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A new framework for high-technology project evaluation and project portfolio selection based on Pythagorean fuzzy WASPAS, MOORA and mathematical modeling

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Abstract

High-technology projects are known as tools that help achieving productive forces through scientific and technological knowledge. These knowledge-based projects are associated with high levels of risks and returns. The process of high-technology project and project portfolio selection has technical complexities and uncertainties. This paper presents a novel two-parted method of high-technology project portfolio selection. In the first part, a new decision-making model under Pythagorean fuzzy set (PFS) uncertainty is introduced that is last aggregation and avoids defuzzification until the last step of the process. In the last step, a new PFS ranking method is used to make crisp and comparable values. Outcomes from this part form the objective function of a new integer programming (IP) of the project portfolio selection. To display the models application, data from a real case study of high-technology project evaluation and selection is presented, and the steps of the approach are illustrated in addition to presenting the efficacy of the model.

Keywords: Project evaluation, project portfolio selection, high-technology projects, group decision-making process, mathematical modeling, Pythagorean fuzzy sets (PFSs).

1 Introduction

Large high-tech mega-projects are referred to projects that require research and development and/or application of technology in addition to a substantial infrastructure and multi-million or even billion dollar budgets. Moreover, their time-horizons are measured in at least years. Despite existence of a large number of technologies, management systems and even proper tools, it is still difficult to successfully implement projects with huge scientific and engineering requirements. The sheer scale of globally funded projects indicates that failure in these sort of projects harms national budgets as well as collaborative reputations [9]. Despite constant investigation of high-technology (high-tech) projects management, this issue is not yet well understood, especially when it comes to applied techniques of the project manager [8].

Project portfolio selection is the periodic and continuous effort of selecting a portfolio of projects that is aligned with organizations stated goals and objectives while considering resources and other constraints. It is a part of project portfolio management process [29]. Involvement of many factors and considerations in this process has made this process complex. Decision makers (DMs) power to perform proper analysis is weakened by existence of high levels of uncertainties and risks or non-existence of proper project information [35, 11, 24]. This complexity and uncertainty is intensified in high-tech projects since factors, such as budget, schedule, safety, reliability and feasibility, are involved in the process [32].

High-tech mega-projects have high levels of risk, ambiguity and uncertainty. Technology, complex groups, and often a dispersed management or infrastructure are some of the main sources of this uncertainty. At the initial phases, uncertainty often impacts performance expectations, political environments, goals and motivations and capabilities [8].

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From the vision of Atkinson et al. [2] uncertainty meant not having all the information which as a matter of fact is not at all unusual in high-tech projects. Thomas and Mengel [34] stated that complex projects have vagueness and ambiguity of the not-yet-known that occur as events that crucially reframe meaning, interpretation, and social significance emerge. Moreover, in these projects, boundaries should be scanned for unwelcomed messengers and the occurrence of situations that reshape understanding of all that has happened before [30]. On the social level, uncertainty is emitted from sources, such as unknown agendas and emergent and divergent strategies [31].

Due to lack of adequate historical data, vagueness and high influence of expert judgment on project selection problems, fuzzy sets theory has been used as an accepted approach in considering project uncertainty [22]. Some of the recent fuzzy sets theory-based studies on project selection are as follows: Oztaysi [25] introduced a group decision-making method, in which fuzzy sets were applied for information system project selection. Khandekar et al. [17] applied fuzzy axiomatic design principles to address project selection. Cheng et al. [7] introduced consistent fuzzy preference relations to deal with project selection. A large number of methods were based on the principles of multi-criteria decision making (MCDM) and multi-criteria analysis [19]. Ivanović et al. [14] studied infrastructure project evaluation and introduced a method based on multi-criteria analysis. Taylan et al. [33] addressed project selection in construction section by introducing a method based on fuzzy MCDM methods. Salehi [28] introduced a fuzzy MCDM approach for project selection. The method was based on combining two well-known MCDM techniques to solve project selection problem. Baysal et al. [3] introduced a two-phase model based on fuzzy MCDM to select projects that need highly complex evaluating processes. They first used fuzzy TOPSIS to find the main group of projects and then applied fuzzy AHP to find the best project. Debnath et al. [10] applied MCDM techniques to present a method for strategic project portfolio selection. Chakraborty and Chakraborty [6] applied TODIM method to address construction project selection.

It can be concluded from the above that most of the studies even in the recent years are based on classic fuzzy sets (FSs). Such sets are characterized by their crisp membership functions in interval [0, 1] which cannot comprehensively address various sorts of uncertainties that occur while expressing numerical values by linguistic methods or while subjectively showing experts judgments [5, 12]. Intuitionistic fuzzy sets (IFSs) [1] are well-known FSs which have been applied in many different areas. They are able to address the degrees of not belonging and hesitation in addition to the levels of belonging. Despite this advantage over classic sets, they still have limitations in denoting the uncertainties. One typical example is when sum of the membership and non-membership degrees surpass the value of 1. To address this issue, Pythagorean fuzzy sets (PFSs) were introduced as an extension of IFSs [36, 37]. In comparison with intuitionistic sets, PFSs are better in addressing uncertainty. In fact, PFSs enable the DMs to express uncertainties which cannot be addressed by applications of intuitionistic sets. In recent years, a number of researches were carried out based on PFSs. Karasan et al. [16] developed a safety and critical effect analysis based on PFSs to address risk assessment. Another example of applying PFSs is using them in the present worth analysis [15]. Mohagheghi et al. [23] developed a decision-making framework based on concepts of the PFS. Moreover, PFS has been applied to address occupational health and safety risk assessment [13]. Another example of PFS application is addressing supplier selection by extending a decision-making process [4]. Despite all the finer points and recent applications of PFSs, they are still new to project and project portfolio related studies.

It can be concluded that high-tech projects have their own special features that call for tailored selection methods. Such projects are highly uncertain since they are dependent on scientific advancements. In these projects, the level of historical data is much lower than other projects, such as conventional research and development (R & D) projects or information technology (IT) projects. In other words, since such projects lead to very high levels of competitive advantages, related data are not available for high-tech projects. Therefore, evaluating such projects is highly dependent on experts opinions. Since these projects require high levels of resources, failure in them would do high damages. Therefore, they need proper project management techniques. Moreover, even success in such projects if they are not properly chosen would be damaging since the firm has spent high levels of its resources on projects that would not mostly contribute to strategic goals. As a result, it is necessary to find the best projects or portfolio of projects when it comes to high-tech projects and failure in doing so would result in damages that are much higher than other projects, such as R&D and IT projects.

Main motivations of proposing this paper are as follows: (1) literature of project selection and projects portfolio selection is very weak when it comes to high-tech projects. In other words, many studies have focused on other projects, such as R&D and IT projects; but, the literature is weak when it comes to high-tech projects; (2) this problem contains very high levels of uncertainty and vagueness, and they are not yet well addressed. When addressing such projects using experts opinions becomes more important and such opinions could be better expressed by fuzzy sets that can address degrees of membership, non-membership and hesitancy. IFSs and their extensions are new to high-tech projects; (3) new fuzzy extensions especially PFSs are very new to this subject. PFSs give more flexibility and power in expressing and calculating uncertainty in comparison with common IFSs and they are new to this environment; (4) the existing decision-making methods do not comprehensively address criteria weighting. In other words, in evaluating high-tech projects one of the most important issues is giving proper levels of importance to selected criteria. This needs using proper methods that incorporate subjective and objective data to form criteria weights in high-tech project and project portfolio selection. This issue is new to high-tech environments; (5) since evaluating such projects is expert dependent, assigning proper weights to them is necessary in properly addressing this group decision-making problem. Decision-making weighting is another issue that is not well addressed in high-tech projects; (6) Using last aggregation methods in project selection problems is still new. In other words, when addressing high-tech project evaluation, it is necessary to use all the gathered information and information loss would result in the loss of effectiveness in the decision. First aggregation methods which are common in project evaluation processes would cause loss of data whereas using last aggregation methods and delaying the aggregation step would result in better use of gathered judgments and avoiding information loss which is also one of the motivations of this paper.

In order to fill the gaps of this important decision-making problem, in this paper a novel two-part model of high-tech project portfolio selection is proposed under uncertain conditions that uses PFSs to model uncertainty. In the first part of the model, a new evaluation and ranking method is presented to score projects based on evaluation of several criteria. This process is last aggregation. It also uses a method to compute the weights of criteria and DMs. Moreover, a new evaluation index is presented that is based on the concept of the weighted aggregated sum product assessment (WASPAS) method and ratio system of the multi-objective optimization on the basis of ratio analysis (MOORA) method. To better model the uncertain environment of high tech projects, defuzzification is done in the ending part of the approach by introducing a novel PFS ranking method. After computing the score of each project, the results are used in a mathematical model in the second part of the method to find the best portfolio of projects. To highlight the main finer points of this study that give it advantage over similar literature works, the following is presented:

- Weights of DMs and criteria are separately used in the process.
- A new decision process is presented that is based on the method of WASPAS and ratio system of MOORA method
- To model and calculate the uncertainty of this problem, PFSs are applied. This novel approach gives the model great advantages over the similar classic fuzzy set-based studies. In other words, uncertainty is expressed, modeled and calculated with a highly sophisticated tool that provides a very high level of flexibility.
- Principles of last aggregation methods are applied in this process. This would result in keeping away from information loss.
- Defuzzification is carried out in the last step of the process. This approach keeps the uncertain characteristics of the data.
- A new PFSs ranking method is introduced and used in the evaluation process. The concept of ideal solutions is applied in this part.

The remainder of this paper is organized as follows. In section 2, a brief introduction to PFSs is presented. Section 3 presents the introduced model. Models application is presented in section 4, and the ending remarks are presented in section 5.

2 A brief introduction to PFSs

In this section, a brief introduction to PFSs is presented to facilitate the comprehension of the introduced approach.

S as an IFS in a universe of discourse (X) can be denoted as follows [26]:

$$S = \{< x, \mu_s(x), \nu_s(x) > \ | \ x \in X\} \tag{1}$$

In Eq. (1), $\mu_s : X \to [0,1]$ is used to address the levels of membership while $\nu_s : X \to [0,1]$ is applied to address the degree of non-membership of element $x \in X$ to the set S, respectively. It is worth mentioning that the values of the abovementioned degrees are subject to the following:

$$0 \leq \mu_s(x) + \nu_s(x) \leq 1 \tag{2}$$

Such sets are identified by another element which is called the degree of indeterminacy $\pi_s(x)$:

$$\pi_s(x) = 1 - \mu_s(x) - \nu_s(x) \tag{3}$$

These sets have been applied in various fields; for instance, sustainable project portfolio selection [20], product lifecycle management [18], product end-of-life scenario selection [22], and supplier selection [41]. Despite the advantages of such sets over classic sets, in times when adding the degrees of satisfaction and dissatisfaction of an alternative like x_i with respect to an attribute, L_j, results in values bigger than 1, intuitionistic sets become inapplicable. To handle such situations, Yager [36, 37] introduced the PFS. The PFSs are introduced in the following:

A PFS like C in a universe of discourse (X) can be expressed by Eq. (4):

$$C = \{< x, \mu_C(x), \nu_C(x) > | x \in X\} \tag{4}$$

In Eq. (4), $\mu_C : X \rightarrow [0, 1]$ shows the degree of membership and $\nu_C : X \rightarrow [0, 1]$ indicates the non-membership degree of $x \in X$ to the set C, respectively. Such values are subject to Eq. (5):

$$0 \leq (\mu_C(x))^2 + (\nu_C(x))^2 \leq 1 \tag{5}$$

Finally, there is another element, called the degree of indeterminacy $\pi_C(x)$, which is denoted by Eq. (6):

$$\pi_C(x) = \sqrt{1 - (\mu_C(x))^2 - (\nu_C(x))^2} \tag{6}$$

Zhang and Xu [38] referred to $(\mu_C(x), \nu_C(x))$ as a Pythagorean fuzzy number (PFN) displayed by $C = (\mu_C, \nu_C)$. An overview of the IFSs and PFSs shows that their constraint condition is the main difference of them. Figure 1 presents a visual description of this difference. Let $p_1 = (\mu_1, \nu_1)$ and $p_1 = (\mu_1, \nu_1)$ be two PFNs and $\rho > 0$; then, the following

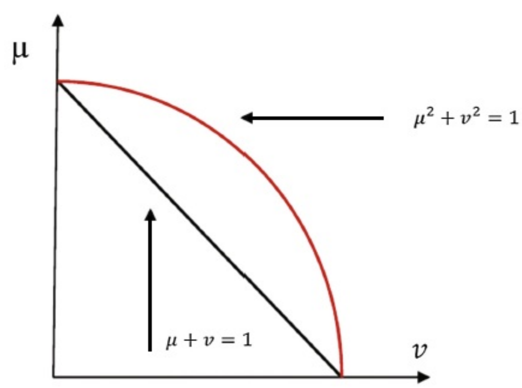

Figure 1: Comparison of IFNs and PFNs

arithmetic operations are applicable [38, 27, 40]:

Addition operator is defined as:

$$p_1 \oplus p_2 = (\sqrt{\mu_1^2 + \mu_2^2 - \mu_1^2\mu_2^2}, \nu_1\nu_2); \tag{7}$$

Multiplication operator is defined as:

$$p_1 \otimes p_2 = (\mu_1\mu_2, \sqrt{\nu_1^2 + \nu_2^2 - \nu_1^2\nu_2^2}); \tag{8}$$

Multiplication of a crisp value (ρ) in a PFS (p_1) is defined as:

$$\rho p_1 = (\sqrt{1 - (1 - (\mu_1)^2)^\rho}, (\nu_1)^\rho) \tag{9}$$

Subtraction of PFSs is defined as follows:

$$p_1 \ominus p_2 = \left(\sqrt{\frac{\mu_1^2 - \mu_2^2}{1 - \mu_2^2}}, \frac{\nu_1}{\nu_2}\right), \text{ if } \mu_1 \geq \mu_2, \nu_1 \leq \min \nu_2, (\nu_2.\pi_1)/\pi_2 \tag{10}$$

Generalized division of PFSs are as follows:

$$\frac{p_1}{p_2}=\left(\frac{\mu_1}{\mu_2},\sqrt{\frac{\nu_1^2-\nu_2^2}{1-\nu_2^2}}\right),\ \text{if}\ \mu_1\leq\min\left\{\mu_2,\frac{\mu_2.\pi_1}{\pi_2}\right\},\ \nu_1\geq\nu_2 \tag{11}$$

A PFS in power of a crisp value (ρ) can be computed by means of Eq. (12):

$$p^{\rho}=\left((\mu^{\rho}),\sqrt{1-(1-\nu^2)^{\rho}}\right),\quad \rho>0 \tag{12}$$

The distance between p_1 and p_2 is defined as follows [38]:

$$d(p_1,p_2)=\frac{1}{2}(|(\mu_{p_1})^2-(\mu_{p_2})^2|+|(\nu_{p_1})^2-(\nu_{p_2})^2|+|(\pi_{p_1})^2-(\pi_{p_2})^2|) \tag{13}$$

Score function of a PFS (p_1) is computable by means of Eq. (14) [40]:

$$s_{p_1}=(\mu_{p_1})^2-(\nu_{p_1})^2 \tag{14}$$

3 Proposed methodology

In order to introduce a new method of high-tech project portfolio selection, this section presents a two-part model of project portfolio selection under PFS uncertainty. The model has two main parts. In the first part, a new decision-making and evaluation method is introduced under PFS uncertainty. Outcome of this section provides the input of the next part of the model in which a mathematical model is used to find the optimal portfolio of projects. This method could best suit high-tech project environments due to the following features: (1) PFSs are used to address uncertainty. This would enable addressing membership, non-membership and hesitancy with more flexibility and less limitations. Such ability could lead to better decision making since highly uncertain elements of high-tech projects can be addressed with proper tools; (2) Using a two-part method would separate analysis of projects based on quantitative and qualitative data and present the main DMs with a better overall view of the studied projects; (3) given the importance of evaluation criteria in assessing high-tech projects, the introduced method for criteria weighting would better address the importance of criteria in this process; (4) given the importance of experts and their roles in high-tech decision making, the presented method applies a group decision-making process that not only attends to weights of DMs, but also applies a last aggregation approach that avoids loss of experts expressed judgments and expertise; (5) Since high-tech projects are involved with high levels of uncertainty, the project evaluation process delays the defuzzification until the last step and this would keep the fuzziness of data and process to better address this uncertain decision-making situation; and (6) in the mathematical modeling of this process, disruption of projects and the maximum level of resistance of firms to disruptions are addressed. In other words, disruptions and delays are not unusual in high-tech projects and firms cannot deal with high levels of disruptions; therefore, their maximum level of resistance to such conditions should be attended to high-tech portfolio optimization.

3.1 Introduced decision-making method

In this part of the methodology, after gathering the judgments, a new project evaluation method is introduced under PFS uncertainty. The method is last aggregation and its defuzzification step is carried out in the last step of the process. Aggregation is carried out through presenting a novel aggregation step based on the concept of the WASPAS method. Additionally, fuzzy entropy is applied in order to better attend to the weights of criteria. The following step-by-step process explains the introduced approach:

1. The initial decision matrices are made by getting the opinions of each DM. This would result in getting the values for matrices displayed in Eqs. (15) and (16):

$$\tilde{D}_k=(\tilde{D}_{ij}^K)_{m\times n}=\begin{bmatrix}(d(\mu_{11}^K,\nu_{11}^K) & \cdots & d(\mu_{1n}^K,\nu_{1n}^K)\\ \vdots & \ddots & \vdots\\ d(\mu_{m1}^K,\nu_{m1}^K) & \cdots & d(\mu_{mn}^K,\nu_{mn}^K)\end{bmatrix} \tag{15}$$

$$\tilde{W}_K=(w(\mu_1^K,\nu_1^K),w(\mu_2^K,\nu_2^K),\ldots,w(\mu_n^K,\nu_n^K)),K\in T \tag{16}$$

where $\tilde{D}_K$ denotes the decision matrix and $\tilde{W}_K$ denotes criteria weights, n shows the number of criteria, m displays the number of alternatives and T shows a group of the DMs.

2. Eq. (17) is applied to obtain the normalized decision matrix ($\widetilde{ND}$):

$$(\widetilde{ND})_{ij} = \begin{cases} d(\mu_{ij}^K, \nu_{ij}^K) & \text{for benefit criterion} \quad C_j \\ (d(\mu_{ij}^K, \nu_{ij}^K))^c & \text{for cost criterion} \quad C_j \end{cases} \tag{17}$$

where $(d(\mu_{ij}^K, \nu_{ij}^K))^c$ is the complement of $d(\mu_{ij}^K, \nu_{ij}^K)$ and is equal to $d(\nu_{ij}^K, \mu_{ij}^K)$. 3. The entropy developed by Zhang and Jiang [39] is applied to attend to the weight of criteria.

3.1. Eq. (18) is applied to calculate entropy of each criterion:

$$E(W_j) = \frac{1}{T}\sum_{k=1}^{T} \frac{\min\{\mu(w_j^k), \nu(w_j^k)\}}{\max\{\mu(w_j^k), \nu(w_j^k)\}} \tag{18}$$

3.2. A novel weight (NW) based on the gathered judgments of importance of criteria and their entropy is obtained by means of Eq. (19):

$$NW_j = (nw(\mu_1^K, \nu_1^K), nw(\mu_2^K, \nu_2^K), \ldots, nw(\mu_n^K, \nu_n^K)) \tag{19}$$

where

$$nw(\mu_1^K, \nu_1^K) = \left(\sqrt{1-(1-(\mu(w_j^k))^2)^{E(W_j)}}, (\nu(w_j^k))^{E(W_j)}\right) \tag{20}$$

4. The resulting matrices are applied to carry out the ranking of alternatives based on judgments of each DM. To do so, these sub-steps are carried out:

4.1. Each DMs judgments are gathered in the following form:

$$\tilde{X}_k = \begin{bmatrix} x(\mu_{11}^K, \nu_{11}^K & \cdots & x(\mu_{1n}^K, \nu_{1n}^K) \\ \vdots & \ddots & \vdots \\ x(\mu_{m1}^K, \nu_{m1}^K) & \cdots & x(\mu_{mn}^K, \nu_{mn}^K) \end{bmatrix} \tag{21}$$

$$x(\mu_{11}^K, \nu_{11}^K) = ND_{ij} \times NW_j = \left(\mu_{ND_{ij}}\mu_{NW_j}, \sqrt{\nu_{ND_{ij}}^2 + \nu_{NW_j}^2 - \nu_{ND_{ij}}^2\nu_{NW_j}^2}\right) \tag{22}$$

4.2. Eq. (23) is applied to get a ranking of alternatives for each DM:

$$S_i^k = \sum_{j\in Bi}\left(\mu_{WD_{ij}}\mu_{NW_j}, \sqrt{\nu_{WD_{ij}}^2 + \nu_{NW_j}^2 - \nu_{WD_{ij}}^2\nu_{NW_j}^2}\right) - \sum_{j\in Ci}\left(\mu_{WD_{ij}}\mu_{NW_j}, \sqrt{\nu_{WD_{ij}}^2 + \nu_{NW_j}^2 - \nu_{WD_{ij}}^2\nu_{NW_j}^2}\right) \tag{23}$$

It should be noted that S_i^k is a PFN.

5. Compute weight of each DM by using their own judgments on alternatives by following the sub-steps:

5.1. Compute the score function of each opinion on rating of alternatives $(S(\tilde{X}_k))$ by means of the following Eq.:

$$S(\tilde{X}_k) = \begin{bmatrix} (\mu_{11}^K)^2 - (\nu_{11}^K)^2 & \cdots & (\mu_{1n}^K)^2 - (\nu_{1n}^K)^2 \\ \vdots & \ddots & \vdots \\ (\mu_{m1}^K)^2 - (\nu_{m1}^K)^2 & \cdots & (\mu_{mn}^K)^2 - (\nu_{mn}^K)^2) \end{bmatrix} \tag{24}$$

5.2. Compute the positive ideal solution ($PIDM_{ij}$) and negative ideal solution ($NIDM_{ij}$) of each DM by using the following:

$$PIDM_{ij} = \frac{\sum_{k=1}^{T}(\mu_{ij}^K)^2 - (\nu_{ij}^K)^2}{T} \tag{25}$$

$$NIDM_{ij} = \max_K(|(\mu_{ij}^K)^2 - (\nu_{ij}^K)^2|) \tag{26}$$

5.3. Compute the distance of each opinion from the positive ($DPIDM_{ij}^k$) and the negative ($DNIDM_{ij}$) ideal solutions to get the following matrices:

$$DPIDM_{ij}^k = \begin{bmatrix} ((\mu_{11}^K)^2-(\nu_{11}^K)^2)-\dfrac{\sum_{k=1}^T(\mu_{11}^K)^2-(\nu_{11}^K)^2}{T} & \cdots & ((\mu_{1n}^K)^2-(\nu_{1n}^K)^2)-\dfrac{\sum_{k=1}^T(\mu_{1n}^K)^2-(\nu_{1n}^K)^2}{T} \\ \vdots & \ddots & \vdots \\ ((\mu_{m1}^K)^2-(\nu_{m1}^K)^2)-\dfrac{\sum_{k=1}^T(\mu_{m1}^K)^2-(\nu_{m1}^K)}{T} & \cdots & ((\mu_{mn}^K)^2-(\nu_{mn}^K)^2)-\dfrac{\sum_{k=1}^T(\mu_{mn}^K)^2-(\nu_{mn}^K)^2}{T} \end{bmatrix} \tag{27}$$

$$DPIDM_{ij}^k = \begin{bmatrix} ((\mu_{11}^K)^2-(\nu_{11}^K)^2)-\max_K(|(\mu_{11}^K)^2-(\nu_{11}^K)^2|) & \cdots & ((\mu_{1n}^K)^2-(\nu_{1n}^K)^2)-\max_K(|(\mu_{1n}^K)^2-(\nu_{1n}^K)^2|) \\ \vdots & \ddots & \vdots \\ ((\mu_{m1}^K)^2-(\nu_{m1}^K)^2)-\max_K(|(\mu_{m1}^K)^2-(\nu_{m1}^K)^2|) & \cdots & ((\mu_{mn}^K)^2-(\nu_{mn}^K)^2)-\max_K(|(\mu_{mn}^K)^2-(\nu_{mn}^K)^2|)) \end{bmatrix} \tag{28}$$

5.4. Compute the overall positive ideal ($OPISk$) and negative ideal score ($ONISk$) of each DM by using the following Eq.:

$$OPIS^k = \sum_{i=1}^{n}\sum_{j=1}^{m}\left|((\mu_{ij}^K)^2-(\nu_{ij}^K)^2)-\frac{\sum_{k=1}^T(\mu_{ij}^K)^2-(v_{ij}^K)^2}{T}\right| \tag{29}$$

$$ONIS^k = \sum_{i=1}^{n}\sum_{j=1}^{m}\left|((\mu_{ij}^K)^2-(v_{ij}^K)^2)-\max_K(\left|(\mu_{ij}^K)^2-(\nu_{ij}^K)^2\right|)\right| \tag{30}$$

5.5. Solve the following mathematical model to get the weight of each DM (WDM^k):

$$\begin{aligned} Z_1 = \min \varepsilon\Bigg(\sum_{k=1}^{T}\Bigg(\sum_{i=1}^{n}\sum_{j=1}^{m}\left|((\mu_{ij}^K)^2-(\nu_{ij}^K)^2)-\frac{\sum_{k=1}^T(\mu_{ij}^K)^2-(\nu_{ij}^K)^2}{T}\right|\Bigg)(WDM^k)\Bigg) \\ +(1-\varepsilon)\Bigg(-\sum_{k=1}^{T}\Bigg(\sum_{i=1}^{n}\sum_{j=1}^{m}\left|((\mu_{ij}^K)^2-(\nu_{ij}^K)^2)-\max_K(|(\mu_{ij}^K)^2-(\nu_{ij}^K)^2|)\right|\Bigg)(WDM^k)\Bigg) \end{aligned} \tag{31}$$

subject to

$$\sum_{k=1}^{T} WDM^k = 1 \tag{32}$$

$$WDM^k \in S, \quad \forall k \in T \tag{33}$$

$$0 \le \varepsilon \le 1 \tag{34}$$

The objective function (Eq. (31)) aims at setting the values of weights in a way that the most important DM would be the one with the least distance from positive ideal opinions and the most distance from ideal negative opinions. Eq. (32) sets the sum of all weights to 1. Eq. (33) ensures that the resulting weights are in the acceptable limitations (S). Finally, Eq. (34) addresses ε which expresses the preferences of DMs in dealing with positive or negative ideal solutions.

6. Aggregate the rankings of DMs by using the following introduced aggregation step based on the concept of WASPAS:

$$\begin{aligned} R_i = &\Bigg(\varphi\Bigg(\sum_{k=1}^{T}\Bigg(\sum_{(} j\in Bi)(\mu_{WD_{ij}}\mu_{NW_j}, \sqrt{\nu_{WD_{ij}}^2+\nu_{NW_j}^2-\nu_{WD_{ij}}^2\nu_{NW_j}^2}\Bigg) \\ &-\sum_{j\in Ci}\Bigg(\mu_{WD_{ij}}\mu_{NW_j}, \sqrt{\nu_{WD_{ij}}^2+\nu_{NW_j}^2-\nu_{WD_{ij}}^2\nu_{NW_j}^2)}\Bigg)WD_k\Bigg)\Bigg) \\ &+(1-\varepsilon)\Pi_{k=1}^{T}\Bigg(\sum_{j\in Bi}\Bigg(\mu_{WD_{ij}}\mu_{NW_j}, \sqrt{\nu_{WD_{ij}}^2+\nu_{NW_j}^2-\nu_{WD_{ij}}^2\nu_{NW_j}^2}\Bigg) \\ &-\sum_{j\in Ci}\Bigg(\mu_{WD_{ij}}\mu_{NW_j}, \sqrt{\nu_{WD_{ij}}^2+\nu_{NW_j}^2-\nu_{WD_{ij}}^2\nu_{NW_j}^2}\Bigg)\Bigg)^{WD_k}\Bigg) \end{aligned} \tag{35}$$

7. In order to make Pythagorean fuzzy values comparable, the following PFS ranking process is introduced:
7.1. Define the ideal solution as $R_{i_{\max}}$ and negative ideal solution as $R_{i_{\min}}$ by using the concept of score function.
7.2. Compute the distance based degree of similarity of each value of R_i and the positive ideal solution ($dbds_i^+$) by using Eq. (36):

$$dbds_i^+(R_i, R_{i,\max}) = \frac{1}{2}\left(\left|(\mu_{R_i})^2 - (\mu_{R_{i,\max}})^2\right| + \left|(\nu_{R_i})^2 - (\nu_{\mu_{R_{i,\max}}})^2\right| + \left|(\pi_{R_i})^2 - (\pi_{\mu_{R_{i,\max}}})^2\right|\right) \tag{36}$$

7.3. Compute the distance based degree of similarity of each value of R_i and the negative ideal solution ($dbds_i^-$) by using Eq. (37):

$$dbds_i^-(R_i, R_{i,\min}) = \frac{1}{2}\left(\left|(\mu_{R_i})^2 - (\mu_{R_{i,\min}})^2\right| + \left|({}_{R_i})^2 - (\nu_{\mu_{R_{i,\min}}})^2\right| + \left|(\pi_{R_i})^2 - (\pi_{\mu_{R_{i,\min}}})^2\right|\right) \tag{37}$$

7.4. Determine the value of project score (PS_i) for each alternative by using the following:

$$PS_i = \left(\frac{dbds_i^-}{dbds_i^- + dbds_i^+}\right), \quad i = 1, 2, \ldots, m \tag{38}$$

7.5. Rank the values of PS_i in decreasing order.

In this step, the DM is provided by an initial ranking of projects. Therefore, in this step by using threshold some projects could be omitted from the process. Moreover, this two-part approach makes it easier to see if any project requires further investigation or attention. The scores achieved in this part form the coefficients of the next step. It is worth noting that the number of projects passing through this part of the model is not the main issue, and the main issue is to find the projects that require further investigation and the projects that are so weak that it is better to avoid considering them in the portfolio optimization process. However, DMs can set thresholds to better address projects that can pass through this part. In the following, three thresholds for this part are introduced based on pessimistic, realistic and optimistic aspects. The values of 0.7, 0.5 and 0.3 are introduced for pessimistic, realistic and optimistic aspects, respectively. In other words, when taking a pessimistic view on the process, the projects with PS_i less than 0.7 are omitted. In the realistic approach, the threshold is 0.5 and in the optimistic approach, threshold is 0.3 meaning that projects with even lower values (i.e., values greater than 0.3) can pass the first part. As a matter of fact, setting these thresholds depending on the way firms deal with risk can vary and each firm can tailor the values according to its own approach.

3.2 Introduced mathematical model of project portfolio selection

In this sub-section, a model is presented that is aiming at obtaining a portfolio of projects that suits all the existing criteria of the process in the best possible way. Notations used in this sub-section are described as follows:

I_i, investment in project i,
$\min_I$, minimum amount of acceptable investment,
$\max_I$, maximum amount of acceptable investment,
PS_i, score of project i obtained in section 3.2,
Hr_i, human resource requirement of project i, $\max_H R$, maximum level of available human resource,
$\min_H R$, minimum level of available human resource,
dis_i, disruption of project i,
$DR_{\max}$, maximum level of disruption resistance,
K, group of projects with mutual exclusiveness relationship,
L, group of projects that must be in the portfolio,
x_i, decision variable which is defined by:

$$x_i = \begin{cases} 0 & \text{if project } i \text{ is rejected} \\ 1 & if project\ i \text{ is selected} \end{cases}$$

$$Z = \max \sum_{i=1}^{m} PS_i x_i \tag{39}$$

Subject to:

$$\min_I \le \sum_{i=1}^{n} I_i x_i \le \max_I \tag{40}$$

$$\min_H R \le \sum_{i=1}^{n} HR_i x_i \le \max_H R \tag{41}$$

$$\sum_{st \in short-term} I_s t x_i \le \frac{\alpha}{\mu} \sum_{i=1}^{N} I_i x_i \tag{42}$$

$$\sum_{mt \in mid-term} I_m t x_i \leq \frac{\beta}{\mu} \sum_{i=1}^{N} I_i x_i \quad (43)$$

$$\sum_{lt \in long-term} I_l t x_i \leq \frac{\gamma}{\mu} \sum_{i=1}^{N} I_i x_i \quad (44)$$

$$\sum_{i=1}^{n} |(\mu_{dis_i})^2 - (\nu_{dis_i})^2| x_i \leq |(\mu_{DR_{\max}})^2 - (\nu_{DR_{\max}})^2| \quad (45)$$

$$\alpha + \beta + \gamma = \mu \quad (46)$$

$$x_i \neq x_i^{"} \quad for\, i = 1, 2, \ldots, n; \quad (i, i^{''}) \in K \quad (47)$$

$$x_i = 1\, for\, i = 1, 2, \ldots, n; \forall i \in L \quad (48)$$

Eq. (40) keeps the amount of investment in the feasible region. Eq. (41) keeps the number of human resource of the entire selected portfolio in the practical area. Eqs. (42), (43) and (44) can be added to the model to provide it with more real-world application capability by helping the managers to plan their short, mid and long term time horizons. Eq. (45) considers the risk of disruptions and delays in a portfolio. In other words, this constraint makes sure that the total level of disruption that a portfolio could have would be limited to the maximum amount of disruption resiliency that a portfolio can handle. Since the disruption level of projects and the maximum level of disruption that a portfolio can handle are mostly based on opinions of experts, PFSs are used and the concept of score function is applied to address them. $((\mu_{dis_i})^2 - (\nu_{dis_i})^2)$ presents score function of the disruption of project i and $((\mu_{DR_{\max}})^2 - (\nu_{DR_{\max}})^2)$ shows the score function of the maximum disruption resistance that a portfolio has. Eq. (47) indicates the mutual exclusiveness relationship of projects. Therefore, it allows only one of the projects to be in the final portfolio. Eq. (48) addresses inclusion of a certain project in the portfolio.

4 Case study in a mining and industrial company

In order to present the applicability of the introduced approach, in this section the approach is applied to find the best project among a number of high-tech projects in mining sector of an Iranian mining and industrial firm. It should be noted that Iran is one of the 10 top countries in the field of mineral reserves. The goals of the studied firm in mining section include exploring and extracting minerals, creating the minerals construction units, providing primary materials of the domestic industrial units, exporting mineral raw and surplus productive materials and presenting consultation services on mining activities, based on modern technology in the country by focusing on scientific ability of the elites and specialists of the mine major with the goal of becoming one of the best mining industry investor companies in the country. What has improved the intensity of this case study is the situation of mining industries in recent years. mining industries are facing with decreasing cash flow, rise of financing costs, decrease in commodities prices and other similar trends. This has put mining industries in a weak financial position and has made them more careful when making investment decisions. The studied firm is faced with four different high-tech proposed projects. These projects are related to exploring, extracting and creating the mineral construction units of Gilsonite, rare earth extractions and Phosphate rock. It should be noted that due to the policies of the firm the information of candidate projects is confidential. Due to confidentiality of the information, it was not possible to present proposed projects in details.

The main motivation for applying fuzzy sets in this case study is the fact that the candidate projects are in new areas and the company lacks historical data of similar projects. On the other hand, given the competitive environment of business, other similar firms did not provide the company with the required data with reasonable prices. Therefore, the firm realized that the most efficient way is to use the expertise of highly experienced experts. Given the fact that opinions and judgments were being used in the process, fuzzy sets were applied as the proper tools to handle the problem.

4.1 First part of the proposed model

After reviewing various factors in high-tech project selection and consulting with the main DMs of the firm, a set of criteria was chosen to evaluate the proposed projects. The evaluation criteria are as follows: Total cost of investment (C_1) which presents the cost of proposed projects; the availability of international cooperation (C_2) which is an important factor due to requirement of foreign technology in all of the projects; technical feasibility (C_3) which shows the technical possibility of successful implementation of the project; ecological impacts (C_4) which measures the reduction in ecological damages that can be achieved by using new and high-technology. Personnel factor (C_5) which shows the availability of skillful personnel at reasonable costs. Eventually, project risk factor (C_6) which shows the risks, such as financial and technical risks, that can affect the project. It should be noted that C_1 and C_6 are cost criteria and the rest are benefit criteria. The proposed high-tech projects are denoted as A_1, A_2, A_3 and A_4. A group consisting of 3 experts (E_1, E_2 and E_3) were asked to express their opinions. They were asked to provide a degree of agreement (μ) and a degree of disagreement (v) while rating each project versus evaluation criteria and when expressing the importance of each criterion. They also had to consider that $0 \leq (\mu)^2 + (v)^2 \leq 1$. Tables 1 and 2 present the rating of each

Table 1: The rating of projects versus criteria

Alternatives	Experts	C_1	C_2	C_3
A_1	E_1	$(0.5, 0.5)$	$(0.5, 0.2)$	$(0.7, 0.4)$
	E_2	$(0.6, 0.3)$	$(0.7, 0.5)$	$(0.7, 0.5)$
	E_3	$(0.7, 0.4)$	$(0.6, 0.5)$	$(0.6, 0.5)$
A_2	E_1	$(0.5, 0.7)$	$(0.8, 0.3)$	$(0.9, 0.3)$
	E_2	$(0.4, 0.8)$	$(0.8, 0.5)$	$(0.8, 0.6)$
	E_3	$(0.3, 0.7)$	$(0.7, 0.6)$	$(0.7, 0.5)$
A_3	E_1	$(0.3, 0.8)$	$(0.55, 0.45)$	$(0.5, 0.5)$
	E_2	$(0.5, 0.75)$	$(0.5, 0.6)$	$(0.5, 0.7)$
	E_3	$(0.4, 0.7)$	$(0.6, 0.8)$	$(0.6, 0.5)$
A_4	E_1	$(0.7, 0.6)$	$(0.7, 0.3)$	$(0.9, 0.2)$
	E_2	$(0.5, 0.5)$	$(0.9, 0.2)$	$(0.92, 0.22)$
	E_3	$(0.5, 0.6)$	$(0.8, 0.4)$	$(0.85, 0.35)$
Alternatives	Experts	C_4	C_5	C_6
A_1	E_1	$(0.7, 0.3)$	$(0.3, 0.7)$	$(0.3, 0.7)$
	E_2	$(0.5, 0.6)$	$(0.6, 0.3)$	$(0.4, 0.8)$
	E_3	$(0.7, 0.6)$	$(0.8, 0.4)$	$(0.5, 0.7)$
A_2	E_1	$(0.7, 0.3)$	$(0.8, 0.4)$	$(0.9, 0.3)$
	E_2	$(0.6, 0.3)$	$(0.9, 0.2)$	$(0.9, 0.2)$
	E_3	$(0.8, 0.3)$	$(0.85, 0.42)$	$(0.8, 0.2)$
A_3	E_1	$(0.6, 0.7)$	$(0.5, 0.7)$	$(0.3, 0.8)$
	E_2	$(0.45, 0.65)$	$(0.5, 0.6)$	$(0.4, 0.8)$
	E_3	$(0.5, 0.5)$	$(0.5, 0.5)$	$(0.5, 0.5)$
A_4	E_1	$(0.85, 0.35)$	$(0.9, 0.3)$	$(0.5, 0.5)$
	E_2	$(0.7, 0.3)$	$(0.8, 0.35)$	$(0.7, 0.5)$
	E_3	$(0.9, 0.4)$	$(0.9, 0.25)$	$(0.5, 0.6)$

Table 2: Importance of each criterion

$Experts$	C_1	C_2	C_3	C_4	C_5	C_6
E_1	$(0.8, 0.2)$	$(0.8, 0.4)$	$(0.7, 0.4)$	$(0.65, 0.55)$	$(0.8, 0.5)$	$(0.8, 0.4)$
E_2	$(0.8, 0.4)$	$(0.9, 0.2)$	$(0.8, 0.4)$	$(0.8, 0.5)$	$(0.5, 0.4)$	$(0.8, 0.3)$
E_3	$(0.9, 0.2)$	$(0.9, 0.4)$	$(0.6, 0.5)$	$(0.6, 0.5)$	$(0.7, 0.3)$	$(0.9, 0.4)$

project versus evaluation criteria and the importance of each criterion, respectively. To obtain the score of each one of the projects, the followings were done:

1. Tables 1 and 2 are used to form the decision matrices.

2. The normalization step is carried out by using Eq. (15). The normalized values are presented in Table 3. 3. To attend to criteria weights, this process is accomplished:

3.1. Eq. (17) is used to compute the entropy of each criterion. The results are presented in Table 4.

3.2. In order to compute the values of NW, Eqs. (18) and (19) are applied. NW is the weight of criteria that is affected by the entropy of importance of each criterion. Table 5 presents the results.

Table 3: Normalized values

Alternatives	Experts	C_1	C_2	C_3
A_1	E_1	$(0.5, 0.5)$	$(0.5, 0.2)$	$(0.7, 0.4)$
	E_2	$(0.3, 0.6)$	$(0.7, 0.5)$	$(0.7, 0.5)$
	E_3	$(0.4, 0.7)$	$(0.6, 0.5)$	$(0.6, 0.5)$
A_2	E_1	$(0.7, 0.5)$	$(0.8, 0.3)$	$(0.9, 0.3)$
	E_2	$(0.8, 0.4)$	$(0.8, 0.5)$	$(0.8, 0.6)$
	E_3	$(0.7, 0.3)$	$(0.7, 0.6)$	$(0.7, 0.5)$
A_3	E_1	$(0.8, 0.3)$	$(0.55, 0.45)$	$(0.5, 0.5)$
	E_2	$(0.75, 0.5)$	$(0.5, 0.6)$	$(0.5, 0.7)$
	E_3	$(0.7, 0.4)$	$(0.6, 0.8)$	$(0.6, 0.5)$
A_4	E_1	$(0.6, 0.7)$	$(0.7, 0.3)$	$(0.9, 0.2)$
	E_2	$(0.5, 0.5)$	$(0.9, 0.2)$	$(0.92, 0.22)$
	E_3	$(0.6, 0.5)$	$(0.8, 0.4)$	$(0.85, 0.35)$
Alternatives	Experts	C_4	C_5	C_6
A_1	E_1	$(0.7, 0.3)$	$(0.3, 0.7)$	$(0.7, 0.3)$
	E_2	$(0.5, 0.6)$	$(0.6, 0.3)$	$(0.8, 0.4)$
	E_3	$(0.7, 0.6)$	$(0.8, 0.4)$	$(0.7, 0.5)$
A_2	E_1	$(0.7, 0.3)$	$(0.8, 0.4)$	$(0.3, 0.9)$
	E_2	$(0.6, 0.3)$	$(0.9, 0.2)$	$(0.2, 0.9)$
	E_3	$(0.8, 0.3)$	$(0.85, 0.42)$	$(0.2, 0.8)$
A_3	E_1	$(0.6, 0.7)$	$(0.5, 0.7)$	$(0.8, 0.3)$
	E_2	$(0.45, 0.65)$	$(0.5, 0.6)$	$(0.8, 0.4)$
	E_3	$(0.5, 0.5)$	$(0.5, 0.5)$	$(0.5, 0.5)$
A_4	E_1	$(0.85, 0.35)$	$(0.9, 0.3)$	$(0.5, 0.5)$
	E_2	$(0.7, 0.3)$	$(0.8, 0.35)$	$(0.5, 0.7)$
	E_3	$(0.9, 0.4)$	$(0.9, 0.25)$	$(0.6, 0.5)$

Table 4: Entropy of criteria

Criteria	C_1	C_2	C_3	C_4	C_5	C_6
Entropy	0.1	0.12	0.2	0.25	0.2	0.14

Table 5: The values of NW

Experts	C_1	C_2	C_3
E_1	$(0.32, 0.84)$	$(0.35, 0.88)$	$(0.36, 0.82)$
E_2	$(0.32, 0.9)$	$(0.43, 0.81)$	$(0.43, 0.82)$
E_3	$(0.4, 0.84)$	$(0.43, 0.88)$	$(0.29, 0.86)$
Experts	C_4	C_5	C_6
E_1	$(0.35, 0.86)$	$(0.42, 0.87)$	$(0.37, 0.87)$
E_2	$(0.47, 0.83)$	$(0.23, 0.83)$	$(0.37, 0.83)$
E_3	$(0.35, 0.83)$	$(0.35, 0.78)$	$(0.46, 0.87)$

4. In order to evaluate the proposed high-tech projects, the following is carried out:

4.1. By employing Eqs. (20) and (21) the decision matrix of each DM is formed. Table 6 presents the corresponding values.

Table 6: The weighted decision matrix

Alternatives	Experts	C_1	C_2	C_3
A_1	E_1	$(0.16, 0.88)$	$(0.17, 0.89)$	$(0.25, 0.85)$
	E_2	$(0.09, 0.94)$	$(0.3, 0.86)$	$(0.3, 0.87)$
	E_3	$(0.16, 0.92)$	$(0.26, 0.91)$	$(0.17, 0.9)$
A_2	E_1	$(0.22, 0.88)$	$(0.28, 0.89)$	$(0.32, 0.84)$
	E_2	$(0.25, 0.92)$	$(0.35, 0.86)$	$(0.34, 0.89)$
	E_3	$(0.28, 0.85)$	$(0.3, 0.93)$	$(0.2, 0.9)$
A_3	E_1	$(0.25, 0.85)$	$(0.19, 0.91)$	$(0.18, 0.87)$
	E_2	$(0.24, 0.93)$	$(0.21, 0.88)$	$(0.21, 0.91)$
	E_3	$(0.28, 0.86)$	$(0.26, 0.96)$	$(0.17, 0.9)$
A_4	E_1	$(0.19, 0.92)$	$(0.24, 0.89)$	$(0.32, 0.83)$
	E_2	$(0.16, 0.93)$	$(0.39, 0.82)$	$(0.4, 0.83)$
	E_3	$(0.24, 0.88)$	$(0.35, 0.9)$	$(0.25, 0.88)$
Alternatives	Experts	C_4	C_5	C_6
A_1	E_1	$(0.25, 0.87)$	$(0.12, 0.93)$	$(0.26, 0.88)$
	E_2	$(0.23, 0.9)$	$(0.14, 0.84)$	$(0.29, 0.86)$
	E_3	$(0.22, 0.9)$	$(0.28, 0.82)$	$(0.32, 0.9)$
A_2	E_1	$(0.25, 0.87)$	$(0.34, 0.89)$	$(0.11, 0.97)$
	E_2	$(0.28, 0.85)$	$(0.21, 0.83)$	$(0.07, 0.97)$
	E_3	$(0.26, 0.85)$	$(0.3, 0.82)$	$(0.09, 0.95)$
A_3	E_1	$(0.21, 0.93)$	$(0.21, 0.93)$	$(0.29, 0.88)$
	E_2	$(0.21, 0.9)$	$(0.11, 0.89)$	$(0.29, 0.86)$
	E_3	$(0.16, 0.88)$	$(0.17, 0.84)$	$(0.23, 0.9)$
A_4	E_1	$(0.3, 0.87)$	$(0.38, 0.88)$	$(0.18, 0.9)$
	E_2	$(0.33, 0.85)$	$(0.18, 0.85)$	$(0.18, 0.92)$
	E_3	$(0.29, 0.86)$	$(0.31, 0.8)$	$(0.27, 0.9)$

4.2. The alternatives are evaluated for each DM by using Eq. (21). It should be reminded that the values denoting ranking score of each alternative are still PFNs and are not aggregated. The results are presented in Table 7. 5. Weight of each DM is

Table 7: The results of alternative evaluation for each decision maker

Alternatives	Experts	S_i
A_1	E_1	$(0.28, 0.23)$
	E_2	$(0.39, 0.18)$
	E_3	$(0.31, 0.18)$
A_2	E_1	$(0.52, 0.19)$
	E_2	$(0.52, 0.20)$
	E_3	$(0.44, 0.22)$
A_3	E_1	$(0.06, 0.21)$
	E_2	$(0.07, 0.19)$
	E_3	$(0.15, 0.15)$
A_4	E_1	$(0.54, 0.24)$
	E_2	$(0.59, 0.22)$
	E_3	$(0.47, 0.23)$

computed by using the following process.
5.1. The values of $S(\tilde{X}_k)$ are computed.
5.2. The positive ideal solution $(PIDM_{ij})$ and negative ideal solution $(NIDM_{ij})$ for each DM are computed.
5.3. The distance of each opinion from the positive $(DPIDM_{ij}^k)$ and the negative $(DNIDM_{ij})$ ideal solutions are calculated.
5.4. The overall positive ideal (OPISk) and negative ideal score (ONISk) of each DM are computed.
5.5. The mathematical model (31-34) is formed and computed. The acceptable limits of weights were considered between 0.25 and 0.48 which are based on the lowest and highest weights that the DMs have considered while expressing their opinions on importance of each other. Table 8 presents the results. In order to do a sensitivity analysis in this step, the values of ε are

changed, and the results are presented in Table 8. $\varepsilon = 1$ indicates that only distance from positive ideal opinion is considered while $\varepsilon = 0$ shows that only distance from negative ideal solution is addressed. $\varepsilon = 0.5$ indicates that both distances are equally accounted in the process. The obtained values in the latter case are used in the rest of the process.

Table 8: Weight of each DM

Experts	$\varepsilon = 1$	$\varepsilon = 0$	$\varepsilon = 0.5$
E_1	0.27	0.27	0.27
E_2	0.48	0.25	0.25
E_3	0.25	0.48	0.48

6. In this step, the PFN values denting the score of each project are aggregated using the WASPAS method. Table 9 displays the results of aggregated project evaluation scores.
7. To make the resulting PFNs comparable, the following is carried out:

Table 9: Aggregated PFN results

Alternatives	A_1	A_2	A_3	A_4
R_i	$(0.32, 0.19)$	$(0.48, 37)$	$(0.11, 0.17)$	$(0.52, 0.23)$

7.1. Score function is used to define the ideal positive and negative solutions. The ideal positive solution is set as (0.52, 0.23) and the negative ideal solution is (0.11, 0.17).
7.2. The distance based degree of similarity between R_i and $R_{i,\max}$ is computed.
7.3. The distance based degree of similarity between R_i and $R_{i,\min}$ is computed.
7.4. The values of PS_i are computed by using Eq. (38).
7.5. PS_i is used to rank the projects. The alternative projects are ranked in increasing order of PS_i. The results of step 7 are presented in Table 10.

The results of the first part of the method are presented in Table 10. The table presents the values of $dbds_i^+$, $dbds_i^-, PS_i$ and ranking. The results indicate that A_3 has the worst evaluation score and A_4 has the best score. As it can be observed the positive and negative ideal solutions are addressed in ranking and the projects are ranked according to them. Based on the first part, the projects are ranked as $A_4 > A_2 > A_1 > A_3$.

Table 10: The results of step 7

Alternatives	A_1	A_2	A_3	A_4
$dbds_i^+$	0.09	0.03	0.14	0
$dbds_i^-$	0.05	0.13	0	0.14
PS_i	0.35	0.80	0	1
Ranking	3	2	4	1

5 The second part of the proposed approach

To find a portfolio of project after evaluating each project, the second part of the model is used. In this step, the values of FR_i obtained from the last part are used to form the model. Table 11 presents the values of input parameters in the mathematical model. It should be noted that HR_i in Table 10 presents the number of highly skilled people needed in the project and is not an indicator of all the people required in the project. Another issue that should be considered is the number of projects. Given the features of high-tech projects the firm was not able to consider a large number of projects in its selection process. To find the best portfolio of projects, the firm has considered several scenarios. In each scenario, the limits of available investment and human resource are different. Lingo 15.0 was used to find the optimum portfolio of projects. Table 12 presents the final results. In order to display the importance of addressing risk of disruption in the method, the optimization process is solved under various scenarios while considering risk of disruption. The disruptions of projects were also expressed using PFSs due to uncertainty of high-tech projects. The PFNs of $(0.46, 0.68), (0.72, 0.41)$ and $(0.87, 0.38)$ denote the disruption risk of projects A_1, A_2 and A_4, respectively. To better show the impacts of risk of disruption, various values are added to scenarios presented in Table 12. The results are depicted in Table 13.

Table 11: The features of proposed projects

Projects	A_1	A_2	A_4
PS_i	0.35	0.8	1
I_i (million \$)	8	6	9
HR_i (persons)	18	12	25

Table 12: Results for different scenarios

Projects	$\min_I I_i$ (million \$)	$\max_I I_i$ (million \$)	$\min_H R$ (persons)	$\max_H R$ (persons)
Scenario 1	5	9	10	20
Scenario 2	8	14	15	40
Scenario 3	8	20	15	50
Scenario 4	5	22	10	54
Projects	A_1	A_2	A_4	Objectives
Scenario 1	0	1	0	0.8
Scenario 2	1	1	0	1.15
Scenario 3	0	1	1	1.8
Scenario 4	0	1	1	1.8

Table 13: Results for different scenarios under risk of disruption

Projects	$\min_I I_i$ (million \$)	$\max_I I_i$ (million \$)	$\min_H R$ (persons)	$\max_H R$ (persons)	$DR_{\max}$
Scenario 1	5	9	10	20	$(0.78, 0.51)$
Scenario 2	8	14	15	40	$(0.25, 0.85)$
Scenario 3	8	20	15	50	$(0.91, 0.31)$
Scenario 4	5	22	10	54	$(0.98, 0.1)$
Projects	A_1	A_2	A_4	Objectives	
Scenario 1	1	0	0	0.35	
Scenario 2	1	1	0	1.15	
Scenario 3	1	1	0	1.15	
Scenario 4	1	0	1	1.35	

5.1 Comparative Analysis

In order to carry out a comprehensive comparative analysis, first the method is compared with IF-based methods to show the advantages of this method in comparison with well-known IF-based methods. Then, the method is compared with the study of Zhang and Xu [38] in order to present the advantages of this method with existing IFS-based studies. Then, in order to show the finer points of this paper in comparison with existing project selection and project portfolio optimization methods, a comparative analysis with them is carried out. In order to compare the method with existing IF-based methods, it is necessary to pay attention to the constraint of IFSs ($0 \leq \mu_s(x) + \nu_s(x) \leq 1$). The initial evaluations presented in Table 1 are checked in Table 14. The bold values in this table present the ones that IFSs cannot handle. A glance at this table clearly shows that IFSs are not able to solve this problem. In other words, since IFSs have more limitations in expressing the degrees of membership and non-membership in comparison with PFSs, it is not always possible to apply IFSs to solve problems that can be solved with PFSs. However, it is obvious that IFS-based problems can be handled with PFS methods because PFSs cover wider ranges of uncertainty. Consequently, PFSs in comparison with IFSs are more flexible and powerful to address uncertainty. This shows that applying such sets in high-tech projects results in better expressing uncertainty.

Table 14: Checking the possibility of using IFSs

Alternatives	Experts	C_1	C_2	C_3
A_1	E_1	$0.5 + 0.5 = 1$	$0.5 + 0.2 = 0.7$	$0.7 + 0.4 = 1.1$
	E_2	$0.6 + 0.3 = 0.9$	$0.7 + 0.5 = 1.2$	$0.7 + 0.5 = 1.2$
	E_3	$0.7 + 0.4 = 1.1$	$0.6 + 0.5 = 1.1$	$0.6 + 0.5 = 1.1$
A_2	E_1	$0.5 + 0.7 = 1.2$	$0.8 + 0.3 = 1.1$	$0.9 + 0.3 = 1.2$
	E_2	$0.4 + 0.8 = 1.2$	$0.8 + 0.5 = 1.3$	$0.8 + 0.6 = 1.4$
	E_3	$0.3 + 0.7 = 1$	$0.7 + 0.6 = 1.3$	$0.7 + 0.5 = 1.2$
A3	E_1	$0.3 + 0.8 = 1.1$	$0.55 + 0.45 = 1$	$0.5 + 0.5 = 1$
	E_2	$0.5 + 0.75 = 1.25$	$0.5 + 0.6 = 1.1$	$0.5 + 0.7 = 1.2$
	E_3	$0.4 + 0.7 = 1.1$	$0.6 + 0.8 = 1.4$	$0.6 + 0.5 = 1.1$
A_4	E_1	$0.7 + 0.6 = 1.3$	$0.7 + 0.3 = 1$	$0.9 + 0.2 = 1.1$
	E_2	$0.5 + 0.5 = 1$	$0.9 + 0.2 = 1.1$	$0.92 + 0.22 = 1.14$
	E_3	$0.5 + 0.6 = 1.1$	$0.8 + 0.4 = 1.2$	$0.85 + 0.35 = 1.2$
Alternatives	Experts	C_4	C_5	C_6
A_1	E_1	$0.7 + 0.3 = 1$	$0.3 + 0.7 = 1$	$0.3 + 0.7 = 1$
	E_2	$0.5 + 0.6 = 1.1$	$0.6 + 0.3 = 0.9$	$0.4 + 0.8 = 1.2$
	E_3	$0.7 + 0.6 = 1.3$	$0.8 + 0.4 = 1.2$	$0.5 + 0.7 = 1.2$
A_2	E_1	$0.7 + 0.3 = 1$	$0.8 + 0.4 = 1.2$	$0.9 + 0.3 = 1.2$
	E_2	$0.6 + 0.3 = 0.9$	$0.9 + 0.2 = 1.1$	$0.9 + 0.2 = 1.1$
	E_3	$0.8 + 0.3 = 1.1$	$0.85 + 0.42 = 1.27$	$0.8 + 0.2 = 1$
A3	E_1	$0.6 + 0.7 = 1.3$	$0.5 + 0.7 = 1.2$	$0.3 + 0.8 = 1.1$
	E_2	$0.45 + 0.65 = 1.1$	$0.5 + 0.6 = 1.1$	$0.4 + 0.8 = 1.2$
	E_3	$0.5 + 0.5 = 1$	$0.5 + 0.5 = 1$	$0.5 + 0.5 = 1$
A_4	E_1	$0.85 + 0.35 = 1.2$	$0.9 + 0.3 = 1.2$	$0.5 + 0.5 = 1$
	E_2	$0.7 + 0.3 = 1$	$0.8 + 0.35 = 1.15$	$0.7 + 0.5 = 1.2$
	E_3	$0.9 + 0.4 = 1.3$	$0.9 + 0.25 = 1.15$	$0.5 + 0.6 = 1.1$

To compare the method with PFS based studies, method of Zhang and Xu [38] is applied. The results are presented in Table 15. Comparing the first part of the method with an existing method provides the following advantages: The first part

Table 15: Results of the introduced method and Zhang and Xu [38]

	A_1	A_2	$A3$	$A4$
Score of the proposed method	0.35	0.80	0	1
Ranking of the proposed method	3	2	4	1
Score of Zhang and Xu [40]	-1.97	-1.52	-2.83	0
Ranking of Zhang and Xu [40]	3	2	4	1

of the approach is last aggregation. In other words, the method first computes the evaluation of each project according to the judgments of each DM, and then, the results are aggregated. This avoids the usual information loss that is possible in averaging step of similar methods; (2) In order to aggregate the results WASPAS method is applied. This aggregation approach provides the method with benefits of WASPAS method; (3) Weight of each criterion is modified by using entropy, and the weight of each DM is used in the aggregation step; (4) Defuzzification is carried out in the last step of the first part. In other words, the fuzziness of data is saved until the last step. However, in most of the similar studies defuzzification is carried out before computing the ranking score of alternatives;

Finally, the method is compared with existing project and project portfolio selection studies (i.e., [32, 23, 7].Table 16 presents the results of comparisons.

6 Conclusions

Project portfolio selection is one of the most important tasks of project oriented organizations. When it comes to high-tech projects, this task becomes even more important. Successful selection and implementation of high-tech projects enables organizations to gain competitive edge over rivals. In order to achieve this success, a formal and well organized process is required. Given this importance, this paper offered a new method of high-tech project evaluation and project portfolio selection. The method applied Pythagorean fuzzy sets (PFSs) to model uncertainty. Given the high flexibility of these sets, the method was able to address the vague condition of high-tech projects. To address the uncertain environment of high-tech projects, fuzziness of data

Table 16: Comparison of the method with existing studies

Comparison aspects	Results
Computational flexibility	Applying PFSs results in providing higher degrees of flexibility in computations due to the fact that uncertain elements can be expressed through degrees of membership, non-membership and hesitancy with more flexibility in comparison with IFSs.
Computational steps	The method applies two main steps to evaluate projects and optimize project portfolios. The two-part approach provides the DMs with a better understanding of projects and makes it possible for them to go through project portfolio selection process with a better vision of how the projects are.
Qualitative analysis	The method investigates the role of qualitative data in high-tech project evaluation through introducing a novel multi-criteria decision making approach. The method applies subjective and objective data to address weights of criteria. Moreover, it uses a last aggregation approach which avoids loss of information. Weights of DMs are computed by introducing a novel approach. Moreover, the entire multi-criteria framework introduced in this method is a new approach designed for addressing high and new technology project evaluation applications.

was saved and defuzzification was carried out in the last step of the process through a new PFS ranking method. Application of the presented method was depicted through demonstration of project evaluation and project portfolio selection in a case study of high-tech project selection in an Iranian mining and industrial firm. For future studies, providing a multi-objective model for the second part of the model with PFS uncertain parameters could result in a new approach with improved real world applicability.

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