35, -0.03)$
C_4	$(0.376, -0.016)$
C_5	$(0.174, -0.072)$

Table 8. Weighted Average Values

Company	η_i
C_1	1.1582
C_2	1.4087
C_3	1.2703
C_4	1.4019
C_5	1.3689

Table 9. Normalized Values

C_2 has maximum normalized value which shows that highest benefits can be obtained by investing in company C_2's shares.

3. Detection of bipolar disorder in children

Bipolar disorder among children is a serious mental illness that causes strange mood changes from happy, stressed, energetic and lazy than usual. This phenomenon is curious that whether children can be diagnosed from bipolar disorder or not.

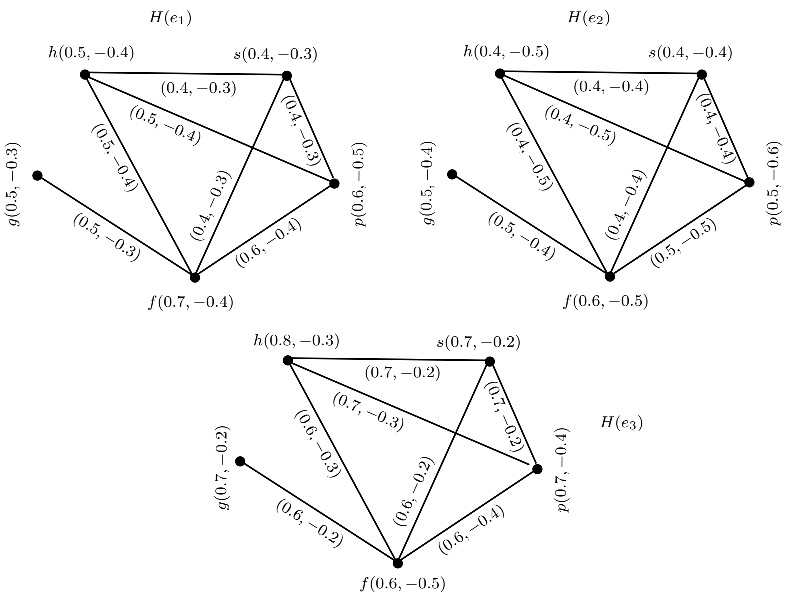

FIGURE 9. Bipolar Disorder in a Child

The detection and diagnosis of bipolar disorder in children is difficult because it is similar to normal children behavior and doctors do not use any lab tests. Bipolar fuzzy soft graphs can be used to detect bipolar disorder in a child using anxiety, co-operation and behavior of a child at school, home, games and with parents and friends. The method for the detection of bipolar disorder in a child is given in Algorithm 4.3.

Algorithm 4.3.

1. *Input the set of parameters* $e_1, e_2, \ldots, e_k$.
2. *Enter the membership values of vertices* {*h=home, s=school, g=games, f=friends, p=parents*} *and edges with respect to parameters.*
3. *Construct the bipolar fuzzy graph* $H(e_1) \cap H(e_2) \cap \ldots \cap H(e_k)$.
4. *Calculate the normalized values of all the vertices as,*
$x' = \sqrt{(x^P)^2 + (1 - x^N)^2}, \qquad i = 1, 2, \ldots, 5.$
5. *Find the average of all normalized values.*
6. *IF average is greater than* 1 *THEN the child is suffering from bipolar disorder.*

An example of a child behavior history with respect to parameters e_1 =co-operation, e_2 =behavior and e_3 =anxiety is shown in Figure 9. The degree of membership of each vertex in $H(e_1)$ represents child co-operation and laziness at home, school, games, with parents and friends. The degree of membership of each vertex in $H(e_2)$ represents the degree of normal and abnormal behavior and in $H(e_3)$, the degree of anxiety and happiness. The bipolar fuzzy graph $H(e_1) \cap H(e_2) \cap H(e_3)$ is shown in Figure 10 and normalized values of all the vertices are shown in Table 10.

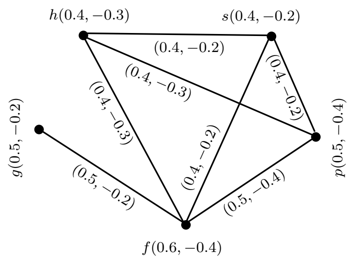

FIGURE 10. $H(e_1) \cap H(e_2) \cap H(e_3)$

x	x'
h	1.3601
s	1.7436
p	1.4866
f	1.5232
g	1.3

TABLE 10. Normalized Values

The average of normalized values is 1.4827 which shows that the child is suffering from bipolar disorder.

5. Conclusions

The fuzzy modeling has been applied in a wide variety of fields, including engineering and management sciences and social sciences to solve a number decision-making problems which involve impreciseness, uncertainty and vagueness in data. In particular, applications of this modeling technique in decision-making problems have remarkable significance. These problems have been tackled using various models, including probability, fuzzy models, intuitionistic fuzzy models, bipolar fuzzy models, which lack in parametrization of the tools due to which they could not be applied successfully to such problems.

Intuitionistic fuzzy soft models and bipolar fuzzy soft models are two different soft computing models which have a promising potential for giving an optimal solution for complex problems. In this paper, we have presented applications of the bipolar fuzzy soft graphs in a multiple criteria decision-making problem. We plan to extend our research of fuzzification to (1) Interval-valued fuzzy soft graphs, (2) Bipolar fuzzy soft hypergraphs, (3) Rough fuzzy soft graphs, and (4) Application of bipolar fuzzy soft graphs to decision support systems.

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