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Fuzzy adaptive tracking control for a class of nonlinearly parameterized systems with unknown control directions

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Abstract

This paper addresses the problem of adaptive fuzzy tracking control for a class of nonlinearly parameterized systems with unknown control directions. In this paper, the nonlinearly parameterized functions are lumped into the unknown continuous functions which can be approximated by using the fuzzy logic systems (FLS) in Mamdani type. Then, the Nussbaum-type function is used to detect the unknown control direction and based on the backstepping technique, the adaptive fuzzy controller is designed. The main advantages of this paper are that (1) in the existing results the separation principle is used to deal with the nonlinearly parameterized functions, unlike them in this paper, the FLS are applied to approximate the nonlinearly parameterized functions, (2) by using the minimal learning parameters (MLP) algorithm, only one parameter needs to be adjusted online in the controller design procedure, which reduces the online computation burden greatly, (3) the Nussbaum-gain technique is introduced to resolve the unknown control direction problems. It is proven that the proposed control scheme renders the closed-loop system stable in the sense of semiglobal uniformly ultimately bounded (UUB). Finally, simulation results are provided to show the effectiveness of the proposed approach.

Keywords: Fuzzy logic system, backstepping technique, nonlinearly parameterized systems, minimal learning parameters algorithm, unknown control directions.

1 Introduction

In recent years, many new control theories have been gradually established because of the controlled plants and the control objectives becoming more and more complex. For the control problem of the complex nonlinear systems, adaptive control approaches using fuzzy systems have received sustained attention. In the existing fuzzy control theory, the fuzzy logic systems (FLS) in Mamdani type control method receives more and more attention from the scholars, for details see [11, 10, 9, 14, 17, 21, 23, 22, 29, 26], and the references therein. The main advantage of the method is that it can combine some experience and knowledge from designers or experts to investigate the stability and the controller design problem of the nonlinear systems. However, the main disadvantage of the method is that due to the characteristic of the FLS such that there is no systematic stability theory analysis method, and this is the challenging of using the FLS control method.

In the field of adaptive control, nonlinear parameterization is very common in biochemical process, fermentation process, and other related fields [2, 3, 7, 12, 15, 24, 28]. From a practical point of view, when designing controller for a system, nonlinear parameterizations problem should be taken into account. On the other hand, from a theoretical view point, adaptive control of nonlinearly parameterized systems is also interesting, because it represents a new challenge to the theory of nonlinear adaptive control. In [12], the nonlinearly parameterized functions are expressed as linear parameterized functions by introducing the separation principle, and then, this design idea was extended to investigate the stability and the controller design problem of the nonlinearly parameterized systems [7, 15, 24, 28]. However, for

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the nonlinearly parameterized systems, how to handle with the nonlinearly parameterized functions without using the separation principle is a challenging problem.

For the actual system, the unknown control direction is very important and almost universal. In this case, Nussbaum-gain technique is an effective way to deal with it, for example [5, 16, 20]. In [16], by using Nussbaum-gain technique, a new adaptive fuzzy control scheme with prescribed performance was developed for a class of feedback linearizable uncertain MIMO nonlinear systems with unknown control direction and external disturbances, in [5], an adaptive fuzzy output feedback control approach was proposed for a class of MIMO uncertain nonlinear systems with unmeasured states and unknown control directions by applying Nussbaum-gain functions, and in [20], the authors investigated the adaptive fuzzy control design problem of MIMO non-strict feedback nonlinear systems with unknown control directions and dead zones by combining with the Nussbaum-gain technique. However, in this paper, the investigated systems there are both the nonlinear parameterization problem and the unknown control directions problem, which make it become much more difficult to design the effective adaptive tracking controller.

It should be pointed out that a main weakness of the fuzzy control methods, such as [19, 22], is that the number of adaptation parameters laws depends on the number of the fuzzy rule bases or the order of the systems. With an increase of fuzzy rules or the order of the systems, the number of parameters to be estimated will increase significantly. As a result, the online learning time becomes prohibitively large. To avoid this disadvantage, the "minimal learning parameters (MLP) algorithm" is proposed in [8], that is to say, the proposed controller should contain as few adjusted parameters as possible such that the online computational burden is reduced, and then, in [6, 27], the controllers contain only one parameter that needs to be adjusted online were proposed for strict-feedback nonlinear systems and for a class of non-affine stochastic nonlinear systems without lower triangular structure, respectively. And for the MIMO nonlinear systems, the authors constructed the controller with only one adjusted parameter for each subsystem in [13]. Nevertheless, for the nonlinearly parameterized systems, how to design the controller with only one online adjusted parameter is worth studying.

Based on the above observation, in this paper, the tracking control problem is revisited for nonlinearly parameterized systems with unknown control directions using fuzzy control. The nonlinearly parameterized functions will not be dealt with by using the separation principle. The unknown control directions will be detected by introducing the Nussbaum-type functions, and then, the FLS are employed to approximate the unknown nonlinear functions. Finally, by utilizing the MLP algorithm and the backstepping technique, the controller containing only one adjusted parameter is designed.

Compared with existing results, for examples [7, 15, 12, 24, 29, 28], our study has the following contributions:

(1) In this paper, the nonlinearly parameterized functions are dealt with as a whole rather than separated into the linear parameterization by using the separation principle.

(2) By using the MLP algorithm, the controller with only one adjusted parameter is constructed, which reduces the online computation burden greatly.

(3) The existence of the unknown control directions makes the controller design procedure much more complex and difficult. In this paper, Nussbaum-gain technique is introduced to resolve the unknown control direction problems.

It can be proven that all the signals in the closed loop system are bounded and the tracking error can converge to a small residual set around the origin in the mean square sense. Simulation results are provided to show the effectiveness of the proposed approach.

2 System description and preliminaries

2.1 System description

Consider the following nonlinearly parameterized systems

$$\begin{cases} \dot{x}_i = g_i(\bar{x}_i)x_{i+1} + f_i\left(\bar{x}_i, \rho\right), \\ \dot{x}_n = g_n(\bar{x}_n)u\left(t\right) + f_n\left(\bar{x}_n, \rho\right), \\ y = x_1, \end{cases} \tag{1}$$

where $\bar{x}_n = [x_1, \ldots, x_n]^T = x \in R^n$, $u \in R$ and $y \in R$ denote state variables, the control input and the output of the system (1), respectively, and for $1 \le i \le n-1$, $\bar{x}_i = [x_1\ , \ldots, x_i]^T$. Functions $f_i\left(\cdot\right)$ and $g_i\left(\cdot\right)$ are unknown smooth functions, and $f_i\left(\bar{x}_i, \rho\right)$ may be nonlinearly parameterized. $\rho = [\rho_1, \ldots, \rho_m]^T$ is a parameter vector with $\rho_1, \ldots, \rho_m$ being unknown parameters.

The main goal of this paper is to design a fuzzy tracking controller for system (1) such that the system output $y\left(t\right)$ tracks a desired reference signal $y_d\left(t\right)$ which is bound, while all the signals remain bounded. To the end, define a vector function as $\bar{y}_{di} = \left[y_d, y_d^{(1)}, \ldots, y_d^{(i)}\right]^T, i = 1, \ldots, n$, where $y_d^{(i)}$ is the ith time derivative of y_d.

Now, the following assumptions are introduced.

Assumption 1. The desired trajectory vectors $\bar{y}_{di}$ are known and continuous, and $\bar{y}_{di} \in \Omega_{di} \subset R^{i+1}$ with Ω_{di} being known compact sets.

Assumption 2. For $1 \leq i \leq n$, the sign of $g_i(\bar{x}_i) \neq 0$ is unknown, and there exist b and c such that $0 < b \leq |g_i(\bar{x}_i)| \leq c < \infty$.

2.2 Preliminaries

Before proposing the results, we introduce the knowledge about Nussbaum-type function and fuzzy logic system.

Definition 2.1. [13] *$N(\zeta) : R \to R$ is an even smooth Nussbaum-type function, if the function has the following properties*

$$\lim_{s\to\infty} \sup \frac{1}{s}\int_0^s N(\zeta) d\zeta = +\infty, \quad \lim_{s\to\infty} \inf \frac{1}{s}\int_0^s N(\zeta) d\zeta = -\infty. \tag{2}$$

Many functions such as $\zeta^2 \cos(\zeta), \exp(\zeta^2)\cos((\pi/2)\zeta)$ can serve as Nussbaum-type functions. In the paper, the even function $\exp(\zeta^2)\cos(\pi/2\zeta)$ are chosen as Nussbaum-type function to carry out the controller design and simulation.

Lemma 2.2. [13] *If $N(\zeta)$ is a Nussbaum-type function, then:*

(1) Given an arbitrary bounded function $g(\cdot) : R^n \to R$, and $|g(\cdot)| \in [\varepsilon_0, g_0)$ in which ε_0 is an arbitrary positive constant and g_0 is an unknown positive constant with $\varepsilon_0 < g_0 < +\infty$, then, $g(\cdot)N(\zeta)$ is also a Nussbaum-type function.

(2) Given an arbitrary function $c(\cdot) \in [-\varepsilon_0, \varepsilon_0] \subset R$, then, $N(\zeta) + c(\cdot)$ is also a Nussbaum-type function.

Lemma 2.3. [13] *Let $V(t) \geq 0$ and $\zeta_i(t), i = 1, 2 \ldots p$ be smooth functions defined on $[0, t_f)$, and $N(\zeta_i(t)), i = 1, 2 \ldots p$ be smooth Nussbaum-type functions. If the following inequality holds:*

$$V(t) \leq c_0 + e^{-Dt} \sum_{i=1}^{p} \int_0^t N(\zeta_i(\tau))\dot{\zeta}_i(\tau) e^{D\tau} d\tau, \tag{3}$$

where p is bounded integer, D is positive constant and c_0 represents some suitable constant, then, $V(t)$, $\zeta_i(t)$, $i = 1, \ldots, p$ and $\int_0^t N(\zeta_i(\tau))\dot{\zeta}_i(\tau) d\tau$ must be bounded on $[0, t_f)$.

Lemma 2.2 has been proven in [4](See pp. 510) and lemma 2.3 has been proven in [13] (See pp. 2750-2752, Appendix A), respectively.

In this paper, the following rules are used to develop the adaptive fuzzy controller

R^l: if x_1 is F_1^l and x_2 is F_2^l and … and x_n is F_n^l , then y is $G^l, l = 1, 2..., Q$, where $x = \begin{bmatrix} x_1, & \ldots, & x_n \end{bmatrix}^T$ and y are the FLS input and output, respectively. Fuzzy sets F_i^l and G^l, associated with the membership function $\mu_{F_i^l}(x_i)$ and $\mu_{G^l}(y)$, respectively. Q is the rule number. Through singleton function, center average defuzzification, the FLS can be expressed as follows

$$y(x) = \frac{\sum_{l=1}^{Q} \Phi_l \Pi_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{Q} \Pi_{i=1}^{n} \mu_{F_i^l}(x_i)}, \tag{4}$$

where $x = [x_1, x_2, \ldots, x_n]^T \in R^n$, $\mu_{F_i^l}(x_i)$ is the membership function of F_i^l, and $\Phi_l = \arg\sup_{y\in R} \mu_{G^l}(y)$. Define $\phi = [\Phi_1, \ldots, \Phi_Q]^T$ and $\xi(x) = [\xi_1(x) \ldots, \xi_Q(x)]^T$ with the fuzzy basis function ξ_l given by

$$\xi_l(x) = \frac{\Pi_{i=1}^{n} \mu_{F_i^l}(x_i)}{\sum_{l=1}^{Q} \Pi_{i=1}^{n} \mu_{F_i^l}(x_i)}. \tag{5}$$

Then, the fuzzy logic system (4) can be rewritten as

$$y(x) = \phi^T \xi(x). \tag{6}$$

Our first choice for the membership function is the Gaussian function $\mu_{F_i^l}(x_i) = \exp\left(-\frac{1}{2}\left(\left(x_i - a_i^l\right)/\sigma_i^l\right)^2\right)$, where σ_i^l and a_i^l are fixed parameters. It has been proven that when the membership functions are chosen as Gaussian functions, the above fuzzy logic system is capable of uniformly approximating any continuous nonlinear function over a compact set with any degree of accuracy. This property is shown by the following lemma.

Lemma 2.4. [18] *Let $f(x)$ be a continuous function defined on compact set Ω. Then, for any constant $\iota > 0$, there exists an FLS (6) such that*

$$\sup_{x\in\Omega} \left| f(x) - \phi^T \xi(x) \right| \leq \iota. \tag{7}$$

3 Controller design

In this section, we will use the recursive backstepping technique to develop the adaptive fuzzy tracking control laws as follows

$$\alpha_i = N(\zeta_i) M_i, \quad \dot{\zeta}_i = e_i M_i, \quad M_i = k_i e_i + \frac{\vartheta \hat{\theta} \xi_i^T(Z_i) \xi_i(Z_i) e_i}{2\eta^2}, \tag{8}$$

$$\dot{\hat{\theta}} = \sum_{i=1}^{n} \vartheta \xi_i^T(Z_i) \xi_i(Z_i) e_i^2 - \sigma \hat{\theta}, \tag{9}$$

where $1 \le i \le n$, $\vartheta > 0$ and $\sigma > 0$ are designed parameters. $\theta = \max\{\theta_1, \ldots, \theta_n\}$ and $\theta_i = \|\phi_i\|^2$, where $\phi_i = [\Phi_i, \ldots, \Phi_{iN}]^T$. $\hat{\theta}$ is the estimation of θ and the estimation error is $\tilde{\theta} = \hat{\theta} - \theta$. $e_i = x_i - \alpha_{i-1}$, α_0 is equal to y_d, and for $i = 1, \ldots, n$, the control gain k_i satisfy $k_i > \frac{1}{\lambda}$ with λ being a positive design parameter, $\xi_i(Z_i)$ is a fuzzy basis function vector with Z_i being the input vector. Note that when $i = n$, α_n is the true control input $u(t)$.

The n-step adaptive fuzzy backstepping tracking control design is based on the following change of coordinates:

$$\begin{cases} e_1 = y - y_d, \\ e_i = x_i - \alpha_{i-1}, i = 2, \cdots, n-1. \end{cases} \tag{10}$$

Now, we propose the following backstepping-based design procedure.

Step 1: Define tracking error as $e_1 = x_1 - y_d$. Then, its time derivative is given by

$$\dot{e}_1 = \dot{x}_1 - \dot{y}_d = g_1(x_1) x_2 + f_1(x_1, \rho) - \dot{y}_d. \tag{11}$$

Choose Lyapunov function candidate as

$$V_{e_1} = \frac{1}{2} e_1^2, \tag{12}$$

differentiating V_{e_1} and then using $x_2 = e_2 + \alpha_1$ give that

$$\dot{V}_{e_1} = e_1(f_1(x_1, \rho) + g_1(x_1) x_2 - \dot{y}_d) = e_1(f_1(x_1, \rho) + g_1(x_1)(e_2 + \alpha_1) - \dot{y}_d) = g_1(x_1) e_1 e_2 + e_1(g_1(x_1)\alpha_1 + F_1(Z_1)), \tag{13}$$

where

$$F_1(Z_1) = f_1(x_1, \rho) - \dot{y}_d, \tag{14}$$

and $Z_1 = [x_1, \dot{y}_d]^T \in \Omega_{Z_1} \subset R^2$ with Ω_{Z_1} being some known compact set in R^2. Notice that $F_1(Z_1)$ is continuous in the compact Ω_{Z_1}, thus, the nonlinear function $F_1(Z_1)$ can be approximated by an FLS $\phi_1^T \xi_1(Z_1)$ such that

$$F_1(Z_1) = \phi_1^T \xi_1(Z_1) + \delta_1(Z_1), \tag{15}$$

where $\delta_1(Z_1)$ is the approximate error of the FLS.

Remark 3.1. *In (14), the nonlinearly parameterized function $F_1(Z_1)$ contains the variable x_1 and the unknown parameter vector ρ, however, ρ is not variable and it is an unknown constant vector, therefore, if the function $F_1(Z_1)$ is continuous about the variables x_1 and $\dot{y}_d$ in some known compact set Ω_{Z_1}, it can be approximated by using the FLS $\phi_1^T \xi_1(Z_1) + \delta_1(Z_1)$ with any degree of accuracy.*

Remark 3.2. *In the existing result* [29], *without using the separation principle, an adaptive fuzzy tracking control approach is proposed for a class of SISO nonlinear systems in which the unknown continuous functions may be nonlinearly parameterized and the control directions are known. In this paper, Unlike the reference* [29], *for the nth SISO nonlinear systems with nonlinear parameterization and unknown control directions which make the controller design procedure much more complex and difficult, by combining with the Nussbaum-gain technique, the effective adaptive fuzzy tracking controller with only one online adjusted parameter is designed.*

Consequently, substituting (14) and (15) into (13), we can get that

$$\dot{V}_{e_1} = g_1(x_1) e_1 e_2 + g_1(x_1) \alpha_1 e_1 + e_1\left(\phi_1^T \xi_1(Z_1) + \delta_1(Z_1)\right).$$

$$\tag{16}$$

For simplicity, let ε_1 be the upper bound of the fuzzy approximation error $\delta_1(\mathrm{Z}_1)$, according to triangle inequality yields that

$$e_1\phi_1^T\xi_1(Z_1) \le \frac{\vartheta\theta_1}{2\eta^2}\xi_1^T(Z_1)\xi_1(Z_1)e_1^2 + \frac{\eta^2}{2\vartheta}, \quad e_1\delta_1(Z_1) \le \frac{e_1^2}{2\lambda} + \frac{\lambda\varepsilon_1^2}{2}, \tag{17}$$

then, substituting (17) into (16) yields that

$$\dot V_{e_1} \le g_1(x_1)(e_2+\alpha_1)e_1 + \frac{\vartheta\theta_1}{2\eta^2}\xi_1^T(Z_1)\xi_1(Z_1)e_1^2 + \frac{e_1^2}{2\lambda} + d_1,$$

where $d_1 = \frac{\lambda\varepsilon_1^2}{2} + \frac{\eta^2}{2\vartheta}$ with λ, η and ϑ being positive design parameters. Now, choosing the virtual control α_1 in (8) and substituting it into above inequality yield that

$$\dot V_{e_1} \le g_1(x_1)e_2e_1 + (g_1(x_1)N(\zeta_1)+1)\dot\zeta_1 - k_1e_1^2 - \frac{\vartheta\hat\theta\xi_1^T(Z_1)\xi_1(Z_1)e_1^2}{2\eta^2} + \frac{e_1^2}{2\lambda} + \frac{\vartheta\theta_1}{2\eta^2}\xi_1^T(Z_1)\xi_1(Z_1)e_1^2 + d_1, \tag{18}$$

it follows from the definition of $\hat\theta$ that

$$-\frac{\vartheta\hat\theta\xi_1^T(\mathrm{Z}_1)\xi_1(\mathrm{Z}_1)e_1^2}{2\eta^2} + \frac{\vartheta\theta_1}{2\eta^2}\xi_1^T(\mathrm{Z}_1)\xi_1(\mathrm{Z}_1)e_1^2 \le -\frac{\vartheta\hat\theta\xi_1^T(\mathrm{Z}_1)\xi_1(\mathrm{Z}_1)e_1^2}{2\eta^2} + \frac{\vartheta\theta}{2\eta^2}\xi_1^T(\mathrm{Z}_1)\xi_1(\mathrm{Z}_1)e_1^2 \le -\frac{\vartheta\tilde\theta\xi_1^T(\mathrm{Z}_1)\xi_1(\mathrm{Z}_1)e_1^2}{2\eta^2},$$

further, (18) becomes

$$\dot V_{e_1} \le -\left(k_1 - \frac{1}{2\lambda}\right)e_1^2 - \frac{\vartheta\tilde\theta}{2\eta^2}\xi_1^T(Z_1)\xi_1(Z_1)e_1^2 + d_1 + g_1(x_1)e_2e_1 + (g_1(x_1)N(\zeta_1)+1)\dot\zeta_1, \tag{19}$$

where $g_1(x_1)e_2e_1$ will be canceled in the next step.

Step i $(1 \le i \le n-1)$: Considering $e_i = x_i - \alpha_{i-1}$, where α_{i-1} defined in (10), the dynamics of e_i-subsystem is given by

$$\dot e_i = \dot x_i - \dot\alpha_{i-1} = f_i(\bar x_i, \rho) + g_i(\bar x_i)x_{i+1} - W_i - \frac{\partial\alpha_{i-1}}{\partial\hat\theta}\dot{\hat\theta},$$

where

$$W_i = \sum_{j=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial x_j}(f_j(\bar x_j,\rho) + g_j(\bar x_j)x_{j+1}) + \frac{\partial\alpha_{i-1}}{\partial\bar y_{d(i-1)}}\dot{\bar y}_{d(i-1)}.$$

We choose the Lyapunov function as

$$V_{e_i} = \frac{1}{2}e_i^2.$$

The time derivative of V_{e_i} is

$$\dot V_{e_i} = e_i\dot e_i = e_i\left(f_i(\bar x_i,\rho) + g_i(\bar x_i)x_{i+1} - W_i - \frac{\partial\alpha_{i-1}}{\partial\hat\theta}\dot{\hat\theta}\right), \tag{20}$$

furthermore, (20) can be rewritten as

$$\dot V_{e_i} = e_i\left(g_i(\bar x_i)x_{i+1} + \hat f_i(\mathrm{Z}_i)\right) - g_{i-1}(\bar x_{i-1})e_ie_{i-1},$$

where $\hat f_i(\mathrm{Z}_i)$ is defined as

$$\hat f_i(\mathrm{Z}_i) = f_i(\bar x_i,\rho) + g_{i-1}(\bar x_{i-1})e_{i-1} - W_i - \frac{\partial\alpha_{i-1}}{\partial\hat\theta}\dot{\hat\theta}_i.$$

Next, we want to approximate $\hat f_i(\mathrm{Z}_i)$ by using the FLS, but according to (9) we know that $\dot{\hat\theta}$ is a function of all state variables, hence, the term $\frac{\partial\alpha_{i-1}}{\partial\hat\theta}\dot{\hat\theta}$ cannot be dealt with directly. To compensate this term, introduce a function Υ_i as follows

$$\Upsilon_i = \sum_{l=1}^{i-1}\frac{\partial\alpha_{i-1}}{\partial\hat\theta}\vartheta\xi_l^T(Z_l)\xi_l(Z_l)e_l^2 - \sigma\hat\theta\frac{\partial\alpha_{i-1}}{\partial\hat\theta} - \vartheta e_i\sum_{l=2}^{i}(n-l+2)\left|e_l\frac{\partial\alpha_{l-1}}{\partial\hat\theta}\right|,$$

(21)

then, define the function $F_i(Z_i)$ as $F_i(Z_i) = \hat{f}_i(\mathrm{Z}_i) + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}} - \Upsilon_i$, where $Z_i = \left[\bar{x}_i^T, \hat{\theta}, \bar{y}_{di}^T\right]^T \in \Omega_{Z_i} \subset R^{2i+2}$ with Ω_{Z_i} being some known compact set. Similar to step 1, using the FLS to approximate $F_i(Z_i)$ gives that

$$F_i(Z_i) = \phi_i^T \xi_i(Z_i) + \delta_i(Z_i).$$

Then, one gets

$$\dot{V}_{e_i} \le g_i(\bar{x}_i) e_i x_{i+1} + e_i\left(\phi_i^T \xi_i(Z_i) + \delta_i(Z_i)\right) + e_i\left(\Upsilon_i - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right) - g_{i-1}(\bar{x}_{i-1}) e_{i-1} e_i. \tag{22}$$

Furthermore, employing $x_{i+1} = e_{i+1} + \alpha_i$ and the inequalities similar to (17) yields that

$$\dot{V}_{e_i} \le g_i(\bar{x}_i) e_{i+1} e_i + g_i(\bar{x}_i) e_i \alpha_i + \frac{\vartheta \theta_i}{2\eta^2}\xi_i^T(Z_i)\xi_i(Z_i) e_i^2 + d_i + \frac{e_i^2}{2\lambda} + e_i\left(\Upsilon_i - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right) - g_{i-1}(\bar{x}_{i-1}) e_{i-1} e_i. \tag{23}$$

where $d_i = \frac{\lambda \varepsilon_i^2}{2} + \frac{\eta^2}{2\vartheta}$ with ε_i being the upper bound of the fuzzy approximation error $\delta_i(\mathrm{Z}_i)$. In addition, we have

$$\frac{\vartheta \theta_i}{2\eta^2}\xi_i^T(Z_i)\xi_i(Z_i) e_i^2 - \frac{\vartheta \hat{\theta} \xi_i^T(Z_i)\xi_i(Z_i) e_i^2}{2\eta^2} \le \frac{\vartheta \theta}{2\eta^2}\xi_i^T(Z_i)\xi_i(Z_i) e_i^2 - \frac{\vartheta \hat{\theta} \xi_i^T(Z_i)\xi_i(Z_i) e_i^2}{2\eta^2} \le -\frac{\vartheta \tilde{\theta} \xi_i^T(Z_i)\xi_i(Z_i) e_i^2}{2\eta^2}, \tag{24}$$

then, substituting (8), (24) into (23) gives that

$$\dot{V}_{e_i} \le g_i(\bar{x}_i) e_{i+1} e_i + \left(g_i(\bar{x}_i) N(\zeta_i) + 1\right)\dot{\zeta}_i - \frac{\vartheta \tilde{\theta}}{2\eta^2}\xi_i^T(Z_i)\xi_i(Z_i) e_i^2 + e_i\left(\Upsilon_i - \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right) + d_i - \left(k_i - \frac{1}{2\lambda}\right) e_i^2 - g_{i-1}(\bar{x}_{i-1}) e_{i-1} e_i, \tag{25}$$

Remark 3.3. *The existence of the unknown control directions problem makes it become much more complex to construct the controller with only one parameter, in this scheme, the term $\frac{\partial \alpha_{i-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}$ cannot be dealt with directly. To compensate this term, we add the terms Υ_i to counteract the effect of this term on the stability of closed-loop systems.*

Step n: In this step, the true control $u(t)$ will be constructed. Considering $e_n = x_n - \alpha_{n-1}$, the dynamics of e_n-subsystem is given by

$$\dot{e}_n = \dot{x}_n - \dot{\alpha}_{n-1} = f_n(\bar{x}_n, \rho) + g_n(\bar{x}_n) u(t) - W_n - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}, \tag{26}$$

where $W_n = \sum\limits_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}\left(f_j(\bar{x}_j, \rho) + g_j(\bar{x}_j) x_{j+1}\right) + \frac{\partial \alpha_{n-1}}{\partial \bar{y}_{d(n-1)}}\dot{\bar{y}}_{d(n-1)}$. Then, choose the following Lyapunov function as

$$V_{e_n} = \frac{1}{2}e_n^2. \tag{27}$$

Differentiating V_{e_n} yields that

$$\dot{V}_{e_n} = e_n\left(f_n(\bar{x}_n, \rho) + g_n(\bar{x}_n) u(t) - W_n - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right). \tag{28}$$

Similar to step i, (28) can be written as

$$\dot{V}_{e_n} \le g_n(\bar{x}_n) e_n u + e_n F_n(Z_n) - g_{n-1}(\bar{x}_{n-1}) e_{n-1} e_n, \tag{29}$$

where $\bar{\mathrm{Z}}_n = \left[\bar{x}_n^T, \hat{\theta}, \bar{y}_{dn}^T\right]^T \in \Omega_{Z_n} \subset R^{2n+2}$ with Ω_{Z_n} being some known compact set and $F_n(Z_n)$ is defined as

$$F_n(Z_n) = f_n(\bar{x}_n, \rho) + g_{n-1}(\bar{x}_{n-1}) e_{n-1} - W_n - \Upsilon_n,$$

therefore, an FLS is utilized to approximate the unknown function $F_n(Z_n)$ such that

$$F_n(Z_n) = \phi_n^T \xi_n(Z_n) + \delta_n(Z_n).$$

Then, combining the above formula with (29) and applying the inequalities similar to (17), we have

$$\dot{V}_{e_n} \le g_n(\bar{x}_n) e_n u(t) - g_{n-1}(\bar{x}_{n-1}) e_{n-1} e_n + \frac{\vartheta \theta_n}{2\eta^2}\xi_n^T(Z_n)\xi_n(Z_n) e_n^2 + d_n + \frac{e_n^2}{2\lambda} + e_n\left(\Upsilon_n - \frac{\partial \alpha_{n-1}}{\partial \hat{\theta}}\dot{\hat{\theta}}\right),$$

$$(30)$$

where $d_n = \frac{\lambda\varepsilon_n^2}{2} + \frac{\eta^2}{2\vartheta}$ with ε_n being the upper bound of the fuzzy approximation error $\delta_n(\mathrm{Z}_n)$. Now, choosing the controller defined in (8) produces that

$$\dot{V}_{e_n} \leq (g_n(\bar{x}_n)N(\zeta_n)+1)\dot{\zeta}_n + d_n - \left(k_n - \frac{1}{2\lambda}\right)e_n^2 - \frac{\vartheta\tilde{\theta}}{2\eta^2}\xi_n^T(Z_n)\xi_n(Z_n)e_n^2 + e_n\left(\Upsilon_n - \frac{\partial\alpha_{n-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\right) - g_{n-1}(\bar{x}_{n-1})e_{n-1}e_n. \tag{31}$$

Finally, the Lyapunov function candidate is chosen as

$$V(t) = \sum_{j=1}^{n} V_{e_j} + \frac{\tilde{\theta}^2}{4\eta^2}, \tag{32}$$

the derivative of $V(t)$ along time is given by

$$\dot{V}(t) \leq \sum_{j=1}^{n}(g_j(\bar{x}_j)N(\zeta_j)+1)\dot{\zeta}_j + \sum_{j=1}^{n}d_j - \sum_{j=1}^{n}\left(k_j - \frac{1}{2\lambda}\right)e_j^2 + \sum_{j=2}^{n}e_j\left(\Upsilon_j - \frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\right) + \frac{1}{2\eta^2}\tilde{\theta}\left(\dot{\hat{\theta}} - \sum_{j=1}^{n}\xi_j^T(Z_j)\xi_j(\mathrm{Z}_j)e_j^2\right). \tag{33}$$

Applying the parameter adaptive law defined in (9) and substituting it into (33) give that

$$\dot{V}(t) \leq -\sum_{j=1}^{n}\left(k_j - \frac{1}{2\lambda}\right)e_j^2 - \frac{\sigma\tilde{\theta}}{2\eta^2}\hat{\theta} + \sum_{j=1}^{n}(g_j(\bar{x}_j)N(\zeta_j)+1)\dot{\zeta}_j + \sum_{j=2}^{n}\left(\Upsilon_j - \frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\right)e_j + \sum_{j=1}^{n}d_j. \tag{34}$$

Now, we will give the following theorem.

Theorem 3.4. *Consider the closed-loop system (1) under assumptions 1-2. Then, under the action of the adaptive fuzzy control laws (8) and the adaptive law (9) with bounded initial conditions $\hat{\theta}(t_0) > 0$ and $\zeta_i(t_0)$, for $1 \leq i \leq n$, the following properties are guaranteed:*
(i) All the signals in the closed-loop system are bounded.
(ii) The error signals, in the mean square sense, eventually converge to the following small neighborhood of the origin

$$\Xi_\chi := \left\{ e \in R^n | e_{r\chi} \leq \frac{C}{k - \frac{1}{2\lambda}} \right\}$$

where $e = [e_1, \ldots e_n]^T$ and $e_{r\chi} = \frac{1}{t}\int_0^t \|e(\tau)\|^2 d\tau$.

Proof. Now, considering the Lyapunov function candidate defined in (32), and then, combining (34) with the following inequality

$$-\frac{\sigma\tilde{\theta}\hat{\theta}}{2\eta^2} \leq -\frac{\sigma\tilde{\theta}^2}{4\eta^2} + \frac{\sigma\theta^2}{4\eta^2}, \tag{35}$$

we can obtain

$$\dot{V} \leq -\sum_{j=1}^{n}\left(k_j - \frac{1}{2\lambda}\right)e_j^2 - \frac{\sigma\tilde{\theta}^2}{4\eta^2} + C + \sum_{j=1}^{n}(g_j(\bar{x}_j)N(\zeta_j)+1)\dot{\zeta}_j + \sum_{j=2}^{n}e_j\left(\Upsilon_j - \frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\dot{\hat{\theta}}\right), \tag{36}$$

where $C = \frac{\sigma\theta^2}{4\eta^2} + \sum\limits_{j=1}^{n} d_j$. In the following, it will be shown that the last term in (36) is non-positive. With the definitions of $\dot{\hat{\theta}}$ and Υ_j, we have

$$\begin{aligned}\sum_{j=2}^{n}e_j\Upsilon_j - \sum_{j=2}^{n}e_j\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\dot{\hat{\theta}} &= \sum_{j=2}^{n}e_j\Upsilon_j - \sum_{j=2}^{n}e_j\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\left(\sum_{l=1}^{j-1}\vartheta\xi_l^T\xi_l e_l^2 + \sum_{l=j}^{n}\vartheta\xi_i^T(\mathrm{Z}_l)\xi_l(\mathrm{Z}_l)e_l^2 - \sigma\hat{\theta}\right)\\ &= -\sum_{j=2}^{n}\vartheta e_j^2\sum_{l=2}^{j}(n-l+2)\left|e_l\frac{\partial\alpha_{l-1}}{\partial\hat{\theta}}\right| - \sum_{j=2}^{n}e_j\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\left(\sum_{l=j}^{n}\vartheta\xi_l^T(\mathrm{Z}_l)\xi_l(\mathrm{Z}_l)e_l^2\right).\end{aligned} \tag{37}$$

Then, similar to [1] (see p.1533), by rearranging the sequence and using the fact $0 < \xi_l^T\xi_l \leq 1$ for all l, we have

$$-\sum_{j=2}^{n}e_j\frac{\partial\alpha_{j-1}}{\partial\hat{\theta}}\left(\sum_{l=j}^{n}\vartheta\xi_l^T(\bar{\mathrm{Z}}_l)\xi_l(\bar{\mathrm{Z}}_l)e_l^2\right) \leq \sum_{j=2}^{n}\vartheta e_j^2\left(\sum_{l=2}^{j}\left|e_l\frac{\partial\alpha_{l-1}}{\partial\hat{\theta}}\right|\right),$$

(38)

consequently, considering $n \geq l$ and $l \geq 2$, one can get

$$\sum_{j=2}^{n} e_j \Upsilon_j - \sum_{j=2}^{n} e_j \frac{\partial \alpha_{j-1}}{\partial \hat{\theta}} \dot{\hat{\theta}} \leq -\sum_{j=2}^{n} \vartheta e_j^2 \sum_{l=2}^{j} (n-l+2) \left| e_l \frac{\partial \alpha_{l-1}}{\partial \hat{\theta}} \right| + \sum_{j=2}^{n} \vartheta e_j^2 \left(\sum_{l=2}^{j} \left| e_l \frac{\partial \alpha_{l-1}}{\partial \hat{\theta}} \right| \right) \leq 0,$$

which implies that the last term in (36) is non-positive. Thus, (36) can be rewritten as

$$\dot{V} \leq \sum_{j=1}^{n} \left(g_j(\bar{x}_j) N(\zeta_j) + 1 \right) \dot{\zeta}_j - \sum_{j=1}^{n} \left(k_j - \frac{1}{2\lambda} \right) e_j^2 - \frac{\sigma \tilde{\theta}^2}{4\eta^2} + C, \tag{39}$$

where k_j satisfy $k_j > \frac{1}{\lambda}$, for $j = 1, \ldots, n$, so we can deduce $k_j - \frac{1}{2\lambda} > 0$. Let $\psi = \min\left(k_1 - 1/(2\lambda), \ldots, k_n - 1/(2\lambda), \sigma\right)$. From (39) we have

$$\dot{V} \leq -\psi V + \sum_{j=1}^{n} \left(g_j(\bar{x}_j) N(\zeta_j) + 1 \right) \dot{\zeta}_j + C, \tag{40}$$

multiplying both side by $\mathrm{e}^{\psi t}$, (40) can be expressed as

$$\frac{dV(t)\,\mathrm{e}^{\psi t}}{dt} \leq C\mathrm{e}^{\psi t} + \mathrm{e}^{\psi t} \sum_{j=1}^{n} \left(g_j(\bar{x}_j) N(\zeta_j) + 1 \right) \dot{\zeta}_j. \tag{41}$$

Integrating (41) over $[0, t)$ shows that

$$0 \leq V(t) \leq \frac{C}{\psi} + \left[V(0) - \frac{C}{\psi} \right] \mathrm{e}^{-\psi t} + \mathrm{e}^{-\psi t} \sum_{j=1}^{n} \int_0^t \left(g_j(\bar{x}_j) N(\zeta_j) + 1 \right) \dot{\zeta}_j \mathrm{e}^{\psi \tau} d\tau. \tag{42}$$

Note that $0 < \mathrm{e}^{-\psi t} < 1$, we have $\left[V(0) - \frac{C}{\psi} \right] \mathrm{e}^{-\psi t} \leq V(0)$. Then, (42) becomes

$$0 \leq V(t) \leq \frac{C}{\psi} + V(0) + \int_0^t \mathrm{e}^{-\psi t} \sum\nolimits_{j=1}^{n} \left(g_j(\bar{x}_j) N(\zeta_j) + 1 \right) \dot{\zeta}_j \mathrm{e}^{\psi \tau} d\tau. \tag{43}$$

From assumption 2, we know that $g_j(\bar{x}_j)$ is a bounded function. With the help of Lemma 2.2, it can be known that $g_j(\bar{x}_j) N(\zeta_j) + 1$, $j = 1, \ldots, n$ is a Nussbaum-type function. Using Lemma 2.3, it can easily conclude that $V(t), \zeta_j(t)$ and $\int_0^t \left(g_j(\bar{x}_j) N(\zeta_j) + 1 \right) \dot{\zeta}_j \, d\tau$, $1 \leq j \leq n$ must be bounded on $[0, t_f)$. Further, in consideration of the continuous of $N(\zeta_j)$ and the boundedness of ζ_j, we can get that $N(\zeta_j)$ is also bounded. In the following, according to the definition of $V(t)$, boundedness of $e_j, j = 1, \ldots, n$ and $\hat{\theta}$ on $[0, t_f)$ is obtained. Moreover, $\tilde{\theta}$ is bounded as θ is a constant. Then, it follows from $e_1 = x_1 - y_d$ and the boundedness of y_d that x_1 is also bounded. Using (9) with $i = 1$, and noting that $N(\zeta_1)$, e_1, $\hat{\theta}$ and $\xi_1(Z_1)$ are all bounded, we can conclude that α_1 and M_1 are bounded. Consequently, it follows from $e_2 = x_2 - \alpha_1$ that x_2 is bounded for $t \in [0, t_f)$. Following the same way, α_{j-1}, u and $x_j, j = 3, \ldots, n$ can be proven to be bounded on $[0, t_f)$. Thus, all the signals in the closed-loop system are bounded on $[0, t_f)$. Therefore, no finite-time escape phenomenon may happen at $t_f = \infty$.

(ii) According to (39) and $-\frac{\sigma \tilde{\theta}^2}{4\eta^2} < 0$, we can deduce

$$\dot{V}(t) \leq -\sum_{j=1}^{n} \left(k_j - \frac{1}{2\lambda} \right) e_j^2 + C + \sum_{j=1}^{n} \left(g_j(\bar{x}_j) N(\zeta_j) + 1 \right) \dot{\zeta}_j. \tag{44}$$

Integrating (44) from 0 to t gives that

$$\frac{1}{t}\left(V(t) - V(0) \right) \leq -\frac{1}{t} \sum_{j=1}^{n} \left(k_j - \frac{1}{2\lambda} \right) \int_0^t e_j^2(\tau) d\tau + \frac{1}{t} \int_0^t \sum_{j=1}^{n} \left(g_j(\bar{x}_j) N(\zeta_j(\tau)) + 1 \right) \dot{\zeta}_j(\tau) \, d\tau + C,$$

defining $k = \min\left[k_j, \ldots, k_n\right]$, we have

$$e_{r\chi} = \frac{1}{t} \int_0^t \|e(\tau)\|^2 d\tau \leq \frac{\frac{V(0)}{t} + \frac{1}{t} \int_0^t \sum\limits_{j=1}^{n} \left(g_j(\bar{x}_j) N(\zeta_j(\tau)) + 1 \right) \dot{\zeta}_j(\tau) \, d\tau}{k - 1/(2\lambda)} + \frac{C}{k - 1/(2\lambda)}. \tag{45}$$

according to the result of (i), we know that $\int_0^t \sum\limits_{j=1}^{n} \left(g_j\left(\bar{x}_j\right) N\left(\zeta_j\left(\tau\right)\right)+1\right) \dot{\zeta}_j\left(\tau\right) d\tau$ is bounded. Without loss of generality, we assume that Δ is the upper bound of $\int_0^t \sum\limits_{j=1}^{n} \left(g_j\left(\bar{x}_j\right) N\left(\zeta_j\left(\tau\right)\right)+1\right) \dot{\zeta}_j\left(\tau\right) d\tau$. Then, from (45), we can get

$$e_{r\chi} = \frac{1}{t}\int_0^t \left\|e\left(\tau\right)\right\|^2 d\tau \le \frac{\frac{V(0)}{t}+\frac{\Delta}{t}+C}{k-\frac{1}{2\lambda}}. \tag{46}$$

Because no finite-time escape phenomenon may happen, we can get

$$\lim_{t\to+\infty} \frac{\frac{V_n(0)}{t}+\frac{\Delta}{t}+C}{k-\frac{1}{2\lambda}} = \frac{C}{k-\frac{1}{2\lambda}}, \tag{47}$$

which means that e eventually converges to the following set:

$$\Xi_\chi := \left\{ e\in R^n \,\middle|\, e_{r\chi} \le \frac{C}{k-\frac{1}{2\lambda}} \right\}. \tag{48}$$

This completes the proof of Theorem 3.4.

□

4 Simulation

In this section, three numerical simulation examples are given to demonstrate the effectiveness of the proposed control method. In Examples 4.1 and 4.2, the systems with unknown nonlinearly parameterized functions are investigated. And, in Example 4.3, the Brusselator model in dimensionless form is considered.

Example 4.1. *Consider the following nonlinearly parameterized system:*

$$\begin{cases} \dot{x}_1 = \left(2+\cos\left(0.1x_1\right)\right)x_2 + \rho_1^{x_1}, \\ \dot{x}_2 = \left(2.5+\cos\left(x_1x_2\right)\right)u\left(t\right) + \ln\left(1+\left(\rho_2x_2\right)^2\right), \\ y = x_1, \end{cases} \tag{49}$$

In the simulation, $\rho_1 = 0.1$, $\rho_2 = 0.1$, the Nussbaum function is chosen as $N\left(\zeta\right) = \exp(\zeta^2)\cos\left(0.5\pi\zeta\right)$ and the initial states are chosen as $x_1\left(0\right) = 0$, $x_2\left(0\right) = 0$, $\hat{\theta}\left(0\right) = 0$, $\zeta_1(0) = 0$ and $\zeta_2\left(0\right) = 0.5$. The simulation objective is to apply the developed adaptive fuzzy controller such that the boundedness of all the signals in the closed-loop system is guaranteed and the system output y *follows the reference signal y_d to a small neighborhood of zero where $y_d = 0.5\sin t + 0.5\sin(0.5t)$. The virtual controller, the true controller and the parameter adaptive laws are chosen as*

$$\alpha_1 = N\left(\zeta_1\right)M_1, \quad \dot{\zeta}_1 = e_1M_1, \quad M_1 = k_1e_1 + \frac{\vartheta\hat{\theta}\xi_1^T\left(Z_1\right)\xi_1\left(Z_1\right)e_1}{2\eta^2}, \tag{50}$$

$$u(t) = N\left(\zeta_2\right)M_2, \quad \dot{\zeta}_2 = e_2M_2, \quad M_2 = k_2e_2 + \frac{\vartheta\hat{\theta}\xi_2^T\left(Z_2\right)\xi_2\left(Z_2\right)e_2}{2\eta^2}, \tag{51}$$

$$\dot{\hat{\theta}} = \sum_{i=1}^{2} \vartheta\xi_i^T\left(Z_i\right)\xi_i\left(Z_i\right)e_i^2 - \sigma\hat{\theta}, \tag{52}$$

select the design parameters as $\eta = 10$, $\sigma = 0.001$, $\vartheta = 0.8$, $k_1 = 1$ and $k_2 = 0.3$.

The simulation results are shown by Figures 1-6. From Figure 1 and Figure 2, it can be seen that good tracking performance is obtained. The boundedness of u and x_2 are illustrated in Figures 3-4, respectively. Adaptive parameter $\hat{\theta}$ is also bounded by Figure 5. We can deduce that parameters ζ_1 and ζ_2 are also bounded from Figure 6.

Example 4.2. *Consider the following nonlinearly parameterized system:*

$$\begin{cases} \dot{x}_1 = 1 - 3x_1^{\rho_1} + x_1^2x_2, \\ \dot{x}_2 = 2x_1^{\rho_2} + x_2^{\rho_2} + (1.5-\sin(x_1))u(t). \end{cases} \tag{53}$$

In the simulation, we assume that $x_1 \ne 0$, $\rho_1 = \rho_2 = 3$. The Nussbaum function is chosen as $N\left(\zeta\right) = \exp(\zeta^2)\cos\left(0.5\pi\zeta\right)$ and the initial states are chosen as $x_1\left(0\right) = 0.8$, $x_2\left(0\right) = 1$, $\hat{\theta}\left(0\right) = 0$, $\zeta_1(0) = 0$ and $\zeta_2\left(0\right) = 0$. The reference signal is $y_d = 0.6 + 0.2\cos\left(0.6t\right)$. The controller and the parameter adaptive laws are chosen as

$$\alpha_1 = N\left(\zeta_1\right)M_1, \quad \dot{\zeta}_1 = e_1M_1, \quad M_1 = k_1e_1 + \frac{\vartheta\hat{\theta}\xi_1^T\left(Z_1\right)\xi_1\left(Z_1\right)e_1}{2\eta^2},$$

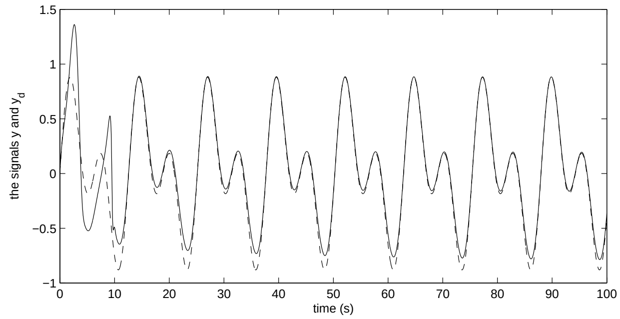

Figure 1: The output y and the reference signal y_d (dashed line).

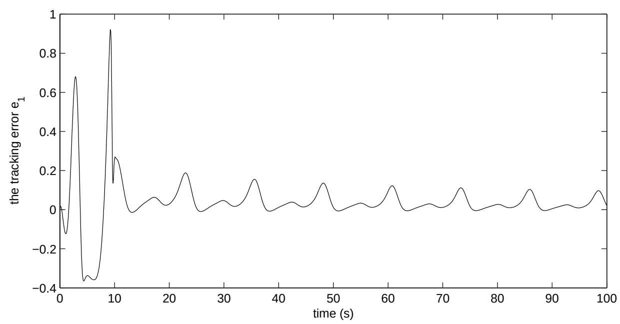

Figure 2: The tracking error e_1.

(54)

$$u(t) = N(\zeta_2) M_2, \quad \dot{\zeta}_2 = e_2 M_2, \quad M_2 = k_2 e_2 + \frac{\vartheta \hat{\theta} \xi_2^T(Z_2) \xi_2(Z_2) e_2}{2\eta^2}, \tag{55}$$

$$\dot{\hat{\theta}} = \sum_{i=1}^{2} \vartheta \xi_i^T(Z_i) \xi_i(Z_i) e_i^2 - \sigma \hat{\theta},$$

(56)

select the design parameters as $\eta = 0.1,\ \sigma = 0.001,\ \vartheta = 1,\ \ k_1 == k_2 = 15$. The simulation results are shown by figures 7-12. Apparently, the simulation results show that under the action of the suggested controller, a good tracking performance has been achieved.

Example 4.3. *The Brusselator model is considered in this section:*

$$\begin{cases} \dot{x}_1 = \rho_1 - (\rho_2 + 1)x_1 + x_1^2 x_2, \\ \dot{x}_2 = \rho_2 x_1 - x_1^2 x_2, \end{cases}$$

(57)

where x_1 and x_2 denote the concentrations of the reaction intermediates: $\rho_1, \rho_2 > 0$ are parameters describing the (constant) supply of "reservoir" chemicals. The Brusselator model is a simplified model describing a certain set of chemical reactions. As a simplified model depicting chemical reactions, the Brusselator model is derived from partial differential equations after a series of approximations. Thus, there must exist modelling errors and other types of unknown nonlinearities in the practical chemical reactions. The controller Brusselator which is considered in [25] is assumed as

$$\begin{cases} \dot{x}_1 = \rho_1 - (\rho_2 + 1)x_1 + x_1^2 x_2, \\ \dot{x}_2 = \rho_2 x_1 - x_1^2 x_2 + (2 + \cos(x_1))\, u, \\ y = x_1, \end{cases}$$

(58)

the nonlinearities $f_1(x_1, \rho) = \rho_1 - (\rho_2 + 1)x_1$, $f_2(\bar{x}_2, \rho) = \rho_2 x_1$, $g_1(x_1) = x_1^2$, $g_2(\bar{x}_2) = 2 + \cos(x_1)$ are assumed unknown functions. In the simulation, we assume that $x_1 \neq 0,\ \rho_1 = 1,\ \rho_2 = 3$. The Nussbaum function is chosen as $N(\zeta) = \exp(\zeta^2)\cos(0.5\pi\zeta)$ and the initial states are chosen as $x_1(0) = 0.8,\ x_2(0) = 1,\ \hat{\theta}(0) = 0,\ \zeta_1(0) = 0$ and $\zeta_2(0) = 0$. The reference signal is $y_d = 0.4 + 0.2\cos(0.6t)$. The controller and the parameter adaptive laws are similar to the Examples 1 and 2, Select the design parameters as $\eta = 0.05,\ \sigma = 0.001,\ \vartheta = 1,\ \ k_1 = k_2 = 1$.

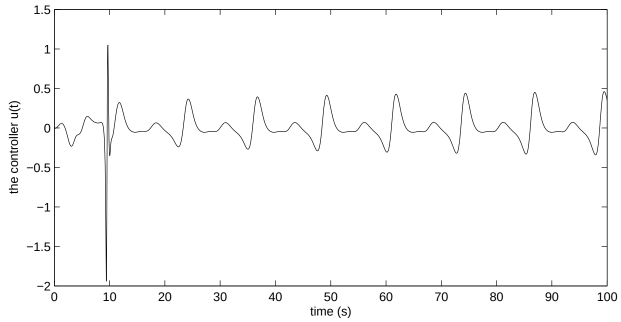

Figure 3: The controller u.

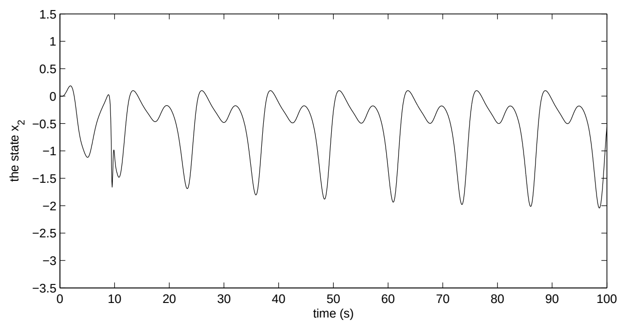

Figure 4: The state x_2.

Remark 4.4. *In the existing results [24, 28, 7, 15], the nonlinearly parameterized functions are separated by using the separation principle, then, the separated out unknown parameters are estimated by designing the parameter adaptive law, which greatly increases the number of the adjusted parameter and the computation burden online. Unlike them in this manuscript, the separation principle is not used, during the controller design process, the nonlinearly parameterized functions are lumped into the continuous functions which can be approximated by using the FLS, and then, by combining the MLP algorithm the controller contains only one adjusted parameter is constructed, and this control scheme can greatly reduces the computation burden online.*

From the simulation results figures 13-18, we can see that the proposed adaptive fuzzy control method can ensure that all the system variables are bounded. And also, the tracking error can be kept in a small neighborhood of the desired trajectory.

Remark 4.5. *From (48) yields that the error signals* $e_1, e_2, \ldots, e_n$ *are all bounded. Moreover, in theory, the tracking error may be made arbitrarily small by appropriately adjusting the design parameters* η, σ, k_1, ϑ *and* k_2. *However, how to choose the optimal parameters to get the optimal tracking performance is still an open problem. In the simulation, the design parameters are set using a trial-and-error method.*

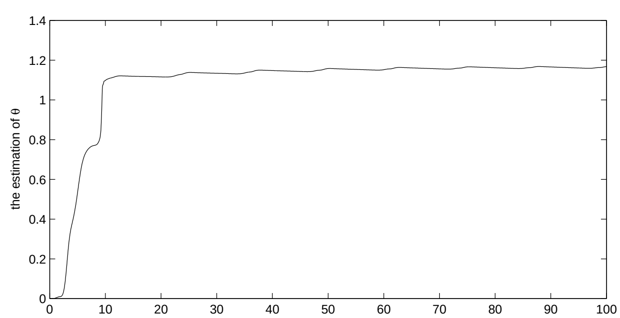

Figure 5: The parameter curve of θ.

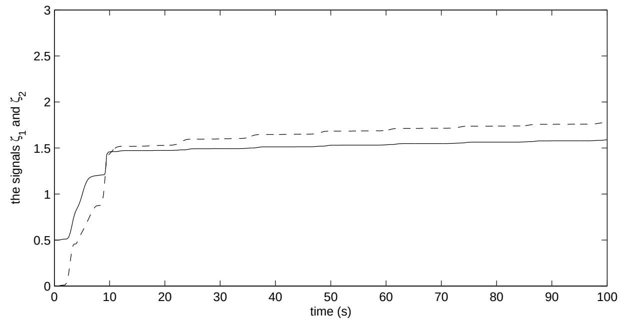

Figure 6: The trajectories ζ_1 and ζ_2 (dashed line).

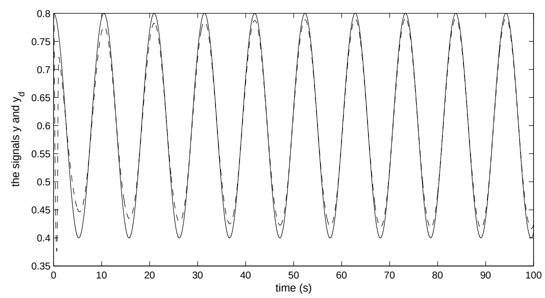

Figure 7: The output y and the reference signal y_d (dashed line).

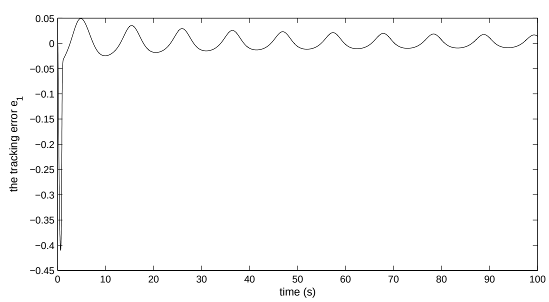

Figure 8: The tracking error e_1.

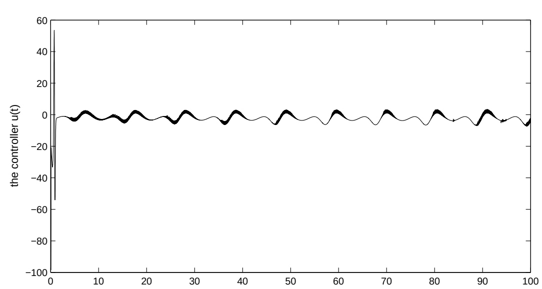

Figure 9: The controller u.

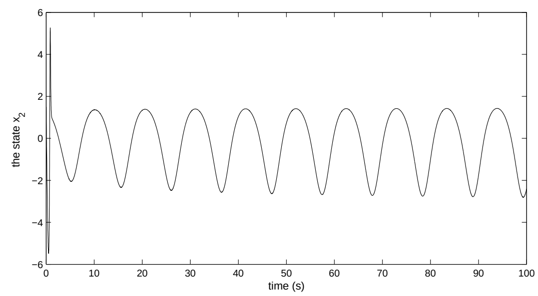

Figure 10: The state x_2.

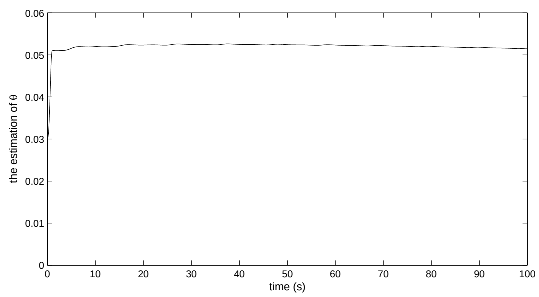

Figure 11: The parameter curve of θ.

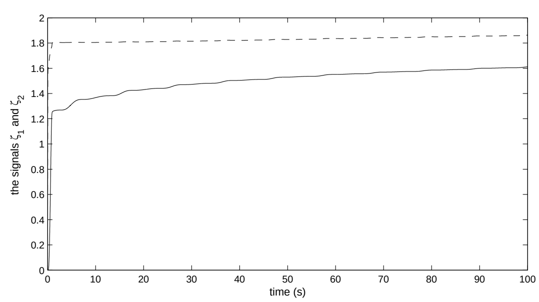

Figure 12: The trajectories ζ_1 and ζ_2 (dashed line).

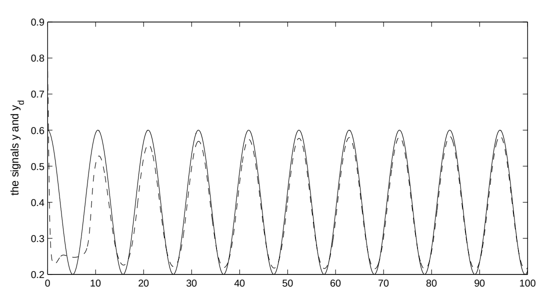

Figure 13: The output y and the reference signal y_d (dashed line).

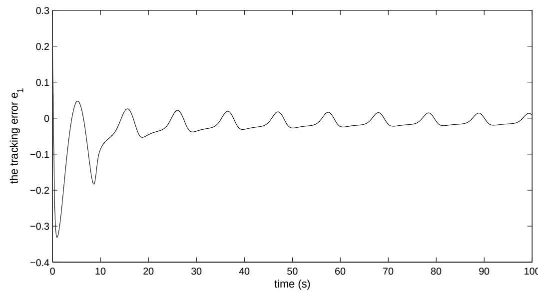

Figure 14: The tracking error e_1.

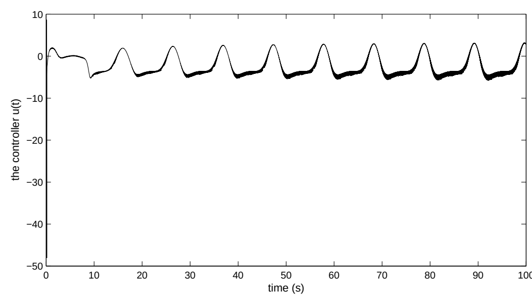

Figure 15: The controller u.

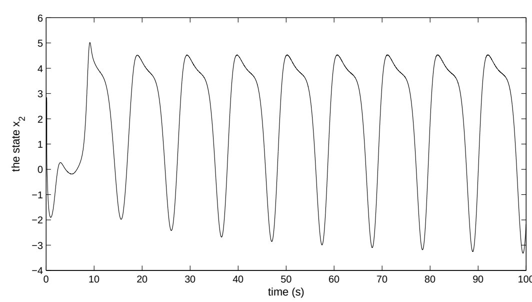

Figure 16: The state x_2.

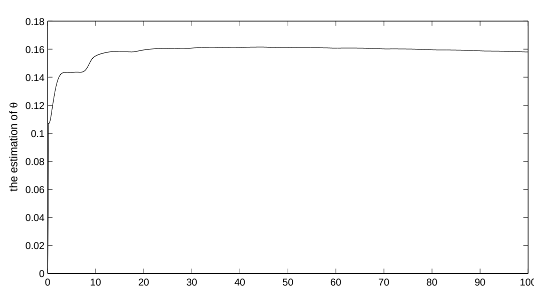

Figure 17: The parameter curve of θ.

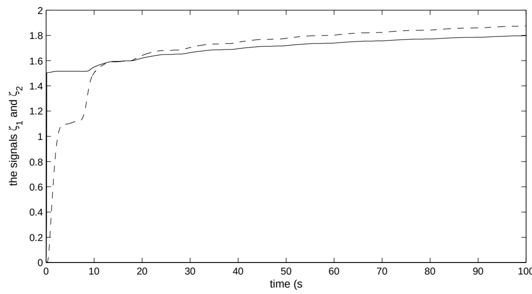

Figure 18: The trajectories ζ_1 and ζ_2 (dashed line).

5 Conclusions

This study has presented an adaptive fuzzy control method for a class of nonlinearly parameterized systems with unknown control directions. Without using the separation principle, the nonlinearly parameterized functions are approximated by using the FLS, and then, the adaptive fuzzy controller is constructed by using the backstepping approach and the Nussbaum-gain technique. Meanwhile, the proposed controller contains only one adaptive parameter. Therefore, the computation burden is significantly reduced and the algorithm is easily realized in practice. The stability of the closed-loop system and the tracking performance of the control system have been demonstrated by using the Lyapunov function theory. Finally, the simulation results have further checked the effectiveness of the proposed control design method and theory. In the future, the problems of adaptive fuzzy control for MIMO nonlinear systems would be considered.

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