Iranian Journal of Fuzzy Systems **Vol. 2, No. 1,** (2005) pp. 49-63

CATEGORY OF $(POM)_L$-FUZZY GRAPHS AND HYPERGRAPHS

M. M. ZAHEDI AND M. R. KHORASHADI-ZADEH

Abstract. In this note by considering a complete lattice L, we define the notion of an L-Fuzzy hyperrelation on a given non-empty set X. Then we define the concepts of $(POM)_L$-Fuzzy graph, hypergraph and subhypergroup and obtain some related results. In particular we construct the categories of the above mentioned notions, and give a (full and faithful) functor form the category of $(POM)_L$-Fuzzy subhypergroups ($(POM)_L$-Fuzzy graphs) into the category of $(POM)_L$-Fuzzy hypergraphs. Also we show that for each finite objects in the category of $(POM)_L$-Fuzzy graphs, the coproduct exists, and under a suitable condition the product also exists.

1. Introduction

Rosenfeld [9] in 1975 defined the notion of a fuzzy graph. Berge studied hypergraphs [1]. Roy and Goetschel gave the notion of fuzzy hypergraphs [10]. Zahedi and Khorashadi-Zadeh in [9] gave some categoric connections between fuzzy hypergraphs, subhypergroups, ... Now we follow [10] and [11]. In this regard we redefine the notion of fuzzy hypergraph. In fact we give a new approach to this notion. To explain this, first we give the notions of fuzzy graph and fuzzy hypergraph which have defined in [9] and [10] respectively.

Definition 1.1. [9] A fuzzy graph is a triple (X, δ, μ), where δ is a fuzzy subset of a finite non-empty set of X and μ is a fuzzy relation on δ, i.e. μ is a fuzzy subset of $X \times X$, and $\mu(x, y) \leq \delta(x) \wedge \delta(y)$, for all $x, y \in X$.

Definition 1.2. [10] Let X be a finite non-empty set and let ξ be a finite family of non-trivial fuzzy subsets on X, i.e. for all μ in ξ, supp$\mu \neq \emptyset$ and $X = \bigcup_{\mu\in\xi}$ supp μ, where by suppμ we mean the set $\{x \in X | \mu(x) > 0\}$. Then the pair $\mathcal{H} = (X, \xi)$ is called a fuzzy hypergraph on X.

Remark 1.3. We expect a hypergraph to be a fuzzy graph if ξ in Definition 1.2 is a singleton. However if δ is a fuzzy subset on a finite nonempty set X and supp $\delta = X$, then the pair $(X, \xi = \{\delta\})$ is a fuzzy hypergraph on X according to Definition 1.2, while (X, δ) is not a fuzzy graph according to Definition 1.1. The problem arises because the fuzzy relation μ has no place in Definition 1.2. Hence this definition is not a generalization of Definition 1.1. So in this paper we work with the Definition 1.2, that is a genuine extension of Definition 1.1.

Received: November 2004; Accepted: February 2005
Key words and phrases: Fuzzy graph, Fuzzy hypergraph, Fuzzy subhypergroup, Partially ordered monoid.

Now in this note first we give the notions of L-Fuzzy hypersubsets, L-Fuzzy hyperrelations on a set X, $(POM)_L$-Fuzzy hyperrelations on an L-Fuzzy hypersubsets of X and $(POM)_L$-Fuzzy hyperrelations on a finite family of L-Fuzzy subsets. Then we present the concepts of $(POM)_L$-Fuzzy (hyper)graphs and construct the categories $(POM)_L - FG_r$ of all $(POM)_L$-Fuzzy graphs and $(POM)_L - FHG_r$ of all $(POM)_L$-Fuzzy hypergraphs. After that we show that there is a full and faithful functor from $(POM)_L - FG_r$ into $(POM)_L - FHG_r$. Finally we define the notion of $(POM)_L$-Fuzzy hypergroups and then construct the category $(POM)_L - FHG_p$ of all $(POM)_L$-Fuzzy hypergroups, and obtain some related results.

Throughout this paper we let L be a complete lattice with the greatest element 1 and the least element 0.

Definition 1.4. [3, 8] Let $T : L \times L \longrightarrow L$ be a binary operation having the properties:

(i) $T(x, 1) = x$
(ii) $T(x, y) = T(y, x)$
(iii) $T(x, y) \leq T(u, y)$ if $x \leq u$
(iv) $T(x, T(y, z)) = T(T(x, y), z)$.

Henceforth (L, T) is a partially ordered commutative monoid [2].

Remark 1.5. In the mathematical tradition of algebra (L, T) is better known as a partially ordered monoid in which the unity coincides with the top element of the lattice, for example in this regard see [2, 5]. However authors in the fuzzy set tradition sometimes call T an L-t-norm, because T is known as a t-norm when L is the unit interval.
It is obvious that if $\{x_\alpha\}_{\alpha \in \Lambda}$ and $\{y_\beta\}_{\beta \in \Lambda'}$ are two families of elements of L, then

$$\bigvee_{\substack{\alpha \in \Lambda \\ \beta \in \Lambda'}} T(x_\alpha, y_\beta) \leq T(\bigvee_{\alpha \in \Lambda} x_\alpha, \bigvee_{\beta \in \Lambda'} y_\beta). \ (*)$$

By an L-Fuzzy subset δ on a set X, we mean a function $\delta : X \longrightarrow L$.

Notation:

(i) We let $F_L(X)$ shows the set of all L-Fuzzy subsets on X, i.e.

$$F_L(X) = \{\delta | \delta : X \longrightarrow L \text{ is a function } \}.$$

(ii) Write $p^*(X) = p(X) \backslash \{\emptyset\}$, i.e. $p^*(X)$ is the set of all non-empty subsets of X.

Definition 1.6. [7] It is well-known that a hyperstructure is a non-empty set H together with a map $o : H \times H \longrightarrow p^*(H)$, called hyperoperation. A hyperstructure (H, o) is called a hypergroup if the following axioms hold:

(i) $(xoy)oz = xo(yoz), \forall x, y, z \in H$,
(ii) $aoH = H = Hoa, \ \forall a \in H$,

where by AoB we mean $\bigcup_{x \in A, y \in B} xoy$, for all subsets A, B of H.

Lemma 1.7. *Let (H, o) be a hyperstructure. Then the following statements are equivalent:*

(i) $aoH = Hoa = H$, $\forall a \in H$,

(ii) *for all* a *and* y *in* H *there exist* u *and* v *in* H *such that* $y \in uoa$ *and* $y \in aov$.

2. $(POM)_L$-Fuzzy (hyper)graphs

Definition 2.1. Let $\delta \in F_L(X)$. Then we say that δ is non-trivial if

$$\delta^* = \text{supp}\delta = \{x \in X | 0 \leq \delta(x), \delta(x) \neq 0\} \neq \emptyset.$$

Definition 2.2. Let $\mu \in F_L(X \times X)$. Then we say that μ is an L-Fuzzy relation on X.

Definition 2.3. Let $\delta \in F_L(X)$ and $\mu \in F_L(X \times X)$. Then μ is said to be a $(POM)_L$-Fuzzy relation on δ if

$$\mu(x, y) \leq T(\delta(x), \delta(y)), \ \forall x, y \in X.$$

Definition 2.4. Let $X \neq \emptyset$ and $\delta \in F_L(p^*(X))$. Then δ is said to be an L-Fuzzy hypersubset of X, if for any finite family $\{A_i\}_{i=1,2,\dots,n}$ of $p^*(X)$ we have

$$\delta(\bigcup_{i=1}^{n} A_i) \leq \bigvee_{i=1}^{n} \delta(A_i) \ .$$

Lemma 2.5. *Let* $\delta \in F_L(X)$. *Then* δ *induces an* L*-Fuzzy hypersubset* δ' *of* X.

Sketch of proof. Define $\delta' \in F_L(p^*(X))$ as follows:

$$\delta'(A) = \bigvee_{a \in A} \delta(a) \quad , \quad \forall A \in p^*(X).$$

Definition 2.6. Let $\mu \in F_L(p^*(X) \times p^*(X))$. Then μ is called an L-Fuzzy hyperrelation on X if:

$$\mu(\bigcup_{i=1}^{n} E_i, \bigcup_{j=1}^{m} F_j) = \bigvee_{i=1}^{n} (\bigvee_{j=1}^{m} \mu(E_i, F_j))$$

for any finite families $\{E_i\}_{i=1,2,\dots,n}$ and $\{F_j\}_{j=1,2,\dots,m}$ of $p^*(X)$.

Remark 2.7. Let μ be an L-Fuzzy hyperrelation on X. Then $A \subseteq B$ and $C \subseteq D$ imply that $\mu(A, C) \leq \mu(B, D)$.

Theorem 2.8. *Let* μ *be an* L*-Fuzzy relation on* X. *Then* μ *induces an* L*-Fuzzy hyperrelation* μ' *on* X.

Sketch of proof. Define $\mu' \in F_L(p^*(X) \times p^*(X))$ as follows:

$$\mu'(A, B) = \bigvee_{a \in A} (\bigvee_{b \in B} \mu(a, b)) \quad , \quad \forall A, B \in p^*(X).$$

Definition 2.9. Let δ be an L-Fuzzy hypersubset of X and μ be an L-Fuzzy hyperrelation on X. Then μ is said to be a $(POM)_L$-Fuzzy hyperrelation on δ if

$$\mu(E, F) \leq T(\delta(E), \delta(F)) \quad , \quad \forall E, F \in p^*(X).$$

Lemma 2.10. *Let μ be a $(POM)_L$-Fuzzy relation on δ, and μ', δ' be such as defined in Theorem 2.8 and Lemma 2.5 respectively. Then μ' is a $(POM)_L$-Fuzzy hyperrelation on δ'.*

Proof. The proof is easy. □

Definition 2.11. Let $X \neq \emptyset$ and $\xi = \{\mu_i\}_{i=1,2,\ldots,n}$ be a family of non-trivial L-Fuzzy subsets of X and $X = \bigcup_{i=1}^{n} \mu_i^*$. Then the L-Fuzzy hyperrelation μ on X is called a $(POM)_L$-Fuzzy hyperrelation on ξ if for all $i, j \in \{1, 2, \ldots, n\}$:

$$\mu(A,B) \leq T(\bigvee_{x \in A} \mu_i(x), \bigvee_{y \in B} \mu_j(y)) \quad , \quad \forall A, B \in p^*(X), A \subseteq \mu_i^*, B \subseteq \mu_j^*.$$

Theorem 2.12. *Let $X = \{x_1, x_2, \ldots, x_n\}$, $\mu \in F_L(p^*(X) \times p^*(X))$ and $\delta \in F_L(p^*(X))$. If μ is a $(POM)_L$-Fuzzy hyperrelation on δ, then there is a family ξ of non-trivial elements of $F_L(X)$ such that μ is a $(POM)_L$-Fuzzy hyperrelation on ξ.*

Sketch of proof. For each $1 \leq i \leq n$, define μ_i as follows:

$$\mu_i : X \longrightarrow L, \ \mu_i(x) = \begin{cases} \delta(\{x_i\}) & \text{if} \quad \delta(\{x_i\}) \neq 0, x = x_i \\ 1 & \text{if} \quad \delta(\{x_i\}) = 0, x = x_i \\ 0 & \text{if} \quad x \neq x_i \end{cases}$$

Now let $\xi = \{\mu_1, \mu_2, \ldots, \mu_n\}$. Then it can be checked that μ is a $(POM)_L$-Fuzzy hyperrelation on ξ.

Theorem 2.13. *Let μ be a $(POM)_L$-Fuzzy hyperrelation on a family $\xi = \{\mu_i\}_{i=1,2,\ldots,n}$ of L-Fuzzy subsets of X. Then there exists an L-Fuzzy hypersubset δ of X such that μ is a $(POM)_L$-Fuzzy hyperrelation on δ.*

Proof. Define the L-Fuzzy hypersubset δ of X as follows:

$$\begin{aligned} \delta : p^*(X) &\longrightarrow L \\ A &\longmapsto \bigvee_{i=1}^{n} (\bigvee_{x \in A \cap \mu_i^*} \mu_i(x)). \end{aligned}$$

It is obvious that δ is well-defined and since each μ_i is non-trivial for $i = 1, 2, \ldots, n$, we conclude that δ is also non-trivial. First we show that δ is an L-Fuzzy hypersubset of X. Let $\{A_j\}_{j=1,2,\ldots,t}$ be a finite family of $p^*(X)$, then

$$\begin{aligned}
\delta\left(\bigcup_{j=1}^{t} A_j\right) &= \bigvee_{i=1}^{n}\left(\bigvee_{x\in(\bigcup_{j=1}^{t} A_j)\cap\mu_i^*} \mu_i(x)\right) = \bigvee_{i=1}^{n}\left(\bigvee_{x\in \bigcup_{j=1}^{t}(A_j\cap\mu_i^*)} \mu_i(x)\right) \\
&= \bigvee_{i=1}^{n}\left(\bigvee_{j=1}^{t}\left(\bigvee_{x\in(A_j\cap\mu_i^*)} \mu_i(x)\right)\right) = \bigvee_{j=1}^{t}\left(\bigvee_{i=1}^{n}\left(\bigvee_{x\in(A_j\cap\mu_i^*)} \mu_i(x)\right)\right) \\
&= \bigvee_{j=1}^{t}(\delta(A_j)) \ .
\end{aligned}$$

Now let $E, F \in p^*(X)$, then

$$\begin{aligned}
\mu(E,F) &= \mu(X\cap E, X\cap F) = \mu\left(\bigcup_{i=1}^{n}\mu_i^*\cap E, \bigcup_{j=1}^{n}\mu_j^*\cap F\right) \\
&= \mu\left(\bigcup_{i=1}^{n}(\mu_i^*\cap E), \bigcup_{j=1}^{n}(\mu_j^*\cap F)\right) \\
&= \bigvee_{i=1}^{n}\left(\bigvee_{j=1}^{n}\mu(\mu_i^*\cap E, \mu_j^*\cap F)\right), \\
&\leq \bigvee_{i=1}^{n}\left(\bigvee_{j=1}^{n} T\left(\bigvee_{x\in\mu_i^*\cap E}\mu_i(x), \bigvee_{y\in\mu_j^*\cap F}\mu_j(y)\right)\right) \\
&\leq T\left(\bigvee_{i=1}^{n}\left(\bigvee_{x\in\mu_i^*\cap E}\mu_i(x)\right), \bigvee_{j=1}^{n}\left(\bigvee_{y\in\mu_j^*\cap F}\mu_j(y)\right)\right), \quad by(*) \\
&= T(\delta(E), \delta(F)) \ .
\end{aligned}$$

□

Definition 2.14. Let $X \neq \emptyset$ be a finite set. Then the triple $H = (X, \delta, \mu)$ is called a $(POM)_L$-Fuzzy graph on X if

(i) $\delta \in F_L(X)$,

(ii) $\mu \in F_L(X \times X)$ and μ is a $(POM)_L$-Fuzzy relation on δ.

Note that if $L = [0, 1] \subseteq \mathbf{R}$ and $T = min$, then a $(POM)_L$-Fuzzy graph is also a fuzzy graph.

Remark 2.15. Let $H = (X, \delta, \mu)$ be a $(POM)_L$-Fuzzy graph. So $\mu(x, y) \leq T(\delta(x), \delta(y))$, for all $x, y \in X$. If $x \notin \delta^*$, then

$$\mu(x, y) \leq T(\delta(x), \delta(y)) = T(0, \delta(y)) = 0 \ ; \ \forall y \in X$$

That is $\mu(x, y) = 0$. So $(x, y) \notin \mu^*$ for all $y \in X$. Now if we put $Y = \delta^* \subseteq X$, then $(Y, \delta|_Y, \mu|_{Y\times Y})$ is a $(POM)_L$-Fuzzy graph, called the saturated $(POM)_L$-Fuzzy subgraph of (X, δ, μ).

From now on we let all $(POM)_L$-Fuzzy graph (X, δ, μ) to be the saturated $(POM)_L$-Fuzzy subgraph of itself, so that $\delta^* = X$.

Definition 2.16. Let $X \neq \emptyset$ be a finite set and $\mathcal{H} = (X, \{\mu_i\}_{i=1,2,\ldots,n}, \mu)$. Then $\mathcal{H}$ is called a $(POM)_L$-Fuzzy hypergraph on X if μ is a $(POM)_L$-Fuzzy hyperrelation on $\{\mu_i\}_{i=1,2,\ldots,n}$.

Theorem 2.17. *Every $(POM)_L$-Fuzzy graph on X, induces (naturally) a $(POM)_L$-Fuzzy hypergraph on X.*

Proof. Let (X, δ, μ) be a $(POM)_L$-Fuzzy graph where $\delta^* = X = \{x_1, x_2, \ldots, x_n\}$. We define δ_i, for all $i = 1, 2, \ldots, n$ as follows:

$$\delta_i : X \to L \ , \ \delta_i(x) = \begin{cases} \delta(x_i) & \text{if} \quad x = x_i \\ 0 & \text{if} \quad x \neq x_i \end{cases}$$

we have $\delta_i^* = \{x_i\}$ and $X = \bigcup_{i=1}^{n} \delta_i^*$. Consider $\mu' \in F_L(p^*(X) \times p^*(X))$ as defined in Theorem 2.8. Now we claim that $(X, \{\delta_i\}_{i=1,2,\ldots,n}, \mu')$ is a $(POM)_L$-Fuzzy hypergraph on X. To see this, since $\delta(x_i) = \delta_i(x_i)$ for all $i = 1, 2, \ldots, n$ we have

$$\begin{aligned} \mu'(\delta_i^*, \delta_j^*) &= \mu'(\{x_i\}, \{x_j\}) = \mu(x_i, x_j) \\ &\leq T(\delta(x_i), \delta(x_j)) = T(\delta_i(x_i), \delta_j(x_j)) \\ &= T(\bigvee_{x\in\delta_i^*} \delta_i(x), \bigvee_{y\in\delta_j^*} \delta_j(y)). \end{aligned}$$

Thus μ is a $(POM)_L$-Fuzzy hyperrelation on $\{\delta_i\}_{i=1,2,\ldots,n}$, and the proof is complete. □

Theorem 2.18. *Let $X = \{x_1, x_2, \ldots, x_n\}$ and $(X, \{\mu_i\}_{i=1,2,\ldots,n}, \mu)$ be a $(POM)_L$-Fuzzy hypergraph on X such that $\mu_i^* = \{x_i\}$, for $i = 1, 2, \ldots, n$. Then μ induces a $(POM)_L$-Fuzzy graph on X.*

Sketch of Proof. Define $\delta \in F_L(X)$ and $\mu' \in F_L(X \times X)$ as follows:

$$\delta : X \to L \ , \ \delta(x_i) = \mu_i(x_i), \ \forall i = 1, 2, \ldots, n$$

and

$$\mu'(x_i, x_j) = \mu(\{x_i\}, \{x_j\}), \ \forall x_i, x_j \in X \ .$$

Then the proof can be completed by some calculations.

Theorem 2.19. *(i) Every (ordinary) graph is a $(POM)_L$-Fuzzy graph.*
(ii) Every (ordinary) hypergraph is a $(POM)_L$-Fuzzy hypergraph.

Sketch of proof. (i) Let $G = (X, E)$ be a graph. Define

$$\delta : X \to L \quad , \quad \delta(x) = 1 \quad , \quad \text{for all} \quad x \in X$$

and

$$\mu : X \times X \to L, \qquad \mu(x, y) = \begin{cases} 1 & \text{if} \quad (x, y) \in E \\ 0 & \text{if} \quad (x, y) \notin E \ . \end{cases}$$

Then we see that (X, δ, μ) is a $(POM)_L$-Fuzzy graph on X.
(ii) Let $\mathcal{H} = (X, \{E_i\}_{i=1,2,\ldots,n})$ be a hypergraph. Define $\mu_i = \chi_{E_i}$, for all $i = 1, 2, \ldots, n$. Then if μ is an arbitrary L-Fuzzy hyperrelation on X, we conclude that $(X, \{\mu_i\}_{i=1,2,\ldots,n}, \mu)$ is a $(POM)_L$-Fuzzy hypergraph on X.

3. Category of $(POM)_L$-Fuzzy hypergraphs

Definition 3.1. Let $(X, \{\mu_i\}_{i=1,2,\ldots,n}, \mu)$ and $(Y, \{\delta_i\}_{i=1,2,\ldots,m}, \delta)$ be two $(POM)_L$-Fuzzy hypergraphs. If

$$\alpha : \{1, 2, \ldots, n\} \longrightarrow \{1, 2, \ldots, m\}$$

and $f : X \longrightarrow Y$ be two functions such that

(i) $f(\mu_i^*) \subseteq \delta^*_{\alpha(i)}$, $i = 1, 2, \ldots, n$,
(ii) $\mu_i(x) \leq \delta_{\alpha(i)}(f(x))$, $i = 1, 2, \ldots, n$, $\forall x \in X$,
(iii) $\mu(E, F) \leq \delta(f(E), f(F))$, $\forall E, F \in p^*(X)$,

then (f, α) is called a homomorphism of $(POM)_L$-Fuzzy hypergraphs.

Category of $(POM)_L$-Fuzzy hypergraphs ($(POM)_L - FHG_r$):

In order to construct the category $(POM)_L - FHG_r$ of all $(POM)_L$-Fuzzy hypergraphs, we consider all $(POM)_L$-Fuzzy hypergraphs as the objects of this category and for any two objects $\mathcal{X} = (X, \{\mu_i\}_{i=1,2,\ldots,n}, \mu)$ and $\mathcal{Y} = (Y, \{\delta_i\}_{i=1,2,\ldots,m}, \delta)$, we define $\mathrm{Hom}(\mathcal{X}, \mathcal{Y})$ as follows:

$$\mathrm{Hom}(\mathcal{X}, \mathcal{Y}) = \{(f, \alpha) | (f, \alpha) \text{ is a homomorphism from } \mathcal{X} to \mathcal{Y}\}.$$

Now let $\mathcal{X} = (X, \{\mu_i\}_{i=1,2,\ldots,l}, \mu)$, $\mathcal{Y} = (Y, \{\delta_i\}_{i=1,2,\ldots,m}, \delta)$, $\mathcal{Z} = (Z, \{\nu_i\}_{i=1,2,\ldots,n}, \nu)$ be three $(POM)_L$-Fuzzy hypergraphs and let $(f, \alpha) : \mathcal{X} \to \mathcal{Y}$ and $(g, \beta) : \mathcal{Y} \to \mathcal{Z}$ be two homomorphisms of $(POM)_L$-Fuzzy hypergraphs. We define the composition of these homomorphisms by

$$(g, \beta) o (f, \alpha) = (gof, \beta o \alpha) \ .$$

Then $(gof, \beta o\alpha)$ is a homomorphism from $\mathcal{X}$ to $\mathcal{Z}$, b ecause for all $i = 1, 2, \ldots, n$ we have

i) $gof(\mu_i^*) = g(f(\mu_i^*)) \subseteq g(\delta^*_{\alpha(i)}) \subseteq \nu^*_{\beta o \alpha(i)}$
ii) $\mu_i(x) \leq \delta_{\alpha(i)}(f(x)) \leq \nu_{\beta o\alpha(i)}(gof(x)), \forall x \in X$
iii) $\mu(E, F) \leq \delta(f(E), f(F)) \leq \nu(g(f(E), g(f(F)), \qquad \forall E, F \in p^*(X)$.

Now it is easy to check that $(POM)_L - FHG_r$ has all properties of a category.

Theorem 3.2. *Let* $(f,\alpha) : (X, \{\mu_i\}_{i=1,2,\ldots,n}, \mu) \longrightarrow (Y, \{\delta_i\}_{i=1,2,\ldots,m}, \delta)$ *be a homomorphism of* $(POM)_L$*-Fuzzy hypergraphs. Then* (f,α) *is an isomorphism in* $(POM)_L - FHG_r$ *if and only if*

(i) $\alpha \in S_n$*, where* S_n *is the permutation group on* $\{1,2,\ldots,n\}$*,*
(ii) f *is one to one and onto,*
(iii) $f(\mu_i^*) = \delta^*_{\alpha(i)}$*,* $i = 1,2,\ldots,n$*,*
(iv) $\mu_i(x) = \delta_{\alpha(i)}(f(x))$*,* $i = 1,2,\ldots,n$*,* $\forall x \in X$*,*
(v) $\mu(E,F) = \delta(f(E), f(F))$*,* $\forall E,F \in p^*(X)$*.*

Proof. $(\Rightarrow)$ Let (f,α) be an isomorphism. Then

(i),(ii): There exists a morphism (g,β) in $(POM)_L - FHG_r$ such that $(g,\beta)o(f,\alpha) = (1_X, 1_{\{1,2,\ldots,n\}})$ and $(f,\alpha)o(g,\beta) = (1_Y, 1_{\{1,2,\ldots,m\}}$. These show that $gof = 1_X, fog = 1_Y, \beta o\alpha = 1_{\{1,2,\ldots,n\}}$ and $\alpha o\beta = 1_{\{1,2,\ldots,m\}}$. This means that f is bijective and $\alpha \in S_n$; moreover $m = n$.

(iii): Let $i \in \{1,2,\ldots,n\}$ and $j = \alpha(i)$. Then $\beta(j) = 1$. So

$$\begin{aligned} g(\delta_j^*) \ &\subseteq\ \mu^*_{\beta(j)} \Rightarrow f(g(\delta_j^*)) \subseteq f(\mu^*_{\beta(j)}) \\ &\Rightarrow\ \delta_j^* \subseteq f(\mu^*_{\beta(j)}) \Rightarrow \delta^*_{\alpha(i)} \subseteq f(\mu_i^*). \end{aligned}$$

On the other hand we have $f(\mu_i^*) \subseteq \delta^*_{\alpha(i)}$. Thus $\delta^*_{\alpha(i)} = f(\mu_i^*)$.

(iv): Let $i \in \{1,2,\ldots,n\}$, $x \in X$ and $\alpha(i) = j$, $f(x) = y$. Then

$$\mu_i(x) \leq \delta_{\alpha(i)}(f(x)) \ .$$

Since $\delta_j(y) \leq \mu_{\beta(j)}(g(y))$ we get that $\delta_{\alpha(i)}(f(x)) \leq \mu_i(x)$. Thus $\mu_i(x) = \delta_{\alpha(i)}(f(x))$.

(v): Let $E,F \subseteq X$, and $A = f(E)$, $B = f(F)$. Thus $g(A) = E$ and $g(B) = F$. Since $\delta(A,B) \leq \mu(g(A), g(B))$ we conclude that

$$\delta(f(E), f(F)) \leq \mu(E,F) \leq \delta(f(E), f(F)),$$

and (v) is proved.

$(\Leftarrow)$ Define $g = f^{-1}$ and $\beta = \alpha^{-1}$, first we show that (g,β) is a morphism in $(POM)_L - FHG_r$ from $(Y, \{\delta_i\}_{i=1,2,\ldots,m}, \delta)$ in to $(X, \{\mu_i\}_{i=1,2,\ldots,n}, \mu)$.

Note that since α is bijective, we must have $m = n$. Let $j \in \{1,2,\ldots,n\}$. Then there exists $i \in \{1,2,\ldots,n\}$ such that $i = \beta(j)$. For i we have $f(\mu_i^*) = \delta^*_{\alpha(i)}$, by (iii) so $\mu_i^* = g(\delta^*_{\alpha(i)})$, and hence $g(\delta_j^*) = \mu^*_{\beta(j)}$. Thus condition (i) of Definition 3.1 holds.

Now let $j \in \{1,2,\ldots,n\}$ and $y \in Y$. Then there exists $i \in \{1,2,\ldots,n\}$ and $x \in X$ such that $i = \beta(j)$ and $x = g(y)$. Now from (iv) we get

$$\mu_i(x) = \delta_{\alpha(i)}(f(x)) \Rightarrow \mu_{\beta(j)}(g(y)) = \delta_j(y) \ .$$

Hence, the condition (ii) of Definition 3.1 holds too. Let $A,B \subseteq Y$. Then there exist $E,F \subseteq X$ such that $E = g(A)$ and $F = g(B)$. For E,F we have

$$\mu(E,F) = \delta(f(E), f(F)) \Rightarrow \delta(A,B) = \mu(g(A), g(B)) \ .$$

Hence (g,β) is a morphism in $(POM)_L - FHG_r$. It is clear that $(g,\beta)o(f,\alpha) = (f,\alpha)o(g,\beta) = (1,1)$. So (g,β) is an isomorphism. □

Definition 3.3. Let $\mathcal{X} = (X, \delta, \mu)$, $\mathcal{Y} = (Y, \delta', \mu')$ be two $(POM)_L$-Fuzzy graphs. If $f : X \longrightarrow Y$ be a function such that:

(i) $\delta(x) \leq \delta'(f(x))$, $\forall x \in X$,
(ii) $\mu(x,y) \leq \mu'(f(x), f(y))$, $\forall (x,y) \in X \times Y$,
then we say that f is a homomorphism from $\mathcal{X}$ to $\mathcal{Y}$.

Category of $(POM)_L$-Fuzzy graphs ($(POM)_L - FG_r$):

We construct the category $(POM)_L - FG_r$ of all $(POM)_L$-Fuzzy graphs. The objects of this category are all $(POM)_L$-Fuzzy graphs, and for any two objects $\mathcal{X} = (X, \delta, \mu)$, $\mathcal{Y} = (Y, \delta', \mu')$, we define $\mathrm{Hom}(\mathcal{X}, \mathcal{Y})$ to be the set of all homomorphism from $\mathcal{X}$ into $\mathcal{Y}$. It is easy to see that $(POM)_L - FG_r$ has all properties of a category.

Theorem 3.4. *In $(POM)_L - FG_r$, coproduct exists, for any finite family of objects.*

Proof. Let $\{(A_i, \delta_i, \mu_i)\}_{i\in I}$ be a finite family of objects of $(POM)_L - FG_r$. For $\{A_i\}_{i\in I}$. It is well-known that $(\overset{0}{\bigcup\limits_{i\in I}} A_i, \lambda_i)$ is a coproduct in the category of sets, where by $\overset{0}{\bigcup\limits_{i\in I}} A_i$ we mean the set $\{(a,i) | (a,i) \in \bigcup\limits_{i\in I}(A_i \times \{i\}), a \in A_i\}$, and for each $i \in I$,

$$\lambda_i : A_i \to \overset{0}{\bigcup_{i\in I}} A_i \quad , \quad \lambda_i(a) = (a,i).$$

Now we define

$$\delta : \overset{0}{\bigcup_{i\in I}} A_i \to L \quad , \quad \delta((a,i)) = \delta_i(a) \qquad \text{for all}(a,i),$$

and

$$\mu : \overset{0}{\bigcup_{i\in I}} A_i \times \overset{0}{\bigcup_{i\in I}} A_i \to L \quad , \quad \mu((a,i),(b,j)) = \begin{cases} \mu_i(a,b) & \text{if} \quad i = j \\ 0 & \text{if} \quad i \neq j \end{cases}$$

Then it is easy to see that μ is a $(POM)_L$-Fuzzy relation on δ. So $(\overset{0}{\bigcup\limits_{i\in I}} A_i, \delta, \mu)$ is an object in $(POM)_L - FG_r$. Now we show that for each $i \in I$, $\lambda_i : (A_i, \delta_i, \mu_i) \to (\overset{0}{\bigcup\limits_{i\in I}} A_i, \delta, \mu)$ is a morphism in $(POM)_L - FG_r$. To see this:
(i) $\delta(\lambda_i(a)) = \delta(a,i) = \delta_i(a)$, for all $a \in A_i$,
(ii) $\mu(\lambda_i(a), \lambda_i(b)) = \mu((a,i),(b,i)) = \mu_i(a,b), \forall a, b \in A_i$.
Thus λ_i is a morphism.

Next we prove that $((\overset{0}{\bigcup\limits_{i\in I}} A_i, \delta, \mu), \{\lambda_i\}_{i\in I})$ is a coproduct for $\{(A_i, \delta_i, \mu_i)\}_{i\in I}$.

Let (S, δ', μ') be a $(POM)_L$-Fuzzy graph and $\{f_i : (A_i, \delta_i, \mu_i) \to (S, \delta', \mu')\}_{i\in I}$ be a family of morphisms in $(POM)_L - FG_r$.

Since $(\bigcup^{0}_{i\in I} A_i, \{\lambda_i\}_{i\in I})$ is the coproduct for $\{A_i\}_{i\in I}$ in the category of sets, we conclude that there exists a unique morphism $\psi : \bigcup^{0}_{i\in I} A_i \to S$ in the category of sets, such that

$$\psi o\lambda_i = f_i \quad , \quad \forall i \in I \ .$$

Now it can be checked that, in fact $\psi : (\bigcup^{0}_{i\in I} A_i, \delta, \mu) \to (S, \delta', \mu')$ is a morphism in $(POM)_L - FG_r$. Moreover it is unique, and this makes commutative the following diagram:

$$\begin{array}{ccc} (\bigcup^{0}_{i\in I} A_i, \delta, \mu) & \overset{\lambda_i}{\longleftarrow} & (A_i, \delta_i, \mu_i) \\ \psi \ \downarrow & & \swarrow \ f_i \\ (S, \delta', \mu') & & \end{array}$$

that is $(\bigcup^{0}_{i\in I} A_i, \delta, \mu)$ is a coproduct.

□

Category of fuzzy subsets (FS) [5]:

The objects in this category are pairs (S, δ), where S is a set and δ is a fuzzy subset on S. A morphism from (S, δ) to (S', δ') is an ordinary function $f : S \longrightarrow S'$ such that $\delta(x) \leq \delta'(f(x))$, $\forall x \in S$. The identity associated with the object (S, δ) is the identity map on the set S.

The composition of maps $f : (S, \delta) \longrightarrow (S', \delta')$ and $g : (S', \delta') \longrightarrow (S'', \delta'')$ is $gof : (S, \delta) \longrightarrow (S'', \delta'')$ where $gof : S \longrightarrow S'$ and $\delta(S) \leq \delta'(f(S)) \leq \delta''(g(f(S))$ for all $s \in S$.

Lemma 3.5. *Let $\{(A_i, S_i)\}_{i\in I}$ be a family of fuzzy subsets. Then the product of this family exists in FS.*

Sketch of Proof. Define the fuzzy subset δ as follows:

$$\delta : \prod_{i\in I} A_i \longrightarrow [0, 1], \quad \delta((a_i)_{i\in I}) = \bigwedge_{i\in I} \delta_i(a_i).$$

Now $\{(\prod_{i\in I} A_i, \delta), \{\Pi_i\}_{i\in I})$ is a product for $\{(A_i, \delta_i)\}_{i\in I}$; for details see [4].

Definition 3.6. [8] Let (L, T) be a partially ordered monoid such that

$$\bigwedge_{\substack{\alpha\in I\\ \beta\in J}} T(x_\alpha, y_\beta) \leq T(\bigwedge_{\alpha\in I} x_\alpha, \bigwedge_{\beta\in J} y_\beta),$$

for any two families $\{x_\alpha\}_{\alpha\in I}$, $\{y_\beta\}_{\beta\in J}$ of elements of L. Then we say that T satisfies the meet-property.

Theorem 3.7. *Let (L, T) be a partially ordered monoid which satisfies the meet-property. Then the product exists for any finite family of objects in $(POM)_L - FG_r$.*

Sketch of Proof. Let $\{(A_i, \delta_i, \mu_i)\}_{i\in I}$ be a finite family of objects of $(POM)_L - FG_r$. Consider the product $(\prod_{i\in I} A_i, \{\Pi_i\}_{i\in I})$ of $\{A_i\}_{i\in I}$ in the category of sets. Define

$$\delta : \prod_{i\in I} A_i \longrightarrow L \quad ; \quad \delta((a_i)_{i\in I}) = \bigwedge_{i\in I} \delta_i(a_i) \ (1)$$

and

$$\mu : \prod_{i\in I} A_i \times \prod_{i\in I} A_i \longrightarrow L \quad ; \quad \mu((a_i)_{i\in I}, (b_i)_{i\in I}) \longrightarrow \bigwedge_{i\in I} \mu_i(a_i, b_i) \ (2)$$

Now we show that μ is a $(POM)_L$-Fuzzy relation on δ. We have

$$\begin{aligned}
\mu((a_i)_{i\in I}, (b_i)_{i\in I}) &= \bigwedge_{i\in I} \mu_i(a_i, b_i) \ ; \text{by } (2) \\
&\leq \bigwedge_{i\in I} T(\delta_i(a_i), \delta_i(b_i)); \text{since } \mu_i \text{ is } L_t - \text{fuzzy relation on } \delta_i, \forall i \in I \\
&\leq T(\bigwedge_{i\in I} \delta_i(a_i), \bigwedge_{i\in I} \delta_i(b_i)) \ ; \text{by meet property of T} \\
&= T(\delta((a_i)_{i\in I}), \delta((b_i)_{i\in I})) \ ; \text{by } (1).
\end{aligned}$$

Thus $(\prod_{i\in I} A_i, \delta, \mu)$ is an object in $(POM)_L - FG_r$.

By considering the family $\{\Pi_i | \Pi_i : (\prod_{i\in I} A_i, \delta, \mu) \longrightarrow (A_i, \delta_i, \mu_i)\}_{i\in I}$ of morphism in $(POM)_L - FG_r$, it is not difficult to prove that $((\prod_{i\in I} A_i, \delta, \mu), \{\Pi_i\}_{i\in I})$ is the product of $\{(A_i, \delta_i, \mu_i)\}_{i\in I}$ in $(POM)_L - FG_r$.

Theorem 3.8. *There exists a full and faithful functor from $(POM)_L - FG_r$ to $(POM)_L - FHG_r$. Hence there exists an embedding from $(POM)_L - FG_r$ into $(POM)_L - FHG_r$.*

Proof. Let (X, δ, μ) be a $(POM)_L$-Fuzzy graph, and $A, B \in p^*(X)$. Define

$$\mu'(A, B) = \bigvee_{a\in A} (\bigvee_{b\in B} \mu(a, b)).$$

Then

$$\begin{aligned}
\mu'(A, B) &= \bigvee_{a\in A} (\bigvee_{b\in B} \mu(a, b)) \\
&\leq \bigvee_{a\in A} (\bigvee_{b\in B} T(\delta(a), \delta(b))) \\
&\leq T(\bigvee_{a\in A} \delta(a), \bigvee_{b\in B} \delta(b)), \text{ by } (*)
\end{aligned}$$

So $(X, \{\delta\}, \mu')$ is a $(POM)_L$-Fuzzy hypergraph. Now define

$$\begin{array}{rcl} F: L_t - FG_r & \longrightarrow & L_t - FHG_r \\ (X, \delta, \mu) & \longmapsto & (X, \{\delta\}, \mu') \\ f & \longmapsto & (f, 1) \end{array}$$

for any morphism $f : (X, \delta, \mu) \longrightarrow (Y, \lambda, \nu)$ in $(POM)_L - FG_r$, where $1 : \{1\} \longrightarrow \{1\}$ is the identity function.

We show that F is a functor.

(i) $f(\delta^*) = f(X) \subseteq Y = \lambda^*$

(ii) Since f is a morphism in $(POM)_L - FG_r$, then

$$\delta(x) \leq \lambda(f(x)),\ \forall x \in X.$$

(iii) $\mu'(E, F) = \bigvee_{a \in E} (\bigvee_{b \in F} \mu(a, b)) \leq \bigvee_{a \in E} \bigvee_{b \in F} (\nu(f(a), f(b))) = \nu'(f(E), f(F))$.

Thus $(f, 1)$ is a morphism in $(POM)_L$-Fuzzy hypergraph. Now if $g : (Y, \lambda, \nu) \longrightarrow (Z, \xi, \rho)$ be a morphism in $(POM)_L - FG_r$, then

$$\begin{array}{rcl} f(gof) & = & (gof, 1) = (gof, 1o1) \\ & = & (g, 1)o(f, 1), \text{ by Definition of } L_t - FHG_r \\ & = & F(g)oF(f). \end{array}$$

It is clear that

$$F(1_{(X,\delta,\mu)}) = (1_X, 1) = 1_{F(X)}.$$

So F is a (covariant) functor.

Now let $\mathcal{X} = (X, \delta, \mu)$ and $\mathcal{Y} = (Y, \lambda, \nu)$ be two arbitrary objects in $(POM)_L - FG_r$. Consider two arbitrary morphisms f, g form $\mathcal{X}$ to $\mathcal{Y}$ such that $F(f) = F(g)$. Thus we have $(f, 1) = (g, 1)$, which implies that $f = g$. That is F is a faithful functor.

Also for the given objects $\mathcal{X}$ and $\mathcal{Y}$, let (f, α) be an arbitrary morphism from $F(\mathcal{X}) = (X, \{\delta\}, \mu')$ to $F(\mathcal{Y}) = (Y, \{\lambda\}, \nu')$ in $(POM)_L - FHG_r$. Then $f : X \longrightarrow Y$ and $\alpha = 1 : \{1\} \longrightarrow \{1\}$ are two functions. Now it is easy to check that f is a morphism from $\mathcal{X}$ to $\mathcal{Y}$ in $(POM)_L - FG_r$, and moreover $F(f) = (f, 1) = (f, \alpha)$. Thus F is a full functor. □

Definition 3.9. Let $(H, *)$ be a hypergroup and $\delta \in F_L(H)$. Then $(H, *, \delta)$ is called a $(POM)_L$-Fuzzy subhypergroup of H if

(i) $T(\delta(x), \delta(y)) \leq \bigwedge_{\alpha \in x*y} \{\delta(\alpha)\}$, $\forall x, y \in H$

(ii) $\forall x, a \in H$, $\exists y \in H$ such that $x \in a * y$ and

$$T(\delta(x), \delta(a)) \leq \delta(y).$$

(ii) $\forall x, a \in H$, $\exists z \in H$ such that $x \in z * a$ and

$$T(\delta(x), \delta(a)) \leq \delta(z).$$

Example 3.10. Let A be a set of n elements, say $\{a_1, a_2, ..., a_n\}$. Then $L = (p(A), \subseteq)$ is a complete lattice which is not a chain. If we consider T as follows:

$T : L \times L \longrightarrow L$

$(B, C) \longmapsto B \bigcap C$

Then (L, T) is a partially ordered monoid. Now let $H = \{1, 2, ..., n\}$. Define the hyperoperation " $\circ$ " on H by

$\circ : H \times H \longrightarrow P^*(H)$

$(i, j) \longmapsto \{i, j\}$

Then it is easy to see that $(H, \circ)$ is a (commutative) hypergroup. Clearly

$\delta : H \longrightarrow L$

$i \longmapsto \{a_1, ..., a_i\}$

is an L-Fuzzy subset on H. Now we can check that $(H, \circ, \delta)$ is an $(POM)_L$-Fuzzy subhypergroup of H. Moreover we can show that for any $k \in N$, $k \leq n$, $(H_k, \circ, \delta)$ is a $(POM)_L$-Fuzzy subhypergroup of H_k.

Remark 3.11. Let $(H, *, \delta)$ be a $(POM)_L$-Fuzzy subhypergroup of H. If $x \in H$ and $x \notin \delta^*$, i.e. $\delta(x) = 0$, then the conditions of Definitions 3.9 always hold. Thus without loss of generality we always suppose that $\delta^* = H$.

Category of $(POM)_L$-Fuzzy subhypergroups ($(POM)_L - FHG_p$):

The objects are all $(POM)_L$-Fuzzy subhypergroups. A morphism form $(H, *, \delta)$ to $(H', *', \delta')$ is a function $f : H \longrightarrow H'$, satisfies

(i) $f(x * y) = f(x) *' f(y)$, $\forall x, y \in H$

(ii) $\delta(x) \leq \delta'(f(x))$, $\forall x \in H$.

Lemma and Definition 3.12 (see [9]). Let $\delta \in F_L(X)$. Define $\mu_\delta \in F_L(X \times X)$ as follows:

$$\mu_\delta(x, y) = T(\delta(x), \delta(y)), \ \forall (x, y) \in X \times Y.$$

Then μ_δ is a $(POM)_L$-Fuzzy relation on δ, and called the strong $(POM)_L$-Fuzzy relation on X.

Proof. The proof is obvious. □

Theorem 3.12. *There exists a functor from $(POM)_L - FHG_p$ to $(POM)_L - FG_r$.*

Proof. Let $(H, *, \delta)$ be a $(POM)_L$-Fuzzy subhypergroup. Define $F((H, *, \delta)) = (H, \delta, \mu_\delta)$. By Lemma 3.12 (H, δ, μ_δ) is a $(POM)_L$-Fuzzy graph. Let $f : (A, *, \delta) \longrightarrow (B, o, \delta')$ be a morphism in $(POM)_L - FHG_p$. Define $F(f) = f$. We have $F(f) : (A, \delta, \mu_\delta) \longrightarrow (B, \delta', \mu_{\delta'})$ such that

i) $\delta(a) \leq \delta'(f(a))$, $\forall a \in A$

ii)
$$\begin{aligned} \mu_\delta(a, b) &= T(\delta(a), \delta(b)) \\ &\leq T(\delta'(f(a), \delta'(f(b))) \\ &= \mu_{\delta'}(f(a), f(b)). \end{aligned}$$

Therefore $F(f)$ is a morphism in $(POM)_L - FG_r$. It is clear that $F(1_{(A,*,\delta)}) = 1_{(A,\delta,\mu_\delta)}$, and $F(gof) = F(g)oF(f)$. Hence F is a functor. □

Theorem 3.13. *There exists a functor from* $(POM)_L - FHG_p$ *to* $(POM)_L - FHG_r$.

Proof. The proof follows from Theorems 3.8 and 3.13. □

Theorem 3.14. *Let* L *be totally ordered. Then every* $(POM)_L$*-Fuzzy hypergraph, induces a* $(POM)_L$*-Fuzzy hypergroup.*

Proof. Let $\mathcal{H} = (X, \{\mu_i\}_{i=1,2,\ldots,n}, \mu)$ be a $(POM)_L$-Fuzzy hypergraph. Define

$$o : p^*(X) \times p^*(X) \longrightarrow p^*(p^*(X))$$

by

$$o(A,B) = o(B,A) = \{C \in p^*(X) | \bigvee_{i=1}^{n} \bigvee_{a \in A} \mu_i(a) \leq \bigvee_{i=1}^{n} \bigvee_{c \in C} \mu_i(c) \leq \bigvee_{i=1}^{n} \bigvee_{b \in B} \mu_i(b)$$

or

$$\bigvee_{i=1}^{n} \bigvee_{b \in B} \mu_i(b) \leq \bigvee_{i=1}^{n} \bigvee_{c \in C} \mu_i(c) \leq \bigvee_{i=1}^{n} \bigvee_{a \in A} \mu_i(a)\}.$$

Thus clearly $A, B \in AoB$, $\forall A, B \in p^*(X)$. (1)

Now we must show that $(p^*(X), o)$ is a commutative hypergroup, to see this let $A, Y \in p^*(X)$ and $U = V = Y$. By (1) we have $Y \in AoV$ and $Y = UoA$. Therefore by Lemma 1.5 we have

$$Aop^*(X) = p^*(X)oA = p^*(X) \quad , \quad \forall A \in p^*(X).$$

Let $A, B, C \in p^*(X)$. Then by considering the totally ordered property of L, it is not difficult to check that $(AoB)oC = Ao(BoC)$. Hence $(p^*(X), o)$ is a commutative hypergroup.

Now define

$$\delta : p^*(X) \longrightarrow L \quad ; \quad \delta(A) = \bigvee_{i=1}^{n} \bigvee_{a \in A} \mu_i(a) \ , \ \forall A \in p^*(X).$$

We claim that $(p^*(X), o, \delta)$ is a $(POM)_L$-Fuzzy subhypergroup. Let $A, B \in p^*(X)$ such that $\delta(A) \leq \delta(B)$, we have

$$\begin{aligned} \inf_{D \in AoB} \{\delta(D)\} &= \inf_{\delta(A) \leq \delta(D) \leq \delta(B)} \{\delta(D)\} \geq \delta(A) \\ &= T(\delta(A), 1) \geq T(\delta(A), \delta(B)) \end{aligned}$$

So condition (i) of Definition 3.9 holds.

Since $B \in AoB = BoA$ and $T(\delta(A), \delta(B)) \leq \delta(A)$, so conditions (ii) and (iii) of Definition 3.9 hold too. Therefore $(p^*(X), o, \delta)$ is a $(POM)_L$-Fuzzy subhypergroup. Note that since $X = \bigcup_{i=1}^{n} \mu_i^*$, hence $\delta^* = p^*(X)$. □

Question: Let L be totally ordered. Then can the object function defined in Theorem 3.14 be completed to a functor?

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M. M. Zahedi, Department of Mathematics, Shahid Bahonar University of Kerman, Kerman, Iran

E-mail address: zahedi_mm@mail.uk.ac.ir

M. R. Khorashadi-Zadeh*, Department of Mathematics, Imam Ali Military University, Tehran, Iran

E-mail address: mr_khorashadi@yahoo.com

*Corresponding author