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Determining appropriate weight for criteria in multi criteria group decision making problems using an Lp model and similarity measure

Z. Eslaminasab[1] and A. Hamzehee[2]

[1,2] *Department of Mathematics, Kerman Branch, Islamic Azad University, Kerman, Iran*

z.esb.math@gmail.com, hamzehee_ali@yahoo.com(a_hamzehee@iauk.ac.ir)

Abstract

Decision matrix in group decision making problems depends on a lot of criteria. It is essential to know the necessity of weight or coefficient of each criterion. Accurate and precise selection of weight will help to achieve the intended goal. The aim of this article is to introduce a linear programming model for recognizing the importance of each criterion in multi criteria group decision making with intuitionistic fuzzy data through similarity measure between each alternative and ideal alternative. Based on this model, decision makers and experts will be prevented from making mistakes in recognizing the weight and shape of standardization of their mental measurement units. By using of determined weights, the alternatives will be ranked according to a new method based on ELECTRE III method. An applied and numerical example is presented and the obtained results are compared with other methods.

Keywords: ELECTRE III method, Intuitionistic Fuzzy Set, Multi Criteria Group Decision Making, Similarity measure, Weight of criteria.

1 Introduction

The basic needs of human beings and their requirements force them to make personal decisions based on their personal necessities. For this reason, they need to use multi criteria decision making (MCDM) to select the best alternative with respect to some criteria in many situation that they have. There are many reasons and criteria that are contradictory in selecting the most appropriate alternative which will be surveyed as a multi criteria group decision making (MCGDM) problem.

Uncertain data have effect on studying and solving decision making and optimization problems. These data contain statistical or random, fuzzy, interval, rough and even a combination of the aforementioned imprecise data. Fuzzy sets theory proposed by Zadeh [36], is an effective tool which deals with imprecision and uncertainity and appears to be successful in different feilds. Atanassov [2] concludes that intuitionistic fuzzy sets (IFSs) are extended from fuzzy sets theory, characterized by a membership function and a non-membership function.

In group decision making (GDM) models, the matrix of decision making has diversity of criteria, so that being aware of the importance coefficient or weight for each of them is a requirement for decision making. Each criterion weight shows its relative importance with respect to other criteria. Chen et al. [6] studied MCGDM problems with IFSs and evidential reasoning method where the weight of criteria and decision makers (DMs) were inductively specified. Singh and Yadav [25] developed an approach for solving intuitionistic fuzzy linear fractional programming problem (IFLFPP). A new VIKOR method as a compromise ranking approach to solve MCDM problems through intuitionistic fuzzy analysis was presented by Shahriari [22]. Liang et al. [14] proposed two extended projection-based preference ranking organization methods for enrichment evaluations for MCDM with hesitant fuzzy linguistic term sets. The MCGDM problems investigated by Li and Chen [12] with trapezoidal intuitionistic fuzzy data, such that they proposed two model for determining the weight of criteria and DMs. Mousavi et al. [16] developed a VIKOR method based on IFSs in multi attribute group decision making (MAGDM) problems. Zhang [37] studied hesitant fuzzy MCGDM

Corresponding Author: A. Hamzehee
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with unknown weight information. Xian et al. [31] offered an operator for linguistic fuzzy data in GDM where the weight of each criterion was predetermined. Cao et al. [4] proposed an approach to interval-valued intuitionistic stochastic MCDM problems using set pair analysis. Such that in order to determine criteria weights, they stablished a mathematical programming model. Chen [5] investigated a MCGDM method using type II interval fuzzy sets where the weight of criteria was inductively introduced. Yue [35] suggested a method for determining weight of DMs with projection method. Wang et al. [29] submitted an approach of MCDM that combines a HFLTS with an outranking method, and specified weight for criteria. A model for GDM problems based on a fuzzy VIKOR approach using linguistic variables proposed by Wu et al. [30], in which the criteria importance was obtained by linguistic variables. Sevastjanov and Dymova [21] ranked alternatives within the Dempster-Shafer theory (DST) using hesitant fuzzy values and the importance of criteria that was predetermined. Zhang et al. [38] developed an innovative method to address MCDM problems with Linguistic intuitionistic fuzzy numbers in which the weight information is completely unknown.

Benayoun [3] proposed the elimination and choice translating reality (ELECTRE) method which is a famous ranking method in decision making problems and then Van Delft and Nijkamp [26] developed it further. In this method, the dominance concept is used implicitly and the alternatives are compared with each other pairwise, and recognition of dominant and nondominant alternatives are surveyed. At last, deletion of weak or nondominant alternatives was done. Different versions of this method have been proposed as its subset because of the complexity of new decision making problems and developing usage of MCDM methods. Since that time, the primary method proposed by Benayoun was considered as ELECTRE I. The principle of all the new methods are the same as ELECTRE I, they were named ELECTRE II, ELECTRE III, ELECTRE IV, ELECTRE IS and ELECTRE TRI. A short brief of these methods presented by Figueira et al. [8]. In Shen et al. models [23, 24], ELECTRE III in MCGDM with intuitionistic fuzzy data, was used for ranking alternative in which the weight of criteria and DMs were previously specified. ELECTRE method in MCGDM was extended by Peng et al. [18] with multi hesitant fuzzy sets and predetermined weights for each criterion. Hashemi et al. [10] studied the MCGDM problems using ELECTRE III with interval-valued intuitionistic fuzzy data, where each criteria weight was defined as interval-valued intuitionistic fuzzy values. An ELECTRE-Based outranking method was developed by Rashid et al.[19] with hesitant intuitionistic fuzzy linguistic term sets in which the weight of criteria was determined through a special method. Wang et al. [28] defined some outranking relations derived by ELECTRE III for MCDM problems with hesitant interval-valued fuzzy sets. Zhou et al. [40] used the extended gray numbers, integrated with discrete gray numbers and interval gray numbers to express the uncertainty of stochastic MCDM problems. In their work, stochastic multicriteria acceptability analysis and ELECTRE III are combined to solve stochastic MCDM problems with uncertain weight information.

One of the useful tools for measuring the degree of resemblance between objects is similarity measure. Many researchers have investigated the concept of similarity measure of fuzzy sets. Also, some new methods for measuring the degree of similarity between IFSs and between elements proposed and their properties were discussed by Liu [15]. Arefi and Taheri [1] suggested a new approach to define the similarity measure between interval-valued fuzzy sets. Mukherjee and Basu [17] used similarity measures in solution of a class of assignment problem with restriction on person cost depending on efficiency/qualification and restriction on job cost where both the costs are considered as intuitionistic fuzzy numbers (IFNs). Ye [34] proposed three vector similarity measures between trapezoidal IFNs (TIFNs) in the vector space and applied them to the fuzzy MCGDM problem, in which the criteria weights and the evaluated values in decision matrix are expressed by TIFNs. Zhang et al. [39] presented some similarity measures between two triangular fuzzy numbers (TFNs) based on the vector similarity measures in vector space, so that it can be used to aggregate the decision information with TFNs and a methodology proposed for MCGDM problems with triangular fuzzy information.

In most of studies for solving decision making problems the importance of each criterion is specified through a weight by the DMs or with one of the defined methods, for example entropy, LINMAP, weighted least squares, eigenvector, etc. In this work, it is tried to assess and recognize the importance of each criterion by solving a linear programming model through similarity measure between each alternative and ideal alternative. Then, an introduction to a ranking model based on ELECTRE III is carried out. Also an MCGDM approach based on intuitionistic information is proposed. Preliminaries and the mathematical formulation is presented in Section 2. In Section 3, the ELECTRE III method and linear programming based on IFNs and similarity measure are briefly described. The proposed algorithmic method is illustrated by a case study in Section 4. Finally, short conclusions and future research directions are given in Section **??**.

2 Preliminaries

In the following, the definition and operations of IFSs are briefly reviewed.

Definition 2.1. [36] *Let X be a reference set. A fuzzy set $\tilde{A}$ in X is defined as $\tilde{A} = \{\langle x, \mu_{\tilde{A}}(x)\rangle | x \in X\}$, where the function $\mu_{\tilde{A}} : X \to [0,1]$ is the membership function of $\tilde{A}$ and the function $\mu_{\tilde{A}}(x)$ is called the grade of membership of* *x in* $\tilde{A}$.

Definition 2.2. [2] *Let* X *be a reference set. An IFS* P *in* X *is defined a* $P = \{\langle x, \mu_P(x), \nu_P(x)\rangle | x \in X\}$*, where the functions* $\mu_P : X \to [0,1]$ *and* $\nu_P : X \to [0,1]$ *are membership function and non-membership function of IFS* P*, respectively. The hesitancy degree of* x *in* P *is* $\pi_P(x) = 1 - \mu_P(x) - \nu_P(x)$.

Definition 2.3. [2] *Let* X *be a reference set,* A *and* B *be two IFSs in* X *and* λ *be a positive scalar. Then addition, scalar mathematical and power operators are defined as follow:*

1. $A \bigoplus B = \{\langle x, \mu_A(x) + \mu_B(x) - \mu_A(x).\mu_B(x), \nu_A(x).\nu_B(x)\rangle \mid x \in X\}$
2. $\lambda A = \{\langle x, 1 - (1 - \mu_A(x))^\lambda, (\nu_A(x))^\lambda\rangle \mid x \in X\}$
3. $A^\lambda = \{\langle x, (\mu_A(x))^\lambda, 1 - (1 - \nu_A(x))^\lambda\rangle \mid x \in X\}$.

Remark 2.4. *For every* $P = \{\langle x, \mu_P(x), \nu_P(x)\rangle | x \in X\}$*, these relations hold,* $0 \le \mu_P(x) \le 1, 0 \le \nu_P(x) \le 1, 0 \le \mu_P(x) + \nu_P(x) \le 1$ *and it can be seen that* $\pi_P(X) \in [0,1]$.

Definition 2.5. [32] *Let* X *be a reference set and* $P = \{\langle x, \mu_P(x), \nu_P(x)\rangle | x \in X\}$ *be an IFS in* X*. An IFN can be named* $(\mu_\alpha, \nu_\alpha, \pi_\alpha)$ *that denoted as* α *where* $0 \le \mu_\alpha \le 1$*,* $0 \le \nu_\alpha \le 1$*,* $0 \le \mu_\alpha + \nu_\alpha \le 1$ *and* $\pi_\alpha = 1 - \mu_\alpha - \nu_\alpha$*. If* $\mu_\alpha + \nu_\alpha = 1$ *then the* $(\mu_\alpha,\ \nu_\alpha,\ \pi_\alpha)$ *reduced to* $(\mu_\alpha,\ 1 - \nu_\alpha)$.

Definition 2.6. [7, 11] *Let* $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ *be an IFN. The score function* $s(\alpha)$ *and the accuracy function* $h(\alpha)$ *which is used to measure the degree of suitability and the accuracy degree of* α*, respectively, are as follow:*

$$s(\alpha) = \mu_\alpha - \nu_\alpha \quad \text{and} \quad h(\alpha) = \mu_\alpha + \nu_\alpha.$$

In many theoretical and practical decision making problems, DMs often need to compare two IFSs or IFNs, in the same universe. Xu and Yager [33] presented some relations between any two IFNs α and β that is stated in the following.

Definition 2.7. [33] *Let* $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ *and* $\beta = (\mu_\beta, \nu_\beta, \pi_\beta)$ *be two IFNs, the score and accuracy degree of* α *and* β *are defined as* $s(\alpha)$ *and* $h(\alpha)$*, respectively. Then:*

- $\alpha \prec \beta$ *if* $s(\alpha) < s(\beta)$
- $\alpha \succ \beta$ *if* $s(\alpha) > s(\beta)$
- *if* $s(\alpha) = s(\beta)$*, then* $\begin{cases} \alpha \prec \beta, & \text{if } h(\alpha) < h(\beta); \\ \alpha \succ \beta, & \text{if } h(\alpha) > h(\beta); \\ \alpha \approx \beta, & \text{if } h(\alpha) = h(\beta). \end{cases}$

Definition 2.8. [32] *Let* $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ *and* $\beta = (\mu_\beta, \nu_\beta, \pi_\beta)$ *be two IFNs, the normalized Hamming distance between the IFNs* α *and* β *is defined as follows:*

$$d_H(\alpha, \beta) = \frac{1}{2}(|\mu_\alpha - \mu_\beta| + |\nu_\alpha - \nu_\beta| + |\pi_\alpha - \pi_\beta|) \tag{1}$$

Lemma 2.9. [32] *Let* $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ *and* $\beta = (\mu_\beta, \nu_\beta, \pi_\beta)$ *be two IFNs, then* $0 \le d_H(\alpha, \beta) \le 1$ *,* $d_H(\alpha, \alpha) = 0$ *and* $d_H(\alpha, \beta) = d_H(\beta, \alpha)$.

Definition 2.10. *Let* $\alpha = (\mu_\alpha, \nu_\alpha, \pi_\alpha)$ *and* $\beta = (\mu_\beta, \nu_\beta, \pi_\beta)$ *be two IFNs. The similarity measure between* α *and* β *is defined as:*

$$s(\alpha, \beta) = \frac{1}{1 + d_H(\alpha, \beta)} \tag{2}$$

where $d_H(\alpha, \beta)$ *is the normalized Hamming distance between two IFNs* α *and* β.

In order to solve the MCGDM problems, the final decision should result from the preferences of a group of DMs. However, it is difficult for the DMs to determine the preferences with crisp values because the information available for DMs is vague and imprecise under uncertain environment. Hence the IFNs can be used to construct decision matrix for each DM. In the MCGDM problems, DMs usually come from different research fields and each DM has his/her unique characteristics with regard to knowledge, skills and experience. They may be actual experts in some of criteria but not in all criteria. Then it is not reasonable to set criteria weights based on DMs information. So in the following a linear programing model is introduced to determine the weight of each criterion, and next alternatives are ranked through a new method based on ELECTRE III method.

3 The proposed MCGDM approach based on IFN data

In the following, a new decision approach to carry out the mentioned MCGDM problems with IFNs is introduced. For a given MCGDM problem under intuitionistic environment the decision matrix can be constructed based on IFNs for each DM. These problems consist of m alternatives and n criteria, that are denoted by $A = \{a_1, a_2, ..., a_m\}$ and $C = \{c_1, c_2, ..., c_n\}$, respectively. Also, let $D = \{d_1, d_2, ..., d_q\}$ be the set of DMs. The decision matrix is defined as $R_k = (r_{ijk})_{m\times n}$ in which $r_{ijk}; (i = 1, 2, ..., m, j = 1, 2, ..., n, k = 1, 2, .., q)$ is an IFN and is provided by DM d_k for the alternative a_i with respect to the criteria c_j.

3.1 Determine the weight for criteria

In MCDM techniques, the optimal alternative is selected. In most techniques, determining weight of criteria is based on DMs' ideas and not through specific methods. Also in other techniques, weight of criteria is calculated through a special model. In this models, there are different alternatives that are evaluated based on several criteria independently. Then, alternatives are ranked using the values. It is essential to know the necessity of weight or coefficient of each criterion. Accurate and precise selection of weight will help to achieve the intended goal. Therefore a linear model is introduced to determine the importance of each criteria in MCGDM problems with intuitionistic fuzzy data through similarity measure between each alternative and ideal alternative. Consider the matrix R' that its members are the average of the members in decision matrices R_k for $k = 1, 2, ..., q$ as:

$$R' = \begin{pmatrix} r'_{11} & \dots & r'_{1j} & \dots & r'_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r'_{i1} & \dots & r'_{ij} & \dots & r'_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ r'_{m1} & \dots & r'_{mj} & \dots & r'_{mn} \end{pmatrix}$$

where $r'_{ij} = \frac{1}{q}[r_{ij1} \bigoplus r_{ij2} \bigoplus ... \bigoplus r_{ijq}]$.
The positive-ideal and negative-ideal solution based on IFNs are defined as following:
Positive ideal: for each criterion c_j with positive aspect in decision matrices $R_k, (1 \le k \le q)$, find:

$$\Gamma_1 := \{(t, l) \mid \mu_{r_{tjl}} = \max_{1\le k\le q}\{\max_{1\le i\le m}(\mu_{r_{ijk}})\}$$

If $|\Gamma_1| = 1$, define $a^*_j = (\mu_{r_{tjl}}, \nu_{r_{tjl}}, \pi_{r_{tjl}})$, $(t, l) \in \Gamma_1$. Else: $a^*_j = (\mu_{r_{tjl}}, \min_{(t,l)\in\Gamma_1}(\nu_{r_{tjl}}), \pi_{r_{tjl}})$.
Negative ideal: for each criterion c_j with negative aspect in decision matrices R_k $(1 \le k \le q)$, find:

$$\Gamma_2 := \{(t, l) \mid \nu_{r_{tjl}} = \max_{1\le k\le q}\{\max_{1\le i\le m}(\nu_{r_{ijk}})\}.$$

If $|\Gamma_2| = 1$, define $a^*_j = (\mu_{r_{tjl}}, \nu_{r_{tjl}}, \pi_{r_{tjl}})$, $(t, l) \in \Gamma_2$. Else: $a^*_j = (\min_{(t,l)\in\Gamma_2}(\mu_{r_{tjl}}), \nu_{r_{tjl}}, \pi_{r_{tjl}})$. Let $S(R', A^*)$ be the similarity measure matrix, where its elements are the similarity measures between the elements of matrix R' and ideal alternatives $A^* = (a^*_1, a^*_2, ..., a^*_n)$ with respect to the criteria with positive or negative aspect, that are obtained using (2).

$$S = S(R', A^*) = \begin{pmatrix} s_{11} & \dots & s_{1j} & \dots & s_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ s_{i1} & \dots & s_{ij} & \dots & s_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ s_{m1} & \dots & s_{mj} & \dots & s_{mn} \end{pmatrix}$$

where s_{ij} is the similarity measure between r'_{ij} and a^*_j for $i = 1, 2, ..., m;\quad j = 1, 2, ..., n$ that is defined as follows:

$$s_{ij} = \frac{1}{1 + d_{ij}} \tag{3}$$

Note that d_{ij} is the normalized Hamming distance between the intuitionistic fuzzy values $r'_{ij} = (\mu_{r'_{ij}}, \nu_{r'_{ij}}, \pi_{r'_{ij}})$ and $a^*_j = (\mu_{a^*_j}, \nu_{a^*_j}, \pi_{a^*_j})$ for $i = 1, 2, ..., m;\quad j = 1, 2, ..., n$, and is defined as:

$$d_{ij} = d(r'_{ij}, a^*_j) = \frac{1}{2}(|\mu_{r'_{ij}} - \mu_{a^*_j}| + |\nu_{r'_{ij}} - \nu_{a^*_j}| + |\pi_{r'_{ij}} - \pi_{a^*_j}|). \tag{4}$$

Now consider the following linear programming model:

$$(P) \qquad \gamma := \max \tfrac{1}{q} \textstyle\sum_{k=1}^{q} \varepsilon_k \;\; \text{s.t.} \;\; \sum_{j=1}^{n} w'_j = 1 s_{ij} w'_j - \varepsilon_k \geq 0; i = 1,2,...,m; j = 1,2,...,n; k = 1,2,...,q. \tag{5}$$

where $\varepsilon = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_q)^T$ and $w' = (w'_1, w'_2, ..., w'_n)^T$ are the decision variables vectors.

Lemma 3.1. *Let γ be the optimal solution of the linear programming model (5). Then $\gamma > 0$.*

Proof. At first, consider the dual of (5), i.e. the problem (6), as follows:

$$\min \alpha \;\; \text{s.t.} \;\; \alpha + \textstyle\sum_{i=1}^{m} \sum_{k=1}^{q} s_{ij} \beta_{ijk} = 0; \;\; -\sum_{i=1}^{m} \sum_{j=1}^{n} \beta_{ijk} = \tfrac{1}{q}; \quad j = 1,...,n, \; k = 1,2,...,q, \; \beta_{ijk} \leq 0; \;\; \forall i,j,k. \tag{6}$$

Now, by contradiction assume that γ not greater than 0, i.e. $\gamma \leq 0$.
If $\gamma = 0$ then due to the dual problem $\alpha = \gamma = 0$. So $\sum_{i=1}^{m} \sum_{k=1}^{q} s_{ij} \beta_{ijk} = 0$. It is contradict with constraints in (6), since $s_{ij} \neq 0$. If $\gamma < 0$ then $\sum_{i=1}^{m} \sum_{k=1}^{q} s_{ij} \beta_{ijk} > 0$. But it is also contradict with constraints in (6), since $s_{ij} > 0$ and $\beta_{ijk} \leq 0$. □

According to the previous explanation, let $s_{ij}; i = 1,2,...,m, \;\; j = 1,2,...,n$ be the similarity measure between alternative $a_i; i = 1,2,...,m$ respect to the criterion $c_j; j = 1,2,...,n$ from the positive/negative ideal alternative $a^*_j; j = 1,2,...,n$, in average matrix R'. Such that γ and $s_{ij}; i = 1,2,...,m, j = 1,2,...,n$ calculated from (5) and (3), respectively.

Now, let $W = \{w_1, w_2, ..., w_n\}$ be a vector for n weights of criteria, in order to obtain this vector, the following linear programming model have to solve for $i = 1,2,...,m$:

$$\max \textstyle\sum_{j=1}^{n} w'_j s_{ij} \;\; \text{s.t.,} \;\; \sum_{j=1}^{n} w'_j = 1 \;\; w'_j \geq \gamma; \quad j = 1,...,n. \tag{7}$$

By solving the linear programming model (7), the weight matrix W' is result as follows:

$$W' = \begin{pmatrix} w'_{11} & \cdots & w'_{1j} & \cdots & w'_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ w'_{i1} & \cdots & w'_{ij} & \cdots & w'_{in} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ w'_{m1} & \cdots & w'_{mj} & \cdots & w'_{mn} \end{pmatrix} \tag{8}$$

where w'_{ij} is the weight of alternative $a_i; i = 1,2,...,m$ with respect to the criterion $c_j; j = 1,2,...,n$ in similarity measure matrix S.

To obtain the vector for n weights of criteria, $W = \{w_1, w_2, ..., w_n\}$, do the following steps:

Step 1 Based on the weight matrix W' and the similarity measure measure matrix S, construct the matrix Y, named aggregated decision matrix, that is shown as follows:

$$Y = [S(W')^T]^T S(W')^T \tag{9}$$

Step 2 Calculate eigenvector of the aggregated decision matrix Y i.e. $\omega = \{\omega_1, \omega_2, ..., \omega_m\}$, then the weight vector of criteria $c_j (j = 1,2,...,n)$ can be obtained by the following formula:

$$W = (W')^T \omega \tag{10}$$

3.2 Ranking the alternatives

In this section, by the weight $W = \{w_1, w_2, ..., w_n\}$, which is obtained based on the proposed model, a new ranking method is introduced according to ELECTRE III method with intuitionistic fuzzy information. Concordance and discordance matrices are presented based on the normalized Hamming distance after constructing the intuitionistic fuzzy decision matrix via DMs and the definition of the thresholds p, q and v. At last, through the minimum Ratio [9], the last ranking of alternatives are achieved.

Step 1. (Create decision matrix) In the first step, DMs are asked to express their opinions about the alternatives $a_i; (i = 1,2,...,m)$ with respect to the criteria $c_j; (j = 1,2,...,n)$. Such that each $R_k; (k = 1,2,...,q)$ is a decision matrix that is constructed by a DM, which their elements are displayed as r_{ijk} in the form of IFNs data.

Step 2. (Define thresholds and assign weights to the criteria) In this step, in order to strengthen of the ability to identify superior alternatives, as well as the DMs involved in the selection process, Rogers and Bruen [20] introduced three new thresholds as follows:

- The preference threshold p_j, above which the DM shows a clear strict criterion preference of one alternative over the other.
- The indifference threshold q_j, beneath which the DM is indifferent to two alternatives on certain criterion.

- The veto threshold v_j, where a discordant difference in favor of one alternative greater than this value requires the DM to negate any possible outranking relationship indicated by the other criteria.

Remark 3.2. *Note that if $g_j(a_i)$ be the Hamminig distance between alternative $a_i; (i = 1, 2, ..., m)$ and ideal alternative with respect to the criteria $c_j; (j = 1, 2, ..., n)$ with positive or negative aspect. Then the concept of the thresholds of preference (p_j), indifference (q_j) and veto (v_j) aims at defining the statement $a_i S a_l, (i, l = 1, 2, ..., m)$ for every couple a_i and a_l of the alternatives set. The statement $a_i S a_l$ means that action a_i outranks action a_l, when a_i is at least as good as a_l in the most of the criteria and never significantly worse in the rest of them. Respectively, the statement $a_i S_j a_l$ is defined for every criterion c_j. Then the criterion c_j is in agreement with $a_i S a_l$ if only $a_i S_j a_l$ and even if $g_j(a_i) > g_j(a_l) - q_j$. The ELECTRE III method is based on the definition of two matrices, the concordance and the discordance matrices, which determine if the statement $a_i S a_l$ is acceptable. The rule of concordance requests that the majority of the criteria, after their relative importance has been taken into account, have to be in favour of the statement $a_i S a_l$, while the rule of discordance requests that no criterion from the minority that does not support the statement $a_i S a_l$ be strongly against it. The discordance matrix can be calculated as long as the veto threshold (v_j); v_j allows the complete rejection of the $a_i S a_l$ statement when the relation $g_j(a_l) > g_j(a_i) + v_j$ is valid for every criterion c_j.*

One of the other important parameters in choosing the appropriate alternative in this step, is weight or degree of importance of each criterion. For this purpose, using the model (7) after solving problem (5), the normalized weight vector W is obtained with relation (10).

Step 3. (Create concordance matrix for each criterion) Concordance matrices $\widetilde{c}_j; (j = 1, 2, ..., n)$ are constructed by every DM, using the decision matrices and the thresholds suggested in previous steps, as follows:

$$\widetilde{c}_j(a_i, a_l) = \begin{cases} 1, & \text{if } g_j(a_i) + q_j \geq g_j(a_l) \\ 0, & \text{if } g_j(a_i) + p_j \leq g_j(a_l) \\ \frac{g_j(a_i) + p_j - g_j(a_l)}{p_j - q_j} & \textit{otherwise} \end{cases} \tag{11}$$

Step 4. (Create the discordance matrix for each criterion) At this level, using the decision matrices and the thresholds set, the discordance matrices $\widetilde{d}_j; (j = 1, 2, ..., n)$ are formed with respect to each criterion by each DM using the following formula:

$$\widetilde{d}_j(a_i, a_l) = \begin{cases} 0, & \text{if } g_j(a_i) + p_j \geq g_j(a_l) \\ 1, & \text{if } g_j(a_i) + \nu_j \leq g_j(a_l) \\ \frac{g_j(a_l) - p_j - g_j(a_i)}{v_j - p_j} & \textit{otherwise} \end{cases} \tag{12}$$

Step 5. (Create the credibility matrix) After creating the intuitionistic fuzzy concordance matrices $\widetilde{c}_j$ and intuitionistic fuzzy discordance matrices $\widetilde{d}_j$, the intuitionistic fuzzy credibility matrix $\rho_k(a_i, a_l); (i, l = 1, 2, ..., m; k = 1, 2, ..., q)$ indicates the credibility degree of the conclusion that a_i outranks a_l and using the following mathematical relationship can be calculated:

$$\rho_k(a_i, a_l) = \sum_{j=1}^{n} w_j [\widetilde{c}_j(a_i, a_l) \times \widetilde{d}_j(a_i, a_l)] \tag{13}$$

Step 6. (Create the net outranking flow matrix) Let A be the set of alternatives, $\forall a_i \in A$ the net outranking flow $\Phi_k(a_i, a_l); (i, l = 1, 2, ..., m; k = 1, 2, ..., q)$, represents the net outranking character of a_i over all remaining alternatives and is defined as follows:

$$\Phi_k(a_i, a_l) = \rho_k(a_i, a_l) - \rho_k(a_l, a_i) \tag{14}$$

Step 7. (The net outranking flow index) In this step, the net outranking flow index $\Gamma_k(a_i); (i = 1, 2, ..., m; k = 1, 2, ..., q)$, i.e. the net outranking alternative a_i is overall to the remaining alternatives, is calculated with the following formula:

$$\Gamma_k(a_i) = \sum_{l=1, l \neq i}^{m} \Phi_k(a_i, a_l) \quad \forall k = 1, 2, ..., q \tag{15}$$

If $\Gamma_k(a'_i); (i = 1, 2, ..., m; k = 1, 2, ..., q)$ be the largest value among all values $\Gamma_k(a_i); (i = 1, 2, ..., m)$, i.e.

$$\Gamma_k(a'_i) = \max_{1 \leq i \leq m} \{\Gamma_k(a_i)\} \tag{16}$$

Then alternative a'_i is the best choice.

Step 8. (The group net outranking flow index) The group net outranking flow index $\Gamma_G(a_i); (i = 1, 2, ..., m)$, is represented as follows:

$$\Gamma_G(a_i) = \sum_{k=1}^{q} \lambda_k \Gamma_k(a_i) \tag{17}$$

where $\lambda_k; (k = 1, 2, ..., q)$, indicate the DMs weights.

Step 9. (The group's satisfaction index) To gain the group's satisfaction index, the minimum Ratio [9] is used between individual and group preorders as fallows:

$$\phi_k = \frac{1}{m} \sum_{i=1}^{m} r_i; \quad k = 1, 2, ..., q \tag{18}$$

where $r_i = \min\{\frac{U(a_i)}{u_k(a_i)}, \frac{u_k(a_i)}{U(a_i)}\}; i = 1, 2, ..., m$ $u_k(a_i)$ is the ranking order of alternative a_i by DM d_k according to the equation (15). The values of $\Gamma_k(a_i); (i = 1, 2, ..., m; k = 1, 2, ..., q)$ are sorted decreasingly for each DM d_k and $U(a_i)$ is the group ranking order that obtained from equation (17) for each alternative which its values are sorted decreasingly. Then the group's satisfaction index is:

$$\phi_G = \sum_{k=1}^{q} \lambda_k \phi_k \tag{19}$$

Remark 3.3. *If $\phi_G \geq \theta$, where $\theta(-1 \leq \theta \leq 1)$ is the threshold of the acceptable group satisfaction level, then ϕ_G is an acceptable group satisfaction degree and if $\phi_G < \theta$, then ϕ_G is an unacceptable group satisfaction degree, such that θ is usually determined by the DMs.*

This satisfaction degree which is very useful in the interactive procedures is required to reach a consensus assignments. The DMs rank the alternatives according to the net outranking flow indices. Therefore if the majority of preferences of the net outranking flow $\Phi_k(a_i, a_l)$ is close and a few of DMs' preferences are in disagreement, it is reasonable to change only these particular preferences.

Step 10. (The weighted deviation degree) Let $\Pi = \{\Phi_1(a_i, a_l), \Phi_2(a_i, a_l), ...,$
$\Phi_q(a_i, a_l)\}$ be the set of DMs' net outranking flow values, where $\Phi_k(a_i, a_l)$ is the net outranking flow value of DM d_k with respect to the pair of alternatives
$(a_i, a_l); (a_i, a_l \in A, i \neq l)$. The weighted deviation degree ξ_{il} is calculated as follows:

$$\xi_{il} = \sqrt{\sum_{k=1}^{q} \lambda_k (\Phi_k(a_i, a_l) - \overline{\Phi}_G(a_i, a_l))^2} \tag{20}$$

where $\overline{\Phi}_G(a_i, a_l) = \sum_{k=1}^{q} \lambda_k \Phi_k(a_i, a_l)$ is the weighted mean of the set Π.
So, the weighted deviation degree matrix ξ is defined as:

$$\xi = \begin{pmatrix} - & \cdots & \xi_{1l} & \cdots & \xi_{1m} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \xi_{il} & \cdots & \xi_{il} & \cdots & \xi_{im} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \xi_{ml} & \cdots & \xi_{ml} & \cdots & - \end{pmatrix} \tag{21}$$

The automatic adjustment of the group satisfaction consists of two processes [23]:

- Choose the maximum element $\xi_{i^* l^*}$ from the matrix ξ, then find the element $\Phi_{k^*}(a_{i^*}, a_{l^*})$ using

$$\lambda_{k^*}(\Phi_{k^*(a_{i^*}, a_{l^*})} - \overline{\Phi}_G(a_{i^*}, a_{l^*}))^2 = \max_{1 \leq k \leq q}\{\lambda_k(\Phi_k(a_{i^*}, a_{l^*}) - \overline{\Phi}_G(a_{i^*}, a_{l^*}))^2\} \tag{22}$$

- Replace $\Phi_{k^*}(a_{i^*}, a_{l^*})$ with a numerical value $\Phi'_{k^*}(a_{i^*}, a_{l^*}) = \overline{\Phi}_G(a_i, a_l)$, and other elements do not change i.e., let

$$\Phi_k^{h+1}(a_i, a_l) = \begin{cases} \overline{\Phi}_G^h(a_i, a_l), & \text{for } i = i^*, l = l^*, and \;\; k = k^* \\ \Phi_k^h(a_i, a_l) & otherwise. \end{cases} \tag{23}$$

Let $\Pi^0(a_i, a_l) = \{\Phi_1^0(a_i, a_l), \Phi_2^0(a_i, a_l), ..., \Phi_q^0(a_i, a_l)\}$ be an original set of the DMs' net outranking flow values and ξ_{il}^0 be its weighted deviation degree, and $\Pi^1(a_i, a_l) = \{\Phi_1^1(a_i, a_l), \Phi_2^1(a_i, a_l), ..., \Phi_q^1(a_i, a_l)\}$ be the corresponding first adjusted set and ξ_{il}^1 be its corresponding weighted deviation degree, obviously then, ξ_{il}^1 is smaller than ξ_{il}^0. Moreover, if one continue to change some elements of the set in this method, the weighted deviation degree approaches zero. Obviously, the recent automatic adjustment strategy is convergent. In fact, the following theorem is stated toward convergence.

Theorem 3.4. *Let $\Pi^h(a_i, a_l) = \{\Phi_1^h(a_i, a_l), \Phi_2^h(a_i, a_l), ..., \Phi_q^h(a_i, a_l)\}$ $(i, l = 1, 2, ...m, i \neq l)$ be a set of the DMs' net outranking flow values after h times, and the above adjustment ξ_{il}^h be the corresponding weighted deviation degree, then*

$$\lim_{h \longrightarrow \infty} \xi_{il}^h = 0 \tag{24}$$

Proof. See [23]. □

The most important advantage of the proposed method for solving MCGDM problems toward some of the existing method is that the weight of criteria is not set beforehand. Which it can enhance the flexibility and practically of the model under uncertain environment. A suitable and informative choice from the weight of criteria can be a great help in obtaining the final result. Also in the paper the weight of criteria is obtained by linear programming model and is not according to the personal idea of DMs. So errors of decision making decrease and the correction of ranking alternatives increase.

4 Applied and Numerical Example

Example 4.1. *Due to the high variety of customer demands, advances in technologies and the increasing importance of communication and information systems in today's highly competitive and global operating environment, companies have been forced to focus on supply chain management. Competitive advantages associated with supply chain management philosophy can be achieved by strategic collaboration with suppliers and service providers. The success of a supply chain is highly dependent on selection of good suppliers or suppliers. Supplier evaluation and selection becomes one of the most vital actions of companies in a supply chain. Particularly for companies who spend a high percentage of their sales revenue on parts and material supplies, and whose material costs represent a larger portion of total costs, savings from supplies are of particular importance. Selecting the wrong supplier could be enough to deteriorate the whole supply chain's financial and operational position, whereas selecting the right suppliers significantly reduces purchasing costs, improves competitiveness in the market and enhances end user satisfaction. Thus supplier selection has drawn considerable attention from researchers and practitioners in supply chain management [13]. In order to elucidate the details of the proposed method in this paper and demonstrate the method in practice, a supplier evaluation problem in a high-tech company is considered.*

In this section, according to an example from Shen et al. [23], a high-technology company which manufactures electronic products intends to rank its five material suppliers $(a_1, a_2, a_3, a_4, a_5)$ is considered. For this important supplier evaluation problem, a single DM may not be able to accurately consider all relevant aspects of the decision problem. Hence, the company manager decided to use GDM to evaluate these five suppliers. A committee of three DMs was established as an engineering expert d_1, a financial expert d_2, and a quality control expert d_3. The expert weight vector was given by $\lambda = (\frac{1}{3}, \frac{1}{3}, \frac{1}{3})^T$. According to the companys development strategy, the company manager decided on four criteria to evaluate the five suppliers; Cost control (c_1), Performance (c_2), Technique (c_3) and Service (c_4). Over the course of the decision evaluation, the experts may not have had a precise or sufficient level of knowledge of the problem, and so they may have been unable to be confident about their judgments. To deal with this fuzziness and hesitation, the experts preference values were expressed using IFNs. After the discussion, the company manager determined the acceptable group satisfaction degree $\theta = 0.6$.

Three experts (DMs) gave their own opinions about the performance of alternative $a_i; (i = 1, ..., 5)$ with respect to each criterion $c_j; (j = 1, ..., 4)$, as R_1, R_2 and R_3 matrices and the thresholds given by three experts are stated in Table 1.

$$R_1 = \begin{pmatrix} (0.45, 0.55, 0.00) & (0.40, 0.55, 0.05) & (0.70, 0.30, 0.00) & (0.70, 0.20, 0.10) \\ (0.50, 0.40, 0.10) & (0.70, 0.30, 0.00) & (0.60, 0.40, 0.00) & (0.60, 0.35, 0.05) \\ (0.70, 0.25, 0.05) & (0.40, 0.55, 0.05) & (0.40, 0.55, 0.05) & (0.90, 0.10, 0.00) \\ (0.40, 0.55, 0.05) & (0.50, 0.50, 0.00) & (0.70, 0.25, 0.05) & (0.60, 0.40, 0.00) \\ (0.40, 0.60, 0.00) & (0.40, 0.50, 0.10) & (0.50, 0.50, 0.00) & (0.50, 0.45, 0.05) \end{pmatrix}$$

$$R_2 = \begin{pmatrix} (0.50, 0.40, 0.10) & (0.40, 0.50, 0.10) & (0.80, 0.20, 0.00) & (0.60, 0.35, 0.05) \\ (0.50, 0.50, 0.00) & (0.60, 0.40, 0.00) & (0.70, 0.30, 0.00) & (0.40, 0.25, 0.05) \\ (0.70, 0.30, 0.00) & (0.40, 0.50, 0.10) & (0.40, 0.60, 0.00) & (0.70, 0.30, 0.00) \\ (0.30, 0.60, 0.10) & (0.60, 0.35, 0.05) & (0.70, 0.25, 0.05) & (0.80, 0.10, 0.10) \\ (0.60, 0.35, 0.05) & (0.50, 0.50, 0.00) & (0.60, 0.30, 0.10) & (0.70, 0.20, 0.10) \end{pmatrix}$$

$$R_3 = \begin{pmatrix} (0.60, 0.30, 0.10) & (0.40, 0.45, 0.15) & (0.90, 0.10, 0.00) & (0.70, 0.25, 0.05) \\ (0.40, 0.55, 0.05) & (0.60, 0.35, 0.05) & (0.65, 0.25, 0.10) & (0.80, 0.15, 0.05) \\ (0.65, 0.25, 0.10) & (0.30, 0.65, 0.05) & (0.50, 0.45, 0.05) & (0.50, 0.20, 0.30) \\ (0.40, 0.50, 0.10) & (0.60, 0.40, 0.00) & (0.40, 0.60, 0.00) & (0.80, 0.15, 0.05) \\ (0.40, 0.60, 0.00) & (0.40, 0.55, 0.05) & (0.60, 0.40, 0.00) & (0.40, 0.60, 0.00) \end{pmatrix}$$

DMs	q_k	p_k	ν_k
d_1	0.05	0.15	0.30
d_2	0.10	0.20	0.35
d_3	0.03	0.13	0.18

Table 1: Thresholds given by three experts

Firstly, according to the formula (10), the criteria weight vector $W = \{w_1, w_2, w_3, w_4\}$ *for four criteria is obtained, these weights are stated in Table 2.*

w_1	w_2	w_3	w_4
0.43	0.19	0.19	0.19

Table 2: Normalized weights of criteria

Then, intuitionistic fuzzy concordance matrices and intuitionistic fuzzy discordance matrices are calculated using equations (11) and (12), respectively. Next, based on equation (13), the credibility matrices ρ_1, ρ_2 *and* ρ_3 *are obtained and in the following, the net outranking flow matrices* Φ_1, Φ_2 *and* Φ_3 *are constructed using equation (14), as follow:*

$$\Phi_1 = \begin{pmatrix} 0 & -0.0633 & 1.6067 & -0.0000 & 0.2533 \\ 0.0633 & 0 & 1.6700 & 0.0633 & 0.3167 \\ -1.6067 & -1.6700 & 0 & -1.5033 & -1.3450 \\ 0.0000 & -0.0633 & 1.5033 & 0 & 0.2533 \\ -0.2533 & -0.3167 & 1.3450 & -0.2533 & 0 \end{pmatrix}$$

$$\Phi_2 = \begin{pmatrix} 0 & 0.0633 & 0.8733 & -0.0000 & 0.2867 \\ -0.0633 & 0 & 0.8100 & -0.0633 & 0.2233 \\ -0.8733 & -0.8100 & 0 & -0.8733 & -0.5867 \\ 0.0000 & 0.0633 & 0.8733 & 0 & 0.2867 \\ -0.2867 & -0.2233 & 0.5867 & -0.2867 & 0 \end{pmatrix}$$

$$\Phi_3 = \begin{pmatrix} 0 & -0.6273 & 1.2913 & -0.5893 & 0.1580 \\ 0.6273 & 0 & 1.8819 & 0.2609 & 0.6523 \\ -1.2913 & -1.8819 & 0 & -1.6033 & -1.1587 \\ 0.5893 & -0.2609 & 1.6033 & 0 & 0.3737 \\ -0.1580 & -0.6523 & 1.1587 & -0.3737 & 0 \end{pmatrix}$$

Next, the net outranking flow index and group net outranking flow index values are calculated using equation (15) until (17) which are abstracted in Table 3.

	Γ_1^0	Γ_2^0	Γ_3^0	Γ_G^0
a_1	1.7967	1.2233	0.2327	1.0842
a_2	2.1133	0.9067	3.4225	2.1475
a_3	-6.1250	-3.1433	-5.9352	-5.0678
a_4	1.6933	1.2233	2.3053	1.7407
a_5	0.5217	-0.2100	-0.0253	0.0954

Table 3: Net outranking flow index and group net outranking flow index values

Table 4 shows the result of the first individual and groups' original preorders on the alternatives based on the values in Table 3. Personal and group's original satisfaction index values are computed using equations (18) and (19), and abstracted in Table 5. According to Remark 3.3 the relation between acceptable group satisfaction degree θ *and* ϕ_G^0*, that is shown in Table 5, i.e.* $\phi_G^0 = \frac{1}{3}[\phi_1^0 + \phi_2^0 + \phi_3^0] = 0.78$*. Since* $0.78 > 0.6 = \theta$*, thus the desired level of group consensus is achieved, and the final group ranking result of the alternatives are given as follow,* $a_2 \succ a_4 \succ a_1 \succ a_5 \succ a_3$*.*

Now, let $\theta = 0.8$ *be the acceptable group satisfaction degree. Since* $\phi_G^0 = 0.78 < 0.8$*, the next stage is going to perform. In the second stage after calculating the value of the individual and group's adjusted preorders, the new result is* $\phi_G^1 = 0.9 > 0.8$ *(the*

u_1^0	u_2^0	u_3^0	u_G^0
2	1	3	3
1	2	1	1
5	4	5	5
3	1	2	2
4	3	4	4

Table 4: The first individual and group's adjusted preorders

ϕ_1^0	ϕ_2^0	ϕ_3^0	ϕ_G^0
0.86	0.47	1	0.78

Table 5: The first personal and group's satisfaction index values

calculations are ignored). Then the desired level of group consensus is achieved, and the final group ranking result of the alternatives are the same as first stage. One way of analyzing the proposed method is the rank reversal test. Since Wang and Luo [27] found that the rank reversal phenomenon occurs in many decision making methods, the rank reversal has become a common criterion to measure the performance of the decision making methods. A reasonable method should avoid the rank reversal by adding or deleting of an alternative. In other words, if an alternative is added or deleted from the problem, then any other two alternatives should keep the same ranking order. To test the rank reversal of the proposed method, the set of alternatives is decomposed into Subproblem 1 with alternatives set $\{a_2, a_3, a_4, a_5\}$ *and Subproblem 2 with alternatives set* $\{a_1, a_2, a_4, a_5\}$*, respectively. Subproblem 1 is obtained by deleting alternative* A_1*, Subproblem 2 is obtained by deleting alternative* A_3 *from the original problem. Using the proposed method, the corresponding computation results are respectively given in Table 6.*

	Original problem	Subproblem 1	Subproblem 2
Satisfaction degree	$\{a_1, a_2, a_3, a_4, a_5\}$	$\{a_2, a_3, a_4, a_5\}$	$\{a_1, a_2, a_4, a_5\}$
$\theta = 0.6$	$a_2 \succ a_4 \succ a_1 \succ a_5 \succ a_3$	$a_2 \succ a_4 \succ a_5 \succ a_3$	$a_2 \succ a_4 \succ a_1 \succ a_5$
$\theta = 0.8$	$a_2 \succ a_4 \succ a_1 \succ a_5 \succ a_3$	$a_2 \succ a_4 \succ a_5 \succ a_3$	$a_2 \succ a_4 \succ a_1 \succ a_5$

Table 6: Ranking alternatives and rank reversal test

There is no any rank reversal for any two alternatives by addition or deletion of an alternative. Thus, the proposed method can well avoid rank reversal. The examination of the rank reversal shows the validity and practicability of the proposed method in this paper. The comparison between proposed method and ranking alternatives based on Shen et al. [23] is shown in Table 7. It can be seen from Table 7 that the ranking order obtained by Shen et al. [23] method is different from whatever obtained with the proposed method. The principal reason is that in Shen et al. [23] method, weight of criteria is based on DMs' ideas and not through specific methods, which may lead to unreasonable decision results. By contrast, the proposed method determines weight of criteria by a linear model through similarity measure between each alternative and ideal alternative, which can avoid the subjective randomness and improve convincingness of decision results.

5 Conclusions

The traditional outranking sorting methods is focused on DMs' opinions for determining the weight of criteria. In this paper, a new method is proposed to solve MCGDM problems by a linear programming model with the similarity measure between each alternative and ideal alternative in the form of intuitionistic fuzzy information. Also this method is suggested to evaluate the importance of each criterion. To solve the problems of MCGDM, the application of ELECTRE III method is proposed with intuitionistic fuzzy data. To overcome the convergence problems, an automatic adjustment of group satisfaction is utilized. So that, the decision making error decreased and the accuracy of alternatives ranking increased based on the weight of criteria which is achieved through proposed model. The proposed outranking method, for the practical application is applied in a high-technology company which manufactures electronic products intends to rank its five material suppliers. Then the obtained results compare with the other procedure. The researcher gets that the results of ranking of alternatives are different from the other suggested method. For further use, this research can be investigated by other types of uncertain environments and also the determination of the importance of DMs' in MCGDM problems can be surveyed.

Alternatives	Proposed method	Shen et al. method
a_1	3	1
a_2	1	2
a_3	5	3
a_4	2	4
a_5	4	5

Table 7: Compare the rank of proposed method with Shen et al. method

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