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A novel parametric ranking method for intuitionistic fuzzy numbers

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Abstract

Since the inception of intuitionistic fuzzy sets in 1986, many authors have proposed different methods for ranking intuitionistic fuzzy numbers (IFNs). However, due to the complexity of the problem, a method which gives a satisfactory result to all situations is a challenging task. Most of them contained some shortcomings, such as requirement of complicated calculations, inconsistency with human intuition and indiscrimination and some produce different rankings for the same situation and some methods cannot rank crisp numbers. For overcoming the above problems, in this paper, a new parametric ranking method for IFNs is proposed. It is developed based on the concept α-cuts and β-cuts and area on left side of IFNs. The proposed ranking method is applied to solve partner selection problem in which the rating of partner on attributes are expressed by using triangular IFNs. The proposed method is much simpler and more efficient than other methods in the literature. Some comparative examples are also given to illustrate the advantages of the proposed method.

Keywords: Intuitionistic fuzzy number, Ranking, α-cuts, β-cuts.

1 Introduction

It is known that there is no unique linear ordering in a family of IFNs. Thus ranking IFNs is one of the fundamental problems of fuzzy arithmetic. Ranking IFN play a key role in decision-making procedures. In fuzzy decision analysis, IFNs are frequently employed to describe the performance of alternatives in modeling a real world problem. The decision makers assess the alternatives with IFNs and the selection of alternatives will eventually lead to the ranking of the corresponding IFNs. Furthermore, the concept of optimum and best choice to come true are also completely based on the ranking or comparison of IFNs. Thus, specific ranking of IFNs is an important procedure for decision-making in a fuzzy environment and generally has become one of the main problems in fuzzy theory.

In order to rank IFNs, one IFN needs to be evaluated and compared to others but this may not be easy. As known, the real numbers in can be linearly ordered by $\geq$; however, IFNs cannot be done in such a way. Since IFNs are represented by Possibility distributions, they can overlap with each other and it is difficult to determine clearly whether one IFN is larger or smaller than the other.

The ranking methods can be classified in three categories. The first category directly transforms each IFN into a crisp real number and the second category compares a IFN to all the other n−1 IFNs to obtain its mapping into a positive real number. The third category differs substantially from the first two. In this category, a method for pair wise ranking or preference for all pairs of IFNs is determined and then based on these pair wise orderings, a final order of the n IFNs is attempted. Two factors play significant roles in fuzzy decision systems:

- Contribution of the decision-maker in the decision making process,
- Simplicity of calculation.

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It is worthwhile to notice that the precision and characteristics of ranking methods have great influence on the decision results. Such influence can be summarized into two aspects: one is the precision of the ranking method and the another is the way that the IFNs are compared.

1.1 The Literature Review

Numerous methods have been proposed in literature to rank IFN. Also, different methods satisfy different desirable criteria. Many of these methods are based on the area measurement with the integral value about the membership function of IFN. In following, we review some exiting methods to ranking IFNs.
Chen and Tan [5] provided a score function to compare IFNs. Hong and Choi [11] pointed out the defects and proposed an improved technique based on the score function and accuracy function. Later, Li [15] gave a series of improved score functions. The mentioned functions are called evaluation functions. By using these evaluation functions, we can obtain certain rank of the IFNs. Grzegorzewski [10] defined two families of metrics in the space of IFNs and proposed a ranking method for IFNs based on these metrics. Mitchell [19] extended the natural ordering of real numbers to triangular IFNs by adopting a statistical view point and interpreting each IFN as ensemble of ordinary fuzzy numbers. Li et al. [16, 17] defined the value and ambiguity of a TIFN and given the lexicographic ranking method and the ratio ranking method for TIFNs, respectively, which are applied to MADM problems in which the ratings of alternatives on attributes are expressed by using TIFNs. Nayagam et al. [29] introduced triangular IFNs of special type and described a method to rank them. Although their ranking method appears to be attractive, the very definition of triangular IFN seems unrealistic. This is because the triangular non-membership function is defined to geometrically behave in an identical manner as the membership function. Su [25] investigated the signed distance method for ranking interval valued fuzzy numbers, which for triangular fuzzy numbers becomes analogous to the centroid method. Nehi [20] put forward a new ordering method for IFNs in which two characteristic values for IFNs are defined by the integral of the inverse fuzzy membership and non-membership functions multiplied by the grade with powered parameter. Rezvani [22] used a value index to rank TRIFNs. A compromise ratio ranking method for TIFNs was developed on the value-index and ambiguity-index of TIFNs in [34]. Xu and Yager [30] proposed score function and accuracy function to rank interval valued IFNs. Ye [31] also proposed a novel accuracy function to rank interval valued IFNs. Zeng al. [33] proposed a value and ambiguity-based ranking method. Wan and Dong [26] developed a new lexicographic method to rank TrIFNs. A new ranking method of TRIFNs was developed by utilizing the concept of centroid point in [8]. Also, new ranking technique based on signed distance is defined by Aggarwal and gupta in [1]. In [12] a linear (total) ordering on the class of trapezoidal intuitionistic fuzzy numbers using axiomatic set of eight different scores was introduced. Nayagam [14] proposed a total order on the entire class of IFN using upper lower dense sequence in the interval [0, 1]. A new ranking method based on the centroid was proposed for TrIFNs and TIFNs by Prakash and et al. [22].

1.2 The Main Aim of This Paper

After analyzing the aforementioned ranking procedure, it is observed that, in some case, they fail to calculate the ranking result correctly. Under these circumstances, the decision maker may not be able to carry out the comparison and recognition properly. This creates a problem in practical application. In order to overcome the shortcoming of the exiting methods, a new method for ranking IFNs is proposed in this paper. We propose an area measurement based method for ranking IFN. This method is very easy to use with respect to the complexity of many other methods. Our method is interested in ranking any IFN (triangular, trapezoidal, etc). In this paper we concentrate our attention on the ranking process based on α-cuts and β-cuts. We will begin by computing area on left side of IFN. Then we will rank IFN according to left side values. Further, this method uses an index of optimism to reflect the decision maker's optimistic attitude and also uses an index of pessimistic that represents the pessimism of the decision maker. In fact, by choosing values close to zero for α and β values close to one the decision maker reflects the pessimism. And by selecting values close to one for α and β values close to zero the decision maker reflects the optimism. This paper is an extension of the work of Shureshjani and Darehmiraki [24].
The remainder of the paper is organized as follows. Section 2 contains preliminaries followed by section 3 which is consisting of proposed ranking method for IFNs. Numerical examples and comparison analysis are given in Section 4. In Section 5 a set of examples is provided to show significance of the proposed method and to compare with other methods. An application of proposed ranking method is presented in Section 6. This paper concludes in Section 7.

2 Preliminaries

As a preparation for introducing our new method, some relevant concepts are illustrated in this section. In the following, let's review some basic concepts and operations, which will be useful in the text sections. Atanassov [3] extended fuzzy set to intuitionistic fuzzy set by adding a non-membership function, which is expressed as following

Definition 2.1. *Let X be a fixed set, an intuitionistic fuzzy set (IFS) A in X is defined as*

$$A = \{(x, \mu(x), \nu(x))|x \in X\}$$

where the functions $\mu_A(x) : X \longrightarrow [0, 1]$, $\nu_A(x) : X \longrightarrow [0, 1]$ are the degrees of membership and nonmembership of an element , respectively, satisfying $0 \leq \mu(x) + \nu(x) \leq 1$ [3].

Definition 2.2. *A fuzzy number u in parametric form is a pair $(\underline{u}(r), \overline{u}(r))$ of functions $\underline{u}(r)$, $\overline{u}(r)$, $0 \leq r \leq 1$, which satisfy the following requirements:*

1. *$\underline{u}(r)$ is a bounded left continuous non-decreasing function over [0,1],*
2. *$\overline{u}(r)$ is a bounded left continuous non-increasing function over [0,1],*
3. *$\underline{u}(r) \leq \overline{u}(r)$, $0 \leq r \leq 1$.*

ω is an arbitrary constant between zero and one $(0 < \omega \leq 1)$. A crisp number "k" is simply represented by $\underline{u}(r) = \overline{u}(r) = k$, $0 \leq r \leq 1$ [18].

Definition 2.3. *An intuitionistic fuzzy subset $A = \{(x, \mu_A(x), \nu_A(x)) : x \in \mathbf{R}\}$ of the real line is called an intuitionistic fuzzy number if*

1. *A is If-convex (i.e. its membership function μ is fuzzy convex and its non membership function ν is fuzzy concave),*
2. *μ_A is upper semi continuous and ν_A is lower semicontinuous,*
3. *$supp(A) = cl(\{x \in X : \nu_A(x) < 1\})$ is bounded.*

Similar to definition (2.1) we could seem parametric form to IFN. A IFN y in parametric form is a $(\underline{u}(r), \overline{u}(r), \underline{v}(r), \overline{v}(r))$ of functions $\underline{u}(r), \overline{u}(r), \underline{v}(r), \overline{v}(r)$, $0 \leq r \leq 1$, which satisfy the above requirements [2].

In special case a trapezoidal fuzzy number is defined as:

Definition 2.4. *A TRIFN $\hat{a} = \langle(a_1, a_2, a_3, a_4); u_{\hat{a}}, w_{\hat{a}}\rangle$ is a special IFN on a set of real number $\mathbb{R}$, whose membership function and non membership function are defined as follows:*

$$\mu_{\hat{a}}(x) \begin{cases} \dfrac{(x - a_1)}{(a_2 - a_1)} w_{\hat{a}}, & a_1 \leq x \leq a_2, \\ w_{\hat{a}} & a_2 \leq x \leq a_3, \\ \dfrac{(a_4 - x)}{(a_4 - a_3)} w_{\hat{a}}, & a_3 \leq x \leq a_4, \\ 0, & a_4 < x,\ x < a_1, \end{cases}$$

$$\begin{cases} \dfrac{(a_2 - x) + u_{\hat{a}}(x - a_1)}{(a_2 - a_1)},\ a_1 \leq x \leq a_2, \\ u_{\hat{a}},\ a_2 \leq x \leq a_3, \\ \dfrac{(x - a_3) + u_{\hat{a}}(x - a_1)}{(a_2 - a_1)},\ a_3 \leq x \leq a_4, \\ 1,\ x > a_4, x < a_1. \end{cases}$$

The values $w_{\hat{a}}$ and $u_{\hat{a}}$ respectively represent the maximum membership degree and the minimum non-membership degree which satisfy the conditions, that is, $0 \leq w_{\hat{a}} \leq 1$, $0 \leq u_{\hat{a}} \leq 1$ and $0 \leq w_{\hat{a}} + u_{\hat{a}} \leq 1$. If $a_2 = a_3$, trapezoidal IFN changed to triangular IFN [17].

Definition 2.5. *A α-cut set of IFN $\hat{a}$ is a crisp subset of* $\mathbf{R}$ *which is defined as*

$$\hat{a}_\alpha = \{x|\mu_{\hat{a}} \geq \alpha\}$$

where $0 \leq \alpha \leq w_{\hat{a}}$[3].

Definition 2.6. *A β-cut set of IFN $\hat{a}$ is a crisp subset of* $\mathbf{R}$ *which is defined as*

$$\hat{a}_\beta = \{x|\nu_{\hat{a}} \leq \beta\}$$

where $u_{\hat{a}} \leq \beta \leq 1$[3].

Definition 2.7. *Let $\hat{a} = \langle(a_1, a_2, a_3, a_4); w_{\hat{a}}, u_{\hat{a}}\rangle$ and $\hat{b} = \langle(b_1, b_2, b_3, b_4); w_{\hat{b}}, u_{\hat{b}}\rangle$ be two TRIFN and λ be a real number. The arithmetical operations are* [4],

$$\hat{a} + \hat{b} = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); min\{w_{\hat{a}}, w_{\hat{b}}\}, max\{u_{\hat{a}}, u_{\hat{b}}\}\rangle$$
$$\hat{a}\hat{b} = \langle(a_1b_1, a_2b_2, a_3b_3, a_4b_4); min\{w_{\hat{a}}, w_{\hat{b}}\}, max\{u_{\hat{a}}, u_{\hat{b}}\}\rangle$$
$$\lambda\hat{a} = \langle(\lambda a_1, \lambda a_2, \lambda_3, \lambda a_4); 1 - (1 - w_{\hat{a}})^\lambda, u_{\hat{a}}^\lambda\rangle, (\lambda \geq 0)$$
$$\hat{a}^\lambda = \langle(a_1^\lambda, a_2^\lambda, a_3^\lambda, a_4^\lambda); w_{\hat{a}}^\lambda, 1 - (1 - u_{\hat{a}})^\lambda\rangle, (\lambda \geq 0).$$

Definition 2.8. *An IFN $\hat{a} = (\mu_A(x), \nu_A(x))$ is called a crisp value IFN if the following holds:*

$$\forall x \quad \mu_A(x) = 1 \ \nu_A(x) = 0.$$

3 A New Ranking Method for Intuitionistic Fuzzy Number

Let $\hat{y} = (\underline{u}(r), \overline{u}(r), \underline{v}(r), \overline{v}(r))$ be an IFN based on mentioned definition of an IFN. Value $Q_{\alpha,\beta}(\hat{y})$ is assigned to $\hat{y}$ for a decision level higher than α and less than β which is calculated as follows:

$$Q_{\alpha,\beta}(\hat{y}) = Q_\alpha - Q_\beta, \quad 0 \leq \alpha \leq w_{\hat{y}}, \ u_{\hat{y}} \leq \beta \leq 1,$$

where

$$Q_\alpha = \int_\alpha^{w_{\hat{y}}} (\underline{u}(r) + \overline{u}(r))dr \ and \ Q_\beta = \int_{u_{\hat{y}}}^\beta (\underline{v}(r) + \overline{v}(r))dr$$

This quantity will be used as a basic for comparing IFNs in decision level higher than α and less than β . As shown

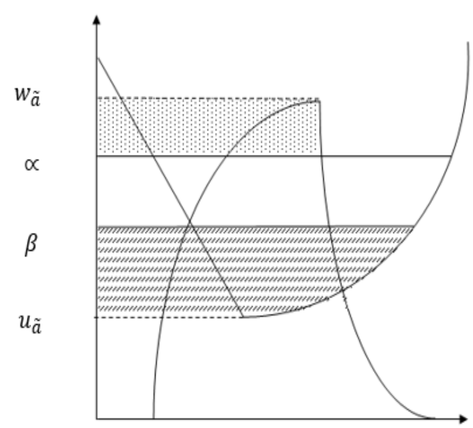

Figure 1: $Q_{\alpha,\beta}(\hat{y})$ Quantity

in Figure 1, the presented quantity is the sum of the dotted area and the cross-hatched area respectively.

Definition 3.1. *If $\hat{a}$ and $\hat{b}$ are two arbitrary IFNs,*

1. $Q_{\alpha,\beta}(\hat{a}) < Q_{\alpha,\beta}(\hat{b}) \longleftrightarrow \hat{a} \preceq_{\alpha,\beta} \hat{b}$.
2. $Q_{\alpha,\beta}(\hat{a}) = Q_{\alpha,\beta}(\hat{b}) \longrightarrow$

- *If* $Q_\alpha(\hat{a}) < Q_\alpha(\hat{b}) \longleftrightarrow \hat{a} \preceq_{\alpha,\beta} \hat{b}$.
- *If* $Q_\alpha(\hat{a}) = Q_\alpha(\hat{b})$, $Q_\beta(\hat{a}) < Q_\beta(\hat{b}) \longleftrightarrow \hat{b} \preceq_{\alpha,\beta} \hat{a}$.
- *If* $Q_\alpha(\hat{a}) = Q_\alpha(\hat{b})$, $Q_\beta(\hat{a}) = Q_\beta(\hat{b}) \longleftrightarrow \hat{b} \approx_{\alpha,\beta} \hat{a}$.

3. $Q_{\alpha,\beta}(\hat{a}) > Q_{\alpha,\beta}(\hat{b}) \longleftrightarrow \hat{b} \preceq_{\alpha,\beta} \hat{a}$.

where $\hat{a} \preceq_{\alpha,\beta} \hat{b}$*, i.e., at decision levels higher than* α *and less than* β*,* $\hat{b}$ *is greater than or equal to* $\hat{a}$ *. If* α *is close to* $w_{\hat{a}}$ *and* β *is close to* $u_{\hat{a}}$*, the pertaining decision is called a "high level decision", in which case only parts of the two IFNs, with membership values between* α *and* $w_{\hat{a}}$ *and nonmembership values between* $u_{\hat{a}}$ *and* β*, will be compared. Likewise, if* α *is close to zero and* β *is close to one, the pertaining decision is referred to as a "low level decision".*
Wang and Kerre [27] proposed some axioms which serve as reasonable properties for working out the rationality of a ranking method for the ordering of fuzzy quantities. In the following, we verify that proposed ranking method is satisfied by these axioms.

Let M be the proposed method in this paper, S the set of IFNs for which the method M can be applied and $\mathfrak{A}$ subset of S. The statement "two elements A and B in $\mathfrak{A}$ satisfy that A has a higher ranking than B when M is applied to the IFNs in $\mathfrak{A}$" will be written as "$A \succeq B$ by M on $\mathfrak{A}$". "$A \approx B$ by M on $\mathfrak{A}$" and "$A \preceq B$ by M on $\mathfrak{A}$" are similarly interpreted.
In following, it is assumed that A, B and C are three IFNs and

$$A = (\underline{u_1}(r), \overline{u_1}(r), \underline{v_1}(r), \overline{v_1}(r)),$$
$$A = (\underline{u_2}(r), \overline{u_2}(r), \underline{v_2}(r), \overline{v_2}(r)),$$
$$A = (\underline{u_3}(r), \overline{u_3}(r), \underline{v_3}(r), \overline{v_3}(r)).$$

Proposition 3.2. *For an arbitrary finite subset* $\mathfrak{A}$ *of* S*, determined* α *and* β *and* $A \in \mathfrak{A}$

- $A \succeq A$ *by* M *on* $\mathfrak{A}$.
- *If* $(A, B) \in \mathfrak{A}^2$*,* $A \preceq B$ *and* $B \preceq A$ *by* M *on* $\mathfrak{A}$ *then we have* $A \approx B$ *by* M *on* $\mathfrak{A}$.
- *If* $(A, B, C) \in \mathfrak{A}^2$*,* $A \preceq B$ *and* $B \preceq C$ *by* M *on* $\mathfrak{A}$ *then we have* $A \preceq C$ *by* M *on* $\mathfrak{A}$.

Proof.

1. This case is very clear.
2. Suppose A and B are two intuitionistic fuzzy numbers. Let $A \preceq B$ then
$$Q_{\alpha,\beta}(A) \leq Q_{\alpha,\beta}(B)$$
Similarly for $B \preceq A$ we have
$$Q_{\alpha,\beta}(B) \leq Q_{\alpha,\beta}(A)$$
From (1) and (2) we have $A \approx B$.
3. Suppose A, B and C are three intuitionistic fuzzy numbers. Let $A \preceq B$ then
$$Q_{\alpha,\beta}(A) \leq Q_{\alpha,\beta}(B)$$
Similarly for $B \preceq C$ we have
$$Q_{\alpha,\beta}(B) \leq Q_{\alpha,\beta}(C)$$
From (3) and (4) we have $Q_{\alpha,\beta}(A) \leq Q_{\alpha,\beta}(C)$ that implies $A \preceq C$.

□

Proposition 3.3. *Let* S *and* S' *be two arbitrary finite sets of IFNs in which* M *can be applied and* A *and* B *are in* $S \cap S'$*. We obtain the ranking order* $A \succ B$ *by* M *on* S' *iff* $A \succ B$ *by* M *on* S*.*

Proof. The final ranking order of A and B is solely dependent on the values $Q_{\alpha,\beta}(A)$ and $Q_{\alpha,\beta}(A$ and fas nothing to do with any other IFN in S or S'. Therefore, when we rank A and B based on S and S', we get the same final ranking order. □

Remark 3.4. *It is worth noticing that proposed method able to rank generalized TrIFN where the position on membership ad non-membership are different. They are defined as* $\hat{a} = \langle((a_1, a_2, a_3, a_4); w_1), ((b_1, b_2, b_3, b_4); w_2)\rangle$ *with parameters* $b_1 \leq a_1$*,* $b_2 \leq a_2 \leq a_3 \leq b_3$*,* $a_4 \leq b_4$*.*

4 Illustrative Examples

In this section several examples to demonstrate the effectiveness of the proposed method and compare it with other methods are presented.

Example 4.1. *In here $Q_{\alpha,\beta}(.)$ for the TRIFNs is computed.*
According to the definition 2.3

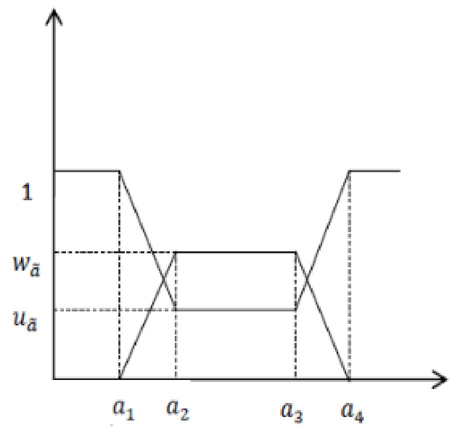

Figure 2: TRIFN

$$\underline{\mu}(r) = a_1 + \frac{1}{w_{\hat{a}}}(a_2 - a_1)r, \quad \overline{\mu}(r) = a_4 + \frac{1}{w_{\hat{a}}}(a_3 - a_4)r,$$
$$\underline{\nu}(r) = \frac{1}{1 - u_{\hat{a}}}(r(a_4 - a_3) + a_3 - u_{\hat{a}}a_4), \quad \overline{\nu}(r) = \frac{1}{u_{\hat{a}} - 1}(r(a_2 - a_1) + u_{\hat{a}}a_1 - a_2).$$

The parametric values assigned to the TRIFN, represented by $Q^{Tra}_{\alpha,\beta}(\hat{a})$, can be calculated as follows:

$$Q^{Tra(\hat{a})}_{\alpha,\beta} = \int_{\alpha}^{w_{\hat{a}}}(\underline{u}(r) + \overline{u}(r))dr - \int_{u_{\hat{a}}}^{\beta}(\underline{v}(r) + \overline{v}(r))dr = \int_{\alpha}^{w_{\hat{a}}}(a_1 + \frac{1}{w_{\hat{a}}}(a_2 - a_1)r$$
$$+ a_4 + \frac{1}{w_{\hat{a}}}(a_3 - a_4)r)dr + \frac{1}{1 - u_{\hat{a}}}\int_{u_{\hat{a}}}^{\beta}((r(a_1 - a_2) - u_{\hat{a}}a_1 + a_2) + (r(a_4 - a_3)$$
$$+ a_3 - u_{\hat{a}}a_4))dr = (a_1(w_{\hat{a}} - \alpha) + \frac{1}{2w_{\hat{a}}}(a_2 - a_1)(w_{\hat{a}} - \alpha)^2 + a_4(w_{\hat{a}} - \alpha) +$$
$$\frac{1}{2w_{\hat{a}}}(a_3 - a_4)(w_{\hat{a}} - \alpha)^2) - \frac{1}{1 - u_{\hat{a}}}((\frac{(\beta - u_{\hat{a}})^2}{2}(a_1 - a_2)$$
$$- u_{\hat{a}}a_1(\beta - u_{\hat{a}}) + a_2(\beta - u_{\hat{a}})) + (\frac{(\beta - u_{\hat{a}})^2}{2}(a_4 - a_3) + (\beta - u_{\hat{a}})(a_3 - a_4u_{\hat{a}}))).$$

Proposition 4.2. *Let $\hat{a} = \langle(a_1, a_2, a_3, a_4); w, v\rangle$ and $\hat{b} = \langle(b_1, b_2, b_3, b_4); w, v\rangle$ be two TrIFNs. Therefore, the following equation is valid:*

$$Q_{\alpha,\beta}(\hat{a} + \hat{b}) = Q_{\alpha,\beta}(\hat{a}) + Q_{\alpha,\beta}(\hat{b}).$$

Proof. Let $(\underline{u}(r), \overline{u}(r), \underline{v}(r), \overline{v}(r))$ be parametric representation of $\hat{a} + \hat{b}$. According to definition 2.7, we have

$$\hat{a} + \hat{b} = \langle(a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4); w, v\rangle.$$

So

$$\underline{u}(r) = a_1 + b_1 + \frac{1}{w}(a_2 + b_2 - a_1 - b_1)r = a_1 + \frac{1}{w}(a_2 - a_1)r + b_1 + \frac{1}{w}(b_2 - b_1)r$$
$$= \underline{u_1}(r) + \underline{u_2}(r),$$
$$\overline{u}(r) = a_4 + b_4 + \frac{1}{w}(a_3 + b_3 - a_4 - b_4)r = a_4 + \frac{1}{w}(a_3 - a_4)r + b_1 + \frac{1}{w}(b_3 - b_4)r$$
$$= \overline{u_1}(r) + \overline{u_2}(r),$$
$$\overline{v}(r) = \frac{1}{v-1}(r(a_2 + b_2 - a_1 - b_1) + v(a_1 + b_1) - a_2 - b_2) = \frac{1}{v-1}(r(a_2 - a_1)$$
$$+ va_1 - a_2) + \frac{1}{v-1}(r(b_2 - b_1) + vb_1 - b_2) = \overline{v_1}(r) + \overline{v_2}(r),$$
$$\underline{v}(r) = \frac{1}{1-v}(r(a_4 + b_4 - a_3 - b_3) + a_3 + b_3 - v(a_4 + b_4)) = \frac{1}{1-v}(r(a_4 - a_3)$$
$$+ a_3 - va_4) + \frac{1}{1-v}(r(b_4 - b_3) + b_3 - vb_4) = \underline{v_1}(r) + \underline{v_2}(r).$$

Now according to definition of $Q_{\alpha,\beta}$ the proof is complete. □

Example 4.3. *Let us take three sets of IFN:*

$\underline{Set\ I:}\ \hat{a} = \langle(0.5, 0.7, 0.9); 0.7, 0.2\rangle,\ \hat{b} = \langle(0.2, 0.3, 0.4); 0.6, 0.4\rangle,\ \hat{c} = \langle(0.4, 0.7, 0.9); 0.6, 0.3\rangle,$

$\underline{Set\ II:}\ \hat{a} = \langle(0.10, 0.19, 0.25, 0.30); 0.7, 0.2\rangle,\ \hat{b} = \langle(0.12, 0.2, 0.23, 0.28); 0.8, 0.1\rangle,$
$\hat{c} = \langle(0.21, 0.27, 0.32, 0.35); 0.6, 0.3\rangle$

$\underline{Set\ III:}\ \hat{a} = \langle(0.2, 0.5, 0.7); 0.7, 0.2\rangle,\ \hat{b} = \langle(0.2, 0.3, 0.9); 0.6, 0.4\rangle,\ \hat{c} = \langle(0.2, 0.4, 0.5, 0.9); 0.5, 0.3\rangle.$

The Table 1 illustrates the ranking of IFN by proposed method.

Set no.	IFN	$Q_{w_{\hat{a}}}$	$u_{\hat{a}}$	$Q_{\alpha,\beta}$	Result
	$\hat{a}$	0.7	0.84	-0.14	
Set I $\alpha = 0.2,\ \beta = 0.8$	$\hat{b}$	0.24	0.24	0	$\hat{c} \preceq \hat{a} \preceq \hat{b}$
	$\hat{c}$	0.53	0.7	-0.17	
	$\hat{a}$	0.2	0.35	-0.15	
Set I $\alpha = 0.55,\ \beta = 0.45$	$\hat{b}$	0.03	0.03	0	$\hat{c} \preceq \hat{a} \preceq \hat{b}$
	$\hat{c}$	0.06	0.21	-0.15	
	$\hat{a}$	0.2	0.26	-0.08	
Set II $\alpha = 0.2,\ \beta = 0.8$	$\hat{b}$	0.246	0.295	-0.049	$\hat{c} \preceq \hat{a} \preceq \hat{b}$
	$\hat{c}$	0.228	0.296	-0.068	
	$\hat{a}$	0.0606	0.0669	-0.0063	
Set II $\alpha = 0.55,\ \beta = 0.35$	$\hat{b}$	0.1011	0.1072	-0.0061	$\hat{a} \preceq \hat{b} \preceq \hat{c}$
	$\hat{c}$	0.28	0.30	-0.002	
	$\hat{a}$	0.467	0.592	-0.125	
Set III $\alpha = 0.2,\ \beta = 0.8$	$\hat{b}$	0.373	0.173	0.2	$\hat{c} \preceq \hat{a} \preceq \hat{b}$
	$\hat{c}$	0.312	0.442	-0.13	
	$\hat{a}$	0.229	0.252	-0.023	
Set III $\alpha = 0.45,\ \beta = 0.45$	$\hat{b}$	0.155	0.0143	0.1407	$\hat{c} \preceq \hat{a} \preceq \hat{b}$
	$\hat{c}$	0.054	0.125	-0.071	

Table 1: Comparison of IFNs in Example (4.3)

5 Significance of Proposed Method

Many researchers have proposed different ranking methods on IFNs, but none of them has covered the entire class of IFNs, and also almost all the methods have disadvantage that at some point of time they ranked two different numbers as the same.

In this section significance of our proposed method is shown by comparing our method with some existing techniques and observed that the ranking results by utilizing the proposed method are reasonable and consistent with human intuition.

Example 5.1. *Consider TIFNs* $\hat{a} = \langle(3, 4, 5); 0.8, 0.2\rangle$ *and* $\hat{b} = \langle(6, 8, 10); 0.4, 0.6\rangle$. *The ranking results by Li's method is* $R(\hat{a}) = R(\hat{b}) = 2.087$. *This implies that TIFNs* $\hat{a}$ *and* $\hat{b}$ *are not comparable although they are not identical. Hence, the result obtained by Li's approach is not satisfactory. But the ranking result by utilizing the proposed method is* $\hat{a} \preceq \hat{b}$.

Example 5.2. *For two TIFNs*

$$\hat{a} = \langle(0.57, 0.73, 0.83); 0.73, 0.2\rangle,$$

$$\hat{b} = \langle(0.58, 0.74, 0.819); 0.72, 0.2\rangle,$$

according to Wei's method, the distances from R^+ *are* $d(A_1, R^+) = d(A_2, R^+) = 0.453$. *This implies that they are not comparable. It is to be noted that they are not identical, so the result obtained by Wei's method is not satisfactory. It is observed that ranking result by using proposed method is* $\hat{b} \preceq \hat{a}$.

Example 5.3. *With respect to Zeng et al. ranking method for two TIFNs* $\hat{a} = \langle(0, 0.25, 0.3); 1, 0\rangle$ *and* $\hat{b} = \langle(0.1, 0.2, 0.4); 1, 0\rangle$, *we obtain* $A_\lambda(\hat{a}) = A_\lambda(\hat{b}) = 0.05$ *which implies that* $\hat{a}$ *and* $\hat{b}$ *are equal which is illogical whereas the ranking result obtained by the proposed method is* $\hat{a} \preceq \hat{b}$.

Example 5.4. *Let* $\hat{a} = \langle(0.4, 0.45, 0.5, 0.55); 0, 1\rangle$ *and* $\hat{b} = \langle(0.3, 0.35, 0.6, 0.65); 0, 1\rangle$ *be two TrIFNs. Then by applying Wan and Jiu approach, we get* $\hat{a} = \hat{b}$. *Therefor these two TrIFNs are indifference according to this approach although they are not identical. The ranking result by utilizing proposed method is* $\hat{a} \preceq \hat{b}$.

Example 5.5. *Consider TrIFNs* $\hat{a} = \langle(1, 2, 2, 3); 0, 1\rangle$ *and* $\hat{b} = \langle(0, 2, 2, 4); 0, 1\rangle$. *The ranking result by applying Das and Guha and Prakash and et al. method is* $\hat{a} = \hat{b}$. *Therefore, these two approaches do not provide a satisfactory result. While the obtained result by the proposed method is* $\hat{a} \preceq \hat{b}$.

Example 5.6. *For two TrIFNs*

$$\hat{a} = \langle(1, 2, 3, 4), (2.5, 2, 3, 3.5); 0.5, 0.7\rangle,$$

$$\hat{b} = \langle(1.75, 2, 3, 3.25), (1.5, 2, 3, 3.5); 0.5, 0.7\rangle,$$

according to Aggrawal and Gupta method, we get $\hat{a} = \hat{b}$ *which is illogical. But the ranking result by proposed method is* $\hat{b} \preceq \hat{a}$.

Example 5.7. *Consider the three IFNs*

$$\hat{a} = \langle(0.3, 0.4, 0.5, 0.6); 0.5, 0.3\rangle,$$

$$\hat{b} = \langle(0.7, 0.8, 0.9, 1); 0.5, 0.3\rangle,$$

$$\hat{c} = \langle(0.2, 0.4, 0.6, 0.8); 0.5, 0.3\rangle.$$

By Xu and Yager which is based on score and accuracy function method the given number are ranked as $\hat{a} \approx \hat{b} \approx \hat{c}$. *But the proposed method indicates that the ranking order among these given set of numbers is* $\hat{a} \preceq_{0.1,0.8} \hat{c} \preceq_{0.1,0.8} \hat{b}$ *in down level and* $\hat{a} \preceq_{0.4,0.4} \hat{b} \preceq_{0.4,0.4} \hat{c}$ *in high level.*

Example 5.8. *Let us take three more IFNs*

$$\hat{a} = \langle(0.7, 0.8, 0.9, 1); 0.2, 0.5\rangle,$$

$$\hat{b} = \langle(0.3, 0.4, 0.5, 0.6); 0.7, 0.1\rangle,$$

$$\hat{c} = \langle(0.5, 0.6, 0.7, 0.8); 0.8, 0.2\rangle.$$

According to proposed distance measure, the ranking order of these three numbers is $\hat{a} \preceq_{0.05,0.85} \hat{b} \preceq_{0.05,0.85} \hat{c}$. *Now negative of these numbers are*

$$-\hat{a} = \langle(-1, -0.9, -0.8, -0.7); 0.2, 0.5\rangle,$$

$$-\hat{b} = \langle(-0.6, -0.5, -0.4, -0.3); 0.7, 0.1\rangle,$$

$$-\hat{c} = \langle(-0.8, -0.7, -0.6, -0.5); 0.8, 0.2\rangle.$$

It is important here to mention that the proposed method obeys natural ordering of numbers i.e., the ranking order of negative of these three numbers is $-\hat{c} \preceq_{0.05,0.85} -\hat{b} \preceq_{0.05,0.85} \hat{-a}$.

Example 5.9. *Let us consider a new set of IFNs with each having same degrees of membership and non-membership as follows:*

$$\hat{a} = \langle (0.3, 0.4, 0.5, 0.6); 0.5, 0.3 \rangle,$$
$$\hat{b} = \langle (0.7, 0.8, 0.9, 1); 0.5, 0.3 \rangle,$$
$$\hat{c} = \langle (0.2, 0.4, 0.6, 0.8); 0.5, 0.3 \rangle.$$

By Novel accuracy function defined by Ye, the given numbers are ranked as $\hat{a} \approx \hat{b} \approx \hat{c}$*. But according to proposed method, the actual ranking order of these numbers become* $\hat{a} \preceq \hat{c} \preceq \hat{b}$*.*

Example 5.10. *Consider the following IFNs:*

$$\hat{a} = \langle (01, 0.4, 0.4, 0.5); 1, 1 \rangle,$$
$$\hat{b} = \langle (0.2, 0.3, 0.3, 0.6); 1, 1 \rangle.$$

Using the proposed method following two IFNs are ranked as $\hat{b} \preceq \hat{a}$*. Hence, it can be said that the proposed method outperforms when there is compensation of areas in the IFNs.*
It should be noted that as stated in [6] *the proposed method in* [24] *also outperforms when there is compensation of areas in the FNs. Therefore, it is reasonable that the extension of it also satisfies this property.*

Example 5.11. *In this example, taken from* [7]*, some of IFNs have been presented to compare the proposed method with the exiting method* [29, 13, 23, 16]*. Table 2 discussed that the proposed ranking method is able to overcome the demerits of the existing ranking methods in it and give more reasonable results for the comparisons of TrIFNs. For example, it is observed that for two TrIFNs*

$$A = \langle (0.57, 0.73, 0.83); 0.73, 0.2 \rangle \quad and \quad B = \langle (0.58, 0.74, 0.819); 0.72, 0.2 \rangle$$

The ranking result by using Wu and Cao approach is $A \approx B$ *which is a unreasonable result. But the ranking result by utilizing the proposed method is* $A \preceq_{0.1,0.8} B$ *and* $B \preceq_{0.6,0.3} A$ *this result is reasonable and consistent with human intuition.*
In rows 2,3, and 5 of Table 2 when values of $Q^A_{\alpha,\beta}$ *and* $Q^B_{\alpha,\beta}$ *are equal for ranking IFNs A and B we refer to values of* Q^A_α *and* Q^B_α *or* Q^A_β *and* Q^B_β *as stated in the Definition 3.1.*

6 Partner Selection

This section contains an illustration of the model, which is used to demonstrate the proposed ranking approach to partner selection for the formation of a new virtual enterprise.
The partner selection is an important decision problem in the formation of a new virtual enterprise (VE). Partner selection for the formation of a VE is a complex decision problem, which usually needs to consider multiple attributes and multiple decision makers (DMs). Furthermore, in many real-world evaluation problems, DMs may prefer to convey their opinions with fuzzy expression, even though there are some quantitative factors. In this instance, the intuitionistic fuzzy theory might be more suitable for evaluating the candidates. Consider the following example, taken from [32].

Example 6.1. *A precision machinery company in Shenzhen, China, catches a market opportunity consisting of several sub-projects. However, the whole task cannot be done by the company itself since it lacks some resources. And then the company initiates the creation of a VE for the market opportunity, and this company becomes the principal company in the VE. To form this new VE, the manufacturing of the product must be appropriately decomposed into a number of manufacturing tasks in considering resource utilization, production planning, quality control, and so on.*
So, this company decides that three core tasks and the manufacturing design would be done by the company itself, and the other tasks would be done by the partners. The partner selection is made based on five main attributes, including

- u_1 : *processing cost.*
- u_2 : *earliest start time.*
- u_3 : *processing time.*
- u_4 : *transportation cost.*

The expressions of existing ranking process	Examples	The proposed method
Wu and Cao *[29]*	Example 1	$Q^{A}_{0.1,0.8} = 0.04$
$d(A, r^{+}) =$	$A = \langle(0.57, 0.73, 0.83); 0.73, 0.2\rangle$	$Q^{B}_{0.1,0.8} = 0.02$
$\frac{1}{8}[\|(1 + u_1 - v_1)a - (1 + u_2 - v_2).1\|$	$A = \langle(0.58, 0.74, 0.819); 0.72, 0.2\rangle$	$Q^{A}_{0.6,0.3} = -0.67$
$+\|(1 + u_1 - v_1)b - (1 + u_2 - v_2).1]\|$	$d(A, r^{+}) = 0.45$	$Q^{B}_{0.6,0.3} = 0.69$
$where\ A = \langle(a, b, bc, d); u, v\rangle\ and$	$d(B, r^{+}) = 0.45$	$B \preceq_{0.1,0.8} A$
$r^{+} = \langle(1, 1, 1, 1); 0, 1\rangle$	$A \approx B$	$A \preceq_{0.6,0.3} B$
$If\ d(A_i, r^{+}) \geq d(A)_j, r^{+})\ then A_j \preceq A_i$		
Jianqiang and Zhong *[13]*	Example 2	$Q^{A}_{0.1,0.8} = 0.1484$
$I(A) = \frac{1}{8}[(a + b + c + d) \times (1 + u - v)]$	$A = \langle(0.56, 0.74, 0.80, 0.90); 0.5, 0.5\rangle$	$Q^{B}_{0.1,0.8} = 0.148$
$S(A) = I(A) \times (u - v)$	$A = \langle(0.5, 0.7, 0.85, 0.95); 0.5, 0.5\rangle$	$Q^{A}_{0.4,0.6} = 0$
$H(A) = I(A) \times (u + v)$	$S(A) = 0$	$Q^{B}_{0.4,0.6} = 0$
$where\ A = \langle(a, b, bc, d); u, v\rangle$	$S(B) = 0$	$B \preceq_{0.1,0.8} A$
$If\ S(A_i) \leq S(A_j)\ then\ A_i \preceq A_j$	$H(A) = 0.3750$	$A \preceq_{0.4,0.6} B$
$If\ S(A_i) = S(A_j)\ then\ A_i \preceq A_j\ If$ $H(A_i) \leq H(A_j)$	$H(B) = 0.3750$	
Rezvani [23]	Example 3	$Q^{A}_{0.2,0.8} = 0$
$\nu(A) = \frac{(a + 2b + 2c + d)}{6}$	$A = \langle(0.55, 0.6, 0.7, 0.75); 1, 0\rangle$	$Q^{B}_{0.2,0.8} = 0$
$where\ A = \langle(a, b, bc, d); u, v\rangle$	$B = \langle(0.45, 0.65, 0.7, 0.75); 0.5, 0.5\rangle$	$Q^{A}_{0.9,0.9} = -1.04$
$If\ \nu(A_i) \leq \nu(A_j)\ then\ A_i \preceq A_j$	$\nu(A) = 0$	$Q^{B}_{0.9,0.9} = -1.024$
	$A \approx B$	$B \preceq_{0.2,0.8} A$
		$A \preceq_{0.9,0.9} B$
Li [16]	Example 4	$Q^{A}_{0.1,0.8} = -0.25$
$R(a, \lambda) = \frac{\nu(a, \lambda)}{1 + A(a, \lambda)}$	$A = \langle(-6, 1, 2); 0.6, 0.4\rangle$	$Q^{B}_{0.1,0.8} = -0.27$
$where\ A = \langle(a, b, bc, d); u, v\rangle$	$A = \langle(-6, 1, 2); 0.7, 0.3\rangle$	$Q^{A}_{0.2,0.6} = -0.2$
$\nu(a, \lambda) = \nu_u(a) + \lambda(\nu_v(a) - \nu_u(a))$	$A \approx B$	$Q^{B}_{0.2,0.6} = -0.28$
$A(a, \lambda) = A_u(a) + \lambda(A_v(a) - A_u(a))$		$B \preceq_{0.1,0.8} A$
$\nu_u(a) = \frac{u(a_1 + 4a_2 + a_3)}{6}$		$B \preceq_{0.6,0.3} A$
$\nu_v(a) = \frac{(1 - v)(a_1 + 4a_2 + a_3)}{6}$		
$A_u = \frac{u(a_3 - a_1)}{3}$		
$A_v = \frac{(1 - v)(a_3 - a_1)}{3}$		
Das and Guha [7]	Example 5	$Q^{A}_{0.2,0.8} = 0$
$X_A = \frac{(3a^2 + b^2 - c^2 - 3d^2)}{2(3a + b - c - 3d)}$	$A = \langle(2, 2, 2, 2); 1, 0\rangle$	$Q^{B}_{0.2,0.8} = 0$
$Y_A = \frac{7(d - a) + 5(c - b)}{18(d - a) + 6(c - b)}$	$A = \langle(4, 4, 4, 4); 1, 0\rangle$	$Q^{A}_{0.8,0.2} = 0$
$where\ A = \langle(a, b, bc, d); 1, 0\rangle$	The method proposed in [7] Not	$Q^{B}_{0.8,0.2} = 0$
$If\ X_A > X_B\ then\ A > B$	applicable.	

Table 2: A comparison of the Proposed Method with the Existent Methods

ht

Linguistic variables	IFNs
Strong satisfaction (SS)	$\langle(5,6,6,7);0.7,0.2\rangle$
Satisfaction (S)	$\langle(4,5,5,6);0.7,0.2\rangle$
Fair (F)	$\langle(3,4,4,5);0.7,0.2\rangle$
Dissatisfaction (D)	$\langle(2,3,3,4);0.7,0.2\rangle$
Strong Dissatisfaction (SD)	$\langle(1,2,2,3);0.7,0.2\rangle$

Table 3: Linguistic Term for Rating the Importance of Attribute and Corresponding Score

Linguistic variables	IFNs
Very important	5
Important	4
Medium	3
Unimportant	2
Very unimportant	1

Table 4: Linguistic Terms for Rating the Candidates with Respect to Attributes and Corresponding IFNs

- u_5 : *transportation time.*

After receiving the bids for each task and the preliminary selection, there are three candidates A_1, A_2 *and* A_3 *as potential partners for further evaluation.*
The committee of five DMs of the core enterprise takes part in the partner selection process, which are composed of

- d_1 : *general manager.*
- d_2 : *financial manager.*
- d_3 : *technical innovation manager.*
- d_4 : *technical development manager.*
- d_5 : *production manager.*

Using the scales of $1 \sim 5$, *the linguistic terms are used to rate each alternative partner with respect to each attribute by five DMs. The linguistic terms and their corresponding IFNs are shown in Table 3. Also using the scales of* $1 \sim 5$, *the linguistic terms are used to rate the importance of attribute by DMs. The linguistic terms and their corresponding grades are shown in Table 4. The assessment grades are summarized in Table 5, in which include the assessment grades of alternative partner with respect to each attribute and the grades of attribute's importance. For operational convenience, we convert the information of alternatives with respect to attributes with linguistic variables in Table 5 into the IFNs, the attributes' grades with linguistic variables in Table 5 into the nonnegative real number, which are shown in Table 6.*

For the grades $w = (w_1, w_2, w_3, w_4, w_5) = (I, VI, M, M, M)$ *of attributes set* $\{u_1, u_2, u_3, u_4, u_5\}$ *given by DMs. According to the Table 5, we first convert the linguistic variables into the form of scores* $(4, 5, 3, 3, 3)$. *Then we can normalize the scores* $(4, 5, 3, 3, 3)$ *to* $v = (v_1, v_2, v_3, v_4, v_5)$ *by* $v_1 = 4/sum = 0.2200$, $v_2 = 5/sum = 0.2700$, $v_3 = 3/sum = 0.1667$, $v_4 = 5/sum = 0.1667$, $v_5 = 5/sum = 0.1667$ *where* $sum = 4 + 5 + 3 + 3 + 3$.

$$min \sum_{j=1}^{5} u_j v_{pj},$$

S.t.

$$\sum_{j=1}^{5} u_j v_{pj} \leq 1, \quad p = 1, 2, 3, \tag{1}$$

Candidates	DMs	u_1	u_2	u_3	u_4	u_5
A_1	d_1	SS	F	F	SS	D
	d_2	S	D	D	F	F
	d_3	S	D	F	F	D
	d_4	F	S	D	S	SD
	d_5	F	S	D	D	D
A_2	d_1	S	SD	S	S	F
	d_2	S	SD	S	F	F
	d_3	S	D	S	S	F
	d_4	F	S	SS	S	SD
	d_5	S	D	F	F	F
A_3	d_1	SS	D	S	SS	D
	d_2	SS	D	S	S	D
	d_3	F	SD	SS	S	D
	d_4	S	D	S	S	F
	d_5	S	D	S	S	F
Grades of attributes	w	I	VI	M	M	M

Table 5: Group Decisions of Candidates and Grades of Attribute's Importance in Linguistic Variables

$$u_k \geq \epsilon, k = 1, \cdots, 5.$$

By using data envelopment analysis model (1), preference order of three potential partners is as follows:

$$A_1 \preceq A_2 \preceq A_3$$

Preference order in high level decision and low level decision is the same.

7 Conclusions

In fuzzy decision-making problems, fuzzy ranking is one of the most-researched areas. The concept of IFS can be viewed as an alternative approach to define a fuzzy set in the case when available information is not sufficient to define a conventional fuzzy set. In this study, an area-based ranking approach is offered for IFNs. The proposed method can rank any kinds of IFNs with different kinds of membership functions and it is simple and easy to understand and implement. The paper herein presents several comparative examples to illustrate the validity and advantages of proposed ranking method. It shows that the ranking order obtained by the proposed approach is more consistent with human intuitions than existing methods. Furthermore, the new method is capable of effectively ranking various types of IFNs and overcome to weakness of previous one. The proposed method can properly rank symmetric fuzzy numbers sharing the same core and different supports (Example 3, Table 2); hence, the proposed method is more efficient than the other methods. Also the proposed method can rank crisp valued IFNs which were ignored in most of the studies (Example 5, Table 2). Apart from that the proposed method can overcome the limitations of the other methods that arise due to compensation of area in IFNs (Example 5.10). Further, IFNs sharing same support and different cores are ranked effectively (Example 4, Table 2). Hence, it is claimed that the proposed method outperforms most of the other ranking methods. Despite that the method has resolved many of the exiting problems in IFNs ranking, yet it has its limitations. The proposed method cannot give a justified ranking order in ranking images of fuzzy numbers. In particular, for arbitrary fuzzy numbers and if the values are equal and then in this case, the decision maker cannot make a logical decision.

References

[1] S. Aggarwal, C. Gupta, *Solving intuitionistic fuzzy solid transportation problem via new ranking method based on signed distance*, International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems, **24(04)** (2016), 483–501.

[2] Z. Ai, Z. Xu, Q. Lei, *Limit properties and derivative operations in the metric space of intuitionistic fuzzy numbers*, Fuzzy Optimization and Decision Making, **161** (2017), 71–87.

Candidates	DMs	u_1	u_2	u_3	u_4	u_5
A_1	d_1	$\langle(5,6,6,7);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$	$\langle(5,6,6,7);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$
	d_2	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$
	d_3	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$
	d_4	$\langle(3,4,4,5);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(1,2,2,3);0.7,0.2\rangle$
	d_5	$\langle(3,4,4,5);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$
A_2	d_1	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(1,2,2,3);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$
	d_2	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(1,2,2,3);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$
	d_3	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$
	d_4	$\langle(3,4,4,5);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(5,6,6,7);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(1,2,2,3);0.7,0.2\rangle$
	d_5	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$
A_3	d_1	$\langle(5,6,6,7);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(5,6,6,7);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$
	d_2	$\langle(5,6,6,7);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$
	d_3	$\langle(3,4,4,5);0.7,0.2\rangle$	$\langle(1,2,2,3);0.7,0.2\rangle$	$\langle(5,6,6,7);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$
	d_4	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$
	d_5	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(2,3,3,4);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(4,5,5,6);0.7,0.2\rangle$	$\langle(3,4,4,5);0.7,0.2\rangle$
Grades of attributes	w	0.2222	0.2777	0.1667	0.1667	0.1667

Table 6: Group Decision of Candidates in IFNs and Weights of Attributes

Candidates	DMs	Aggregate weighted attribute	$Q_{0.8,0.3}$	$Q_{0.2,0.9}$
A_1	d_1	$\langle(3.6111, 4.6111, 4.6111, 5.6111); 0.7, 0.2\rangle$	1.844	12.911
	d_2	$\langle(2.7778, 3.7778, 3.7778, 4.7778); 0.7, 0.2\rangle$	1.511	10.557
	d_3	$\langle(2.7778, 3.7778, 3.7778, 4.7778); 0.7, 0.2\rangle$	1.511	10.557
	d_4	$\langle(2.9443, 3.9443, 3.9443, 4.9443); 0.7, 0.2\rangle$	1.577	11.044
	d_5	$\langle(3.111, 4.111, 4.111, 5.111); 0.7, 0.2\rangle$	1.644	10.510
A_2	d_1	$\langle(3.002, 4.002, 4.002, 5.002); 0.7, 0.2\rangle$	1.600	11.205
	d_2	$\langle(2.8335, 3.8335, 3.8335, 4.8335); 0.7, 0.2\rangle$	1.533	10.733
	d_3	$\langle(3.2779, 4.2779, 4.2779, 5.2779); 0.7, 0.2\rangle$	1.677	11.729
	d_4	$\langle(3.6111, 4.6111, 4.6111, 5.6111); 0.7, 0.2\rangle$	1.785	12.476
	d_5	$\langle(2.9445, 3.9445, 3.9445, 4.9445); 0.7, 0.2\rangle$	1.569	10.982
A_3	d_1	$\langle(3.5001, 4.5001, 4.5001, 5.5001); 0.7, 0.2\rangle$	1.749	12.227
	d_2	$\langle(3.3334, 4.3334, 4.3334, 5.3334); 0.7, 0.2\rangle$	1.695	11.85
	d_3	$\langle(2.778, 3.778, 3.778, 4.778); 0.7, 0.2\rangle$	1.515	10.609
	d_4	$\langle(3.2779, 4.2779, 4.2779, 5.2779); 0.7, 0.2\rangle$	1.677	11.729
	d_5	$\langle(3.2779, 4.2779, 4.2779, 5.2779); 0.7, 0.2\rangle$	1.677	11.729

Table 7: Weighted Group Decisions of Three Candidates

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